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PRICE, QUALITY AND WELFARE CONSEQUENCES OF ALTERNATIVE CLUB OBJECTIVES IN A PROFESSIONAL SPORTS LEAGUE

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Abstract

A 2-club professional sports league model is presented where gate revenues are the revenue source and expenditure on players ("quality") is the only cost. Clubs choose quality and the ticket price for a match at their stadium (of given capacity). Performance of clubs and leagues is studied under three club objectives, profit, quality and fan welfare maximization, the last two subject to non-negative profits. Results suggest that fan welfare maximization is interesting positively (in explaining black markets for tickets and empirically observed price inelasticities) and normatively. Profit maximization does not find strong normative support.

1. INTRODUCTION

The paper presents a basic industrial organization model of a professional sports league with two professional clubs[1] fielding teams that play each other twice, once at their home stadium and once away. Clubs make decisions on expenditure on the perfectly elastic supply of playing talent (“quality”), and on the ticket price for admission to their home match where attendance is bounded above by the given stadium capacity. The resulting gate revenues are the only source of club income, and expenditure on playing talent is the only cost. Each club has an exogenously given set of “fans” who derive utility from attending its home match (fans do not travel to away games, so a club is a monopolistic seller of tickets to its fans). The consequences for ticket price, team quality and social welfare of alternative club objectives are addressed by studying club best responses and Nash equilibria in leagues where clubs operate independently but with the same type of objective, namely either profit maximization, or quality maximization (equivalent to maximizing the probability that the club wins the league) or fan welfare maximization (short for maximization of the aggregate utility of their fans), the last two subject to a non-negative profit constraint. Thus we investigate how the performance of clubs and leagues varies with the type of governance within clubs.

As far as we know, this is the first formal model in the sports literature to address the club objective of fan welfare maximization[2], an objective suggested by club governance that allows a significant influence from fans, for example members clubs in European soccer where fans elect club officials, and the emerging and growing role of supporters trusts in UK soccer[3]. Existing analyses of the industrial organization of professional sports leagues have mostly assumed profit maximizing clubs, with some attention to the quality maximization alternative, and almost all of these suppress any analysis of the ticket price decision, taking as the primitive concept a club revenue function that depends only on qualities of the participating teams. In this literature there are numerous papers that look at the consequences of regulatory policies, gate-sharing in particular, for team qualities and competitive balance in leagues of (usually) profit-maximizing clubs[4]. However detailed utility microfoundations, and welfare analysis, are absent from almost all of these, as are considerations of match attendances and stadium capacity constraints. An exception to “almost all” is Falconieri et al. (2004) who study a model where the only revenues are from TV with focus on collective versus individual bargaining between clubs and the broadcaster, using a utility and welfare specification that is broadly similar to that used here. We bring this broad framework to bear on a more basic, non-cooperative setting, providing analyses of ticket prices, team qualities, social welfare, and the resulting match attendances relative to stadium capacities not found in the existing studies of similar settings, and our regulatory focus is purely on the club governance question[5].

When capacities are large, so stadiums are never full, an analogy is suggested that provides the perspective taken here behind the modelling of the leagues. We think of a match as a pure excludable public good, excludable since stadium entry can be controlled, and plausibly non-rival (at least as a first approximation – the addition of an extra fan to the crowd allows all to enjoy the match as before the addition[6]). The literature on excludable public goods is large, with suggested applications to the economics of galleries, museums, parks, the performing arts, transport, zoos, and so on (e.g. Brito and Oakland (1981), Cornes and Sandler (1996), Fraser (1996,2000), Traub and Missong (2005)). However the difference between these applications and the match in a sports league is that whereas the owners of a zoo (say) do not need the animals from another zoo to make their facility desirable, the analogue is not true in a sports league. There is a payoff complementarity/externality whereby one club's output (its team) is only of value when combined with that of another club in a match; without this our model would be exactly a monopoly excludable public good model.

Sections 2-5 restrict attention to the large capacity case, section 3 focusing on price and quality best responses of the three different owner types, section 4 applying these findings to study social welfare in the simpler, underlying excludable public good model and section 5 deriving and comparing Nash equilibria in the leagues. Section 6 looks at binding capacity constraints and section 7 concludes.

2. THE FRAMEWORK

Two clubs and their teams comprise the professional sports league. The exogenous league rules are that each team plays the other twice, once at home and once away. Club i=1,2 has a stadium where its team plays its home match; the stadium has a given "large" (for now) capacity k_i, and we abstract from all stadium costs. Clubs hire players and $Q_i \geq 0$ denotes the expenditure on talent by team *i*; Q_i is alternatively referred to as the quality of team *i*[7].

Club *i* sets the ticket admission price p_i for its home match and receives all gate revenue from this match; no price discrimination is possible. There are disjoint sets $F_i, i = 1,2$ of fans of *i*, who feel an (exogenously given) affinity to club *i* and are the potential spectators for *i's* home match. We can think of club *i* as located in region i, and F_i consists of people living in that region with an interest in watching the region's team, and with travel to away games infeasible. Fans in region i are located at distances $x \in [0, c_i]$ from the stadium in i, and are uniformly distributed over $[0, c_i]$ with density μ_i. c_i is "large", indeed we assume throughout that $k_i < \mu_i c_i$ so that the total number of fans exceeds stadium capacity.

A fan of i located at distance x from its stadium derives (ex ante) utility from attending i's home match given by $v(Q_i, Q_j) - p_i - x$, where x is referred to as the heterogeneity parameter and $v(Q_i, Q_j)$ is the (common) valuation function[8]. It is assumed throughout that $v(Q_i, Q_j)$ is C^2 and strictly increasing in both arguments, reflecting the desire of fans to see better quality matches[9]. Also $v(Q_i, Q_j)^2$ appears in the objective function of many of the subsequent optimization problems and we assume that it (and hence $v(Q_i, Q_j)$ itself) is strictly concave and satisfies the Inada conditions. For some of our arguments we need to be more specific with the Cobb-Douglas valuation function, $v(Q_i, Q_j) = Q_i^{\alpha} Q_j^{\beta}$ where $\alpha, \beta > 0$ and $\alpha + \beta < 1/2$. Then $e = \alpha + \beta$ measures a fan's elasticity of willingness to pay for a match ticket with respect to (linear increases in) team qualities. Also $f = \alpha / e$ measures the fraction of a given amount of talent that a fan would allocate to their own team for their optimal match; f is referred to as the degree of fan bias, the usual bias suggesting $f \geq 1/2$, with $f = 1/2$ the case of completely non-partisan fans and $f = 1$ the completely partisan case. For their TV audience, Falconieri et al. assume $f = 1/2$ (with the Cobb-Douglas valuation function).

The fan of i at x will demand a ticket if $x \leq v(Q_i, Q_j) - p_i$ so that i's ticket demand (=match attendance with large capacities) is $D_i(Q_i, Q_j, p_i) = \mu_i[v(Q_i, Q_j) - p_i]$ yielding gate revenues $p_i D_i(Q_i, Q_j, p_i)$.

Each club makes independent decisions about hiring of talent (Q_i) and ticket prices (p_i) to fulfil their objectives, to be specified later. Once talent has been hired and tickets sold, matches are played and a winner emerges. Ex ante, before the play of matches, the probability that i is the winner is some function $P(Q_i, Q_j)$, increasing in Q_i and decreasing in Q_j, perhaps the contest success function $Q_i/(Q_i + Q_j)$, although the exact specification is irrelevant for our purposes.

The timing of the interactions, and further discussion of the assumptions are as follows.

Stage I Clubs hire talent $Q_i \geq 0$ simultaneously, incurring costs Q_i. These are the only costs, abstracting from all stadium costs, for instance.

Stage II Clubs set prices $p_i \in [0, v(Q_i, Q_j)]$ and earn gate revenues of $p_i \mu_i [v(Q_i, Q_j) - p_i]$. These are the only revenues; we abstract from TV, sponsorship, merchandise, gate-sharing and prize revenues, for instance. Notice that there is no direct strategic interaction in ticket price setting; club i is a monopolistic seller of tickets to its fans.

Once these stage I and II decisions on team quality and ticket prices have been made (according to the owner objectives) the matches are played and a winner emerges. To complete the 2-stage game specification we need to describe owner objectives, and we study 3 alternatives.

PROFIT MAXIMIZATION

The owner payoff function is;

$$\Pi_i(Q_i, Q_j, p_i) = p_i \mu_i [v(Q_i, Q_j) - p_i] - Q_i$$

This is the most common assumption in the existing literature.

QUALITY MAXIMIZATION

The owner payoff is now Q_i. The owners wish to produce the best team, or equivalently (as it is a monotone increasing transformation) they wish to maximize the probability that they win the league, $P(Q_i, Q_j)$. In Fort and Quirk's (2004) terminology this objective is maximization of the win percentage (WPM, where PM is the previous profit maximization). Of course there has to be a budget constraint on the achievement of this objective which we take to be $\Pi_i(Q_i, Q_j, p_i) \geq 0$.[10]

FAN WELFARE MAXIMIZATION

Again there is a budget constraint taken to be $\Pi_i(Q_i, Q_j, p_i) \geq 0$. Subject to this constraint, the owner now wishes to maximize the aggregate utility of their fans, or;

$$W_i(Q_i, Q_j, p_i) = \int_0^{v(Q_i Q_j)-p_i} \mu_i[v(Q_i, Q_j) - p_i - x]dx$$

$$= \mu_i [v(Q_i, Q_j) - p_i]^2 / 2$$

Notice that this is a monotone transformation of $v(Q_i, Q_j) - p_i$, which is what each fan of club *i* would choose to maximize if they were in control of club choices, restricted by non-negative profits. If the club governance structure allowed fans to vote for a representative to influence decisions about p_i and Q_i (subject to non-negative profits), then fan welfare maximization would be an unbeatable platform in the election of this representative.[11]

3. PRICE AND QUALITY BEST RESPONSES

We consider the price (p_i) and quality (Q_i) best responses of a club (*i*) with given characteristics (μ_i, k_i, c_i) to the quality (Q_j) chosen by the other team (*j*) in the league, and how this varies with the club's objective (profit (Π), quality (Q) or fan welfare (W) maximization). The other team's price (p_j) does not affect the answer in any of the 3 cases (p_j has no affect on *i*'s payoff or constraints). Best responses for $X = \Pi, Q, W$ are denoted $p_{iX}(Q_j)$, $Q_{iX}(Q_j)$. Capacity constraints are ignored in this section.

