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On Intertemporal Poverty: Affluence-Dependent Measures*

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Abstract: This paper proposes a class of intertemporal poverty measures based on a sequence-specific weighted average of an individual's snapshot poverty measures. The weight assigned to the level of poverty in each time period is determined by the number of periods of relative affluence directly preceding that poor period. These can have a short-lived mitigating effect on a subsequent poverty period. The properties of the measures are elaborated and an axiomatic foundation is provided.

Keywords: Aggregation, Intertemporal poverty measurement, Equity.

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1 Introduction

The important question of how poverty should best be measured has generated wide interest both in policy circles and academia leading to a large discourse on the level of poverty and the different poverty measures. Most of these poverty indices, however, capture poverty only at any given point in time (Watts, 1968, Sen, 1976, Clark, Hemming and Ulph, 1981, Chakravarty, 1983, Foster, Greer and Thorbecke (1984); see also Zheng 1997). An increasing number of studies, however, indicate that measuring poverty at any single point of time is inadequate for capturing the true level of poverty since a far greater proportion of people may experience poverty when observed over a longer term (Baulch and Hoddinott, 2000).

In this paper we develop a class of poverty measures which takes into account previous poverty experiences of individuals in the society. We focus on the dynamics of poverty and, as such, the paper adds to a recent literature with notable contributions by Foster (2007), Hoy and Zheng (2008) and Bossert, Chakravarty and D'Ambrosio (2010), among others.[1] These papers essentially attempt to address the question of distinguishing between two individuals who may have the same level of poverty in the current period, but had different levels of poverty, or were poor at different times from one another, in the past. In this sense all these papers measure ex-post poverty but they differ in their treatment of capturing the spread and depth of poverty over time.

In Foster (2007), the spread of poor episodes over time is of no importance; only the proportion of spells in which the individual is in poverty is taken into account. Overall poverty for each individual is a simple average of the generalized poverty gaps in each period. Bossert et al. (2010), on the other hand, assign particular importance to the extent to which individuals are in a state of poverty for consecutive periods. They argue for a principle in which people facing periods of poverty which are interrupted by relatively affluent periods are deemed as being able to manage more easily than those who are exposed to longer consecutive periods of poverty (even if the total number of periods

[1] See also Cruces (2005), Calvo and Dercon (2007), Grab and Grimm (2007), Carter and Ikegami (2007), Porter and Quinn (2008), Foster and Santos (2009) and Gradín, del Río and Cantó (2010).

of poverty and non-poverty are the same in each case). Hoy and Zheng (2008) contend that poverty early in life is more detrimental than poverty later in life and, further, that a person who is poor in periods which are either consecutive or separated by only short spells of relative wellbeing is worse off than a person with similar incomes but more widely dispersed poverty episodes. A common theme in many of the existing measures of intertemporal poverty is that periods of poverty which are closely bunched together lead to a higher level of poverty.

The measures proposed in this paper are motivated by the relevance of consecutive poverty periods, but we differ in our interpretation of the dynamics which cause closely bunched spells to be debilitating. Our approach is motivated by the observation that the longer the spell of relative affluence is which individuals have experienced prior to becoming poor, the better equipped they are to deal with that period of poverty. Thus we discount the impact of a poor period according to the number of non-poor periods directly preceding it. Poor periods which are bunched closely together are particularly bad precisely because this means that there must have been few opportunities to accumulate mitigating resources that would have helped during those poor periods. This way we add to the distributional aspect of Bossert et al. (2010) the characteristic of mitigation by preceding non-poverty periods.

The accumulated resources of individuals in non-poor periods can be of a material, physiological or social nature. Our measure is general and does not specify the mitigating attributes explicitly (e.g., income as in Hoy and Zheng, 2008). However, we acknowledge the realistic scenario that such mitigating resources are short lived. Consider, for instance, social networks which are regarded by Woolcock and Narayan (2000) as being one of the primary resources the poor have for managing risk and vulnerability. Beall (2001) points out how social resources can be quickly eroded by poverty. Citing evidence from Kumasi, in Ghana, it is argued that desperately poor individuals borrowing and begging from relatives, with no realistic possibility of repayment, can pose big problems for their relatives who may already be in precarious situations themselves. Citing further evidence from Johannesburg and Vizakhpatnam, Beall (2001) observes that in the absence of basic resources and security, it is, at best, extremely difficult for poor families to sustain self-help and mutual assistance. Thus in order to build up the resources that help to mitigate poverty, an individual requires spending some time in non-poverty. The downside of poverty is that such accumulated resources are nearly instantly exhausted. The intuition that mitigating resources take a long time to build up yet are quickly eroded is captured in our classes of poverty measures.

An advantage of our approach is easily illustrated via the following simple stylized example. Consider two individuals who both live over four time periods and are poor in two periods. The first person is poor in the first and third period, while the second person is poor in the first and last period. Otherwise they are non-poor. Assume that, whenever they are poor, this happens to a similar extent. In our view, there is a strong case for arguing that the first individual seems worse off, because following the initial poor episode, he has fewer opportunities to recover and accumulate resources before he must face the second poor episode. While other recent measures, e.g., Foster (2007) and Bossert et al. (2010), regard both individuals as equally poor, our class of measures differentiates between such individuals.[2] Due to this additional sensitivity, our measures allow for a richer, more diversified ordering of profiles in terms of chronic poverty.

