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Michal Król

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The University of Manchester
Manchester M13 9PL

More price competition can benefit spatial duopolists when the consumer preferences are uncertain*

Michał Król†

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ABSTRACT. This note considers the effect of the demand faced by Hotelling duopolists varying in both location and responsiveness to price changes. The latter may increase on average, but still make the firms better off due to the equilibrium product differentiation increasing sufficiently to relax the intensified second-stage price competition. Furthermore, a social planner could always improve welfare *via* a proportional transport tax schedule contingent on the realization of the uncertainty.

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†michal.krol@manchester.ac.uk, Economics, The University of Manchester, M13 9PL, UK

1 Introduction

The problem of introducing demand uncertainty into the spatial duopoly framework has been present in the literature for over a decade. In pursuit of this goal, some authors use a modified Hotelling setting. For instance, Balvers and Szerb [1] study the effect of random shocks to the products' desirability under fixed prices, while Harter [4] examines the uncertainty in the form of a uniformly distributed random shift of the (uniform) customer distribution, where the firms locate sequentially. Other authors, such as Casado-Izaga [2], remain true to the original Hotelling framework, applying the same form of uncertainty as Harter to a game in which players move simultaneously. More recently, the last approach was generalized by Meagher and Zauner in two separate ways: by parametrizing the support of the uniform distribution of the random shock [6] and by considering a shock arbitrarily, rather than uniformly, distributed on a fixed interval ([5], henceforth, MZ).

Nevertheless, what has been so far ignored by the existing literature is a possibility of random factors causing not only a parallel shift of the demand function each firm faces (*via* re-locating the uniform consumer preferences), but also a change in the price elasticity of demand. In the classic Hotelling framework [3], the demand's responsiveness to price changes is determined by the scale of (quadratic) transport costs. However, the corresponding parameter has no effect on location decisions, although higher costs increase the second-stage equilibrium prices and profits. Therefore, on the one hand, omitting transport cost uncertainty seems justified, because the optimal location decisions should be the same regardless of the eventual resolution of the corresponding random variable, while higher average costs would simply increase the expected equilibrium profits. On the other hand, this ignores the potential interaction between the two types of demand uncertainty, which is explored in the present note, based on the setting of MZ with the added uncertainty about the transport cost parameter, where the joint distribution of the two random variables is still arbitrary[1].

It turns out that the modified model is equivalent to the original MZ framework for an appropriately constructed one-variable probability distribution, with interesting consequences for the role of the transport cost uncertainty. In particular, costs can decrease on average, but still make the duopolists better off if the scale of this change is greater for the central demand locations. This is because the resulting increase in the equilibrium product differentiation may offset the intensification of second-stage price competition due to smaller costs. For the same reason, the equilibrium level of social welfare, measured by the expected total transport costs incurred by the consumers, could increase despite the costs being on average smaller. Finally, a social planner can always take advantage of this effect and improve welfare *via* a proportional transport tax schedule contingent on the realization of the uncertainty.

[1] A possibility of the exact distribution being unknown to the duopolists is considered in M Krol *Product differentiation decisions under demand location ambiguity*, The University of Manchester, mimeo 2010

2 Model

As indicated above, the setting is almost the same as in MZ. In the first stage of the game, two firms simultaneously choose locations x_1, x_2 (without loss of generality let $x_1 \leq x_2$) and then proceed to simultaneous setting of their respective prices p_1, p_2 in the second stage. As usual, a consumer located at x chooses to buy a unit of the good from firm $i \in \{1,2\}$, so as to minimize the total purchase cost of $p_i + t(x_i - x)^2$, where $t > 0$ is the transportation cost parameter. The good costs nothing to produce and the consumers are uniformly distributed on the interval $\left[M - \frac{1}{2}, M + \frac{1}{2}\right]$, where the duopolists get to know the value of M, as well as t, once they choose their locations, but before setting prices.
Initially, all they know is that the joint probability distribution of the two random factors is characterized by a continuous probability density function $f(M,t)$ with support $[-1/2, 1/2] \times [0, \infty]$.[2]
Consequently, the second-stage of the game is no different from the classic Hotelling model. Thus, similarly to MZ, the payoff in the reduced first-stage location game is given by the expected value of the second-stage equilibrium profits with respect to $f(\cdot)$, i.e.:

$$\Pi_i(x_1, x_2) = \int_{-1/2}^{1/2} \int_0^{\infty} \pi_i^*(x_1, x_2, M, t) \times f(M,t)\; dt\, dM$$

where $\pi_i^*(\cdot)$ is the second-stage unique Nash Equilibrium profit associated with a particular realization of the uncertainty and a given location-pair.

3 Results

To begin with, consider the second-stage subgame associated with particular values of x_1, x_2, M and t. Let $\overline{x} = (x_1 + x_2)/2$ and let $\widetilde{x}$ denote the value of x that solves:

$$p_1 + t(x_1 - x)^2 = p_2 + t(x_2 - x)^2$$

Then the second-stage payoff of firm $i \in \{1,2\}$ equals its price p_i multiplied by the corresponding demand D_i, where:

$$D_i = \frac{1}{2} + (-1)^i [M - \widetilde{x}] = \frac{1}{2} + (-1)^i \left[M + \frac{p_1 - p_2}{2t(x_2 - x_1)} - \overline{x}\right]$$

as long as the prices are such that $\widetilde{x} \in \left[M - \frac{1}{2}, M + \frac{1}{2}\right]$ (no firm captures the entire market). It is apparent that, while a change in M shifts the firms' respective demand curves in opposite directions, variations in t affect the demand's responsiveness to prices.

[2] The assumption $M \in [-1/2, 1/2]$ was imposed in MZ for the purpose of mathematical tractability and is equally useful here. The difference from MZ is that the latter assumes $t = 1$.

More precisely, smaller transport costs increase the additional demand acquired through a decrease in one's own price, thereby making the firms' second-stage interaction more competitive. Indeed, the unique second-stage Nash Equilibrium profits, obtained *via* a simple re-parametrization of the classic Hotelling model [3], are given by:

$$\pi_i^*(x_1,x_2,M,t) = \begin{cases} t(x_2-x_1)\left[1+2(-1)^i(M-\overline{x})\right] & (-1)^i(M-\overline{x}) \geq 3/2 \\ t(x_2-x_1)\left[3(-1)^i+2(M-\overline{x})\right]^2/18 & (M-\overline{x}) \in (-3/2,3/2) \\ 0 & \text{otherwise} \end{cases}$$

and are clearly increasing in t.
One can now re-write the expected profit of firm i as:

$$\Pi_i(x_1,x_2) = \int_{-1/2}^{1/2}\int_0^{\infty} \pi_i^*(x_1,x_2,M,t) \times f(M,t)\; dt\, dM =$$

$$\int_{-1/2}^{1/2}\int_0^{\infty} t \times \pi_i^*(x_1,x_2,M,1) \times f(M,t)\; dt\, dM =$$

$$\int_{-1/2}^{1/2} \pi_i^*(x_1,x_2,M,1)\left[\int_0^{\infty} t \times f(M,t)\; dt\right] dM$$

and since multiplying payoffs by a positive constant would not change the equilibrium locations, we may equally well consider a game with a payoff function given by:

$$\widehat{\Pi}_i(x_1,x_2) = \overline{t} \times \Pi_i(x_1,x_2)\,,\ \text{where } \overline{t} = \int_{-1/2}^{1/2}\int_0^{\infty} t \times f(M,t)\; dt\, dM$$

so that we have:

$$\widehat{\Pi}_i(x_1,x_2) = \int_{-1/2}^{1/2} \pi_i^*(x_1,x_2,M,1) \times g(M)\; dM,\ \text{where } g(M) = \left[\int_0^{\infty} t \times f(M,t)\; dt\right] / \;\overline{t}$$

This means that the equilibrium locations must be the same as in the MZ model (which has $t = 1$) for a probability distribution of M characterized by a density function g. This is legitimate, since $t \geq 0$ implies that $g(M)$ is non-negative and we have:

$$\int_{-1/2}^{1/2} g(M)\, dM = 1$$

leading to the following statement.

