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Regarding Preferences

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# Distributive Politics with Other-Regarding Preferences

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## Abstract

We extend upon the results of Lindbeck and Weibull [*Public Choice* 52 (3), (1987)] to study distributive politics when voters have not only self-interested preferences, but also other-regarding concerns. We consider a broad family of other-regarding behavior (including *fairness preferences*, *income-dependent altruism*, and *inequality aversion*), for which results on equilibrium existence and optimality have not been established yet. We provide a sufficient condition for smooth and non-smooth payoffs that generalizes Lindbeck and Weibull’s condition, and guarantees the existence of a unique Nash equilibria in pure strategies. In addition, we determine conditions under which the equilibrium results in an income distribution that can be rationalised as the outcome of maximizing a mixture of a “self-regarding utilitarian” social welfare function and society’s other-regarding preferences.

**Keywords:** Redistribution; Other-Regarding Preferences; Fairness; Altruism; Inequality Aversion; Non-smooth Optimization.

**JEL Classification Codes:** C72, D72, D78.

## 1 Introduction

Models of political economy, particularly of income redistribution, typically assume that individuals are selfish and care only about their material interests. In the literature on behavioral economics, however, there is mounting evidence that say otherwise, suggesting that people also express concern with the well-being of other individuals in society.<sup>1</sup>

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<sup>1</sup>This evidence has been documented in a large number of experimental and neuro-imaging studies, including among many others the work of Fehr and Schmidt (1999), Engelmann and Strobel (2004), Dawes et al. (2007) and (2012), Tabibnia et al. (2008), Fehr (2009), Almás et al. (2010), Tricomi et al. (2010), Zaki and Mitchell (2011), and Rilling and Sanfey (2011).

The implications of these behavioral theories of individual preferences over payoffs have just started to be examined in political economy. The aim of this paper is to contribute to this new literature by extending the canonical model of distributive politics due to Lindbeck and Weibull (1987) to accommodate a *broad* family of other-regarding behavior, which includes among others inequality aversion (Fehr and Schmidt 1999), fairness concern (Alesina and Angeletos 2005a), and income-dependent altruism (Dimick, Rueda and Stegmueller 2017).

The model laid out in Section 2 shares the usual features of probabilistic electoral competition. There are two political parties competing in a single election for the main office. Voters are grouped into different socio-economic groups and they have stochastic and policy-independent preferences (ideology) over the parties. The political candidates offer to the electorate a balanced budget redistributive policy from a multidimensional policy space. Voters evaluate these policies taking into account their selfish utility and their ideological bias. In a clear departure from earlier work, in this paper voters also express concern about how these policies affect the well-being of other members of society. To be precise, voters are endowed with an other-regarding utility which is continuous, concave, but not necessarily smooth.<sup>2</sup> Section 3 offers a few important examples that match this description. During the campaign, parties choose simultaneously their distributive policies to maximize the expected vote share, but they care also about voters' other-regarding preferences. The latter implies that the payoff functions of the parties are *not necessarily smooth* on the strategy space.

The main results of the paper are displayed in Section 4 and can be summarized as follows. First, the paper generalizes the Lindbeck and Weibull's (1987) sufficient condition for equilibrium existence, adapting it conveniently to accommodate the other-regarding preferences of the electorate and the resulting non-smooth framework described above. This condition, together with the assumptions on the utility functions, namely, continuity and concavity, shape the expected vote share and the parties' payoffs. To start, the gradient of the expected vote share is shown to be monotone decreasing on the differentiable subset of distributive policies (Lemma 1).

Since the latter does not always constitute a convex set, the previous result is not enough to prove concavity. Thus, as a preliminary step it is shown that the expected vote share of each party has a support almost everywhere (Lemma 2). Finally, using the fundamental theorem on the support of a concave function, Lemma 3 states that the expected vote share is concave on the whole strategy space. This together with the concavity of the other-regarding utilities guarantee that the party payoff functions

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<sup>2</sup>For instance, Fehr and Schmidt's (1999) inequality aversion preferences are not differentiable at the individual's reference point (see equation 4).

are concave as well. The existence of Nash equilibrium in pure strategies follows then immediately from the classical Debreu-Glicksberg-Fan's result for games with continuous and quasi-concave payoffs (Theorem 1).

Second, the paper studies the properties of the Nash equilibria when the parties hold symmetric electoral goals, meaning that they care equally about winning the election. Using the necessary conditions for the existence of a maximum, Theorem 2 characterizes the equilibrium policies of each party, which are shown to be unique and the same for both. In addition, the Theorem also proves that these policies are "optimal", in the sense that they can be rationalised as the outcome of maximizing a mixture of a "self-regarding utilitarian" social welfare function and society's other-regarding preferences. The optimality result in Lindbeck and Weibull (1987) is derived as a special case under the assumption that society is purely selfish (Corollary 1). Finally, third, by strengthening a bit the assumption on the shape of the other-regarding utility, namely, by assuming *strict* concavity, the paper shows in Theorem 3 that the uniqueness result stated in Theorem 2 holds more generally, and not just under symmetric party motivations, provided that the condition for equilibrium existence is in place.

With regard to the literature most closely related to this article, preferences for redistribution that goes beyond those motivated by the agents' own economic benefits have been studied in Galasso (2003), Alesina and Angeletos (2005a,b), Tyran and Sausgruber (2006), Dhami and al-Nowaihi (2010a,b), Lutten and Valfort (2012), and Flamand (2012). These papers differ from the current work primarily because they focus on the Meltzer and Richard's (1981) median voter framework of redistributive politics, instead of the probabilistic voting (swing voter) model. A robust result coming out from this body of research is that the presence of other-regarding preferences leads not only to different predictions concerning the extent of redistribution, but also the link between inequality and redistribution.<sup>3</sup>

In the context of probabilistic electoral competition, to the our knowledge the only two articles that incorporates other-regarding preferences into the analysis are Alesina, Cozzi, and Mantovan (2012) and Debowicz, Saporiti, and Wang (2017). The first paper analyzes a dynamic extension of the Lindbeck-Weibull model to explain how different perceptions of fairness of the market outcomes can lead to different steady states of redistribution and growth. Meanwhile, the second paper, that is, Debowicz et al. (2017), studies the consequences of different distributions of policymaking power over distributive policies and income inequality in the presence of fairness concern. In contrast with the current work, these two papers focus only on fairness, and they do not provide equilibrium

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<sup>3</sup>For example, in the Meltzer-Richard model with social preferences, redistribution depends not only on the mean to median income ratio, but also on the variance of the income distribution.

existence and optimality results for a broad family of other-regarding preferences, which is the precisely the main objectives of the coming sections.

