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Efficient emissions reduction

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Efficient Emissions Reduction*

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Abstract

We propose a simple mechanism capable of achieving international agreement on averting the threat of global warming. It employs a contest creating incentives among participating nations to exert both efficient productive efforts and efficient emissions reductive efforts. Participation in the scheme is voluntary and turns out to be individually rational if the alternative is no agreement at all. The scheme requires no principal or enforcing penalties. All rules are mutually agreeable and are unanimously adopted if proposed. (JEL *C7, D7, H4, Q5.* Keywords: *Climate policy, Contests, Efficiency.*)

1 Introduction

The disappointing 2009 Copenhagen Accord[1] has highlighted the international impasse in preventing further global warming. Yet recent reports such as Mitrovica, Gomez, and Clark (2009) suggest taking action now in order to prevent warming beyond the agreed 2°C limit. This paper provides a strikingly simple answer to one the most pressing questions in this context: How to provide incentives for participating nations to reduce harmful emissions to their socially efficient levels while not infracting upon productive efficiency? Our answer involves a contest among nations rewarding the countries with the highest reductive efforts with some share of global output. This contest provides incentives for the efficient provision of both productive and reductive efforts. Participation is individually rational, that is, players find it more profitable to jointly participate in the reductive contest than to stay out. No enforcing sanctions are required, as the players first commit their output share in order to participate in the contest and only then choose their efforts. Once an agreement is reached and players subscribe to the contest, the unique equilibrium involves the efficient choice of efforts along both dimensions.

Greenhouse gases are widely seen as the main contributing factor to global warming. Emitted by one country, they are disseminated around the globe regardless of where they were produced.

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[1] The UN draft decision is available at http://unfccc.int/resource/docs/2009/cop15/eng/l07.pdf.

Thus a reduction of emissions benefits all countries but the costs of such reductions are carried individually. This generates a classic free rider problem, in which each country would like the threat of global warming removed but no one is ready to pay the cost, relying instead on others to take appropriate measures and reduce emissions.[2] An example for how we think about productive and reductive efforts is the (simultaneous) investment decision into a power plant's generation capacity and emissions filters. For instance, the recent investments of Brandon Shores generation station in emissions reduction are documented in Maryland Department of Natural Resources (2007). Other well publicised examples are the design tradeoffs between engine thrust and emissions in Boeing's new 787 Dreamliner or Airbus Industry's A380 aircrafts.

There are at least three existing approaches to overcome the emissions free-riding problem: command-and-control regulation, quantity-oriented market approaches, and tax-or-pricing regimes. The approach adopted by the 187 signatories of the Kyoto protocol is the quantity-oriented market approach targeting a reduction based on developed countries' emissions in 1990. The treaty, however, failed to obtain ratification by major players including, most prominently, the United States. Moreover, the concern was expressed that developing countries might have ratified the treaty without the intention of keeping emissions in check. This mars the current emissions reduction reality with the dual frustrations of insufficient participation and diluted objectives.

Nordhaus (2006), among others, argues that emissions fees or taxes are likely to be more efficient than quantitative quotas given the considerable uncertainty on climate change. As alternative mechanism, Nordhaus proposes a harmonised carbon tax leading to efficient reduction efforts in the general spirit of Pigouvian taxation. There are many details like the necessary sanctions, taxation location, trade treatments and transfers to developing countries which are subject to negotiation under such a scheme. A system based on our approach, by contrast, is self-enforcing and easier to negotiate while still sharing the benefits of a taxation approach.

1.1 Related literature

The idea that in many circumstances a prize awarded on the basis of a rank order among competitors' efforts induces efficiency is due to Lazear and Rosen (1981). This idea has found numerous applications and extensions, for instance in the work of Green and Stokey (1983), Nalebuff and Stiglitz (1983), Dixit (1987), Moldovanu and Sela (2001), or Siegel (2009). To our knowledge, however, the present paper is the only analysis of contests with multi-dimensional efforts except for the orthogonal Münster (2007) who is concerned with the possibility of sabotage, ie. wholly destructive efforts directed at the opponents. Our analysis is close to Gershkov, Li, and Schweinzer (2009) who analyse the efficient single-dimensional effort choice in partnership problems. While our setup is more complicated, many of their insights still apply and we follow much of their methodology. Morgan (2000) is the only existing analysis of public good provision relating to contests that we are aware of. He studies a lottery which uses some of the proceeds obtained from ticket sales for the

[2] One may advocate the view that some countries could climatically benefit from global warming. However, the impact on the world economy and consequences in terms of migration make us pessimistic about the likelihood of emerging net beneficiaries.

provision of a public good. Contrasting with our market design analysis he is not concerned with achieving efficiency. A detailed survey of the contests literature is made redundant by the recent Konrad (2008).[3]

Our team setup seems to be vindicated by the universally accepted property of international environmental agreements (IEA) to be self-enforcing. Indeed, there is no supranational principal to enforce such arrangements between countries. Nevertheless, this feature is absent from the IEA literature (see Finus (2008) for a summary of the main results). Its self-enforcing branch, including Barrett (1994), has found that IEA are either unlikely to consist of many participants, or if they do, are similarly unlikely to produce substantial benefits. Moreover, Diamantoudi and Sartzetakis (2006) show that no more than four countries will find it profitable to form a coalition regardless of the number of countries participating in the negotiations. Kolstad (2007) demonstrates that the size of IEAs decrease as uncertainty grows. Besides, the outcome of such non-cooperative coalition formation games depends on specific membership rules. Carraro, Marchiori, and Oreffice (2009) show that the introduction of a minimum participation rule increases the number of signatories. More recently Barrett (2006) studies an alternative to the Kyoto protocol in proposing a system of two treaties, one promoting cooperative R&D investments and the other encouraging collective adoption of new technology emerging from this R&D. This solution, however, may not be cost-effective. Carbone, Helm, and Rutherford (2006) argue that even if countries pursue only their self-interest, an international system of trading permits can achieve substantial emission reduction. Our analysis shows, however, that it is unlikely to achieve efficiency.

We depart from the existing literature in two key aspects. First, we design an incentive mechanism which is self-enforcing and therefore induces full participation. Second, our scheme is taking into account both productive and reductive efforts. Our key result is, of course, to obtain efficiency in both dimensions for a broad class of model specifications. Following the model definition in section 2, we present the idea of our mechanism through an illustrative example in section 3.1. Although highly stylised, this simple example conveys much of the intuition of our general results presented in section 4. Proofs and details are relegated to the appendix.