It is helpful to start with quality maximization, where the best response problem is:

$$\max_{p_i, Q_i} Q_i \text{ s.t. } \mu_i p_i[v(Q_i, Q_j) - p_i] - Q_i \geq 0 \qquad (3.1)$$

Figure 3.1 illustrates the feasible set for (3.1) as the shaded region.

The constraint for (3.1) can be written $p_i^2 - p_i v(Q_i, Q_j) + Q_i / \mu_i \geq 0$, and the roots of the quadratic are:

$$p_{iL}(Q_i, Q_j) = \frac{1}{2} v(Q_i, Q_j) - \frac{1}{2}\sqrt{v(Q_i, Q_j)^2 - 4Q_i / \mu_i}$$

$$p_{iH}(Q_i, Q_j) = \frac{1}{2} v(Q_i, Q_j) + \frac{1}{2}\sqrt{v(Q_i, Q_j)^2 - 4Q_i / \mu_i}$$

The roots are real if $Q_i \in [0, \overline{Q}(Q_j)]$, where $\overline{Q}(Q_j)$ is the unique positive solution in Q_i (given the strict concavity and Inada properties of *v*) to $v(Q_i, Q_j) = 4Q_i / \mu_i$. We refer to $p_{iL}(Q_i, Q_j)$ as *the low break-even price*, and $p_{iH}(Q_i, Q_j)$ as *the high break-*

even price, with graphs shown by L,H respectively in Figure 3.1. Notice that $p_{iH}(Q_i, Q_j)$ is strictly concave under our assumptions. Between the L,H branches in Figure 3.1, labelled as M, we have *the monopoly price* $p_M(Q_i, Q_j) = v(Q_i, Q_j)/2$, which maximizes gate revenue (given Q_i, Q_j). Recall that elasticity of ticket demand

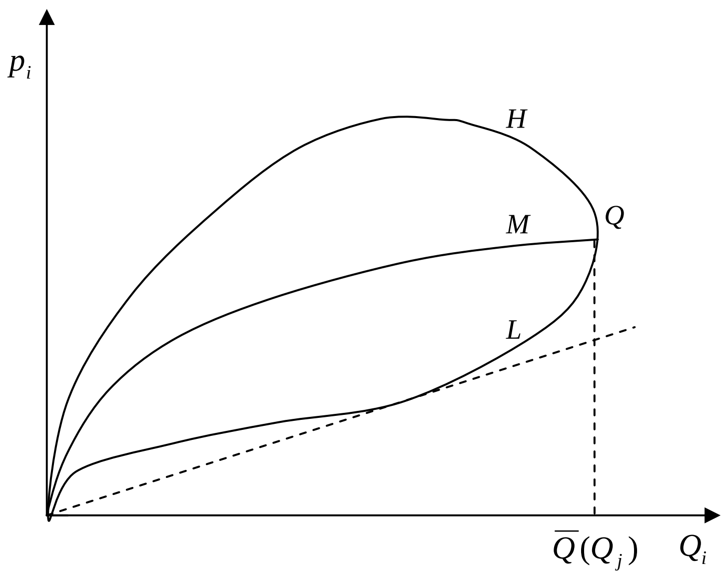

Figure 3.1

is unity along M, higher than one (elastic) above M and lower than one (inelastic) below M. The solution to (3.1) is clear from Figure 3.1, at *Q*:

<u>Lemma 3.1</u> In the absence of capacity constraints the best price and quality responses of a quality maximizing club *i* are:

(a) $Q_{iQ}(Q_j) = \overline{Q}(Q_j)$ where $\overline{Q}(Q_j)$ is the unique positive solution in Q_i to $v(Q_i, Q_j) = 4Q_i / \mu_i$;

(b) $p_{iQ}(Q_j) = \frac{1}{2} v(\overline{Q}(Q_j), Q_j)$.

With profit maximization, the best response problem becomes:

$$\max_{p_i, Q_i} \mu_i p_i [v(Q_i, Q_j) - p_i] - Q_i \tag{3.2}$$

The solution is found in 2 steps. First, given Q_i (and Q_j) the best choice of p_i for the objective is always the monopoly price $p_i = p_M(Q_i, Q_j)$. Secondly the problem (3.2) now reduces to:

$$\max_{Q_i} \frac{1}{4} \mu_i v(Q_i, Q_j)^2 - Q_i \tag{3.3}$$

Given the strict concavity and Inada properties of v[2], and denoting $v_i(Q_i, Q_j) = \partial v / \partial Q_i (Q_i, Q_j)$, (3.3) has a unique positive solution in Q_i defined by marginal revenue equals marginal cost:

$$MR_i(Q_i, Q_j) = \frac{1}{2}\mu_i v(Q_i, Q_j) v_i(Q_i, Q_j) = 1 \tag{3.4}$$

<u>Lemma 3.2</u> In the absence of capacity constraints, the best price and quality responses of a profit-maximizing club *i* are:

(a) $Q_{i\Pi}(Q_j)$ defined by the solution in Q_i to (3.4);

(b) $p_{i\Pi}(Q_j) = p_M(Q_{i\Pi}(Q_j), Q_j)$.

Figure 3.2 illustrates, where we have added the locus $\partial \Pi_i / \partial Q_i = \mu_i p_i v_i(Q_i, Q_j) - 1 = 0$, an upward sloping curve going from the origin to the maximum point of the H curve in Figure 3.1, and crossing M at the overall profit maximum (shown as Π). C is a typical profit contour for a profit level between 0 and the optimal value.

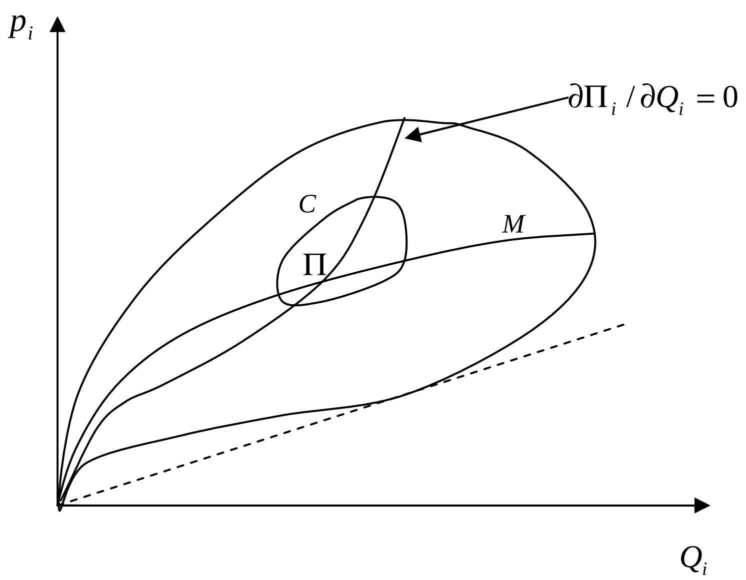

Figure 3.2

The fan welfare maximizer's best response problem can be written:

$$\max_{p_i, Q_i} v(Q_i, Q_j) - p_i \quad \text{s.t.} \quad \mu_i p_i [v(Q_i, Q_j) - p_i] - Q_i \geq 0 \tag{3.5}$$

Feasibility demands $Q_i \in [0, \overline{Q}(Q_j)]$. Proceeding again in 2 steps, for $Q_i \in [0, \overline{Q}(Q_j)]$ the optimal price is clearly $p_i = p_{iL}(Q_i, Q_j)$. As $v(Q_i, Q_j) - p_{iL}(Q_i, Q_j) = p_{iH}(Q_i, Q_j)$, the problem reduces to:

$$\max_{Q_i} \; p_{iH}(Q_i, Q_j) \quad \text{s.t. } Q_i \in [0, \overline{Q}(Q_j)] \tag{3.6}$$

Given the strict concavity of v^2, the objective function in (3.6) is strictly concave (as the sum of strictly concave functions) with derivatives $+\infty$ as $Q_i \to 0$ and $-\infty$ as

$Q_i \to \overline{Q}(Q_j)$, from the Inada assumption. The solution to (3.6) is therefore characterised by the condition $\partial p_{iH} / \partial Q_i = 0$, which produces:

<u>Lemma 3.3</u> In the absence of capacity constraints, the best price and quality responses of a fan welfare maximizing club *i* are:

(a) $Q_{iW}(Q_j)$ is the solution in Q_i to; $\mu_i v v_i - \mu_i Q_i {v_i}^2 = 1$,

or, $\quad 2MR_i(Q_i, Q_j) - 4Q_i^2 AR_i(Q_i, Q_j) = 1$

where $AR_i(Q_i, Q_j) = \frac{1}{4}\mu_i v(Q_i, Q_j)^2 / Q_i$;

(b) $p_{iW}(Q_j) = p_{iL}(Q_{iW}(Q_j), Q_j)$.

<u>Proof</u> (Omits arguments (Q_i, Q_j) of v_i and v)

$\partial p_{iH} / \partial Q_i = \frac{1}{2} v_i + \frac{1}{4}(v^2 - 4Q_i / \mu_i)^{-\frac{1}{2}}(2vv_i - 4) = 0$ if and only if

$v_i (v^2 - 4Q_i / \mu_i)^{\frac{1}{2}} = \frac{2}{\mu_i} - vv_i$, which holds if and only if $\mu_i v v_i - \mu_i Q_i {v_i}^2 = 1$, as claimed. The definitions of MR_i, AR_i produce the alternative statement. *Q.E.D.*

The following observations facilitate diagrammatic representation of the solution:

(1) Contours of the fan welfare maximizer's objective have equations $v(Q_i, Q_j) - p_i = \bar{u}$. Let $\bar{u}*$ denote the value of $\bar{u}$ at the solution to (3.5). For $\bar{u} \in [0, \bar{u}*]$ the utility contour intersects the zero profit contour in Figure 3.1 where the following hold:

$$v(Q_i, Q_j) - p_i = \bar{u} \text{ and } \mu_i p_i (v(Q_i, Q_j) - p_i) - Q_i = 0$$

At any intersection, $\mu_i p_i \bar{u} = Q_i$ and all such intersections are collinear with the origin.

(2) At $\bar{u} = 0$ there is a unique intersection, the origin.

(3) For $\bar{u} \in (0, \bar{u}*)$, intersections are where $\mu_i(v(Q_i, Q_j) - \bar{u})\bar{u} = 0$, and the strict concavity and Inada properties of *v* ensure 2 intersections.

(4) At $\bar{u}*$ there must be tangency between the utility contour and the zero profit contour, and the common tangent goes through the origin.

