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Collective Bargaining in a Basic North American Sports League Model

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Abstract Earlier general firm/trade union bargaining literature is brought to bear on a specific North American sports league model, where talent supply is fixed and profit-maximizing clubs receive local (gate) revenue plus an equal share of league broadcasting revenue. Club and player representatives negotiate a Collective Bargaining Agreement on the levels of local revenue sharing, salary cap and salary floor. Results, *inter alia*, show how increases in broadcasting market size affect the Nash bargaining solution for player salaries, competitive balance, player salary share of league revenue, the ratio of salary floor to cap and the extent of local revenue sharing.

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I Introduction

The "basic" North American sports league model of the title entails a set of profit-maximizing clubs whose revenues and costs depend on the aggregate playing talent they each hire, and an aggregate supply of talent to the league that is perfectly inelastic, reflecting the fact that the major North American leagues (MLB, NBA, NFL, NHL) face little competition for talent from rival leagues. There is an extensive literature on such models with various approaches to finding their solutions for the talent allocation between clubs (and hence competitive balance), and for the talent wage (the assumed uniform wage per unit of talent which then determines player salaries) under laissez-faire, where no restrictions are imposed on club or player behaviour; for instance there are no revenue sharing arrangements between clubs, there are no salary caps on club payrolls, and implicitly there is complete "free agency" for players to negotiate contracts with any club. The literature also includes many papers that address the impact of exogenous imposition of a restriction such as revenue sharing or a salary cap on the talent allocation and wage.

The 21st century reality for all four major leagues is that typically many restrictions[2] are in operation simultaneously, and decisions on them are jointly made by a club representative (perhaps the league commissioner) and a player representative (perhaps an executive of the players' trade union), leading to a Collective Bargaining Agreement (CBA) document which becomes the framework within which subsequent club and player economic activity takes place. The objective of this paper is to extend the basic North American model to include such collective bargaining, drawing on the earlier literature on general firm-trade union bargaining, and in particular the seminal paper by McDonald and Solow (1981), where bargaining is between a firm and its trade union over the levels of worker wages and employment, and where the firm and the trade union have utility functions defined over wages and employment (the firm's typically as profits, which depend on wages and employment). The set of efficient bargains and other solution concepts from axiomatic bargaining theory can be analysed[3]; in particular Nash bargaining solutions[4] can provide a unique wage and employment collective bargaining solution.

In our model we make what seem to be the most obvious and simple assumptions on the utility functions of the club and player representatives. The club representative has a utility function which is simply the sum of individual club profits, which in turn are functions of the talent allocation and wage. Unlike McDonald and Solow (1981), there can be no variation in employment (all talent will be employed), and the player representative has a utility function which is simply the talent wage. Ultimately the representatives are thought of as bargaining

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[2] Restrictions vary across the leagues, and include the previously mentioned revenue sharing and salary caps, as well as salary floors on payrolls, luxury taxes, agreements on the fraction of the league revenue that will go to players, a procedure for drafting rookie players to clubs with restrictions on the salaries they can be paid, restrictions on when during their careers players can achieve free agency status, restrictions on club roster size, and several more.

[3] There are other approaches to firm/trade union bargaining in the earlier literature, in some of which inefficient outcomes emerge; see for instance Manning (1987). These alternatives might also provide interesting insights if taken to the sports league collective bargaining context – there is no presumption here that the McDonald-Solow route is the only route.

[4] These solutions originated in Nash (1953). Osborne and Rubinstein (1990, chapter 2) provides a more recent discussion which includes in section 2.4.2, p. 19-20 an example generated by McDonald-Solow.

over multiple restrictions such as revenue sharing, salary caps etc., and not directly over the talent allocation and wage, again unlike the analogue in McDonald and Solow (1981). To progress to a full collective bargaining model for our context, we need to specify a set of restrictions and understand how the restrictions map into solutions for the talent allocation and wage using one of the approaches for finding such solutions, so that the representatives' utilities, efficient bargains and Nash bargaining solutions can be analysed; thus, compared to McDonald and Solow (1981), we face an additional step in the required modelling.

The only paper known to the author that studies the impact of (exogenous) imposition of *multiple* restrictions is Dietl et al. (2011) who focus on three restrictions: an arrangement for revenue sharing between clubs (RS for short), a salary cap on club payrolls (SC), and a salary floor on club payrolls (SF). We follow, in that in our model the club and player representatives bargain over the levels of the same three restrictions (RS, SC and SF); our objective becomes endogenous determination of these levels *via* the collective bargaining analysis. Dietl et al. (2011) use a standard two-club model to trace the consequences of the multiple restrictions for the talent allocation and wage solutions, using mainly the Walrasian fixed-supply conjecture approach to finding such solutions. Instead we use the Walrasian large league approach of Madden (2010, 2015), with a large number of two types of club; the consequences are similar to the Walrasian fixed-supply conjecture analysis in Dietl et al. (2011), but more convenient at some points of our extended agenda.

The revenue specification in our model also takes an important lead from existing literature, namely Peeters (2015), in that there are two revenue sources for the league and its clubs. First, as is standard in most models, each club earns a local revenue, thought of as gate revenue from its home games attended by its own "hard-core" fans who have relatively little preference for "uncertainty of outcome", and a relatively strong preference to see their team win. Secondly, and less standard, there is broadcasting revenue, now a major revenue source in all the major North American leagues. Since 1961 it has been legal for the leagues to negotiate collectively with broadcasters regarding sale of TV rights and to distribute proceeds equally between clubs. This is the practice in all four of the major leagues, and we assume so throughout the paper. Since the TV audience will be more neutral than the stadium audience (see Peeters (2015)), with a relatively strong preference for uncertainty of outcome, we assume that the broadcasting revenue coming into the league is larger the more competitively balanced is the league. All this broadly follows Peeters (2015), who uses a different model of the talent market to study the endogenous level of local revenue sharing that would emerge if the level was chosen to maximize league profits, as would be chosen by our club representative if they had no need to negotiate with the union. In reality, because of anti-trust law, the league can impose restrictions which can affect salaries only if they are agreed *via* collective bargaining with the union – hence our collective bargaining model.

Sadler and Sanders (2016) also present a collective bargaining model, a two period non-cooperative game between owners and players, motivated by the 2011 NBA lockout. Based on asymmetric information about the size of league revenue, they show how lockout might emerge as equilibrium. They have nothing to say about collective bargaining *agreements*, the nature and content of which will be our focus. Equally we have nothing to say about lockouts or strikes, which would need some dynamic development of our essentially static story.

Ours is not the only paper to study theoretical issues in sports leagues using Nash bargaining solutions. Solow and Krautmann (2011) study bargaining over salary between a single (elite) player and one or more clubs separately, using the Nash bargaining solution concept. Our collective bargaining context is different, with a representative of all players bargaining with a representative of all clubs, over RS, SC and SF.

Results are derived for our full collective bargaining model in two stages. First we study a hypothetical model where the representatives are able to bargain directly over the talent allocation and wage ignoring the "additional step" needed for the full model mentioned above, and we derive the hypothetical model efficient bargains and Nash bargaining solution; in particular the player salary share of league revenue is always 50%. Secondly, taking the "additional step", we show that under a parameter assumption which, in particular, puts an upper bound on the size of the broadcasting market relative to the clubs' local revenue markets, the talent allocation and wage which are the Nash bargaining solution for the full model are the same as those for the hypothetical model (including its 50% player share); this is supplemented by analysis of the levels of RS, SC and SF which would appear in the CBA document. Comparative static effects of parameter changes on the Nash bargaining solution are studied, but, given the huge increase in broadcasting revenues over the last 40 years, we focus only on the effects of increases in the size of the broadcasting market. Findings include:

(a) Increases in broadcasting market size would have no effect on talent allocations or wages under laissez-faire; each (negligible in size) individual clubs will not be able to influence the league broadcasting revenue, or its (equal) share of it, and marginal revenues (and talent demands, and so talent allocations and wages) are independent of the size of broadcasting revenue, proceeds simply going into the pockets of the profit-maximizing owners. However if our collective bargaining is the league modus operandi, then we show that increases in broadcasting market size will indeed increase the talent wage (and hence player salaries).

(b) The Nash bargaining solution will always produce a talent allocation that maximizes league revenue (which is the sum of revenues from the broadcasting market and the local club markets). As the broadcasting market size increases competitive balance at that talent allocation also increases because of the increasing importance of the broadcasting consumers and their preference for uncertainty of outcome, i.e. for increased competitive balance. Thus our prediction is that competitive balance will increase as the broadcast market size increases.