2 The model

There is a set $\mathcal{N}$ of $n \geq 2$ risk-neutral players. Each player $i \in \mathcal{N}$ can exert efforts along two dimensions: productive effort $e_i \in [0, \infty]$ and reductive effort $f_i \in [0, \infty]$. We assume that neither effort type is verifiable and denote the full vectors of efforts by $\mathbf{e} = e_1, \dots, e_n$ and $\mathbf{f} = f_1, \dots, f_n$, respectively. These combined efforts cost the individual $c(e_i, f_i)$. This function is assumed to be strictly convex in either argument, additively separable and define costless zero efforts for either component.[4] Productive efforts generate strictly concave individual gains of $y(e_i)$ and cause strictly convex global emissions of $m(\max\{0, \sum_h e_h - \sum_h f_h\})$—only depending on the difference between

[3] The contest model we develop may be applied to problems other than climate change (in both the global and local varieties). For instance, the problem of enforcing monetary discipline among a group of trading nations seems to be similarly applicable.

[4] The full separability is not necessary. All we need is for the cross-partial derivatives not to add local maxima.

global productive and reductive efforts—of which player i suffers a known share s_i.[5] We primarily interpret these shares as physical pollution and assume that $\sum_h s_h = 1$ to introduce a public bad team problem.[6]

As means to alleviate this problem we use an incentive system which ranks individual reductive efforts and awards the top-ranked players prizes. The total prize pool is taken to be the sum of fraction $1 - \alpha$ of each participant's individual output. Thus the mechanism's budget balances by definition. From the total prize pool, a fraction β^1 is awarded to the winner, β^2 to the player coming second, and so on. We assume that all $\beta^h \in [0, 1]$ and $\sum_h \beta^h = 1$. We assume that some noisy and partial ranking of the players' reductive efforts is observe- and verifiable. It gives player i's probability $p_i^h(\mathbf{f})$ of being awarded prize h as a function of the imperfectly monitored reductive efforts of all participants. We assume that $p_i(\mathbf{f})$ is strictly increasing in f_i, strictly decreasing in all other arguments, equal to $^1/_n$ for identical arguments, twice continuously differentiable, and zero for $f_i = 0$ and at least one $f_j > 0$, $j \neq i \in \mathcal{N}$.

Timing

Since the players' expected payoffs are symmetric, we can think of a simple proposal game in which the design parameters are proposed by one player and the game is played iff all others agree to the proposed parameters. More precisely, we propose a two-stage mechanism at the first stage of which an arbitrary potential participant is chosen to propose the balanced budget mechanism $\langle \alpha, \beta, p(\mathbf{f}) \rangle$. We call this proposer player 1. All other players then decide whether to accept the proposal or not. If they accept, the reductive contest is set up, players commit their shares $(1 - \alpha)y(e_i)$ and the game proceeds to the next stage. If at least one player rejects, the game ends and each player obtains his reservation utility (which we specify in proposition 3). At the second stage, conditional on the formation of the agreement, players choose their efforts simultaneously to maximise their own expected utility. The noisy ranking of reductive efforts specifies a winner, second, etc, and final output realises. The prize pool is then redistributed among the participants: the winner obtains the share $(1 - \alpha)\beta^1$ of total output, the player coming second gets $(1 - \alpha)\beta^2$, and so on.

One of the main issues with an International Environmental Agreement is the participants' commitment. Countries may sign the agreement but no supranational entity is there to enforce it in case of defection.[7] Thus countries can always choose not to exert the efficient effort and free ride once the agreement is signed, hoping that other countries will not. In our setup, this issue is avoided as countries first pay to participate in the contest and only then choose their efforts. But once they commit their participation share of output, the only rational effort choice is efficient efforts. Thus there is no enforcement issue subsequent to the initial agreement.

[5] Requiring non-negative differences in the damage function $m(\cdot)$ ensures that reductive efforts cannot substitute productive efforts. Since this requirement is fulfilled for most of our analysis, we redefine $m := m(\max\{0, \sum_h e_h - \sum_h f_h\})$ and only make the non-negative argument explicit when necessary.

[6] In principle, our results also apply to the more general case of $0 < T = \sum_h s_h < n$ in which the individual shares s_i could be interpreted as eg. perceived pollution. Depending on this precise interpretation the planner's objective (1) may change to $\sum_i (y(ei) - c(e_i, f_i)) - Tm(\sum_i (e_i - f_i))$.

[7] A simple way of deterring this kind of free-riding on the agreement may be to grant most favoured 'green' trading terms only to participating nations.

3 Efficiency benchmark

The full intuition of our results can be understood from the simple two players case on which the body of the paper rests. The appendix presents the general setting. For the two players case we write $i = 1, 2$ and $j = 3 - i$. We define the efficient levels of both productive and reductive efforts (e^*, f^*) as those maximising social welfare absent of incentive aspects

$$\max_{(\mathbf{e},\mathbf{f})} u(\mathbf{e},\mathbf{f}) = 2y(e) - m(2e - 2f) - 2c(e,f) \Leftrightarrow \begin{cases} y'(e^*) = m'(2e^* - 2f) + c_e(e^*, f), \\ m'(2e - 2f^*) = c_f(e, f^*). \end{cases} \tag{1}$$

In the absence of an incentive scheme, a participating player $i = 1, 2$ individually maximises

$$y(e_i) - s_i m(e_i + e_j - f_i - f_j) - c(e_i, f_i) \Leftrightarrow \begin{cases} y'(e^*) = s_i m'(2e^* - 2f) + c_e(e^*, f), \\ s_i m'(2e - 2f^*) = c_f(e, f^*). \end{cases} \tag{2}$$

where s_i is player i's local share of global emissions. Since $s_i + s_j = 1$, the individual focs cannot both equal those in (1).[8] Introducing an endogenised rank-order emissions reduction reward scheme attaching weights β and $1 - \beta$ to the players coming first and second, respectively, the individual problem changes to

$$\max_{(e_i,f_i)} \alpha y(e_i) + \sum_h p^h(\mathbf{f})\beta^h(1-\alpha)\left(y(e_i) + y(e_j)\right) - s_i m(e_i + e_j - f_i - f_j) - c(e_i, f_i), \tag{3}$$

where the prize pool is taken as a share $1-\alpha$ from individual productive output. We define individual rationality as the requirement that the utility from efficient effort provision in (3) exceeds the utility from both non-formation of the agreement (2) and of free-riding on the others' reductive efforts *within* the agreement. Our mechanism cannot deter free-riding given an existing agreement and therefore the proposal game which endogenises the choice of design parameters must take commitments seriously.[7] The reductive efforts determining the contest outcome can be easily normalised with respect to, for instance, the individual (perceived) emission consumption share s_i.[9]

[8] This, in a nutshell, is the argument used by Holmström (1982) to show that efficient unverifiable efforts are impossible in a partnership production problem. Battaglini (2006) derives multi-dimensional efficiency in an unrelated setup.