Figure 3.3 illustrates the solution at W, and contours for $\bar{u}$=0, $\bar{u} \in (0, \bar{u}*)$ and $\bar{u}*$. We can now compare the price and quality choices of quality, profit and fan welfare maximizers when faced by the same club characteristics (μ_i, k_i, c_i) and the same choice by the rest of the league (Q_j). We can also compare match attendances that would result, $A_{iX}(Q_j) = \mu_i[v(Q_{iX}(Q_j), Q_j) - p_{iX}(Q_j)]$, $X = Q, \Pi, W$, noting that attendance is a monotone increasing transformation of fan welfare, so the attendance contour map is merely a re-labelling of that for fan welfare in Figure 3.3.

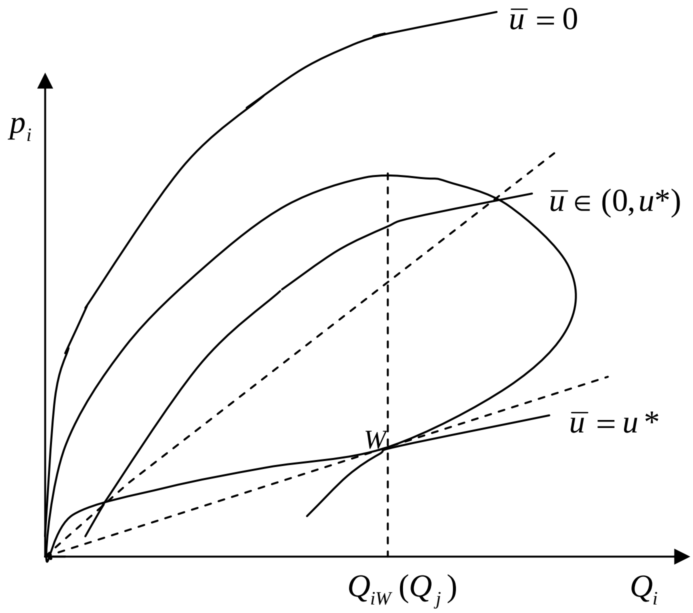

Figure 3.3

From Figures 3.1 and 3.3 ticket price and team quality are higher for a quality maximizer (Q in Figure 3.1) than for a fan welfare maximizer (W in Figure 3.3). The quality maximizer's price and quality are also higher than a profit maximizer's (Π in Figure 3.2). In addition the quality chosen by a fan welfare maximizer is higher than that of a profit maximizer (compare Π in Figure 3.2 and W in Figure 3.3). On the other hand attendance is highest under fan utility maximization and lowest with profit maximization (the attendance/fan welfare contour through Q in Figure 3.1 has slope $v_i(Q_i, Q_j)$ which exceeds the slope of M at that point). And with respect to elasticity of ticket demand at the solutions, both quality and profit maximizer's price at unit elasticity, whilst the fan welfare maximizer generates inelastic pricing. We have proved:

<u>Theorem 3.1</u> In the absence of capacity constraints, best responses of a club *i* with the same characteristics (μ_i, k_i, c_i) and facing the same Q_j, depend on the club's objective, with the following consequences:

(a) $Q_{iQ}(Q_j) > Q_{iW}(Q_j) > Q_{i\Pi}(Q_j)$;

(b) $p_{iQ}(Q_j) > p_{i\Pi}(Q_j)$ and $p_{iQ}(Q_j) > p_{iW}(Q_j)$;

(c) $A_{iW}(Q_j) > A_{iQ}(Q_j) > A_{i\Pi}(Q_j)$;

(d) ticket demand is unit elastic at the best response of a profit maximizer or a quality maximizer, and inelastic at the best response of a fan welfare maximizer.

The missing comparison in Theorem 3.1 is between $p_{i\Pi}(Q_j)$ and $p_{iW}(Q_j)$, and indeed this is generally ambiguous – it may be that $Q_{i\Pi}(Q_j)$ is so far below $Q_{iW}(Q_j)$ that the profit maximizer's price is lower than the fan welfare maximizer's price, despite their monopoly (rather than low break-even) pricing.[12]

The reason for reporting part (d) of Theorem 3.1 is that there is considerable empirical evidence consistent with inelastic pricing of tickets for sports matches – see Fort (2004) for an extensive account of this literature (and see Alexander (2001) for an alternative view). Fort (2004) argues that profit maximization can be consistent with such observations (and discusses related earlier arguments) provided one leaves our gate-revenue (only) scenario[13]. An investigation as to which objective provides the "better" explanation of the facts is certainly beyond the scope of our paper. However within the confines of our model, our result shows that the fan welfare maximizing objective provides the only explanation. Moreover this result is strengthened in the later capacity constrained analysis, where both profit and quality maximizers choose prices strictly in the elastic part of the demand curve (Theorem 6.1).

4. THE UNDERLYING MONOPOLY EXCLUDABLE PUBLIC GOOD MODEL

The sports league perspective of this paper has the 2 clubs as monopoly providers of matches involving its team to its fans, where a match is an excludable public good. The model is not merely one of separate monopoly public good provision problems, because of the externality that *i*'s match is only of value to the fans when both clubs participate. In this section we explore the underlying monopoly public good problem, abstracting from the externality. We do so because the analysis produces clear statements about social welfare consequences of alternative monopoly objectives, providing a benchmark for the later welfare analysis of the sports league.

Consumers (instead of fans) again make a dichotomous choice as to whether or not to "visit" the "facility" at (uniform) entry price *p*. *Q* denotes owner expenditure on quality of the facility, and consumers, with heterogeneity *x* (distributed as earlier with density μ) derive utility from a visit of $v(Q)-p-x$, where $v(Q)$ is strictly increasing and $v(Q)^2$ is strictly concave and satisfies Inada conditions. We study the consequences of the 3 previous owner objectives, which generate the following optimization problems (again ignoring capacity constraints) for, respectively, profit, quality and (now) consumer welfare maximization;

$$\max_{p,Q} \pi(p,Q) = \mu(v(Q)-p)-Q \tag{4.1}$$

$$\max_{p,Q} Q \quad s.t.\ \pi(p,Q) \geq 0 \tag{4.2}$$

$$\max_{p,Q} v(Q)-p \quad s.t.\ \pi(p,Q) \geq 0 \tag{4.3}$$

These optimization problems parallel completely those studied in the last section, and their solutions $(Q_\Pi, p_\Pi),(Q_Q, p_Q),(Q_W, p_W)$ are characterised by the following, where $v'(Q)$ denotes the derivative of *v*:

$$p_\Pi = \frac{1}{2}v(Q_\Pi) \text{ and } MR(Q_\Pi) = \frac{1}{2}\mu v(Q_\Pi)v'(Q_\Pi) = 1 \tag{4.4}$$

$$p_Q = \frac{1}{2}v(Q_Q) \text{ and } AR(Q_Q) = \frac{1}{4}\mu v(Q_Q)^2 / Q_Q = 1 \tag{4.5}$$

$$p_W = \frac{1}{2}v(Q_W) - \frac{1}{2}\sqrt{v(Q_W)^2 - 4Q_W/\mu} \text{ and } 2MR(Q_W) - 4Q_W^2 AR(Q_W) = 1 \tag{4.6}$$

Figures 3.1, 3.2 and 3.3 continue to serve, replacing (p_i, Q_i) with *(p,Q)*, $v(Q_i, Q_j)$ with *v(Q)* and $Q_{iX}(Q_j)$ with $Q_X, X = \Pi, Q, W$. Hence $Q_Q > Q_W > Q_\Pi$ and $p_Q > p_\Pi, p_Q > p_W$, exactly analogously.

Our main interest is in ranking the social welfare performances of profit (S_Π), quality (S_Q) and consumer welfare (S_W) maximization, all of which produce second-best outcomes[14]. It is fairly obvious in the present context that S_W will be the largest of the three, as follows. With quality *Q* and entry price *p*, aggregate surplus is

$$S(Q,p) = \int_0^{v(Q)-p} \mu[v(Q) - p - x]dx + p\mu[v(Q) - p] - Q \tag{4.7}$$

$$= \int_0^{v(Q)-p} \mu[v(Q) - x]dx - Q \tag{4.8}$$

The first term in (4.7) is consumer surplus, the rest producer surplus. With low break-even pricing, *p*= $p_L(Q) = \frac{1}{2}v(Q) - \frac{1}{2}\sqrt{v(Q)^2 - 4Q/\mu}$, producer surplus is zero and, since maximization of consumer surplus equates to the consumer welfare maximizer's objective, it follows immediately that $S_W > S_Q$, since quality maximization produces low break-even pricing also. In addition S_W exceeds the value of social welfare at Q_Π when $p = p_L(Q_\Pi)$; but this latter exceeds S_Π since the higher monopoly price of the profit maximizer, given Q_Π, further reduces social welfare from (4.8). Thus $S_W > S_\Pi$ also.

However, the ranking of S_Q and S_Π is ambiguous:

<u>Theorem 4.1</u> In the monopoly excludable public good model with no capacity constraints:

(a) social welfare is always highest under consumer welfare maximizing provision of the good, i.e. $S_W > S_Q$ and $S_W > S_\Pi$;

(b) if $v(Q) = Q^e, 0 < e < \frac{1}{2}$, $S_Q - S_\Pi$ has the sign of $e - 1/4$.

<u>Proof</u> (a) has been shown in the text.

(b) With $v(Q) = Q^e$ and $p = v(Q)/2$, (4.8) becomes;

$$\int_0^{v(Q)/2} \mu[v(Q) - x]dx - Q = \frac{3}{8}\mu v(Q)^2 - Q = \frac{3}{8}\mu Q^{2e} - Q,$$

and (4.4) and (4.5) produce;

$Q_\Pi = (\frac{1}{2}\mu e)^{\frac{1}{1-2e}}$ and $Q_Q = (\frac{1}{4}\mu)^{\frac{1}{1-2e}}$.

