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## Human Capital Accumulation in a Stochastic Environment: Some New Results on the Relationship Between Growth and Volatility

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# Human Capital Accumulation in a Stochastic Environment: Some New Results on the Relationship Between Growth and Volatility

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## Abstract

We study the relationship between growth and volatility in a simple analytical model, where human capital accumulation depends on both deliberate and non-deliberate learning, and where stochastic fluctuations arise from both preference and technology shocks. We derive a number of new results which challenge some of the results in the existing literature. First, we show that the optimal allocations of time to working and learning are both pro-cyclical. Second, we identify a preference parameter (other than the coefficient of relative risk aversion) that is potentially crucial for governing the effect of volatility on growth. Third, we demonstrate how the correlation between growth and volatility can be either positive or negative under each type of learning. Fourth, we also reveal how the sign of the correlation may be different for the two types of shock. Our results may be seen as providing further explanation for the lack of robust evidence on the relationship.

**Keywords:** Human capital, learning, growth, volatility.

**JEL classification:** E32, O41.

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# 1 Introduction

An important issue in macroeconomics is the extent to which short-term fluctuations in an economy might influence long-term trends in economic performance. Evidence that such an influence exists is provided by a number of empirical studies which report statistically significant correlations between output growth and output volatility using various cross-section and time series data. What is notable about these correlations is not that they are quantitatively different in size (which is obviously to be expected), but that they can be qualitatively different in sign as well. Thus, whilst positive correlations are identified in some studies (e.g., Caporale and McKiernan 1996; Grier and Tullock 1989; Kormendi and Meguire 1985), negative correlations are detected in others (e.g., Kneller and Young 2001; Martin and Rogers 2000; Ramey and Ramey 1995). These conflicting results may reflect differences in methodologies and data (such as the measurement of volatility, the choice of conditioning variables and the coverage of the sample), and it has recently been shown how one can arrive at opposite conclusions depending on what group of countries is included in the cross-section (e.g., Fatas 2002) and what level of disaggregation is applied to the data (e.g., Imbs 2002).

Alongside the empirical literature there is a fairly substantial body of theoretical work on the relationship between growth and volatility (or growth and uncertainty, as is often the expression).<sup>1</sup> This research is based on the construction of stochastic dynamic general equilibrium models in which growth occurs endogenously through endogenous changes in technology and productivity. A key feature of these models is that temporary shocks can have permanent effects on the level of output because of their impact on variables that govern technological progress and productivity improvements. More significantly, the models share the distinction of determining the average growth rate of output as a function of the structural characteristics (the amplitude, frequency and persistence) of fluctuations. In this way, one establishes a relationship between long-term growth and short-term variability. The sign of this relationship is seen to be sensitive to a number of factors, which may help to explain the ambiguous empirical evidence.

In one class of models it is attitudes towards risk that matter for the effects of volatility on growth (e.g., de Hek 1999; Jones *et al.* 2005; Smith 1996). For sufficiently high (low) degrees of risk aversion, an increase in

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<sup>1</sup>Strictly speaking, volatility and uncertainty are two different concepts that refer to two different phenomena: by the former is meant fluctuations in a variable, while by the latter is meant unpredictability of these fluctuations. Of course, to the extent that the two phenomena usually go hand-in-hand, it is common practice to use the concepts interchangeably.

volatility causes an increase (decrease) in precautionary investments in physical or human capital, implying an increase (decrease) in long-run growth.<sup>2</sup> In another class of models it is the mechanism of technological change that is important for the results (e.g., Aghion and Saint-Paul 1998a, 1998b; Blackburn and Galindev 2003; Martin and Rogers 2000). If this mechanism is based on internal (external) learning, then an increase in volatility leads to an increase (decrease) in the amount of learning that takes place, generating an increase (decrease) in trend growth.<sup>3</sup> The impulse source of fluctuations in these models is either a technology or a preference shock. Elsewhere, some authors have studied the growth implications of volatility arising from policy - specifically, fiscal policy - shocks.<sup>4</sup> In Aizenman and Marion (1993) and Hopenhayn and Muniaguria (1996) it is shown how an increase in uncertainty about investment taxes and subsidies can have positive or negative growth effects depending on other characteristics of policy changes (e.g., their persistence), as well as the source of greater uncertainty (e.g., an increase in the amplitude or frequency of fluctuations). In Varvarigos (2006) it is shown how an increase in the variability of productive public expenditures can have similarly ambiguous effects depending on the nature of these expenditures (whether they function as inputs to output or human capital production) and the parameters governing technologies (the elasticity of output with respect to public inputs).<sup>5</sup>

Given the above, the purpose of the present paper is to identify new channels through which economic fluctuations in the short-run can influence economic prospects in the long-run. Our analysis is based on a stochastic endogenous growth model that is similar in some respects, but dissimilar in others, to existing models. The mechanism of endogenous growth is the accumulation of human capital through which endogenous improvements in

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<sup>2</sup>Other analyses that stress the role of precautionary investments include Canton (2002) and Dotsey and Sarte (2000).

