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The Tyranny of Rules: Fiscal Discipline, Productive Spending, and Growth

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The Tyranny of Rules: Fiscal Discipline, Productive Spending, and Growth

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Abstract

The performance of alternative fiscal rules is examined in an endogenous growth model with public capital and debt. In addition to investing in infrastructure, the government spends on maintenance and health. Infrastructure affects the production of both commodities and health services. The performance of a balanced budget rule, as well as standard and modified (including and excluding productive spending) golden rules and primary surplus rules are compared numerically. Under a range of plausible parameter configurations and spending shares, and as long as the debt-related risk premium is not too elastic, a primary surplus rule that excludes productive spending is shown to perform better than alternative rules in response to a variety of shocks. As a practical policy implication, we propose the definition of a transparent Core Productive Expenditure Program.

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> The policy of rendering the investment budget elastic while stabilizing the current budget—which follows from the acceptance of borrowing in the one and rejection of borrowing in the other case—may easily result in an undesirable curtailment of useful "ordinary" expenditures.
>
> Richard A. Musgrave (1939, p. 263).

1 Introduction

There has been much debate in recent years on whether explicit fiscal frameworks may help to achieve and maintain fiscal discipline. Fiscal rules, in particular, have taken the form of maintaining fixed targets for the deficit (variously defined) and/or public debt ratios to GDP. Such rules have been used in industrial and developing countries alike. In the euro area, the commitment was made under the Stability and Growth Pact to limit the deficit to 3 percent of GDP. Brazil introduced a Fiscal Responsibility Law in May 2000 that prohibits financial support operations among different levels of government and requires that limits on the indebtedness of each level of government be set by the Senate.

A common criticism of standard deficit rules (including balanced budget rules) is that they are inflexible (to the extent that they are defined irrespective of the cyclical position of the economy) and tend to be pro-cyclical. Studies based on fiscal constraints in US states, such as Fatás and Mihov (2006), have shown indeed that while balanced budget rules have proved effective in limiting the size of deficits and the volatility of spending, they have also imposed costs to the states' economies because of the large (downward) adjustment in government spending that is required during recessions.[1] Similar results have been shown to hold in cross-country studies of industrial countries (see Lane (2003)).

In response, deficit rules have been refined and are now often applied either to a cyclically adjusted deficit measure (such as the structural budget deficit) or an average over the economic cycle. Chile, for instance, introduced

[1] The evidence provided by Canova and Pappa (2005), which is also based on the experience of US states, does not support the existence of a close link between fiscal constraints and (lack of) volatility—the reason being, in their view, that these constraints have not been properly enforced.

in early 2000 a structural surplus rule (of 1 percent of GDP) that allows for limited deficits during recessions.[2] By doing so, advocates claim, these rules may allow the operation of automatic stabilizers and possibly provide some room for discretionary policy within the cycle. However, this increased flexibility comes at a cost, because the benchmark against which fiscal performance is to be judged is made more complicated—especially if estimates of potential output are revised, as is often the case. In turn, this increases the scope to bypass the rules, making them potentially harder to enforce and undermining their credibility. In countries with a poor (or uneven) track record of policy consistency, lack of credibility may lead to higher interest rates (as in Agénor, Jensen, Yeldan, and Verghis (2006)), thereby exacerbating debt sustainability problems.

Another criticism of deficit rules is that they discourage public investment. Some economists have advocated a "golden rule" approach to budgetary policy, whereby the focus is on maintaining a balance or surplus on the current fiscal account (that is, current revenues less current expenditures), with net capital expenditure financed from government savings and borrowing. However, this rule has also been criticized on a number of grounds; critics have pointed out, among other arguments, its vulnerability to creative accounting, and the fact that a preferential treatment of physical investment could bias expenditure decisions against spending on other potentially productive components (such as education and health), and stress that what matters to the economy, and the growth process in particular, is the overall capital stock—regardless of whether it is private or public (see, for instance Buti, Eijffinger, and Franco (2003)).

Moreover, components of recurrent expenditure (such as maintenance spending on infrastructure, schools, and hospitals) may be equally important to maintain the quality of the services produced by public and private capital. In a growth context, therefore, the question that arises is where should one draw the line in imposing a deficit rule. This is the question that we address in this paper, in the context of an endogenous growth model with public capital and debt. In addition to investing in infrastructure, the government spends on maintenance (which improves the durability of public capital) and health services (which raise labor productivity). Infrastructure, in turn, affects the production of both commodities and health services. We examine

[2] The budget is adjusted not only for the effects of the business cycle on public finances, but also for fluctuations in the price of copper—Chile's main export commodity.

the performance (measured in terms of their impact on growth and fiscal indicators) of several alternative fiscal rules: a balanced budget rule, a golden rule, and four alternative primary surplus rules. Because of the complexity of the model, these rules are compared numerically, based on a calibration that reflects some key characteristics of middle-income countries—where, presumably, the productivity effects of public capital are potentially the largest. In addition, there is growing evidence suggesting that in these countries, externalities associated with public infrastructure may be more important that commonly thought. Indeed, it has been found that infrastructure may have a sizable impact on health and education outcomes.[3] For instance, access to clean water and sanitation helps to improve health and thereby productivity. By reducing the cost of boiling water, and reducing the need to rely on smoky traditional fuels for cooking, access to electricity also helps to improve hygiene and health. Availability of electricity is essential for the functioning of hospitals and the delivery of health services. Better transportation networks also contribute to easier access to health care, particularly in rural areas. There is also evidence of direct linkages between infrastructure and education. Electricity allows for more studying and greater access to learning technologies. Enrollment rates and the quality of education tend to improve with better transportation networks, particularly in rural areas. Greater access to sanitation and clean water in schools also tend to raise attendance rates. Although we do not attempt to model explicitly the impact of infrastructure on education, and its implications for the design of fiscal rules, our focus on health is sufficient to illustrate the potential implications of adding a learning technology.

The remainder of the paper is organized as follows. Section II discusses in more detail the rationale and limitations of the golden rule. Section III presents the model. Section IV examines the nature of the equilibrium growth path with a balanced budget, which we view as our "benchmark" case. The properties of the model under that rule are fully characterized. Section V specifies four alternative fiscal rules: standard primary surplus and golden rules, and two alternative rules that exclude all productive spending (including not only infrastructure investment but also spending on maintenance and health). Because the resulting dynamic system cannot be solved analytically, we resort to numerical simulations for comparative analysis. Section

[3] See Agénor and Moreno-Dodson (2006) and Agénor and Neanidis (2006*a*) for a more detailed discussion.