The proposed class of intertemporal poverty measures also allows for a range of possible judgements as to the overall impact of a poor period that is preceded by a non-poor spell. When no significance is attached to the relatively affluent periods, our measures reduce to the simple average of per period (static) poverty measures advocated by Foster (2007). In this sense we provide axiomatic foundations for a class of measures that encompasses the latter.[3] In the extreme case where an individual is poor in all time periods, our measures do not discount any of the poor episodes because none of them have been preceded by non-poor periods in which the individual had an opportunity to accumulate

[2] With the choice of a weighting function which is strictly decreasing in time, Hoy and Zheng (2008) may also conclude that the first individual in our example is poorer in these circumstances. This follows because, in their framework, the extent to which each indvidual is wealthy during the non-poor periods can also have an impact. As Hoy and Zheng focus on lifetime poverty their non-increasing weights result from an "Early Poverty" axiom. In our framework such an axiom may be difficult to justify unless the time periods considered span a very long period of life time.

[3] Strictly speaking, for the generalization to hold, our measure and that of Foster (2007) must use the same static FGT index at the individual level. As the paper goes on to discuss, we advocate using one particular FGT index, the normalized poverty gap.

resources. Our measures then will again lead to a simple average of per period poverty as in Foster (2007). By putting some weight, however small, on relatively affluent periods, our proposed measures provide different rankings.[4]

The remainder of the paper is organized as follows. Section 2 introduces our notation and the basic framework. Section 3 formally introduces our classes of individual intertemporal poverty measures and provides axiomatic characterizations. Section 4 concludes. All proofs are deferred to the Appendix.

2 Notation and Basic Framework

In a society composed of finitely many individuals, we focus on the measurement of an individual's poverty over finitely many time periods. This requires the determination of a static poverty index for each time period and an aggregation of the latter across time. Subsequently, one can construct measures of poverty for the entire society by aggregating across individuals. We focus on the former two steps.

For $T \in \mathbb{N}$ let $t \in \{1, \ldots, T\}$ denote a particular time period. An individual has income $x_t \geq 0$ in each period $t = 1, ..., T$. The income profile $\mathbf{x} = (x_1, \ldots, x_T)$ is a vector in $\mathbb{R}^T_+$. As usual, there is an exogenously determined poverty line z_t for each time period t, where $0 < z_t < \infty$, and $\mathbf{z} = (z_1, \ldots, z_T) \in \mathbb{R}^T_{++}$ denotes the profile of poverty lines. The individual is *poor* in period $t \in \{1, \ldots, T\}$ if $x_t < z_t$ with *snapshot poverty* level p_t in period t being the well-known normalized *poverty gap*:

$$p_t := \begin{cases} \dfrac{z_t - x_t}{z_t}, \text{ if } x_t < z_t, \\ 0, \text{ otherwise.} \end{cases}$$

[4] Our methodology has slight echoes of the approach of Lee (2008) and Basu and Lee (2009) in the context of measuring literacy. Previous literacy measures, for example that of Basu and Foster (1998), had evaluated 'effective literacy' by supplementing the actual literacy rate with a positive factor to account for so-called 'proximate illiterates', that is those who whilst illiterate themselves nevertheless have access to a literate person. The novel approach of Lee (2008) and Basu and Lee (2009) was to regard proximate illiteracy as the norm for an illiterate person and evaluating effective literacy by subtracting from the actual literacy rate a component for 'isolated illiterates', those who are illiterate and without access to a literate person. In a similar way, our measures treat the simple average of static measures as the maximum possible level of inter-temporal poverty for a given combination of incomes and adjust this downwards according to the ordering in which the poor episodes occur.

As a result of this normalization we have $p_t \in [0,1]$ for all $t \in \{1,\ldots,T\}$, with 1 representing the maximum level of poverty, while 0 means that the individual is not poor in the respective period. This static measure of an individual's poverty has some appealing properties. It is decreasing in x_t and is scale invariant since for any $\lambda \neq 0$, $(\lambda z_t - \lambda x_t)/\lambda z_t = (z_t - x_t)/z_t$. It also has a money-metric interpretation. When denormalized it can be interpreted as the minimum cost to society of removing the individual from poverty.[5]

The individual's poverty profile is $\mathbf{p} = (p_1, .., p_T)$, representing the poverty levels that the individual faces in each of the T time periods. Thus, a poverty profile is a T-vector where $\mathbf{p} \in [0,1]^T$. We use $\mathbf{0}^T$ to represent the poverty profile in which there is no poor period, i.e. $p_t = 0$, for all $t \in \{1, .., T\}$. Further, a T-period poverty profile with only one poor period such as $\mathbf{p} = (0, .., 0, p_s, 0, .., 0)$, $1 \leq s \leq T$, is represented as $\mathbf{p} = p_s \cdot \mathbf{e}_s^T$, where $\mathbf{e}_s^T$ is the profile with $e_t = 0$ for all $t \in \{1, ..., T\} \setminus \{s\}$ and $e_s = 1$.