Hence $S_Q = \frac{1}{8}(\frac{1}{4})^{\frac{2e}{1-2e}}\mu^{\frac{1}{1-2e}}$ and $S_\Pi = \mu^{\frac{1}{1-2e}}[\frac{3}{8}(\frac{1}{2}e)^{\frac{2e}{1-2e}} - (\frac{1}{2}e)^{\frac{1}{1-2e}}]$. After some manipulation, $S_Q > S_\Pi$ if and only if $1 > (3-4e)(2e)^{\frac{2e}{1-2e}}$, which is true if and only if *e>1/4.* *Q.E.D.*

Thus, naturally in this model, consumer welfare maximization performs best, and, as is also intuitive, quality maximization is better than profit maximization if the elasticity of willingness to pay for quality is high, the reverse when it is low.[15]

5. PRICE, QUALITY AND WELFARE IN LEAGUE NASH EQUILIBRIA

In this section we assume the Cobb-Douglas valuation function $v(Q_i, Q_j) = Q_i^\alpha Q_j^\beta$ (with elasticity $e = \alpha + \beta$ and fan bias $f = \alpha / e$), and investigate Nash equilibria in 3 games; a Π-league (2 profit maximizing clubs), a Q-league (2 quality maximizers) and a W-league(with 2 fan welfare maximizers). Again there are no capacity constraints and club best responses follow from the general analysis in Section 3 :

<u>Lemma 5.1</u> In the absence of capacity constraints and with the Cobb-Douglas valuation function, best responses are:

(a) $Q_{i\Pi}(Q_j) = (\frac{1}{2}\alpha\mu_i Q_j^{2\beta})^{\frac{1}{1-2\alpha}}$, $p_{i\Pi}(Q_j) = \frac{1}{2} Q_{i\Pi}(Q_j)^\alpha Q_j^\beta$

(b) $Q_{iQ}(Q_j) = (\frac{1}{4}\mu_i Q_j^{2\beta})^{\frac{1}{1-2\alpha}}$, $p_{iQ}(Q_j) = \frac{1}{2} Q_{iQ}(Q_j)^\alpha Q_j^\beta$

(c) $Q_{iW}(Q_j) = [\alpha(1-\alpha)\mu_i Q_j^{2\beta}]^{\frac{1}{1-2\alpha}}$, $p_{iW}(Q_j) = \alpha\, Q_{iW}(Q_j)^\alpha Q_j^\beta$

<u>Proof</u> Quality formulae follow from Lemmas 3.1, 3.2 and 3.3 with $v(Q_i, Q_j) = Q_i^\alpha Q_j^\beta$. The prices in (a) and (b) are the monopoly prices from Lemmas 3.1 and 3.2. For the fan welfare maximizer price is $p_i = p_{iL}(Q_i, Q_j) = \frac{1}{2} Q_i^\alpha Q_j^\beta - \frac{1}{2}\sqrt{Q_i^{2\alpha} Q_j^{2\beta} - 4Q_i / \mu_i}$

$= \frac{1}{2} Q_i^\alpha Q_j^\beta [1 - \sqrt{1 - 4Q_i / \mu_i Q_i^{2\alpha} Q_j^{2\beta}}\,]$. But with $Q_i = Q_{iW}(Q_j)$,

$4Q_{i_i} / \mu_i\, Q_i^{2\alpha} Q_j^{2\beta} = 4\alpha(1-\alpha)$ and so $p_i = \frac{1}{2} Q_i^\alpha Q_j^\beta [1 - (1-2\alpha)]$ *Q.E.D.*

Notice that all 3 games entail global strategic complementarity – as *j*'s quality increases, *i*'s best response is to increase its own expenditure on talent.[16] All 3 games have a unique Nash equilibrium which can now be computed (with qualities, prices and attendances denoted $Q_{iX}, p_{iX}, A_{iX}, i = 1{,}2, X = \Pi, Q, W$). The conditions for all 3 equilibria can be written:

$$Q_{1X}^{1-2\alpha} = r_X \mu_1 Q_{2X}^{2\beta}, \quad Q_{2X}^{1-2\alpha} = r_X \mu_2 Q_{1X}^{2\beta} \tag{5.1}$$

for X= Π, Q, W with $r_\Pi = \frac{1}{2}\alpha$, $r_Q = \frac{1}{4}$, $r_W = \alpha(1-\alpha)$.

Hence, in all Nash equilibria we have:

$$Q_{1X} / Q_{2X} = (\mu_1 / \mu_2)^{\frac{1}{1-2\alpha+2\beta}} \tag{5.2}$$

Q_{1X} / Q_{2X} measures the league equilibrium competitive imbalance. (6.2) shows that this imbalance is the same in all 3 leagues, and in favour of the club with the bigger fan-base ($\mu_1 > \mu_2 \Leftrightarrow Q_{1X} > Q_{2X}$); the bigger club will have the higher quality team and be more likely to win the league. Substituting (5.2) into (5.1) gives the following equilibrium qualities:

<u>Lemma 5.2</u> In the absence of capacity constraints and with the Cobb-Douglas valuation function, league Nash equilibrium team qualities are, for $i = 1,2, i \neq j$:

(a) $Q_{i\Pi} = (\frac{1}{2}\alpha\mu_i^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_j^{\frac{2\beta}{1-2\alpha+2\beta}})^{\frac{1}{1-2\alpha-2\beta}}$;

(b) $Q_{iQ} = (\frac{1}{4}\mu_i^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_j^{\frac{2\beta}{1-2\alpha+2\beta}})^{\frac{1}{1-2\alpha-2\beta}}$

(c) $Q_{iW} = [\alpha(1-\alpha)\mu_i^{\frac{1-2\alpha}{1-2\alpha+2\beta}} \mu_j^{\frac{2\beta}{1-2\alpha+2\beta}}]^{\frac{1}{1-2\alpha-2\beta}}$

We now compare equilibrium outcomes, both within and between leagues. Within leagues we have:

<u>Theorem 5.1</u> In the absence of capacity constraints and with the Cobb-Douglas valuation function, for $X = \Pi, Q, W$:

(a) $Q_{1X} - Q_{2X}$ has the sign of $(\mu_1 - \mu_2)$;

(b) $p_{1X} - p_{2X}$ has the sign of $(f - 1/2)(\mu_1 - \mu_2)$;

(c) $A_{1X} - A_{2X}$ has the sign of $(\mu_1 - \mu_2)$.

<u>Proof</u> (a) follows from (6.2), as already remarked.

(b)For X= Π,Q,W, from Lemma 6.1;

$$\frac{p_{1X}}{p_{2X}} = \frac{v(Q_{1X}, Q_{2X})}{v(Q_{1X}, Q_{2X})} = \frac{Q_{1X}^\alpha Q_{2X}^\beta}{Q_{2X}^\alpha Q_{1X}^\beta} = (\frac{Q_{1X}}{Q_{2X}})^{\alpha-\beta} = (\frac{\mu_1}{\mu_2})^{\frac{e(2f-1)}{1+2e(1-2f)}}$$

The result follows.

(c) $A_{iX} = \mu_i[v(Q_{iX}, Q_{jX}) - p_{iX}]$ and for X= Π,Q,W;

$$\frac{A_{1X}}{A_{2X}} = \frac{\mu_1}{\mu_2}(\frac{Q_{1X}}{Q_{2X}})^{\alpha-\beta} = (\frac{\mu_1}{\mu_2})^{\frac{1+e(1-2f)}{1+2e(1-2f)}}$$

This ensures the result. *Q.E.D.*

Thus the bigger club in the Π, Q or W-league will not only have the better team but will also have larger match attendances and (with the usual fan bias of $f \geq 1/2$) charge the higher admission price. For the between league comparisons we have first:

Theorem 5.2 In the absence of capacity constraints and with the Cobb-Douglas valuation function, for i=1,2:
(a) $Q_{iQ} > Q_{iW} > Q_{i\Pi}$;
(b) $p_{iQ} > p_{iW}$ and $p_{iQ} > p_{i\Pi}$;
(c) $A_{iQ} > A_{i\Pi}$, $A_{iW} > A_{i\Pi}$ and $A_{iQ} > A_{iW}$ if and only if $1 > 2(ef)^{1-e}(ef)^{e}$.

Proof (a) follows from Lemma 5.2 since $\frac{1}{4} > \alpha(1-\alpha) > \frac{1}{2}\alpha$ when $\alpha \in (0,1/2)$.

(b)Using Lemma 5.2 and the abbreviation $\hat{\mu}_i = [\mu_i^{\frac{\alpha-2\alpha^2+2\beta^2}{1-2\alpha+2\beta}} \mu_j^{\frac{\beta}{1-2\alpha+2\beta}}]^{\frac{1}{1-2\alpha-2\beta}}$, we have;

$$p_{iQ} = \frac{1}{2}v(Q_{iQ}, Q_{jQ}) = \frac{1}{2}(\frac{1}{4})^{\frac{\alpha+\beta}{1-2\alpha-2\beta}}\hat{\mu}_i,$$
$$p_{i\Pi} = \frac{1}{2}v(Q_{i\Pi}, Q_{j\Pi}) = \frac{1}{2}(\frac{1}{2}\alpha)^{\frac{\alpha+\beta}{1-2\alpha-2\beta}}\hat{\mu}_i,$$
$$p_{iW} = \alpha v(Q_{iW}, Q_{jW}) = \alpha[\alpha(1-\alpha)]^{\frac{\alpha+\beta}{1-2\alpha-2\beta}}\hat{\mu}_i.$$

$p_{iQ} > p_{iW}$ and $p_{iQ} > p_{i\Pi}$ follow immediately since $\alpha \in (0,1/2)$.

(c) $A_{iQ} = \frac{1}{2}\mu_i v(Q_{iQ}, Q_{jQ})$, $A_{i\Pi} = \frac{1}{2}\mu_i v(Q_{i\Pi}, Q_{j\Pi})$ and $A_{iW} = (1-\alpha)\mu_i v(Q_{iW}, Q_{jW})$.
$A_{iQ} > A_{i\Pi}$ immediately from (a). From Lemma 5.2, $A_{iW} > A_{i\Pi}$ is;

$$(1-\alpha)[\alpha(1-\alpha)]^{\frac{\alpha+\beta}{1-2\alpha-2\beta}} > \frac{1}{2}(\frac{1}{2}\alpha)^{\frac{\alpha+\beta}{1-2\alpha-2\beta}}$$

This becomes $2(1-\alpha) > 1$, which always holds as $\alpha < 1/2$. Again from Lemma 5.2, $A_{iQ} > A_{iW}$ is;

$$\frac{1}{2}(\frac{1}{4})^{\frac{\alpha+\beta}{1-2\alpha-2\beta}} > (1-\alpha)[\alpha(1-\alpha)]^{\frac{\alpha+\beta}{1-2\alpha-2\beta}}$$

which rearranges into the claimed inequality substituting $\alpha + \beta = e$ and $\alpha = ef$
Q.E.D.