VI presents the calibration procedure. Experiments are reported in Section VII, where we consider both the stability properties of the model and the speed of convergence (in response to various shocks) to the steady state. We examine, in particular, whether there exist some plausible parameter configurations (pertaining, most notably, to the degree of efficiency of public investment in infrastructure) under which a primary surplus rule that excludes both maintenance and health spending yields higher steady-state growth and more rapid convergence than alternative rules. The final section offers some concluding remarks.

2 The Debate on the Golden Rule

As noted earlier, a common criticism of budget rules that take the form of strict limits on the overall fiscal deficit-to-GDP ratio is that they may end up discouraging public investment. A fiscal rule that caps the overall budget deficit puts both current and investment spending on an equal footing in the measurement of the deficit. The danger, then, is that whenever the rule becomes binding, the government will choose to cut those spending categories that are politically less costly to alter. If the political cost of postponing or abandoning investment projects is lower than the political cost of constraining current expenditure—as is often the case in practice—an overall deficit rule will entail a built-in bias against public investment. In the presence of a close complementarity effect between public capital and private investment (as one would expect for infrastructure), there could be a significant adverse effect on growth.

The existence of this bias has led a number of economists, most notably Blanchard and Giavazzi (2004), to advocate reliance on a "golden rule", whereby the focus is on maintaining a current balance (that is, current revenues less current expenditures) or surplus, with capital expenditures being financed from government savings and borrowing. Under the Blanchard-Giavazzi rule, governments should borrow in net terms on a continuous basis only to the extent that this net borrowing finances net public investment, that is, gross investment less capital depreciation (which counts as current spending).[4] This rule therefore would allow gross borrowing for the purpose

[4]Musgrave (1939) was an early proponent of a rule aimed at excluding capital outlays from the operating budget, while including depreciation of the government capital stock. However, as indicated in the quote at the beginning of this paper, he was also aware of

of refinancing maturing debt, thereby leaving net debt unaffected. Moreover, to the extent that public investment boosts the economy's production capacity on a permanent rather than just temporary basis, it would affect welfare of not only the present generation but also future generations. Thus, intergenerational equity provides a rationale—as pointed out by Musgrave (1939, pp. 269-70) early on—to spread the costs of public investment over both current and future generations, by financing investment through government borrowing instead of current tax revenues. By implementing a golden rule—or, more specifically, by allowing no new borrowing in net terms to finance current spending—the outstanding debt stock of any country would, over time, become fully backed by the public capital stock.

Despite its intuitive appeal, the golden rule has attracted much criticism. First, advocates of the golden rule have generally emphasized the need to exclude capital expenditure on *infrastructure* from the fiscal deficit rule. In countries with large infrastructure gaps, certain projects (such as roads, ports, or airports) may indeed have rates of return that are so high, and a degree of complementarity with private investment that is so tight, that they justify receiving priority in the design of a public investment program. However, in other countries (particularly those with low income), investment in health and human capital may be an equally important priority, in part because it may have a larger impact on growth. Excluding public investment in "core" infrastructure only (as opposed to investment in schools and hospitals) from fiscal targets would create a bias against these other components of public investment.

Second, if applied to *gross* public investment—as opposed to *net* investment, as advocated by Blanchard and Giavazzi (2004)—the golden rule could turn into an obstacle to deficit and debt reduction. Given the ratio of public investment as a share of GDP, the long-run equilibrium level of government debt could be quite high, especially in an environment of low inflation. This, in turn, could push interest rates and debt service to unsustainable levels. To prevent this from happening, some countries have also considered a limit on public debt. The United Kingdom, for instance, adopted the golden rule of borrowing only to finance capital spending, and accompanied it by the sustainable investment rule, which limits net debt to 40 percent of GDP over the cycle.

Third, the golden rule is not a good guide to fiscal policy if some com-

the potential "displacement effects" that such a rule could lead to.

ponents of current expenditure—such as on operation and maintenance that keeps existing infrastructure in good condition, or spending that contributes to health outcomes and the accumulation of human capital—can promote growth more effectively than capital expenditure *per se*. This may be particularly so if the golden rule creates a moral hazard problem, in the sense that the possibility of borrowing without limits to finance investments lowers the care that must be taken when evaluating the costs and benefits of each project. Put differently, components of recurrent expenditure such as spending on schools, and hospitals may be equally important to maintain the quality of the services produced by the capital stock in those categories.

Fourth, rules focusing on current spending may entail a bias against operations and maintenance, the current public spending required to keep the existing physical infrastructure in good working order. A depressingly routine fact in developing countries is that new projects are undertaken while the existing infrastructure is left to deteriorate.[5] The deterioration of infrastructure may retard growth more than new capital projects add to growth, if for instance maintaining public capital raises the durability of private capital, as discussed for instance by Agénor (2005*b*) and Agénor and Moreno-Dodson (2006).

A broad implication of the foregoing discussion is that alternative rules may have an ambiguous effect on growth and fiscal performance. The question that arises, therefore, is where should one draw the line in imposing a deficit rule. As noted earlier, current spending on education and health enhances human capital. Excluding them from, say, a primary surplus rule is all the more important in countries where vast amounts of flow spending in infrastructure are wasted and turn only partly into public capital, and if public capital in infrastructure has a small complementarity effect with private investment. In such conditions, singling out public investment from other budget items makes little sense; a tax reform that alleviates distortions and translates into a lowers tax burden on firms may lead to higher private in-

[5] The quality of existing infrastructure will tend to deteriorate if corruption leads to cutbacks on operation and maintenance expenditure. Tanzi and Davoodi (1998) have claimed to find evidence that countries with high corruption tend to have poor quality infrastructure. Because direct cross-country data on operation and maintenance expenditures are not available, their analysis uses as a proxy a measure of "expenditures on other goods and services," taken from the IMF's *Government Financial Statistics*, which include operation and maintenance expenditures. However, this is a very unreliable measure. Note also that both quantity and quality of the public capital stock may be affected by corruption.

vestment (and higher growth) and may be preferable to an increase in public investment. At the same time, however, public capital in infrastructure may have (as noted in the introduction) a sizable impact on health and education outcomes. If these effects are sufficiently strong, a rule that entails some bias toward investment in infrastructure only may still lead to higher growth rates—despite some degree of inefficiency in the investment process itself. Our analytical framework, and subsequent numerical simulations, attempt to shed some light on the importance of these various effects.