[9] The Jesuit missionary Paul Le Jeune documents the convexity of damage cost for his 1635 experiences during a winter hunting party on which he accompanied native Canadians. The bitter cold made it necessary to light fires inside the Indians' hunting cabin, "but, as to the smoke, I confess to you that it is martyrdom. It almost killed me, and made me weep continually, although I had neither grief nor sadness in my heart. It sometimes grounded all of us who were in the cabin; that is, it caused us to place our mouths against the earth in order to breathe. For, although the Savages were accustomed to this torment, yet occasionally it became so dense that they, as well as I, were compelled to prostrate themselves, and as it were to eat the earth, so as not to drink the smoke. I have sometimes remained several hours in this position, especially during the most severe cold and when it snowed; for it was then the smoke assailed us with the greatest fury, seizing us by the throat, nose, and eyes."

3.1 Example of the efficient mechanism

We use a simple symmetric example to demonstrate the basic idea behind our model. In this framework, substituting into the planner's objective (1) gives

$$\max_{(e,f)} 2e^{1/2} - (2e-2f)^2 - 2(e^2+f^2) \;\Leftrightarrow \begin{cases} e^* = \dfrac{\left(\frac{3}{5}\right)^{2/3}}{2\times 2^{1/3}} \approx 0.2823, \\ f^* = \dfrac{1}{5^{2/3}6^{1/3}} \approx 0.1882. \end{cases} \tag{4}$$

The corresponding individual problem is to

$$\max_{(e_i,f_i)} e_i^{\frac{1}{2}} - s_i(e_i+e_j-f_i-f_j)^2 - (e_i^2+f_i^2) \quad \Leftrightarrow \begin{cases} \hat{e}_i &= \dfrac{\left(\frac{1+2s_i}{1+4s_i}\right)^{2/3}}{2\times 2^{1/3}} > e^*, \\ \hat{f}_i &= \dfrac{s_i}{(2+20s_i+64s_i^2+64s_i^3)^{1/3}} < f^*. \end{cases}$$

For the present example we assume that the probability of winning the reduction award is given by the Tullock success function.[10] Then, under our incentive scheme, the individual problem is to max

$$\alpha e_i^{\frac{1}{2}} + \frac{f_i^r}{f_i^r+f_j^r}\beta(1-\alpha)(e_i^{\frac{1}{2}}+e_j^{\frac{1}{2}}) + \frac{f_j^r}{f_i^r+f_j^r}(1-\beta)(1-\alpha)(e_i^{\frac{1}{2}}+e_j^{\frac{1}{2}}) - s_i(e_i+e_j-f_i-f_j)^2 - (e_i^2+f_i^2)$$

which gives the set of simultaneous focs as

$$16e_i = 8f_i + \frac{1+\alpha}{\sqrt{e_i}},\; 2e_i = 4f_i + \frac{\sqrt{e_i}r(\alpha-1)(2\beta-1)}{2f_i}.$$

Setting $e = e_i = e^*$, $f = f_i = f^*$ and solving for symmetric efforts (under $s_i = 1/2$) gives the efficiency inducing

$$\alpha^* = \frac{3}{5},\; \beta^* = \frac{1}{2} + \frac{1}{6r}. \tag{5}$$

Notice that β^* only depends on r. Thus, the rewards scheme—and in particular the relative size of the prizes paid to the winner and loser—can be designed as seen fit and allowed by equilibrium existence. Figure 1 shows that participating in the contest gives higher utility than staying out and free-riding on the other's effort. It confirms $(\alpha^*, \beta^*, e^*, f^*)$ as unique equilibrium in pure strategies.[11]

The economics behind this result is simple: An increase in productive efforts e_i causes individual output $y(e_i)$ and global pollution $m(\sum_h e_h - \sum_h f_h)$ to rise of which the player retains the shares α and s_i, respectively. An increase in reductive efforts f_i enlarges the player's chance to win the prize share β in the reduction contest and simultaneously decreases global pollution. Trading off α against β allows us to fine-tune efforts to their efficient levels.

[10] The particular monitoring technology is not important as we generalise over the set of applicable success functions in the appendix. What is important is that the success function incorporates enough randomness in its outcome. If the ranking is too precise (as is the case with the all-pay auction) then equilibria in pure strategies typically fail to exist. This would be problematic as our contest strives to implement the efficient pure effort choices.

[11] Since both partial derivatives are strictly negative, it is sufficient to investigate optimality of the two-dimensional problem along both effort dimensions. That this is admissible is shown in the proof of proposition 2.

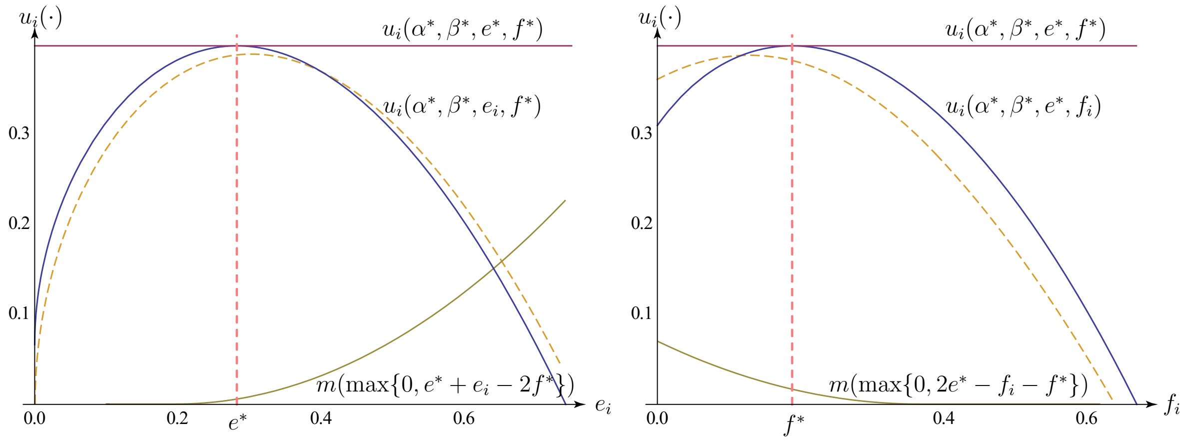

Figure 1: The top line is the equilibrium utility from $(\alpha^*, \beta^*, e^*, f^*)$. The curves below show the utility from unilaterally deviating in either effort dimension. Notice the positive utility from free-riding at zero efforts. The dashed lines give the (outside) utility from no agreement formation.

