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Angelos Angelopoulos

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Economics
School of Social Sciences
The University of Manchester
Manchester M13 9PL

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ANGELOS ANGELOPOULOS

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School of Social Sciences
University of Manchester
Oxford Road
Manchester M13 9PL
United Kingdom

ABSTRACT: We assume an exchange economy with cooperating asymmetrically informed agents, who face uncountably infinite states and extreme ambiguity. Upon this economy we define the maximin (efficient incentive compatible) value allocation. We then prove existence of this notion.

KEYWORDS: value allocation; asymmetric information; ambiguity; uncountable states; existence.

JEL Classification: D5 · D81 · D82 · D86

1. Introduction

Consider an economy with asymmetrically (i.e., privately) informed agents. Assume, specifically, that each agent's private information is a partition of the economy's state space or, interchangeably, the σ - algebra generated by this partition. Ambiguity arises naturally in such an economy and agents lose sensibly their Bayesian identity. This can be easily understood, by means of the following simple example of an economy with a finite number of states:

Say that $\Omega = \{\omega_1, \omega_2, \omega_3\}$ and fix an agent α in this economy. For α, assume that $\Pi_\alpha = \{\{\omega_1\}, \{\omega_2, \omega_3\}\}$, so that $\mathcal{F}_\alpha = \sigma(\Pi_\alpha) = \{\{\omega_1\}, \{\omega_2, \omega_3\}, \emptyset, \Omega\}$. It is reasonable to assume that since α is privately informed, the prior belief (additive probability measure) q_α of α is informationally restricted, i.e., that $q_\alpha : \mathcal{F}_\alpha \to [0, 1]$. Then, although, for example, α assigns a probability to $\{\omega_2, \omega_3\}$, he is unable to attach a probability to ω_2 and ω_3. That is, $q_\alpha(\omega_2)$ and $q_\alpha(\omega_3)$ are unknown to α; α has ambiguity concerning the probability of occurrence of the states ω_2 and ω_3. Agent α is non Bayesian, since his Bayesian (or subjective) expected utility cannot be defined.

The (non Bayesian) agent α is specifically said to face extreme ambiguity, if he ignores the probability of occurrence of all the (non trivial) events in his informational algebra. In our example, if for all $A \in \mathcal{F}_\alpha$, with $\emptyset \neq A \subset \Omega$, it holds that $q_\alpha(A)$ is not unambiguously known to α. Clearly, an agent with extreme ambiguity concerning his probabilities does not have a prior belief at all. No probability measure makes sense to be assigned to him in the first place.

Whichever the case is, it is an experimental fact[1] that individuals are ambiguity averse. By assuming that agents are, in particular, maximin ambiguity averse, the gain in the properties of general equilibrium outcomes is tremendous (see in de Castro and Yannelis, 2009, de Castro et al., 2011, 2012, He and Yannelis, 2013, Angelopoulos and Koutsougeras, 2014 and Angelopoulos, 2014).

The value allocation under ambiguity (namely, the maximin value allocation) is motivated along this analytical line. The private (information) maximin value allocation, specifically, was introduced in de Castro and Yannelis (2009). It was revisited and discussed in Angelopoulos and Koutsougeras (2014) as well, but only as a special

[1]First recognized by Ellsberg, 1961.

case of an "informationally generalized" maximin value allocation notion. Indeed, Angelopoulos and Koutsougeras (2014) introduce maximin value allocations where the informational partition (or algebra) that individuals use inside coalitions is either the initial one they are endowed with, or some (any) other, depending on the coalition they are into and the underlying information exchange protocol within it.

