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# Fan welfare maximization as a club objective in a professional sports league

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# FAN WELFARE MAXIMIZATION AS A CLUB OBJECTIVE IN A PROFESSIONAL SPORTS LEAGUE

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**Abstract** Motivated by aspects of European soccer club governance (members' clubs supporters' trusts), a first formal analysis of fan welfare maximization as a club objective in a sports league is provided, with comparisons to objectives studied previously (profit and win maximization). Positive comparisons focus on team qualities, ticket prices, attendances and the impact of capacity crowds; empirically observed ticket black markets and inelastic pricing are consistent only with fan welfare maximization. Normatively, social welfare (aggregate league surplus) is well-served by a league of fan welfare maximizers, or sometimes win maximizers, but not profit maximizers; leagues should not normally make profits.

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## 1. INTRODUCTION

Existing theoretical analysis of the professional sports league industry has focused on leagues where the objective of individual clubs[1] is either profit maximization, or, subject to a budget constraint, maximization of win percentage (equivalent to maximizing relative team quality)[2]. The conventional view is that profit maximization may serve reasonably well the major North American sports leagues where clubs seem largely to have been run on the lines of businesses in other industries, but win maximization may be more prevalent in European soccer, where wealthy club owners have seemingly been prepared to forego profit to produce champion teams[3]. However, particularly in the European context, there is a pressing case to examine a third type of objective, what we call "fan welfare maximization", again subject to a budget constraint, where fans (or supporters) have a particular allegiance to a club, are the consumers of its products, and directly influence club policies.

Historically German soccer clubs have been constituted as members' clubs (Verein), whereby fans elect club officials to make the important business strategy decisions. Although the prevalence of this governance mechanism has declined to a small extent recently, it remains the case that almost all the clubs in the top division of German soccer (Bundesliga) are still governed in this way. Similar members' club constitutions exist in Spain's Primera Liga. Absent political economy problems in the electoral process, one would expect fan welfare maximizing policies to influence winning platforms in the elections of club officials, and hence club strategy. There is also a growing role for "supporters' trusts" in UK soccer[4], where these associations of fans of a club are acquiring increasing representation on club boards, and so increasing influence on the club decision process. Indeed, in the wake of UEFA's 10-year strategy statement (UEFA (2004)), the EU commissioned an independent report (Arnaut (2005)) to tackle specific issues faced by European soccer, and a broad recommendation for greater supporter involvement in governance emerged[5].

The aim of the paper is to provide a first formal analysis of fan welfare maximization as a club objective in a professional sports league, with a particular view to comparing its consequences (both normative and positive) to those of the profit and win maximization alternatives. Accordingly we take a simplified, basic framework similar to that used in previous literature for the study of profit and win maximization, and

[1] We use the following terms solely with their sporting meanings; club, team, match, player. However games refer to their usual meaning in economic models.

[2] Coverage of the literature can be found in the major surveys of Fort and Quirk (1995) and Szymanski (2003), the handbook of Andreff and Szymanski (2006), the textbooks by Fort (2006) and Sandy et al. (2004), and in the materials for the increasing numbers of courses on Sports Economics being taught around the world. The book by Kesenne (2007) provides a full account of existing results on both profit and win maximization.

[3] Based on data from English and Spanish soccer leagues, Garcia-del-Barrio and Szymanski (2009) have recently argued that win maximization does indeed provide a better fit than profit maximization.

[4] Michie et al. (2006) provide information on governance of UK soccer clubs, and the role of supporters' trusts in particular. Although many professional soccer clubs in the hierarchy of English leagues have active supporter's trusts, influence via board representation is currently limited to lower levels in the hierarchy – League 1 (3rd tier) and below.

[5] "The authors of the Review believe that properly structured supporter involvement will help to contribute to improved governance and financial stability (as well as other benefits)", Arnaut (2005, p.81).

add the fan welfare maximization analysis. With European soccer and its relatively fierce inter-league competition for players in mind, we follow this established framework and assume a perfectly elastic supply of playing talent to the league[6], which consists of two clubs that play each other twice over the season, once at home and once away in stadiums of given capacity. Clubs earn revenue from attendance by their fans at their home match (with a fan utility microfoundation that generalises Falconieri et al. (2004)) and incur the costs of hiring playing talent. Club decisions (best responses) on match ticket prices and expenditure on playing talent are analysed and compared, as are the consequences for match attendances and the effects of binding stadium capacity constraints; further questions are whether the observed phenomena of black markets for tickets and inelastic ticket pricing are consistent with all or any of the club objectives. We also study league Nash equilibria in leagues of profit maximizers, win maximizers and fan welfare maximizers, with a view to comparing the performance of these three leagues both positively and normatively, the latter to decide which governance mechanism performs best from the social (aggregate surplus) viewpoint[7]. Some remarks are offered comparing the German Bundesliga with the English Premier League, in the light of the theoretical findings.

Section 2 sets out the broad framework for the analysis when stadium capacity constraints are slack, and Section 3 investigates individual club decisions under the three objectives. Section 4 looks at Nash equilibria of the three leagues indicated above, and compares the consequences for aggregate surplus. Section 5 allows the stadium capacity constraint to become binding. Section 6 compares the Bundesliga and the Premier League, and Section 7 concludes.

## 2. THE FRAMEWORK

Two clubs and their teams comprise the professional sports league. The exogenous league rules are that each team plays the other twice, once at home and once away. Club $i$=1,2 has a stadium where its team plays its home match; the stadium has a given capacity, sufficiently large so as to be never binding on match attendance (until Section 5), and we abstract from stadium costs as is usual in the literature. Clubs hire players and $Q_i \geq 0$ denotes the expenditure on playing talent by team $i$. Following the established treatment with a European soccer league in mind, talent is in perfectly elastic supply at a wage normalised to 1, so $Q_i$ is also the quantity of playing talent, alternatively referred to as the quality of team $i$.

Club $i$ sets the ticket price $p_i$ for admission to its home match and receives all gate revenue from this match; no price discrimination is possible. There are disjoint sets

[6] In contrast, the major North American sports leagues face relatively little competition from other leagues for their specialised playing talent, and an inelastic supply of talent to the league is typically assumed. However the resulting literature has overlooked the oligopsony power that clubs should then possess (see Madden (2009a,b)). Our "European", perfectly elastic supply assumption overrides this oligopsony power, and the issues raised in Madden (2009a,b) are not applicable here.

[7] The policy focus in existing literature has been on the effects of revenue sharing and salary caps on competitive balance in leagues of profit maximizers (or win maximizers), typically without a welfare base. Here the focus is on the club governance question, with answers rooted in standard welfare economics. The normative question addressed is therefore; "How *should* a soccer club be governed?".

of fans of each club *i,* who feel an (exogenously given) affinity to club *i* and are assumed to be the only potential spectators for *i's* home match – fans do not travel to away matches. Fans of *i* are heterogeneous in their willingness to pay for tickets, denoted $v(Q_i, Q_j) - x$ where the heterogeneity parameter is $x \geq 0$ and $v(Q_i, Q_j)$ is the maximum valuation[8]. It is assumed that $x$ is uniformly distributed over $[0, c]$ with density $\mu_i$, and $c$ is sufficiently large that the total number of fans $\mu_i c$ exceeds stadium capacity; $\mu_i$ is a measure of the number of fans of club *i*, its "fanbase". It is also assumed that $v(Q_i, Q_j)$ is $C^2$ and strictly increasing in both arguments, reflecting the desire of fans to see better quality matches. Since $v(Q_i, Q_j)^2$ appears in the objective function of many of the subsequent optimization problems we assume that it (and hence $v(Q_i, Q_j)$ itself) is strictly concave and satisfies the Inada conditions. If $v(Q_i, Q_j)$ is symmetric, fans are non-partisan and would divide a given amount of talent equally between the 2 teams for their optimal match. In our context, an asymmetry leading to more talent going to the home team for a fan's optimal match is appropriate, to reflect the previously mentioned "affinity". This fan bias is captured by the assumption that $v(Q_i, Q_j) > v(Q_j, Q_i)$ if $Q_i > Q_j$; in the extreme limiting case of completely home partisan fans $v(Q_i, Q_j)$ depends only on $Q_i$[9]. A useful example will be the Cobb-Douglas case, $v(Q_i, Q_j) = Q_i^{\alpha} Q_j^{\beta}$ with $\alpha, \beta > 0$ and $\alpha + \beta < 1/2$; here $f = \alpha/(\alpha + \beta)$ provides a natural measure of fan bias, as the fraction of a given amount of talent that a fan would allocate to their own team for their optimal match, ranging from non-partisan fans ($f = 1/2$), as assumed by Falconieri et al. (2004) for their TV audience, to the completely home partisan limit ($f = 1$). Also $e = \alpha + \beta$ measures a fan's elasticity of willingness to pay for a match ticket with respect to (linear) increases in team qualities, quality elasticity for short.

A fan with heterogeneity parameter $x$ will demand a ticket if $x \leq v(Q_i, Q_j) - p_i$ so that $i$'s (linear in price) ticket demand (=match attendance with large capacities) is $D_i(Q_i, Q_j, p_i) = \mu_i[v(Q_i, Q_j) - p_i]$ giving revenues $p_i D_i(Q_i, Q_j, p_i)$, and profits $\Pi_i(Q_i, Q_j, p_i) = p_i D_i(Q_i, Q_j, p_i) - Q_i$.

Once talent has been hired and tickets sold, matches are played and a winner emerges. Ex ante, before the play of matches, the probability that $i$ is the winner is some function $W(Q_i, Q_j)$, increasing in $Q_i$ and decreasing in $Q_j$; this could be one of the popular contest success functions (see Skaperdas (1996)), although the exact specification is irrelevant for our purposes.

[8] Implicitly we are assuming that the full fan utility function is quasi-linear, defined over a numeraire (endowment $y$ and large) and the match ticket. Full utility is then $y$ without the ticket and $y - p_i + v(Q_i, Q_j) - x$ with the ticket.

[9] One could strengthen the fan "affinity" by assuming that $v(Q_i, Q_j)$ is strictly increasing only on some large enough cone in the $(Q_i, Q_j)$ plane, but this adds nothing to the analysis and is not pursued.

The clubs make independent decisions about hiring of talent ($Q_i$) and ticket prices ($p_i$) to fulfil their objectives, leading to the following decision problems (in the absence of capacity constraints).

**PROFIT MAXIMIZATION**

The club decision problem is;

$$\max_{p_i, Q_i} \Pi_i(Q_i, Q_j, p_i) \qquad (\Pi\text{MAX})$$

This is the most common assumption in the existing literature.

**WIN MAXIMIZATION**

The objective function is now $W(Q_i, Q_j)$ - the club wishes to produce a team of the highest quality relative to the rival, or to maximize its probability of winning or its win percentage[10]. Of course there has to be a budget constraint on the achievement of this objective which we take to be $\Pi_i(Q_i, Q_j, p_i) \geq 0$, so the decision problem is;

$$\max_{p_i, Q_i} W(Q_i, Q_j) \text{ subject to } \Pi_i(Q_i, Q_j, p_i) \geq 0 \qquad (\text{WMAX})$$

Notice that the objective can be replaced merely by $Q_i$, since $W(Q_i, Q_j)$ is increasing in $Q_i$.