## 2 The Model

There is a society with a continuum of voters divided into  $n$  disjoint groups, noted  $N = \{1, 2, \dots, n\}$ , where  $n_i \in (0, 1)$  indicates the size of group  $i \in N$ , and  $\sum_{i \in N} n_i = 1$ . The initial (finite) gross income of each voter of group  $i \in N$  is given by  $w_i > 0$ . Let  $w = \sum_{i \in N} n_i w_i$  be the total income of the economy, and denote the set of all possible distributions of  $w$  by  $Y = \{\mathbf{y} \in \mathbb{R}_+^N \mid \sum_{i \in N} n_i y_i = w\}$ .

The preferences of each voter  $i \in N$  over  $Y$  are additively separable. To be precise, voter  $i$ 's utility associated with each income distribution  $\mathbf{y} \in Y$  is defined as

$$U_i^h(\mathbf{y}) = u_i(y_i) + \alpha_i \sigma^h(\mathbf{y}), \quad (1)$$

where  $u_i : \mathbb{R}_+ \rightarrow \mathbb{R}$  is a *self-regarding utility* over disposable income  $y_i$ , and the function  $\sigma^h : \mathbb{R}_+^N \rightarrow \mathbb{R}$  represents voter  $i$ 's *other-regarding utility*, parameterized by  $\alpha_i \in \mathbb{R}_+$ , with the index  $h$  denoting the specific other-regarding hypothesis under consideration, to be discussed in Section 3. These utility functions are assumed to satisfy the following standard assumptions of microeconomic theory:

**A1.**  $u_i(\cdot)$  is twice continuously differentiable on  $\mathbb{R}_+$ , with  $u_i'(\cdot) > 0$  and  $u_i''(\cdot) < 0$ .

**A2.**  $\sigma^h(\cdot)$  is continuous and concave on  $Y$ .

As the examples in Section 3 point out, full differentiability of  $\sigma^h(\cdot)$  is not always guaranteed under the different models of other-regarding preferences that this paper aims to accommodate. A case in point is inequality aversion, where the utility has a kink and it is not differentiable at the individual's reference point (own payoff). To deal with these cases, assume that the other-regarding utility verifies the following assumption:

**A3.**  $\sigma^h(\cdot)$  is twice continuously differentiable on  $Y_d \subset Y$ , except possibly on a subset  $\bar{Y}_d = Y \setminus Y_d$  of Lebesgue measure zero.

Moving to the political setup, there are two political parties, indexed by  $C = A, B$ , that compete in a single election proposing simultaneously a tax-and-transfer distributive policy  $\mathbf{x}_C \in X = \{\mathbf{x}'_C \in \mathbb{R}^N \mid \sum_{i \in N} n_i x'_{iC} = 0 \text{ and } \forall i \in N, x'_{iC} \geq -w_i\}$ . Given that the initial income of each group is held fixed during the analysis, define accordingly from **A3** a subset of policies  $X_d \subset X$  where the party payoff functions (yet to be defined) are smooth, with  $\bar{X}_d = X \setminus X_d$  denoting the subset where they are not. This notation will be used in Section 4 along the proofs of the main results.

A voter in group  $i \in N$  votes for party  $A$  if  $U_i^h(\mathbf{x}_A) \geq U_i^h(\mathbf{x}_B) + \theta_i$ ,<sup>4</sup> where  $\theta_i \in \mathbb{R}$  denotes voter  $i$ 's policy-independent preference bias towards party  $B$ , drawn from a twice continuously differentiable distribution function  $F_i$ , with density  $f_i$  positive everywhere over the interval that includes all possible values of the utility differences  $t_i^h(\mathbf{x}_A, \mathbf{x}_B) = U_i^h(\mathbf{x}_A) - U_i^h(\mathbf{x}_B)$ .<sup>5</sup> The (expected) vote share of party  $A$  is given by  $v_A^h(\mathbf{x}_A, \mathbf{x}_B) = \sum_{i \in N} n_i F_i(U_i^h(\mathbf{x}_A) - U_i^h(\mathbf{x}_B))$ . Assuming no voter abstention, party  $B$ 's vote share is  $v_B^h = 1 - v_A^h$ .

The payoff functions of the parties, viz.  $\Pi_C^h$ , express the interests of the politicians, who campaign to maximize their vote share (expected plurality). In addition, the payoffs reflect the views of regular party members, who see the party as a vehicle to promote not just their own interest, but also the well-being of others in society. Formally, the payoff function of party  $C$  is defined as  $\Pi_C^h(\mathbf{x}_A, \mathbf{x}_B) = v_C^h(\mathbf{x}_A, \mathbf{x}_B) + \alpha_C \sigma^h(\mathbf{x}_C)$ , where  $\alpha_C \in [0, \infty)$  is the *relative* value that party  $C$  assigns to other-regarding concerns.<sup>6</sup>

Let  $\mathcal{G}^h = (X, \Pi_C^h)_{C=A,B}$  denote the ***distributive election game*** determined by the model sketched above. The timing of  $\mathcal{G}^h$  is as follows. First, parties  $A$  and  $B$  choose simultaneously and non-cooperatively  $\mathbf{x}_A$  and  $\mathbf{x}_B$ , respectively. At this stage, parties know the initial income of the groups, voters' preferences over the income distributions, and the group-specific cumulative distributions of the preference bias. Second, the actual values of  $\theta_i$  are realized. Third, voters cast their vote for one of the parties. Fourth, plurality rule determines the winning party (with ties broken by a fair lottery) and its policy platform is implemented. Finally, fifth, parties and voters receive their payoffs.

A pure-strategy Nash equilibrium of  $\mathcal{G}^h = (X, \Pi_C^h)_{C=A,B}$  is a policy profile  $(\hat{\mathbf{x}}_A, \hat{\mathbf{x}}_B) \in X \times X$  such that  $\Pi_C^h(\hat{\mathbf{x}}_C, \hat{\mathbf{x}}_{-C}) \geq \Pi_C^h(\mathbf{x}', \hat{\mathbf{x}}_{-C})$  for all  $\mathbf{x}' \in X$  and  $C = A, B$ , where the index  $-C$  denotes  $B$  if  $C = A$  and  $A$  if  $C = B$ .