In this paper, attention is specifically drawn to the private maximin value allocation for various reasons. Any maximin value allocation is a cardinal (Shapley, 1969) value allocation, hence, a fair cooperative general equilibrium concept. Indeed, the level of the contribution of an agent in his coalitions reflects on the level of the utility this agent is assigned with. The private maximin value allocation, in particular, extends the private Bayesian value allocation of Krasa and Yannelis, 1994, 1996 (see in Angelopoulos and Koutsougeras, 2014 and Angelopoulos, 2014). Thereby, it inherits all the desirable properties of the latter notion and principally the fact that the informational superiority of an agent is rewarded (in consumption, hence, utility terms). At the same time, its chief advantages over the private Bayesian value allocation are two (see, again, in Angelopoulos and Koutsougeras, 2014 and Angelopoulos, 2014): (i) it exists without private information measurable net trades; thus, allows for informationally unconstrained (first best) Pareto efficiency and (ii) it is less[2] informationally constrained efficient incentive compatible.

Maximin value allocations are not necessarily viable in economies with a non - finite number of states[3]. Such economies, on the other hand, arise naturally in real life. In Angelopoulos (2014), existence of a private maximin value allocation was proved with countably infinite many states. In this paper the same is done with an uncountably infinite state space.

[2]In the sense that private information measurable initial endowments only (and not consumption) need to assumed.

[3]Indeed, agents' maximin utilities (minimized over the states) are not well defined to begin with.

2. Preliminaries

Let $\mathcal{E}$ be a two (interim - ex post) period exchange economy. The state space $\Omega = \mathbb{R}^k$, $k < \infty$, models the underlying state contingent uncertainty, state dependent randomness and informational structure of $\mathcal{E}$. $I = \{1, 2, ..., n\}$ is the finite set of agents of $\mathcal{E}$ and an $S \in \mathcal{P}(I)$ is a coalition of agents. $\mathbb{R}^l$, $l < \infty$, is the economy's commodity space. Agents of $\mathcal{E}$ trade by writing (coalitionally) consumption contracts. $\mathcal{F} = \mathcal{B}(\mathbb{R}^k)$ is the Borel σ - algebra of Ω and $(\Omega, \mathcal{F})$ is a Borel space.

Π_i is the informational partition of the agent i. Π_i is assumed to be a measurable partition of $\mathbb{R}^k$. $\mathcal{F}_i = \sigma(\Pi_i) \subset \mathcal{B}(\mathbb{R}^k)$ is the informational σ - algebra of the same agent. Agents of $\mathcal{E}$ trade in the economy's interim period, in which they receive market signals regarding the ex post realized state. They are, therefore, endowed with advanced information: $\Pi_i(\omega) \in \Pi_i$ contains the actual (realised in the second period) state ω. $\Pi_i(\omega) \subseteq \mathbb{R}^k$ [and $\Pi_i(\omega) \in \mathcal{B}(\mathbb{R}^k)$], for any $\omega \in \Omega$, becomes now the new, refined informational set of the i agent, onto which he focuses.

The (Borel) probability measure $q_i : \mathcal{F}_i \to [0, 1]$ is the informationally restricted private prior of the i agent, satisfying (by definition) the following incompleteness property: $q_i(B_i)$ may be unknown for a $\emptyset \neq B_i \subset A_i \in \mathcal{F}_i$, even though $q_i(A_i)$ is provided (known) by q_i. That is, the economy's agents may be unable to completely form a prior belief. To put it differently, agents of $\mathcal{E}$ face ambiguity. Since for any $\omega \in \Omega$ the event $\Pi_i(\omega)$ and all its subevents are, actually, the only events that "matter" for the i agent, agent i accumulates the probability distribution of his prior q_i to $\Pi_i(\omega) \in \mathcal{F}_i$, or (w.l.o.g.) to $\Pi_i(\omega) \in \Pi_i$. Then, for any $i \in I$ and for any $\omega \in \Omega$, $q_i(B_i)$ is unknown for any B_i that satisfies (i) $\emptyset \neq B_i \subset \Pi_i(\omega)$ and (ii) B_i has the cardinality of the continuum[4], even though $q_i(\Pi_i(\omega))$ is known $[q_i(\Pi_i(\omega)) = 1]$. Thus, the non Bayesian agents of $\mathcal{E}$ face extreme ambiguity and lose their priors.

