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Carlo Reggiani

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School of Social Sciences
The University of Manchester
Manchester M13 9PL

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Abstract

The spokes model allows addressing non-localized spatial competition between firms. In a spatial context firms can personalize their product and price discriminate using location-contingent pricing. Non-localized competition implies that neighbouring effects are not relevant to firms. This paper analyses spatial price discrimination and location choices in the spokes model. Highly asymmetric location patterns are a very likely outcome: firms either supply a generally appealing product line or focus on a specific niche. Moreover, multiple equilibriums can arise so that the location patterns do not always globally minimize the sum of transportation costs.

JEL code: D43, L11, L13.

Keywords: spatial price discrimination, spokes model, optimal location.

1 Introduction

Price discrimination is a pervasive practice in many markets: it takes place both in highly concentrated markets and also in more competitive ones, in which several firms are active. Price discrimination also arises in markets strongly characterized by a spatial dimension. A feature of these markets is that competition is not necessarily localized: firms compete to attract a consumer not only with neighbouring firms but

*School of Social Sciences, University of Manchester, Manchester M13 9PL, UK. E-mail: carlo.reggiani@manchester.ac.uk. I wish to thank Hemant Bhargava, Anindya Bhattacharya, John Bone, Ramon Caminal, Bipasa Datta, Norman Ireland, Mario Pezzino, Alejandro Saporiti, Peter Simmons, Klaus Zauner and seminar participants in Braga, Manchester, York, the IO Workshop in Lecce, the EARIE Annual Meeting in Ljubljana and the SAET Conference in Faro for helpful discussion and comments. The usual disclaimer applies.

also with more distant ones. A number of spatial non-localized markets exist where price discrimination takes place; if location is interpreted as the space of characteristics, examples include wines, beers and other alcoholic and soft drinks but also sports equipment, shoes and clothes: in all these markets, IT and marketing innovation are leading to wider and more personalized product lines as firms try to better match consumers' tastes and extract surplus from consumers in different segments.

A key strategic decision in these markets is firms' location and, hence, which segment of the market to be targeted. A very relevant question is whether firms hit on a specific niche of the market or adapt their product line in a way that makes it appealing to a wider segment of consumers, i.e. they supply a "general purpose" product line.

The spokes model (Chen and Riordan, 2007a) provides a framework to analyse markets characterized by spatial but non-localized competition between firms: the model naturally extends the Hotelling (1929) approach to the case of several segments and an arbitrary number of firms by modelling the market as a collection of spokes with a common core. Consumers can buy from whichever firm they like: if the firm is not located on their own spoke, however, either the customer or the delivering firm have to travel through the centre of the market. The spokes model is an important alternative to the circular city model (Salop, 1979) when the neighbouring effects of competition are not particularly relevant.

This paper addresses the question of what segments firms target by analysing optimal location in the spokes model. The analysis shows that, in presence of product personalization and price discrimination, the market is likely to be characterized by one firm whose product line is appealing to consumers in all segments of the market; the remaining competitors target only part of their market segment, focusing on a niche of customers with a strong preference for the varieties supplied by the firm. In devising the optimal location/product line patterns in the spokes model with price discrimination, it is also found that multiple equilibriums may arise. In that case, one of the outcomes may not globally minimize the sum of transportation costs.

The spokes model has been introduced relatively recently but has already been widely used in the literature. Chen and Riordan (2007a) show that the model captures Chamberlin's original idea of monopolistic competition; moreover, strategic interaction between firms may lead to price increasing competition. Caminal and Claici (2007) use the model to show how the business stealing effect makes loyalty-rewarding schemes mostly pro-competitive. Chen and Riordan (2007b) show the joint relevance of vertical integration and exclusive dealing in foreclosing the upstream market and increasing downstream prices. Germano (2009) and Germano and Meier (2010) adopt the model to address issues related to the bias of information in media markets. Caminal (2010) analyses the supply of content in different languages in bilingual contexts. Caminal and Granero (forthcoming) look at the provision of variety by multi-product firms in the spokes model. Mantovani and Ruiz-Aliseda (2011) analyse cooperative innovation activity of firms producing complementary products. A unifying characteristic of the recalled literature is the focus on pricing and entry aspects of the interaction between firms. This paper is, to the best of our knowledge, the first addressing the issue of location in the spokes model.