And the novel objective of this paper is:

**FAN WELFARE MAXIMIZATION**

Again there is a budget constraint taken to be $\Pi_i(Q_i, Q_j, p_i) \geq 0$. Subject to this constraint, the club aims to maximize the aggregate utility of their fans, or;

$$F_i(Q_i, Q_j, p_i) = \int_0^{v(Q_i Q_j) - p_i} \mu_i [v(Q_i, Q_j) - p_i - x] dx$$

$$= \mu_i [v(Q_i, Q_j) - p_i]^2 / 2$$

Thus the decision problem is:

$$\max_{p_i, Q_i} F_i(Q_i, Q_j, p_i) \text{ subject to } \Pi_i(Q_i, Q_j, p_i) \geq 0 \qquad (\text{FMAX})$$

Notice that the objective function is a monotone transformation of $v(Q_i, Q_j) - p_i$, which is what each fan of club $i$ would choose to maximize if they were in control of club choices, restricted by non-negative profits. If the club governance structure allowed fans to vote for a representative to influence decisions about $p_i$ and $Q_i$ (subject to non-negative profits), then fan welfare maximization would be an unbeatable platform in the election of this representative.

Notice also that the objective is a monotone transformation of attendance $\mu_i[v(Q_i, Q_j) - p_i]$ – the area of the fan surplus triangle is proportional to its base. Thus, in the absence of capacity constraints, fan welfare maximization is equivalent to maximizing match attendance[11, 12].

[10] Win maximization is the established terminology; see Fort and Quirk (2004), Kesenne (2007).

[11] The modelling of all three objectives is of course idealised. Just as profit maximization (for instance) abstracts from agency and other difficulties that a football company may face in delivering on the objective, so too for the new fan welfare hypothesis, which assumes that the club solves all political economy and other problems associated with implementing its objective. Dietl and Franck (2007) and

# 3. ANALYSIS OF CLUB DECISIONS

In the continued absence of capacity constraints, we consider the price ( $p_i$ ) and quality ( $Q_i$ ) best responses of a club with fanbase $\mu_i$ to the quality ($Q_j$) chosen by the other team in the league, and how these (and the resulting match attendance) vary with the club's objective (profit ( $\Pi$ ), win (W) or fan welfare (F) maximization), and with its fanbase. The other team's price ( $p_j$ ) does not affect any of the answers ( $p_j$ has no affect on *i*'s payoff or constraints – each club sells tickets only to its own fans). Best responses are denoted $p_{iX}(Q_j)$ , $Q_{iX}(Q_j)$ , and the resulting attendances are $A_{iX}(Q_j)$ , $X = \Pi, W, F$ .

We look first at the nature of the non-negative profit constraint, shown in Figure 3.1.

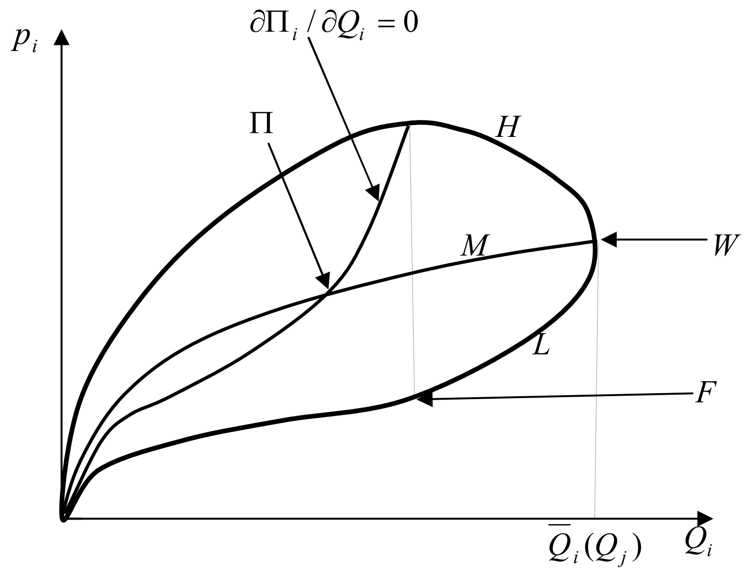


**Figure 3.1; The bubble-shaped non-negative profit region**

The zero profit contour is $p_i^2 - p_i v(Q_i, Q_j) + Q_i / \mu_i = 0$ , with roots of the quadratic;

$$p_{iL}(Q_i, Q_j) = \tfrac{1}{2} v(Q_i, Q_j) - \tfrac{1}{2}\sqrt{v(Q_i, Q_j)^2 - 4Q_i / \mu_i}$$

$$p_H(Q_i, Q_j) = \tfrac{1}{2} v(Q_i, Q_j) + \tfrac{1}{2}\sqrt{v(Q_i, Q_j)^2 - 4Q_i / \mu_i}$$

The roots are real if $Q_i \in [0, \overline{Q}_i(Q_j)]$, where $\overline{Q}_i(Q_j)$ is the unique positive solution in $Q_i$ (given the strict concavity and Inada properties of *v*) to $v(Q_i, Q_j)^2 = 4Q_i / \mu_i$ . We refer to $p_{iL}(Q_i, Q_j)$ as *the low break-even price*, and $p_{iH}(Q_i, Q_j)$ as *the high break-*

Franck (2009) provide interesting accounts of governance problems faced by members clubs in German soccer.

[12] In an early and now well-known paper in the literature, Sloane (1971) in fact suggests attendance as an argument in a club's multi-objective function, although neither attendance nor the equivalent fan welfare seems to have received a formal analysis in the interim.

*even price*, with graphs shown by L,H respectively in Figure 3.1. Notice that the roots sum to $v(Q_i,Q_j)$, and that $p_{iH}(Q_i,Q_j)$ is strictly concave under our assumptions. Between the L,H branches, labelled as M, we have *the monopoly price* $p_M(Q_i,Q_j)=v(Q_i,Q_j)/2$ which maximizes gate revenue (given $Q_i,Q_j$).
The bubble-shaped region in Figure 3.1, bounded above by H and below by L, depicts the non-negative profit constraint. The solution to WMAX is now obvious, at W in Figure 3.1. Hence;

Lemma 3.1 The best price and quality responses of a win maximizing club *i* are:
(a) $Q_{iW}(Q_j)=\overline{Q}(Q_j)$ where $\overline{Q}(Q_j)$ is the unique positive solution in $Q_i$ to $v(Q_i,Q_j)^2=4Q_i/\mu_i$;
(b) $p_{iW}(Q_j)=p_M(\overline{Q}(Q_j),Q_j)$.

For $\Pi$MAX the first order condition with respect to price implies monopoly prices, and that with respect to quality gives[13] $\partial\Pi_i/\partial Q_i=\mu_i p_i v_i(Q_i,Q_j)-1=0$, both upward sloping loci in Figure 3.1 with unique intersection at $\Pi$. That $\Pi$ is indeed the unique global profit maximum follows since substitution of monopoly prices into the objective reduces it to $\frac{1}{4}\mu_i v(Q_i,Q_j)^2-Q_i$, which is strictly concave[14] with a unique global maximum where marginal revenue ($MR_i(Q_i,Q_j)=\frac{1}{2}\mu_i v(Q_i,Q_j)v_i(Q_i,Q_j)$) = marginal cost (= 1);

Lemma 3.2 The best price and quality responses of a profit-maximizing club *i* are:
(a) $Q_{i\Pi}(Q_j)$ defined by $MR_i(Q_i,Q_j)=1$;
(b) $p_{i\Pi}(Q_j)=p_M(Q_{i\Pi}(Q_j),Q_j)$.

For FMAX and any $Q_i\in[0,\overline{Q}(Q_j)]$ the optimal price is clearly $p_i=p_{iL}(Q_i,Q_j)$. As low and high break-even prices sum to $v(Q_i,Q_j)$, the optimal quality solves:

$$\max_{Q_i} p_{iH}(Q_i,Q_j) \text{ subject to } Q_i\in[0,\overline{Q}(Q_j)] \qquad (3.1)$$

The solution in $Q_i$ is defined by the intersection in Figure 3.1 of H and $\partial\Pi_i/\partial Q_i=0$, and with the corresponding low break-even price, F is the solution, characterised precisely as follows (see appendix for proof);

Lemma 3.3 The best price and quality responses of a fan welfare maximizing club *i* are:
(a) $Q_{iF}(Q_j)$ is the solution in $Q_i$ to $\mu_i v(Q_i,Q_j)v_i(Q_i,Q_j)-\mu_i Q_i v_i(Q_i,Q_j)^2=1$;
(b) $p_{iF}(Q_j)=p_{iL}(Q_{iF}(Q_j),Q_j)$.

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[13] $v_i(Q_i,Q_j)$ denotes $\partial v/\partial Q_i(Q_i,Q_j)$ and $v_{ii}(Q_i,Q_j)$, $v_{ij}(Q_i,Q_j)$ will be used later to denote second partial derivatives.

[14] As the bubble-shaped non-negative profit region in Figure 3.1 shows, $\Pi_i(Q_i,Q_j,p_i)$ is neither globally concave nor globally quasi-concave as a function of $(Q_i,p_i)$. The 2 stage argument shows however that the stationary point is a global maximum. Such 2 stage arguments are used for this purpose throughout the paper

Thus, facing the same choice by the rival club, it follows from Figure 3.1 that win maximizers produce (unsurprisingly) the best quality team and the highest ticket prices, and profit maximization leads to the lowest quality team. Since fan welfare maximization is equivalent to attendance maximization, they will draw the biggest crowds, and since attendance contours have twice the slope of M, profit maximization also leads to the lowest attendance. All this is summarised in;

Theorem 3.1 Best responses of club $i$ (with fanbase $\mu_i$) vary with the club's objective, as follows:
(a) $Q_{iW}(Q_j) > Q_{iF}(Q_j) > Q_{i\Pi}(Q_j)$;
(b) $p_{iW}(Q_j) > p_{i\Pi}(Q_j)$ and $p_{iW}(Q_j) > p_{iF}(Q_j)$ [15];
(c) $A_{iF}(Q_j) > A_{iW}(Q_j) > A_{i\Pi}(Q_j)$;

There is considerable empirical evidence consistent with inelastic pricing of tickets for sports matches – see Fort (2004) for an account of this literature. Recalling that price elasticity of ticket demand is unity along M in Figure 3.1, higher than one (elastic) above M and lower than one (inelastic) below M, it follows that win and profit maximization lead to unit elasticity whilst fan welfare maximization produces inelastic ticket pricing:

Corollary to Theorem 3.1 Best response ticket prices are at a unit elastic point on the ticket demand curve for a win or profit maximizer, but in the inelastic part of the ticket demand curve for a fan welfare maximizer.

Thus in the context of our model fan welfare maximization is the only objective consistent with the empirical evidence[16].

Changes in the fanbase also affect behaviour, as follows (see appendix for proof);

Theorem 3.2 Best responses of club $i$ (with fanbase $\mu_i$) vary with the club's fanbase so that $Q_{iX}(Q_j)$, $p_{iX}(Q_j)$ and $A_{iX}(Q_j)$ increase as $\mu_i$ increases, $X = \Pi, W, F$, with the exception that the price of a fan-welfare maximizer may not increase as $\mu_i$ increases.