For a profile $\mathbf{p}$ we define n_t to be the number of consecutive non-poor periods immediately prior to a poor period t, and let m_t denote the length of the spell of consecutive poor periods which includes the poor period t. Formally,

$$n_t \; : \; = \begin{cases} 0, & \text{if } t = 1, \\ t - \min\{s : s < t \text{ and } p_s = \cdots = p_{t-1} = 0\}, & \text{otherwise,} \end{cases}$$

and

$$m_t \; : \; = \begin{cases} \max\{r : 0 < p_s, \ldots, p_{s+r}\}, & \text{if } s \leq t \leq s + r, \\ 0, & \text{otherwise.} \end{cases}$$

For example, for $T = 5$, the poverty profile $\mathbf{p} = (p_1, 0, p_3, p_4, 0)$ has $n_1 = 0$, $m_1 = 1$, $n_3 = 1$, $m_3 = 2$, and $n_4 = 0$, $m_4 = 2$. Later on we use these numbers to illustrate how the static poverty gap in each poor period may contribute to an overall intertemporal poverty measure for an individual. It will then

[5] Note that we deliberately avoid using more sophisticated, distribution sensitive, measures. The usual justification for using such measures is related to the manner in which they treat the distribution of poverty among the poor. As we focus on a single individual's intertemporal poverty index, prior to distributional concerns among the poor, it seems more appropriate to retain a simple money-metric measure of poverty. The appealing distributional properties, leading to more sophisticated poverty measures, would need to be incorporated at the aggregation stage across individuals.

become clear that for non-poor periods we do not need to define such variables, since $p_t = 0$ for these periods.

3 Individual Intertemporal Poverty Indices

An intertemporal poverty measure for an individual is a function that assigns to each poverty profile a nonnegative number. Thus, $P : \cup_{T\in\mathbb{N}}[0,1]^T \longrightarrow \mathbb{R}_+$. The class of individual intertemporal poverty measures that we consider are close in structure to the measures of Foster (2007) and Bossert et al. (2010). They can be expressed as

$$P(\mathbf{p}) = \frac{1}{T}\sum_{t=1}^{T} w_t h(p_t), \tag{1}$$

where $h : [0,1] \to \mathbb{R}_+$ can be any static measure of poverty and $w_t \in [0,1]$ denotes the weight attached to period t, $t = 1,\ldots,T$.

In Bossert et al. (2010), $w_t = m_t$, the length of a spell of ongoing poor periods including t. These weights serve to intensify the impact of poor periods in the aggregation process across time. In Foster (2007), provided that the proportion of time periods for which $p_t > 0$ exceeds some threshold $\tau \in [0,1]$, the static measure is $h(p_t) = (p_t)^\theta$, $\theta \geq 0$, otherwise $h(p_t) = 0$, and further, the weight $w_t = 1$ for all t. It is clear that these weights do not serve to either intensify or diminish the impact of poor periods according to any notion of bunching, respectively, mitigating affluence.

Note that the measures of Bossert et al. (2010) and Foster (2007) have an additively separable structure. This will also be the case for our measures. Before introducing our general class of affluence-dependent intertemporal poverty measures, we clarify our approach and our understanding of the extent to which non-poor periods can act to mitigate subsequent poor periods. For this purpose, we start our discussion with a specific functional.

The *constant-relative affluence-dependent intertemporal poverty measure* P_β is defined as

$$P_\beta(\mathbf{p}) = \frac{1}{T}\sum_{t=1}^{T} w_t p_t \text{ where } w_t = \left(\frac{1}{1+n_t}\right)^\beta \text{ and } \beta \geq 0. \tag{2}$$

The parameter β can be interpreted as an index representing how much the social planner chooses to discount the impact of an individual's poor episodes by according to preceding uninterrupted periods of non-poverty. Clearly, where no significance is attached to the ability of relatively affluent periods to mitigate the impact of subsequent poverty, $\beta = 0$ will be chosen. In that case, this measure reduces to the simple average of static poverty measures advocated by Foster (2007). The larger β is, the more the social planner allows the effects of genuine poverty to be discounted by the mitigating prior non-poverty.

The critical question that needs to be addressed is why the social planner should, for period of relative affluence, discount only in the first period of poverty but no subsequent poor periods. Our motivation is as follows. Although, when not poor, individuals and households accumulate resources that can help mitigate the potential subsequent deprivation, we think that these resources will be exhausted quickly during a poor spell. Just a few weeks spent in poverty may have devastating effects on one's health, assets and social networks. Davis (2006) and Davis and Baulch (2009), for example, provide empirical evidence that in Bangladesh, while improvements in life conditions of individuals typically occur slowly, over long periods of time, they suddenly decline following shocks.

When accounting for the mitigating effect of affluence on poverty, we follow the traditional approach where all that matters is the fact that an individual is out of poverty — the extent to which their income is above the poverty line is irrelevant. Thus, our approach agrees in spirit with the well-known focus axiom (Sen 1976). In a static framework, this axiom requires that the measure of poverty be independent of the income level of the non-poor. This approach is common also in the literature on intertemporal poverty measurement.[6] For example, Foster (2007) argues that the focus axiom is

[6] Hoy and Zheng (2008) is a notable exception. There, the extent to which an individual's income is above the poverty line has a mitigating impact providing his average level of income across all time periods is below the poverty line, that is providing $p(\overline{x}; z) > 0$.

important in the present context on the basis that each period's income is not directly aggregated with the following one. This argument is perhaps particularly convincing when consumption rather than income is the variable of interest. In that case, it can be argued that even if an individual was very wealthy in the recent past, being unable to consume enough to meet his basic needs here and now is enough evidence that recent wealth is no longer of assistance.

Next we proceed to some examples, which illustrate some additional properties of P_β. In each example we evaluate an individual's intertemporal poverty level for a four period poverty profile (p_1, p_2, p_3, p_4), and with and $\beta = 1$, the linear weighting function.