The contribution of this paper is also related to the literature on spatial price discrimination and endogenous location choice. Thisse and Vives (1988) observed that price discrimination in a spatial market is detrimental to firms' profits: firms exploit their information on consumers' locations and can match any offer made by a rival firm, unless this is lower than the cost of delivering the good. The classical paper of Lerner and Singer (1937) established that the optimal location configuration on a line is transport cost minimizing. Lederer and Hurter (1986) endogenize location and establish the existence of price-location equilibriums in a duopolistic spatial framework. They confirm under quite general assumptions (e.g. two-dimensional space, generic consumers' distribution) that the profit maximizing locations chosen by firms correspond to the ones that minimize the overall transport costs afforded by firms. However, in presence of multiple equilibriums, the location configuration may not be globally minimizing the sum of transport costs. MacLeod, Norman and Thisse (1988) consider the price-location equilibriums of an n-firms spatial model where the number of firms is endogenously determined by the fixed costs. Free entry might lead to either a too large or a too small number of varieties. Vogel (2011) analyses spatial price discrimination and the location of heterogeneous firms in the circle finding that more efficient firms are relatively more isolated in equilibrium. Anderson and De Palma (1988) question Lederer and Hurter's results by introducing product heterogeneity: the equilibrium location pattern minimizes the overall transport costs only in presence of homogeneous or very heterogeneous products. Konrad (2000) shows that in presence of a contest for consumers, in which firms afford sunk costs, the equilibrium locations are not minimizing overall transport costs. Gupta (1992) considers sequential entry in a linear city with discriminatory pricing; his results crucially depend on the number of firms (two or three) entering in the market. As Lederer and Hurter (1986), Anderson and De Palma (1988) and Konrad (2000), also this paper shows that price discrimination may not lead to locations profiles that minimize the overall transport cost: the market features and structure (number of firms and segments), however, determine the possible rise of inefficiency in the spokes model. Differently from MacLeod, Norman and Thisse (1988) and Vogel (2011) our focus is on location and targeting of different segments of the market rather than on issues related to entry and the heterogeneity of firms.

The rest of the paper is structured as follows. Section 2 introduces the spokes model and the game played by price discriminating firms. Section 3 characterizes the outcomes of competition. Section 4 discusses and provides an interpretation of the results. Section 5 concludes.

2 The spokes model with price discrimination

The market is constituted of a set of spokes with a common core. There is a fixed number of spokes N and each spoke has constant length, normalized to $l_s = 1/2$, $s = 1...N$. Customers are distributed along each spoke according to a uniform distribution function $f(x_s)$ so that on each spoke there are $\frac{2}{N}$ customers[1]. Each customer has a valuation v for the good and can demand at most one unit of it; v is assumed to be high enough so that the market is covered.

Every firm i is assigned a spoke i and can choose a location along this spoke, with $i = 1...n$. Consistently with Chen and Riordan (2007a) it is assumed that the number of firms does not exceed the number of spokes, i.e. $n \leq N$. The good supplied is homogeneous at the source but can be adapted to consumers' tastes as the firms deliver the product and bear the cost of the distance that separates them from the consumers. A generic firm i can locate on any point of its spoke l_i which is denoted by y_i; so that $y_i \in [0,\ 1/2]$.

[1] The uniform distribution is assumed for expositional convenience; many of the results, however, are qualitatively unaffected if any non degenerate distribution function is employed. The intuition is the following: in computing both firms' profits and social cost functions, each location has to be considered independently; this is due to the assumption of price discrimination. Local competition implies that the shape of the distribution function affects the optimal location but does not affect the properties of it.

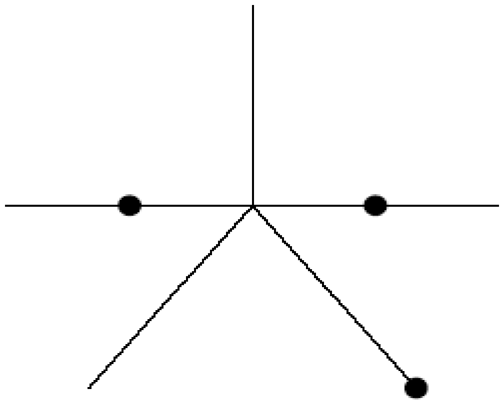

Figure 1: Spokes model with endogenous location with $n = 3$ and $N = 5$.

Figure 1 illustrates the spokes model in case two firms are located in the interior of their spoke while one is at the extreme. The two remaining spokes are not occupied by any firm although consumers are uniformly distributed over all the spokes.

Firms price discriminate customers according to their location over the spokes: as they deliver the product and so they know their taste. A generic customer located on a spoke s is identified by x: consumers in $x = 0$ are located at the extreme of the considered spoke while consumers at $x = 1/2$ are exactly at the centre of the market. The location of each consumer is fully identified by x_s. The distance between firm i, located at y_i and the customer located at x_s is denoted as $d(y_i, x_s)$ and it is also spoke-dependent. In particular, if the firm and the customer are both located on the i-th spoke, then distance can be written as:

$$d(y_i, x_s) = |y_i - x_s| \qquad s = i$$

If the firm is located on a different spoke with respect to customer x_s the distance is:

$$d(y_i, x_s) = \left(\frac{1}{2} - y_i\right) + \left(\frac{1}{2} - x_s\right) = 1 - y_i - x_s \qquad \forall s \neq i$$

as the firms always have to travel towards the centre of the market to deliver the product to consumers located over different segments.

The unit transportation cost is denoted by t captures the disutility of adapting the good to consumers tastes and it is identical for all firms and customers. Each firm produces the good at a unit and marginal cost c[2].

[2]The assumption of cost homogeneity is convenient to simplify the presentation of the main

The timing of the game played by the $n \leq N$ firms:

1. Nature assigns to each of the n firms a spoke i.
2. Location stage: each firm chooses its location $y_i \in [0, 1/2]$ on its spoke;
3. Pricing stage: given the location y_i, the firm chooses the price schedule $p_i(x_s|y_i)$.