### 3 Other-Regarding Preferences

This section offers a few important examples of other-regarding behavior that fit well into the model of Section 2. Consider first Alesina and Angeletos' (2005a) *fairness preferences* (FP) hypothesis. The distinctive feature of this hypothesis is that individuals distinguish

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<sup>4</sup>To save on notation and given that the initial income  $w_i$  is fixed, the utility  $U_i^h$  is written simply as a function of  $\mathbf{x}_C$ , instead of the disposable incomes  $\mathbf{y}_C = (y_{iC})_{i \in N} \in Y$ , where  $y_{iC} = w_i + x_{iC}$ . When there is no risk of confusion, the same notation is adopted for other functions that also depend on  $\mathbf{y}_C$ .

<sup>5</sup>Instead of being additive, the preference bias can be a multiplicative factor on the utility of policy, implying that party  $A$  is preferred by  $i$  if  $U_i^h(\mathbf{x}_A) \geq \theta_i U_i^h(\mathbf{x}_B)$ . Given that the logarithm of  $U_i^h(\cdot)$  is also a utility, the results obtained for the additive case extend directly to the multiplicative model.

<sup>6</sup>Alternatively,  $\alpha_C$  can be seen in some cases as the reputation cost for the party of campaigning on distributive policies perceived by the electorate as "socially insensible" (i.e., the cost of building the image of being a "nasty party" that only cares about the privileged few and not the many, as the British Conservative Prime Minister, Theresa May, put it in her 2002 party conference speech).

between *fair* and *unfair* income inequality, and they express dislike and concern only for the second. To be more precise, suppose the initial income of voter  $i \in N$  is given by  $w_i = e_i + \eta_i$ , where  $e_i$  denotes his fair (earned) income, received in compensation for talent and effort, and  $\eta_i$  indicates his unfair (unearned) income, obtained through lucky or illicit transactions. Assume  $\eta_i$  is distributed independently from  $e_i$  with zero mean.

In the presence of fairness concern, the other-regarding utility corresponding to any income distribution  $\mathbf{y} \in Y$  takes the form

$$\sigma^{FP}(\mathbf{y}) = - \sum_{i \in N} n_i (y_i - e_i)^2, \quad (2)$$

which captures that only unfair income comes at a utility cost to the individuals.

A second hypothesis of other-regarding behavior corresponds to the model proposed by Dimick, Rueda and Stegmueller (2017), named *income-dependent altruism* (IDA). The main assumption is that individuals are concerned with aggregate social welfare. To be concrete, under this hypothesis the other-regarding utility of any income distribution  $\mathbf{y} \in Y$  takes the form of the standard utilitarian social welfare function,

$$\sigma^{IDA}(\mathbf{y}) = \sum_{i \in N} n_i u_i(y_i), \quad (3)$$

which is the sum of individuals' self-regarding utilities, each weighted by the group size.

Finally, the third hypothesis discussed here is the (reference-dependent) *inequality aversion* (IA) model of Fehr and Schmidt (1999). The key feature of it is that individuals evaluate inequality differently depending on the position of their own payoff relative to the others. For any  $\mathbf{y} \in Y$ , the other-regarding utility of voter  $i$  is

$$\sigma^{IA}(\mathbf{y}) = -\gamma \sum_{j \neq i} n_j \max\{y_j - y_i, 0\} - \beta \sum_{j \neq i} n_j \max\{y_i - y_j, 0\}, \quad (4)$$

where  $\beta \leq \gamma$  and  $\beta \in [0, 1)$ .<sup>7</sup> The first (resp., second) term in the right-hand side of equation (4) represents group  $i$ 's disadvantageous (resp., advantageous) inequality, weighted by  $\gamma$  (resp.,  $\beta$ ). The assumption is that individuals are more selfish than altruistic, and consequently that they are more concerned with disadvantageous inequality.<sup>8</sup>

Notice that the examples of other-regarding preferences given above are associated

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<sup>7</sup>Dhami and al-Nowaihi (2010a) and (2010b) consider a generalization of Fehr and Schmidt's (1999) model where payoff comparisons are not made directly in terms of monetary payoffs, but in utility terms. A major drawback of that model is that the other-regarding utility is not necessarily concave.

<sup>8</sup>Inequality aversion preferences are self-centered, because individuals use their payoff as a reference point with which everyone else is compared to. However, people are not concerned with inequality per se. This stands in opposition with experimental evidence, which shows that in simple distribution games people also consider differences among others in their utility functions (Engelmann and Strobel 2004).

with continuous and concave utility functions, which are differentiable everywhere except possibly on a set of points of Lebesgue measure zero. There are other examples of other-regarding behavior relevant for distributive politics which also share the properties of assumptions **A2** and **A3**, including maximin and quasi-maximin preferences, efficiency concerns, Bolton and Ockenfels' (2000) inequality aversion model, etc.<sup>9</sup> The next section explores equilibrium existence and optimality within the class of social preferences that satisfy these restrictions and guarantee the concavity of the parties' conditional payoff functions.

## 4 Results

To start the analysis, define below a condition, denoted  $\mathbb{C}^h$ , that generalizes Lindbeck and Weibull's (1987) sufficient condition (see the discussion at the end), conveniently adapted for the framework laid out in Section 2. Fix any  $\mathbf{x}_{-C} \in X$  and let  $t_i^h(\mathbf{x}_C, \mathbf{x}_{-C}) = t_i^h(\mathbf{x}_C)$ .

**Condition  $\mathbb{C}^h$ :** For all  $i \in N$ ,

$$\inf_{\mathbf{x}, \hat{\mathbf{x}} \in X_d} \left( \frac{f_i(t_i^h(\mathbf{x})) - f_i(t_i^h(\hat{\mathbf{x}}))}{f_i(t_i^h(\mathbf{x}))} \cdot \frac{\sum_{j \in N} \frac{\partial U_i^h(\hat{\mathbf{x}})}{\partial x_j} \cdot (x_j - \hat{x}_j)}{\sum_{j \in N} \left( \frac{\partial U_i^h(\mathbf{x})}{\partial x_j} - \frac{\partial U_i^h(\hat{\mathbf{x}})}{\partial x_j} \right) \cdot (x_j - \hat{x}_j)} \right) \geq -1.$$

The next three propositions illustrate how  $\mathbb{C}^h$  together with **A1**, **A2** and **A3** shape the conditional payoffs of the parties. Beginning with the gradient of the expected vote shares, these are shown to be monotone decreasing on the differentiable subset of distributive policies, a result that follows immediately from condition  $\mathbb{C}^h$ .