3 The Framework

The economy that we consider is closed and populated by an infinitely-lived representative household-producer (or household, for short). A single traded commodity, which can be used for consumption or investment, is produced with labor and capital. The price of the good is constant and normalized to unity. The government has no access to seigniorage but can issue bonds to finance its deficit. It collects a proportional tax on output, invests in infrastructure, and spends on maintenance and health services. It also services its debt and provides lump-sum transfers to households. Infrastructure and health services (which are produced by the government) are provided free of charge.

3.1 Production

Commodities are produced, in quantity Y, with private capital, K_P, public capital in infrastructure, K_I, and effective labor, defined as the product of the quantity of labor and productivity, A. Population growth is zero. Normalizing the population size to unity and assuming that the technology is Cobb-Douglas yields[6]

$$Y = K_I^{\alpha} A^{\beta} K_P^{1-\alpha-\beta}, \tag{1}$$

where $\alpha, \beta \in (0, 1)$.

Productivity depends solely on the availability of health services, H.[7] For simplicity, we assume that the relationship between A and H is one of strict

[6] In what folllows, time subscripts are omitted for simplicity, and a dot over a variable is used to denote its time derivative.

[7] See Agénor and Neanidis (2006b) for a more general specification of productivity.

proportionality, so that $A = H$. Using this result with (1) yields

$$Y = (\frac{K_I}{K_P})^{\alpha}(\frac{H}{K_P})^{\beta}K_P. \tag{2}$$

From standard conditions for profit maximization, the gross (pre-tax) wage rate, ω, and the direct pre-tax rental rate on capital, r_K, are given by

$$\omega = \beta Y/H, \qquad r_K = \eta Y/K_P, \tag{3}$$

where $\eta \equiv 1 - \alpha - \beta$.

As noted earlier, access to public infrastructure is provided at no cost to users. As in Agénor and Neanidis (2006*b*), we will assume in what follows that the implicit rent corresponding to the marginal return on public capital, $\alpha Y/K_I$, accrues to private capital. The *effective* rate of return on private physical capital, r, exceeds therefore the direct marginal product given in equation (3). Indeed, from the identity $Y = \omega H + rK_P$, as well as the condition on ω, r is given by

$$r = \frac{(1-\beta)Y}{K_P} > r_K. \tag{4}$$

Production of health services requires combining government spending on health, G_H, and public capital in infrastructure. Assuming also a Cobb-Douglas technology yields

$$H = K_I^{\mu}G_H^{1-\mu}, \tag{5}$$

where $\mu \in (0, 1)$.

3.2 Household Optimization

The household's optimization problem can be specified as

$$\max_C V = \int_0^{\infty} \frac{C^{1-1/\sigma}}{1-1/\sigma}\exp(-\rho t)dt, \quad \sigma \neq 1, \tag{6}$$

where C is consumption, ρ the discount rate, and σ is the elasticity of intertemporal substitution.

The household's resource constraint is given by

$$\dot{W} = \dot{K}_P + \dot{B} = (1-\tau)(\omega H + rW) + T - C, \tag{7}$$

where $W = K_P + B$ is total assets, consisting of private physical capital and government bonds, in quantity B, T is lump-sum transfers (taken as given by the household), $\tau \in (0, 1)$ is the tax rate on income. Taxes are levied on interest-inclusive income, with interest income consisting not only of the (effective) return to capital but also of the return to government bonds. Through standard (after-tax) arbitrage conditions, the rate of return on both categories of assets is identical and equal to r. For simplicity, we assume that private capital does not depreciate.[8]

The household takes public policies and the depreciation rate as given when choosing the optimal sequence of consumption. Using (1), (6), and (7), the current-value Hamiltonian for problem (6) can be written as

$$L = \frac{C^{1-1/\sigma}}{1-1/\sigma} + \lambda[(1-\tau)(\omega H + rW) + T - C],$$

where λ is the co-state variable associated with constraint (7). From the first-order condition $dH/dC = 0$ and the co-state condition $-dH/dW = \dot{\lambda} - \rho\lambda$, optimality conditions for this problem take the familiar form

$$C^{-1/\sigma} = \lambda, \tag{8}$$

$$\dot{\lambda} = \lambda[\rho - (1-\tau)r], \tag{9}$$

together with the budget constraint (7) and the transversality condition

$$\lim_{t\to\infty} \lambda W \exp(-\rho t) = 0. \tag{10}$$

3.3 Government

The government invests in infrastructure capital, G_I, and spends on maintenance, G_M, health services, G_H, unproductive items, G_U, transfers, T, and interest payments, rB. As noted earlier, it also collects a proportional tax τ on output. Thus, the government budget constraint is given by

$$\dot{B} = \sum_{h=I,H,M,U} G_h + T + rB - \tau(\omega H + rW). \tag{11}$$

[8] With private capital depreciating at the rate δ_P, the interest arbitrage condition would take the form $r_B = r - \delta_P/(1-\tau)$, where r_B is the rate of return on government bonds.

We begin first with the assumption that all components of spending are fixed fractions of total tax revenues:

$$G_h = \upsilon_h \tau(\omega H + rW), \;\; h = I, H, M, T, U \tag{12}$$

Using these equations, the government budget constraint, equation (11), can be rewritten as

$$\dot{B} = rB - (1 - \sum_{h=I,H,M,T,U} \upsilon_h)\tau(\omega H + rW). \tag{13}$$

Using (12), the stock of public capital in infrastructure evolves over time according to

$$\dot{K}_I = \varphi \upsilon_I \tau(\omega H + rW) - \delta_I K_I, \tag{14}$$

where $\delta_I \in (0,1)$ is a constant depreciation rate and $\varphi \in (0,1)$ an efficiency parameter that measures the extent to which investment flows translate into actual accumulation of public capital. The case $\varphi < 1$ reflects the view that investment outlays are subject to inefficiencies, which tend to limit their positive impact on the public capital stock.[9]

As in Agénor (2005*b*), the rate of depreciation of public capital in infrastructure, δ_I, is taken to depend negatively on the ratio of maintenance expenditure to the public stock of capital in infrastructure:

$$\delta_I = 1 - \varepsilon(\frac{G_M}{K_I})^{\zeta}, \tag{15}$$

where $\varepsilon \in (0,1)$ and $\zeta > 0$.[10] Thus, maintenance expenditure enhances the durability of public infrastructure capital.