Example 1 For $(1/2, 0, 1/4, 0)$ we have

$$P_1(\frac{1}{2}, 0, \frac{1}{4}, 0) = \frac{1}{4}(1 \cdot \frac{1}{2} + 0 + \frac{1}{2} \cdot \frac{1}{4} + 0) = \frac{5}{32}(\approx 0.156)$$

and for $(1/2, 0, 0, 1/4)$ we have

$$P_1(\frac{1}{2}, 0, 0, \frac{1}{4}) = \frac{1}{4}(1 \cdot \frac{1}{2} + 0 + 0 + \frac{1}{3} \cdot \frac{1}{4}) = \frac{7}{48}(\approx 0.146).$$

□

The profiles in this example represent individuals, who are poor during two of the four time periods and they are similarly poor except for the timing of the second poverty period. The measure P_1 differentiates between these two profiles by attaching lower weight to the snapshot poverty level $1/4$ in the second profile, thereby indicating that the second profile represents less intertemporal poverty. The measures of Foster (2007) and Bossert et al. (2010) rank the two profiles as equally poor, on the basis that they are each poor for the same proportion of periods, respectively, on the basis that each individual's poor episodes form part of sequences of consecutive poor episodes of the same length.

Example 2 For $(0, 3/4, 1/2, 3/4)$ we have

$$P_1(0, \frac{3}{4}, \frac{1}{2}, \frac{3}{4}) = \frac{1}{4}(0 + \frac{1}{2} \cdot \frac{3}{4} + 1 \cdot \frac{1}{2} + 1 \cdot \frac{3}{4}) = \frac{13}{32},$$

for $(0, 1/2, 3/4, 3/4)$ we have

$$P_1(0, \frac{1}{2}, \frac{3}{4}, \frac{3}{4}) = \frac{1}{4}(0 + \frac{1}{2} \cdot \frac{1}{2} + 1 \cdot \frac{3}{4} + 1 \cdot \frac{3}{4}) = \frac{14}{32},$$

and for $(3/4, 1/2, 3/4, 0)$ we have

$$P_1(\frac{3}{4}, \frac{1}{2}, \frac{3}{4}, 0) = \frac{1}{4}(1 \cdot \frac{3}{4} + 1 \cdot \frac{1}{2} + 1 \cdot \frac{3}{4} + 0) = \frac{16}{32}.$$

□

For similar reasons as in the previous example, the three profiles considered here would be equal under the measures of Foster (2007) and Bossert et al. (2010). Due to the weight for period $t = 2$ in P_1, it matters whether the individual starts of with a low or high snapshot poverty level when we compare the first two profiles. The worst case is the third profile, where the individual has no opportunity to build up any resources and enters instantly into a prolonged spell of poverty.

Having illustrated the additional sensitivity of the measure P_β in ranking poverty profiles, we introduce the more general class of intertemporal poverty measures. The *affluence-dependent intertemporal poverty measure* P_f is defined as

$$P_f(\mathbf{p}) = \frac{1}{T} \sum_{t=1}^{T} w_t p_t, \tag{3}$$

where $w_t = f(n_t)$ for a function $f : \mathbb{Z}_+ \to [0, 1]$ such that $f(0) = 1$ and $f(n_t + 1) \leq f(n_t)$.

The weights $f(n_t)$ ensure that poverty is discounted whenever there are directly preceding periods of affluence. They are inversely related to the length of the preceding uninterrupted non-poverty periods, leading to higher discounting for longer spells of affluence. In the extreme case, where there are no preceding periods of affluence, there is no discounting and the maximum weight of 1 is applied to all snapshot poverty gaps. In this case, the intertemporal poverty measure reduces to the simple average of the static poverty levels in each period.

Next we turn to the axiomatic foundations for the proposed affluence-dependent intertemporal poverty measures.

3.1 A Foundation for P_β

Our first requirement for an individual intertemporal poverty measure is that in trivial cases, where there is only one time period, the individual intertemporal poverty measure is identical to the individual's snapshot poverty measure (see also Bossert et al., 2010).

Axiom 1 *Single period equivalence holds if $P(p) = p$ for all $p \in [0, 1]$.*

The second axiom considers the possibility of partitioning a longer poverty profile into shorter ones and the relation of the subprofile measures with the overall measure. This is an additive separability condition. It is clear from our objective, that some periods of poverty will enter with a different weight into the aggregate poverty measure. Hence, only a restricted version of separability into specific subprofiles is permitted and the timing of the periods which have a positive poverty gap is critical.

Axiom 2 *Time decomposability holds if for all lengths of periods $T \in \mathbb{N}\backslash\{1\}$, all poverty profiles $\mathbf{p} \in [0, 1]^T$ and all $t \in \{1, \ldots, T\}$ we have that, if $p_t > 0$ then*

$$P(\mathbf{p}) = \frac{t}{T} P(p_1, \ldots, p_t) + \frac{T-t}{T} P(p_{t+1}, \ldots, p_T).$$

Similar restrictions of separability were used in Bossert et al. (2010). There, decomposability means that intertemporal poverty must be equal to a weighted average of two subprofiles, where the weights are proportional to the lengths of the two subprofiles. The restrictions follow as the separation is applied only to situations in which either the first subprofile ends with a non-poor period or the second sub-profile starts with a non-poor period. In contrast, our axiom explicitly excludes instances in which the final period of the first subprofile is non-poor.

The next requirement fixes the location of our measurement scale. It concerns only the desirable case of non-poverty profiles.