**Lemma 1** Suppose assumptions **A1**–**A2** hold. Under condition  $\mathbb{C}^h$ , for each  $C = A, B$  and all  $\mathbf{x}_{-C} \in X$ , the gradient  $\nabla v_C^h(\cdot, \mathbf{x}_{-C})$  is monotone decreasing on  $X_d$ .

**Proof.** Fix any  $\mathbf{x}_{-C} \in X$ . The gradient of party  $C$ 's expected vote share  $\nabla v_C^h(\cdot, \mathbf{x}_{-C})$  is monotone decreasing on  $X_d$  if for all  $\mathbf{x}^1, \mathbf{x}^2 \in X_d$ ,

$$[\nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) - \nabla v_C^h(\mathbf{x}^2, \mathbf{x}_{-C})] \cdot (\mathbf{x}^1 - \mathbf{x}^2) \leq 0. \quad (5)$$

Recall that  $v_C^h(\mathbf{x}_C, \mathbf{x}_{-C}) = \sum_{i \in N} n_i v_{iC}^h(\mathbf{x}_C, \mathbf{x}_{-C})$ , where  $v_{iC}^h(\mathbf{x}_C, \mathbf{x}_{-C}) = F_i(U_i^h(\mathbf{x}_C) - U_i^h(\mathbf{x}_{-C}))$ . Thus, inequality (5) holds if for all  $i \in N$ ,

$$\frac{f_i(t_i^h(\mathbf{x}^1)) - f_i(t_i^h(\mathbf{x}^2))}{f_i(t_i^h(\mathbf{x}^1))} \cdot \frac{\sum_{j \in N} \frac{\partial U_i^h(\mathbf{x}^2)}{\partial x_j} (x_j^1 - x_j^2)}{\sum_{j \in N} \left[ \frac{\partial U_i^h(\mathbf{x}^1)}{\partial x_j} - \frac{\partial U_i^h(\mathbf{x}^2)}{\partial x_j} \right] (x_j^1 - x_j^2)} \geq -1,$$

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<sup>9</sup>See Engelmann and Strobel (2007), Alesina and Giuliano (2010), Clark and D'Ambrosio (2015), Dhami (2016) and the references therein for alternative theories of redistribution preferences.



which is implied by condition  $\mathbb{C}^h$ .  $\blacksquare$

Due to the presence of non-differentiable points, the policy subset  $X_d$  is not necessarily convex. Hence, Lemma 1 is not enough to prove the concavity of  $v_C^h(\cdot, \mathbf{x}_{-C})$ . To do so, we need another preliminary result, which ensures that the expected vote share of each party has a support on  $X_d$ .

**Lemma 2** Suppose assumptions **A1–A3** hold. Under condition  $\mathbb{C}^h$ , for each  $C = A, B$  and all  $\mathbf{x}_{-C} \in X$ , the expected vote share  $v_C^h(\cdot, \mathbf{x}_{-C})$  has a support at each  $\mathbf{x} \in X_d$ .

**Proof.** Fix any  $\mathbf{x}_{-C} \in X$ . The expected vote share  $v_C^h(\cdot, \mathbf{x}_{-C})$  has a support at  $\mathbf{x}^1 \in X_d$  if there exists a vector  $\mathbf{a}(\mathbf{x}^1) \in \mathbb{R}^N$  such that for any  $\mathbf{x}^2 \in X_d$ ,

$$v_C^h(\mathbf{x}^2, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) + \mathbf{a}(\mathbf{x}^1) \cdot (\mathbf{x}^2 - \mathbf{x}^1). \quad (6)$$

Consider any  $\mathbf{x}', \mathbf{x}'' \in X$ , and let  $S(\mathbf{x}', \mathbf{x}'') = \{\delta \mathbf{x}' + (1 - \delta) \mathbf{x}'' \in X, \text{ with } \delta \in (0, 1)\}$ . Fix *any*  $\mathbf{x}^2 \in X_d$ . There are three cases to study.

**Case 1.** Suppose  $S(\mathbf{x}^1, \mathbf{x}^2) \cap \overline{X}_d = \emptyset$ . Then, the function  $v_C^h(\cdot, \mathbf{x}_{-C})$  is differentiable on  $S(\mathbf{x}^1, \mathbf{x}^2)$ . Taking the gradient  $\nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C})$  as the support vector  $\mathbf{a}(\mathbf{x}^1)$ , inequality (6) holds because  $S(\mathbf{x}^1, \mathbf{x}^2)$  is open and convex, and consequently Lemma 1 implies that  $v_C^h(\cdot, \mathbf{x}_{-C})$  is concave on it.

**Case 2.** Assume  $S(\mathbf{x}^1, \mathbf{x}^2) \cap \overline{X}_d = \{\mathbf{z}^k, k = 1, \dots, K\}$ , where  $K$  is a finite positive integer. For each  $k$ , let  $\mathbf{z}^k = \lambda^k \mathbf{x}^1 + (1 - \lambda^k) \mathbf{x}^2$  for some  $\lambda^k \in (0, 1)$ . Without loss of generality, assume  $\lambda^1 < \dots < \lambda^K$ . Consider the open and convex subsets  $S(\mathbf{x}^1, \mathbf{z}^1), S(\mathbf{z}^1, \mathbf{z}^2), \dots, S(\mathbf{z}^K, \mathbf{x}^2)$ . The expected vote share  $v_C^h(\cdot, \mathbf{x}_{-C})$  is smooth on each of these subsets. Using the argument of Case 1 on the first subset  $S(\mathbf{x}^1, \mathbf{z}^1)$ , it follows from (6) that

$$v_C^h(\mathbf{z}^1, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) + \nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) \cdot (\mathbf{z}^1 - \mathbf{x}^1). \quad (7)$$