Proposition 1 *Consider a variant of the Hotelling duopoly game in which the joint distribution of transport costs and the median of the (uniform) consumer preferences is characterized by density function $f(M,t)$ with support $[-1/2,1/2]\times[0,\infty]$. Suppose the firms choose locations so as to maximize the expected value of their second-stage equilibrium profits. Then the unique equilibrium locations are given by:*

$$x_1^* = -\frac{3}{4}+\mu-\frac{\sigma^2}{3},\; x_2^* = \frac{3}{4}+\mu+\frac{\sigma^2}{3}$$

where μ and σ^2 represent mean and variance of a one-variable probability distribution with support $[-1/2,1/2]$ and density function:

$$g(M) = \left[\int_0^\infty t\times f(M,t)\;dt\right] / \left[\int_{-1/2}^{1/2}\int_0^\infty t\times f(m,t)\;dt\,dm\right]$$

The intuition for the above result is simple. If a particular realization of the customer distribution coincides with the transport costs being, on average, relatively high, then with little price competition the potential second-stage equilibrium profits are significant. This provides an incentive for the firms to compete for this demand realization location-wise, i.e. to locate relatively close to it in the first stage of the game, which is what they would also opt for if this instance of the customer distribution was more likely to occur. In a sense, locating under uncertainty is similar to designing a product to be sold in many distinct markets (and then price-discriminating between them), where each market is characterized by specific consumer preferences and a certain degree of price competition. It is therefore natural for the firms to target those markets which are 'more valuable', either because they are more likely to materialize, or because the consumers care more about the characteristics of a product than about its price.

Consequently, product differentiation increases as the extreme preference realizations (M close to $\pm 1/2$) become more likely, but also as they begin to coincide with higher transportation costs, because both of these changes would increase σ^2. Conversely, if the 'central' demand locations are more probable / associated with less price competition, then the firms will choose to locate closer together.

As for the corresponding changes in the duopolists' equilibrium expected profits, one must account for the fact that in order to establish the equivalence with MZ, the actual first-stage payoffs $\Pi_i(x_1,x_2)$ were divided by the average transportation costs $\bar{t}$. Thus, the actual equilibrium payoffs in the present model are equal to the ones in the corresponding MZ model multiplied by $\bar{t}$.

The equivalence with MZ is no longer complete in the second stage of the game, since the equilibrium prices will depend on the exact value of t. However, a simple re-parametrization of the classic Hotelling model shows that:

$$\widetilde{x} = \max\left\{\min\left\{(2M+\overline{x})/3, M+1/2\right\}, M-1/2\right\}$$

i.e. the location of the 'indifferent consumer' (and hence each firm's share of the market) does not change with re-scaling the transport costs. Consequently, the fact that $\widetilde{x} \in (M - 1/2, M + 1/2)$ in MZ implies that it must also be true in the present model (since the equilibrium locations are the same), i.e. for all values of M and t the resulting equilibrium prices are such that no firm captures the entire market. Furthermore, just like the equilibrium profits π_i^*, the equilibrium prices are a multiple of t. Hence, an argument similar to the one which established the expected profit equivalence with MZ can be used to show that the expected prices in the present model are equal to the ones in the corresponding MZ model multiplied by $\bar{t}$. Finally, the same logic applies to the total transport costs incurred by the consumer, which constitute the appropriate measure of social welfare in the Hotelling framework. That is to say, the expected welfare in the present framework equals $\bar{t}$ times the one in the corresponding MZ model. The above observations are summarized below in a supplement to Proposition 1.