The government cannot run a Ponzi scheme, which implies that it is subject to the transversality condition

$$\lim_{t \to \infty} B_t \exp[-\int_0^t r_u du] = 0,$$

[9] Arestoff and Hurlin (2005), for instance, estimate the value of φ to vary between 0.4 and 0.6 for a group of developing countries. As argued by Tanzi and Davoodi (1998) poor efficiency of public investment may be the consequence of a high degree of corruption.

[10] These restrictions on ε and ζ are sufficient to ensure that $\delta_I \in (0,1)$ as long as $G_M/K_I \leq 1$.

or, using (11),

$$B_0 + \int_0^\infty (G+T)\exp[\int_0^t r_u du]dt = \int_0^\infty Z\exp[\int_0^t r_u du]dt.$$

where $G = \sum_{h=I,H,M,U} G_h$ and $Z = \tau(\omega H + rW)$.[11] Equivalently, using (7) yields

$$B_0 = (1 - \Sigma\upsilon_h)\tau \int_0^\infty (\omega H + rW)\exp[\int_0^t r_u du]dt. \tag{16}$$

From (7), (11), and the identity $Y = \omega H + rK_P$, the economy's consolidated budget constraint (or equivalently, the goods market equilibrium condition) can be derived as

$$Y = C + \dot{K}_P + G. \tag{17}$$

It should be noted that although we focus solely on the efficiency of infrastructure investment in this paper, there are equally important issues associated with the degree of efficiency of current spending on health and maintenance. Accounting for partial efficiency could be introduced by replacing G_H by $\varphi_H G_H$ in (5), and G_M by $\varphi_M G_M$ in (15), with $\varphi_H, \varphi_M \in (0,1)$ representing efficiency parameters. Sensitivity tests similar to those reported later with respect to changes in φ could therefore be performed with respect to changes in φ_H and φ_M as well.

4 Benchmark Case: Balanced Budget Rule

As a benchmark case, let us consider the case of a zero deficit (or balanced budget) rule. We denote this rule BBR, and we implement it by imposing $\dot{B} = 0$ in (11) and solving the government budget constraint for lump-sum transfers, T. Equivalently, setting the constant value of B equal to zero as well, the model determines endogenously the share of spending on transfers, υ_T:

$$\upsilon_T = 1 - \sum_{h=I,H,M,U} \upsilon_h. \tag{18}$$

[11] Note that economies that have unsustainable policies in the medium run may have a sustainable public debt for $t \to \infty$.

This rule ensures that the transversality condition (16) is satisfied, given that it entails $B_0 = 0$. It is a particular case of the rule $\dot{B} = \gamma_B B$, where $\gamma_B \in (0, 1)$ is a constant growth rate.

Determining the balanced growth path (BGP) associated with BBR proceeds in three steps. First, note that from (4), (12), (14), and (15), noting that $\omega H + rW = \omega H + rK_P = Y$, and that $G_M/K_I = \upsilon_M \tau Y/K_I$ with $\zeta = 1$ for simplicity,[12]

$$\frac{\dot{K}_I}{K_I} = \varphi \upsilon_I \tau (\frac{Y}{K_I}) - \delta_I = \tau(\varphi \upsilon_I + \varepsilon \upsilon_M)(\frac{Y}{K_P}) k_I^{-1} - 1, \tag{19}$$

where $k_I = K_I/K_P$. From (1), (5), and (12),

$$Y = k_I^{\alpha}(\frac{K_I^{\mu} G_H^{1-\mu}}{K_P})^{\beta} K_P = k_I^{\alpha+\mu\beta}(\upsilon_H \tau)^{(1-\mu)\beta}(\frac{Y}{K_P})^{(1-\mu)\beta} K_P.$$

This expression can be rewritten as

$$\frac{Y}{K_P} = (\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega}, \tag{20}$$

where $\Omega \equiv 1 - (1-\mu)\beta > 0$.

Combining this result with (19) and (15) yields

$$\frac{\dot{K}_I}{K_I} = \tau^{1/\Omega}(\varphi \upsilon_I + \varepsilon \upsilon_M) \upsilon_H{}^{(1-\mu)\beta/\Omega} k_I^{-\eta/\Omega} - 1. \tag{21}$$

Second, from the budget constraint (7) with $\dot{B} = B_0 = 0$, together with (12) and (18),

$$\dot{K}_P = qY - C,$$

where $q \equiv 1 - \tau \sum_{h=I,H,M,U} \upsilon_h$, so that $q \in (0, 1)$. Substituting (1) and (20) in this expression yields

$$\frac{\dot{K}_P}{K_P} = q(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} - c, \tag{22}$$

where $c = C/K_P$.

[12] Although in the calibration we take ζ different than unity to get reasonable estimates of depreciation of public infrastructure in the simulations, this simplification does not alter the saddlepath properties as long as $\zeta > 0$.

Third, taking logs of (8) and differentiating with respect to time yields $\dot{C}/C = -\sigma(\dot{\lambda}/\lambda)$. Substituting (9) in this expression yields, setting $s \equiv (1-\tau)(1-\beta)$,

$$\frac{\dot{C}}{C} = \sigma[s(\frac{Y}{K_P}) - \rho], \tag{23}$$

that is, using (20),

$$\frac{\dot{C}}{C} = \sigma s(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} - \sigma\rho. \tag{24}$$

Equations (21), (22), and (24) can be further condensed into a first-order nonlinear differential equation system in $c = C/K_P$ and $k_I = K_I/K_P$:

$$\frac{\dot{c}}{c} = (\sigma s - q)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c - \sigma\rho, \tag{25}$$

$$\frac{\dot{k}_I}{k_I} = \tau^{1/\Omega} \chi {\upsilon_H}^{(1-\mu)\beta/\Omega} k_I^{-\eta/\Omega} - q(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c - 1, \tag{26}$$

where $\chi \equiv \varphi\upsilon_I + \varepsilon\upsilon_M$.