<sup>3</sup>By internal learning is meant the deliberate acquisition of knowledge and skills by individuals through their own purposeful actions. By external learning is meant the acquirement of these attributes as a by-product of individuals' shared experiences in productive activities, or as a consequence of other factors that are beyond individuals' own control (such as public policy).

<sup>4</sup>For analyses that deal with the growth effects of monetary policy variability, see Blackburn and Pelloni (2004) and Dotsey and Sarte (2000). For some early and more recent empirical work in the area, see Aizenman and Marion (1993), Brunetti (1998) and Easterly and Rebelo (1993).

<sup>5</sup>From a different perspective, other authors have studied how the growth effects of volatility may lead to important implications for the desirability of policies intended to stabilise fluctuations (e.g., Blackburn 1999; Blackburn and Pelloni 2005; Cassou and Lansing 1997; Martin and Rogers 1997).

productivity occur. Human capital accumulation depends, in general, on the amount of time (or effort) that agents devote to learning and acquiring new skills for themselves, and on any expenditures that the government makes on public goods and services which enhance such activities (e.g., expenditures on education, training, health and infrastructure). Accordingly, the model allows for factors in the production of knowledge that are both internal and external to individual decision making. Which of these factors is the more or less important is determined by the parameters of the human capital production technology. The sources of stochastic fluctuations in the model are preference and technology shocks <sup>6</sup>

Our analysis yields a number of results that have not, to our knowledge, been established before. First, we show that the optimal allocations of time to output production and human capital accumulation are both pro-cyclical. This is in contrast to the usual prediction that time spent learning is counter-cyclical because of opportunity cost effects - that is, booms (recessions) are events which induce agents to spend less (more) time on learning and more (less) time on working because of the higher (lower) returns to the latter activity (e.g., Aghion and Saint-Paul 1998; Blackburn and Galindev 2003; Saint-Paul 1993, 1997).<sup>7</sup> Second, we identify a preference parameter (other than the coefficient of relative risk aversion) that is potentially crucial for governing the effect of volatility on growth. When the value of this parameter - which determines the disutility of total effort devoted to non-leisure activities - is relatively low (high), the effect is more likely to be positive (negative). Third, we demonstrate how the correlation between growth and volatility can be of either sign under both internal learning and external learning. Again, this is in contrast to the usual finding that the correlation is positive in the case of the former but negative in the case of the latter (e.g., Aghion and Saint-Paul 1998a, 1998b; Blackburn and Galindev 2003; Martin and Rogers 2000). Fourth, we also reveal how the sign of the correlation in the case of technology shocks can depend on the relative importance of the two types of learning (or the relative importance of private and public inputs in human capital formation). The more (less) important is external learning and the less (more) important is internal learning, the wider is the range of

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<sup>6</sup>We indicate later how the model may be extended straightforwardly to incorporate policy shocks with similar implications to those of the present analysis.

<sup>7</sup>Empirical evidence on the issue is scarce and mixed, with human capital investment being usually proxied by some measure of educational status (e.g., school enrolment). In two recent studies, Dellas and Sakellaris (2003) detect counter-cyclical behaviour in this variable, whilst Sakellaris and Spilimbergo (2000) find that the result is sensitive to the cross-section of countries with the variable displaying pro-cyclical behaviour in a large sample of countries.

parameter values under which the correlation is positive (negative); this also implies that, for a given configuration of parameter values, the correlation can be different for the two different shocks.

As well its contribution to the literature on growth and volatility, our analysis may be viewed in connection with some contemporary research on the changing pattern of business fluctuations in actual economies. For example, there is evidence to suggest that the cyclical volatility of output and its components has declined significantly in the US since the early 1980s. Several authors have offered explanations for this, the most recent being Arias *et al.* (2006) who contend that much of the decline in output volatility can be accounted for by a decline in the volatility of productivity shocks within a standard real business model. Whatever the explanation, our analysis (and others like it) of possible linkages between short-run fluctuations and long-run growth introduces additional dimensions with potentially important implications. Naturally, for these linkages to be identified, it is necessary to work within an endogenous, rather than exogenous, growth framework. Thus, notwithstanding the general debate about the relative merits of these frameworks, we choose the former as being the most appropriate for the purposes at hand.

The remainder of the paper is organised as follows. In Section 2 we set out the model. In Section 3 we solve for the dynamic general equilibrium. In Section 4 we study the relationship between growth and volatility. In Section 5 we make a few concluding remarks.

## 2 The Model

Time is discrete and indexed by  $t = 0, \dots, \infty$ . There is a constant population of identical, infinitely-lived agents with a measure of unit mass. Each agent is both a producer and consumer of a single, perishable commodity. Each agent is also an investor in human capital, the accumulation of which provides the engine of growth in the economy. Agents pay taxes to the government which makes expenditures on public goods and services that may contribute to human capital accumulation. Technology and preference shocks create uncertainty for agents and cause stochastic fluctuations in the economy. All markets are perfectly competitive. In more detail the model is described as follows.