(c) Although the Nash bargaining solution for the talent allocation and wage is unique, there are multiple ways of choosing the levels of RS, SC and SF which would produce this talent allocation and wage. Phrasing this differently, there are in general multiple solutions for the content of the CBA document that will support the Nash bargaining solution, offering a rich set to explore for consistency with actual CBAs. As the size of the broadcasting market increases, we show there will be CBA documents in which: (i) the ratio of SF to SC increases, and so does the level of SF; (ii) the extent of local revenue sharing stays the same; (iii) in an augmented model where a final RS decision is delegated by the player representative to negotiations between the club representative and the clubs themselves, then, instead of (ii), the extent of local revenue sharing decreases.

(d) There is some very tentative evidence in support of conclusions in (a), (b) and (c) above, drawn from recent NFL experience.

Section II of the paper sets out the initial framework for the analysis, including (II.1) the cost and revenue specifications, (II.2) the resulting league revenue maximizing talent allocation and (II.3) the hypothetical collective bargaining model. Section III addresses the full model, including (III.1) its more extended framework to include the "additional step" and (III.2) discussion of its efficient bargains and Nash bargaining solution with derivation of the results behind the findings (a), (b) and (c) above and with pointers for (d). Section IV discusses some possible extensions, including changes which will allow the otherwise rigid 50% player share to vary, and which will relax the parameter assumption's upper bound on broadcasting market size. Section V concludes.

II The framework

II.1 Costs and revenues

We adopt the "large league" specification used in Madden (2010, 2015). The league consists of two types of profit-maximizing club, with a continuum of mass 1 of each type. $t_{ic} \geq 0, c \in [0,1], i = 1,2$ will denote the aggregate playing talent on the roster of club c of type i. $w \geq 0$ denotes the wage per unit of talent, assumed uniform across the league; wt_{ic} is the payroll of club ic. The aggregate talent supply to the league is $S(w) \geq 0$ and is assumed perfectly inelastic; normalising, $S(w) = 1$ for all $w \geq 0$, and the implicit uniform reservation wage is 0. If all clubs of type i have the same talent then t_i will denote that common allocation. In all Walrasian large league solutions clubs of the same type will have the same talent, and the talent market will clear; $t_1 + t_2 = 1$.

The league and its clubs receive revenues of two different types, broadcasting and local gate revenue.

Suppose $t_1 + t_2 = 1$ and one club, ic say, deviates from t_i. Think of the local gate revenue for ic as stemming from purchase of season tickets[5] for their club's home games by "hard-core" fans, and that the willingness-to-pay for such tickets, and hence revenue, depends on the talent level of their own team t_{ic}, and the average talent level of all teams in the league, $\bar{t}$ say; $\bar{t} = \frac{1}{2}$ and, because club ic is negligibly small in the continuum setting, variation in t_{ic} will not change $\bar{t}$, which can also be thought of as the average talent level of all visiting teams to club ic. Suppose that the hard-core fans would prefer to see their team with greater talent

[5] Although not in the continuum setting, the idea of a season ticket league is due to Fort and Quirk (2011); fans buy a season ticket ahead of the season, allowing them subsequent entry to all home games.

than the league, or rest of league, average ($\bar{t} = \frac{1}{2}$), but not too much greater when the boredom of one-sided games eventually reduces the interest of even the hard-core, and assume that this leads to revenue for club ic which is increasing in t_{ic} up to some level in excess of $\bar{t} = \frac{1}{2}$, after which it starts to decline; in the common parlance there is some (but not complete) preference for uncertainty of outcome. It is convenient for the purposes of this paper to assume the following quadratic form for the local gate revenue function of club ic when it deviates from t_i;

$$R_i(t_{ic}) = m_i(t_{ic} - \tfrac{1}{2}t_{ic}^2) \quad (1)$$

Here $m_i > 0$ is a measure of the size of the local market ("fanbase") for each club of type i, and it is assumed throughout that $m_1 > m_2 > 0$, so that type 1 clubs are the big market clubs.

Revenue in (1) would be maximized when $t_{ic} = 2\bar{t} = 1$, and club ic has twice the talent level of the league average[6], an arithmetically convenient feature of (1).

Regarding broadcasting revenue, as mentioned earlier, all four major North American leagues sell league TV rights collectively, distributing proceeds equally between all clubs. There is evidence that armchair consumers behind this revenue have a relatively strong preference for uncertainty of outcome, at least compared to hard-core fans (see Peeters (2015)). If $m_B \geq 0$ is a measure of the size of the league's broadcasting market then we assume that broadcasting revenue is;

$$R_B(t_1) = m_B(t_1 - t_1^2) \quad (2)$$

This reflects a strong uncertainty of outcome preference, the revenue being maximized when $t_1(= t_2 = \bar{t}) = \frac{1}{2}$ and all clubs have equal talent, and again imposes a helpfully simple quadratic specification. With equal distribution to all (mass 2) clubs, each club receives;

$$b = \tfrac{1}{2}m_B(t_1 - t_1^2) \quad (3)$$

<u>REMARK</u> There are pros and cons associated with adopting the large league modelling approach compared with any of the approaches to modelling two-club leagues in the literature[7]. Two-club leagues overestimate the extent of strategic interaction between individual clubs since each team has only one rival whilst the actual number in reality is considerably greater. On the other hand, the large league underestimates the extent of such strategic interactions between individual clubs – there are none. There seems to be no *a priori* reason to dismiss either type of model – just as duopoly and perfect competition models have

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[6] (1) could generalise and retain the desired qualitative properties with $R_i(t_{ic}) = m_i\left(t_{ic} - \frac{1}{2z}t_{ic}^2\right), z > \frac{1}{2}$. Local revenue is then maximized when $t_{ic} = z > \bar{t}$. As stated, the choice $z = 1$ is arithmetically convenient.

[7] These approaches are: Walrasian fixed-supply conjectures (Fort and Quirk (1995), Vrooman (1995)); Contest-Nash (Szymanski (2004), Szymanski and Kesenne (2004)); strategic market games (Madden (2011)); Cournot (Driskill and Vrooman (2016, 2017)); wage schedules (Burguet and Sakovics (2018)).

both enhanced knowledge in industrial economics generally, so it should be in sports league economics too. Large leagues, like perfect competition, are often more easily analysed than the alternative, which is one reason for their adoption here.

II.2 The league revenue maximizing talent allocation

Consider the problem of allocating the available one unit of talent to clubs so that the sum of broadcasting and all local revenues is maximized. This allocation plays an important role in our analysis, and is described in Lemma 1 below.

Note first that the strict concavity of (1) means that all clubs of the same type will get the same allocation. Thus the problem becomes finding the maximum with respect to $t_1(= 1 - t_2 \in [0,1])$ of;

$$R(t_1) \equiv R_B(t_1) + R_1(t_1) + R_2(1 - t_1) \tag{4}$$

Lemma 1

(a) The league revenue maximizing talent allocation is $t_1^* = \frac{m_1+m_B}{m_1+m_2+2m_B} (> \frac{1}{2})$;

(b) The maximum league revenue is $R^* = \frac{(m_1+m_2+m_B)^2 - m_1m_2}{2(m_1+m_2+2m_B)}$ (> 0).

Proof From (1), (2) and (4), $R(t_1) = m_B(t_1 - t_1^2) + m_1\left(t_1 - \frac{1}{2}t_1^2\right) + \frac{1}{2}m_2(1 - t_1^2)$ and is strictly concave. Its maximum is where $R'(t_1) = m_B(1 - 2t_1) + m_1(1 - t_1) - m_2t_1 = 0$ which gives $t_1 = t_1^*$; $t_1^* > \frac{1}{2}$ as $m_1 > m_2$ Substituting in $R(t_1)$ and simplifying, $R(t_1^*) = R^* > 0$.■

There are three market size parameters in the model, m_1, m_2 and m_B, and comparative static effects of changes in these values can be traced. However since the most obvious big changes over the last forty years have been increases in m_B, the comparative statics focus in the rest of the paper is on these changes, starting with:

Lemma 2

(a) $\frac{\partial t_1^*}{\partial m_B} < 0$, with t_1^* declining from $t_1^* = \frac{m_1}{m_1+m_2}$ when $m_B = 0$ towards $t_1^* = \frac{1}{2}$ as $m_B \to \infty$;

(b) $\frac{\partial R^*}{\partial m_B} > 0$.