The game is solved by backward induction to identify strategies which are undominated and constitute a sub-game perfect equilibrium. The following analysis closely parallels Lederer and Hurter (1986): analogies and differences will be highlighted.

3 Results

3.1 The pricing stage

Given the selected location y_i over their spoke, firms choose a price schedule: $p_i(x_s|y_i)$ $\forall i = 1...n$. Customers at location x_s choose to buy from the firm providing the good at the lowest price[3]. Naming X as the set of locations over all the N spokes, from the point of view of firm i the market X can be partitioned as follows:

$$D_i(p_i, p_{-i}) = \{x \in X \,\text{s.t.}\, p_i(x|y_i) < \min\{p_{-i}(x|y_{-i}\}\}$$

$$D_S(p_i, p_{-i}) = \{x \in X \,\text{s.t.}\, p_i(x|y_i) = \min\{p_{-i}(x|y_{-i}\}\}$$

The set D_i is the segment of demand served by the i-th firm individually while D_S is shared with one or more rival firms. A cost-advantage (or efficient) sharing rule completes the definition of firm i's demand schedule, i.e. a function $r(y_i, p_i, y_{-i}, p_{-i}, x)$ that in case of a price tie allocates the demand to the lowest net cost producer. The profit function of firm i can then be written as:

$$\pi_i^r(y_i, p_i, y_{-i}, p_{-i}, x) = \frac{2}{N} \int_{D_i} [p_i(x|y_i) - td(y_i, x) - c]\, dx$$
$$+ \frac{2}{N} \int_{D_S} [p_i(x|y_i) - td(y_i, x) - c]\, r(y_i, p_i, y_{-i}, p_{-i}, x) dx$$

arguments; however, the results can be generalized to a heterogeneous distribution of production costs.

[3] When no ambiguity is possible, the notation x is used from now on instead of x_s.

Consistently with Lederer and Hurter (1986), weakly dominated strategies and, hence, possible equilibriums in which weakly dominated strategies are played are ruled out. This implies:

$$p_i(x|y) \geq c + td(y_i, x) \quad \forall x \in X$$

and:

$$p_i(x|y) \leq v \quad \forall x \in X.$$

Proposition 1 characterizes the unique pure strategy equilibrium of the pricing stage and it is a straight generalization of Lederer-Hurter (1986) to the spokes model and n firms:

Proposition 1 *Given the set of locations $y = (y_1, ..., y_i, ..., y_n)$, the unique equilibrium of the pricing stage is:*

$$p_i^*(x|y) = \max \{c + td(y_i, x), \min \{c + td(y_{-i}, x)\}\} \tag{1}$$

The equilibrium price schedule is closely related to the cost structure. As a consequence of undercutting, the price at a generic location x is either the firm's cost of delivering the product or, if the firm is the lowest cost provider, the cost of the firm that is the second most efficient in delivering the good. This result constitutes the foundation of the ensuing analysis of firms' location decisions.

3.2 The location stage

The equilibrium price schedule identified by (1) implies that the profit function for firm i can be written as:

$$\pi_i(y_i, p_i, y_{-i}, p_{-i}, x) = \frac{2}{N} \int_{D_i} \min\{c + td(y_{-i}, x)\} - (c + td(y_i, x))dx$$

The Nash equilibrium of the location stage is defined as:

$$y_i^* \in \arg\max_{y_i \in X} \pi_i(y_i, p_i^*, y_{-i}^*, p_{-i}^*) \quad \forall i = 1...n$$

Consistently with Chen and Riordan (2007a), two cases are analyzed: if $n = N$ there are as many firms as spokes; if $n < N$ some spokes are not assigned to any firm.

3.2.1 The $n = N$ case

The number of firms on the market equals the number of spokes. In this context, social cost is defined as the sum of transport costs borne by firms to supply the good to all customers on the market in a cost minimizing way. Given a vector of locations $y = (y_1, ..., y_i, ...y_n)$, social cost is then:

$$SC(y) = \frac{2}{N} \int_X \min_{\forall i} \{c + td(y_i, x)\} dx$$

Social cost is a continuous function of y over the support X. The social cost function and the profits of a generic firm are closely related:

$$\pi_i(y_i, p_i, y_{-i}, p_{-i}, x) = \frac{2}{N} \int_{D_i} \min\{c + td(y_{-i}, x)\} - (c + td(y_i, x))dx =$$

$$= \frac{2}{N} \left[\int_X \min\{c + td(y_{-i}, x)\} dx - \int_X \min_{\forall j} \{c + td(y_j, x)\} dx \right] =$$

$$= \frac{2}{N} \int_X \min\{c + td(y_{-i}, x)\} dx - SC(y) \tag{2}$$

The profits of firm i consist of two elements. The first is positive and it is obtained in the region D_i where the firm is the lowest cost provider: in that region, by definition, the firm concurs to the social cost. According to (1) the profits on D_i are the differential between the firm's delivery cost and the second most efficient firm's cost. The other part is constituted by the rest of the market X on which the firm is not the lowest cost provider and, as such, does not concur to the social cost; however, it does not make any profit either. Hence, profits consist of the difference, on all the market X, between the lowest cost rival and the social cost, which in region D_i corresponds to the firm's cost while outside D_i it corresponds to the cost of the most efficient rival. Relation (2) leads to:

Proposition 2 *In the spokes mode with price discrimination and* $n = N$ *the vector of locations* $y^* = (y_1^*, ..., y_i^*, ...y_n^*)$ *is an equilibrium if and only if:*

$$SC(y_i^*, y_{-i}^*) \leq SC(y_i, y_{-i}^*) \qquad \forall y_i \in X \quad \forall i = 1...n \tag{3}$$

In the spokes model with price discrimination and $n = N$, if the equilibrium outcome is unique then it is minimising the social cost function. The result can be interpreted as follows. The competitive pressure between firms drives prices down to cost; in case of a price tie, the most efficient firm wins the consumer as by the sharing rule. As the cost of the second most efficient firm is not affected by the firm's location, all that matters to the choice of location is to minimize cost over the firm's own turf; this is in line with minimizing the social cost function. The result extends Lederer and Hurter (1986), Theorem 3, to the spokes model[4]: under price discrimination in a spokes market structure and in presence of competition between n firms, an equilibrium vector of location minimizes the social cost function.

The next result characterizes the outcomes of the game and relates them to the number of firms on the market:

Proposition 3 *Price discrimination in the spokes model with $n = N$ firms leads to the following outcomes:*

(i) the symmetric location configuration $y_i^* = \frac{1}{4}$, $\forall i = 1...n$ is an equilibrium if $n \leq 3$.

(ii) the symmetric location configuration $y_i^* = \frac{1}{4}$, $\forall i = 1...n$ and an asymmetric one $y_i^* = \frac{1}{2}$, $y_j^* = \frac{1}{6}$ $\forall j \neq i$ are both equilibriums if $n = 4, 5$.

(iii) the asymmetric outcome $y_i^* = \frac{1}{2}$, $y_j^* = \frac{1}{6}$ $\forall j \neq i$ is an equilibrium if $n \geq 6$.

The result is consistent with the outcome of spatial price discrimination in the Hotelling model with homogeneous firms (Lerner and Singer, 1937; Kats, 1987): if $n = 2$, firms locate at half of their own spoke. This symmetric location configuration constitutes an equilibrium when the market structure is not too competitive ($n \leq 5$). The reason is that when the number of spokes (and, consequently, competitors) is relatively small a move towards the centre is not profitable; however, as the number of spokes increases ($n \geq 6$), the gains of a small deviation are multiplied sufficiently to compensate for the inframarginal losses that the firm makes on its captive market. As the market becomes competitive enough ($n \geq 4$), an asymmetric location configuration is also an equilibrium. One firm occupies the central location of the market while other firms locate at one third of their spoke. Hence, if the market features four or five firms both location configurations constitute an equilibrium. The equilibriums can be compared in terms of profits: in a symmetric configuration firms get $\pi_i\left(\frac{1}{4}, ..., \frac{1}{4}\right) = \frac{3t}{8n}$ while in the asymmetric configuration the firm located in the centre

[4] This result is at the same time a special case of Theorem 3 in Lederer-Hurter[16] as the location in the spokes model is constrained to one-dimension.

gets $\pi_i\left(\frac{1}{2},\frac{1}{6},...,\frac{1}{6}\right)=\frac{t}{18n}(n+5)$ while all other firms get $\pi_j\left(\frac{1}{6},...,\frac{1}{6},\frac{1}{2}\right)=\frac{t}{6n}$. The comparison of the expressions for the relevant values of n leads to conclude that:

$$\pi_i\left(\frac{1}{2},\frac{1}{6},...,\frac{1}{6}\right)>\pi_i\left(\frac{1}{4},...,\frac{1}{4}\right)>\pi_j\left(\frac{1}{6},...,\frac{1}{6},\frac{1}{2}\right)$$

Although the two equilibrium configurations are equally likely in a simultaneous setting, if location choice was sequential the first firm would occupy the centre. The case of $n=4,5$ firms is also interesting for the implications of multiplicity on social cost. Social cost in the symmetric configuration is $SC\left(\frac{1}{4},...,\frac{1}{4}\right)=c+\frac{1}{8}t$ while in the asymmetric one is $SC\left(\frac{1}{2},\frac{1}{6},...,\frac{1}{6}\right)=c+\frac{n+2}{12n}t$; hence, social cost is identical in both configurations if $n=4$ while $SC\left(\frac{1}{2},\frac{1}{6},...,\frac{1}{6}\right)<SC\left(\frac{1}{4},...,\frac{1}{4}\right)$ if $n=5$. The asymmetric configuration is then the global minimizer of the social cost. Lederer and Hurter (1986) also showed through an example how equilibrium location chosen by two firms in the space may not be globally minimizing social cost; our example shows that an equilibrium which does not minimize globally social cost can take place even when competition takes place in one-dimension as in the spokes model. Finally, when the number of firms and spokes is high enough, the asymmetric configuration is the only equilibrium.