The representative agent produces  $y_t$  units of output with  $l_t$  units of raw labour and  $h_t$  units of (previously-accumulated) human capital according to

$$y_t = Y(v_t, l_t h_t), \tag{1}$$

where  $v_t$  is a positively-valued, bounded random variable (a technology shock) that is identically and independently distributed with constant mean  $\mu_v$  and constant variance  $\sigma_v^2$ . The term  $l_t h_t$  is a composite production input that measures effective (or efficiency) units of time spent working. Naturally, we assume that the function  $Y(\cdot)$  satisfies  $Y_1(\cdot) > 0$  and  $Y_2(\cdot) > 0$ . If one makes the further assumption of constant returns to  $l_t h_t$ , then  $Y_{22}(\cdot) = 0$  and output would always grow at the same rate as human capital in the absence of exogenous shocks.

An agent's accumulation of human capital depends, in general, on factors that are both internal and external to his decision making. The former, denoted  $s_t$ , is the amount of time (or effort) that the agent devotes to learning and acquiring new skills for himself. The latter, denoted  $g_t$ , is the government's provision of public goods and services that enhance such activities, that are accessible to all agents and that are taken as given by each agent.<sup>8</sup> The dynamic process governing human capital is summarised by

$$h_{t+1} = H(s_t h_t, g_t). \quad (2)$$

Like before,  $s_t h_t$  measures effective (or efficiency) units of time that an agent spends on learning. The function  $H(\cdot)$  is assumed to be linearly homogeneous and to display some configuration of the following properties:  $H_i(\cdot) \geq 0$ ,  $H_{ii}(\cdot) \leq 0$  and  $H_{ij}(\cdot) \geq 0$  ( $i, j = 1, 2$ ). As we shall see, the specific assumption in this case of constant returns ensures endogenous growth, whilst the less specific nature of other restrictions allows us to study both general and special cases of interest.

An agent derives utility from his consumption of output,  $c_t$ , and incurs disutility from the total effort he devotes to non-leisure activities,  $l_t + s_t$ . The expected lifetime utility of an agent is given by

$$u = E_0 \sum_{t=0}^{\infty} \beta^t U[\nu_t c_t - h_t V(l_t + s_t)], \quad (3)$$

where  $E_0$  is the conditional expectations operator,  $\beta \in (0, 1)$  is a discount factor and  $\nu_t$  is a positively-valued, bounded random variable (a preference shock) that is identically and independently distributed with constant mean  $\mu_\nu$  and constant variance  $\sigma_\nu^2$ . The functions  $U(\cdot)$  and  $V(\cdot)$  are assumed to

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<sup>8</sup>An alternative way of incorporating externality effects is to appeal to the notion of learning-by-doing, whereby an agent acquires knowledge as a by-product of his own production experiences and the production experiences of others. This is typically formalised by specifying human capital accumulation to depend directly on some measure of aggregate economic activity, such as aggregate output. In our model this dependence emerges indirectly through public expenditures. The results in both cases are the same.

satisfy  $U'(\cdot) > 0$ ,  $U''(\cdot) < 0$ ,  $V'(\cdot) > 0$  and  $V''(\cdot) > 0$ .<sup>9</sup> This specification of preferences is new to the literature on growth and volatility, but is fairly common in other areas of macroeconomic research (e.g., Cassou and Lansing 1998, 2004; Collard 1999; Hercowitz and Sampson 1991). One of its immediate implications is that the marginal rate of substitution between consumption and labour is independent of the level of consumption. This means that the optimal amount of labour can be determined separately from intertemporal consumption decisions.<sup>10</sup> The term  $h_t V(\cdot)$  may be interpreted as a quality-adjusted measure of the disutility of foregone leisure, where leisure serves as an input to home production activities and where the efficiency of this input is enhanced by human capital. Under such circumstances, an increase in human capital causes not only an increase in the productivity of non-leisure occupations (working and learning), but also an increase in the marginal disutility (or opportunity cost) of these occupations (because of the increase in productivity in home activities). The convexity of the function  $V(\cdot)$  can be thought of as capturing a fatigue effect that reduces effective leisure time, whilst the linearity in  $h_t$  ensures that the optimal time allocations are stationary along the balanced growth path. The latter result occurs because of parallel improvements in the returns to all uses of time as a consequence of these returns sharing the same stochastic trend.<sup>11</sup>

The budget constraint facing an agent states simply that consumption is equal to disposable income. The latter is given by output less taxes, where taxes are levied at the constant proportional rate  $\tau \in (0, 1)$ . Thus

$$c_t = (1 - \tau)y_t \quad (4)$$

Revenues from taxation are used by the government to finance the provision of public goods and services. Assuming that the government runs a continuously balanced budget, we have

$$g_t = \tau y_t \quad (5)$$

This completes our description of the economy. The remainder of our analysis is based on a parameterised version of the model using specifications of technologies and preferences that admit closed-form solutions. These

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<sup>9</sup>Writing  $\tilde{U}(c_t, l_t + s_t) = U[\cdot]$ , these restrictions imply the standard properties  $\tilde{U}_1(\cdot) > 0$ ,  $\tilde{U}_{11}(\cdot) < 0$ ,  $\tilde{U}_2(\cdot) < 0$  and  $\tilde{U}_{22}(\cdot) < 0$ .