Applying the same reasoning on the second subset, i.e.,  $S(\mathbf{z}^1, \mathbf{z}^2)$ , and invoking the continuity of the expected vote share  $v_C^h(\cdot, \mathbf{x}_{-C})$ , we have that

$$v_C^h(\mathbf{z}^2, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{z}^1, \mathbf{x}_{-C}) + \lim_{\delta \rightarrow 0} \nabla v_C^h(\delta \mathbf{z}^2 + (1 - \delta) \mathbf{z}^1, \mathbf{x}_{-C}) \cdot (\mathbf{z}^2 - \mathbf{z}^1). \quad (8)$$

Notice in the above inequality that since  $\mathbf{z}^1 \in \overline{X}_d$ ,  $\lim_{\delta \rightarrow 0} \nabla v_C^h(\delta \mathbf{z}^2 + (1 - \delta) \mathbf{z}^1, \mathbf{x}_{-C})$  represents the superdifferential  $\partial_S v_C^h(\mathbf{z}^1, \mathbf{x}_{-C})$  of  $v_C^h(\cdot, \mathbf{x}_{-C})$  at  $\mathbf{z}^1$ , and that (8) holds for

each supergradient vector in  $\partial_S v_C^h(\mathbf{z}^1, \mathbf{x}_{-C})$ .<sup>10</sup> Adding up (7) and (8) and using the fact that by Lemma 1,

$$\left[ \nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) - \lim_{\delta \rightarrow 0} \nabla v_C^h(\delta \mathbf{z}^2 + (1 - \delta) \mathbf{z}^1, \mathbf{x}_{-C}) \right] \cdot (\mathbf{z}^2 - \mathbf{z}^1) \geq 0,$$

it follows that

$$v_C^h(\mathbf{z}^2, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) + \nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) \cdot (\mathbf{z}^2 - \mathbf{x}^1). \quad (9)$$

Finally, the desired result is obtained by repeating the previous argument over all the remaining subsets, which proves that inequality (6) holds strictly on  $S(\mathbf{x}^1, \mathbf{x}^2)$  with support vector  $\mathbf{a}(\mathbf{x}^1) = \nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C})$ .

**Case 3.** Suppose  $S(\mathbf{x}^1, \mathbf{x}^2) \cap \overline{X}_d = \cup_{k=1}^K A^k$ , where each  $A^k$  is a closed and convex subset, and  $K$  is a finite positive integer. If  $A^k$  is a singleton for every  $k$ , then this case coincides with Case 2. Otherwise, there must exist some  $k$  and  $\underline{\delta}, \overline{\delta} \in (0, 1)$  such that the subset  $A^k = \{\delta \mathbf{x}^1 + (1 - \delta) \mathbf{x}^2 \in \overline{X}_d, \text{ with } \delta \in [\underline{\delta}, \overline{\delta}]\}$ . By **A3**, for all  $\epsilon > 0$  there exists  $\hat{\mathbf{x}}^2 \in B_\epsilon(\mathbf{x}^2)$  such that  $S(\mathbf{x}^1, \hat{\mathbf{x}}^2) \cap \overline{X}_d$  is a finite set. Using the argument of Case 2, note that equation (6) holds strictly on  $S(\mathbf{x}^1, \hat{\mathbf{x}}^2)$ , with support vector  $\mathbf{a}(\mathbf{x}^1) = \nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C})$ . Applying the continuity of the function  $v_C^h(\cdot, \mathbf{x}_{-C})$  gives

$$v_C^h(\mathbf{x}^2, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) + \nabla v_C^h(\mathbf{x}^1, \mathbf{x}_{-C}) \cdot (\mathbf{x}^2 - \mathbf{x}^1), \quad (10)$$

which concludes the argument for  $A^k$  and completes the proof that  $v_C^h(\cdot, \mathbf{x}_{-C})$  has a support at  $\mathbf{x}$ . ■

We are finally ready to show the concavity of the expected vote share.

**Lemma 3** Suppose assumptions **A1–A3** hold. Under condition  $\mathbb{C}^h$ , for each  $C = A, B$  and all  $\mathbf{x}_{-C} \in X$ , the expected vote share  $v_C^h(\cdot, \mathbf{x}_{-C})$  is concave on  $X$ .

**Proof.** Fix any  $\mathbf{x}_{-C} \in X$ . By the fundamental theorem on the support of a concave function,  $v_C^h(\cdot, \mathbf{x}_{-C})$  is concave on  $X$  if and only if it has support at each interior point of  $X$ . By Lemma 2,  $v_C^h(\cdot, \mathbf{x}_{-C})$  has support on  $X_d$ . The rest of the proof is based on the following two claims.

**Claim 1** For each  $\mathbf{x} \in X_d$ , the support vector of  $v_C^h(\cdot, \mathbf{x}_{-C})$  at  $\mathbf{x}$  holds for all  $\mathbf{x}^0 \in \overline{X}_d$ .

<sup>10</sup>Recall that the superdifferential of a function  $f : X \subset \mathbb{R}^N \rightarrow \mathbb{R}$  at  $\mathbf{x} \in X$  is the set of vectors  $\partial_S f(\mathbf{x}) = \{\mathbf{a} \in \mathbb{R}^N \mid f(\hat{\mathbf{x}}) \leq f(\mathbf{x}) + \mathbf{a} \cdot (\hat{\mathbf{x}} - \mathbf{x}), \text{ for all } \hat{\mathbf{x}} \in X\}$ .

Fix  $\mathbf{x} \in X_d$  and consider any  $\mathbf{x}^0 \in \overline{X_d}$ . By **A3**, for all  $\epsilon > 0$  there exists  $\mathbf{x}' \in B_\epsilon(\mathbf{x}^0) \cap X_d$ . By Lemma 2, the function  $v_C^h(\cdot, \mathbf{x}_{-C})$  has support over  $X_d$ , meaning that  $v_C^h(\mathbf{x}', \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}, \mathbf{x}_{-C}) + \nabla v_C^h(\mathbf{x}, \mathbf{x}_{-C}) \cdot (\mathbf{x}' - \mathbf{x})$ . Since  $v_C^h(\cdot, \mathbf{x}_{-C})$  is continuous on  $X$ , taking the limit of the previous inequality as  $\mathbf{x}' \rightarrow \mathbf{x}^0$  gives

$$v_C^h(\mathbf{x}^0, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}, \mathbf{x}_{-C}) + \nabla v_C^h(\mathbf{x}, \mathbf{x}_{-C}) \cdot (\mathbf{x}^0 - \mathbf{x}),$$

which provides the desired result.