These two equations, together with the initial condition $k_I^0 > 0$ and the transversality condition (10), characterize the dynamics of the economy.

The BGP is a set of functions $\{c, k_I\}_{t=0}^{\infty}$ such that equations (25) and (26), the budget constraint (18), and the household's transversality condition (10), are satisfied, and consumption and the stocks of public and private capital, all grow at the same constant rate γ.[13] Along the BGP, the household's transversality condition is always satisfied because C/K_P is constant.

As shown in the Appendix, the economy is saddlepath stable in the neighborhood of the BGP. Moreover, the BGP is unique. Thus, the model is locally determinate.[14] From (21) and (24), the steady-state growth rate is given by

$$\gamma = \sigma s(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \sigma\rho, \tag{27}$$

$$\gamma = \tau^{1/\Omega}(\varphi\upsilon_I + \varepsilon\upsilon_M){\upsilon_H}^{(1-\mu)\beta/\Omega} \tilde{k}_I^{-\eta/\Omega} - 1, \tag{28}$$

where $\tilde{x}$ denotes the stationary value of x.[15]

[13] γ is also the rate of growth of output of commodities and health services, given the assumption of constant returns to scale.

[14] To ensure that $\delta_I > 0$ in the steady state requires $1 - \varepsilon\upsilon_M \tau^{1/\Omega} {\upsilon_H}^{(1-\mu)\beta/\Omega} \tilde{k}_I^{-\eta/\Omega} > 0$. This imposes, in principle, a restriction on the admissible values of the shares υ_M and υ_H, or ε.

[15] A diagrammatic analysis of the adjustment path, as well as experiments associated

5 Alternative Fiscal Rules

We now consider several alternative fiscal rules and their implications for growth dynamics. We begin with the golden rule, given the attention that it has received in the recent literature. Next, we consider two alternative primary surplus rules, a standard rule that includes all components of current spending, and an alternative rule, which excludes all productive spending categories. Finally, we consider a modified golden rule, whereby borrowing is used to finance not only infrastructure investment but also productive components of current spending.

5.1 The Golden Rule

As discussed earlier, funding capital expenditure from current revenues would imply a disincentive to undertake projects producing deferred economic benefits but entailing upfront political costs; this disincentive may be particularly high during periods of fiscal consolidation. Moreover, the golden rule allows spreading the burden of capital projects over the different generations of taxpayers benefiting from them. But to do so, as emphasized by Blanchard and Giavazzi (2004), the rule needs to be defined in terms of net spending. Indeed, it is only the net addition to public capital (or net capital spending) that should be financed via borrowing while the part that covers depreciation (or maintenance) should remain tax financed.

Formally, the golden rule (denoted GR) can be implemented in our framework by requiring that the sum of (current) government spending on health, maintenance, transfers, and interest payments, must be equal to a fraction, θ, of tax revenues:

$$G_U + G_H + G_M + T + rB = \theta\tau(\omega H + rW), \tag{29}$$

where $\theta \in (0, 1)$. We will also assume that all spending shares, other than transfers, continue to be fixed fractions of total revenues, as indicated in (12). Thus, equation (29) determines residually lump-sum transfers as

$$T = \tau(\theta - \upsilon_H - \upsilon_M - \upsilon_U)(\omega H + rW) - rB. \tag{30}$$

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with changes in the spending shares υ_H, υ_I, and υ_M, can be performed as in Agénor (2005*a*, 2005*b*, 2005*c*).

Combining (11) and (29) implies also that debt accumulates as a result of excess investment in infrastructure over the remaining tax revenues:

$$\dot{B} = G_I - (1-\theta)\tau(\omega H + rW). \tag{31}$$

Using (12), equation (31) becomes

$$\dot{B} = -(1-\upsilon_I-\theta)\tau[\omega H + r(B+K_P)]. \tag{32}$$

Dividing (32) by B yields

$$\frac{\dot{B}}{B} = -(1-\upsilon_I-\theta)\tau b^{-1}[\omega h + r(1+b)], \tag{33}$$

where $h = H/K_P$ and $b = B/K_P$. From (5) and (12),

$$h = k_I^{\mu}(\frac{G_H}{K_P})^{1-\mu} = (\upsilon_H\tau)^{1-\mu}k_I^{\mu}[\omega h + r(1+b)]^{1-\mu},$$

whereas equations (3) imply

$$\omega = \beta k_I^{\alpha}h^{\beta-1}, \qquad r = (1-\beta)k_I^{\alpha}h^{\beta}. \tag{34}$$

Substituting these expressions in the preceding equation yields

$$h = (\upsilon_H\tau)^{(1-\mu)/\Omega}g^{(1-\mu)/\Omega}k_I^{\Phi/\Omega}, \tag{35}$$

where $\Phi \equiv \mu + \alpha(1-\mu) > 0$ and $g = 1 + (1-\beta)b$.

Equations (34) can also be used in (33) to give

$$\frac{\dot{B}}{B} = -(1-\upsilon_I-\theta)b^{-1}\tau g k_I^{\alpha}h^{\beta}, \tag{36}$$

which implies that debt increases over time only if $\upsilon_I + \theta > 1$.