Axiom 3 *Normalization holds if for all $T \in \mathbb{N}$ we have $P(\mathbf{0}^T) = 0$.*

Next we focus on axioms which reflect our motivation that in exceptional cases poverty in some periods can be mitigated by preceding periods of affluence. Recall Examples 1 and 2, which demonstrated how an additional preceding period of non-poverty leads to smaller weights being given to an adjacent period of poverty. Our first condition requires that the discounting of the poor period's poverty gap is proportional to the length of the immediately preceding non-poverty spell.

Axiom 4 *Linear poverty mitigation holds if for all $T \in \mathbb{N}\backslash\{1\}$ and all $p \in (0,1]$ we have $P(p\mathbf{e}_t^T) = P(p\mathbf{e}_1^T)/t$ for any $t \in \{1,\ldots,T\}$.*

Axiom 4, implicit in our examples above, may be seen as too rigid. A more flexible property is presented next.

Axiom 5 *Constant relative poverty mitigation holds if for all $T \in \mathbb{N}\backslash\{1\}$ and all $p \in (0,1]$ we have $P(p\mathbf{e}_t^T) = P(p\mathbf{e}_1^T)/t^\beta$ for $\beta > 0$ and any $t \in \{1,\ldots,T\}$.*

Combining the above axioms we obtain the following result.

Theorem 1 *An intertemporal poverty measure satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3) and constant-relative poverty mitigation (Axiom 5) if and only if it is the constant relative affluence-dependent intertemporal poverty measure P_β.* □

The proof of this theorem is presented in the appendix. We conclude this section with a corollary.

Corollary 2 *An intertemporal poverty measure satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3) and linear poverty mitigation (Axiom 4) if and only if it is the affluence-dependent intertemporal poverty measure P_1.* □

3.2 A Foundation for P_f

In the preceding subsection, we provided axioms that characterize a specific class of intertemporal poverty measures, where the weight attached to each poverty gap is generated by a positive power function. In this subsection, we allow the weights to be generated by a general function over consecutive non-poor periods. Except for a monotonicity requirement in n_t, the number of affluent periods immediately preceding a period of poverty t, no further restrictions apply.

Recall that for the measures characterized in Theorem 1, Axiom 5 played a crucial role. To obtain the desired relaxation for the manner in which a poor period can be discounted if preceded by periods of affluence, we need to replace this condition with a more flexible one, while maintaining the monotonicity requirement implicit in the choice of $\beta \geq 0$.

Our next axiom is a monotonicity condition. It considers two poverty profiles of length T, each with only one episode of poverty of level p. In one profile the poverty episode occurs in period $t \leq T$ and in the other profile the poverty episode takes place in period $(t-1) \geq 1$. Note that the static level of poverty in the poor period is the same in each profile. The axiom stipulates that $t-1$ directly preceding periods of affluence have a greater mitigating impact than $t-2$ periods.

Axiom 6 *Monotonic poverty mitigation holds if for all $T \in \mathbb{N} \backslash \{1\}$ and all $p \in (0,1]$ we have $P(p\mathbf{e}^T_{t-1}) \geq P(p\mathbf{e}^T_t)$ for any $t \in \{2, \ldots, T\}$.*

We now turn to a second aspect that was incorporated in Axiom 5. We require the weight for the poverty gap in period t, while depending on the number of directly preceding non-poor periods, to be independent of the level of poverty p_t. Therefore, considering two poverty profiles of length T, if poverty occurs in periods t and $t-s$, respectively, but the overall poverty measure is the same for the two profiles, then re-scaling the level of poverty in those periods maintains the equality of the overall poverty in the resulting profiles.

Axiom 7 *Poverty level scale invariance holds if for all* $T \in \mathbb{N}\backslash\{1\}$ *and all* $p, q \in (0,1]$ *we have that if* $P(p\mathbf{e}_t^T) = P(q\mathbf{e}_s^T)$ *for some* $1 \leq s < t \leq T$*, then* $P(\lambda p\mathbf{e}_t^T) = P(\lambda q\mathbf{e}_s^T)$ *whenever* λ *is such that* $\lambda p, \lambda q \in (0,1]$.

The following theorem formally characterizes our affluence dependent intertemporal poverty measures.

Theorem 3 *An intertemporal poverty measure satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), monotonic poverty mitigation (Axiom 6) and poverty level scale invariance (Axiom 7) if and only if it is the affluence-dependent intertemporal poverty measure* P_f. □

The proof of the theorem is presented in the Appendix.

4 Conclusions

This paper has proposed and characterized new classes of individual intertemporal poverty measures. The central innovation relies on accounting for poverty mitigating effects through sustained periods of affluence. Like the measures of Foster (2007) and Bossert et al. (2010), our measures incorporate an intertemporal version of the focus axiom in order to obtain a weighted average over individual static per period poverty measures. As a result, the weight attached to each period's poverty index is determined by the length of the immediately preceding and uninterrupted episode of non-poverty. A family of measures with weights generated by a one-parameter function, similar to a parametric discounting function, is proposed and a more general class of measures where a non-parametric discount function determines the weights. Interestingly, no explicit continuity property is required to derive the latter class. It can be inferred from the proofs that Axiom 1 provides sufficient structural richness and is critical in this respect.