Corollary 2 *Given locations x_i^* as specified in Proposition 1, the corresponding expected second-stage equilibrium profits, prices and total transport costs incurred by the consumers are equal to the average value of the transport cost parameter multiplied by $\left(9 + 4\sigma^2\right)^2/108$, $\left(3/2 + 2\sigma^2/3\right)$ and $\left(13/48 + \sigma^2/2 - 7\sigma^4/27\right)$ respectively.*

This relatively simple result has some rather interesting consequences. In particular, if the duopolists knew the exact value of M, any uncertainty about t would have no effect on location decisions and the expected profits would be proportional to $\bar{t}$ (since $\sigma^2 = 0$). However, for M and t *both* uncertain, changes in average costs affect the outcome of the game in a more complicated way. In general, much depends on how the costs change in those states of nature in which M is relatively close to $\pm 1/2$, compared with those in which the demand is more centrally located. For instance, costs might on average become higher, but much more so for the 'central' values of M, thus reducing the relative weight of the peripheral demand realizations, reflected in a decrease of σ^2. This will result in less equilibrium product differentiation and more competitive pricing, offsetting (at least partially) the relaxation of price competition due to increased transport costs.

Remark 3 *It is possible for a change in the average transport costs to cause an opposite sign change in the equilibrium expected profits, prices and social welfare.*

This becomes immediately clear upon observing that σ^2 and $\bar{t}$ can vary independently. That is to say, the distribution of t over all states of nature (and the associated density function g) could change in such a way that σ^2 is affected, but not $\bar{t}$, or the other way round (for instance, if the cost parameter in every state is multiplied by a constant, leaving g unaltered). Consequently, a change in σ^2 can cause equal sign changes in the equilibrium product differentiation, expected prices, profits and welfare, more than offsetting the effects of an associated opposite sign change in $\bar{t}$, as long as the latter change is sufficiently small.

A common feature of the spatial duopoly framework is that the equilibrium product differentiation tends to exceed the socially-optimal level. This continues to be an issue in MZ, and hence also in the present model. In fact, the presence of uncertainty only makes the situation worse, because it is more costly to serve the consumers when their preferences are more variable. However, the current discussion suggests that a social planner might influence the firms' location decisions by imposing a transport tax schedule contingent on the realization of M. This could be based on a proportion of the costs paid by a consumer, affecting the distribution of t across demand locations.

Remark 4 *It is always possible for a social planner to improve welfare using a transport tax schedule contingent on the outcome of the uncertainty.*

In particular, by taxing the 'central' demand realizations more heavily than the 'peripheral' ones, the planner should be able to reduce the excessive product differentiation, making it arbitrarily close to the one associated with certainty ($\sigma^2 = 0$). Note that the tax would not affect the social welfare for a given location-pair, because the extra costs due to taxation do not constitute a social loss[3]. For this reason, the planner's commitment to the schedule is credible ex-ante: there is no incentive to violate it once the locations are set and the outcome of the uncertainty is known. Furthermore, only uncertainty about M (but not about t) is required to implement the scheme.

4 Concluding remarks

This note considered the effect of the demand faced by spatial duopolists varying in both location and responsiveness to price changes, as captured by the scale parameter of the quadratic transport costs. The model was shown to be equivalent to the Meagher and Zauner [5] framework for an appropriately constructed one-variable distribution (of the demand's location), with interesting consequences for the role of the transport cost uncertainty. In particular, the duopolists can become better off when the costs decrease on average, but more so for the demand realizations closer to the centre of the market. This is because, unlike in the 'certainty' Hotelling model, the scale of transport costs affects the location decisions, as the distribution of the associated parameter interacts with that of the demand's location. Consequently, an increased product differentiation due to relatively less price-responsive peripheral demand realizations may offset the intensification of second-stage price competition due to costs being smaller in general. For the same reason, the expected total transport costs incurred by the consumers (reflecting the level of social welfare), can move in the opposite direction to the average value of the cost parameter. Finally, a social planner can always improve welfare *via* a proportional transport tax schedule contingent on the realization of the uncertainty.

[3] The proceeds from the tax could be re-distributed among the firms and consumers. As an alternative, a proportional subsidy on transport costs could be used, financed by lump-sum taxes.

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