<sup>10</sup>One of the earliest analyses to exploit this property was that of Greenwood *et al.* (1988) in the context of real business cycle theory.

<sup>11</sup>As indicated above, our specification of preferences is a novel aspect of our analysis, as compared to the existing literature on growth and volatility. Aside from its technical merits, the specification takes seriously the view (reflected in other macroeconomic models) that home production is an important activity for individuals' (perhaps very important in less developed economies).



specifications are given as follows.

$$Y(v_t, l_t h_t) = v_t l_t h_t, \quad (6)$$

$$H(s_t h_t, g_t) = \Theta (s_t h_t)^\theta g_t^{1-\theta} \quad (7)$$

$$U[\nu_t c_t - h_t V(l_t + s_t)] = \log[\nu_t c_t - h_t (l_t + s_t)^\phi] \quad (8)$$

As indicated earlier, the linearity of  $Y(\cdot)$  with respect to  $l_t h_t$  in (6) ensures that, in the absence of exogenous shocks, output would always grow at the same (constant) rate as human capital, implying that the economy would always be on a steady state balanced growth path. The parameter  $\theta \in [0, 1]$  in (7) governs the relative importance of the two potential inputs - personal effort and public expenditures - in human capital production. The higher (lower) is the value of  $\theta$ , the more (less) productive is the former relative to the latter. In the limiting case of  $\theta = 1$  ( $\theta = 0$ ) public expenditures (personal effort) are wholly unproductive, implying that human capital accumulation takes place solely through factors that are internal (external) to an agent's decision making. The parameter  $\phi > 1$  in (8) governs the disutility of total effort spent on non-leisure activities. As will become clear, the inverse of this parameter,  $\frac{1}{\phi}$ , measures the elasticity of total effort with respect to the returns to working. These returns are determined by both of the exogenous shocks in the model,  $v_t$  and  $\nu_t$ . The logarithmic form for  $U(\cdot)$  implies a unit coefficient of relative risk aversion that would normally negate any impact of these shocks on time allocations because of offsetting income and substitution effects. This does not arise with the present structure of preferences (for which income effects vanish), and the consequence of either type of shock is to cause optimal adjustments in total time spent both working and learning.<sup>12</sup>

### 3 Solution of the Model

We solve our model economy for a stochastic dynamic competitive equilibrium that describes the aggregate behaviour of variables consistent with the optimality of agent's decisions and the feasibility of resource allocations. More precisely, we have

**Definition 1** *A stochastic dynamic competitive equilibrium is a sequence of quantities,  $\{l_t, s_t, h_{t+1}, c_t, y_t, g_t, v_t, \nu_t\}_{t=0}^\infty$ , such that, for a given  $h_0$ , the following conditions are satisfied:*

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<sup>12</sup>As indicated earlier, the model can be extended straightforwardly to allow for policy shocks by treating the tax rate,  $\tau$ , as a random variable. Doing this yields few additional insights since the effects of such shocks are similar to the effects of preference shocks. For this reason, we confine our attention to the latter.

- i)  $\{l_t, s_t, h_{t+1}, c_t, y_t\}_{t=0}^{\infty}$  solves the representative agent's optimisation problem, given  $\{g_t, v_t, \nu_t\}_{t=0}^{\infty}$ ;
- ii)  $l_t$  and  $s_t$  are stationary;
- iii) the government's budget constraint is satisfied each period,  $g_t = \tau y_t$  for  $t = 0, 1, \dots, \infty$ ;
- iv) the goods market clears each period,  $c_t + g_t = y_t$  for  $t = 0, 1, \dots, \infty$ .

Given the above, we proceed with our solution as follows.