The result mirrors the earlier best response comparison (Theorem 3.1), with one exception. It can now be that $A_{iQ} > A_{iW}$, and this has important consequences for welfare. The following is the general formula for aggregate surplus in a league equilibrium with qualities and prices $Q_i, p_i, i = 1,2$:

$$S(Q_1, Q_2, p_1, p_2) = \int_0^{v(Q_1,Q_2)-p_1} \mu_1[v(Q_1,Q_2) - p_1 - x]dx +$$
$$\int_0^{v(Q_2,Q_1)-p_2} \mu_2[v(Q_2,Q_1) - p_2 - x]dx$$
$$+ p_1\mu_1[v(Q_1,Q_2) - p_1] + p_2\mu_2[v(Q_2,Q_1) - p_2] - Q_1 - Q_2 \qquad (5.3)$$

Here the first 2 terms are aggregate consumer (fan) surplus and the remaining terms are aggregate producer (club) surplus. Consider first the comparison between the Q-league and the W-league equilibrium. In both cases producer surplus is zero (low break-even pricing) and writing attendance as $A_i = \mu_i[\ v(Q_i, Q_j)\ - p_i]$, (5.3) abbreviates to:

$$S(A_1, A_2) = A_1^2 / 2\mu_1 + A_2^2 / 2\mu_2$$

Thus the league which produces the higher attendances produces the greater social welfare. Writing $S_X, X = Q,W$ (or Π later) as the value of $S(Q_1, Q_2, p_1, p_2)$ at the X-league equilibrium, Theorem 5.2(c) produces:

<u>Theorem 5.3</u> In the absence of capacity constraints and with the Cobb-Douglas valuation function, $S_Q > S_W$ if and only if $1 > 2(1-ef)^{1-e}(ef)^e$.

We return later to the condition on *e,f* in the last 2 theorems but note now that, unlike the monopoly excludable public good model, the fan (consumer) welfare maximizing owner objective no longer necessarily produces the best outcome of the three under investigation. In Section 4 the consumer welfare maximizer's objective reduced to aggregate consumer surplus, so such an owner automatically generated the highest social welfare with low break-even pricing, dominating the choice of a quality maximizer in particular. Now in the W-league each club's objective reduces to maximizing consumer surplus *of their own fans*, but they overlook the beneficial impact of increases in their team quality on fans of the other club. There is no corresponding difference in the Q-league, and the Q-league equilibrium can now be socially superior as a result. Nevertheless, the W-league qualities are still higher than those of the Π-league and the W-league equilibrium is always socially superior to the Π-league. The following Lemma facilitates the remaining comparisons in Theorem 5.4 (see appendix for proofs):

<u>Lemma 5.3</u>

(a) $S_\Pi = Q_{1\Pi}(\dfrac{3}{4\alpha} - 1)[1 + (\dfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]$

(b) $S_W = \dfrac{1}{2} Q_{1W}(\dfrac{1}{\alpha} - 1)[1 + (\dfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]$

(c) $S_Q = \dfrac{1}{2} Q_{1Q}[1 + (\dfrac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]$

<u>Theorem 5.4</u> In the absence of capacity constraints and with the Cobb-Douglas valuation function:

(a) $S_W > S_\Pi$;

(b) $S_Q > S_\Pi$ if and only if $1 > (3-4ef)^{1-2e}(2ef)^{2e}$.

Figure 5.1 summarizes the welfare comparisons.

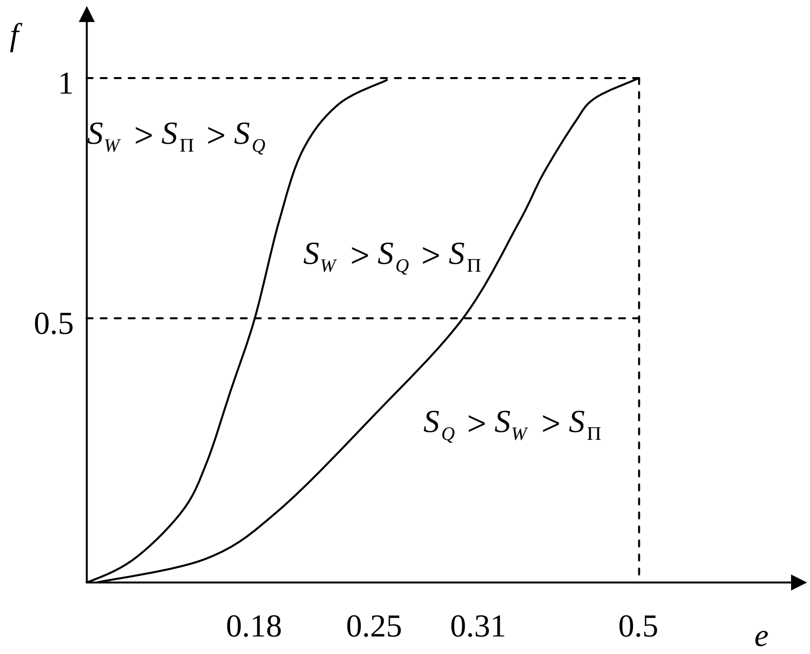

Figure 5.1

Thus the lower quality and (relative) high prices of the Π-league continues to mean that it produces always socially inferior outcomes to the W-league, but it may dominate the Q-league when the elasticity of willingness to pay for quality is low, similar qualitatively to the excludable public good conclusion of section 4. Returning to the comparison between the W-league and the Q-league in the light of Figure 5.1, notice that when *f*=1 the sports league externality disappears, and we are effectively back in the excludable public good model; as in section 4, S_W is then always the largest. Although not realistic, it is helpful to consider the other extreme where *f*=0. Here fans have an interest only in the quality of the away team, and there is no incentive for clubs in a W-league to invest in talent, leading to zero quality equilibrium – see Lemma 5.2(c). But the quality maximizer still has incentives to invest and the Q-league (Lemma 5.2(b)) produces a non-degenerate positive quality equilibrium which is now always socially the best of the three. As *f* falls from 1 towards 0 and the externality impact of away team quality on home fan utility increases, the social attractiveness of the W-league compared to the Q-league gradually diminishes.

6. CAPACITY CONSTRAINTS

Suppose $k_i < A_{iQ}(Q_j)$, so that the unconstrained best response of a quality maximizer is infeasible. Figure 6.1 illustrates the feasible set now facing the quality maximizer. In the downward shaded region, profits are non-negative and ticket demand does not exceed capacity ($v(Q_i, Q_j) - p_i = k_i / \mu_i$ is the contour where ticket demand equals capacity). In the upward shaded region, ticket demand is above capacity and the k_i available tickets would have to be rationed amongst fans (in some way that is irrelevant for now); the region is bounded below by the capacity non-negative profit constraint, $p_i k_i \geq Q_i$.

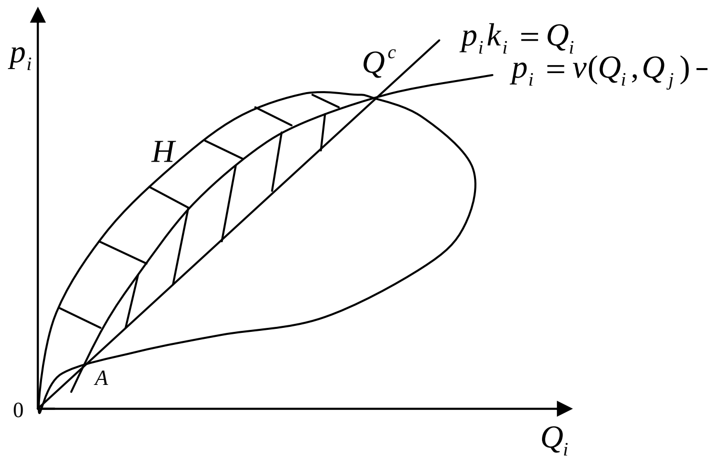

Figure 6.1

The solution is clearly at Q^c shown, on the capacity ticket demand contour, so there is no rationing of ticket demand, and the optimal choice is on H, so chosen prices are now in the elastic part of the ticket demand curve.

Consider next the constrained profit maximizer, and suppose the configuration shown in Figure 6.1 precludes the profit maximizer's unconstrained optimum also. Profit contours below the capacity ticket demand contour are straight lines parallel to OAQ^c, and the solution will occur again on the capacity ticket demand contour, now at a tangency with a linear profit contour, shown as Π^c in Figure 6.2.

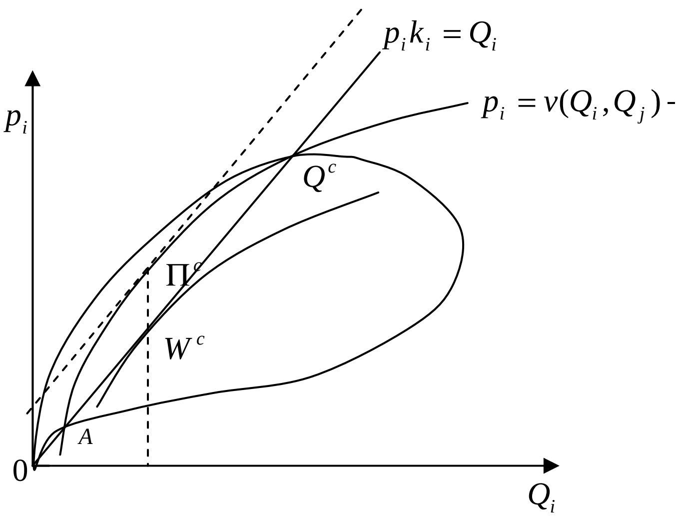

Figure 6.2

The solution is characterised by the conditions $p_i = v(Q_i, Q_j) - k_i / \mu_i$ and $k_i v_i(Q_i, Q_j) = 1$, and again involves no rationing of ticket demand and prices in the elastic section of the ticket demand curve.[17]

Detailed calculations of capacity constrained league equilibria (as in section 5's unconstrained results) are intractable. However the above constrained best response analysis allows the following statement:

Theorem 6.1 In an equilibrium of the Q-league or the Π-league in which capacity constraints are binding on both clubs, there is no rationing of ticket demand, and prices are in the elastic section of the ticket demand curve.

In the constrained welfare maximization case we need to be precise about the nature of demand rationing when it occurs, and the obvious first assumption is:

Efficient Rationing The k_i tickets (price $p_i < v(Q_i, Q_j) - k_i / \mu_i$ so there is excess demand) are allocated to fans with the greatest valuation, namely those with $x \in [0, k_i / \mu_i]$. Fans with $x \in [k_i / \mu_i, v(Q_i, Q_j) - p_i]$ would like a ticket but receive none.