The (surjective) function $x_i : \Omega \to X_i \subset \mathbb{R}^l_+$ gives a random state dependent (r.s.d.) consumption plan $\{x_i(\omega) : \omega \in \Omega\}$ of the i agent. The agent's i r.s.d. consumption set is identified with the set of functions

[4] Clearly, condition (ii) is not needed when $\Pi_i(\omega)$ is finite or countable.

$$L_{X_i} = \{\ x_i \mid x_i : \Omega \to X_i\ \},$$

or (equivalently) with the class of sets

$$\mathbb{X}_i = \{\ x_i(\Omega) = X_i : X_i \subset \mathbb{R}^l_+\ \},$$

which is equivalent to the set of vectors

$$\mathbb{X}_i = \{\ \{x_i(\omega) : \omega \in \Omega\}\ \}=\{x_i(\omega) : x_i(\omega) \in \mathbb{R}^l_+\} \subset \mathbb{R}^l_+.$$

The set $\mathbb{X}_i$ (or the set L_{X_i}) contains the feasible consumption of the i agent. The function $e_i \in L_{X_i}$ gives the r.s.d. initial endowment plan of the i agent.

The agent's i preferences are represented by the r.s.d. utility function

$$u_i : \Pi_i(\omega) \times \mathbb{X}_i \to \mathbb{R}_+, \text{ for any } \omega \in \Omega.$$

Agents are utility maximizers. They also are maximin ambiguity averse. Thus, the agent's i aforementioned preferences give rise to the same agent's (interim) maximin preferences, represented by the (interim) maximin utility map $\underline{u}_i : \Omega \times L_{X_i} \to \mathbb{R}_+$. According to de Castro and Yannelis (2009), $\underline{u}_i$ is given for any $(\omega, x_i) \in \Omega \times L_{X_i}$ by the formula

$$\underline{u}_i(\omega, x_i) = \min_{\omega' \in \Pi_i(\omega)} u_i(\omega', x_i(\omega')).$$

The interpretation of the above formulation is the following: *The i agent, considering the worst possible state, chooses the best possible utility.* In de Castro and Yannelis (2009), the agent's i (interim) maximin (non expected) utility is established as above, when (and because) Ω is finite. With an infinite Ω, however, the previous expression is not well defined, since the minimum may fail to exist. Towards overcoming this analytical obstacle, it seems natural to redefine the agent's i $\underline{u}_i$ as

$$\underline{u}_i(\omega, x_i) = \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in \mathbb{X}_i} u_i(\omega', x_i(\omega')).$$

This is an equivalent reformulation of the agent's i (interim) maximin utility, since the new formula carries the following interpretation: *The i agent minimizes (with* *respect to the states) his maximum (with respect to his consumption) utility.* More importantly, the previous reformulation will allow us to well define the agent's i (interim) maximin utility format. When the agent i maximizes his utility u_i, then the same agent maximizes his (interim) maximin utility $\underline{u}_i$ as well. The agents of $\mathcal{E}$, therefore, are (interim) maximin utility maximizers. Additionally, agents are assumed to have monotone/increasing (interim) maximin preferences[5].

Concluding, we define the economy

$$\mathcal{E}=\{\ \mathbb{R}^l\ ;\ (\mathbb{R}^k, \mathcal{B}(\mathbb{R}^k))\ ;\ (\ [\ \Pi_i, \mathbb{X}_i, e_i, \underline{u}_i(u_i)\ ] : i \in I\ )\ \},$$

for which the list of assigned (to all the economy's agents) functions

$$x = (x_1, x_2, ..., x_i, ..., x_s) \in L_X = \prod_{i \in I} L_{X_i},$$

$$\text{satisfying} \quad \sum_{i \in I} x_i = \sum_{i \in I} e_i \iff \sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega), \text{ for all } \omega \in \Omega,$$

is a feasible r.s.d. allocation (contract), i.e., a non free disposal general equilibrium.