The objective of the representative agent is to choose a sequence of actions,  $\{l_t, s_t, h_{t+1}, c_t, y_t\}_{t=0}^{\infty}$ , so as to maximise his expected lifetime utility in (3), subject to the technologies and constraints in (1), (2) and (4), and taking as given the sequence of non-choice variables,  $\{g_t, v_t, \nu_t\}_{t=0}^{\infty}$ . The first-order conditions for solving this problem are

$$\frac{\nu_t}{\nu_t c_t - h_t(l_t + s_t)^\phi} = \zeta_t, \quad (9)$$

$$\frac{\phi h_t(l_t + s_t)^{\phi-1}}{\nu_t c_t - h_t(l_t + s_t)^\phi} = \zeta_t(1 - \tau)v_t h_t, \quad (10)$$

$$\frac{\phi h_t(l_t + s_t)^{\phi-1}}{\nu_t c_t - h_t(l_t + s_t)^\phi} = \xi_t \theta \Theta s_t^{\theta-1} h_t^\theta g_t^{1-\theta}, \quad (11)$$

$$\begin{aligned} \xi_t = & \beta E_t[\xi_{t+1} \theta \Theta s_{t+1}^\theta h_{t+1}^{\theta-1} \bar{y}_{t+1}^{1-\theta}] + \beta E_t[\zeta_{t+1}(1 - \tau)v_{t+1}l_{t+1}] \\ & - \beta E_t \left[ \frac{(l_{t+1} + s_{t+1})^\phi}{\nu_{t+1}c_{t+1} - h_{t+1}(l_{t+1} + s_{t+1})^\phi} \right]. \end{aligned} \quad (12)$$

where  $\zeta_t$  and  $\xi_t$  are the Lagrange multipliers associated with (4) and (2), respectively. The condition in (9) equates the marginal utility of consumption with the shadow value of wealth. The conditions in (10) and (11) equate the marginal benefits and marginal costs of extra hours spent working and learning, respectively. The marginal cost in each case reflects the reduction in current leisure time. The marginal benefit of working derives from the increase in current output, whilst the marginal benefit of learning arises from the increase in future human capital. The condition in (12) shows that the marginal value of additional human capital comprises the expected discounted marginal value of extra human capital in the future, the expected discounted marginal value of extra consumption in the future and the expected discounted marginal cost of extra effort in the future.

The above conditions may be written more compactly by eliminating  $\zeta_t$  and making use of (1), (2) and (4) to arrive at

$$\phi(l_t + s_t)^{\phi-1} = (1 - \tau)\nu_t v_t, \quad (13)$$

$$\frac{\phi(l_t + s_t)^{\phi-1}}{(1 - \tau)\nu_t v_t l_t - (l_t + s_t)^\phi} = \frac{\theta \xi_t h_{t+1}}{s_t}, \quad (14)$$

$$\xi_t h_{t+1} = \beta \theta E_t(\xi_{t+1} h_{t+2}) + \beta. \quad (15)$$

One may also note the transversality condition,

$$\lim_{T \rightarrow \infty} \beta^T E_t(\xi_{t+T} h_{t+T+1}) = 0. \quad (16)$$

The expression in (13) reflects the property, alluded to earlier, that the marginal rate of substitution between consumption and labour is independent of the level of consumption. As is evident, the total amount of time devoted to non-leisure activities,  $l_t + s_t$ , is determined immediately from this expression as a function of the exogenous shocks,  $v_t$  and  $\nu_t$ . Together with (14), one may then conjecture solutions for  $l_t$  and  $s_t$  that are each functions of  $v_t$  and  $\nu_t$ , as well as the term  $\xi_t h_{t+1}$ . This term is determined according to (15) which describes an expectations difference equation that satisfies the terminal condition in (16). Based on this equation, one may also conjecture a solution for  $\xi_t h_{t+1}$  that is time-invariant. These observations lead us to

**Lemma 1** *There exists a unique stochastic dynamic competitive equilibrium with time allocations  $l_t = \mathcal{L}(v_t, \nu_t) > 0$  and  $s_t = \mathcal{S}(v_t, \nu_t) > 0$ .*

**Proof.** From (13) and (14),  $s_t = \Xi_t l_t$  and  $\phi(1 + \Xi_t)^{\phi-1} l_t^{\phi-1} = (1 - \tau)\nu_t v_t$ , where  $\Xi_t = \frac{(\phi-1)\theta \xi_t h_{t+1}}{\phi + \theta \xi_t h_{t+1}}$ . Solving (15) forwards in time, and applying (16), gives  $\xi_t h_{t+1} = \frac{\beta}{1 - \beta \theta}$  for all  $t = 0, 1, \dots, \infty$ . Hence  $\Xi_t = \frac{(\phi-1)\beta \theta}{\phi(1 - \beta \theta) + \beta \theta}$  for all  $t = 0, 1, \dots, \infty$  as well. It follows that  $l_t = \mathcal{L}(v_t, \nu_t)$  and  $s_t = \mathcal{S}(v_t, \nu_t)$ . ■

Obviously, since both  $v_t$  and  $\nu_t$  are governed by stationary processes, so too are both  $l_t$  and  $s_t$ . The precise expressions for  $\mathcal{L}(\cdot)$  and  $\mathcal{S}(\cdot)$  are

$$\mathcal{L}(v_t, \nu_t) = \left( \frac{1 - \tau}{\phi^\phi} \right)^{\frac{1}{\phi-1}} [\phi(1 - \beta \theta) + \beta \theta] (v_t \nu_t)^{\frac{1}{\phi-1}} \equiv L(v_t \nu_t)^{\frac{1}{\phi-1}}, \quad (17)$$

$$\mathcal{S}(v_t, \nu_t) = \left( \frac{1 - \tau}{\phi^\phi} \right)^{\frac{1}{\phi-1}} (\phi - 1) \beta \theta (v_t \nu_t)^{\frac{1}{\phi-1}} \equiv S(v_t \nu_t)^{\frac{1}{\phi-1}}. \quad (18)$$