4 Results

Recall that under our award scheme, an individual $i = 1, 2$ chooses a pair of efforts (e_i, f_i) to max

$$\alpha y(e_i) + (1-\alpha)\left(\beta p(\mathbf{f})\sum_j y(e_j) + (1-\beta)(1-p(\mathbf{f}))\sum_j y(e_j)\right) - s_i m\left(e_i + e_j - f_i - f_j\right) - c(e_i, f_i)$$

where $p(\mathbf{f})$ is the probability of coming first in a ranking of reductive efforts f. We require that $y' > 0$, $y'' \leq 0$, $m' > 0$, $m'' \geq 0$, and finally $c'_{1,2} > 0$, $c''_{1,2} \geq 0$. We moreover assume that $m(\cdot)$ only depends on the difference of total productive minus reductive efforts and that the cost function is additively separable in both types of efforts. Taking derivatives wrt both effort types, we obtain the simultaneous pair of focs defining individually optimal efforts (e_i, f_i) as

$$\begin{aligned} c_e(e_i, f_i) + s_i m'(e_i + e_j - f_i - f_j) &= (1 - \beta + \alpha\beta + (1-\alpha)(2\beta - 1)p(f_i, f_j))y'(e_i) \\ c_f(e_i, f_i) + (\alpha - 1)(2\beta - 1)(y(e_i) + y(e_j))p'(f_i, f_j) &= s_i m'(e_i + e_j - f_i - f_j). \end{aligned}$$

Assuming that a symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$, $s_i = {}^1\!/_2$ exists, this simplifies to

$$\begin{aligned} 2c_e(e, f) + m'(2e - 2f) &= (\alpha + 1)y'(e) \\ 2c_f(e, f) - m'(2e - 2f) &= 4(1-\alpha)(2\beta - 1)p'(\mathbf{f}^*)y(e) \end{aligned} \tag{6}$$

where $\mathbf{f}^* = (f^*, f^*)$. Equating these efforts to those in (1), we obtain

$$4p'(\mathbf{f}^*)(2\beta - 1) = \frac{y'(e^*)}{y(e^*)} \Leftrightarrow \begin{cases} c_e(e^*, f) = \alpha y'(e^*), \\ c_f(e, f^*) = 4(1-\alpha)(2\beta - 1)p'(\mathbf{f}^*)y(e^*). \end{cases} \tag{7}$$

Efficiency can be obtained as we know that there exists an $\alpha \in [0, 1]$ to satisfy the first equation from (1). Substituting this α in the second equation determines $\beta \in [{}^1\!/_2, 1]$ for a suitably chosen ranking p. Without further restrictions on the design parameters—and in particular the slope of the ranking technology $p(\cdot)$ in equilibrium—(7) can always be accomplished. Taking equilibrium existence as given, the following proposition establishes the precise criteria on the parameters for both productive and reductive efficiency to obtain simultaneously for any number of players $n \geq 2$.

Proposition 1. *For appropriately chosen $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ specifying prize shares $\beta = \left(\beta^1, \frac{1-\beta^1}{n-1}, \ldots, \frac{1-\beta^1}{n-1}\right)$, player $i \in \mathcal{N}$ chooses efficient productive as well as reductive efforts (e^*, f^*) in*

$$\max_{(e_i, f_i)} \alpha y(e_i) + (1-\alpha) \sum_h \left(\beta^h p^h(\mathbf{f}) \sum_j y(e_j) \right) - s_i m \left(\sum_h e_h - \sum_h f_h \right) - c(e_i, f_i). \quad (8)$$

All proofs can be found in the appendix. It is straightforward to show that proposing $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ at the first stage of the game maximises player 1's expected utility, given that players' efforts are functions of the proposed mechanism $(e(\alpha, \beta, p), f(\alpha, \beta, p))$. This is unsurprising as payoffs are symmetric and, hence, whatever maximises welfare must also maximise the proposer's utility.

A consequence of the previous result is that full efficiency in the symmetric n-player model can be obtained with just two different prizes: one for the winner and another for everyone else. As one only needs to check for a winner, such a scheme is easy to monitor. Since the general objective is not necessarily well behaved, we proceed to show that equilibria exist for the subclass of problems governed by the Tullock success function in the following proposition.

Proposition 2. *The provision of efficient efforts (e^*, f^*) is globally optimal for player $i \in \mathcal{N}$ provided that reduction costs are sufficiently convex. In particular, we derive a sufficient existence threshold (17) for contests governed by the Tullock success function.*

Equilibrium existence implies that free-riding is not attractive once a nation has joined the agreement. As the number of participants in the mechanism n goes up, the utility from free-riding increases as the disutility from pollution $m(\sum_h (e_h - f_h))$ approaches the efficient level. Hence the only leverage left is the contest on the pre-committed output share of $(1-\alpha)$. Our contest cannot deter free-riding on the reductive efforts of the participants once an agreement is in place. Thus participation in the agreement is individually rational in the sense that it is optimal to agree to commit one's output share at the proposal stage. Players cannot, however, be granted the option to leave the agreement once it is formed.

Proposition 3. *For a sufficiently large number of participants n, a player's expected utility from participation in mechanism (8) is at least as high as the utility when no agreement is reached.*

For the simple quadratic setup of the example in section 3.1, $n = 2$ is enough to encourage agreement formation. Finally, we show that our efficiency result is not an artifact of our symmetry assumptions. The result is presented for two players as the extension to n-players case is trivial.