When $p_i < v(Q_i, Q_j) - k_i / \mu_i$, the fan welfare maximizer's objective function changes to:

$$W_i = \int_0^{k/\mu_i} \mu_i [v(Q_i, Q_j) - p_i - x] dx = k_i [v(Q_i, Q_j) - p_i] - \frac{1}{2} k_i^2 / \mu_i \qquad (6.1)$$

However, this is still a monotone increasing transformation of $v(Q_i, Q_j) - p_i$, and the contour map remains as in section 3 (Figure 3.3). In Figure 6.2 the profit maximizer wanted to shift the line AQ^c vertically up, to tangency with the capacity ticket demand contour. The fan welfare maximizer wants to shift the capacity ticket demand contour vertically down to tangency with AQ^c, as indicated in Figure 6.2. It is clear therefore that fan welfare and profit maximizers choose the same quality when similarly capacity constrained, but the fan welfare maximizer's solution (W^c in Figure 6.2) involves a lower price, and hence demand rationing.
The precise conditions characterising W^c are $k_i v_i(Q_i, Q_j) = 1$ (as for Π^c) and $p_i = Q_i / k_i$, and the solution still entails inelastic pricing,[18] as in the unconstrained case. Quite differently from Theorem 6.1, we have:

Theorem 6.2 Under the assumption of efficient rationing, in an equilibrium of the W-league in which capacity constraints are binding on both clubs, there is rationing of ticket demand, and price is in the inelastic section of the ticket demand curve.

The efficient rationing assumption precludes active black markets in tickets, common at many matches, since the only fans prepared to pay more than the official ticket price receive tickets. In addition, it is not clear that it is a realistic assumption – certainly one would not expect clubs to be able to acquire the information to allocate tickets in this way. If tickets are sold at a stadium ticket office, or on-line, one would expect the allocation would be more random amongst applicants, opening up the possibility of a black market. An alternative rationing assumption is:

<u>Random Rationing with a Black Market</u> At stage 1 the k_i tickets (sold at the official price $p_i < v(Q_i, Q_j) - k_i / \mu_i$ so there is excess demand) are allocated randomly to applicants. At stage 2 there is a black market where applicants may re-trade their initial allocation at a price b_i. If $b_i > p_i$ is anticipated, the set of applicants at stage 1 would be large. To simplify notation (it makes no qualitative difference) we assume clubs allocate tickets at stage 1 only to applicants who are fans (precluding others, professional ticket touts maybe). But still the entire set F_i would apply at stage 1, with intentions of attending the match or making a black market profit. Of the fans with heterogeneity parameter $x(\in [0, c_i])$, k_i/c_i would receive tickets in the random stage 1 allocation, and the remaining $\mu_i - k_i/c_i$ would be frustrated. At stage 2 fans without a ticket buy on the black market if $v(Q_i, Q_j) - b_i - x \geq 0$, or $x \in [0, v(Q_i, Q_j) - b_i]$; black market demand is $B_i^D = (\mu_i - \frac{k_i}{c_i})[v(Q_i, Q_j) - b_i]$. Fans with tickets sell if $v(Q_i, Q_j) - p_i - x \leq b_i - p_i$, or $x \in [0, v(Q_i, Q_j) - b_i]$ giving a black market supply $B_i^S = [c_i - (v(Q_i, Q_j) - b_i)]\frac{k_i}{c_i}$.

The black market clearing price is $b_i = v(Q_i, Q_j) - k_i / \mu_i$, and tickets end up with all fans with $x \in [0, k_i/\mu_i]$, as under efficient rationing, k_i/c_i of them paying p_i and $\mu_i - k_i/c_i$ paying b_i ; a black market profit of $b_i - p_i$ accrues to k_i/c_i of the fans with $x \in [k_i / \mu_i, c_i]$.

Under this assumption, when $p_i < v(Q_i, Q_j) - k_i / \mu_i$ the fan welfare maximizer's objective becomes:

$$W_i = \int_0^{k/\mu_i} k_i / c_i [v(Q_i, Q_j) - p_i - x]dx \quad + \quad \int_0^{k/\mu_i} (\mu_i - k_i / c_i)[v(Q_i, Q_j) - p_i - x]dx \quad + \int_{k_i/\mu_i}^{c_i} k_i / c_i [b_i - p_i]dx = \int_0^{c_i} k_i / c_i [b_i - p_i]dx + \int_0^{k/\mu_i} \mu_i [v(Q_i, Q_j) - b_i - x]dx$$

$$= k_i [v(Q_i, Q_j) - p_i] - \frac{1}{2} k_i^2 / \mu_i \tag{6.2}$$

This is exactly the same as in (6.1) and so the alternative rationing assumption does not change the best response, which remains as in Figure 6.2. Hence:

<u>Corollary to Theorem 6.2</u> Under the assumption of random rationing with a black market, the conclusions of Theorem 6.2 hold. In addition the W-league equilibrium is now characterised by an active black market.

We start the welfare analysis with capacity constraints for the underlying excludable public good model. If facility capacity is low enough to be binding on both a profit and consumer welfare maximizer ($(k < A_\Pi (< A_W))$, the above results (Figure 6.2) imply that both these objectives produce the same level of quality (defined by

$kv'(Q) = 1$, so $dQ/dk = -v'(Q)/v''(Q) > 0$), attendances at capacity and different prices. Given capacity attendance, formulae for social welfare in either case are:

$$S(Q, p, k) = \int_0^{k/\mu} \mu[v(Q) - p - x]dx + \mu p[v(Q) - p] - Q = \int_0^{k/\mu} \mu[v(Q) - x]dx$$

Since qualities are the same it follows that $S_W = S_\Pi$ now (the different prices change the distribution of aggregate surplus but not its level), and since Q increases with k, S_W and S_Π do so also. When $A_\Pi \le k < A_W$ the capacity constraint binds only on the consumer welfare maximizer, and again the above formulae show that S_W increases with k in this range. Hence, from Theorem 4.1, $S_W > S_\Pi$ if $A_\Pi < k$, and $S_W = S_\Pi$ if $k \le A_\Pi$, as shown in Figure 6.3(a).

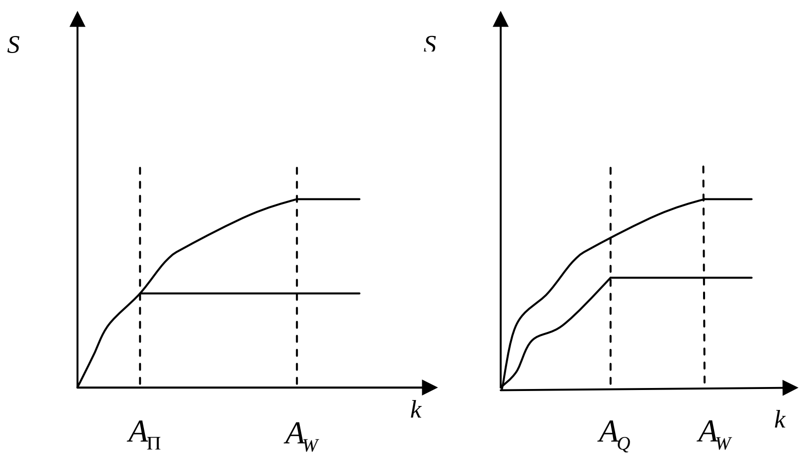

Figure 6.3(a) Figure 6.3(b)

Social welfare can also be written $S = k[v(Q) - p] - k^2/2\mu$ when p is a low break-even price, and again this is a monotone increasing transformation of $v(Q) - p$. It is clear from Figure 6.3 that $S_W > S_Q$ continues to hold (as it did in Theorem 4.1 when $k \ge A_W$) when $k < A_Q$ and capacity is binding on both the quality and consumer welfare maximizer. For $k \in [A_Q, A_W)$, S_W is increasing in k, as above, and Figure 6.3(b) follows.

Thus, whether capacity constraints bind or not, the outcome under consumer welfare maximization at least weakly dominates the other 2 possibilities in the excludable public good setting.

In a Π -league and a W-league (under either rationing regime) where capacity constraints bind on both teams in both leagues, social welfare can be written:

$$S(Q_1, Q_2, p_1, p_2, k_1, k_2) = \int_0^{k_1/\mu_1} \mu_1[v(Q_1, Q_2) - p_1 - x]dx +$$
$$\int_0^{k_2/\mu_2} \mu_2[v(Q_2, Q_1) - p_2 - x]dx + p_1 k_1 - Q_1 + p_2 k_2 - Q_2$$
$$= \int_0^{k_1/\mu_1} \mu_1[v(Q_1, Q_2) - x]dx + \int_0^{k_2/\mu_2} \mu_2[v(Q_2, Q_1) - x]dx \qquad (6.3)$$

In both leagues equilibrium qualities will be the same since both are defined by the same 2 equations (namely $k_i v_i(Q_i, Q_j) = 1, i = 1,2, j \neq i.$ It follows that $S_W = S_\Pi$.

Theorem 6.3 Social welfare is the same in the equilibrium of a W-league and a Π -league if capacity constraints bind on both clubs in both leagues, and if there is efficient rationing or random rationing with a black market.

A full picture of the relation between S_W , S_Π and k_1, k_2 is intractable, as is capacity constrained analysis of the Q-league. However the following does provide some more information about the former (see appendix for proof)

Theorem 6.4 In a Π -league and a W-league, assume capacities are proportional to capacity unconstrained league equilibrium attendances ($k_1 / k_2 = A_{1X} / A_{2X}, X = W, \Pi$) , assume the Cobb-Douglas valuation function and assume either efficient rationing or random rationing with a black market. Then:

(a) If $A_{iW} > k_i > A_{i\Pi}, i = 1,2$, then social welfare in the W-league equilibrium decreases with k_i and is strictly greater than in the Π -league equilibrium;

(b) If $A_{i\Pi} \geq k_i, i = 1,2$ then social welfare in the W-league equilibrium is the same as in the Π -league equilibrium, and decreases with k_i .

Coupled with Theorem 5.4(a), this result shows that Figure 6.3(b) can be reinterpreted with k as k_i , A_Π as $A_{i\Pi}$ and A_W as A_{iW} , to show the relation between W and Π -league social welfare under the restrictions of Theorem 6.4.
In the results so far Π -league equilibria are always at least weakly socially dominated by the W-league equilibria. We mention finally a case where this reverses.

Random Rationing The ticket allocation mechanism is just stage 1 of the previous mechanism – now there is no possibility of ticket re-sale.
The objective function now becomes:

$$W_i = \int_0^{v(Q_i, Q_j) - p_i} \frac{k_i}{v(Q_i, Q_j) - p_i} [v(Q_i, Q_j) - p_i - x]dx = k_i [v(Q_i, Q_j) - p_i]/2 \qquad (6.4)$$

This is now lower than the value in (6.1) and (6.2), because of the inefficiency and loss of consumer surplus of random rationing, not now rescued by the black market. However, (6.4) is still a monotone increasing transformation of $v(Q_i, Q_j) - p_i$, and constrained best responses are the same as under the other rationing regimes. The consequence of these two observations is:

Theorem 6.5 Social welfare is higher in the Π-league equilibrium than in the W-league equilibrium if capacity constraints bind on both clubs in both leagues, and if there is random rationing.