Accordingly, we have

**Proposition 1** *The equilibrium allocations of time to both working and learning increase (decrease) in response to positive (negative) technology and preference shocks.*

**Proof.** From (17) and (18),  $\mathcal{L}_1(\cdot), \mathcal{S}_1(\cdot) > 0$  and  $\mathcal{L}_2(\cdot), \mathcal{S}_2(\cdot) < 0$ . ■

A positive technology or preference shock raises the return to working, leading to higher consumption and higher income. The latter effect has no bearing on the marginal condition for working because of parallel changes in the marginal benefit and marginal cost of this activity. This leaves only the former effect to work its way through to adjustments in total non-leisure hours, as revealed in (12). By virtue of (13), this adjustment entails an increase in both hours spent working and hours spent learning in order to preserve equality between the marginal benefits of each activity.<sup>13</sup> An immediate implication of this is

**Corollary 1** *The equilibrium allocations of time to both working and learning are pro-cyclical.*

This result stands in contrast to the prediction of other models in the literature, where one observes learning to display counter-cyclical behaviour because of opportunity cost effects. In our case an increase in the returns to output production (be it from an improvement in technology or an increase in desire for consumption) leads agents to optimally adjust their time allocations in such a way that implies an increase in both labour and learning, rather than a substitution from the latter to the former.<sup>14</sup> As indicated previously, the cyclical nature of learning remains an unresolved empirical issue.

Given the above, we may now establish

**Lemma 2** *The equilibrium growth rate of human capital is given by  $\frac{h_{t+1}}{h_t} = \mathcal{H}(v_t, \nu_t) > 0$ .*

**Proof.** Substituting (5) and (6) into (7), gives  $h_{t+1} = \Theta s_t^\theta l_t^{1-\theta} (\nu_t \tau)^{1-\theta} h_t$ . Eliminating  $l_t$  and  $s_t$  using (17) and (18) then yields  $\frac{h_{t+1}}{h_t} = \mathcal{H}(v_t, \nu_t)$ . ■

Human capital grows endogenously and stochastically such that output and other variables (consumption and government expenditures) follow similar non-stationary processes. As is familiar, the endogeneity of growth is due

<sup>13</sup>This equality is given by  $\frac{\phi}{(\phi-1)l_t - s_t} = \frac{\theta \xi_t h_{t+1}}{s_t}$ , where  $\xi_t h_{t+1} = \frac{\beta}{1-\beta\theta}$  (as determined above). It follows that any increase in total effort,  $l_t + s_t$ , must be associated with an increase in each component, both  $l_t$  and  $s_t$ , if the condition is to remain satisfied.

<sup>14</sup>The absence of a substitution effect in our model is made evident by the fact that  $s_t = \Xi l_t$ , where  $\Xi = \frac{(\phi-1)\beta\theta}{\phi(1-\beta\theta) + \beta\theta}$  (as established above). This condition shows immediately that  $s_t$  and  $l_t$  are positively (not negatively) correlated, and will be adjusted in the same (not the opposite) direction in response to exogenous shocks.

to the property that, in equilibrium, there are constant returns to human capital accumulation.

The precise expression for  $\mathcal{H}(\cdot)$  is

$$\mathcal{H}(v_t, \nu_t) = \Theta S^\theta (\tau L)^{1-\theta} v_t^{\frac{1}{\phi-1} + 1 - \theta} \nu_t^{\frac{1}{\phi-1}}. \quad (19)$$

This yields

**Proposition 2** *The equilibrium growth rate of human capital increases (decreases) in response to positive (negative) technology and preference shocks.*

**Proof.** From (19),  $\mathcal{H}_1(\cdot) > 0$  and  $\mathcal{H}_2(\cdot) > 0$ . ■

A positive technology or preference shock causes agents to devote more of their time to both learning and working. The increase in learning generates an increase in human capital production directly, whilst the increase in working does so indirectly through higher government expenditures afforded by higher tax revenues as a result of higher output.<sup>15</sup>

## 4 Growth and Volatility

Our principle objective is to examine how the long-run (trend) rate of growth of an economy might be influenced by the volatility in economic activity arising from exogenous shocks. To do this, we exploit the following well-known result.