Proposition 4. *Let $i = 1, 2$ and $j = 3 - i$. For appropriately chosen $\langle \alpha, \beta, p(\mathbf{f}) \rangle$, efficient solutions exist to player i's asymmetric problem*

$$\max_{(e_i, f_i)} \quad \begin{aligned} &\alpha y_i(e_i) + (1-\alpha)\left(\beta_i p(\mathbf{f})(y_i(e_i) + y_j(e_j)) + (1-\beta_i)(1-p(\mathbf{f}))(y_i(e_i) + y_j(e_j))\right) \\ &-s_i m\left(e_i + e_j - f_i - f_j\right) - c_i(e_i, f_i). \end{aligned} \tag{9}$$

Although we demonstrate most of our results in a simplified symmetric setup, the previous proposition shows that this can be done without loss of generality. A small example in the appendix shows that our results are robust to the choice of contest success function. A further short example details the working of an asymmetric contest. Both show that our proposed mechanism indeed solves the problem of reducing emissions to their efficient level in a general setup.

5 Concluding remarks

We show that a simple contest organised among nations implements both efficient productive and reductive efforts. Desirable generalisations are in the realism and welfare implications of our assumptions. Which share of global (per capita) GDP would have to be redistributed—in reality—to the country with the highest emissions reduction in order to implement our results? Is the resulting wealth redistribution one we would like to see? These questions are to a large extent empirical and all have huge policy implications. At any rate we do not feel qualified to answer these questions now. What we do provide, however, are firm results showing that an incentive mechanism along the lines we indicate can *in principle* solve the world's emission problems.

Appendix

Proof of proposition 1. Efficient efforts are extending (1) as the pair (e, f) solving

$$y'(e) = m'(ne - nf) + c_e(e, f), \; m'(ne - nf) = c_f(e, f). \tag{10}$$

Let $P = (1-\alpha)\sum_{h=1}^{n} y(e_h)$. Since we are only interested in deviations from symmetric equilibrium, we set $e_j = e_{-i}$. Rewriting (8) for our 2-prize structure $\left(\beta^1, \frac{1-\beta^1}{n-1}, \ldots, \frac{1-\beta^1}{n-1}\right)$ results in

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \sum_{h=2}^{n} \frac{1-\beta^1}{n-1} p_i^h(\mathbf{f})P - s_i m\left(e_i + (n-1)e_j - f_i - (n-1)f_j\right) - c(e_i, f_i)$$

which simplifies to

$$\alpha y(e_i) + \beta^1 p_i^1(\mathbf{f})P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\mathbf{f}))P - s_i m\left(e_i + (n-1)e_j - f_i - (n-1)f_j\right) - c(e_i, f_i).$$

The symmetric $e = e_i = e_j$, $f = f_i = f_j$, focs for this problem are

$$\begin{aligned} c_e(e,f) + s_i m'(ne - nf) &= \frac{1-\beta+\alpha(n+\beta^1-2)+(1-\alpha)(n\beta-1)}{n-1} p(f) y'(e), \\ c_f(e,f) &= s_i m'((e-f)n) + \frac{n(1-\alpha)(n\beta^1-1)}{n-1} p'(f) y(e). \end{aligned} \tag{11}$$

Plugging in (10) and imposing $s_i = 1/n$, one obtains

$$\alpha^* = 1 - \frac{y'(e^*) - c_e(e^*, f^*)}{y'(e^*)} \text{ and } \beta^* = \frac{1}{n} + \frac{(n-1)^2 y'(e^*)}{n^3 y(e^*) p'(\mathbf{f}^*)} \tag{12}$$

which can always be done by picking a suitably steep ranking technology $p(\mathbf{f}^*)$. □

Proof of proposition 2. We tentatively assume that we can split the problem into two independent problems along the respective effort dimensions. As it turns out that both partial derivatives are strictly increasing to the left of the efficient efforts and strictly decreasing to the right, no mixture between the two can constitute a beneficial deviation either. Setting $P = (1-\alpha)\sum_{h=1}^{n} y(e_h)$, the two separate problems are

$$\begin{aligned} \alpha y(e_i) + \beta^1 p_i^1(\mathbf{f}^*) P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\mathbf{f}^*))P - s_i m\left(e_i + (n-1)e^* - n f^*\right) - c(e_i, f^*), \\ \alpha y(e_i^*) + \beta^1 p_i^1(\mathbf{f}) P + \frac{1-\beta^1}{n-1}(1 - p_i^1(\mathbf{f}))P - s_i m\left(ne^* - f_i - (n-1)f^*\right) - c(e^*, f_i). \end{aligned} \tag{13}$$

1) We show that exerting productive effort $e_i = e^*$ gives a global maximum. As players are symmetric and we are looking for a profitable deviation from the efficient level we set $\mathbf{f}^* = (f_1 = f^*, \ldots, f_n = f^*)$ implying that the probability of wining is $p_i^1(\mathbf{f}^*) = 1/n$. Thus the problem simplifies to

$$\alpha y(e_i) + \frac{1}{n} P - s_i m\left(e_i + (n-1)e^* - (n) f^*\right) - c(e_i, f^*) \tag{14}$$

giving the foc for productive effort e_i as[12]

$$\underbrace{y'(e_i)\left(\alpha + \frac{1}{n}(1-\alpha)\right)}_{\searrow} = \underbrace{s_i m'\left(\max\{0, e_i + (n-1)e_j^* - (n)f^*\}\right)}_{\nearrow} + \underbrace{c_{e_i}(e_i, f^*)}_{\nearrow}.$$

Notice that output is strictly increasing in e_i and is strictly concave. Thus $y''(e_i) < 0$ and $y'(e_i)$ is decreasing. Both cost functions are increasing and convex, therefore $s_i m''(\cdot) + c''(e_i) > 0$ and the rhs is increasing. As $y'(0) > s_i m'(\max\{0, (n-1)e^* - nf^*\}) + c'(0)$,[13] this confirms single crossing of rhs and lhs and ensures the existence of a unique equilibrium.

2) We now demonstrate global optimality of $f_i = f^*$. Assuming efficient productive effort

[12] It is routine to verify that both focs identify a maximum.