The household budget constraint (7) can be rewritten as

$$\frac{\dot{K}_P}{K_P} = -\frac{\dot{B}}{B}b + (1-\tau)[\omega h + r(1+b)] + z - c,$$

where $z = T/K_P$. Substituting (36) in this expression and using (34) yields

$$\frac{\dot{K}_P}{K_P} = [(1-\upsilon_I-\theta)\tau + (1-\tau)]\, g k_I^{\alpha}h^{\beta} + z - c. \tag{37}$$

From (14) and (15), and given that $G_M/K_I = \upsilon_M \tau(\omega H + rW)/K_I$, the growth rate of public capital in infrastructure is given by

$$\frac{\dot{K}_I}{K_I} = k_I^{-1}(\varphi\upsilon_I + \varepsilon\upsilon_M)\tau[\omega h + r(1+b)] - 1,$$

that is, using (34),

$$\frac{\dot{K}_I}{K_I} = k_I^{-1}(\varphi\upsilon_I + \varepsilon\upsilon_M)\tau g k_I^{\alpha} h^{\beta} - 1. \tag{38}$$

Finally, dividing (30) by K_P yields

$$z = (\theta - \upsilon_H - \upsilon_M - \upsilon_U)\tau[\omega h + r(1+b)] - rb,$$

that is, using (34),

$$z = [(\theta - \upsilon_H - \upsilon_M - \upsilon_U)\tau g - (1-\beta)b]\, k_I^{\alpha} h^{\beta}. \tag{39}$$

Equations (23), (35), (36), (37), (38), and (39) describe the economy's dynamics. These equations can be further manipulated to produce a first-order differential equation system in c, k_I, and b:

$$\frac{\dot{c}}{c} = \{\Pi\tau g + \sigma s - 1\}\,(\upsilon_H \tau g)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} - \sigma\rho + c, \tag{40}$$

$$\frac{\dot{k}_I}{k_I} = \left\{\left[\Pi + k_I^{-1}(\varphi\upsilon_I + \varepsilon\upsilon_M)\right]\tau g - 1\right\}(\upsilon_H \tau g)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c - 1, \tag{41}$$

$$\frac{\dot{b}}{b} = \left\{\left[\Pi + b^{-1}(\upsilon_I + \theta - 1)\right]\tau g - 1\right\}(\upsilon_H \tau g)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c, \tag{42}$$

where $\Pi \equiv \sum_{h=I,H,M,U} \upsilon_h < 1$.

The steady-state growth rate is given again by (27), or alternatively, from (36) and (38), under the assumption that $\upsilon_I + \theta > 1$,

$$\gamma = (\upsilon_I + \theta - 1)\tilde{b}^{-1}\tau\tilde{g}\tilde{k}_I^{\alpha}\tilde{h}^{\beta}, \tag{43}$$

$$\gamma = (\varphi\upsilon_I + \varepsilon\upsilon_M)\tau\tilde{g}\tilde{k}_I^{\alpha-1}\tilde{h}^{\beta} - 1, \tag{44}$$

with $\tilde{g} = 1 + (1-\beta)\tilde{b}$.[16]

[16] In the simulation results reported below, we set $\theta = 1$, which implies indeed that $\upsilon_I + \theta > 1$. Note also that equation (37) provides a fourth alternative form of γ; but given the steady-state solution for c obtained by setting $\dot{c} = 0$ in (37), it is in fact equivalent to (43).

5.2 Primary Surplus Rules

A fixed primary surplus rule can be specified in the present framework in several alternative ways. We first consider the "standard" case where all components of current spending (including transfers) are included. We then consider an alternative case, excluding all productive spending categories, namely infrastructure, maintenance, and health.

5.2.1 General Form

Under a general primary surplus rule (denoted PSR), the constraint linking current expenditure and tax revenues is given by

$$G_U + G_H + G_M + G_I + T = \theta\tau(\omega H + rW), \tag{45}$$

which indicates that all noninterest spending (including transfers), must be financed by a fraction θ of total tax revenues.

Combining (11) and (45) yields

$$\dot{B} = rB - (1-\theta)\tau(\omega H + rW), \tag{46}$$

which implies that interest payments, to the extent that they are not covered by a residual fraction $1-\theta$ of tax revenues, must be financed by borrowing.

Using (12) and dividing by B, we get

$$\frac{\dot{B}}{B} = -(1-\theta)\tau b^{-1}[\omega h + r(1+b)] + r,$$

which, using (34), and the definition of g, can be written as

$$\frac{\dot{B}}{B} = \left[-(1-\theta)b^{-1}\tau g + (1-\beta)\right] k_I^{\alpha} h^{\beta}. \tag{47}$$

As in the previous section, dividing the household budget constraint (7) by K_P gives

$$\frac{\dot{K}_P}{K_P} = -\frac{\dot{B}}{B}b + (1-\tau)[\omega h + r(1+b)] + z - c,$$

where $z = T/K_P$. Using (12) and (45) implies

$$T = (\theta - \upsilon_H - \upsilon_M - \upsilon_U - \upsilon_I)\tau[\omega h + r(1+b)]. \tag{48}$$

Dividing (48) by K_P, together with (34), yields

$$z = (\theta - \upsilon_H - \upsilon_M - \upsilon_U - \upsilon_I)\tau g k_I^{\alpha} h^{\beta}. \tag{49}$$

Substituting (47) and (49) in the above equation and using (34), the growth rate of private capital becomes

$$\frac{\dot{K}_P}{K_P} = (1 - \Pi\tau g)k_I^{\alpha} h^{\beta} - c. \tag{50}$$

The growth rates of consumption, $\dot{C}/C$, and public capital in infrastructure, $\dot{K}_I/K_I$, are as defined in (38) and (35), respectively. Equation (23) defines h as a function of b and k_I. Together with (47) and (50), these equations can be rearranged to define the dynamics of the economy under the general primary surplus rule. The dynamic equations driving c, k_I are given, as before, by (40) and (41), whereas the equation of motion for b is now given by

$$\frac{\dot{b}}{b} = \left\{ \left[\Pi + (\theta - 1)b^{-1}\right] \tau g - \beta \right\} (\upsilon_H \tau g)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c. \tag{51}$$

The steady-state growth rate is given in equivalent forms by (27), (44), or, from (47),[17]

$$\gamma = \left[-(1-\theta)b^{-1}\tau\tilde{g} + (1-\beta)\right] \tilde{k}_I^{\alpha}\tilde{h}^{\beta}. \tag{52}$$

5.2.2 A Modified Rule

We now define a fixed primary surplus rule (denoted MPSR) that excludes all productive spending, that is infrastructure, maintenance and health. The constraint linking current expenditure and tax revenues therefore takes the form

$$G_U + T = \theta\tau(\omega H + rW). \tag{53}$$

Combining this equation with (11) yields

$$\dot{B} = G_I + G_H + G_M + rB - (1-\theta)\tau(\omega H + rW). \tag{54}$$

[17] In the simulation results reported later, we set $\theta = 1$; this assumption ensures that $\gamma > 0$ in equation (52).