**Theorem 1** *Let  $F(x)$  be some function, where  $x$  is a random variable. If  $F(\cdot)$  is convex (concave), then the expected value of  $F(\cdot)$  is increased (decreased) by a mean preserving spread in the distribution of  $x$ .*

**Proof.** See Rothschild and Stiglitz (1971). ■

Our measure of trend growth is the average (or expected) rate of growth of human capital. Recalling that  $\mu_v$  ( $\mu_\nu$ ) and  $\sigma_v^2$  ( $\sigma_\nu^2$ ) are the mean and variance of  $v_t$  ( $\nu_t$ ), respectively, we may establish

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<sup>15</sup>Of course, a positive technology shock has a direct impact on output as well. As a slight digression, one may also note (using the expressions for  $S$  and  $L$ ) the non-monotonic relationship between growth and taxes implied by our model - a relationship that is positive at low levels of taxes, but negative at high levels of taxes. This is essentially the result of Barro (1980) who derives the growth-maximising tax rate in a deterministic endogenous growth model. The equivalent tax rate in our case would be  $\tau = \frac{(1-\theta)(\phi-1)}{(1-\theta)(\phi-1)+1}$ .

**Lemma 3** *The trend rate of growth of human capital is given by  $E\left(\frac{h_{t+1}}{h_t}\right) = \mathbf{H}(\sigma_v^2, \sigma_\nu^2)$ .*

**Proof.** Taking a second-order Taylor series approximation of  $\mathcal{H}(\cdot)$  in (19) gives  $\mathcal{H}(\cdot) \approx \mathcal{H}(\mu_v, \mu_\nu) + \mathcal{H}_1(\mu_v, \mu_\nu)(v_t - \mu_v) + \mathcal{H}_2(\mu_v, \mu_\nu)(\nu_t - \mu_\nu) + \frac{1}{2}\mathcal{H}_{11}(\mu_v, \mu_\nu)(v_t - \mu_v)^2 + \frac{1}{2}\mathcal{H}_{11}(\mu_v, \mu_\nu)(\nu_t - \mu_\nu)^2$ . The expected value of this is  $E[\mathcal{H}(\cdot)] = \mathcal{H}(\mu_v, \mu_\nu) + \frac{1}{2}\mathcal{H}_{11}(\mu_v, \mu_\nu)\sigma_v^2 + \frac{1}{2}\mathcal{H}_{22}(\mu_v, \mu_\nu)\sigma_\nu^2 \equiv \mathbf{H}(\sigma_v^2, \sigma_\nu^2)$ . ■

Observe that  $\mathbf{H}_i(\cdot) = \frac{1}{2}\mathcal{H}_{ii}(\cdot)$  ( $i = 1, 2$ ). Thus, in accordance with the above, an increase in  $\sigma_v^2$  ( $\sigma_\nu^2$ ) will increase (decrease)  $E\left(\frac{h_{t+1}}{h_t}\right)$  depending on whether  $\mathbf{H}_1(\cdot) \geq 0$  ( $\mathbf{H}_2(\cdot) \geq 0$ ). Intuitively, when  $\mathcal{H}(\cdot)$  is convex (concave), the gain in learning as a result of a favourable shock more (less) than compensates the loss in learning as a result of an unfavourable shock so that, on average, growth is increased (decreased) by a mean preserving spread in the distribution of the shock. By computing  $\mathbf{H}_i(\cdot)$ , we are able to determine precise restrictions on parameter values under which different outcomes will occur. It is straightforward to verify that

$$\text{sgn}\mathbf{H}_1(\cdot) = \text{sgn}\left(\frac{1}{\phi - 1} - \theta\right), \quad (20)$$

$$\text{sgn}\mathbf{H}_2(\cdot) = \text{sgn}\left(\frac{1}{\phi - 1} - 1\right). \quad (21)$$

In analysing these expressions, it is instructive to consider, in turn, the case in which growth is driven solely by agents' own actions (i.e., public expenditures are non-productive) and the case in which growth depends also (or possibly exclusively) on government actions (i.e., public expenditures are productive). These scenarios are captured by the parameter restrictions  $\theta = 1$  and  $0 \leq \theta < 1$ , respectively.

We begin with the following result.

**Proposition 3** *When public expenditures are non-productive, an increase in the volatility of either technology or preference shocks increases (decreases) trend growth if  $\phi < 2$  ( $\phi > 2$ ).*

**Proof.** For  $\theta = 1$ , both (20) and (21) yield  $\mathbf{H}_i(\cdot) \geq 0$  according to  $\phi \leq 2$  ( $i = 1, 2$ ). ■

This result identifies a preference parameter (other than the coefficient of relative risk aversion) that can be crucial in determining the relationship

between growth and volatility. If this parameter - which determines the disutility of effort spent on non-leisure activities (and which measures the elasticity of these activities with respect to the returns from working) - is lower (higher) than a critical value of 2, then the relationship is positive (negative). This stands in contrast to the usual finding that growth and volatility are positively correlated when productivity improvements are due solely to internal learning. The role of  $\phi$  in our model is to determine the curvature of  $\mathcal{H}(\cdot)$  (through the non-linearities in  $\mathcal{L}(\cdot)$  and  $\mathcal{S}(\cdot)$ ) and, with this, the asymmetric effects of positive and negative shocks on human capital accumulation. When  $\phi < 2$  ( $\phi > 2$ ),  $\mathcal{H}(\cdot)$  is convex (concave) in both  $v_t$  and  $\nu_t$ , implying that the effects of positive (negative) shocks are more (less) pronounced than the effects of negative (positive) shocks so that, on average, human capital accumulation increases (decreases) with an increase in the volatility of shocks.