Proof $\frac{\partial t_1^*}{\partial m_B} = \frac{m_2 - 2m_1}{(m_1+m_2+2m_B)^2} < 0$ since $m_1 > m_2$;

$$\frac{\partial R^*}{\partial m_B} = \frac{4(m_1m_2 + m_B(m_1+m_2) + m_B^2)}{(m_1+m_2+2m_B)^2} > 0. \quad ■$$

Obviously the revenue generating capacity of the league increases with m_B, and so does the maximum revenue, as in (b). The effect on t_1^* is a natural consequence of the increasing importance for league revenue generation of the armchair consumers and their strong taste for uncertainty of outcome; phrasing (a) alternatively, increases in m_B imply that the league revenue maximizing competitive balance increases, approaching perfect balance as $m_B \to \infty$.

II.3 A hypothetical collective bargaining model

We consider bargaining between a player representative and a club representative. The utilities of the representatives depend on the talent allocation and wage. The utility for the player representative is simply the wage (per unit of talent);

$$U_P(t_1, w) = w(\geq 0) \tag{5}$$

And the utility of the club representative is simply the aggregate profit;

$$U_C(t_1, w) = R(t_1) - w(\geq 0) \tag{6}$$

These linear in income utilities are thought of as the Bernoulli utility functions of risk-neutral expected utility maximizers. The zero lower bounds on utilities will be the disagreement utilities in our bargaining games[8].

In the first of these bargaining games (hypothetical – section III will look at the "full" model of main interest) we think of the two representatives as bargaining over the talent allocation $t_1(= 1 - t_2 \in [0,1]\,)$ and wage w where the feasible bargaining agreements are those in the following set[9];

$$\mathcal{F} = \{(t_1 \in [0,1], w \geq 0): R(t_1) - w \geq 0\} \tag{7}$$

$\mathcal{F}$ is shown in Figure 1 below, which also shows typical indifference curves of the player representative (the dashed horizontal line) and the club representative (the dashed curve), tangential whenever $t_1 = t_1^*$ and $w \in [0, R^*]$; thus the efficient bargains are the vertical locus where $t_1 = t_1^*$ and $w \in [0, R^*]$.

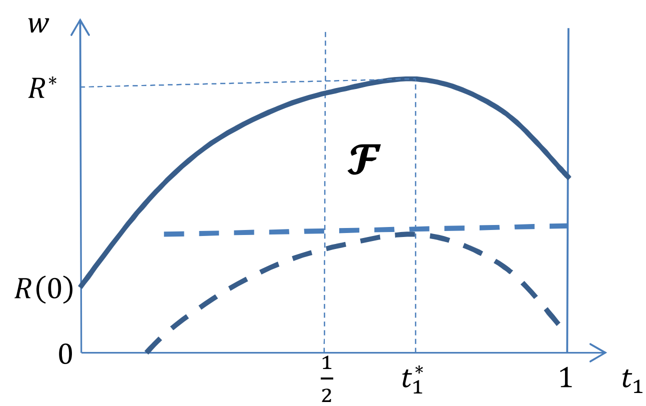

Figure 1; The set $\mathcal{F}$

[8] In the event of disagreement it is natural to assume that the league will become inactive; no players are hired, no games are played and no revenues are earned. Hence the zero utility disagreement specification.

[9] For the later full model, this feasible set will reduce to include only talent allocations and wages which are Walrasian large league solutions for some levels of RS, SC and SF; that is, the full model will incorporate the "additional step" mentioned in the introduction.

From (5) and (6), agreement on $(t_1, w) \in \mathcal{F}$ produces representative utilities where $U_C + U_P = R(t_1)$. There is then a function that maps $\mathcal{F}$ onto the following attainable utility set $\mathcal{U}$, shown in figure 2:

$$\mathcal{U} = \{(U_C \geq 0, U_P \geq 0): R^* \geq U_C + U_P \geq R(0)\} \quad (8)$$

Disagreement produces (0,0) utilities, and the efficient bargains correspond to utilities on the north-east boundary of $\mathcal{U}$.

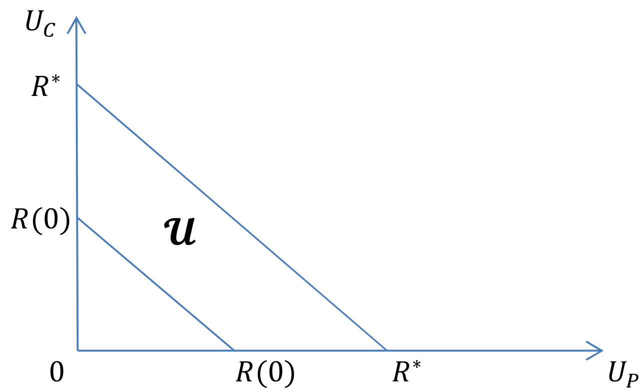

Figure 2; The set $\mathcal{U}$

From the celebrated result of Nash (1953), if the bargaining satisfies the four axioms of Efficiency, Independence of Irrelevant Alternatives, Symmetry and Invariance to Positive Affine Transformations then there is a unique utility outcome. If, generally, disagreement utilities generally are d_C, d_P and the attainable utility set following agreement $\mathcal{U}$ is compact and convex, the outcome maximizes the Nash product $(U_C - d_C)(U_P - d_P)$ on $\mathcal{U}$.[10] In our case $d_C = d_P = 0$, and the Nash bargaining solution for utilities is clear from figure 2 – maximization of the symmetric function $U_C U_P$ on $\mathcal{U}$ occurs at the midpoint of its north-east boundary where $U_C = U_P = \frac{1}{2}R^*$. The corresponding agreement in $\mathcal{F}$ has talent allocation t_1^* and wage $\frac{1}{2}R^*$, and is more informative than the utilities solution;

Proposition 1 For the hypothetical collective bargaining model:

(a) The efficient bargains are (t_1, w) where $t_1 = t_1^*$ and $w \in [0, R^*]$.

(b) The Nash bargaining solution for the talent allocation and wage is $t_1^{NB} = t_1^*$, $w^{NB} = \frac{1}{2}R^*$.

Thus the (hypothetical) Nash bargain solution prediction is that collective bargaining will lead to the talent allocation that maximizes league revenue, and that the player salary share of that revenue will be 50%.

[10] Most expositions of the proof of the Nash bargaining theorem assume that $(d_C, d_P) \in \mathcal{U}$ (which we do not have here) with some $(U_C, U_P) \in \mathcal{U}$ where $U_C > d_C, U_P > d_P$. However it is sufficient to assume (as we do have here) that for every $U_C, U_P \in \mathcal{U}$, $U_C \geq d_C, U_P \geq d_P$ and $(U_C, U_P) \neq (d_C, d_P)$, with $U_C > d_C, U_P > d_P$ for some $(U_C, U_P) \in \mathcal{U}$.

The effect of increasing m_B on that prediction is immediate from Lemma 2:

Proposition 2 For the Nash bargaining solution of the hypothetical collective bargaining model, increases in m_B lead to increased competitive balance and increased player salaries; as $m_B \to \infty$, $t_1^{NB} \to \frac{1}{2}$ and $w^{NB} \to \infty$.

III The full collective bargaining model

III.1 The extended framework

The utilities of the representatives are again as in (5) and (6) .The change from section II.3 is that the only (t_1, w) that are feasible are those that are Walrasian large league solutions for some levels of the RS, SC and SF restrictions, where we follow the Dietl et al. (2011) specifications.

(RS) A standard textbook format is assumed, whereby all clubs retain only a fraction $\alpha \in [0,1]$ of their local revenue, the rest going into a central pool which is then distributed equally between all clubs. It is a format that has been in place for some time in the NFL.

(SC) The (hard) salary cap is denoted $C(\geq 0)$ and restricts all clubs to payrolls that satisfy $wt_{ic} \leq C$.

(SF) Similarly the salary floor is hard, $F(\geq 0)$, and restricts all clubs to payrolls $wt_{ic} \geq F$. It must be that $F \leq C$, otherwise clubs cannot feasibly respect both the cap and floor restrictions.