Our individual level measures have much in common with the measures of Bossert et al. (2010), which also evaluate individual intertemporal poverty as a weighted average across time of snapshot poverty. The similarity concerns the idea that the weight attached to each poor spell is determined by a concern for some notion of chronic poverty. We introduce additional sensitivity by accounting for periods of affluence, although we recognize that any such benefits will decay quickly. This additional feature can be useful for policy purposes as it allows for a more diversified approach when it comes to allocation of resources. How precisely a redistribution of wealth or consumption should be implemented in an intertemporal framework, among the poor but also between the time periods, is an important issue to which a solution is not immediately apparent. This issue is left for future research.

Appendix: Proofs

Proof of Theorem 1: We concentrate on the "only if" part of the proof, as it is immediate to verify that P_β satisfies the axioms stated in Theorem 1.

Suppose that P satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3) and constant relative poverty mitigation (Axiom 5). We need to show that for any time period $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0,1]^T$ we have $P(\mathbf{p}) = P_\beta(\mathbf{p})$ for an exogenously determined $\beta \geq 0$. So, take any $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0,1]^T$.

Suppose $T = 1$. In this case single period equivalence holds and $P(\mathbf{p}) = P(p) = P_\beta(p)$ for some $p \in [0,1]$. Hence, $P(\mathbf{p}) = P_\beta(\mathbf{p})$ follows.

Assume now that $T > 1$. If $\mathbf{p} = \mathbf{0}^T$, then by normalization we obtain $P(\mathbf{p}) = P(\mathbf{0}^T) = 0 = P_\beta(\mathbf{0})$. Thus, $P(\mathbf{p}) = P_\beta(\mathbf{p})$ follows.

Next we proceed by induction on the number of poor periods, thus when $p_t > 0$. Consider the case where there is exactly one poor period. Without loss of generality, we can write $\mathbf{p} = p \cdot \mathbf{e}_t^T$ where

$p > 0$ and $t \in \{1, ..., T\}$. Thus $P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T)$. Then

$$\begin{aligned}
P(p \cdot \mathbf{e}_t^T) &= \frac{t}{T}P(p \cdot \mathbf{e}_t^t) + \frac{T-t}{T}P(\mathbf{0}^{T-t}), \text{ by time decomposability,} \\
&= \frac{t}{T}P(p \cdot \mathbf{e}_t^t), \text{ by normalization,} \\
&= \frac{t}{T}\frac{P(p \cdot \mathbf{e}_1^t)}{t^\beta}, \text{ by constant relative poverty mitigation,} \\
&= \frac{1}{Tt^{\beta-1}} \cdot [\frac{1}{t}P(p) + \frac{t-1}{t}P(\mathbf{0}^{t-1})], \text{ by time decomposability,} \\
&= \frac{P(p)}{Tt^\beta}, \text{ by normalization,} \\
&= \frac{p}{Tt^\beta}, \text{ by single period equivalence.}
\end{aligned}$$

Hence, $P(\mathbf{p}) = P_\beta(\mathbf{p})$ is obtained.

Suppose now that $P(\hat{\mathbf{p}}) = P_\beta(\hat{\mathbf{p}})$ whenever $\hat{\mathbf{p}}$ contains m poor periods, for some $m \in \{1, ..., T-1\}$. Let $\mathbf{p} \in [0,1]^T$ be any poverty profile, such that the number of poor periods is $m+1$. Let $t \in \{2, ..., T\}$ be such that the final poor period is period t. Thus $t = \max\{s : 2 \leq s \leq T, p_s > 0\}$. From time decomposability we derive

$$\begin{aligned}
P(\mathbf{p}) &= P(p_1, ..., p_t, ..., p_T) \\
&= \frac{t}{T}P(p_1, ..., p_t) + \frac{T-t}{T}P(p_{t+1}, ..., p_T).
\end{aligned}$$

Now t being the final poor period means that by normalization we get

$$P(\mathbf{p}) = \frac{t}{T}P(p_1, ..., p_t). \tag{4}$$

Let $s \neq t$ be maximal with $p_s > 0$. So s is the last poor period prior to t. Then by time decomposability we obtain

$$P(p_1, ..., p_s, ..., p_t) = \frac{s}{t}P(p_1, ..., p_s) + \frac{t-s}{t}P(p_{s+1}, ..., p_t). \tag{5}$$

Now $(p_1, ..., p_s)$ contains m poor periods. Thus, by the induction hypothesis, we have

$$P(p_1, ..., p_s) = P_\beta(p_1, ..., p_s) = \frac{1}{s}\sum_{i=1}^{s} w_i p_i \text{ where } w_i = \frac{1}{(1+n_i)^\beta}. \tag{6}$$

Further, $P(p_{s+1}, ..., p_t) = P(p_t \cdot \mathbf{e}_{t-s}^{t-s})$ since $p_i = 0$ for all $i \in \{s+1, ..., t-1\}$. Applying single period equivalence, time decomposability, normalization and constant relative poverty mitigation we obtain, similar to the first induction step, that

$$P(p_{s+1}, ..., p_t) = \frac{p_t}{(t-s)^{1+\beta}}. \tag{7}$$

Substituting (6) and (7) into (5) yields

$$P(p_1, ..., p_s, ..., p_t) = \left[\frac{1}{t}\sum_{i=1}^{s}\frac{p_i}{(1+n_i)^\beta}\right] + \frac{p_t}{t(t-s)^\beta}. \tag{8}$$

Further, substituting (8) into (4) we obtain

$$P(\mathbf{p}) = \frac{1}{T}\left[\left(\sum_{i=1}^{s}\frac{p_i}{(1+n_i)^\beta}\right) + \frac{p_t}{(t-s)^\beta}\right].$$