Our next result is stated as

**Proposition 4** *When public expenditures are productive, an increase in the volatility of technology shocks increases (decreases) trend growth if  $\phi < \frac{1+\theta}{\theta}$  ( $\phi > \frac{1+\theta}{\theta}$ ), whilst an increase in the volatility of preference shocks increases (decreases) trend growth if  $\phi < 2$  ( $\phi > 2$ ).*

**Proof.** For  $0 < \theta < 1$ , (20) implies that  $\mathbf{H}_1(\cdot) \geq 0$  according to  $\phi \leq \frac{1+\theta}{\theta}$ , whilst (21) implies that  $\mathbf{H}_2(\cdot) \geq 0$  according to  $\phi \leq 2$ . ■

As before, and for the same reason as before, this result shows that the relationship between growth and volatility depends importantly on the disutility of effort parameter in agents' preferences. Unlike before, however, the critical value of this parameter is different for the two different shocks. Specifically, the critical value is greater in the case of technology shocks than in the case of preference shocks (i.e.,  $\frac{1+\theta}{\theta} > 2$ ). An implication of this is

**Corollary 2** *When public expenditures are productive, the range of parameter values for which growth and volatility are positively (negatively) correlated is larger (smaller) in the case of technology shocks than in the case of preference shocks.*

This finding suggests another reason why the relationship between growth and volatility can be tenuous - namely, that the relationship may depend on the source of fluctuations. Thus, there is a range of parameter values,  $\phi \in (2, \frac{1+\theta}{\theta})$ , for which the relationship will be positive if technology shocks predominate, but negative if preference shocks predominate.

Our final result is given as

**Proposition 5** *When public expenditures are the only input to human capital accumulation, an increase in the volatility of technology shocks increases trend growth under all parameter configurations, whilst an increase in the volatility of preference shocks increases (decreases) trend growth if  $\phi < 2$  ( $\phi > 2$ ).*

**Proof.** For  $\theta = 0$ , (20) implies that  $\mathbf{H}_1(\cdot) > 0$  always, whilst (21) implies that  $\mathbf{H}_2(\cdot) \geq 0$  according to  $\phi \leq 2$ . ■

This scenario is the opposite of the case considered earlier (where human capital accumulation was driven solely by agent's own actions), and the above result, like the result obtained before, provides an important qualification to conventional wisdom - which is that the correlation between growth and volatility is negative when productivity improvements depend exclusively on factors that are external to agents' decision making. In our case this is not necessarily true (and perhaps is not likely to be true). Like our other previous finding, the result also demonstrates how the correlation may be different for different shocks, being positive (negative) in the case of technology (preference) shocks when  $\phi > 2$ .<sup>16</sup>

The foregoing investigations reveal that, in all but one of the cases considered, the parameter  $\phi$  can be crucial in determining the effects of volatility on growth. Significantly, empirical estimates and calibrated values of this parameter vary over a wide range, from as low as 1.3 to as high as 6.0 (e.g., Cassou and Lansing 2004; Greenwood *et al.* 1988; Hercowitz and Sampson 1991). Accordingly, there is nothing in either theory or practise to rule out any of our results, the basic message of which is that the growth effects of volatility are generally ambiguous and context-specific.

## 5 Conclusions

The purpose of this paper has been to make a theoretical contribution to the literature on the interactions between growth and volatility. Our analysis has been based on a dynamic general equilibrium model in which growth occurs endogenously through human capital accumulation and stochastic fluctua-

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<sup>16</sup>The assumption that human capital accumulation depends solely on public expenditures may seem somewhat extreme, but need not be taken too literally. Rather, the assumption may be seen as mimicing the types of model that focus on learning-by-doing as the engine of growth. As indicated in n.8, we would obtain the same results were we to use such a model.



tions arise because of exogenous shocks. The model allows for both individual effort and economy-wide externalities to influence productivity improvements, and also allows for both technology and preference shocks as sources of random variability. Within this framework, we have studied the relationship between growth and volatility, deriving a number of new results which provide additional insights into this relationship and which challenge some of the results in the existing literature. From an empirical perspective, our findings may be seen as offering further explanations for the lack of robust evidence on the relationship.

Naturally, our analysis has been simplified in a number of respects with the view to maintaining tractability and facilitating exposition in terms of the clarity and intuition of the results. We have no reason to believe that the basic message of the analysis would be altered in a more complicated model with more general specifications of preferences and technologies. Nevertheless, it may be interesting to construct and calibrate such a model for the purpose of conducting a quantitative investigation using numerical simulations.

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