To progress we need to develop understanding of the (t_1, w) that are Walrasian large league solutions for the various possible levels of the restrictions. Start with the case of laissez-faire, where there are no restrictions ($\alpha = 1$, with $C = +\infty, F = 0$ for instance). The profits of club $c \in [0,1]$ of type $i = 1,2$ are;

$$\pi_i(t_{ic}) = m_i\left(t_{ic} - \frac{1}{2}t_{ic}^2\right) - wt_{ic} + b \qquad (9)$$

Because of the large league setting, the choice of t_{ic} will have no effect on w, which adjusts to clear the talent market in the usual Walrasian fashion. In addition, for the same large league reason, the choice of t_{ic} will have no effect on league broadcast income or on each club's share b. Thus w and b are parameters in the individual club profit maximization problem whose first-order condition is the familiar marginal revenue condition; $mr_i \equiv m_i(1 - t_{ic}) = w$ if $w \leq m_i$ and $t_{ic} = 0$ otherwise. Talent demand is the same for all clubs of the same type, t_i say, and talent market clearance produces the Walrasian large league solution under laissez-faire;

$$t_1^{LF} = \frac{m_1}{m_1+m_2} \quad w^{LF} = \frac{m_1 m_2}{m_1+m_2} \qquad (10)$$

Profits of each club (now independent profit-maximizers) have to be non-negative since, otherwise, we assume that the league would fail. Using (3), (9) and (10) profits of the two types of firm are indeed positive:

$$\pi_1 = \frac{m_1^3 + m_1 m_2 m_B}{2(m_1+m_2)^2} > 0 \qquad \pi_2 = \frac{m_2^3 + m_1 m_2 m_B}{2(m_1+m_2)^2} > 0 \tag{11}$$

It follows that broadcasting revenue has no effect on either the talent allocation or wage in (10). It simply goes into owners' pockets, boosting profits (the right hand sides in (11) increase with m_B). This of course would not happen with win-maximizing clubs, who would attempt to use the extra cash to boost talent demand, and hence player salaries. But it is an inevitable and natural consequence of the profit-maximizing owners assumed here, under laissez-faire.

Notice also that $t_1^{LF} = t_1^*$ if $m_B = 0$, but $t_1^{LF} > t_1^*$ otherwise. Thus under laissez-faire competitive balance is worse than that which maximizes league revenue, except in the absence of broadcasting revenue.

Suppose now there is just one restriction on laissez-faire, namely our local revenue sharing arrangement; $0 \le \alpha < 1$ (with say $C = +\infty, F = 0$). Once again the large league setting has an important consequence - as with broadcasting revenue, individual club talent demand decisions have no effect on the aggregate revenue sharing pool, or their share of it which will be denoted r. Thus the previous profit expression (9) becomes;

$$\pi_i(t_{ic}) = \alpha m_i \left(t_{ic} - \frac{1}{2}t_{ic}^2\right) - w t_{ic} + r + b \tag{12}$$

The first-order marginal revenue condition is: $mr_i(\alpha) \equiv \alpha m_i(1 - t_{ic}) = w$ if $w \le \alpha m_i$ and $t_{ic} = 0$ otherwise. Again talent demand is the same for all clubs of the same type, t_i say, and talent market clearance produces the Walrasian solution for revenue sharing:

$$t_1^{RS} = \frac{m_1}{m_1+m_2} \quad w^{RS} = \alpha \frac{m_1 m_2}{m_1+m_2} \tag{13}$$

Unsurprisingly, this is also the well-known outcome of the Walrasian fixed supply conjecture solution concept for a league with just two clubs – revenue sharing invariance since there is no effect on competitive balance, its only impact being a reduction of player salaries.[11]

The size of the pool in our solution is $(1-\alpha)[R_1(t_1^{RS}) + R_2(1 - t_1^{RS})]$ and so each of the mass 2 clubs receive the share $r = \frac{1}{2}(1-\alpha)[R_1(t_1^{RS}) + R_2(1 - t_1^{RS})]$. Substituting this and b in (2) into (12) shows that profits are again positive:

$$\pi_1 = \frac{(1+\alpha)m_1^3 + (1-\alpha)m_2^3 + 2(1-\alpha)m_1^2 m_2 + 2(1-\alpha)m_1 m_2^2 + 2m_1 m_2 m_B}{4(m_1+m_2)^2} > 0 \tag{14}$$

$$\pi_2 = \frac{(1-\alpha)m_1^3 + (1+\alpha)m_2^3 + 2(1-\alpha)m_1^2 m_2 + 2(1-\alpha)m_1 m_2^2 + 2m_1 m_2 m_B}{4(m_1+m_2)^2} > 0 \tag{15}$$

[11] However there are differences between the two "Walrasian" approaches; in particular with just two clubs our first-order marginal revenue condition $\alpha m_i(1 - t_{ic}) = w$ changes to involve m_j too. Our large league approach simplifies as a result.

In the familiar and very useful Quirk-Fort diagram[12], figure 3 shows mr_1, mr_2 and, for some $\alpha \in (0,1)$, $mr_1(\alpha)$, $mr_2(\alpha)$;[13] the laissez-faire solution is at point a, and the locus of revenue sharing solutions is the vertical line ab.

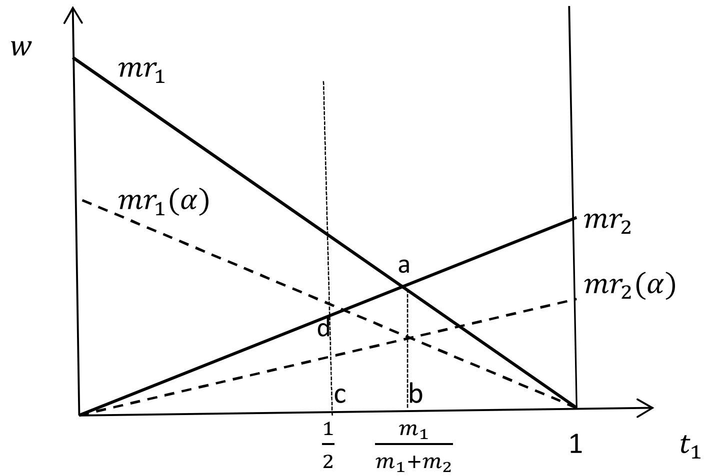

Figure 3: Some Walrasian large league solutions

<u>REMARK</u> Some different revenue sharing arrangements have also been used, whereby only big clubs put into the pool and only small clubs receive a share from it, namely in the NHL and MLB.[14] Although we shall continue in the rest of the paper with the previous textbook specification, this remark notes that such alternatives have a very different impact, a point which may be of independent interest. Assume that only the big clubs contribute a share $(1-\alpha) \in [0,1)$ of their local revenue into the pool, and each small club (only) receives an equal share of that pool. Talent demand from the big clubs is given again by $\alpha m_1(1-t_1) = w$, but because again of the large league setting that for the small clubs is just given by $m_2(1-t_2) = w$. Thus the solution is now $t_1 = \frac{\alpha m_1}{\alpha m_1+m_2}$, $w = \frac{\alpha m_1 m_2}{\alpha m_1+m_2}$, and the effect of this revenue sharing arrangement is to *increase* competitive balance, whilst still reducing the talent wage.

A complete account of the talent allocations and wages produced by the other (SC and SF) single restrictions, and of all possible double or triple restrictions, would be very lengthy, but is not needed in what follows. Lemma 3 below is what is needed, for which we start the explanation by defining the following subset of $\mathcal{F}$, which corresponds to the interior of the region bounded by abcd in figure 3 plus its boundary where $t_1 = \frac{m_1}{m_1+m_2}$; $\hat{\mathcal{F}} \equiv \{(t_1, w) \in \mathcal{F}: \frac{1}{2} < t_1 \leq \frac{m_1}{m_1+m_2}, 0 < w < m_2 t_1\}$. Consider any $(t_1, w) \in \hat{\mathcal{F}}$ where $t_1 = \frac{m_1}{m_1+m_2}$. From the RS discussion above (see (13)) we know this will be the Walrasian large league solution if there is just one restriction, RS with $\alpha = \frac{m_1+m_2}{m_1 m_2} w$.

[12] The diagram originated in Quirk and Fort (1992).
[13] The big club marginal revenues kink up to become vertical at $t_1 = 0$; similarly for the small clubs when $t_1 = 1$. The vertical sections are not indicated in figure 3.
[14] See Vrooman (2009, p. 18 and 21), Peeters (2015, p. 1275).