Thus bigger clubs (in the sense of a larger fanbase, ceteris paribus) will tend to have better quality teams, larger attendances and (with the possible exception of fan

[15] The missing comparison in Theorem 3.1(b) is between $p_{i\Pi}(Q_j)$ and $p_{iF}(Q_j)$, and indeed this is generally ambiguous – it may be that $Q_{i\Pi}(Q_j)$ is so far below $Q_{iF}(Q_j)$ that the profit maximizer's price is lower than the fan welfare maximizer's price, despite their monopoly (rather than low break-even) pricing. The following can be shown (details omitted): (i) with a Cobb-Douglas maximum valuation, it is always the case that $p_{i\Pi}(Q_j) > p_{iF}(Q_j)$, but (ii) if $v(Q_i, Q_j)^2$ is separable and dependence on $Q_i$ is piecewise linear and concave, the ranking is reversed eventually as this dependence approaches linearity; one can "smooth" this to fit our assumptions.

[16] Fort (2004) argues that profit maximization can be consistent with such evidence (and discusses related earlier arguments) provided one leaves our gate-revenue (only) scenario.

welfare maximizers) higher ticket prices[17]. For fan welfare maximizers increases in the fanbase allow the club to increase team quality and attendance but with lower ticket prices, which is sometimes the best response. For each objective, increases in fanbase lead to increases in team quality and so win percentage, consistent with the empirical findings for English soccer of Buraimo et al. (2007).

**Remark 1** The results of this section generalise (details omitted) to the case where the budget constraint facing win and fan welfare maximizers is $\Pi_i(Q_i, Q_j, p_i) + B \geq 0$, where $B$ is positive (e.g. broadcasting income, donation from a wealthy fan) or negative (e.g. some kind of fixed cost). Theorems 3.1 and 3.2 continue to hold provided $|B|$ is not too large; for instance, increasing $B$ then leads to higher quality and price at the solution to WMAX and higher quality (with ambiguous price effects) for FMAX.

## 4. LEAGUE EQUILIBRIA AND AGGREGATE SURPLUS

The focus now is on the behaviour and performance of three entire leagues, namely the F-league (with two fan welfare maximizing clubs), the W-league (two win maximizers) and the $\Pi$-league (two profit maximizers). The vector of fanbases of the 2 clubs $(\mu_1, \mu_2)$ and all other league characteristics are held constant, so the nature of club governance is the only difference between the leagues. Again stadium capacity constraints are ignored. We adopt the usual sum of consumer (fan) and producer surplus as the measure of aggregate surplus created by the league. For each club $i = 1,2$ (with $j \neq i$) define the sum of fan and producer surplus for that club to be:

$$S_i(Q_i, Q_j, p_i) = \int_0^{v(Q_i,Q_j)-p_i} \mu_i[v(Q_i, Q_j) - p_i - x]dx + p_i\mu_i[v(Q_i, Q_j) - p_i] - Q_i \qquad (4.1)$$

Aggregate surplus is then $S_1(Q_1, Q_2, p_1) + S_2(Q_2, Q_1, p_2)$[18]. $S_{1X}, S_{2X}, S_X$ will denote the surplus values at the $X$-league equilibrium, $X = \Pi, W, F$.

### 4.1 A benchmark; completely home partisan fans

A useful benchmark is provided by the limiting special case where $v(Q_i, Q_j)$ depends only on $Q_i$; fans are then completely home partisan, with their optimal division of a given amount of talent providing no talent to the away team. There is then no

[17] In a somewhat different model Kesenne (2006, 2007) also reaches the price and quality conclusions of Theorems 3.1 and 3.2 for the profit and win maximizers.

[18] Notice that our assumption of a perfectly elastic supply of playing talent means that players gain no extra surplus from playing in our league, and so do not enter the social welfare evaluation. Given the supply assumption, this seems appropriate, but differs from the social welfare specifications analysed in Falconieri et al. (2004) who also have the perfectly elastic supply assumption. Also missing are preferences of wealthy individuals who own clubs which (one suspects) may differ from those of fans (see Franck (2009) on this point) – since such individuals are negligible in number compared to the mass of fans they can and should be ignored in the aggregate measure. Finally, with quasi-linear utility for fans (footnote 5), maximization of this aggregate surplus equates in the usual way to Pareto efficiency, legitimising the use of $S_1(Q_1, Q_2, p_1) + S_2(Q_2, Q_1, p_2)$ as the appropriate welfare criterion.

strategic interaction at all between clubs in a league, and the league equilibrium is just the union of the isolated decisions of the two clubs, as studied in Section 3.

From Theorem 3.2 it follows that within each league the club with the larger fanbase will have the better quality team and so be more likely to win the league; its ticket prices (except possibly in the F-league) and attendances will also be larger than those of the smaller club. The between league comparisons of Theorem 3.1 show that the vector of team qualities, and certainly the aggregate quality, is highest in the W-league and lowest in the $\Pi$-league. Also ticket prices are highest in the W-league. Attendances are largest in the F-league, and smallest in the $\Pi$-league.

In the F-league and in the W-league, all producer surpluses are zero. Moreover in the F-league each club chooses quality to maximize the surplus accruing to its fans, given the low-break-even pricing on L in Figure 3.1, which includes point W on its boundary; thus the resulting fan surplus must exceed that at W. It follows that aggregate surplus is unambiguously higher in the F-league than in the W-league. By similar reasoning $S_{iF}$ exceeds the surplus that would accrue to club *i* if it chose $Q_{i\Pi}$ with the associated low-break-even price, which in turn exceeds $S_{i\Pi}$ (where $Q_{i\Pi}$ is chosen with the associated monopoly price), because of the extra surplus lost from the higher monopoly price and lower attendance. Hence;

<u>Theorem 4.1</u> With completely home partisan fans, $S_F > S_W$ and $S_F > S_\Pi$.

Thus the unambiguous socially dominant mode of club governance is fan welfare maximization[19].

**<u>Remark 2</u>** In this extreme case of completely home partisan fans, the decision problems can be re-interpreted as those facing monopoly providers of an excludable public good; the provision of the performing arts, zoos, art galleries, museums are common examples. The owners of a theatre (say) chooses expenditure on performers $Q_i$ (which equates to performance quality) and can control access to theatre-goers (fans) wishing to see the performance, charging entry price $p_i$. In the absence of capacity constraints in the theatre (and any other congestion effects), and ignoring theatre costs, the decision problems above become those of the optimal choice of quality and entry price for the monopoly provision, and do relate to models in the excludable public good literature. Indeed our fan welfare objective, although new in the context of a sports model, is common in the excludable public good literature – see, for instance, Fraser (1996, 2000), Traub and Missong (2005). The results of this section can be applied to this context, interpreting WMAX as the desire to produce the best quality performance and FMAX the objective of maximizing surplus accruing to the audience ("lobby welfare maximization" in Traub and Missong (2005)): WMAX will produce the best quality performance with the highest entry price, profit maximization the worst quality; audience size will be smallest under $\Pi$MAX and largest with FMAX; FMAX produces socially the best of the three outcomes.

[19] The ranking of $S_W$ and $S_\Pi$ is ambiguous in general. Later, Theorem 4.5 (and Figure 4.2) show that if $v(Q_i, Q_j) = Q_i^\alpha$ then $S_W > S_\Pi$ if and only if the quality elasticity ($e = \alpha$) exceeds 0.25.

## 4.2 The general case

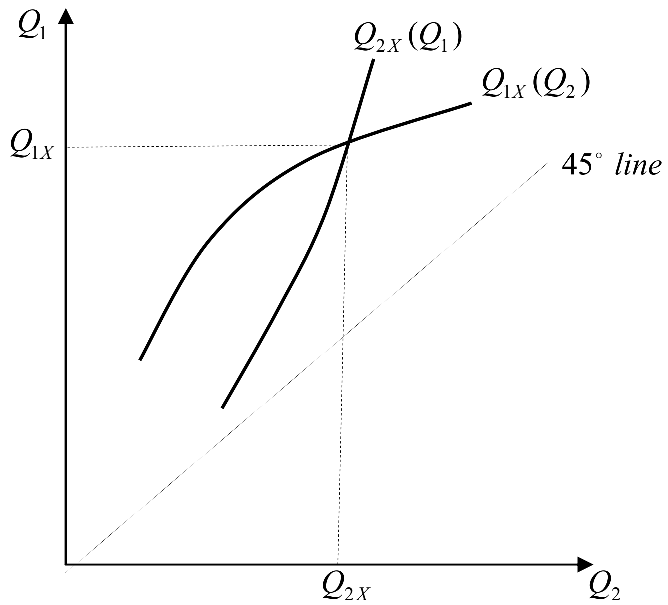
Leaving the extreme case of section 4.1, $v(Q_i, Q_j)$ is now increasing in both arguments so there is an externality effect between clubs. There is now strategic interaction between clubs in team qualities, and we study Nash equilibrium (NE) in the three leagues[20]. Consider first strategic complementarity/substitutability in the three games. From Lemma 3.1(a) it is easy to check that $\partial Q_{iW}(Q_j)/\partial Q_j > 0$, and the W-league game always exhibits global strategic complementarity. A similar conclusion emerges in the two other leagues (straightforwardly from part (a) of Lemmas 3.2 and 3.3) if we assume;

<u>Assumption 1 (A1)</u> $v_{ij}(Q_i, Q_j) \geq 0$.

This seems a natural assumption on fan preferences – increases in rival team quality increase the amount a fan is willing to pay for an increase in the quality of their team. It is certainly satisfied in the Cobb-Douglas case, which (as is seen later) also satisfies:

<u>Assumption 2 (A2)</u> For each league there is a unique, strictly positive Nash equilibrium which is stable in the usual best response dynamic.

Team qualities in league NE are denoted $Q_{iX}$ with prices $p_{iX}$ and attendances $A_{iX}$, $X = \Pi, W, F$. Figure 4.1 illustrates a typical pair of best responses and the resulting NE under (A1) and (A2).




**<u>Figure 4.1; Best response graphs under strategic complementarity</u>**

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[20] Formally we study three normal form games each with strategy sets $Q_i \geq 0$, and with payoffs : $\frac{1}{4}\mu_i v(Q_i, Q_j)^2 - Q_i$ in the $\Pi$-league; $\frac{1}{4}\mu_i p_{iH}(Q_i, Q_j)^2$ if $v(Q_i, Q_j)^2 \geq 4Q_i/\mu_i$ and 0 otherwise in the F-league; and in the W-league, $P(Q_i, Q_j)$ if $v(Q_i, Q_j)^2 \geq 4Q_i/\mu_i$, 0 otherwise. Equilibrium prices and attendances follow from the NE qualities in these games via formulae from section 3.

If $\mu_1 = \mu_2$, the NE in each league would be symmetric with $Q_{1X} = Q_{2X}, X = \Pi, W, F$, along the $45^\circ$ line in Figure 4.1. If $\mu_1$ (say) increases so $\mu_1 > \mu_2$, the best response graph of club 1 shifts upwards in Figure 4.1 from Theorem 3.2, so $Q_{1X} > Q_{2X}$ in the new NE, as shown. Thus the club with the bigger fanbase will end up with the better quality team in the $\Pi -$league, the W-league and the F-league. In the $\Pi -$league and the W-league, prices are $p_{iX} = \frac{1}{2} v(Q_{iX}, Q_{jX})$ so $p_{1X} - p_{2X} = \frac{1}{2}[v(Q_{1X}, Q_{2X}) - v(Q_{2X}, Q_{1X})]$, which is positive with the usual fan bias; in these two leagues attendances are $A_{iX} = \frac{1}{2}\mu_i v(Q_{iX}, Q_{jX})$, and again the usual fan bias, reinforced by $\mu_1 > \mu_2$, ensures that $A_{1X} > A_{2X}$. In the F-league, although price comparisons are generally ambiguous, club objectives are equivalent to attendance maximization and since the smaller club best responses are feasible for the larger club it must be that $A_{1F} > A_{2F}$. Hence:

Theorem 4.2 Assume (A1) and (A2). Then in the equilibrium of the *X*–league, for $X = \Pi, W, F$, $Q_{1X} - Q_{2X}$, $p_{1X} - p_{2X}$ and $A_{1X} - A_{2X}$ have the sign of $(\mu_1 - \mu_2)$, with the possible exception of $p_{1F} - p_{2F}$.