3. The maximin value allocation

Let a Shapley (1953) - value - solvable (interim) maximin transferable utility game $\Gamma = (I, V_{\lambda,\underline{u},\omega}, Sh)$, within which the players' payoffs are identified with (interim) maximin utilities. Γ is a coalitional game with side payments, played by the finitely many players $1, 2, ..., n \in I$. An $S \subseteq I$ is a coalition of players. If Ω is the state space of Γ, $\omega \in \Omega$ is the actual state. $\underline{u}$ is the set of all the players' (interim) maximin utility functions. The players' (interim) maximin utilities $\underline{u}_i(\cdot)$, $i \in I$, become common scaled (hence, interpersonally comparable) and transferable by a personal r.s.d. factor $\lambda_i : \Omega \to \mathbb{R}_{++}$, assigned to each player i. λ is the set of all the players' factors. $V(\lambda, \underline{u}, \omega) := V_{\lambda,\underline{u},\omega} : 2^I \to \mathbb{R}_+$ is a (monotone, superadditive and becoming zero for the null coalition - interim) maximin characteristic function of Γ. If $\mathcal{V}$ is the class of all the $V_{\lambda,\underline{u},\omega}$ of Γ, $Sh : \mathcal{V} \to \mathbb{R}^n_+$ is the (interim) maximin Shapley value function of Γ, which solves Γ by assigning:

[5]The pointwise partial ordering is assumed on both L_{X_i} and $\mathbb{X}_i$.

(i) to Γ the (interim) maximin Shapley (1953) value $Sh(V_{\lambda,\underline{u},\omega}) \in \mathbb{R}^n_+$ and
(ii) to each player i of Γ the respective coordinate $Sh_i(V_{\lambda,\underline{u},\omega}) \in \mathbb{R}_+$ of the previous vector, where in particular

$$Sh_i(V_{\lambda,\underline{u},\omega}) = \sum_{S\subseteq I, i\in S} \tfrac{(|S|-1)!(|I|-|S|)!}{|I|!}[V_{\lambda,\underline{u},\omega}(S) - V_{\lambda,\underline{u},\omega}(S \setminus \{i\})],\ |I| = n.$$

Sh satisfies both group rationality [$\sum_{i\in I} Sh_i(V_{\lambda,\underline{u},\omega}) = V_{\lambda,\underline{u},\omega}(I)$] and individual rationality [$Sh_i(V_{\lambda,\underline{u},\omega}) \geq V_{\lambda,\underline{u},\omega}(\{i\})$, for all $i \in I$].

We now define the (interim private) maximin value allocation for $\mathcal{E}$, of Angelopoulos and Koutsougeras (2014), by associating $\mathcal{E}$ with Γ.

Definition An allocation $x \in L_X$ of $\mathcal{E}$ is said to be an (*interim private*) *maximin value allocation* if the following two conditions are satisfied for any $\omega \in \Omega$:

1. $\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega)$,

2. for all $i \in I$, we have that $\lambda_i(\omega) \min_{\omega'\in\Pi_i(\omega)} \max_{x_i(\omega')\in\mathbb{X}_i} u_i(\omega', x_i(\omega')) = Sh_i(V_{\lambda,\underline{u},\omega})$, where

$\lambda_i(\omega) > 0$ for all i and $V_{\lambda,\underline{u},\omega}$ is defined by

$$V_{\lambda,\underline{u},\omega}(S) = \max_{x_i(\omega)\in\mathbb{X}_i} \sum_{i\in S} \lambda_i(\omega) \min_{\omega'\in\Pi_i(\omega)} \max_{x_i(\omega')\in\mathbb{X}_i} u_i(\omega', x_i(\omega')),$$

$$\text{subject to } \sum_{i\in S} x_i(\omega) = \sum_{i\in S} e_i(\omega). \quad \square$$

Remark 1 Side payments are not assumed within any (interim private) maximin value allocation. The $V_{\lambda,\underline{u},\omega}$ of Γ as specified in the definition above is (indeed) monotone, superadditive and becomes zero for the empty coalition. Group and individual rationality of Sh of Γ imply the (interim) maximin Pareto efficiency and individual rationality of the (interim private) maximin value allocation. (Interim) maximin efficiency and private information measurable initial endowments secure transfer (interim) maximin coalitional incentive compatibility[6] for the (interim private) maximin value allocation. The proof of this statement is essentially the same with the one of Theorem 2 of Angelopoulos and Koutsougeras (2014). Nothing changes with uncountable infinitely many states. $\square$