[13] As output is concave and the sum of cost functions is convex in e_i, the slope of the output derivative at zero $y'(e_i = 0)$ is clearly bigger than that of the cost $s_i m''(\max\{0, (n-1)e^* - nf^*\}) + c''(e_i = 0)$.

provision, the foc for reductive effort is

$$\underbrace{n\,y\,(e^{*})\,(1-\alpha)\,(\beta\,n-1)\,p'(f_i,f^{*})}_{=B}=\underbrace{c_{f_i}(f_i,e^{*})}_{=C\nearrow}-\underbrace{s_i m'(\max\{0,ne^{*}-(n-1)f^{*}-f_i\})}_{=A\searrow}. \quad (15)$$

Notice that the rhs is strictly increasing as we know that wrt f_i, $s_i m''(\cdot) \leq 0$ and thus that A is decreasing and the cost function is convex. Without further assumptions on the monitoring technology $p(\cdot)$ we cannot sign the slope of B. Notice, however, that increasing $c_{f_i}(f_i, e^{*})$ sufficiently guarantees single crossing and thus equilibrium uniqueness whatever the precise specification of $p(\cdot)$.

3) We now show that (15) identifies a global maximum for the Tullock success function.[14] Again, $s_i m(\max\{0, ne^{*} - (n-1)f^{*} - f_i\}) > 0$ for $f_i = 0$ while $p'(f_i, f^{*}) = 0$ and thus the lhs of (15) is zero at $f_i = 0$ while the rhs is negative. Single crossing is immediate for the case of $r \in (0,1]$ as B is (weakly) decreasing. In the general case of

$$p_i(\mathbf{f}) = \frac{f_i^r}{\sum_{j=1}^{n} f_j^r},\ r>1, \quad (16)$$

the function B has a single critical point and is decreasing when $f_i \geq f^{*}\left(\frac{(n-1)(r-1)}{r+1}\right)^{1/r}$.

To get single crossing if the two curves are increasing we need to ensure either strict concavity or convexity for the lhs and strict convexity for the rhs and prove that if $f_i = 0$, lhs is larger than the rhs. As we have not specified anything about our functions regarding the third derivative we illustrate this point using the specific $c_{f_i}(f_i, e^{*}) = bf^{b-1}$ and $s_i m(\max\{0, ne^{*} - (n-1)f^{*} - f_i\}) = s_i(\max\{0, ne^{*} - (n-1)f^{*} - f_i\})^b$. We find that both curves have an inflection point, thus we need to find a condition to ensure single crossing.

We first show that the rhs starts out negative and eventually becomes positive as for $f_i = 0$ we have $C = -s_i m'(\max\{0, ne^{*} - (n-1)f^{*}\}) < 0$. Therefore, as long as the rhs is positive and the lhs negative, the two curves cannot cross. We find that $C - A < 0$ for $f_i < f^{*}\frac{2}{n^{\frac{1}{b-1}}+1}$ because

$$C-A=\frac{-(\overbrace{ne-(n-1)f^{*}}^{=2f^{*}}-f)^{b-1}b}{n}+f^{b-1}b=0\ \Leftrightarrow (2f^{*}-f)^{b-1}=nf^{b-1}.$$

Moreover, for the rhs, the inflection point occurs when the curve is negative, and it is first concave and then convex. Thus we can conclude than when the curve is above zero, it is strictly convex. We find that $(C-A)'' < 0$ for $f_i < f^{*}\frac{2}{n^{\frac{1}{b-3}}+1}$ and $f_i < f^{*}\frac{2}{n^{\frac{1}{b-3}}+1} < f^{*}\frac{2}{n^{\frac{1}{b-1}}+1}$ because

$$C-A=\frac{-(2f^{*}-f)^{b-3}(b-2)(b-1)b}{n}+f^{b-3}(b-2)(b-1)b=0\ \Leftrightarrow (2f^{*}-f)^{b-3}=nf^{b-3},$$

$$\Leftrightarrow f^{*}\frac{2}{n^{\frac{1}{b-1}}+1}-f^{*}\frac{2}{n^{\frac{1}{b-3}}+1}=-2\,\frac{f^{*}\left(-n^{(b-3)^{-1}}+n^{(-1+b)^{-1}}\right)}{\left(n^{(-1+b)^{-1}}+1\right)\left(n^{(b-3)^{-1}}+1\right)}\geq 0.$$

[14] A nearly identical argument can be made for any other ratio-based success function. In that more general case, however, we cannot derive an explicit existence threshold.

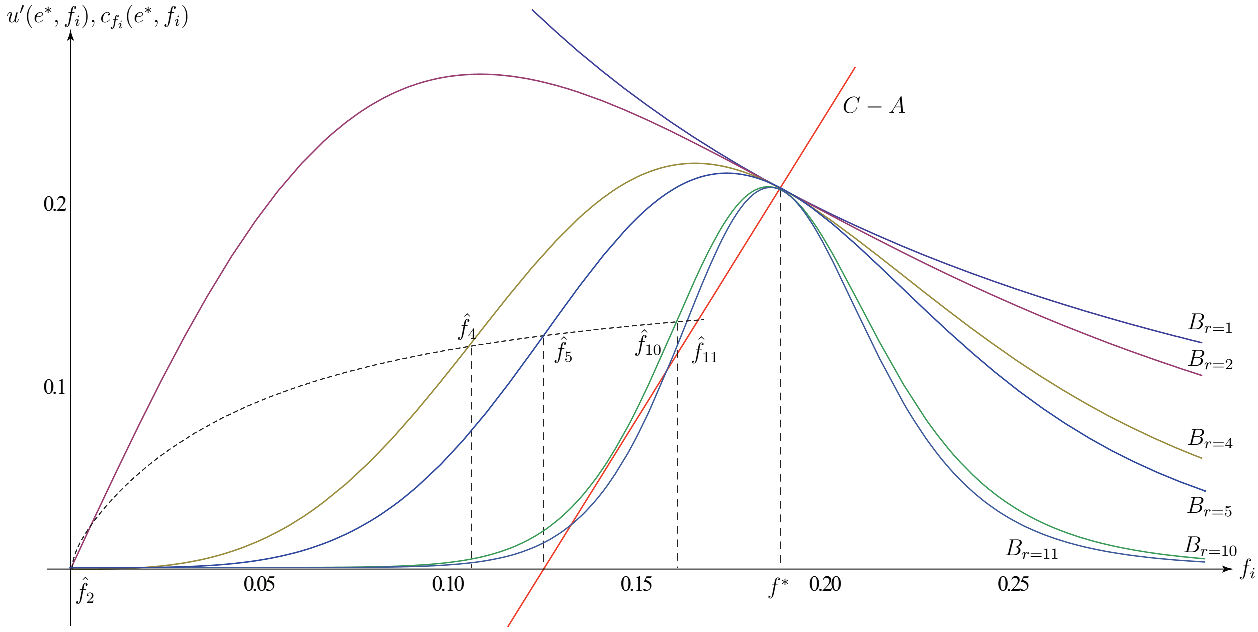

Figure 2: Single crossing in equation (15) ensures a unique global maximum at $f_i = f^*$ for the example setup of section 3.1. The dotted line gives the location of inflection points $\hat{f}$ for different r.