For the between league comparisons, suppose Figure 4.1 depicts the $\Pi -$league best responses and NE. From Theorem 3.1(a), best response graphs for both clubs in the F-league are higher and so the NE entails higher team quality for both the big and small club than in the $\Pi -$league. Similarly team qualities are uniformly higher in the W-league than in the F-league, and the individual club comparisons in Theorem 3.1(a) translate into corresponding league comparisons under (A1) and (A2). Corresponding to Theorem 3.1(b), in the $\Pi -$league and the W-league prices are $p_{iX} = \frac{1}{2} v(Q_{iX}, Q_{jX})$, and since $v(Q_{iX}, Q_{jX})$ is increasing in both arguments, $p_{iQ} > p_{i\Pi}, i = 1,2$, and prices are uniformly higher in the W-league than in the $\Pi$ -league. Since $p_{iF} = \frac{1}{2} v(Q_{iF}, Q_{jF}) - \frac{1}{2}\sqrt{v(Q_{iF}, Q_{jF})^2 - 4\frac{Q_{iF}}{\mu_i}}$, $p_{iW} > p_{iF}, i = 1,2$ since $v(Q_{iX}, Q_{jX})$ is increasing in both arguments, completing the parallel to Theorem 3.1(b). Turning to Theorem 3.1(c), $A_{iW} - A_{i\Pi} = \frac{1}{2}\mu_i[v(Q_{iW}, Q_{jW}) - v(Q_{i\Pi}, Q_{j\Pi})] > 0$ and $A_{iF} - A_{i\Pi} = \frac{1}{2}\mu_i[v(Q_{iF}, Q_{jF}) + \sqrt{v(Q_{iF}, Q_{jF})^2 - 4\frac{Q_{iF}}{\mu_i}} - v(Q_{i\Pi}, Q_{j\Pi})] > 0$, the signs following from the quality rankings since $v(Q_{iX}, Q_{jX})$ is increasing in both arguments. The remaining comparison between $A_{iW}$ and $A_{iF}$ is in fact ambiguous, for interesting reasons explored later in this section. However we have established:

Theorem 4.3 Assume (A1) and (A2). Comparing team quality, ticket price and attendance for club *i*=1,2 in the equilibrium of the $\Pi -$league, W-league and F-league gives:

(a) $Q_{iW} > Q_{iF} > Q_{i\Pi}$;

(b) $p_{iW} > p_{i\Pi}$ and $p_{iW} > p_{iF}$

(c) $A_{iW} > A_{i\Pi}$ and $A_{iF} > A_{i\Pi}$.

Thus the W-league produces the highest (vectors of) team qualities and ticket prices, and the $\Pi -$league produces the lowest team qualities and attendances.

Turning to the aggregate surplus generated in the league equilibria, the following chain of reasoning establishes that $S_F > S_\Pi$ continues to hold in the general setting of (A1) and (A2) as it did with completely home partisan fans (Theorem 4.1), although the other half of Theorem 4.1 ($S_F > S_W$) is now problematic.

(1) $S_{i\Pi} < S_i(Q_{i\Pi}, Q_{j\Pi}, p_{iL}(Q_{i\Pi}, Q_{j\Pi}))$. The only change from the left-hand side to the right is a lowering of ticket price from $p_M(Q_{i\Pi}, Q_{j\Pi})$ to $p_{iL}(Q_{i\Pi}, Q_{j\Pi})$ which increases $S_i$ ($\frac{\partial S_i}{\partial p_i} < 0$).
(2) $S_i(Q_{i\Pi}, Q_{j\Pi}, p_{iL}(Q_{i\Pi}, Q_{j\Pi})) < S_i(Q_{iF}(Q_{j\Pi}), Q_{j\Pi}, p_{iL}(Q_{iF}(Q_{j\Pi}), Q_{j\Pi}))$. The change from left to right is that $Q_{i\Pi}$ is replaced by $Q_{iF}(Q_{j\Pi})$. Since the fan-welfare maximizer wishes equivalently to maximize $S_i$ with low break-even prices, the right-hand side is the maximum value of $S_i$ with low break-even prices and $Q_{j\Pi}$, and the left-hand side is another attainable value of $S_i$ with low break-even prices and $Q_{j\Pi}$. Hence the inequality.
(3) $S_i(Q_{iF}(Q_{j\Pi}), Q_{j\Pi}, p_{iL}(Q_{iF}(Q_{j\Pi}), Q_{j\Pi})) < S_i(Q_{iF}(Q_{j\Pi}), Q_{jF}, p_{iL}(Q_{iF}(Q_{j\Pi}), Q_{j\Pi}))$. The only change is in the middle argument which goes up (Theorem 4.3(a)) from $Q_{j\Pi}$ on the left to $Q_{jF}$ on the right. The inequality follows since $\frac{\partial S_i}{\partial Q_j} > 0$.
(4) $S_i(Q_{iF}(Q_{j\Pi}), Q_{jF}, p_{iL}(Q_{iF}(Q_{j\Pi}), Q_{j\Pi})) < S_i(Q_{iF}(Q_{j\Pi}), Q_{jF}, p_{iL}(Q_{iF}(Q_{j\Pi}), Q_{jF}))$. The change is in the last argument of $p_{iL}$ which now also goes up from $Q_{j\Pi}$ on the left to $Q_{jF}$ on the right. The inequality follows since $\frac{\partial p_{iL}}{\partial Q_j} < 0$ and $\frac{\partial S_i}{\partial p_i} < 0$.
(5) $S_i(Q_{iF}(Q_{j\Pi}), Q_{jF}, p_{iL}(Q_{iF}(Q_{j\Pi}), Q_{jF})) < S_{iF}$. Similarly to (2), the right-hand side is the maximum value of $S_i$ with low break-even prices and $Q_{jF}$, and the left-hand side is another attainable value of $S_i$ with low break-even prices and $Q_{jF}$, hence the inequality and the following:

<u>Theorem 4.4</u> Assume (A1) and (A2). Then $S_F > S_\Pi$.

We have already remarked that two of the results for the completely home partisan case ($A_{iF} > A_{iW}$ and $S_F > S_W$) become ambiguous in the general case. Note first that these two statements are equivalent – since producer surplus is zero in both the F-league and the W-league and since attendance is a monotone transformation of fan surplus, $A_{iF} > A_{iW}$ if and only if $S_{iF} > S_{iW}$ (hence $S_F > S_W$). The reason for their potential reversal in the general case is as follows. With completely home partisan fans, and since win and fan welfare maximizers face the same budget constraint, it is immediate that $A_{iF} > A_{iW}$, since maximizing attendance is equivalently the objective of the fan welfare maximizer. But with the strategic interaction created by less than completely home partisan fans, a fan welfare maximizer fails to take account of the affect increases in its own team quality has on the rival team's home attendance, and the outcome will not produce maximal attendances given the budget constraints. Intuitively, if fans' willingness to pay for quality is large enough, it could then be that W-league attendances (and so social welfare) are higher than those in the F-league.

We explore further using the Cobb-Douglas maximum valuation $v(Q_i, Q_j) = Q_i^{\alpha} Q_j^{\beta}$ (with quality elasticity $e = \alpha + \beta$ and fan bias $f = \alpha / e$), and with $\beta > 0$ so fans are not completely home partisan. Using the results of section 3 one can compute best responses and NE (see appendix for proofs)[21, 22];

<u>Lemma 4.1</u> With the Cobb-Douglas maximum valuation, best responses are:

(a) $Q_{i\Pi}(Q_j) = (\tfrac{1}{2}\alpha\mu_i Q_j^{2\beta})^{\frac{1}{1-2\alpha}}$, $p_{i\Pi}(Q_j) = \tfrac{1}{2}\, Q_{i\Pi}(Q_j)^{\alpha} Q_j^{\beta}$

(b) $Q_{iW}(Q_j) = (\tfrac{1}{4}\mu_i Q_j^{2\beta})^{\frac{1}{1-2\alpha}}$, $p_{iW}(Q_j) = \tfrac{1}{2}\, Q_{iW}(Q_j)^{\alpha} Q_j^{\beta}$

(c) $Q_{iF}(Q_j) = [\alpha(1-\alpha)\mu_i Q_j^{2\beta}]^{\frac{1}{1-2\alpha}}$, $p_{iF}(Q_j) = \alpha\, Q_{iF}(Q_j)^{\alpha} Q_j^{\beta}$

<u>Lemma 4.2</u> With the Cobb-Douglas maximum valuation, league Nash equilibrium team qualities are, for $i = 1,2, i \neq j$:

(a) $Q_{i\Pi} = (\tfrac{1}{2}\alpha\mu_i^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_j^{\frac{2\beta}{1-2\alpha+2\beta}})^{\frac{1}{1-2\alpha-2\beta}}$;

(b) $Q_{iW} = (\tfrac{1}{4}\mu_i^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_j^{\frac{2\beta}{1-2\alpha+2\beta}})^{\frac{1}{1-2\alpha-2\beta}}$;

(c) $Q_{iF} = [\alpha(1-\alpha)\mu_i^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_j^{\frac{2\beta}{1-2\alpha+2\beta}}]^{\frac{1}{1-2\alpha-2\beta}}$.

One can then also compute and compare $S_F, S_W$ and $S_\Pi$ which leads to the following additions to Theorem 4.4:

<u>Theorem 4.5</u> With the Cobb-Douglas maximum valuation, $S_W > S_F$ if and only if $1 > 2(1-ef)^{1-e}(ef)^{e}$ and $S_W > S_\Pi$ if and only if $1 > (3-4ef)^{1-2e}(2ef)^{2e}$.

Figure 4.2 illustrates the findings. Here *f*=1 is the completely home partisan case, and fan welfare maximization is always the socially dominant form of club governance (Theorem 4.1). But for lower values of *f*, as the previous intuition suggested, the F-league is overtaken by the W-league in terms of aggregate surplus if the fans' quality elasticity *e* is large enough[23]. The diagram also shows that the $\Pi$-league may socially dominate the W-league when the quality elasticity is low, and confirms the general result that the $\Pi$-league is always socially inferior to the F-league.

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[21] It follows from Lemma 4.1 that best responses are increasing concave functions going through the origin. So the Cobb-Douglas case does satisfy (A1) and (A2), and is a special case of the general model.

[22] It follows from Lemmas 4.1 and 4.2 that a special feature of the Cobb-Douglas case NE is that $Q_{1X} / Q_{2X}$ has the same value for $X = \Pi, W, F$, as do $\frac{p_{1X}}{p_{2X}}$ and $\frac{A_{1X}}{A_{2X}}$; the last of these is used in Section 5.

[23] To enhance understanding, the diagram has been extended to include the surely unrealistic cases of away partisan fans where *f*<1/2. When *f*=0 and fans are completely away partisan (caring only about the qualityof the visiting team), fan welfare maximization and profit maximization produce zero quality and attendance (since attendance and revenue depend only on the other team quality) and aggregate surplus, but win maximization still produces positive team qualities and attendances, and positive aggregate surplus. Hence the W-league is now socially dominant for all *e*.