Now consider any $(t_1, w) \in \hat{\mathcal{F}}$ where $t_1 < \frac{m_1}{m_1+m_2}$, e.g. as shown in Figure 4. This point will lie on some $mr_2(\alpha)$ line with $\alpha \in (0,1)$, α_H say, as shown; $\alpha_H = \frac{w}{m_2 t_1}$. Thus, given this RS, small club talent demand at w would be $1 - t_1 (< \frac{1}{2})$. But given the RS big club talent demand would exceed t_1; $mr_1(\alpha_H)$ (not shown) intersects $mr_2(\alpha_H)$ at $t_1 = \frac{m_1}{m_1+m_2}$ because of the revenue sharing invariance result, and big club talent demand at w in fact exceeds $\frac{m_1}{m_1+m_2}$. However big club talent demand would reduce to t_1 if there was a SC with $C = wt_1$; this SC will not affect small club talent demand since $C = wt_1 > w(1 - t_1)$. Thus (t_1, w) is the Walrasian large league solution if there are two restrictions; RS and SC with $\alpha = \alpha_H = \frac{w}{m_2 t_1}$ and $C = wt_1$.

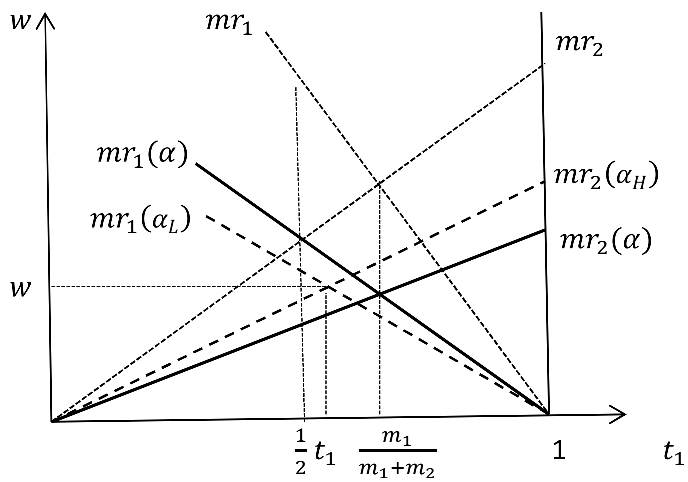

Figure 4: RS,SC and SF for $(t_1, w) \in \mathcal{F}$

The point (t_1, w) will also lie on some $mr_1(\alpha)$ line with (a different) $\alpha \in (0,1)$, α_L say where $\alpha_L < \alpha_H$, as shown; $\alpha_L = \frac{w}{m_1(1-t_1)}$. Given this RS, big club talent demand at w would be $t_1 (> \frac{1}{2})$. But given the RS small club talent demand would be less than $1 - t_1$,[15] and would increase to $1 - t_1$ if there was a SF with $F = w(1 - t_1)$; this SF will not affect big club talent demand since $F = w(1 - t_1) < wt_1$. Thus (t_1, w) is again the Walrasian large league solution with two restrictions, now RS and SF with $\alpha = \alpha_L = \frac{w}{m_1(1-t_1)}$ and $F = w(1 - t_1)$.

Now suppose that $\alpha \in (\alpha_L, \alpha_H)$, with the $mr_1(\alpha)$ and $mr_2(\alpha)$ lines[16] shown in figure 4. Given this RS, small club talent demand at w would be less than $1 - t_1 (< \frac{1}{2})$, and would increase to $1 - t_1$ with a salary floor $F = w(1 - t_1)$. And given this RS big club talent demand would exceed $t_1 (> \frac{1}{2})$, and reduce to t_1 if there was a SC with $C = wt_1$. The SF will

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[15] Similar to the α_H case above, $mr_2(\alpha_L)$ (not shown) intersects $mr_1(\alpha_L)$ at $t_1 = \frac{m_1}{m_1+m_2}$, and small club talent demand at w is less than $1 - \frac{m_1}{m_1+m_2}$.

[16] Because of the RS invariance these lines intersect at $t_1 = \frac{m_1}{m_1+m_2}$.

not affect big club talent demand since $F = w(1 - t_1) < wt_1$, and the SC will not affect small club talent demand since $C = wt_1 > w(1 - t_1)$. Thus (t_1, w) is also the Walrasian large league solution if there are all three restrictions in place with $\alpha \in (\alpha_L, \alpha_H)$, $C = wt_1$ and $F = w(1 - t_1)$.

Hence we have shown;

Lemma 3

(a) The point $(t_1, w) \in \hat{\mathcal{F}}$ where $t_1 = \frac{m_1}{m_1+m_2}$ is the Walrasian large league solution if there is just one restriction, namely RS with $\alpha = \frac{m_1+m_2}{m_1 m_2} w$.

(b) The point $(t_1, w) \in \hat{\mathcal{F}}$ where $t_1 < \frac{m_1}{m_1+m_2}$ is the Walrasian large league solution in any of the following three cases:

  (i) There are two restrictions, namely RS with $\alpha = \alpha_L = \frac{w}{m_1(1-t_1)}$ and SF with $F = w(1 - t_1)$;

  (ii) There are two restrictions, namely RS with $\alpha = \alpha_H = \frac{w}{m_2 t_1}$ and SC with $C = wt_1$;

  (iii) There are three restrictions, namely RS with $\alpha \in (\alpha_L, \alpha_H)$, SF with $F = w(1 - t_1)$ and SC with $C = wt_1$.

Two aspects of this result are worthy of comment.

First suppose that a unique $(t_1, w) \in \hat{\mathcal{F}}$ where $t_1 < \frac{m_1}{m_1+m_2}$ has been singled out, for instance (as will be the case in the next section) suppose that (t_1, w) is the unique Nash bargaining solution for the talent allocation and wage. Then, despite the uniqueness of (t_1, w) there is a continuum of possible restrictions (since all $\alpha \in [\alpha_L, \alpha_H]$ are possible) between which the representatives are indifferent. Putting this differently, there are multiple CBA documents that could accompany (t_1, w). On the one hand this provides flexibility to allow selection from the multiplicity in a way that attempts to match actual CBAs.

Secondly, on the other hand, there is an interesting adjunct to the model which reduces the continuum of possible CBA documents to a singleton, as follows. As in the previous paragraph suppose that a unique $(t_1, w) \in \hat{\mathcal{F}}$ where $t_1 < \frac{m_1}{m_1+m_2}$ has been identified, perhaps the Nash bargaining solution for the talent allocation and wage. The two representatives will get the same utilities from any of the continuum of restrictions in lemma 3(b) that could accompany (t_1, w) in the CBA document. In particular the player representative would be happy to delegate to the club representative choice of (α, C, F) from any in the set defined by lemma 3(b). However although the club representative would also be indifferent between all elements in this set, this is not true of the big clubs separately (they would prefer higher α and less sharing of revenue) or the small clubs separately (they prefer lower α). A plausible response by the club representative is to enter negotiations with a representative of the big clubs and a representative of the small clubs, which might be modelled as a Nash bargaining game over $\alpha \in [\alpha_L, \alpha_H]$; again plausibly, failure to agree by the small club representative leads to $\alpha = \alpha_H$, and disagreement by the big club representative triggers $\alpha = \alpha_L$. Since the sum of big and small club profits is the same at all $\alpha \in [\alpha_L, \alpha_H]$, the disagreement payoffs imply that $\alpha = \frac{1}{2}\alpha_L + \frac{1}{2}\alpha_H$ at this secondary Nash bargaining solution. We refer to this augmented model as *the full collective bargaining model with revenue sharing delegation*. The augmented model thus produces a unique CBA document to accompany the assumed Nash bargaining solution (t_1, w); namely, there are three restrictions with $\alpha = \frac{1}{2}\alpha_L + \frac{1}{2}\alpha_H$, $C = wt_1$ and $F = w(1 - t_1)$.

III.2 Efficient bargains and the Nash bargaining solution

For the full collective bargaining model a subset of the set of efficient bargains is immediately identifiable. In the earlier hypothetical model the efficient bargains were where $t_1 = t_1^*$ and $w \in [0, R^*]$. In the full model, from lemma 3, the set of talent allocations and wages where $t_1 = t_1^*$ and $w \in [0, m_2 t_1^*]$ remains feasible, and since indifference curves and their tangency points are the same in the full and hypothetical models, it follows that;

Lemma 4 For the full collective bargaining model $\{(t_1, w): t_1 = t_1^* \text{ and } w \in [0, m_2 t_1^*]\}$ is a subset of the set of efficient bargains.