**Claim 2**  $v_C^h(\cdot, \mathbf{x}_{-C})$  has support at each  $\mathbf{x}^0 \in \overline{X_d}$ .

Fix  $\mathbf{x}^0 \in \overline{X_d}$ . By **A3**, for sufficient small  $\epsilon > 0$ , there exists  $\mathbf{x}' \in B_\epsilon(\mathbf{x}^0) \cap X_d$  such that  $\{\delta\mathbf{x}' + (1 - \delta)\mathbf{x}^0 : \delta \in (0, 1)\} \subset X_d$ . Using the support of  $v_C^h(\cdot, \mathbf{x}_{-C})$  at  $\mathbf{x}'$  and taking the limit as  $\mathbf{x}' \rightarrow \mathbf{x}^0$ , for all  $\mathbf{x} \in X$ ,

$$v_C^h(\mathbf{x}, \mathbf{x}_{-C}) \leq v_C^h(\mathbf{x}^0, \mathbf{x}_{-C}) + \lim_{\delta \rightarrow 0} \nabla v_C^h(\delta\mathbf{x}' + (1 - \delta)\mathbf{x}^0, \mathbf{x}_{-C}) \cdot (\mathbf{x} - \mathbf{x}^0).$$

Hence,  $v_C^h(\cdot, \mathbf{x}_{-C})$  has support at  $\mathbf{x}^0$ .  $\blacksquare$

The next theorem generalizes Lindbeck and Weibull's (1987) existence result for probabilistic electoral competition, establishing the existence of a Nash equilibrium in pure strategies when voters' other-regarding concerns permit a large degree of preference interdependence, which may imply that the payoff functions of the parties are not smooth on the strategy space. The proof follows immediately from Debreu-Glicksberg-Fan's existence result. Indeed, note first that the strategy space  $X$  is non-empty, compact, and convex.<sup>11</sup> Second, each payoff function  $\Pi_C^h(\mathbf{x}_A, \mathbf{x}_B)$  is continuous on  $(\mathbf{x}_A, \mathbf{x}_B) \in X \times X$ . Finally, third, Lemma 3 together with assumption **A2** guarantee that the conditional payoff functions  $\Pi_C^h(\cdot, \mathbf{x}_{-C})$  are concave in the party's own strategy  $\mathbf{x}_C$ .

**Theorem 1 (Existence)** Suppose assumptions **A1–A3** hold. Under condition  $\mathbb{C}^h$ , the election game  $\mathcal{G}^h = (X, \Pi_C^h)_{C=A,B}$  has a pure strategy Nash equilibrium.

The results stated above guarantees the existence of Nash equilibrium in pure strategies in a broad family of income redistribution games. This includes games with and without social preferences, with voters and parties displaying several patterns of other-regarding behavior, and also with *symmetric* (i.e.,  $\alpha_A = \alpha_B$ ) and *asymmetric* (i.e.,  $\alpha_A \neq \alpha_B$ ) other-regarding concerns in the parties' payoff functions.

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<sup>11</sup>In this paper, the policy space  $X$  is determined by the resource constraint and the groups' non-negative income constraints. However, the proof of Theorem 1 applies more generally, provided that non-emptiness, compactness, and convexity are preserved. That includes other typical restrictions on  $X$ , such as non-income sorting among different socio-economic groups (cf. Debowicz et al. 2017).

While the theorem constitutes an essential part of the equilibrium analysis, existence *per se* is only the first step. To use the model for predictive purposes requires being able to spell the properties of the policies played in equilibrium. The rest of this section deals with this matter. In particular, it focuses on the conditions under which the equilibrium is unique and it results in an “optimal” after-tax income distribution, in the sense that it can be rationalised as the outcome of maximizing a “sound” social welfare function.

Define the weighted (self-regarding) utilitarian social welfare function as  $W(\mathbf{x}) = \sum_{i \in N} n_i f_i(0) u_i(w_i + x_i)$ . Let  $X^0 = \{\mathbf{x} \in X : w_i + x_i > 0 \text{ for all } i \in N\}$ .<sup>12</sup> The next result yields the following equilibrium characterization.

**Theorem 2 (Characterization)** Suppose assumptions **A1–A3** hold. Let  $(\hat{\mathbf{x}}_A, \hat{\mathbf{x}}_B) \in X^0 \times X^0$  be the Nash equilibrium of  $\mathcal{G}^h$ . If  $\alpha_A = \alpha_B \equiv \bar{\alpha}$ , then  $\hat{\mathbf{x}}_A = \hat{\mathbf{x}}_B \equiv \hat{\mathbf{x}}$ , and

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in X^0} W(\mathbf{x}) + \Sigma^h(\mathbf{x}), \quad (11)$$

where  $\Sigma^h(\hat{\mathbf{x}}) = \sum_{i \in N} \psi_i \sigma^h(\hat{\mathbf{x}})$ , with  $\psi_i \equiv n_i f_i(0) \alpha_i + \bar{\alpha}$ .

**Proof.** Fix the equilibrium strategy  $\hat{\mathbf{x}}_{-C}$  and consider the constrained optimization problem of party  $C$ , which consists in maximizing with respect to  $\mathbf{x}_C \in \mathbb{R}^N$  the function  $\Pi_C^h(\mathbf{x}_C, \hat{\mathbf{x}}_{-C}) = v_C^h(\mathbf{x}_C, \hat{\mathbf{x}}_{-C}) + \alpha_C \sigma^h(\mathbf{x}_C)$ , subject to  $\sum_{i \in N} n_i x_{iC} = 0$  and  $w_i + x_{iC} > 0$ , all  $i \in N$ . By assumption **A3**,  $\Pi_C^h(\cdot, \hat{\mathbf{x}}_{-C})$  is twice continuously differentiable almost everywhere on  $X$ . Using the Karush-Kuhn-Tucker optimality conditions, it follows that a necessary condition for a maximum requires that for each group  $i \in N$ , there exists supergradient vector  $p(\hat{\mathbf{x}}_C) \in \partial_S \sigma^h(\hat{\mathbf{x}}_C)$  such that

$$n_i f_i(t_i^h(\hat{\mathbf{x}}_C, \hat{\mathbf{x}}_{-C})) \left( \frac{\partial u_i(w_i + \hat{x}_{iC})}{\partial x_{iC}} + \alpha_i p(\hat{\mathbf{x}}_C) \cdot \mathbf{i} \right) + \alpha_C p(\hat{\mathbf{x}}_C) \cdot \mathbf{i} + n_i \lambda_C = 0, \quad (12)$$

where  $\lambda_C \geq 0$  is the Lagrange multiplier on the party’s resource constraint and  $\mathbf{i} \in \mathbb{R}^N$  is the unit vector in the direction of group  $i$ ’s income.