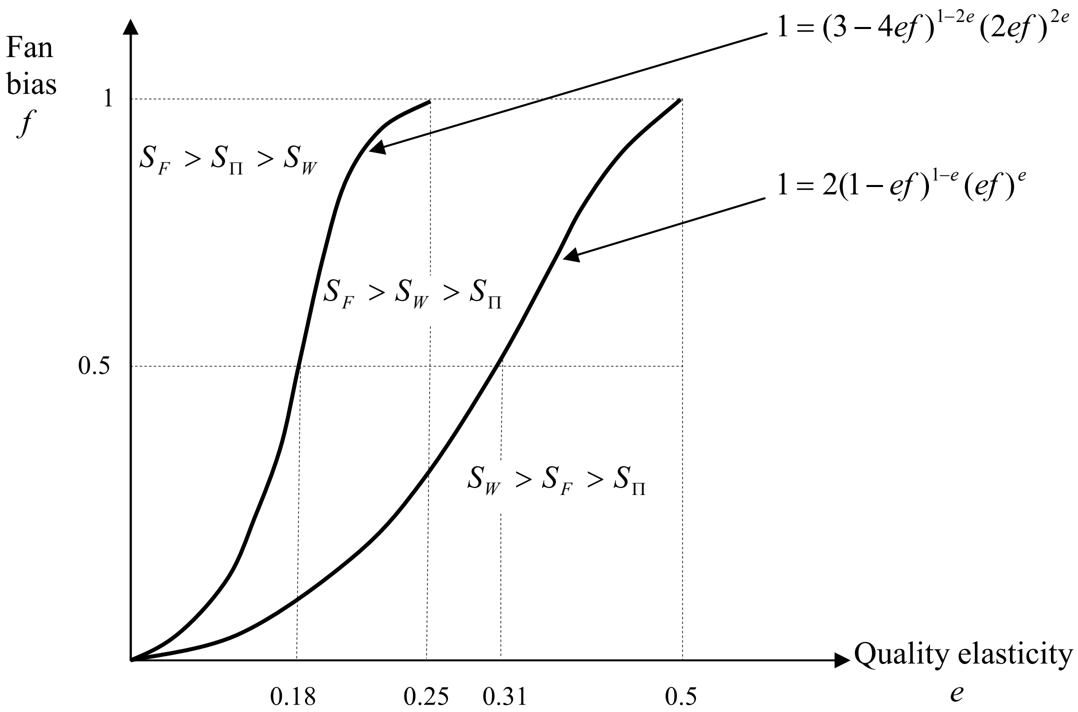


**Figure 4.2; League aggregate surplus rankings**

# 5. CAPACITY CONSTRAINTS

In this section club $i$ has a stadium capacity constraint $k_i$, which is an upper bound on match attendance. Previous sections have assumed that this constraint was never binding, an assumption that is relaxed now. We revisit various aspects of the earlier analysis of club decisions, league equilibria and social welfare. However a new issue now enters the agenda, as follows. Capacity crowds are indeed quite common at many clubs across European soccer, but often seem to be accompanied by an active black market for match tickets. Tickets are distributed initially by the club at the official ticket price, the stock sells out and tickets are then seen changing hands at above the official price on a black market[24]. The additional question is: can the behaviour of profit, win or fan welfare maximizing clubs explain the emergence of such active black markets?

## 5.1 Analysis of club decisions

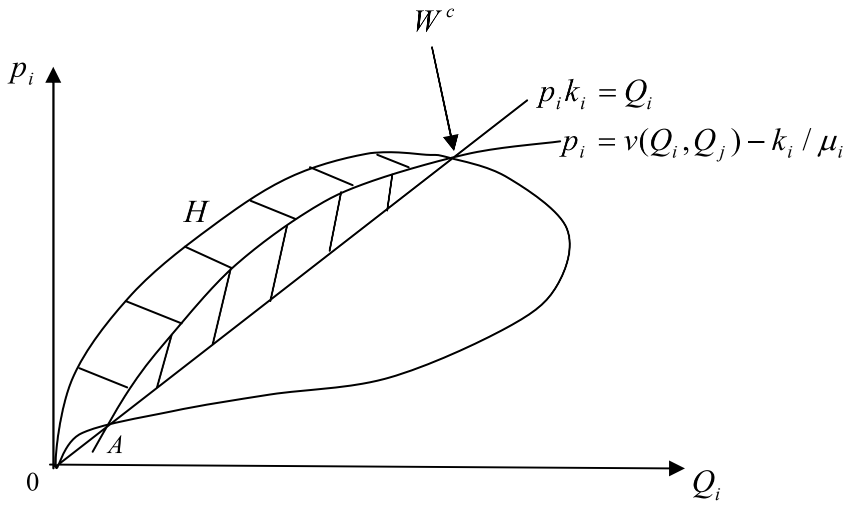
Consider first the effect a capacity constraint has on individual club decisions. Suppose that $k_i < A_{iW}(Q_j)$, so that the unconstrained best response of a win maximizer is infeasible. Figure 5.1 illustrates the shaded truncated feasible set.

[24] Whilst information on the number of sellouts over a season is easily available for major North American sports leagues, the same is not true for European soccer. And since profitable ticket resale is typically illegal, black market activity is subject to the same lack of hard information. Some informal pointers towards the prevalence of both are: (i) extrapolating from the proximity of attendances to stadium capacity, Appendix II provides some rough estimates of the proportion of sellout matches in the English Premier League 2007-8 season, indicating an overall fraction probably safely between 10% and 45%; (ii) despite the illegality, visits to the streets around the stadiums of the biggest European soccer clubs on matchdays, and to certain web-sites, does indicate profitable black market activities.

$p_i = v(Q_i, Q_j) - k_i / \mu_i$ is the capacity attendance contour, where ticket demand exactly equals stadium capacity[25]. In the downward shaded region, profits are non-negative and ticket demand does not exceed capacity. Below the capacity attendance contour, attendance is constrained to $k_i$ and profit is $p_i k_i - Q_i$ (hence with linear profit contours). Thus in the upward shaded region, profits are non-negative but ticket demand does exceed capacity, and the $k_i$ available tickets would have to be rationed amongst fans (in some way that is irrelevant for now). The solution is clearly at $W^c$ shown, on the capacity ticket demand contour. So there is no rationing of ticket demand, and high break-even pricing in the elastic section of the ticket demand curve. Formally;

<u>Lemma 5.1</u> Capacity constrained best responses of a win maximizing club $i$ are characterised by $p_{iH}(Q_i, Q_j) = Q_i / k_i$ and $p_i = Q_i / k_i$.

Consider next the constrained profit maximizer, and suppose the configuration shown in Figure 5.1 precludes the profit maximizer's unconstrained optimum also. Profit contours below the capacity ticket demand contour are straight lines parallel to $OAW^c$, and the solution will occur again on the capacity ticket demand contour, at the tangency shown as $\Pi^c$ in Figure 5.2.




**<u>Figure 5.1; Capacity constrained non-negative profit region</u>**

Again there is no rationing of ticket demand, with a price between the monopoly and high break-even levels in the elastic section of the ticket demand curve. Formally:

<u>Lemma 5.2</u> Capacity constrained best responses of a profit-maximizing club $i$ are characterised by $k_i v_i(Q_i, Q_j) = 1$ and $p_i = v(Q_i, Q_j) - k_i / \mu_i$.

If there is no rationing of ticket demand, all fans whose ticket valuation exceeds the price set by the club (the official price) will receive tickets, and there is no possibility of buying tickets for profitable resale. Rationing of ticket demand is a necessary

[25] It is straightforward to check that the intersections of the capacity attendance contour with the zero profit bubble are collinear with the origin, as shown in Figure 5.1.

condition for the emergence of an active black market, which therefore cannot occur under the win or profit maximization objectives.

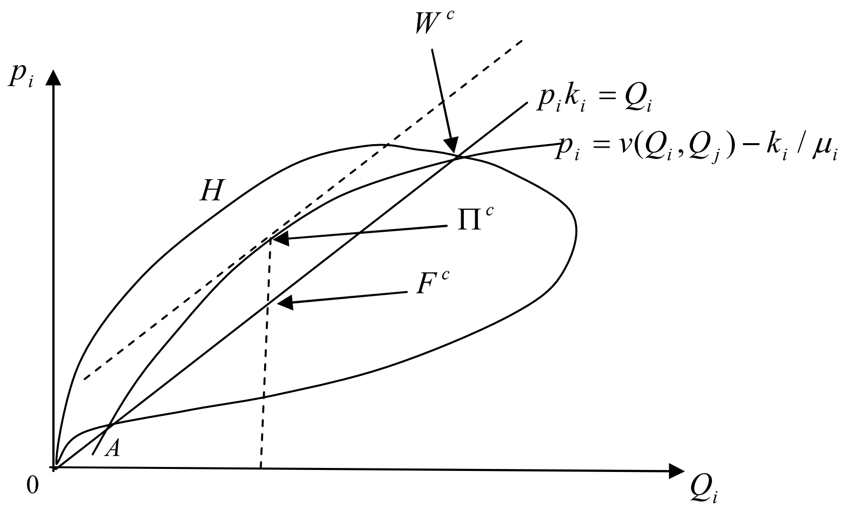



**Figure 5.2; Capacity constrained best responses**

In the fan welfare maximization case we need to be precise about the nature of ticket distribution if demand rationing occurs, and the obvious first assumption is:

Efficient Rationing The $k_i$ tickets (price $p_i < v(Q_i, Q_j) - k_i / \mu_i$ so there is excess demand) are allocated to fans with the greatest valuation, namely those with $x \in [0, k_i / \mu_i]$. Fans with $x \in [k_i / \mu_i, v(Q_i, Q_j) - p_i]$ would like a ticket but receive none.

When $p_i < v(Q_i, Q_j) - k_i / \mu_i$, the fan welfare maximizer's objective function changes to:

$$F_i = \int_0^{k/\mu_i} \mu_i [v(Q_i, Q_j) - p_i - x] dx = k_i [v(Q_i, Q_j) - p_i] - \tfrac{1}{2} k_i^2 / \mu_i \qquad (5.1)$$

However, this is still a monotone increasing transformation of $v(Q_i, Q_j) - p_i$, and the objective contours will be vertical displacements of the capacity attendance contour, leading to the solution shown at $F^c$, with prices now between the low break-even and monopoly levels in the inelastic section of the ticket demand curve. It is clear therefore that fan welfare and profit maximizers choose the same quality when similarly capacity constrained, but the fan welfare maximizer's solution involves a lower price and demand rationing[26]. The formal conditions are:

$$k_i v_i (Q_i, Q_j) = 1 \text{ and } p_i = Q_i / k_i \qquad (5.2)$$

[26] In terms of areas under demand curves to the left of capacity demand, for a given team quality the fan welfare maximizer's objective is the sum of the profit maximizer's profit (rectangle) plus the fan surplus (triangle) created by the profit maximizer. But this triangle has an area which is invariant to team quality (given the capacity constraint), so choosing quality to maximize profit is equivalent to choosing quality to maximize fan surplus.