Again, following the same procedure as before, the dynamic system driving c, k_I and b consists of (40) and (41), as well as

$$\frac{\dot{b}}{b} = \left\{(\Pi + \Gamma b^{-1})\tau g - \beta\right\} (\upsilon_H \tau g)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c, \tag{55}$$

where $\Gamma \equiv \Sigma_{h=I,H,M}\upsilon_h + \theta - 1$.

Following the same calculations as before, it can be shown that the steady-state growth rate is given equivalently by (27), (44), or

$$\gamma = \left[\Gamma \tilde{b}^{-1}\tau\tilde{g} + (1-\beta)\right] \tilde{k}_I^{\alpha}\tilde{h}^{\beta}.$$

5.3 A Modified Golden Rule

We finally define a modified golden rule (denoted MGR) where the government can issue bonds to finance not only infrastructure but also maintenance and health spending, while remaining constrained to borrow for interest payments. The government budget constraint is now represented by

$$G_U + T + rB = \theta\tau(\omega H + rW),$$

with debt accumulation evolving according to

$$\dot{B} = G_I + G_H + G_M - (1-\theta)\tau(\omega H + rW). \tag{56}$$

Solving as before, the dynamics of consumption and infrastructure capital with this specification are given by (40) and (41) whereas the equation of motion for bonds now becomes

$$\frac{\dot{b}}{b} = \left\{(\Pi + \Gamma b^{-1})\tau g - 1\right\} (\upsilon_H \tau g)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c. \tag{57}$$

Again, following the same calculation procedure as before, the steady-state growth rate can be shown to be given equivalently by (27), (40), or

$$\gamma = \Gamma\tilde{b}^{-1}\tau\tilde{g}\tilde{k}_I^{\alpha}\tilde{h}^{\beta}.$$

We therefore have five dynamic systems to consider: the BBR, given by equations (25) and (26), the GR, consisting of (40)-(42), the PS, consisting of (40), (41) and (51), the MPSR, consisting of (40), (41) and (55) and the MGR represented by (40), (41) and (57). We were able to establish formally earlier that the BBR system is saddlepath stable in the vicinity of the BGP. For all the others, however, saddlepath stability (even in a local sense) is not guaranteed, given their high degree of nonlinearity and the complexity of the relevant conditions.[18] To examine whether stability holds under plausible values for the parameters, and to study how the speed of convergence to the steady state (following a shock) depends on the specification of the rule, we turn to numerical simulations.

6 Calibration

As our starting point, we dwell to a significant extent on the calibration exercise performed in Agénor (2005c), who provides a more detailed motivation for our choices and a comparison between alternative sources. As in that study, parameters are chosen to roughly match some "stylized" facts about middle-income developing countries with a small initial public debt. We will also consider an alternative benchmark, that of a highly-indebted country.

We consider an economy with a relatively low stock of public capital to begin with. Specifically, with output Y normalized to $1,000$, the initial public capital stock is set at 600, implying an initial public capital-output ratio of 0.6. The private capital stock is set at $2,000$, implying that the initial private capital-output ratio is 2 and the private-public capital ratio is about 3.3. Thus, of the two components of physical capital, public capital is the relatively scarce factor, with potentially a large (marginal) impact on growth.

The elasticities of production of commodities with respect to public capital and effective labor, α and β respectively, are set equal to 0.1 and 0.6. These estimates imply a share of private capital in output equal to 0.3. For the health technology, an appropriate value for the coefficient μ is more difficult to pin down, because the empirical evidence is essentially microeconomic in nature. At the same time, as noted earlier, assessing the impact of infrastructure on growth and stability is a key purpose of the model. Ac-

[18]In all systems other than BBR, c can jump whereas k_I and b are predetermined. Saddlepath stability requires therefore one unstable (positive) root. The Routh-Hurwicz conditions require that the determinant of the Jacobian matrix of partial derivatives of each dynamic system be positive (in order to exclude one or three negative roots), whereas its trace must be negative (in order to guarantee at least one negative root). These conditions are very difficult to establish in an unambiguous way.

cordingly, we chose an initial value of $\mu = 0.1$ and perform some sensitivity tests.

The rate of time preference, ρ, is set at 4 percent, a fairly conventional choice. Private consumption, C, which is determined from the goods market equilibrium condition, represents about 85 percent of output. The intertemporal elasticity of substitution is set at 0.2. This is consistent with the evidence for many low- and middle-income developing countries, as discussed by Agénor and Montiel (2006).

The tax rate on output, τ, is set at 0.2. This value is in line with actual ratios for many developing countries, where taxation (which is essentially indirect in nature) provides a more limited source of revenue than in industrial countries. The initial shares of government spending on infrastructure services and health services, υ_I and υ_H, are both set at 0.25. The share of spending on maintenance, υ_M, is set at 0.05, whereas the share of "unproductive" spending, υ_U (which includes also public wages and salaries, although they are not explicitly accounted for in our framework) is set at 0.15. Thus, in the benchmark BBR case, the share of transfers, υ_T, is 0.3, as implied by the budget constraint (18).

The initial stock of public debt, B is set at 400. We therefore consider the case of a country where the debt burden burden, while significant (at 40 percent of output), is not too high. In the results reported below, two alternative values are chosen for the parameter characterizing the efficiency of infrastructure investment. Specifically, we consider a "high efficiency" case, where $\varphi = 0.8$, and a "lower efficiency" case, where $\varphi = 0.5$. A new baseline is, of course, computed for each set of parameters. The coefficient ε is set at 0.3,and ζ as 0.01 initially, to yield annual depreciation rates of around 4 percent for infrastructure capital. We also experimented with a higher value of ζ, to investigate the case where the effect of maintenance spending on the depreciation of infrastructure capital is larger. The coefficient θ is set at 1, both for computational simplicity and to ensure that in the absence of dynamic adjustment in the model, the government does not become a net creditor in the steady state.

Calibration of the model around these initial values and parameters (which involves also determining appropriate multiplicative constants in the production functions for commodities and educated labor) produces the baseline solution. Given the values described above, initial ratios of k_I, and b are, respectively 0.31, and 0.21, whereas the initial steady-state growth rate is equal to 2.5 percent.