Using the above expression for both parties and after some algebraic manipulation the following condition characterizes the equilibrium policies:

$$\frac{\lambda_A}{\lambda_B} = \frac{n_i f_i(t_i^h(\hat{\mathbf{x}}_A, \hat{\mathbf{x}}_B)) \left( \frac{\partial u_i(w_i + \hat{x}_{iA})}{\partial x_{iA}} + \alpha_i p(\hat{\mathbf{x}}_A) \cdot \mathbf{i} \right) + \alpha_A p(\hat{\mathbf{x}}_A) \cdot \mathbf{i}}{n_i f_i(t_i^h(\hat{\mathbf{x}}_B, \hat{\mathbf{x}}_A)) \left( \frac{\partial u_i(w_i + \hat{x}_{iB})}{\partial x_{iB}} + \alpha_i p(\hat{\mathbf{x}}_B) \cdot \mathbf{i} \right) + \alpha_B p(\hat{\mathbf{x}}_B) \cdot \mathbf{i}}. \quad (13)$$

It is easy to see that if  $\alpha_A = \alpha_B$ , then a solution to (13) is given by  $\hat{\mathbf{x}}_A = \hat{\mathbf{x}}_B$  and  $\lambda_A = \lambda_B$ . In fact, there is no other solution with  $\hat{\mathbf{x}}_A = \hat{\mathbf{x}}_B$  and  $\lambda_A \neq \lambda_B$ . Therefore,

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<sup>12</sup>Lindbeck and Weibull (1987) also assume that each voter’s disposable income is strictly positive.

any other critical point must be such that  $\hat{\mathbf{x}}_A \neq \hat{\mathbf{x}}_B$ . Without loss of generality, let  $\hat{x}_{iA} < \hat{x}_{iB}$  for some  $i \in N$ . By the resource constraint, there exists a  $j \in N$  such that  $\hat{x}_{jA} > \hat{x}_{jB}$ . By the strict concavity of the self-regarding utility,  $\frac{\partial u_i(w_i + \hat{x}_{iA})}{\partial x_{iA}} > \frac{\partial u_i(w_i + \hat{x}_{iB})}{\partial x_{iB}}$ . By the definition of supergradient vector for non-smooth concave functions,  $p(\hat{\mathbf{x}}_A) \cdot \mathbf{i} \geq \frac{\sigma^h(\hat{\mathbf{x}}_B) - \sigma^h(\hat{\mathbf{x}}_A)}{\hat{x}_{iB} - \hat{x}_{iA}} \geq p(\hat{\mathbf{x}}_B) \cdot \mathbf{i}$ . Since  $f_i(t_i^h(\hat{\mathbf{x}}_A, \hat{\mathbf{x}}_B))$  are positive and the same for both parties, equation (13) implies that  $\lambda_A > \lambda_B$ . Repeating the argument for group  $j \in N$ , with  $\frac{\partial u_j(w_j + \hat{x}_{jA})}{\partial x_{jA}} < \frac{\partial u_j(w_j + \hat{x}_{jB})}{\partial x_{jB}}$  and  $p(\hat{\mathbf{x}}_A) \cdot \mathbf{j} \leq p(\hat{\mathbf{x}}_B) \cdot \mathbf{j}$ , it follows that  $\lambda_A < \lambda_B$ , a contradiction. Hence,  $\hat{\mathbf{x}}_A = \hat{\mathbf{x}}_B$  is the only solution to (13), and consequently  $f_i(t_i^h(\hat{\mathbf{x}}_A, \hat{\mathbf{x}}_A)) = f_i(0)$ .

Finally, equation (11) follows by applying the Karush-Kuhn-Tucker optimality conditions to the object function  $W(\mathbf{x}) + \Sigma^h(\mathbf{x})$ , and realizing that the resulting necessary conditions for a maximum coincide with the expression in (12). ■

The previous theorem shows that when parties are symmetric, in the sense that they value power similarly, their equilibrium distributive policies coincide. This result is driven by the fact that parties' constraint optimization problems share the same necessary conditions. The theorem points out that these conditions also characterize the solution of the social planner's wealth allocation problem, provided that its objective consists in maximizing some weighted (self-regarding) utilitarian social welfare function plus an "aggregate" of individuals' and parties' other-regarding preferences.

In the special case of a purely selfish society, Theorem 2 offers as a corollary the following well-known result due to Lindbeck and Weibull (1987).

**Corollary 1 (Lindbeck-Weibull)** Under the hypotheses of Theorem 2, if  $\alpha_i = \bar{\alpha} = 0$ , then  $\hat{\mathbf{x}}$  maximizes the weighted (self-regarding) utilitarian social welfare function, i.e.,

$$\hat{\mathbf{x}} = \arg \max_{\mathbf{x} \in X^0} W(\mathbf{x}).$$

Notice that the result in Theorem 2 offers a unique equilibrium prediction for the distributive election game. This is actually preserved in a more general family of distribution games which are not necessarily symmetric. To elaborate, let's assume that the other-regarding utility satisfies the following stronger version of assumption **A2**, as is the case with Alesina and Angeletos' (2005a) fairness preferences and Dimick, Rueda and Stegmueller's (2017) income-dependent altruism.

**A2\***.  $\sigma^h(\cdot)$  is continuous and *strictly* concave on  $Y$ .

The assumption above in conjunction with the other conditions already employed allow to state the last result of the paper.

**Theorem 3 (Uniqueness)** If condition  $\mathbb{C}^h$  and assumptions **A1**, **A2\***, and **A3** hold, then the equilibrium of  $\mathcal{G}^h = (X, \Pi_C^h)_{C=A,B}$  is unique.