The results of this section relate to the wider literatures on rationing and on black markets. Finding explanations for why optimizing agents with market power over prices would make choices that lead to rationing on the other side of the market is difficult when agents have "standard" objectives. The best known such story is the efficiency wage explanation for involuntary unemployment in the labour market context (see also Kaas and Madden (2002), Madden and Silvestre (1991, 1992)). Here the explanation is simple and obvious – agents set prices so low that consumers are rationed because they care about the welfare of the consumers. The fact that black markets can rescue the inefficiency of "official" non-market-clearing prices is certainly well-known. For instance, Polterovich (1993) has provided an extensive general equilibrium study, with exogenous prices, of the properties of our black market mechanism and other mechanisms for dealing with such disequilibria. The results of this section are of interest per se, in bringing together an argument showing how the strategic interaction of optimising agents can lead to equilibria with rationing and active black markets, in a context where such markets are seen in reality.

7. CONCLUSIONS

In a theoretical industrial organisation model of a professional sports league, the paper has introduced the owner objective of fan welfare maximization, and studied its consequences for club and league performance, comparing with the more commonly studied profit and quality maximization objectives. Whilst the model is basic in a number of ways and begs further developments in a number of directions[19], it does begin to address the issues of ticket pricing, match attendances, stadium capacity constraints and social welfare, which are absent from almost all previous models. Some of the findings suggest that fan welfare maximization can provide interesting explanations for active black markets for match tickets, and for the empirically observed inelastic pricing of match tickets, phenomena that are not similarly explicable in our model under profit or quality maximization. Of the three governance mechanisms, fan welfare maximization provides socially the most desirable outcome sometimes (when the fan bias, f, is high and/or the elasticity of willingness to pay for tickets, e, is low), and quality maximization is the best sometimes (f low, e high). Profit maximization does not perform socially as well. The only case found where it is not at least weakly dominated by fan welfare maximization is when stadium capacities are small and the fan welfare maximizer allocates tickets inefficiently.

The paper has suggested both positive and normative reasons as to why profit maximization may be a less interesting objective than others in the context. It is perhaps not too surprising that an industry (as viewed here) characterised by imperfectly competitive firms producing outputs which have public good features with between firm externalities should not be well served by profit maximizing firms. Future research maybe should focus more on alternative objectives.

APPENDIX

Proof of Lemma 5.3

With monopoly pricing, the formula (5.3) for social welfare becomes;

$$S=\frac{3}{8}\mu_1 v(Q_1,Q_2)^2+\frac{3}{8}\mu_2 v(Q_1,Q_2)^2-Q_1-Q_2 \qquad \text{(A1)}$$

Introducing the Cobb-Douglas valuation function and (5.2) into (A1) gives:

$$S=\frac{3}{8}\mu_1 Q_1^{2\alpha}Q_2^{2\beta}+\frac{3}{8}\mu_2 Q_1^{2\beta}Q_2^{2\alpha}-Q_1-Q_2$$

$$=\frac{3}{8}\mu_1 Q_1^{2\alpha+2\beta}\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\beta}{1-2\alpha+2\beta}}+\frac{3}{8}\mu_2 Q_1^{2\alpha+2\beta}\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\alpha}{1-2\alpha+2\beta}}-Q_1\left(1+\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right)$$

$$=Q_1\left\{Q_1^{2\alpha+2\beta-1}\frac{3}{8}\left[\mu_1\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\beta}{1-2\alpha+2\beta}}+\mu_2\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\alpha}{1-2\alpha+2\beta}}\right]-1-\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right\} \qquad \text{(A2)}$$

(a) Substituting Lemma 5.2(a) into (A2) gives;

$$S_\Pi=Q_{1\Pi}\left\{\frac{\frac{3}{8}\left[\mu_1\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\beta}{1-2\alpha+2\beta}}+\mu_2\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\alpha}{1-2\alpha+2\beta}}\right]}{\frac{1}{2}\alpha\mu_1^{\frac{1-2\alpha}{1-2\alpha+2\beta}}\mu_2^{\frac{2\beta}{1-2\alpha+2\beta}}}-1-\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right\}$$

$$=Q_{1\Pi}\left\{\frac{3}{4\alpha}\left(1+\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right)-1-\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right\}$$, which rearranges as required.

(b)Substituting Lemma 5.2(b) into (A2) gives;

$$S_Q=Q_{1Q}\left\{\frac{\frac{3}{2}\left[\mu_1\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\beta}{1-2\alpha+2\beta}}+\mu_2\left(\frac{\mu_2}{\mu_1}\right)^{\frac{2\alpha}{1-2\alpha+2\beta}}\right]}{\mu_1^{\frac{1-2\alpha}{1-2\alpha+2\beta}}\mu_2^{\frac{2\beta}{1-2\alpha+2\beta}}}-1-\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right\}$$

$$=Q_{1Q}\left\{\frac{3}{2}\left(1+\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right)-1-\left(\frac{\mu_2}{\mu_1}\right)^{\frac{1}{1-2\alpha+2\beta}}\right\}$$, as required.

(c)With low break-even pricing and the Cobb-Douglas valuation function, the social welfare formula (5.3) becomes;

$$S=\frac{1}{4}\mu_1 Q_1^{\alpha}Q_2^{\beta}\left[Q_1^{\alpha}Q_2^{\beta}+\sqrt{Q_1^{2\alpha}Q_2^{2\beta}-4Q_1\mu_1}\right]+$$

$$\frac{1}{4}\mu_2 Q_1^{\beta}Q_2^{\alpha}\left[Q_1^{\beta}Q_2^{\alpha}+\sqrt{Q_1^{2\beta}Q_2^{2\alpha}-4Q_2\mu_2}\right]-\frac{1}{2}Q_1-\frac{1}{2}Q_2 .$$

Using (5.2) and rearranging gives;

$$S=\frac{1}{4}Q_1\left\{\mu_1(\frac{\mu_2}{\mu_1})^{\frac{2\beta}{1-2\alpha+2\beta}}Q_1^{2\alpha+2\beta-1}[1+\sqrt{1-\frac{4}{\mu_1}Q_1^{1-2\alpha-2\beta}(\frac{\mu_1}{\mu_2})^{\frac{2\beta}{1-2\alpha+2\beta}}}]\right\}+$$

$$\frac{1}{4}Q_1\left\{\mu_2(\frac{\mu_2}{\mu_1})^{\frac{2\alpha}{1-2\alpha+2\beta}}Q_1^{2\alpha+2\beta-1}[1+\sqrt{1-\frac{4}{\mu_2}Q_1^{1-2\alpha-2\beta}(\frac{\mu_2}{\mu_1})^{\frac{1-2\alpha}{1-2\alpha+2\beta}}}]\right\}-\frac{1}{2}Q_1-\frac{1}{2}Q_1(\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}$$

Substituting Lemma 5.2(c) gives;

$$S_W=\frac{1}{4}Q_{1W}\left\{\frac{1}{\alpha(1-\alpha)}[1+\sqrt{1-4\alpha(1-\alpha)}]+\frac{1}{\alpha(1-\alpha)}(\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}[1+\sqrt{1-4\alpha(1-\alpha)}]-2-2(\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}\right\}$$

$$=\frac{1}{4}Q_{1W}\left\{\frac{2}{\alpha}[1+(\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}]-2-2(\frac{\mu_2}{\mu_1})^{\frac{1}{1-2\alpha+2\beta}}\right\}$$, which becomes the required formula. *Q.E.D.*

<u>Proof of Theorem 5.4</u> (a) From Lemma 5.3, $S_W > S_\Pi$ if and only if $Q_{1W}(1-\alpha) > Q_{1\Pi}(\frac{3}{2}-2\alpha)$. From Lemma 5.2 this requirement becomes $2(1-\alpha)^{2-2\alpha-2\beta} > (\frac{3}{2}-2\alpha)^{1-2\alpha-2\beta}$, which, with $\alpha = ef$ and $e=\alpha+\beta$, in turn becomes;

$$\Psi(e,f)=2(1-ef)^{2-2e}-(\frac{3}{2}-2ef)^{1-2e}>0\,.$$

Consider first the case where $f=1, e\in(0,1/2)$. $\Psi(e,1)>0$ is equivalent to;

$$2(\frac{3}{2}-2e)^{2e}(1-e)^{2-2e}>\frac{3}{2}-2e$$

But $2(\frac{3}{2}-2e)^{2e}(1-e)^{2-2e}>2(1-e)^{2e}(1-e)^{2-2e}=2(1-e)^2$ (since $\frac{3}{2}-2e>1-e$ as $e<1/2$), and $2(1-e)^2>\frac{3}{2}-2e$ is equivalent to $(\frac{1}{2}-e)^2>0$ which is true. Thus the required inequality holds when $f=1, e\in(0,1/2)$. For any $f\in[\frac{1}{2},1], e\in(0,1/2)$, consider now $\partial\Psi/\partial f$;

$$\partial\Psi/\partial f=-4(1-e)e(1-ef)^{1-2e}+(1-2e)2e(\frac{3}{2}-2ef)^{-2e}<0 \text{ if and only if}$$

$$1-2e<(1-e)(1-ef)^{1-2e}(\frac{3}{2}-2ef)^{2e}$$

But this last inequality holds since $\frac{3}{2}-2ef>1-ef, (ef<1/2)$ and $1-2e<(1-e)(1-ef)=1+e^2f-e-ef$, or $e(1-f)+e^2f>0$. Thus $\partial\Psi/\partial f<0$ for any $f\in[\frac{1}{2},1], e\in(0,1/2)$, and so $\Psi(e,f)>0$ for any such *e,f*.