We conclude that the rhs is strictly increasing and convex when it is positive.

For the lhs, there are two inflection points: one in the increasing part and the other in the decreasing part. In the increasing part we find a condition which implies that the inflection occurs if the rhs is negative.[15] A sufficient condition for equilibrium uniqueness is therefore that

$$\frac{2^r (f^*)^r}{\left(n^{\frac{1}{b-1}} + 1\right)^r} \geq \underbrace{\frac{(n-1)\left(2(f^*)^r(r^2-1) - \sqrt{3}\sqrt{(f^*)^{2r}r^2(r^2-1)}\right)}{2+3r+r^2}}_{=:\hat{f}}. \tag{17}$$

Thus if the rhs of (15) is positive, it is also strictly convex. If (17) is respected, the lhs is strictly concave or convex. Notice also that at the inflection point, the rhs is positive and the lhs is negative and therefore the lhs is larger than the rhs. The geometric intuition of (17) is shown in figure 2 for the setup of the example section 3.1. The figure shows a family of curves B for $r \in \{1, 2, 4, 10, 11\}$ with inflection points labelled $\hat{f}_2$, $\hat{f}_4$, $\hat{f}_{10}$, and $\hat{f}_{11}$, respectively. Condition (17) is fulfilled as long as the red cost curve $C - A$ is negative at the respective inflection point. This is true for $r = 2$ and $r = 4$ soon after which (17) starts failing. Uniqueness, however, is actually only lost for $r > 10$. $\square$

Proof of proposition 3. Player i's equilibrium participation utility for $P = (1-\alpha)ny(e^*)$ in the efficiency-inducing mechanism $\langle \alpha, \beta, p(\mathbf{f}) \rangle$ from (8) is

$$\begin{aligned} u_i^*(e^*, f^*) &= \alpha y(e^*) + \tfrac{1}{n}\beta P + (n-1)\tfrac{1}{n}\tfrac{1-\beta}{n-1}P - s_i m\,(ne^* - nf^*) - c(e^*, f^*) \\ &= \alpha y(e^*) + \tfrac{1}{n}(1-\alpha)ny(e^*) - s_i m\,(ne^* - nf^*) - c(e^*, f^*) \\ &= y(e^*) - s_i m\,(ne^* - nf^*) - c(e^*, f^*). \end{aligned} \tag{18}$$

[15] The inflection point in the decreasing part does not matter. As long as one curve is increasing and the other is decreasing they can only cross once.

Non-participation gives

$$u_i^s(e_i, f_i) = y(e_i) - s_i m(e_i + (n-1)e^* - f_i - (n-1)f^*) - c(e_i, f_i). \quad (19)$$

Notice that the latter formulation requires $n > 2$ as the contest can only produce efficient incentives if at least two players participate in the contest. The inherent incentives make it impossible to deter free-riding once an agreement is operational. It is, however, individually rational to join the agreement if the alternative is no agreement at all because disagreement utility is

$$u_i^d(e_i, f_i) = y(e_i) - s_i m(e_i + (n-1)e^d - f_i - (n-1)f^d) - c(e_i, f_i) \quad (20)$$

implying both $e^d > e^*$ and $f^d < f^*$ compared with (10). Since the cost differential

$$m(ne^d - nf^d) - m\left(ne^* - nf^*\right) \quad (21)$$

is increasing in n, there is an agreement size which makes everybody join. Thus, for a large enough populace, every player finds it individually rational to join the reductive contest. □

Proof of proposition 4. Analogous to (1), let the player's asymmetric efficient efforts be given by

$$y_i'(e_i^*) = m'(2e_i^* - 2f_i) + c_{e_i}(e_i^*, f_i) \text{ and } m'(2e_i - 2f_i^*) = c_{f_i}(e_i, f_i^*). \quad (22)$$

Let the winner's shares be identity-dependent, ie. a winning player i gets share β_i and a winning j gets share β_j of the prize pool $P = (1-\alpha)(y_i(e_i) + y_j(e_j))$. Thus, taking e_j^*, f_j^* as given, player i maximises (9) simplified to

$$\alpha y_i(e_i) + \beta_i p(\mathbf{f})P + (1-\beta_i)(1-p(\mathbf{f}))P - s_i m\left(e_i + e_j - f_i - f_j\right) - c_i(e_i, f_i).$$

Taking derivatives wrt e_i, f_i and inserting (22), gives player i's best response as determined by

$$\begin{aligned} (1-s_i)m'(e_i + e_j - f_i - f_j) &= (\alpha - 1)((2\beta_i - 1)p(f_i, f_j) - \beta_i)y_i'(e_i), \\ \frac{2\beta_i - 1}{c_{f_i}(f_i^*)}p'(f_i, f_j)P &= 1 - s_i. \end{aligned} \quad (23)$$

Solving for player j's best response to $e_i = e_i^*, f_i = f_i^*$ gives two more equations. Solving the resulting system gives parameters $\langle \alpha, \beta_i, \beta_j, p(\mathbf{f}) \rangle$ eliciting asymmetric efficient efforts. □

Robustness with respect to the choice of success function

Consider a n-player extension of the problem of subsection 3.1 with prize structure $\left(\beta, \frac{1-\beta}{n-1}, \ldots, \frac{1-\beta}{n-1}\right)$. The present example shows that efficiency can be obtained in proposition 2 for the 'difference-form' success function. Difference-form success functions have been widely used in the literature, for instance by Che and Gale (2000), but suffer from the lack of a generally accepted extension to more than two players. We define player i's probability of winning as[16]

$$p_i(\Delta) = \frac{\exp^{\Delta_i^r}}{\sum_{j=1}^n \exp^{\Delta_j^r}}, \text{ where } \Delta = (\Delta_1, \ldots, \Delta_n),\ \Delta_i = f_i - \frac{\sum_{j \neq i} f_j}{n-1}, \text{ and } r > 0. \tag{24}$$