To elaborate on the attainable utility set for the full model, recall that the hypothetical model's attainable utility set corresponded to representative utilities that were attainable via some agreement in $\mathcal{F}$; the convex set $\mathcal{U}$ was the outcome which would be the same if we extended to allow agreements which were lotteries over $\mathcal{F}$(with expected utilities). However a problem for the full model is that the attainable utility set is not in general globally convex unless we do extend to allow agreements which are lotteries over $\mathcal{F}$, which we now do. Then the attainable (expected) utility set for the full model, $\hat{\mathcal{U}}$ say, is a convex (and compact) subset of $\mathcal{U}$. Moreover, lemma 4 ensures that $\hat{\mathcal{U}}$ must contain in its north-east frontier the truncation of the north-east frontier of $\mathcal{U}$, namely the slope -1 line joining $(U_P = R^* - m_2 t_1^*, U_C = m_2 t_1^*)$ to $(U_P = R^*, U_C = 0)$.

We now introduce a parameter assumption (PA) which ensures that this truncated line (call it L) contains $(U_P = \frac{1}{2}R^*, U_C = \frac{1}{2}R^*)$, namely the utilities at the hypothetical model's Nash bargaining solution;

Parameter assumption (PA): $1 < \frac{m_1}{m_2} < \frac{1}{2}(3 + \sqrt{5}) \cong 2.618$ and $0 \le m_B < m_2 - m_1 + \sqrt{m_1 m_2}$.

Lemma 5 If (PA) is satisfied then $w^{NB} = \frac{1}{2}R^* < m_2 t_1^* = m_2 t_1^{NB}$.

Proof Using the definitions of R^* and t_1^* the inequality $\frac{1}{2}R^* < m_2 t_1^*$ becomes, after some rearrangement; $(\frac{m_1}{m_2})^2 + 1 + (\frac{m_B}{m_2})^2 - 3\frac{m_1}{m_2} + 2\frac{m_1}{m_2}\frac{m_B}{m_2} - 2\frac{m_B}{m_2} < 0$. Treating the left hand side as a quadratic expression in $\frac{m_B}{m_2}$ and equating to 0 reveals a positive root $\frac{m_B}{m_2} = 1 - \frac{m_1}{m_2} + \sqrt{\frac{m_1}{m_2}}$ (or $m_B = m_2 - m_1 + \sqrt{m_1 m_2}$) if $1 < \frac{m_1}{m_2} < \frac{1}{2}(3 + \sqrt{5})$. The result follows since the quadratic expression is decreasing in $\frac{m_B}{m_2}$ when $\frac{m_B}{m_2} < 1 - \frac{m_1}{m_2} + \sqrt{\frac{m_1}{m_2}}$ (or $m_B < m_2 - m_1 + \sqrt{m_1 m_2}$) ■

The Nash bargaining theorem can now be applied to the full collective bargaining model with its attainable (expected) utility set $\hat{\mathcal{U}}$. Since $\hat{\mathcal{U}}$ contains the line segment L in its north-east boundary (lemma 4), and since L contains the point $(U_P = \frac{1}{2}R^*, U_C = \frac{1}{2}R^*)$ (lemma 5) where the Nash product contour is tangential to L (as the hypothetical model solution), it follows that the Nash bargaining solution for the full model is the same as for the hypothetical model:

Proposition 3 Assume that (PA) is satisfied. Then the Nash bargaining solution for the hypothetical model is also the Nash bargaining solution of the full collective bargaining model: $t_1^{NB} = t_1^*$ and the talent allocation maximizes league revenue; $w^{NB} = \frac{1}{2}R^*$ and the player salary share is 50% of league revenue.

(PA) requires that local revenue market sizes are not too dissimilar ($\frac{m_1}{m_2} < 2.618$) and that m_B is not too large. Section IV.1 will show how the addition of fixed costs to the model allows relaxation of both these parameter constraints, in particular allowing m_B to reach the very high levels more appropriate for current reality. For the rest of this section (PA) is assumed. As m_B varies in this range, the hypothetical model Nash bargaining solution t_1^{NB}, w^{NB} belongs to $\hat{\mathcal{F}}$ from lemma 5, and we also know from proposition 2 that t_1^{NB} decreases and w^{NB} increases as m_B increases. From proposition 3 the same is true of the full collective bargaining model, and Figure 5 indicates (in bold) a typical locus of its Nash bargaining solutions.

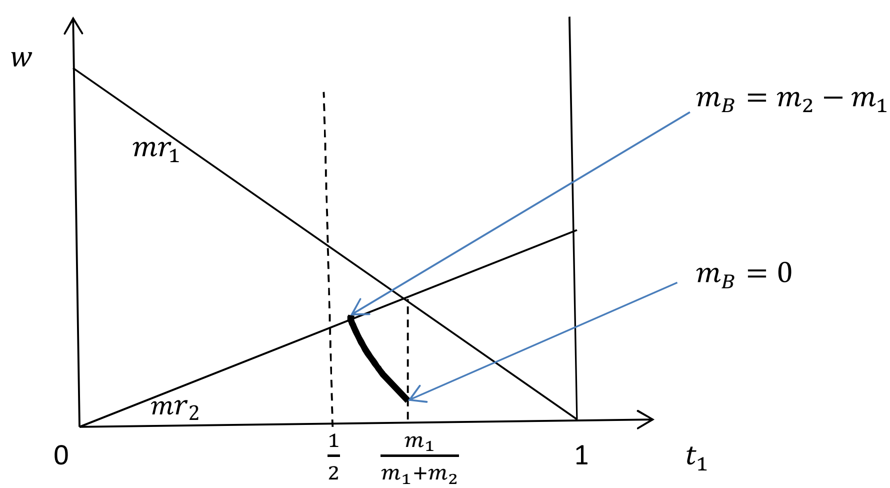

Figure 5: Nash bargaining solution locus (in bold) as m_B varies

Immediate consequences of increases in broadcasting market size on this solution are:

Proposition 4 Assume that (PA) is satisfied. As the size of the broadcasting market increases effects on the Nash bargaining solution for the full collective bargaining model are:

(a) The player salary share of league revenue does not change and is always 50%
(b) Player salaries increase
(c) Competitive balance increases.

Making claims that these findings are in line with any recent CBA reality in the major North American leagues is perhaps premature. Nevertheless, we do offer some pointers regarding the conclusions of proposition 4 (and proposition 5 below) from recent NFL experience - it is only the NFL which might fit the bill, since it is the only league which has adopted our specification for RS, SC and SF. It is certainly the case that NFL broadcasting revenue has grown massively recently – table 2.1 in Vrooman (2011) shows that annual TV rights fees went from $476 million (1987-89) to $4,065 million (2012-13) – and one might hope to see some indications within this period of the comparative static effects in proposition 4 (and 5).

Table 2.5 in Vrooman (2011) shows how the ratio of total player cost to total NFL revenue has fallen gradually from 56.5% in 2000 to 50.6% in 2009. Whilst the 50% number in proposition 4(a) is in the right ballpark, it is a quite rigid number. In section IV.2 we will recall that it stems from the assumed linearity in income of our representative's utility functions, and that introducing strict concavity (or risk-aversion) allows other numerical player salary shares to be possible.

It was seen earlier that the laissez-faire regime exhibits an invariance to changes in broadcasting market size; as m_B increases neither player salaries nor competitive balance change. From (b) and (c) in proposition 4 this is no longer true if the regime is instead our full collective bargaining model. Regarding (b), table 2.4 of Vrooman (2011) shows how player costs (salaries and benefits) have indeed increased dramatically from $535 million (1989) to $4,577 million (2009). Regarding (c), Vrooman (2009, p. 38-40 and figure 5) reports increased NFL competitive balance post-1998, with upward and downward movements but an upward trend over the longer period.

We now assume that the Nash bargaining solution for the talent allocation and wage for our full model is accompanied by a CBA document which imposes restrictions as described in lemma 3(biii), so that all three of our restrictions appear in the document. Suppose that (PA) is satisfied with $m_B > 0$ so that the Nash bargaining solution for the talent allocation and wage lies on the bold curve in figure 5. The following describes how the CBA document could change as m_B increases.