Remark 2 When agents have monotone (interim) maximin preferences, the following property holds for every (feasible and interim maximin individually rational Pareto optimal) interim private maximin value allocation of $\mathcal{E}$: *Every coalition maximizes its (interim) maximin utility subject to its consumption constraints* if and only if *every agent in a coalition independently maximizes his (interim) maximin utility subject to the feasibility of consumption within this coalition.* $\square$

4. Existence

The theorem that follows provides the conditions both for the well definition of the agent's i (interim) maximin utility and for the existence of the corresponding (interim private) maximin value allocation in $\mathcal{E}$.

Theorem If for each agent i and for any state ω the following assumptions hold:
(A_1) $\Pi_i(\omega)$ and $\mathbb{X}_i$ are compact in $\mathbb{R}^k$ and $\mathbb{R}^l$ respectively,
(A_2) u_i is continuous on $\mathbb{R}^k \times \mathbb{R}^l$,
then $\min\limits_{\omega' \in \Pi_i(\omega)} \max\limits_{x_i(\omega') \in \mathbb{X}_i} u_i(\omega', x_i(\omega'))$ and an (interim private) maximin value allocation exist in $\mathcal{E}$.

Proof. For the whole proof: (i) Assume the standard topology and the pointwise ordering on any finite dimensional Euclidean space and (ii) fix an agent i and a state ω. Wlog, define u_i as $u_i : \mathbb{X}_i \times \Pi_i(\omega) \to \mathbb{R}_+$. Consider the correspondence $\phi_i : \Pi_i(\omega) \to \mathbb{X}_i$, defined by $\phi_i(\omega') = \mathbb{X}_i$. This is a constant correspondence, hence a continuous correspondence (both upper and lower hemicontinuous). Also, ϕ_i is nonempty valued (since $e_i \in L_{X_i}$) and compact valued [from (A_1)]. From (A_2),

[6] See in Angelopoulos and Koutsougeras (2014) for the definition of this notion.

u_i is continuous on $\mathbb{X}_i \times \Pi_i(\omega)$, which is a subset of the Euclidean space $\mathbb{R}^l \times \mathbb{R}^k$. Then, from Berge's (1963, p. 116) Maximum Theorem, it follows that the maximum function $f_i : \Pi_i(\omega) \to \mathbb{R}_+$ exists, is defined by

$$f_i(\omega') = \max\{u_i(x_i(\omega'), \omega') : x_i(\omega') \in \phi_i(\omega')\} =$$

$$= \max_{x_i(\omega') \in \phi_i(\omega')} u_i(x_i(\omega'), \omega') = \max_{x_i(\omega') \in \mathbb{X}_i} u_i(x_i(\omega'), \omega')$$

and is continuous on $\Pi_i(\omega)$, which [according to (A_1)] is compact. This means that, from the Weierstrass' Extreme Value Theorem, f_i attains its minimum value over $\Pi_i(\omega)$, i.e., that the

$$\min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in \mathbb{X}_i} u_i(x_i(\omega'), \omega') \text{ exists in } \mathcal{E}.$$

We now verify the validity of condition 1 of the definition in section 3, i.e., we show that a feasible allocation exists in $\mathcal{E}$. Since $e_i \in L_{X_i}$, it holds that

$$\sum_{i \in I} x_i(\omega)\ ,\ \sum_{i \in I} e_i(\omega) \in \sum_{i \in I} \mathbb{X}_i \neq \emptyset.$$