Note that $n_t = t - s - 1$ since period s is the last poor period prior to t. Therefore, we have

$$P(\mathbf{p}) = \frac{1}{T}\left[\left(\sum_{i=1}^{s}\frac{p_i}{(1+n_i)^\beta}\right) + \frac{p_t}{(1+n_t)^\beta}\right].$$

Finally, since $p_i = 0$ for all $i \in \{s+1, ..., t-1\}$ and all $i \in \{t+1, ..., T\}$, we have

$$P(\mathbf{p}) = \frac{1}{T}\sum_{i=1}^{T}\frac{p_i}{(1+n_i)^\beta} = P_\beta(\mathbf{p}).$$

This concludes the proof for the case of $m+1$ poor periods, and by induction it follows that

$P(\mathbf{p}) = P_\beta(\mathbf{p})$ for any poverty profile $\mathbf{p}$. This completes the proof of Theorem 1. $\square$

PROOF OF THEOREM 3: We concentrate on the "only if" part of the proof, as it is immediate to verify that P_f satisfies the axioms stated in Theorem 3.

Suppose that P satisfies single period equivalence (Axiom 1), time decomposability (Axiom 2), normalization (Axiom 3), monotonic poverty mitigation (Axiom 6) and poverty level scale invariance (Axiom 7). We need to show that for any time period $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0,1]^T$ we have $P(\mathbf{p}) = P_f(\mathbf{p})$ for a monotonically decreasing function $f : \mathbb{Z}_+ \to [0,1]$ such that $f(0) = 1$ and $f(n_t + 1) \leq f(n_t)$.

So, take any $T \in \mathbb{N}$ and any poverty profile $\mathbf{p} \in [0,1]^T$. Suppose $T = 1$. In this case single period equivalence holds and $P(\mathbf{p}) = P(p) = P_f(p)$ for some $p \in [0,1]$. Hence, $P(\mathbf{p}) = P_f(\mathbf{p})$ follows in this case.

Assume now that $T > 1$. If $\mathbf{p} = \mathbf{0}^T$, then by normalization we obtain $P(\mathbf{p}) = P(\mathbf{0}^T) = 0 = P_f(\mathbf{0})$. Thus, $P(\mathbf{p}) = P_f(\mathbf{p})$ follows.

Next, we proceed by induction on the number of poor periods, thus when $p_t > 0$. Consider the case where there is exactly one poor period. Without loss of generality, we can write $\mathbf{p} = p \cdot \mathbf{e}_t^T$ where $p \in (0,1]$ and $t \in \{1,...,T\}$. Thus $P(\mathbf{p}) = P(p \cdot \mathbf{e}_t^T)$. If $t = 1$ then, applying single period equivalence, time decomposability and normalization, this gives

$$P(p \cdot \mathbf{e}_1^T) = \frac{1}{T}p.$$

For $t > 1$ applying time decomposability and normalization, we obtain

$$P(p \cdot \mathbf{e}_t^T) = \frac{t}{T}P(p \cdot .\mathbf{e}_t^t).$$

By repeated application of monotonic poverty mitigation we know that $P(p \cdot \mathbf{e}_1^T) \geq P(p \cdot \mathbf{e}_t^T)$, hence, using the previous two equalities, we obtain

$$\frac{p}{T} \geq \frac{t}{T} P(p \cdot \mathbf{e}_t^t).$$

Therefore, there exists $r_t(p) \in (0, p]$ such that

$$\frac{r_t(p)}{p} P(p \cdot \mathbf{e}_1^t) = tP(p \cdot \mathbf{e}_t^t) \Leftrightarrow P(r_t(p) \cdot \mathbf{e}_1^T) = P(p \cdot \mathbf{e}_t^T).$$

We show that $r_t(p)/p$ is a constant, independent of p. By poverty level scale invariance we know that

$$P(r_t(p) \cdot \mathbf{e}_1^T) = P(p \cdot \mathbf{e}_t^T) \Leftrightarrow P(\lambda r_t(p) \cdot \mathbf{e}_1^T) = P(\lambda p \cdot \mathbf{e}_t^T)$$

whenever $\lambda r_t(p), \lambda p \in (0, 1]$. Hence,

$$\begin{aligned}
P(\lambda r_t(p) \cdot \mathbf{e}_1^T) = P(\lambda p \cdot \mathbf{e}_t^T) \quad &\Leftrightarrow \quad \frac{1}{T} \lambda r_t(p) = \frac{t}{T} P(\lambda p \cdot \mathbf{e}_t^t) \\
&\Leftrightarrow \quad \lambda r_t(p) = \frac{r_t(\lambda p) t}{\lambda p} P(\lambda p \cdot \mathbf{e}_1^t) \\
&\Leftrightarrow \quad \lambda r_t(p) = \frac{r_t(\lambda p) t}{\lambda p} \frac{\lambda p}{t} \\
&\Leftrightarrow \quad \frac{r_t(p)}{p} = \frac{r_t(\lambda p)}{\lambda p},
\end{aligned}$$

which proves that $r_t(p)/p$ is independent of p. We define $w_t := r_t(p)/p$.