Setting $P = (1-\alpha)(e_i^o + (n-1)e_j^o)$, $o \in (0,1)$, $m, b > 1$ and all $j \neq i$ equal, player i's individual problem is to

$$\max_{(e_i, f_i)} \alpha e_i^o + p_i(\Delta)\beta P + (1 - p_i(\Delta))\frac{1-\beta}{n-1}P - s_i(e_i + (n-1)e_j - f_i - (n-1)f_j)^m - (e_i^b + f_i^b)$$

which, in symmetric equilibrium $e = e_i = e_j$, $f = f_i = f_j$ gives for any $p_i(\Delta)$

$$\alpha = \frac{e^{-o}\left((e-f)\left(be^b n - e^o o\right) + em((e-f)n)^m s_i\right)}{(e-f)(n-1)o},$$

$$\beta = \frac{e^{-o}\left(-b(e-f)f^b(n-1)n + f\left(m(n-1)((e-f)n)^m s_i + e^o(e-f)n^2(\alpha-1)p_i'(0)\right)\right)}{(e-f)fn^3(\alpha-1)p'(0)}$$

where $\Delta = 0$ is the equilibrium vector of deviations. Plugging in the efficient efforts from (4), employing (24), and returning to the example setup from section 3.1: $n = 2$, $o = 1/2$, $b = m = 2$, and $s_i = 1/2$, this results in a very similar efficient mechanism as under the Tullock success function

$$\alpha^* = \frac{3}{5},\ \beta^* = \frac{1}{2} + \frac{r + (5/6)^{2/3}}{2r}$$

where $\beta^* \in (.5, 1]$ is ensured for $r \geq (5/6)^{2/3}$. A picture nearly identical to figure 1 confirms, for instance, $(\alpha^*, \beta^*, r = 2)$ as equilibrium. The precise form of ranking technology employed is thus immaterial to our results.

Asymmetries

This subsection extends the two-player setup of section 3.1 with unequal relative damage shares $s_i \in (0,1)$, $i = 1, 2$. Since shares sum to 1, both efficient effort types are still given by (4). Player $i's$ problem is unchanged and results in the focs

$$1 + \alpha = 8\sqrt{e}(e + 2es_i - 2fs_i),\ f^2(4 + 8s_i) + \sqrt{e}r(-1+\alpha)(-1+2\beta) = 8efs_i.$$

Imposing efficiency (4) we obtain the shares

$$\alpha^* = \frac{1}{5}(1 + 4s_i),\ \beta^* = \frac{1}{2} + \frac{1}{6r}. \tag{25}$$

Notice that only α^* turns out to dependent on the player's identity (class), the efficiency-inducing prize structure β is identical to the symmetric case. As to be expected, the share $1 - \alpha^*$ of

[16] This formulation is justified by Schweinzer and Segev (2010) who also provide the equilibrium analysis for a broad class of difference-form success functions and the corresponding existence results.

output which has to be committed to the contest gets arbitrarily small when the public bad problem disappears as s_i approaches 1. On the other extreme, a player who does not suffer from the effects of global warming at all must be asked to commit close to $4/5$ of her output to the contest in order to induce efficient efforts on her behalf. A numerical example taking relative damage shares of $s_1 = {}^1\!/_4$, $s_2 = {}^3\!/_4$ requires $\alpha_1 = 0.4$ and $\alpha_2 = 0.8$ in order to implement efficiency.

Next consider the general case of $n > 2$ players with damage shares parameterised by

$$s_i = \frac{2i}{n+n^2},\ i = 1, 2, \ldots, n \quad \text{with} \quad \sum_{i=1}^{n} s_i = 1.$$

Efficient efforts are then given by

$$4e(1+n) = \frac{1}{\sqrt{e}} + 4fn, en = f(1+n) \ \Leftrightarrow \begin{cases} e^* = \dfrac{1+n}{2 \times 2^{1/3}\left((1+n)(1+2n)^2\right)^{1/3}}, \\ f^* = \dfrac{n}{2 \times 2^{1/3}\left((1+n)(1+2n)^2\right)^{1/3}}. \end{cases}$$

Solving the n-player individual asymmetric problem under our example contest and the two-part price structure $\left(\beta, \frac{1-\beta}{n-1}, \ldots, \frac{1-\beta}{n-1}\right)$ employed previously

$$\alpha e_i^{1/2} + \frac{f_i^r}{f_i^r + (n-1)(f^*)^r}\beta P + \left(1 - \frac{f_i^r}{f_i^r + (n-1)(f^*)^r}\right)\left(\frac{1-\beta}{n-1}\right)P - \\ s_i(e_i + (n-1)e^* - f_i - (n-1)f^*)^2 - (e_i^2 + f_i^2)$$

for $P = (1-\alpha)(e_i^{1/2} + (n-1)(e^*)^{1/2})$ results in the intimidating but straightforwardly interpreted efficiency inducing shares

$$\alpha^* = \frac{\sqrt{\frac{1+n}{((1+n)(1+2n)^2)^{1/3}}}(1+n+ns_i)n - \left((1+n)(1+2n)^2\right)^{1/3}}{(n-1)\left((1+n)(1+2n)^2\right)^{1/3}},$$

$$\beta^* = \frac{(n-1)n(1+2n)^2\left(\frac{1+n}{((1+n)(1+2n)^2)^{1/3}}\right)^{3/2} + n\left(n^2-1\right)(1+n+ns_i) + 2(1+n)^2 r(2+n(3+s_i))}{2n(1+n)^2 r(2+n(3+s_i))}.$$

A numerical example for $n = 187$, $r = 3$ gives $\alpha^* = 0.501333$, implying for 'type' $s_i = 1/n$ an efficiency-inducing redistribution vector of

$$\left(\beta^* = 0.170241, \frac{1-\beta^*}{n-1} = 0.00446107, \ldots, \frac{1-\beta^*}{n-1} = 0.00446107\right)$$

which compares to the flat $1/n = 0.00534759$. Under the contest, type $s_i = 1/n$ gives up roughly 50% of her output but gets back 41.6% even if losing the contest. She gets almost 16 times her output if she wins. Notice that if equilibrium existence allows this can be further equalised by employing a more precise ranking and thereby increasing r.

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