3.2.2 The $n<N$ case

Suppose there are more market segments than firms, i.e. the number of firms n in the market is smaller than the number of spokes N. The unique pure strategy equilibrium of the pricing stage in Proposition 1 still applies: on the spokes occupied by one firm, the lowest cost firm serves consumers, pricing at the cost of the second most efficient competitor; on spokes that are not occupied by any firm, a firm with a cost advantage in delivering would capture all the customers by pricing at the most efficient rival's delivered cost; if there is not a most efficient firm, all competitors price equally at the common delivered cost. The equilibrium price schedules are then given by (1).

Turning to the location stage, first it can be ruled out that any symmetric location configuration constitutes an equilibrium. Then, the equilibrium is characterized. The main difference with the previous case ($n=N$) is the presence of empty spokes. The consumers on parts of the market not served by a firm do not have a strongly favourite brand available on the market (or a local supplier in the geographical interpretation); hence, all firms on the market are on even grounds when trying to attract consumers from the empty segments to their product. This feature impacts on the equilibrium as no symmetric configuration is now possible.

Lemma 1 *In the spokes model with price discrimination and $n < N$ firms a symmetric pure strategy equilibrium of the location stage does not exist.*

The intuition for this result is the following. Suppose first that the centre, where all the spokes join, is the symmetric equilibrium location of all firms. In that case, firms obtain no profit and they have a unilateral incentive to deviate to a location internal to their own spokes. However, if the location equilibrium is a vector of points internal to the spokes, then a firm has an incentive to move towards the centre to undercut all competitors and serve a larger share of the market, comprising the empty spokes. A symmetric location, then, can not be an equilibrium.

The next proposition characterizes the equilibrium configuration in the $n < N$ case.

Proposition 4 *In the spokes model with price discrimination and $n < N$ firms the equilibrium location configuration is:*

$$y_i^* = \frac{1}{2} \qquad y_j^* = \frac{1}{6} \qquad \forall j \neq i$$

and price schedules are given by (1). This location configuration minimizes the social cost.

The asymmetric configuration of Proposition 3 is also an equilibrium in case not all spokes are occupied, $n < N$. The intuition for the result is also similar to the previous case: no more than one firm can locate in the centre of the market, otherwise all firms would get zero profits. So only one firm locates in the centre and serves consumers on all the segments of the market. The remaining firms optimally specialize in serving only part of their own spokes. Optimal locations are independent of the number of firms and the number of spokes. Social cost is also minimized when one firm locates at the centre of the market: the total transport costs decrease with the firm location and the centre provides the limit. All other firms choose a location in the interior of their spokes and the cost minimising one coincides with the profit maximizing, $y_j^* = \frac{1}{6}$.

3.3 Discussion

One result stands out in both of the cases we analysed ($n = N$ and $n < N$): highly asymmetric location patterns arise as a result of price discrimination in the spokes model. Notice that the result arises in a context in which firms are homogeneous and the spatial structure is perfectly symmetric. The conclusion is that price discriminating firms that face non-localized competition in a segmented market are likely to locate so that one firm occupies the central spot and serves consumers on all segments of the market. The other firms narrow their focus to their own segment.

MacLeod, Norman and Thisse (1988) propose an interpretation for spatial price discrimination in the characteristics space. In standard spatial models transportation costs can be thought as a measure of disutility and location is a product characteristic; in presence of price discrimination, instead, the situation can be interpreted as firms trying to personalize and adapt their product lines to the demand expressed by consumers. Relationship marketing and one-to-one marketing have become established practices in the last decade. The trend has been enhanced and favoured by the diffusion of online shopping, which guarantees to firms access to an unprecedented amount of data about their customers besides innumerable chances to offer more and more personalized products. In this context, some firms become specialist in providing a wide range of products to a specific segment while other may target several segments of the same market. A number of examples can be provided to illustrate this pattern, which is reminescent of our main result. Consider sports equipment and sport shoes. Nike is one of the leaders in the sector and provides products for most of the existing sports; customers can personalize their products by choosing different colours, prints and sometimes even technical characteristics. Important competitors on the same market, instead, tend to target a more limited segment: Umbro, for example, provides extensive lines of products for football (soccer); Asics is very well established in volleyball and athletics equipment; Sergio Tacchini is a familiar name to tennis players and fans; Spalding is a leading brand when it comes to basketball while Speedo provides highly technical swimming equipment. Another illustration comes from alcoholic beverages and spirits. One of the world largest groups in the sector is Diageo, featuring an extensive product line of wines, beers and spirits. Anheuser-Busch InBev is the established leader in the beers segment but smaller groups and local producers also tend to specialize in one segment or even more limited sub-segments. In UK the group Marks&Spencer offers a wide line of products in segments like food, clothes, furniture and many others in its supermarkets and stores. Other large groups are more specialized: Iceland is known for frozen food, Tesco, Sainsbury's, ASDA and Morrison's core business are grocery supermarkets, Home Base and Ikea specialize in furniture while House of Fraser, Debenhams and Primark in the department store segment.