Therefore, it can be the case that $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$. We finally prove condition 2 of the same definition, to conclude that an (interim private) maximin value allocation exists in $\mathcal{E}$. For this, we first have to show that the $V_{\lambda,\underline{u},\omega}$ of $\Gamma = (I, V_{\lambda,\underline{u},\omega}, Sh)$[7] exists (is well defined). By Remark 2, we have for any $S \subseteq I$ and for $\lambda_i(\omega) > 0$, for all $i \in S$, that

$$V_{\lambda,\underline{u},\omega}(S) = \sum_{i \in S} \lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in \mathbb{X}_i} u_i(\omega', x_i(\omega')), \text{ subject to } \sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega),$$

which leads us to the expression

$$V_{\lambda,\underline{u},\omega}(S) = \sum_{i \in S} \lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega')), \text{ where } e = \sum_{i \in S} e_i(\omega) \in \mathbb{R}^l_+.$$

[7]In the way it was specified in the definition of section 3.

The rectangle $[0, e]$ is compact on $\mathbb{R}^l$ and since u_i is continuous on $\mathbb{R}^k \times \mathbb{R}^l$, it follows that u_i is continuous on $\mathbb{R}^k \times [0, e]$ as well. Then, by following the same argumentation as in the first part of the proof, it is implied that for any $i \in S$ the

$$\min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega'))$$

exists, hence $V_{\lambda,\underline{u},\omega}(S)$ exists for any S. Next, returning again to the fixed agent i, for any $\lambda_i(\omega) > 0$ of this agent and relying on Remark 2, we have (for any S) that

$$V_{\lambda,\underline{u},\omega}(S) - V_{\lambda,\underline{u},\omega}(S \setminus \{i\}) = \sum_{i \in S} \lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega')) \text{ -}$$

$$\text{- } \sum_{i \in S \setminus \{i\}} \lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega')) = \lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega')).$$

Then, $\quad Sh_i(V_{\lambda,\underline{u},\omega}) = \sum_{S \subseteq I, i \in S} \frac{(|S|-1)!(|I|-|S|)!}{|I|!} \lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega'))=$

$$=\lambda_i(\omega) \min_{\omega' \in \Pi_i(\omega)} \max_{x_i(\omega') \in [0,e]} u_i(\omega', x_i(\omega')) \frac{\sum_{S \subseteq I, i \in S} (|S|-1)!(|I|-|S|)!}{|I|!}, \text{ where } |I| = n.$$

We finally verify that $\sum_{S \subseteq I, i \in S} (|S|-1)!(|I|-|S|)! = n!$. By definition,

$$\sum_{S \subseteq I, i \in S} (|S|-1)!(|I|-|S|)! = 0!(n-1)![\binom{n}{1} - \binom{n-1}{1}] + 1!(n-2)![\binom{n}{2} - \binom{n-1}{2}]+$$

$$2!(n-3)![\binom{n}{3} - \binom{n-1}{3}] + ... + (k-1)!(n-k)![\binom{n}{k} - \binom{n-1}{k}] + ... + (n-1)!,$$

where $3 < k < n$ and the quantity $[\binom{n}{k} - \binom{n-1}{k}]$ expresses the number of coalitions of cardinality $k \in \mathbb{N}$ that agent i participates in. Now, each term of the previous expanded sum is equal to $(n-1)!$. We can verify that with the general term, that is

$$(k-1)!(n-k)![\binom{n}{k} - \binom{n-1}{k}] = (k-1)!(n-k)![\frac{n!}{(n-k)!k!} - \frac{(n-1)!}{(n-k-1)!k!}] =$$

$$= \frac{n!}{k} - \frac{(n-k)(n-1)!}{k} = \frac{n!}{k} - \frac{n(n-1)!-k(n-1)!}{k} = \frac{n!}{k} - \frac{n!-k(n-1)!}{k} = (n-1)!$$

This, finally, means that $\sum_{S \subseteq I, i \in S} (|S|-1)!(|I|-|S|)! = n(n-1)! = n!$. $\square$

5. Conclusions

The proof of the theorem provided in section 4 relies heavily on the (Berge's, 1963) Maximum Theorem. Therefore, the analysis (and the theorem of the paper) can be easily generalized if we use ordered topological spaces to model the state space and the commodity space of the economy (instead of Euclidean spaces).

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