Proposition 5 Assume that (PA) is satisfied, that $m_B > 0$ and that (α, C, F) in the CBA document is as given in lemma 3(biii) where (t_1, w) is the Nash bargaining solution for the talent allocation and wage in the full collective bargaining model. As broadcasting market size increases effects on the CBA document are:

(a) F increases.
(b) The ratio F/C increases.
(c) α can remain unchanged if the change in m_B is not too large.
(d) If the model is augmented to include delegated revenue sharing then α increases.

Proof

(a) Since increases in broadcast market size increase w from proposition 4(b) and increase $1 - t_1$ from proposition 4(c), $F = w(1 - t_1)$ also increases. (b) $F = w(1 - t_1)$ and $C = wt_1$ imply $\frac{F}{C} = \frac{1}{t_1} - 1$ which increases as broadcast market size increases from proposition 4(c). (c) Invoking lemma 3(biii), the CBA document that accompanies the Nash bargaining solution for (t_1, w) with the initial broadcast market size will have $\alpha \in (\alpha_L, \alpha_H)$, $F = w(1 - t_1)$ and $C = wt_1$. Small changes in m_B will produce small changes in the Nash bargaining solution (t_1, w), and hence in α_L, α_H. If the changes are small enough, the original α will remain in the open interval (α_L, α_H), and will be part of a new CBA document that can accompany the new Nash bargaining solution. (d) With delegated revenue sharing, $\alpha = \frac{1}{2}\frac{w}{m_1(1-t_1)} + \frac{1}{2}\frac{w}{m_2 t_1}$. Substituting from lemma 1 into proposition 3 gives t_1^{NB} and w^{NB} in terms of m_B, m_1 and m_2, and hence α can be written:

$$\alpha = \frac{1}{4}[(m_1 + m_2 + m_B)^2 - m_1 m_2][\frac{1}{m_2(m_1+m_B)} + \frac{1}{m_1(m_2+m_B)}]$$

From this an expression for $\frac{\partial \alpha}{\partial m_B}$ can be derived, and Mathematica confirms that $\frac{\partial \alpha}{\partial m_B} > 0$ whenever (m_1, m_2, m_B) satisfies (GPA)[17]. ∎

Part (b) predicts that that the salary floor as a percentage of the cap will increase as m_B increases. For the NFL this percentage certainly increased between 2006 and 2011 from 84% to 90% (Vrooman (2011, p.11), and, as in part (a), the absolute size of the floor also increased, as did the cap. Throughout this century the NFL local revenue sharing arrangement has had α at 0.6, a constancy which is consistent with part (c); part (d) suggests that an upward movement in α (and therefore less extensive revenue sharing) may be the direction of eventual change.

<u>REMARK</u> As noted there is extensive literature which reports results on how exogenous changes in restrictions affect, in particular, competitive balance, with special interest on how this might increase. The proof of proposition 5(b) indicates a very simple result that might be of independent interest. Adopt any of the known approaches to finding solutions for the talent allocation and wage (for a large league as here, or for a two-club league), and suppose that F is a binding constraint on small club(s) and C is a binding constraint on big club(s). Since $\frac{F}{C} = \frac{1}{t_1} - 1$, changes in F and C that increase F/C will increase competitive balance. In particular, reductions in F and C which nevertheless increase F/C will increase competitive balance.

[17] I am grateful to Mario Pezzino for this last point.

IV Some extensions

IV.1 Relaxing the parameter assumption

Figure 6 shows the limitation imposed by (PA) on $\frac{m_B}{m_2}$ and $\frac{m_1}{m_2}$ (recalling that $m_B = m_2 - m_1 + \sqrt{m_1 m_2}$) is the same as $\frac{m_B}{m_2} = 1 - \frac{m_1}{m_2} + \sqrt{\frac{m_1}{m_2}}$);

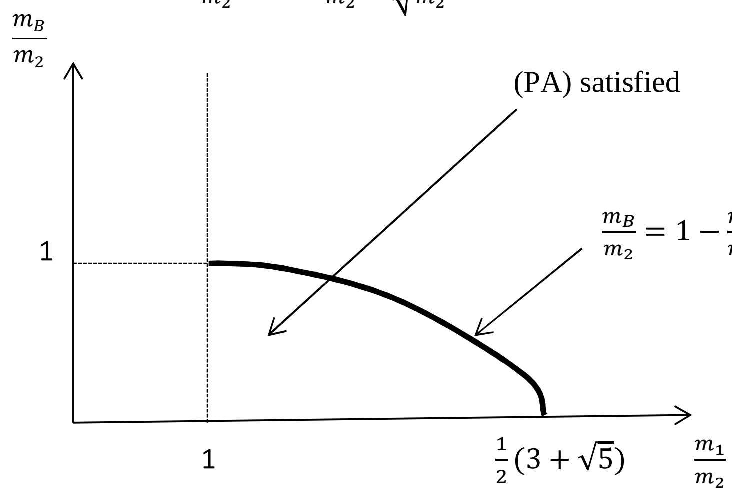

Figure 6: Limitations imposed by (PA)

The most recent reality, e.g. for NFL, is that broadcasting revenue exceeds 50% of total league revenue and is not captured by (PA) which implies $m_B < m_2$ whereas $m_B > m_1 + m_2$ is needed. However we now show that the addition of fixed costs to the model can solve this problem. Let $k_i > 0$ be fixed costs each club of type $i = 1,2$, and let $k = k_1 + k_2$. The inequality $R^* \geq k$ rearranges to:

$$(\frac{m_B}{m_2})^2 + 2\frac{m_B}{m_2}\left(1 + \frac{m_1}{m_2} - 2\frac{k}{m_2}\right) + [1 + \frac{m_1}{m_2} + (\frac{m_1}{m_2})^2 - 2\frac{k}{m_2}\left(1 + \frac{m_1}{m_2}\right)] \geq 0 \qquad (16)$$

With $\frac{m_1}{m_2} = 1$ this is true for all $\frac{m_B}{m_2} \geq 0$ if $\frac{k}{m_2} \leq \frac{3}{4}$. Moreover the partial derivative of the left hand side of (16) with respect to $\frac{m_1}{m_2}$ is $2\frac{m_B}{m_2} + 1 + 2\frac{m_1}{m_2} - 2\frac{k}{m_2}$ which is positive for all $\frac{m_1}{m_2} \geq 1$ if $\frac{k}{m_2} \leq \frac{3}{4}$. Thus (16) holds everywhere if $\frac{k}{m_2} \leq \frac{3}{4}$

The inequality $\frac{1}{2}(R^* - k) < m_2 t_1^*$ rearranges to:

$$(\frac{m_B}{m_2})^2 + 2\frac{m_B}{m_2}\left(\frac{m_1}{m_2} - 1 - 2\frac{k}{m_2}\right) + [1 - 3\frac{m_1}{m_2} + \left(\frac{m_1}{m_2}\right)^2 - 2\frac{k}{m_2}\left(1 + \frac{m_1}{m_2}\right)\Big] < 0 \qquad (17)$$

This quadratic inequality in $\frac{m_B}{m_2}$ is equivalent to:

$$\frac{m_B}{m_2} < 1 - \frac{m_1}{m_2} + 2\frac{k}{m_2} + \sqrt{\frac{m_1}{m_2} - 2\frac{k}{m_2}\frac{m_1}{m_2} + 4(\frac{k}{m_2})^2 + 6\frac{k}{m_2}} \qquad (18)$$

Or: $m_B < m_2 - m_1 + 2k + \sqrt{m_1 m_2 - 2km_1 + 4k^2 + 6km_2}$ (19)

The right hand side of (18) is strictly concave and decreasing in $\frac{m_1}{m_2}$, and $\frac{m_B}{m_2} = 0$ when $\frac{m_1}{m_2} = \frac{1}{2}(3 + 2k + \sqrt{5 + 20k + 4k^2})$. Hence the generalised parameter assumption (GPA), shown in figure 7.