The main result of the paper may suggest an interpretation in terms of "general purpose" products; the analogy, however, is not perfectly fitting and our contribution can be contrasted with the existing literature (Von Ungern-Sternberg, 1988; Hendel and Neiva de Figuereido, 1996; Doraszelski and Draganska, 2006). Firstly, unlike in Von Ungern-Sternberg (1988) and Hendel and Neiva de Figuereido (1996), firms supply product lines rather than individual products; moreover, general purpose and niche product lines co-exist in equilibrium. Finally, in our model the nature of the product is determined by the personalization allowed by price discrimination rather than lower transport costs (Von Ungern-Sternberg, 1988; Hendel and Neiva de Figuereido, 1996).

4 Conclusions

This paper analyses product line choices of firms in the spokes model. The spokes model provides an ideal approach to spatial non-localized competition, i.e. markets characterized by several segments in which neighbouring effects are not extremely relevant. The analysis sheds light on the effects of increased personalization and, consequently, more effective price discrimination in this context. The results suggest that under price discrimination firms' location choices are related to the competitiveness of the market. As the number of firms increases, the more likely it is to observe outcomes characterized by large asymmetries in location. In particular, one firm supplies a product line that is appealing to all segments of the market; other competitors, instead, supply specialized product lines targeted to a specific segment of the market. Such a location pattern is observed in many real world markets constituted of different segments. For intermediate values of the number of firms, multiplicity of equilibrium location patterns arise; this implies that the location configuration may not be minimizing the overall transport costs to serve the market.

The model and the results obtained can be generalized and extended in several directions. First, the outcomes devised imply that only one firm supplies a generally appealing product line. An interesting development would be to find conditions under which two or more firms opt for a product line that attracts consumers from several segments while other specialize on niches. A second interesting extension involves devising and comparing the location equilibriums in case firms adopt a uniform non-discriminatory price. The latter case proposes non-negligible technical challenges and may be solvable only in special cases. Finally, firms may not have complete information on consumers' location. Hence, a more realistic assumption would be that firms only know the distribution of consumers' location and can use that information to price discriminate.

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A Appendix

Proof of Proposition 1

The set of (weakly) undominated prices is:

$$p_i(x|y) \in [\, c + td(y_i, x),\ v\,]$$

For a given y, given the sharing rule r firm i can match any offer of a rival firm j as long as it is the most efficient at serving customer x.
Consider firm i. First the claim for which, in equilibrium, the price $p_i^*(x|y)$ is identical for all firms $i = 1...n$. Having defined above: $D_{S_i} = \{x \in D_S | r_i = 1\}$ the subset of the market region D_S in which firm i has a cost advantage. Then, assuming *ad absurdum* that:

$$\forall x \in D_{S_i},\ p_i(x|y) - p_j(x|y) = \epsilon > 0,\ j \neq i$$

then firm i loses all the customers located in x. On the other hand, proceeding again *ad absurdum*:

$$\forall x \in D_{S_i},\ p_j(x|y) - p_i(x|y) = \epsilon > 0,\ j \neq i$$

then firm i can raise its price and increase the profit margin on customers located at x. Then, the only possibility left is that: $p_i(x|y) = p_j(x|y)$. The reasoning can be repeated for all $j \neq i$ and for all $i = 1...n$.
Second, $p_i^*(x|y) = \max\{c + td(y_i, x), \min\{c + td(y_{-i}, x)\}\}$. Suppose instead that, for $x \in D_{S_i}$, the following holds:

$$p_i^*(x|y) - \max\{c + td(y_i, x), \min\{c + td(y_{-i}, x)\}\} = \epsilon > 0$$

In that case, the second most efficient firm, say j, can choose the price $p_j(x|y) = p_i^* - \xi$ and for sufficiently small ξ raise its profit, which contradicts the definition of equilibrium. The reasoning can be repeated for all $j \neq i$, all $i = 1...n$.
Analogous reasoning allows to establish the result for the subsets of X over which $r_i = r$ and $r_i = 0$. *Q.E.D.*

Proof of Proposition 2

If $y = (y_1^*, .., y_i^*, .., y_n^*)$ is a vector of equilibrium locations, then:

$$\pi_i(y_i^*, p_i^*, y_{-i}^*, p_{-i}^*) \geq \pi_i(y_i, p_i^*, y_{-i}^*, p_{-i}^*) \quad \forall y_i \in X \quad \forall i = 1...n$$

which, by (2), can be written as:

$$\frac{2}{N}\int_X \min\{c+td(y_{-i}, x)\}dx - SC(y_i^*, y_{-i}^*) \geq \frac{2}{N}\int_X \min\{c+td(y_{-i}, x)\}dx - SC(y_i, y_{-i}^*)$$

from which (3) follows immediately. The vector $y = (y_1^*, .., y_i^*, .., y_n^*)$ is then a price-location equilibrium of the spokes model when $n = N$. *Q.E.D.*