(b) Combining Lemmas 5.2 and 5.3 as in (a), $S_Q > S_\Pi$ if and only if $1>(2\alpha)^{2\alpha+2\beta}(3-4\alpha)^{1-2\alpha-2\beta}$, which becomes the condition claimed with $\alpha=ef, e=\alpha+\beta$. *Q.E.D.*

Proof of Theorem 6.4 (a) An equilibrium of the W-league with capacity constraints binding on both clubs is characterised by the best response conditions; $k_i v_i(Q_i, Q_j) = 1$, $p_i = Q_i / k_i$, for i=1,2, $j \neq i$. With the Cobb-Douglas valuation function equilibrium qualities are defined by; $Q_i^{1-\alpha} = \alpha k_i Q_j^{\beta}$, i=1,2, $j \neq i$, giving

$$Q_i^{1-\alpha-\beta} = k_i^{\frac{1-\alpha}{1-\alpha+\beta}} k_j^{\frac{\beta}{1-\alpha+\beta}} \qquad \text{(A1)}$$

Notice $\partial Q_i / \partial k_i, \partial Q_i / \partial k_j > 0$. The capacity constraints bind if $\mu_i [v(Q_i, Q_j) - p_i] > k_i$, i=1,2, $j \neq i$, which after substitution, rearrangement and use of the assumed capacity restrictions become, for i=1,2, $j \neq i$;

$$k_i^{1-2\alpha-2\beta} < \alpha^{\alpha+\beta} (1-\alpha)^{1-\alpha-\beta} \mu_i^{\frac{1+2\alpha^2-3\alpha-2\beta^2}{1-2\alpha+2\beta}} \mu_j^{\frac{\beta}{1-2\alpha+2\beta}} .$$

Using Lemma 5.2 to compute unconstrained equilibrium attendance shows that the above inequality is the same as $k_i < A_{iW}$. Thus under the capacity assumptions (A1) there is indeed an equilibrium described by (A1) in which capacity constraints are binding on both clubs in the W-league.

The social welfare formula in (6.3) becomes;

$S = k_1 v(Q_1, Q_2) - k_1^2 / 2\mu_1 + k_2 v(Q_2 Q_1) - k_2^2 / 2\mu_2 - Q_1 - Q_2$.

Hence;

$\partial S / \partial k_i = v(Q_i, Q_j) + k_i v_i (Q_i, Q_j)[\partial Q_i / \partial k_i + \partial Q_i / \partial k_j] - k_i / \mu_i - \partial Q_i / \partial k_i - \partial Q_j / \partial k_i$

$+ v(Q_2, Q_1) + k_j v_j (Q_j, Q_i)[\partial Q_i / \partial k_i + \partial Q_j / \partial k_i] - k_j / \mu_j$

$= v(Q_i, Q_j) - k_i / \mu_i + v(Q_2, Q_1) - k_j / \mu_j + [\partial Q_i / \partial k_i + \partial Q_j / \partial k_i] > 0$.

Thus (generally) in equilibria where capacity constraints bind on both clubs social welfare increases with the capacity constraints. We know S_W and S_Π are constant and $S_W > S_\Pi$ if $k_i \geq A_{iW}$ (from Theorem 5.4), that S_W is increasing in k_i if $k_i < A_{iW}$, that S_Π is constant if $k_i \geq A_{i\Pi}$ and that $S_\Pi = S_W$ if $k_i \leq A_{i\Pi}$ (from Theorem 6.3). This ensures (a) and (b). *Q.E.D.*

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FOOTNOTES

[1] We use the following terms solely with their sporting meanings; club, team, match, player. However games and agents refer to their usual meanings in economic models.

[2] In one of the earlier papers in the literature Sloane (1971) suggested multi-dimensional club objective functions, with match attendance as one of the possible arguments. As will be seen, in some cases (non-binding capacity constraints) this is equivalent in fact to fan welfare.

[3] Barcelona F.C. is the largest of the members' clubs with over 100,000 members, and such a constitution is pervasive in English professional cricket. Holt et al. (2005) provide much detailed information on governance of UK soccer clubs, and the role of supporters trusts in particular.

[4] For instance, Atkinson, Stanley and Tschirhart (1988), Feess and Muehlheusser (2003), Fort and Quirk (1995, 2004), Hoehn and Szymanski (1999), Kesenne (2000,2004,2005), Marburger (1997), Quirk and El Hodiri (1974), Sanderson. and Siegfried (2003), Szymanski (2003,2004), Szymanski and Kesenne (2004). A focal issue has been the "invariance principle" whereby gate-sharing has no influence on the distribution of playing talent. Fort and Quirk (2004) compare (inconclusively) competitive balance under profit maximization and quality maximization.

[5] In our focus on the governance question we follow Fort and Quirk (2004).

[6] A second step would be to generalise to incorporate "atmosphere" effects, whereby fans perhaps get more enjoyment if attendance is larger.

[7] Essentially talent is perfectly elastically supplied at constant marginal cost normalised to unity. This assumption is more natural in (e.g.) the context of European soccer which is relatively "open" in that players can move freely between leagues, rather than in the major US sports leagues which are much more "closed", more or less the sole employers of the specialised playing talent. Relaxing our assumption would endow clubs with market power on both the input side (buying talent) and on the output side (selling tickets). Existing sports literature has studied both our open (European) supply and the closed (US) alternative, although the latter have largely ignored the resulting input market power of clubs. An exception is Palomino & Sakovics (2004), who pick up on the further fact that highly talented players also have labour market power. Hoehn and Szymanski (1999, and the subsequent discussions by C. Matutes and P.Seabright) and Pomfret (2006) provide wide-ranging analyses of European/US differences.

[8] Implicitly we are assuming that the full fan utility function is quasi-linear, defined over a numeraire (endowment y and large) and the match ticket. Full utility is then y without the ticket and $y - p_i + v(Q_i, Q_j) - x$ with the ticket. In the usual way, the use of aggregate fan surplus as the appropriate welfare measure is then legitimised. Notice that our assumption of a perfectly elastic supply of playing talent (the "open" league assumption in footnote 7) means that players gain no extra surplus from playing in our league, and so do not enter the social welfare evaluation. Given the supply assumption,

this seems appropriate, but differs from the social welfare specification of Falconieri et al. (2004).

[9] The assumption that $v(Q_i, Q_j)$ is increasing in Q_i *and* Q_j for a fan of *i* seems natural in our static, 2-club setting. A fan of a poor team would probably pay more in a one-off decision to see their team play highly talented opposition than a similar low-level team. However (in a multi-club, dynamic setting) it is not so clear that this fan would pay more for tickets through a season of persistent defeats against highly talented opposition than if their team was in a more balanced, low-level league.
[10] In some of the underlying excludable good literature our "regions" are "municipalities" or "jurisdictions" with powers to tax/subsidise residents, and possibly in receipt of transfers from the rest of the economy. Such powers do not seem natural in our context, and we assume that the club has to be self-financing.

[11] This kind of objective, although new in the context of a sports model, is common in the excludable public good literature – see, for instance, Fraser (1996, 2000), Traub & Missong (20005).

[12] The following can be shown (details omitted); (i) with a Cobb-Douglas valuation function, it is always the case that $p_{i\Pi}(Q_j) > p_{iW}(Q_j)$, but (ii) if $v(Q_i, Q_j)^2$ is separable and dependence on Q_i is piecewise linear and concave, the ranking is reversed eventually as this dependence approaches linearity; one can "smooth" this to fit our assumptions.

[13] Indeed Fort (2004, footnote 1) notes the possibility of alternative explanations of these facts, along the lines of our story, and cites earlier references making a similar point. Our contribution is a formal model of an objective which confirms this possibility.

[14] The first best outcome is as follows. Whatever quality Q is chosen, it will be socially optimal for all consumers to visit the facility for whom $x \le v(Q)$. Thus the maximum aggregate surplus from quality Q is:
$S(Q) = \int_0^{v(Q)} \mu(v(Q) - x)dx - Q = \frac{1}{2}\mu v(Q)^2 - Q$. *S(Q)* is a strictly concave function and the Inada assumption ensures a unique, socially optimal quality Q_S, characterised by: $\mu v(Q_S)v'(Q_S) = 1$, or $MR(Q_S) = 1/2$ .The ranking $Q_S > Q_W (> Q_\Pi)$ now follows. However Q_Q may be larger than Q_S . For instance, if $v(Q) = Q^e, 0 < e < 1/2$, then $Q_Q = (\mu/4)^{\frac{1}{1-2e}}$ and $Q_S = (\mu e)^{\frac{1}{2-2e}}$, so $Q_Q > Q_S$ if and only if *e<1/4*; if consumers' elasticity of willingness to pay for facility quality is low, the quality maximizer produces too high quality relative to the social optimum.

[15] As a model per se the scenario of this section probably fits best the zoo example. Zoos are typically fairly spread out geographically with plausibly non-intersecting catchment areas of potential visitors; they do have the major expenditure of acquisition and care of the animals, which reasonably equates to quality; and entry is usually uniformly priced.

[16] This property is a feature of the Cobb-Douglas specification. Whilst it is generally true that the Q-league equilibrium will exhibit strategic complementarity at least in the neighbourhood of equilibrium, the Π -league can have equilibria with local strategic substitutability (e.g. with suitable CES valuation functions). In our sport league context the complementarity seems more plausible.

[17] The Lagrangean is $L = \mu_i p_i [v(Q_i, Q_j) - p_i] - Q_i + \lambda [k_i - \mu_i (v(Q_i, Q_j) - p_i]$, and a necessary Lagrangean condition for a solution is that $\partial L / \partial p_i = 0$, which implies $p_i - v(Q_i, Q_j)/2 = \lambda / 2$. So if the constraint is binding, price exceeds the monopoly value.

[18] Since $v(Q_i, Q_j)^2$ is strictly concave in Q_i, $v(Q_i, Q_j)^2 / Q_i > 2v(Q_i, Q_j) v_i (Q_i, Q_j)$, so $Q_i v_i (Q_i, Q_j) < v(Q_i, Q_j)/2$. The conditions characterising W^c imply $p_i = Q_i / k_i = Q_i v_i (Q_i, Q_j)$, which is therefore below the monopoly price.

[19] For instance: generalisation to more than 2 clubs, and the dynamic issues associated with sequential play of matches and the evolving nature of fan utility; a deeper investigation of the nature of fan utility, incorporating ex post feelings of happiness/sadness after victories/defeats; endogenisation of fan affiliation and analysis of competition between clubs for fans; incorporation of stadium costs and endogenous choice of stadium capacity; generalisation of the current “zero-infinity” congestion costs whereby additional fans at less than capacity have no impact on others utilities; relaxation of the non-negative profit constraint to encompass the behaviour of wealthy benefactors of clubs; study of “hybrid” leagues where objectives are mixtures (weighted averages, e.g.) of those studies here, or with differing intra-league club objectives; labour market imperfections generally, “closed” leagues (as in footnote 7) in particular; distributional issues intra-fans, and between fans, owners and players; regulatory policies other than the nature of club governance, such as revenue sharing, salary caps, ticket price controls; modelling the regulatory authority.