Nevertheless active black markets are still precluded – under efficient rationing, as with no rationing above, the only fans prepared to pay more than the official ticket price do receive tickets and there is no basis for profitable resale. But it is not at all clear that efficient rationing is a realistic assumption – certainly one would not expect clubs to be able to acquire the information to allocate tickets in this way. If tickets are sold at a stadium ticket office, or on-line, one would expect that the allocation would be more random amongst applicants, perhaps opening up the possibility of a black market. An alternative distribution mechanism is:

Random Rationing with Possible Resale At stage 1 the $k_i$ tickets (sold at the official price $p_i < v(Q_i, Q_j) - k_i / \mu_i$ so there is excess demand) are allocated randomly to applicants. At stage 2 there is the possibility of resale in a possibly active black market where applicants may re-trade their initial allocation at a price $b_i$.

Here the initial allocation is random, and nothing is done to block ticket resale.
If $b_i > p_i$ is anticipated under this mechanism, the set of applicants at stage 1 would be large. To simplify notation (it makes no qualitative difference) we assume clubs allocate tickets at stage 1 only to applicants who are fans (precluding others, professional ticket touts maybe). But still the entire set of fans would apply at stage 1, with intentions of attending the match or making a black market profit. Of the fans with heterogeneity parameter $x(\in[0,c])$, $k_i/c$ would receive tickets in the random stage 1 allocation, and the remaining $\mu_i - k_i/c$ would be frustrated. At stage 2 fans without a ticket buy on the black market if $v(Q_i, Q_j) - b_i - x \geq 0$, or $x \in [0, v(Q_i, Q_j) - b_i]$ ; black market demand is $B_i^D = (\mu_i - \frac{k_i}{c})[v(Q_i, Q_j) - b_i]$. Fans with tickets sell if $v(Q_i, Q_j) - p_i - x \leq b_i - p_i$, or $x \in [0, v(Q_i, Q_j) - b_i]$ giving a black market supply $B_i^S = [c - (v(Q_i, Q_j) - b_i)]k_i / c$. The black market clearing price is $b_i = v(Q_i, Q_j) - k_i / \mu_i$, the capacity market-clearing price, and tickets end up with all fans with $x \in [0, k_i/\mu_i]$, as under efficient rationing, $k_i/c$ of them paying $p_i$ and $\mu_i - k_i/c$ paying $b_i$; a black market profit of $b_i - p_i$ accrues to $k_i/c$ of the fans with $x \in [k_i / \mu_i, c]$. Thus this mechanism produces exactly the same level of aggregate fan surplus as under efficient rationing, and the fan welfare maximizer continues to maximize (5.1)[27], leading again to the solution shown in Figure 5.2, characterised by (5.2)[28];

Lemma 5.3 Under efficient rationing or random rationing with possible resale, best responses of a fan welfare maximizing club *i* are characterised by (5.2).

The main findings of this section are summarised in:

[27] To spell this out, when $p_i < v(Q_i, Q_j) - k_i / \mu_i$ the fan welfare maximizer's objective becomes:

$$W_i = \int_0^{k/\mu_i} k_i / c[v(Q_i, Q_j) - p_i - x]dx + \int_0^{k/\mu_i} (\mu_i - k_i / c)[v(Q_i, Q_j) - p_i - x]dx + \int_{k_i/\mu_i}^{c} k_i / c[b_i - p_i]dx =$$

$$\int_0^c k_i / c[b_i - p_i]dx + \int_0^{k/\mu_i} \mu_i[v(Q_i, Q_j) - b_i - x]dx = k_i[v(Q_i, Q_j) - p_i] - \frac{1}{2} k_i^2 / \mu_i .$$

[28] Now in Figure 5.2 the price at $F^c$ is the official price and that at $\Pi^c$ the black market price.

Theorem 5.1 If stadium capacity constraints are binding, best responses are such that: (a) for a profit or win maximizing club, there will be no rationing of ticket demand and no possibility of an active black market for tickets; (b) for a fan welfare maximizing club, there will be demand rationing and an active black market for tickets for that club's home game if its ticket distribution mechanism is random rationing with possible resale; (c) ticket prices are in the elastic part of the ticket demand curve for a win or profit maximizer, but in the inelastic part for a fan welfare maximizer.

Thus only the fan welfare maximization objective is consistent with the observed active black markets for match tickets and their inelastic pricing, the latter reinforcing the earlier Corollary to Theorem 3.1.

## 5.2 League equilibria and aggregate surplus

We consider leagues of 2 clubs each with the same fanbase vector $(\mu_1, \mu_2)$ and the same vector of capacity constraints $(k_1, k_2)$; as before the only difference between leagues is in the nature of club governance. Since the W-league is somewhat intractable even in the Cobb-Douglas case, and anyway to focus on the only unambiguous welfare comparison in Section 3.2, we concentrate on the F-league and $\Pi-$league to see, in particular, if any ambiguity in their welfare ranking emerges in the presence of capacity constraints.

From Section 5.1 we know that if capacities are binding on both clubs in both equilibria then equilibrium qualities will be the same in both league equilibria under either of the earlier demand rationing scenarios, with lower prices in the F-league. Since ticket demand curves are the same for club $i$ in each equilibrium, aggregate surplus is the same in both equilibria, the F-league merely distributing more of this aggregate towards fan rather than producer surplus. Hence:

Theorem 5.2 If capacity constraints bind on both clubs in equilibria of the F-league and the $\Pi$-league, and if there is efficient rationing or random rationing with possible resale, then aggregate surplus is the same in both equilibria.

In all the results so far $\Pi$-league equilibria are always at least weakly socially dominated by the F-league equilibria. However there is a case where this reverses.

Random Rationing The ticket allocation mechanism is just stage 1 of the previous mechanism – now there is no possibility of ticket re-sale.

The objective function of a fan welfare maximizer now becomes:

$$F_i = \int_0^{v(Q_i,Q_j)-p_i} \frac{k_i}{v(Q_i,Q_j)-p_i}[v(Q_i,Q_j)-p_i-x]dx = k_i[v(Q_i,Q_j)-p_i]/2 \qquad (5.3)$$

This is now lower than the value in (5.1), because of the inefficiency and loss of consumer surplus from random rationing, not now rescued by the black market. However, (5.3) is still a monotone increasing transformation of $v(Q_i,Q_j)-p_i$, and constrained best responses are the same as under the other rationing regimes, as described in Lemma 5.3. And of course nothing changes for the profit maximizer. The consequence of these observations is:

Theorem 5.3 If capacity constraints bind on both clubs in equilibria of the F-league and the $\Pi$-league, and if there is random rationing, then aggregate surplus is lower in the F-league than in the $\Pi$-league.

Neither Theorem 5.2 nor Theorem 5.3 provides a complete statement of the type found in Section 3.2. A final and fuller result is available for the Cobb-Douglas case, when capacities of the 2 clubs are proportional to their equilibrium attendances in the absence of capacity constraints. This restriction implies (see footnote 21) that in each league either capacity constraints bind on both clubs or on neither, and avoids the again intractable case of capacity constraints binding on just one club in a league:

Theorem 5.4 In a $\Pi$-league and a F-league, assume capacities are proportional to capacity unconstrained league equilibrium attendances ($k_1 / k_2 = A_{1X} / A_{2X}, X = \Pi, F$), and assume the Cobb-Douglas maximum valuation. Then in each league there is a unique, strictly positive Nash equilibrium which is stable in the usual best response dynamic. Moreover:

(a) Assume either efficient rationing or random rationing with possible resale. If $A_{iF} > k_i > A_{i\Pi}, i = 1,2$, then aggregate surplus in the F-league equilibrium increases with $k_i$ and is strictly greater than in the $\Pi$-league equilibrium; if $A_{i\Pi} \geq k_i, i = 1,2$ then aggregate surplus in the F-league equilibrium is the same as in the $\Pi$-league equilibrium, and increases with $k_i$.

(b) Assume random rationing. If $A_{i\Pi} \geq k_i, i = 1,2$, then aggregate surplus in the F-league equilibrium is lower than in the $\Pi$-league equilibrium.

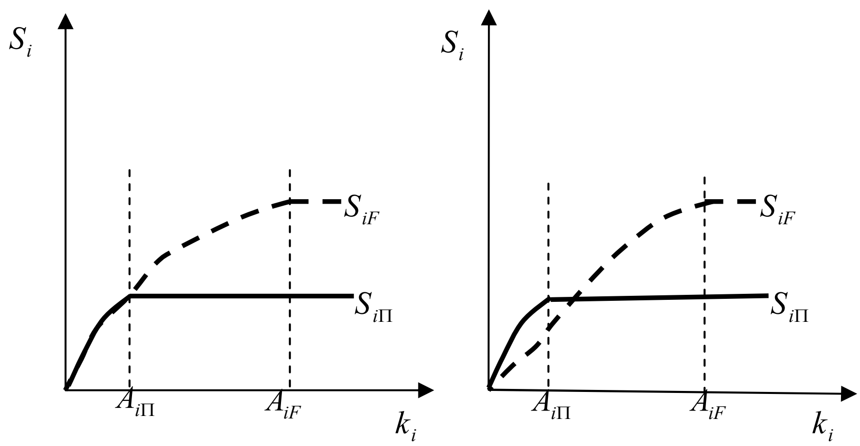



**Figure 5.3(a)** **Figure 5.3(b)**

**Figure 5.3; Capacity constrained league surpluses**

Figure 5.3 illustrates. Figure 5.3(a) shows the relation between aggregate surplus generated by club $i$ in the $\Pi$-league equilibrium ($S_{i\Pi}$) and in the F-league equilibrium ($S_{iF}$) implied by Theorem 5.4(a), and similarly for Theorem 5.4(b) in Figure 5.3(b).

The only possible welfare support for profit maximization is very weak, resting upon the failure of black markets to overcome inefficient (random) ticket distribution by fan welfare maximizers (Figure 5.3(b)). If fan welfare maximizers distribute tickets efficiently (directly or with the help of a black market) then profit maximization is always at least weakly dominated (Figure 5.3(a)). And of course if stadium capacity constraints are never binding the F-league always produces an outcome which strictly dominates the $\Pi$-league (Figure 5.3(a,b) when $k_i > A_{iF}$).

**Remark 3** The results of Section 5 relate to two aspects of the wider literatures on rationing and on black markets. First, the fact that black markets can rescue the inefficiency of "official" non-market-clearing prices is certainly well-known. For instance, Polterovich (1993) has provided an extensive general equilibrium study, with exogenous prices, of the properties of our black market mechanism and other mechanisms for dealing with such disequilibria. Secondly, finding explanations for why optimizing agents with market power over prices would make choices that lead to rationing on the other side of the market has proved elusive, particularly when agents have "standard" objectives. The best known such story[29] is the efficiency wage explanation for involuntary unemployment in the labour market context, where firms set high wages to increase worker productivity and so create rationing of labour supply (involuntary unemployment). Here the explanation is quite straightforward – firms (clubs) set low ticket prices and consumers (fans) are demand rationed because the clubs care about the aggregate welfare of their fans. The results of this section are therefore of interest per se, in bringing together an argument showing how the strategic interaction of optimising agents with market power on one side of a market can lead to equilibria with rationing on the other side of the market, and to active black markets in a context where such outcomes are seen in reality.

## 6. GERMAN BUNDESLIGA VERSUS ENGLISH PREMIER LEAGUE

In the light of the preceding theoretical analysis, it is of interest to try to compare the actual performance of European soccer leagues. Whilst it is true that a non-trivial proportion of matches in at least some of the major 5 leagues[30] are capacity constrained sellouts, it is also very probable that the majority of such matches are not so constrained[31]. So we look for predictions from Section 3 and 4 rather than Section 5. Moreover although it is not at all clear that any of these leagues fits the $\Pi$-league scenario, the following cases can be made for regarding the German Bundesliga (BL) as a F-league, and the English Premier League (PL) as a W-League.