Proof of Proposition 3

- To prove point (i) and (ii) consider first outcomes in which firms choose a location internal to their spoke, $y_i \in [0, \frac{1}{2}[, \forall i = 1..n$. The profit of a given firm i is:

$$\pi_i = \frac{2}{N}\left[\int_0^{\frac{1}{2}} \min_{\forall j \neq i} \{c + td(x, y_j)\} - [c + td(x, y_i)]\, dx + \sum_{\forall j \neq i} \int_{x_{ij}}^{\frac{1}{2}} [c + td(x, y_j)] - [c + td(x, y_i)]\, dx\right]$$

  if $x_{ij} = \frac{1-y_i+y_j}{2} \geq \frac{1}{2}$. Maximizing firm i's profits leads to find $y_i^* = \frac{1}{4}$. The same outcome is obtained in case $x_{ij}^* \leq \frac{1}{2}$.Suppose firm i considers a deviation from $\frac{1}{4}$ to a location $\frac{1}{4} + \delta$. In that case, it would get a profit $\pi_i^D\left(\frac{1}{4} + \delta, \frac{1}{4}, ..., \frac{1}{4}\right)$, $0 < |\delta| < \frac{1}{4}$; this is:

$$\pi_i^D\left(\frac{1}{4} + \delta, \frac{1}{4}, ..., \frac{1}{4}\right) = \frac{1}{8N}\left(3t - 20t\delta^2 + 4nt\delta^2\right)$$

  In case the deviation is profitable, the expression:

$$\pi_i^D - \pi_i^*$$

  would have a positive sign. This happens if: $t\delta^2(-20 + 4n) > 0$, implying there is a possible deviation if and only if $n > 5$. Hence, the vector $y_i = \frac{1}{4}$, $\forall i = 1...n$ is an equilibrium only if $n \leq 5$.

- To prove point (ii) and (iii) consider asymmetric outcomes. Suppose firm i locates at $y_i = \frac{1}{2}$. The profit of a given rival firm $j \neq i$ is:

$$\pi_j = \frac{2}{N}\left[\int_0^{x_{ji}^*} \left[c + td\left(x, \frac{1}{2}\right)\right] - \left[c + td(x, y_j^*)\right] dx\right]$$

  with $x_{ji} = \frac{1}{4} + \frac{y_j}{2}$. Maximizing firm j's profits leads to find $y_j^* = \frac{1}{6}$. Firm i's profits are:

$$\pi_i^*\left(\frac{1}{2}, y_j^*\right) = \frac{t}{18N}(n + 5)$$

Suppose firm i consider a deviation from $\frac{1}{2}$ to a location $\frac{1}{2}-\delta$. In that case, it would get a profit $\pi_i^D\left(\frac{1}{2}-\delta,\frac{1}{6}\right)$, $0<\delta<\frac{1}{2}$; this is:

$$\pi_i^D\left(\frac{1}{2}-\delta,\frac{1}{6}\right)=\frac{t}{18N}\left[(9n-45)\delta^2+(24-6n)\delta+(n+5)\right]$$

In case the deviation is profitable, the expression:

$$\pi_i^D-\pi_i^*$$

would have a positive sign. This happens if: $\delta^2(9n-45)-\delta(24-6n)>0$, implying there is a possible deviation if and only if $n\leq 3$. Hence, the vector $\left(y_i=\frac{1}{2},y_k=\frac{1}{6}\right)$, $\forall k\neq i$ is an equilibrium only if $n\geq 4$.

Q.E.D.

Proof of Lemma 1

Suppose first that the vector of equilibrium locations is $y^*=(\frac{1}{2},...,\frac{1}{2})$, i.e. the centre of the market. In this case all firms obtain zero profits, as no one has cost advantage in delivering the product:

$$c+td(x,y_i^*)=\min\{c+td(x,y_{-i}^*)\}\quad\forall\ x\in X$$

which is implying that:

$$p_i(x|y)=c+td(x,y_i^*)\quad\forall\ x\in X$$

so that $\pi_i=0\,\forall i=1..n$. However, this implies that each firm has a unilateral incentive to deviate from $y_i^*=\frac{1}{2}$ and choose an internal location on its spoke $y_i\in[0,\frac{1}{2}[$. If the deviation is $\delta>0$, then:

$$c+td(x,y_i^*-\delta)<\min_{\forall j\neq i}\{c+td(x,y_j^*)\}\quad\forall\ x\in D_i$$

where D_i, the market served by firm i, is now constituted by consumers on its own spoke with a location such that i faces the lowest cost in delivering to them, i.e. $D_i=\{x\in X_i|x\in[0,\frac{1}{2}-\frac{\delta}{2}]\}$. This implies that firm i makes a positive mark-up on the market served and has a strictly positive profit:

$$\pi_i=\int_{D_i}\min_{\forall j\neq i}\{c+td(x,y_j^*)\}-[c+td(x,y_i^*-\delta)]dx>0$$

This proves that firms have a unilateral incentive to deviate, so the location profile can not be a Nash equilibrium. Then $y^* = (\frac{1}{2}, ..., \frac{1}{2})$ can not be an equilibrium.