Thus for any $t > 1$, we have

$$\begin{aligned}
P(p \cdot \mathbf{e}_t^T) &= \frac{t}{T} P(p \cdot \mathbf{e}_t^t) \\
&= \frac{t}{T} w_t P(p \cdot \mathbf{e}_1^t) \\
&= \frac{w_t p}{T} \\
&= w_t P(p \cdot \mathbf{e}_1^T).
\end{aligned}$$

Note that by construction $w_t \in (0, 1]$ and by monotonic poverty mitigation it follows that $w_s \geq w_{s+1}$ for all $1 \leq s \leq t-1$, $t = 2, \ldots, T$. Further, we have $n_t = t - 1$. Therefore, we define $f(n_t) = w_t$. Obviously, when $n_t = 0$ it must be that $t = 1$. In this case $f(0) = w_1 = 1$ follows trivially. We conclude that

$$P(p \cdot \mathbf{e}_t^T) = \frac{f(n_t)p}{T}$$

for the case that the poverty profile $\mathbf{p}$ contains exactly one positive entry. Thus, $P(\mathbf{p}) = P_f(\mathbf{p})$ in this case.

Suppose now that $P(\hat{\mathbf{p}}) = P_f(\hat{\mathbf{p}})$ whenever $\hat{\mathbf{p}}$ contains m poor periods, for some $m \in \{1, ..., T-1\}$. Let $\mathbf{p} \in [0, 1]^T$ be any poverty profile, such that the number of poor periods is $m + 1$. The proof that $P(\mathbf{p}) = P_f(\mathbf{p})$ follows by similar arguments to those used in the induction step of the proof of Theorem 1. It therefore follows that $P = P_f$, which completes the proof of Theorem 3. $\square$

References

Baulch, B. and Hoddinott, J. (2000), "Economic Mobility and Poverty Dynamics in Developing Countries," Journal of Development Studies, 36, 1–24.

Basu, K. and Foster, J. (1998), "On Measuring Literacy," Economic Journal 108, 1733–1749.

Basu, K. and Lee, T. (2009), "A New and Easy-to-use Measure of Literacy, its Axiomatic Properties and an Application," Social Choice and Welfare, 32, 181–196.

Beall, J. (2001), "Valuing Social Resources or Capitalising on Them? Limits to Pro-poor Urban Governance in Nine Cities of the South," International Planning Studies 6, 357–375.

Bossert, W., Chakravarty, S. and D'Ambrosio, C. (2010), "Poverty and Time," Discussion Paper, CIREQ, University of Montreal.

Calvo, C. and Dercon, S. (2007), "Chronic Poverty and All That: The Measurement of Poverty Over Time," CPRC Working Paper No. 89. Manchester, UK: Chronic Poverty Research Centre (CPRC).

Carter, M. and Ikegami, M. (2007), "Looking Forward: Theory-based Measures of Chronic Poverty and Vulnerability," CPRC Working Paper No. 94. Manchester, UK: Chronic Poverty Research Centre (CPRC).

Chakravarty, S. (1983), "A New Index of Poverty," Mathematical Social Sciences, 6, 307–313.

Clark, S., Hemming, R. and Ulph, D. (1981), "On Indices for the Measurement of Poverty," The Economic Journal 91, 515–526.

Cruces, G. (2005), "Income Fluctuations, Poverty and Well-Being Over Time: Theory and Application to Argentina," London School of Economics, Discussion paper No. DARP 76.

Davis, P. (2006), “Poverty in Time: Exploring Poverty Dynamics from Life History Interviews in Bangladesh,” CPRC Working Paper No. 69. Manchester, UK: Chronic Poverty Research Centre (CPRC).

Davis, P. and Baulch, B. (2009), “Parallel Realities: Exploring Poverty Dynamics Using Mixed Methods in Rural Bangladesh,” CPRC Working Paper No. 142. Manchester, UK: Chronic Poverty Research Centre (CPRC).

Foster, J. (2007), “A Class of Chronic Poverty Measures,” Working Paper No. 07-W01, Department of Economics, Vanderbilt University.

Foster, J., Greer, J. and Thorbecke, E. (1984), “A Class of Decomposable Poverty Measures,” Econometrica 52, 761–766.

Foster, J. E. and M. E. Santos, (2009), “Measuring Chronic Poverty,” mimeo, Vanderbilt University.

Grab, J. and Grimm, M. (2007), “Robust Multiperiod Poverty Comparisons,” Ibero-America Institute for Economic Research Working Paper No. 160.

Gradín, C., del Río, C. and Cantó, O. (2010), “Measuring Poverty Accounting For Time,” Paper Prepared for the 31st General Conference of The International Association for Research in Income and Wealth, St. Gallen, Switzerland.

Hoy, M. and Zheng, B. (2008), “Measuring Lifetime Poverty,” Department of Economics, University of Guelph, Discussion Paper No. 2008-14.

Lee, T. (2008), “Benchmarking the Effective Literacy Rate,” Mathematical Social Sciences, 56, 233–239.

Porter, C. and Quinn, N. (2008), “Intertemporal Poverty Measurement: Tradeoffs and Policy Options,” CSAE WPS/2008-21.

Sen, A. (1976), “Poverty: An Ordinal Approach to Measurement,” Econometrica, 44, 219–231.

Watts, H. (1968), “An Economic Definition of Poverty,” in: D.P Moynihan ed., On Understanding Poverty. Basic Books, New York.

Woolcock, M. and Narayan, D. (2000), “Social Capital: Implications for Development Theory, Research and Policy,” The World Bank Research Observer 15, 225–249.

Zheng, B. (1997), Aggregate Poverty Measures. Journal of Economic Surveys, 11: 123–162.