**Proof.** Suppose, by contradiction, that  $(\mathbf{x}'_A, \mathbf{x}'_B)$  and  $(\mathbf{x}''_A, \mathbf{x}''_B)$  are two Nash equilibria of  $\mathcal{G}^h = (X, \Pi_C^h)_{C=A,B}$ . Without loss of generality, let  $\mathbf{x}'_A \neq \mathbf{x}''_A$ . If  $(\mathbf{x}''_A, \mathbf{x}'_B)$  is a Nash equilibrium, then  $\Pi_A^h(\mathbf{x}'_A, \mathbf{x}'_B) = \Pi_A^h(\mathbf{x}''_A, \mathbf{x}'_B)$ . By assumption **A2\*** and Lemma 3, for each  $C = A, B$  and all  $\mathbf{x}_{-C} \in X$ ,  $\Pi_C^h(\cdot, \mathbf{x}_{-C})$  is strictly concave on  $X$ . Thus, for all  $\delta \in (0, 1)$ ,  $\Pi_A^h(\mathbf{x}_A^\delta, \mathbf{x}'_B) > \Pi_A^h(\mathbf{x}'_A, \mathbf{x}'_B)$ , with  $\mathbf{x}_A^\delta = \delta \mathbf{x}'_A + (1 - \delta) \mathbf{x}''_A$ , contradicting that  $(\mathbf{x}'_A, \mathbf{x}'_B)$  is a Nash equilibrium. Therefore,  $\Pi_A^h(\mathbf{x}'_A, \mathbf{x}'_B) > \Pi_A^h(\mathbf{x}''_A, \mathbf{x}'_B)$ ; and by the same token,  $\Pi_A^h(\mathbf{x}''_A, \mathbf{x}''_B) > \Pi_A^h(\mathbf{x}'_A, \mathbf{x}''_B)$ .

Adding up the above inequalities, it follows that

$$\Pi_A^h(\mathbf{x}'_A, \mathbf{x}'_B) + \Pi_A^h(\mathbf{x}''_A, \mathbf{x}''_B) > \Pi_A^h(\mathbf{x}''_A, \mathbf{x}'_B) + \Pi_A^h(\mathbf{x}'_A, \mathbf{x}''_B). \quad (14)$$

Repeating the argument for party  $B$ ,

$$\Pi_B^h(\mathbf{x}'_A, \mathbf{x}'_B) + \Pi_B^h(\mathbf{x}''_A, \mathbf{x}''_B) > \Pi_B^h(\mathbf{x}'_A, \mathbf{x}''_B) + \Pi_B^h(\mathbf{x}''_A, \mathbf{x}'_B). \quad (15)$$

It is easy to show from (14) that

$$v_A^h(\mathbf{x}'_A, \mathbf{x}'_B) + v_A^h(\mathbf{x}''_A, \mathbf{x}''_B) > v_A^h(\mathbf{x}''_A, \mathbf{x}'_B) + v_A^h(\mathbf{x}'_A, \mathbf{x}''_B). \quad (16)$$

Multiplying (16) by  $-1$ , and adding 2 on both sides,

$$(1 - v_A^h(\mathbf{x}'_A, \mathbf{x}'_B)) + (1 - v_A^h(\mathbf{x}''_A, \mathbf{x}''_B)) < (1 - v_A^h(\mathbf{x}''_A, \mathbf{x}'_B)) + (1 - v_A^h(\mathbf{x}'_A, \mathbf{x}''_B)). \quad (17)$$

Finally, adding  $\alpha_B \sigma(\mathbf{x}'_B)$  and  $\alpha_B \sigma(\mathbf{x}''_B)$  to both sides of (17),

$$\Pi_B^h(\mathbf{x}'_A, \mathbf{x}'_B) + \Pi_B^h(\mathbf{x}''_A, \mathbf{x}''_B) < \Pi_B^h(\mathbf{x}''_A, \mathbf{x}'_B) + \Pi_B^h(\mathbf{x}'_A, \mathbf{x}''_B), \quad (18)$$

which stands in contradiction with (15). Hence, the equilibrium is unique. ■

Motivated once again by Alesina and Angeletos (2005a) and Dimick, Rueda and Stegmüller (2017), suppose that the other-regarding utility is smooth and the welfare effect for voter  $i$  of a marginal change in his disposable income is invariant to the income of the others.<sup>13</sup> Then, under assumption **A2\***, condition  $\mathbb{C}^h$  takes a much simpler form, which relates easily to the Lindbeck–Weibull condition.

To elaborate, define for each group  $i \in N$ , the index  $\left(\sum_{j \in N} \xi_{ij}^h(\mathbf{x})\right)^{-1}$ , which measures the overall concavity of the utility function  $U_i^h(\cdot)$  at  $\mathbf{x} \in X$ , where  $\xi_{ij}^h(\mathbf{x}) = -\frac{[\partial U_i^h(\mathbf{x})/\partial x_j]^2}{\partial^2 U_i^h(\mathbf{x})/\partial x_j \partial x_j}$ . Likewise, given a strategy profile  $(\mathbf{x}_A, \mathbf{x}_B) \in X \times X$ , define the log-

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<sup>13</sup>Technically,  $\frac{\partial^2 \sigma^h(\mathbf{y})}{\partial y_i \partial y_j} = 0$  for all  $i \neq j$ ,  $i, j \in N$ , and all  $\mathbf{y} \in Y$ .

arithmetic rate of change of the probability density  $f_i$  as the ratio  $r_i(t_i^h) = \frac{f_i'(t_i^h)}{f_i(t_i^h)}$ .<sup>14</sup> It is easy to see that condition  $\mathbb{C}^h$  requires that for all  $i \in N$ ,

$$\sup_{t_i^h} r_i(t_i^h) \leq \inf_{\mathbf{x} \in X} \left( \sum_{j \in N} \xi_{ij}^h(\mathbf{x}) \right)^{-1}. \quad (19)$$

Notice from (19) that if voters are purely selfish, that is, if  $\alpha_i = 0$  for all  $i \in N$ , then condition  $\mathbb{C}^h$  reduces simply to Lindbeck and Weibull's (1987) sufficient condition, namely,  $\sup r_i(t_i^h) \leq \inf (\xi_{ii}^h(\mathbf{x}_C))^{-1}$ . The reason is the second-order cross derivatives of the vote shares are all null without other-regarding utility, which simplifies greatly the Hessian matrix of the function  $v_C$ . By contrast, in the presence of other-regarding concern, the marginal increase in the percentage of votes that one party obtains by changing group  $i$ 's transfers varies with the transfers allocated to group  $j \neq i$ , making the cross derivatives nonzero.

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<sup>14</sup>In the uniform case, for instance, this ratio is equal to zero, meaning that changes in the utility differential affect the marginal vote-returns of each party at a constant rate.

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