[29] See also Kaas and Madden (2004), Madden and Silvestre (1991, 1992).
[30] The "big 5" in most economic and sporting senses are in England (Premier League), France (Ligue 1), Germany (Bundesliga), Italy (Serie A) and Spain (Primera Liga).
[31] As remarked earlier, there seems to be no definite data on this. Appendix II contains some rough estimates for the English Premier League.

In Germany, the continuing Verein structure clearly endows fans with some power over club governance, and, as convincingly argued by Franck (2009), precludes the entry of wealthy owners with a view to using their wealth to achieve "winning" for the club; possible examples of the latter are, in England, Mr. Abramovich at Chelsea, Mr. Al-Fayed at Fulham and Sheikh Mansour at Manchester City. Also, as a non-profit organisation, the Verein does not approximate $\Pi$MAX at all.

In England, the previous three anecdotal examples (and several others) come to mind as possible indicators of WMAX but, more scientifically, there is the evidence of Garcia-del-Barrio and Szymanski (2009) that indeed WMAX fits PL data better than $\Pi$MAX. Moreover, there is certainly no evidence of fan power in the governance of any PL club – the growing role for supporter's trusts in English soccer noted earlier is only impacting on governance via board representation at lower league levels (3$^{rd}$ tier and below). Thus there is a case for regarding the BL as a F-League and the PL as a W-League.

The following are some data[32] for the BL and the PL for 2007-8.

| | **Bundesliga** | **Premier League** |
|---|---|---|
| **Average ticket price (in Euros)** | **25.93** | **51.71** |
| **Average wage expenditure (in millions of Euros)** | **40.3** | **75.5** |
| **Average attendance** | **42,600** | **35,600** |

**Table 1; Bundesliga and Premier League data for season 2007-8**

Assuming the fanbases in the 2 leagues are roughly comparable[33], the rankings on ticket price and wage expenditure are consistent with the BL as a F-League and the PL as a W-League, with the PL significantly higher on both[34]. The opposite ranking on attendance is also consistent with this assignment[35], and suggests that the BL is doing a better job with respect to aggregate surplus[36].

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[32] Source; Deloitte Annual Review of Football Finance, 2009, p. 10-22. Average attendance; p. 14. Average wage expenditure; total league wage expenditure, p. 18, divided by the number of clubs. Average ticket price; total league matchday income, p. 13, divided by the product of the number of clubs (n, say) with n-1 (number of home games) and the average attendance.

[33] On the one hand Germany has a larger population and a larger number of registered players (Franck (2009)), but perhaps England has more "very big" clubs. There is no obvious scientific case either way.

[34] The PL has significantly higher broadcast income, whose effect should be to increase further prices and wages (see Remark 1 and Figure 4.1).

[35] The corresponding data for Spain's Primera Liga (SpPL) are 42.19 (ticket price), 45.0 (wages) and 24,500 (attendance). If, following Garcia-del-Barrio and Szymanski (2009), the SpPL is viewed as a W-League (rather than a $\Pi$-League) then the same consistencies and conclusions follow from the BL/SpPL comparison as did for BL/PL. However it is not so easy to discount an influence for FMAX in the SpPL, where member's clubs are still prevalent.

[36] The possibility emerges that the BL Verein might be a role model for soccer club governance. However, without strong international regulation, it seems more likely that the Verein will disappear in Germany, let alone spread elsewhere – see Franck (2009).

## 7. CONCLUSIONS

In a theoretical model of a professional sports league, the paper has introduced the club objective of fan welfare maximization, and investigated its consequences for club and league performance, comparing with the more commonly studied profit and win maximization objectives.

On the positive economics side, extensive comparisons have been made of how the club objective and the size of its fanbase affect team qualities, ticket prices and attendances, both with and without binding stadium capacity constraints. Big clubs (in terms of fanbase) tend to have better quality teams and higher match attendances, whilst profit maximizers produce the lowest quality teams and lowest attendances. Of particular interest given the fan welfare focus here are the conclusions that the optimal behaviour of only the fan welfare maximizer is consistent with the empirically observed ticket black markets and inelastic pricing, suggesting that the current reality does indeed involve some element of fan welfare maximization.

On the normative side, a league of fan welfare maximizers (F-league) unambiguously and strictly dominates a league of profit maximizers (in terms of aggregate surplus) at least when match attendances are below capacity, because of the greater team qualities and attendances that the F-league creates. Although this ranking is weaker and nuanced when sellout matches occur, there is no credible welfare case to recommend profit maximization. The failure of profit maximization to produce a socially valuable outcome should not be too surprising in the context, since the nature of the good supplied by the industry entails two well-known causes of market failure in general. First there is a public good aspect, whereby the addition of an extra fan to a below capacity crowd will involve an element of non-rivalry and perhaps very little cost. Secondly, there is a between firm (club) externality, whereby one firm's payoff is affected positively by another firm's product (team) quality choice[37]. Moreover the F-league is also often socially superior to a league of win maximizers – if the fan bias toward the home team is high enough or the quality elasticity of revenue (reflecting fan's willingness to pay for better quality matches) is low enough this will certainly be so. However it is of interest that this will not always be the case. The reason is the inter-firm positive externality. Fan welfare maximizers will overlook the affect increases in their team quality will have on the overall quality of their away matches and hence on the welfare of rival fans, leading to insufficient team qualities in the league equilibrium. Win maximizers will produce higher team qualities, which may be enough to generate a socially superior outcome if fans' willingness to pay for quality is high.

[37] In the context of the performing arts where only the first type of failure occurs in an analogous fashion, Keynes expressed eloquently a clear negative view on the value of the profit motive; "Even more important…are the…ephemeral ceremonies, shows and entertainments in which the common man can take his delight and recreation after his work is done…Our experience has demonstrated plainly that these things cannot be successfully carried out if they depend on the motive of profit and financial success. The exploitation and incidental destruction of the divine gift of the public entertainer by prostituting it to the purpose of financial gain is one of the…crimes of present-day capitalism." (J.M.Keynes, *Art and the State, 1936*).

This first exploration of fan welfare maximization as a professional sports club objective thus indicates both positive and normative reasons for its recognition as a relevant and important alternative to profit and win maximization; certainly there seems very little to recommend profit maximization, or indeed positive profits, in the context. An increased general focus on club governance issues in European soccer and supporter involvement in this governance seems appropriate, in line with recent UEFA leads.

# APPENDIX I

<u>Proof of Lemma 3.3</u> (Omits arguments $(Q_i, Q_j)$ of $v_i$ and $v$)

Given the strict concavity of $v^2$, the objective function in (3.1) is strictly concave (as the sum of strictly concave functions) with derivatives $+\infty$ as $Q_i \to 0$ and $-\infty$ as $Q_i \to \overline{Q}(Q_j)$, from the Inada assumption. The solution to (3.1) is therefore characterised by the condition $\partial p_{iH} / \partial Q_i = 0$, which produces:

$\partial p_{iH} / \partial Q_i = \frac{1}{2} v_i + \frac{1}{4}( v^2 - 4Q_i / \mu_i)^{-\frac{1}{2}}(2vv_i - \frac{4}{\mu_i}) = 0$ if and only if

$v_i (v^2 - 4Q_i / \mu_i)^{\frac{1}{2}} = \frac{2}{\mu_i} - vv_i$, which holds if and only if $\mu_i vv_i - \mu_i Q_i v_i^{\,2} = 1$, as claimed.

<u>Proof of Theorem 3.2</u> Differentiating the conditions defining quality best responses in part (a) of Lemmas 3.1, 3.2, 3.3 with respect to $\mu_i$, treating $Q_i$ as a function of $\mu_i$, gives, (omitting arguments of functions);

$v^2 + [2\mu_i vv_i - 4]\frac{dQ_{iW}}{d\mu_i} = 0 \Rightarrow \frac{dQ_{iW}}{d\mu_i} > 0$ since the square bracket is negative at $\overline{Q_i}(Q_j)$ because $v^2$ is strictly concave.

$vv_i + \mu_i[vv_{ii} + v_i^2]\frac{dQ_{i\Pi}}{d\mu_i} = 0 \Rightarrow \frac{dQ_{i\Pi}}{d\mu_i} > 0$ since the square bracket is negative because $v^2$ is strictly concave.

$\frac{1}{\mu_i} + \mu_i[vv_{ii} - 2Q_i v_i v_{ii}]\frac{dQ_{iF}}{d\mu_i} = 0 \Rightarrow \frac{dQ_{iF}}{d\mu_i} > 0$ since the square bracket has the sign of $-[v^2 - 2Q_i vv_i]$ which is negative because $v^2$ is strictly concave.

Since the prices of both a win maximizer and a profit maximizer are $\frac{1}{2}v(Q_i, Q_j)$ and do not depend separately on $\mu_i$, the quality results ensure that $\frac{dp_{iW}}{d\mu_i}, \frac{dp_{i\Pi}}{d\mu_i} > 0$. For both these objectives attendance is given by $\frac{1}{2}\mu_i v(Q_i, Q_j)$ and the quality results ensure $\frac{dA_{iW}}{d\mu_i}, \frac{dA_{i\Pi}}{d\mu_i} > 0$. $A_{iF} = \mu_i p_{iH}$, so $\frac{dA_{iF}}{d\mu_i} = p_{iH} + \mu_i \frac{\partial p_{iH}}{\partial Q_{iF}}\frac{dQ_{iF}}{d\mu_i} = p_{iH} > 0$.

<u>Proof of Lemma 4.1</u> Quality formulae follow from Lemmas 3.1, 3.2 and 3.3 with $v(Q_i, Q_j) = Q_i^\alpha Q_j^\beta$. The prices in (a) and (b) are the monopoly prices from Lemmas 3.1 and 3.2. For the fan welfare maximizer price is $p_i = p_{iL}(Q_i, Q_j) = \frac{1}{2} Q_i^\alpha Q_j^\beta - \frac{1}{2}\sqrt{Q_i^{2\alpha} Q_j^{2\beta} - 4Q_i / \mu_i}$

$= \frac{1}{2} Q_i^\alpha Q_j^\beta [1 - \sqrt{1 - 4Q_i / \mu_i Q_i^{2\alpha} Q_j^{2\beta}}\,]$. But with $Q_i = Q_{iF}(Q_j)$,

$4Q_{i_i} / \mu_i\, Q_i^{2\alpha} Q_j^{2\beta} = 4\alpha(1-\alpha)$ and so $p_i = \frac{1}{2} Q_i^\alpha Q_j^\beta [1 - (1 - 2\alpha)]$

<u>Proof of Lemma 4.2</u> Using Lemma 4.1, the conditions for all 3 equilibria can be written: $Q_{1X}^{1-2\alpha} = r_X \mu_1 Q_{2X}^{2\beta}$, $Q_{2X}^{1-2\alpha} = r_X \mu_2 Q_{1X}^{2\beta}$ for X=$\Pi$,W,F with $r_\Pi = \frac{1}{2}\alpha$, $r_W = \frac{1}{4}$, $r_F = \alpha(1-\alpha)$. Hence, in all Nash equilibria we have $Q_{1X} / Q_{2X} = (\mu_1 / \mu_2)^{\frac{1}{1-2\alpha+2\beta}}$, which provides the equilibrium quality formulae when substituted into the earlier expressions.