Suppose, then, the equilibrium vector y^* is such that $y_i^* \in [0, \frac{1}{2}[\ \forall i = 1..n$. The profits received by firms are:

$$\pi_i(y^*) = \int_0^{\frac{1}{2}} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^*)]dx$$

Suppose firm i moves in the direction of the centre of the market by $\delta > 0$. In that case the profits of firm i are:

$$\pi_i(y_i^* + \delta, y_{-i}^*) = \int_{D_i} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^* + \delta)]dx$$

which can be re-expressed as:

$$\pi_i(y_i^* + \delta, y_{-i}^*) = \int_0^{\frac{1}{2}} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^* + \delta)]dx +$$
$$+(N-n)\int_0^{\frac{1}{2}} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^* + \delta)]dx +$$
$$+(n-1)\int_{\frac{1}{2}-\frac{\delta}{2}}^{\frac{1}{2}} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^* + \delta)]dx$$

The profit differential then is:

$$\Delta\pi_i(y^*, \delta) = \pi_i(y_i^* + \delta, y_{-i}^*) - \pi_i(y^*)$$

or, more explicitly:

$$\Delta\pi_i(y^*, \delta) = -\int_0^{\frac{1}{2}} [c + td(x, y_i^* + \delta)] - [c + td(x, y_i^*)]dx +$$
$$+(N-n)\int_0^{\frac{1}{2}} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^* + \delta)]dx +$$
$$+(n-1)\int_{\frac{1}{2}-\frac{\delta}{2}}^{\frac{1}{2}} \min_{\forall j \neq i}\{c + td(x, y_j^*)\} - [c + td(x, y_i^* + \delta)]dx$$

Substituting the expressions for distance and after some simplifications it is found:

$$\Delta\pi_i(y^*, \delta) = \pi_i(y_i^* + \delta, y_{-i}^*) - \pi_i(y^*) = \frac{1}{4}t\delta[8y_i^* - 2 + 2(N-n) + \delta(n-1)]$$

As $N-n>0$ and $n-1\geq 0$ it follows that $\Delta\pi_i(y^*,\delta)>0$ $\forall y^*\in[0,\frac{1}{2}[$; hence, $\pi_i(y_i^*+\delta,y_{-i}^*)>\pi_i(y^*)$ and firms have a unilateral incentive to deviate from y^*. *Q.E.D.*

Proof of Proposition 4

Suppose first that the equilibrium configuration is $y_i=\frac{1}{2}$ for all firms i; in this case, all firms obtain zero profits.This implies that only one firm chooses $y_i=\frac{1}{2}$ and all other firms choose a location belonging to the interior of their spoke. The problem faced by firms is then:

$$\max_{y_j}\pi_j(y_j,y_i)=\frac{2}{N}\int_0^{x_{ij}}[c+td(y_i,x)]-[c+d(y_j,x)]dx \qquad y_i,y_j\in\left[0,\frac{1}{2}\right]$$

$$\begin{aligned}\max_{y_i}\pi_i(y_i,y_j)=&\frac{2}{N}\int_0^{\frac{1}{2}}\min_{\forall k\neq i}\{c+td(y_k,x)\}-[c+td(y_i,x)]dx+\\&+\frac{2}{N}\sum_{k\neq i}\int_{x_{ik}}^{\frac{1}{2}}[c+d(y_k,x)]-[c+td(y_i,x)]dx+\\&+\frac{2}{N}(N-n)\int_0^{\frac{1}{2}}\min_{\forall k\neq i}\{c+td(y_k,x)\}-[c+td(y_i,x)]dx\end{aligned}$$

where:

$$x_{ik}=\frac{1-y_i+y_j}{2}$$

represents the consumer on j-th spoke which is indifferent between firm j and firm i. Were the maximization unconstrained, firm i had an incentive to choose a location $y_i>\frac{1}{2}$:

$$\left.\frac{\partial\pi_i(y_i,y_j)}{\partial y_i}\right|_{y_i=\frac{1}{2}}=\frac{2N-(n+3)}{2(n-1)}-y_j>0$$

implying, that under our assumptions on $N>n$ and $y_j\in[0,\frac{1}{2}]$, the optimal choice is $y_i^*=\frac{1}{2}$. This implies that the problem for firm j has an internal solution given by $y_j^*=\frac{1}{6}$, $\forall j\neq i$.

The location profile obtained is also minimizing the social cost; this can be written as:

$$\begin{aligned}SC(y_i,y_j)=&\frac{2}{N}\sum_{k\neq i}\int_0^{x_{ik}}[c+t|y_k-x|]dx+\frac{2}{N}\int_0^{\frac{1}{2}}[c+t|y_i-x|]dx+\\&+\frac{2}{N}(N-n)\int_0^{\frac{1}{2}}[c+t(1-y_i-x)]dx+\frac{2}{N}\sum_{k\neq i}\int_{x_{ik}}^{\frac{1}{2}}[c+t(1-y_i-x)]dx\end{aligned}$$

The problem is:

$$\min_{y_i, y_j} SC(y_i, y_j)$$
$$\text{s.t. } y_i, y_j \in \left[0, \frac{1}{2}\right]$$

The unconstrained minmization would suggest that firm i should choose location $y_i > \frac{1}{2}$ as:

$$\left.\frac{\partial SC(y_i, y_j)}{\partial y_i}\right|_{y_i=\frac{1}{2}} = -t + \frac{3n+1}{4N}t + \frac{(nty_j + 4)}{2N} < 0$$

holds for all possible $y_j \in [0, \frac{1}{2}]$. Given the constraints, the optimal choice is then $y_i^* = \frac{1}{2}$; the problem for firm j has an internal solution given by $y_j^* = \frac{1}{6}$, $\forall j \neq i$. *Q.E.D.*