Generalised Parameter Assumption (GPA): $1 < \frac{m_1}{m_2} < \frac{1}{2}(3 + 2k + \sqrt{5 + 20k + 4k^2})$ and

$0 \leq m_B < m_2 - m_1 + 2k + \sqrt{m_1 m_2 - 2km_1 + 4k^2 + 6km_2}$

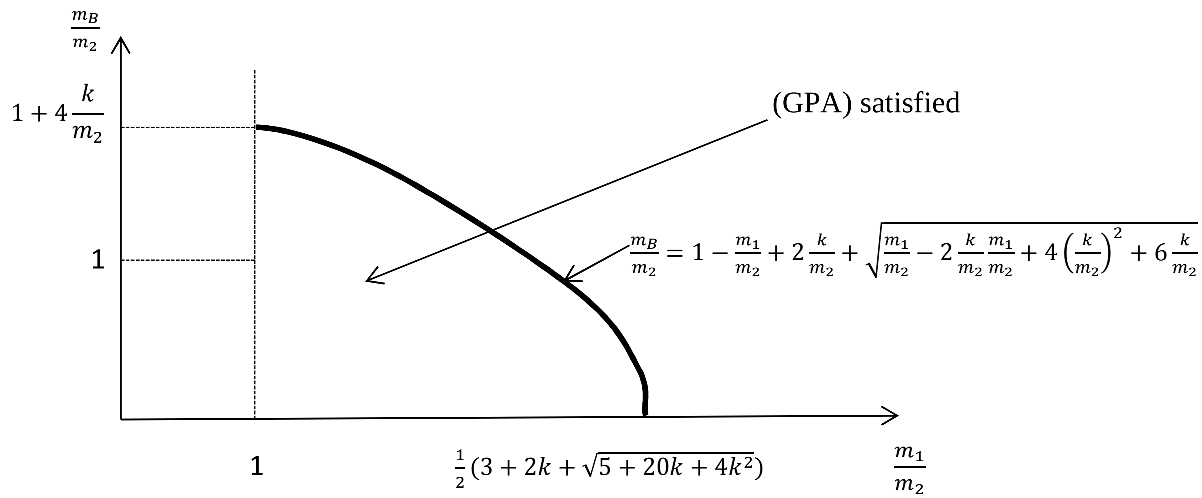

Figure 7: Limitations imposed by (GPA)

Figure 7 goes back to figure 6 when $k = 0$ and the curve in figure 7 moves north-east as k increases.

Hence with fixed costs where $\frac{k}{m_2} \leq \frac{3}{4}$ the hypothetical model will have a positive wage Nash bargaining solution which will remain feasible for the full collective bargaining model if (GPA) is satisfied, and the total fixed cost k is apportioned between clubs so that both types are profitable. In particular, $m_B > m_1 + m_2$, or $\frac{m_B}{m_2} > 1 + \frac{m_1}{m_2}$ is now possible if $\frac{k}{m_2} > \frac{1}{4}$, since when $\frac{m_1}{m_2} = 1$, $\frac{m_B}{m_2} = 1 + 4\frac{k}{m_2} > 2$.

IV.2 Varying the players' share from 50%

A well-known feature of Nash bargaining solutions is that if one starts from a Nash bargaining game with two risk-neutral parties (as we have), and one of the parties becomes risk-averse then the solution pay-off for that party goes down; increasing the risk-aversion worsens the payoff[18]. For our context a simple example is as follows.

Suppose that the club representative remains risk-neutral, as earlier, but the player representative utility function becomes $U_P(t_1, w) = w^\rho$ where $0 < \rho < 1$ instead of the earlier $\rho = 1$. The Nash product is now $w^\rho(R^* - w)$ and its maximum, the Nash bargaining

[18] See for instance, section 2.4.1, p.17-19 of Osborne and Rubinstein (1990).

solution, gives $\frac{w}{R^*} = \frac{\rho}{1+\rho} < \frac{1}{2}$. Thus the players' share of league revenue is now less than 50%, and worsens as their representative becomes more risk-averse (ρ gets smaller).

Alternatively suppose that the player representative remains risk-neutral, as earlier, but the club representative utility function becomes $U_C(t_1, w) = [R(t_1) - w]^\rho$ where $0 < \rho < 1$ instead of the earlier $\rho = 1$. The Nash product is $w[R^* - w]^\rho$ and the solution gives $\frac{w}{R^*} = \frac{1}{1+\rho} > \frac{1}{2}$; player shares above 50% now emerge.

Thus a player share of exactly 50% is not an inevitable consequence of the type of collective bargaining model proposed here.

IV Conclusions

Motivated by McDonald and Solow (1981) and the literature on firm/trade union bargaining in general, the paper has developed a model of Collective Bargaining Agreements (CBAs) for the specific context of a North American sports league where such agreements nowadays are central to the league economic outcomes; analysis of efficient bargains and (mainly) the Nash bargaining solution have been the focus.

The model draws significantly also on various aspects of the sports league literature: its starting point is a standard textbook framework where aggregates of playing talent are all that matters and the total talent available is fixed with an endogenous and uniform wage per unit of talent, and where clubs earn local revenues from their fans; given its increasing importance over the last 40 years, we have taken a lead from Peeters (2015), and introduced league broadcasting revenue which responds more positively to increases in competitive balance than individual club local revenues, and which is collectively negotiated by the league with broadcasters, equal shares going to clubs; the model has also borrowed from Dietl et al. (2011) in selecting three particular restrictions (local revenue sharing, a salary cap and a salary floor, respectively for short RS, SC and SF) over which a player representative and a club representative bargain, the agreed outcome to appear in the CBA document.

Compared to McDonald and Solow (1981) an additional modelling step is needed. The player and club representatives have utility functions that depend on the talent allocation (captured by t_1 in the paper) and wage (w) that result in the league, but the only such (t_1, w) that the parties can attain, with the corresponding utilities, are those that ensue from choice of the available restrictions. Understanding the mapping from RS, SC and SF to solutions for (t_1, w) (and hence utilities) is the additional step needed to allow investigation of efficient bargains and Nash bargaining solutions. To facilitate this analysis we have used the Walrasian large league approach of Madden (2011, 2015) to finding how solutions for (t_1, w) depend on the three restrictions.

Some of the findings are independent of the collective bargaining main focus. (a) The incorporation of broadcasting revenue in a large league led to an invariance result. As this revenue increases each clubs (equal) share of it goes up but is independent of individual club decisions since they have a negligible influence in a large league; the revenue increase has no impact on individual club marginal revenues and, as a result under laissez-faire, it will have no impact on (t_1, w), merely boosting club profits. A first question for the collective bargaining alternative to laissez-faire is whether it predicts something more realistic, particularly regarding player salaries. (b) In our local revenue sharing arrangement, all clubs contribute the same share of their local revenue to a pool, which is then shared equally between all clubs. Whilst this mirrors the current NFL arrangement, other leagues have adopted an alternative whereby only big clubs put a percentage of their local revenue into the pool and only small clubs receive a (equal) share of it. The impact of (exogenous) introduction of such an alternative arrangement on laissez-faire is to *increase* competitive balance. (c) For the large league, or indeed for any of the two-club league modelling approaches in the literature, if a salary cap (C, exogenous) is a binding constraint on big clubs and a salary floor (F, also exogenous) is binding on small clubs, then (exogenous) increases in the ratio F/C will increase competitive balance, even if F and C decrease.

The collective bargaining analysis faced the technical problem that the attainable utility set was not globally convex, a problem not found in McDonald and Solow (1981). An assumption on parameters overcame the problem, very loosely by forcing the Nash bargaining solution into a convex region of the set. At that solution: (d) the talent allocation is that which maximizes league revenue, and (e) the player salary share of league revenue is 50%. Further results reported on how increases in the size of the broadcasting market (captured by m_B) would affect competitive balance, player salaries (i.e. t_1 and w) and the levels of RS, SC and SF that would appear in the CBA document, as follows. (f) The player salary share of total league revenue is 50% and does not change with m_B. (g) Player salaries and competitive balance both increase, unlike the laissez-faire case in (a) above. (h) The ratio of the salary floor to the salary cap goes up as in (c) above, but so does the salary floor itself. (i) The fraction of local revenue provided by each club to the RS pool may remain constant if the increase in m_B is not too great; however that fraction goes down if the model is augmented to allow a final decision on the fraction to be delegated by the player representative to negotiation between the club representative and the clubs themselves.

Whilst some pointers have been offered to suggest that the results of the previous paragraph may have plausibility regarding recent NFL CBA history, and whilst suggestions have been made as to how the currently rigid 50% player salary share and the parameter assumption might be relaxed, it is premature to make any serious claims that our results “fit the facts”. As far as the author knows this is the first paper that attempts to predict CBA content, and it begs subsequent development in a number of directions. For instance, the model has assumed that the specific RS, SC and SF restrictions are what is on the bargaining table. What if further restrictions are added, or substituted? Is it possible to provide endogenous determination by the representatives of what is on the table? Are there explanations of why the different North American sports leagues use different restrictions? It is hoped that the broad methodology of this paper may allow answers to these and other questions.

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