<u>Proof of Theorem 4.5</u> We first derive the following useful formulae;

(a) $S_\Pi = Q_{1\Pi}(\frac{3}{4\alpha} - 1)[1 + (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]$; (b) $S_F = \frac{1}{2} Q_{1F}(\frac{1}{\alpha} - 1)[1 + (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]$;

(c) $S_W = \frac{1}{2} Q_{1W} [1 + (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]$

With monopoly pricing, the formula for aggregate surplus is, from (4.1);

$$S = \tfrac{3}{8}\mu_1 v(Q_1,Q_2)^2 + \tfrac{3}{8}\mu_2 v(Q_1,Q_2)^2 - Q_1 - Q_2 \qquad \text{(A1)}$$

Introducing the Cobb-Douglas maximum valuation into (A1) gives:

$$S = \tfrac{3}{8}\mu_1 Q_1^{2\alpha} Q_2^{2\beta} + \tfrac{3}{8}\mu_2 Q_1^{2\beta} Q_2^{2\alpha} - Q_1 - Q_2$$

$$= \tfrac{3}{8}\mu_1 Q_1^{2\alpha+2\beta} (\tfrac{\mu_2}{\mu_1})^{\frac{2\beta}{1-2\alpha+2\beta}} + \tfrac{3}{8}\mu_2 Q_1^{2\alpha+2\beta} (\tfrac{\mu_2}{\mu_1})^{\frac{2\alpha}{1-2\alpha+2\beta}} - Q_1 (1 + (\tfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}})$$

$$= Q_1 \{ Q_1^{2\alpha+2\beta-1} \tfrac{3}{8} [\mu_1 (\tfrac{\mu_2}{\mu_1})^{\frac{2\beta}{1-2\alpha+2\beta}} + \mu_2 (\tfrac{\mu_2}{\mu_1})^{\frac{2\alpha}{1-2\alpha+2\beta}}] - 1 - (\tfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} ) \} \qquad \text{(A2)}$$

For (a), substituting Lemma 4.2(a) into (A2) gives;

$$S_\Pi = Q_{1\Pi} \left\{ \frac{\tfrac{3}{8}[\mu_1 (\tfrac{\mu_2}{\mu_1})^{\frac{2\beta}{1-2\alpha+2\beta}} + \mu_2 (\tfrac{\mu_2}{\mu_1})^{\frac{2\alpha}{1-2\alpha+2\beta}}]}{\tfrac{1}{2}\alpha \mu_1^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_2^{\frac{2\beta}{1-2\alpha+2\beta}}} - 1 - (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} \right\}$$

$= Q_{1\Pi} \left\{ \frac{3}{4\alpha} (1 + (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}) - 1 - (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} \right\}$, which rearranges as required.

(b)Substituting Lemma 4.2(b) into (A2) gives;

$$S_W = Q_{1W} \left\{ \frac{\tfrac{3}{2}[\mu_1 (\tfrac{\mu_2}{\mu_1})^{\frac{2\beta}{1-2\alpha+2\beta}} + \mu_2 (\tfrac{\mu_2}{\mu_1})^{\frac{2\alpha}{1-2\alpha+2\beta}}]}{\mu_1^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_2^{\frac{2\beta}{1-2\alpha+2\beta}}} - 1 - (\tfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} \right\}$$

$= Q_{1W} \left\{ \frac{3}{2} (1 + (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}) - 1 - (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} \right\}$, as required.

(c)With low break-even pricing and the Cobb-Douglas valuation function, aggregate surplus becomes;

$$S = \tfrac{1}{4}\mu_1 Q_1^{\alpha} Q_2^{\beta} [Q_1^{\alpha} Q_2^{\beta} + \sqrt{Q_1^{2\alpha} Q_2^{2\beta} - 4Q_1\mu_1}] +$$
$$\tfrac{1}{4}\mu_2 Q_1^{\beta} Q_2^{\alpha} [Q_1^{\beta} Q_2^{\alpha} + \sqrt{Q_1^{2\beta} Q_2^{2\alpha} - 4Q_2\mu_2}] - \tfrac{1}{2}Q_1 - \tfrac{1}{2}Q_2 .$$

Hence;

$$S = \tfrac{1}{4} Q_1 \left\{ \mu_1 (\tfrac{\mu_2}{\mu_1})^{\frac{2\beta}{1-2\alpha+2\beta}} Q_1^{2\alpha+2\beta-1} [1 + \sqrt{1 - \tfrac{4}{\mu_1} Q_1^{1-2\alpha-2\beta} (\tfrac{\mu_1}{\mu_2})^{\frac{2\beta}{1-2\alpha+2\beta}}}] \right\} +$$

$$\tfrac{1}{4} Q_1 \left\{ \mu_2 (\tfrac{\mu_2}{\mu_1})^{\frac{2\alpha}{1-2\alpha+2\beta}} Q_1^{2\alpha+2\beta-1} [1 + \sqrt{1 - \tfrac{4}{\mu_2} Q_1^{1-2\alpha-2\beta} (\tfrac{\mu_2}{\mu_1})^{\frac{1-2\alpha}{1-2\alpha+2\beta}}}] \right\} - \tfrac{1}{2} Q_1 - \tfrac{1}{2} Q_1 (\tfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}$$

Substituting Lemma 4.2(c) gives;

$$S_F = \tfrac{1}{4} Q_{1F} \left\{ \tfrac{1}{\alpha(1-\alpha)} [1 + \sqrt{1 - 4\alpha(1-\alpha)}] + \tfrac{1}{\alpha(1-\alpha)} (\tfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} [1 + \sqrt{1 - 4\alpha(1-\alpha)}] - 2 - 2(\tfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} \right\}$$

$= \frac{1}{4} Q_{1F} \left\{ \frac{2}{\alpha} [1 + (\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}] - 2 - 2(\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}} \right\}$, which becomes the required formula.

Using these formulae, $S_W > S_F$ if and only if $Q_{1W} > \frac{1-\alpha}{\alpha} Q_{1F}$. From Lemma 4.2 this requirement becomes $\frac{1}{4} > (1-\alpha)^{2-2\alpha-2\beta}\alpha^{2\alpha+2\beta}$, which, with $\alpha = ef$ and $e = \alpha + \beta$, in turn becomes the claimed inequality.

Similarly using the above formulae in conjunction with Lemma 4.2 shows that $S_W > S_\Pi$ if and only if $1 > (2\alpha)^{2\alpha+2\beta}(3-4\alpha)^{1-2\alpha-2\beta}$, which becomes the condition claimed with $\alpha = ef, e = \alpha + \beta$.

<u>Proof of Theorem 5.4</u> Under any of the rationing regimes, an equilibrium of the F-league with capacity constraints binding on both clubs is characterised by the best response conditions; $k_i v_i(Q_i, Q_j) = 1$, $p_i = Q_i / k_i$, for $i$=1,2, $j \neq i$. With the Cobb-Douglas maximum valuation, the first of these conditions becomes $Q_i^{1-\alpha} = \alpha k_i Q_j^{\beta}$. Quality best response functions are thus increasing concave functions that generate a unique strictly positive Nash equilibrium that is stable in the usual best response dynamic, defined by:

$$Q_i^{1-\alpha-\beta} = k_i^{\frac{1-\alpha}{1-\alpha+\beta}} k_j^{\frac{\beta}{1-\alpha+\beta}} \qquad \text{(A3)}$$

Notice $\partial Q_i / \partial k_i, \partial Q_i / \partial k_j > 0$. The capacity constraints bind if $\mu_i [v(Q_i, Q_j) - p_i] > k_i$, $i$=1,2, $j \neq i$, which after substitution, rearrangement and use of the assumed capacity restrictions become, for $i$=1,2, $j \neq i$;

$$k_i^{1-2\alpha-2\beta} < \alpha^{\alpha+\beta}(1-\alpha)^{1-\alpha-\beta} \mu_i^{\frac{1+2\alpha^2-3\alpha-2\beta^2}{1-2\alpha+2\beta}} \mu_j^{\frac{\beta}{1-2\alpha+2\beta}}.$$

Using Lemma 4.2 to compute unconstrained equilibrium attendance shows that the above inequality is the same as $k_i < A_{iF}$. Thus if $k_i < A_{iF}, i = 1,2$, there is indeed an equilibrium described by (A3) in which capacity constraints are binding on both clubs in the F-league, and of course if $k_i \geq A_{iF}, i = 1,2$ the equilibrium has no binding capacity constraints, as in Lemma 4.2.

The above argument is easily repeated (details omitted) for the $\Pi$ -league; if $k_i < A_{i\Pi}, i = 1,2$, there is an equilibrium described by (A3) in which capacity constraints are binding on both clubs in the $\Pi$ -league, and if $k_i \geq A_{i\Pi}, i = 1,2$ the equilibrium has no binding capacity constraints, as in Lemma 5.2.

With a binding capacity constraint the earlier formula (4.1) for the sum of fan and producer surplus for club $i$ becomes $S_i = k_i v(Q_i, Q_j) - k_i^2 / 2\mu_i - Q_i$. Hence, in a capacity constrained equilibrium for either the F-league or the $\Pi$ -league, and using the common equilibrium condition $k_i v_i(Q_i, Q_j) = 1$;

$$\partial S_i / \partial k_i = v(Q_i, Q_j) - k_i / \mu_i + k_i v_j(Q_i, Q_j) \partial Q_j / \partial k_i > 0$$

$$\partial S_i / \partial k_j = k_i v_j(Q_i, Q_j) \partial Q_j / \partial k_j > 0$$

Thus (generally) in equilibria of either the F-league or the $\Pi$ -league where capacity constraints bind on both clubs, aggregate surplus increases with the capacity constraints. The result now follows from Theorems 5.2 and 5.3.

## APPENDIX II

The Deloitte Annual Review of Football Finance 2009 (Appendix 9) provides data for each English Premier League club's stadium capacity C, highest attendance H, lowest attendance L, and hence the spread S=H-L. To provide an estimate of the proportion of sellout matches, assume that attendances are uniformly distributed over [L,H], and that an attendance exceeding xC indicates a sellout where x is some fraction close to 1. Then the predicted fraction of sellout matches for a club is (H-xC)/S, producing the following for x=0.99 and x=0.95.

Predicted fraction of sellouts, (H-xC)/S

| | x=0.99 | x=0.95 |
|---|---|---|
| Arsenal | 0.561 | 1 |
| Aston Villa | 0.04 | 0.206 |
| Birmingham City | 0 | 0.094 |
| Blackburn Rovers | 0 | 0.065 |
| Bolton Wanderers | 0 | 0 |
| Chelsea | 0.173 | 0.874 |
| Derby County | 0 | 0.403 |
| Everton | 0.035 | 0.233 |
| Fulham | 0.029 | 0.251 |
| Liverpool | 0 | 0.635 |
| Manchester City | 0.001 | 0.22 |
| Manchester United | 0.588 | 1 |
| Middlesborough | 0 | 0.06 |
| Newcastle United | 0.188 | 1 |
| Portsmouth | 0.059 | 0.298 |
| Reading | 0.081 | 0.406 |
| Sunderland | 0 | 0.12 |
| Tottenham Hotspur | 0.343 | 1 |
| West Ham United | 0.022 | 1 |
| Wigan Athletic | 0.022 | 0.112 |
| League average | 0.107 | 0.448 |

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