7 Numerical Simulations

We now examine the stability and convergence properties of the models associated with the various rules specified earlier. To do so we consider both the base calibration and the alternative cases described in the previous section. With consumption being a forward-looking variable, we use the "extended path" method of Fair and Taylor (1984) as our solution procedure. This iterative procedure is quite convenient (once a discrete-time approximation of the model is written) because it allows one to solve perfect foresight models in their nonlinear form. The terminal condition imposed on consumption is that its growth rate at the terminal horizon (t+50 periods here) must be equal to the growth rate of the private capital stock, given the condition that $c = C/K_P$ must be constant along the balanced growth path.

For all experiments, we report the behavior of the ratio of public debt to output, B/Y, the economy's growth rate, measured as the growth rate of output, and the ratio of public debt to the stock of public capital in infrastructure, B/K_I. This last ratio can be viewed as an indicator of the net liability position of the public sector, with a ratio lower (greater) than unity corresponding to a case where net liabilities are negative (positive). Under BBR, of course, the stock of debt is constant over time, and the net liability position moves in opposite direction to changes in the stock of public infrastructure.

7.1 Increase in Spending Shares

We first experiment with increasing the government spending shares on investment in infrastructure, health, and maintenance. In order to contrast the growth and debt effects, we assume in this section that increases in these shares are financed through borrowing where the government is not restricted by the rule to finance the additional spending by issuing bonds, and through a fall in transfers where the rule prevents the government from doing so.

7.1.1 Infrastructure

We first consider a 5 percentage-point increase in υ_I from 0.25 to 0.3. The additional spending on infrastructure is financed by bonds in GR, MGR and MPSR as defined by (31),(54) and (56), whereas transfers are reduced by 5 percentage points under the PSR. Consequently, the increase in spending on infrastructure does not yield to debt accumulation in PSR and the dynamics of the debt-output ratio are entirely driven by the initial productivity of infrastructure and private capital.

The results are reported in Figure 1. Given the assumption of relative scarcity of infrastructure capital with respect to private capital in the calibration, the debt-output ratio displays a large initial fall under PSR, although it stabilizes at a lower value (relative to the baseline solution) in the steady state as diminishing returns set in. In GR, MGR and MPSR, the effect of the increase in infrastructure spending on debt accumulation is twofold: an increase in the debt stock due to higher borrowing, and a subsequent increase in interest payments as the marginal productivity of private capital, r, rises in parallel with the shock. The increase in interest payments further translates into a higher tax base in all these rules (given the definition of taxable income), but it also raises borrowing for maintenance and health under MGR and MPSR because all government spending is proportional to the tax base.[19] However, higher borrowing also leads to crowding out of private capital and therefore the overall effect on output depends on the relative productivity of these factors in production. Owing to high productivity of health especially, the positive effect of additional productive government spending dominates the negative effect associated with the crowding out of private capital; in the long run, the debt-output ratio increases by less, and output grows by more, under MPSR and MGR than under GR.[20] The MPSR displays the largest effect on growth because the financing of interest payments through borrowing yields faster debt accumulation and larger increases in the tax base than the MGR. But at the same time, the debt-output ratio is more sensitive to movements in interest rates under MPSR. Swings in the debt-output ratio are therefore sharper under MPSR, and the long-run increase in that ratio is larger than under MGR. The BBR and the PSR (under which the increase in υ_I is financed by a reduction in transfers) display the lowest

[19] Unlike Greiner and Semmler (2000), who assume that infrastructure investment is determined residually from available tax revenues, in our framework we assume that it is transfers (or equivalently lump-sum taxes) that adjust in response to changes in interest payments under GR as well as MGR. Therefore, the "internal crowding-out effect" associated with an increase in interest payments on productive spending, as identified by Greiner and Semmler (2000), is not present in our case.

[20] The ordering of the rules is invariant to the length of the simulation period, and MPS always stabilizes at a lower value than GR in the base calibration. However, simulations with different configurations of spending shares show that MPS may not yield convergence due to excessive debt accumulation.

effects on output growth. In the PSR rule especially, although the return on bonds increases, the overall fall in the debt-output ratio translates into a lower share of interest income in the tax base to finance investment in health and maintenance, and therefore deviations of output growth from baseline are even smaller than under BBR.

Regarding the net liability position of the government, the MPSR and MGR rules deliver the largest drops from baseline despite the fact that additional spending is financed through borrowing in these regimes—as opposed to the PSR rule where it is carried out through a reduction in transfers. In essence, these rules perform better than the Golden Rule (despite the fact that they lead to more borrowing initially) because there is higher public capital accumulation and faster growth.

7.1.2 Maintenance

The dynamics associated with a 5 percentage-point increase in υ_M are displayed in Figure 2. Unlike the shock to υ_I in the previous section, the debt-ouput ratios increase steadily under the MPSR and the MGR. Although the increase in maintenance spending stimulates higher investment in infrastructure and health under these rules (as in the previous case), the relatively low productivity of maintenance itself yields an initial increase in the debt-output ratio that stabilizes over time. For the same reason, maintenance cannot compensate for the drop in transfers under PSR, and the debt-output ratio displays only a small reduction—before stabilizing at a value slightly above baseline. At the same time, under GR, where interest payments do not affect debt accumulation, the debt-output ratio falls slightly at first and converges to the baseline value in the long run.

Regarding the growth rate of output, the only difference with the previous experiment is that PSR now performs better than BBR and MGR because faster debt accumulation, as indicated by the positive deviation from baseline of the debt-output ratio, generates more productive government spending. Finally, the net liability position of the government behaves just like the debt-output ratio in this case, because maintenance has indirect effects on the stock of infrastructure capital and growth. Given our initial calibration, the positive affect of additional maintenance spending on the rate of depreciation of public infrastructure does not compensate for the additional borrowing required to finance this spending, and the net liability position of the government worsens under the MGR and MPSR.

7.1.3 Health

Next, we consider a 5 percentage-point increase in υ_H, financed through borrowing under MGR and MPSR, and through a reduction in transfers under GR and PSR. The results for the main variables of interest are reported in Figure 3. As shown in the upper panel, the debt-output ratio exhibits an initial drop under each rule, because unlike infrastructure investment and maintenance expenditures that are predetermined, health spending enters the production function for health services (and thus productivity) contemporaneously. The increase in υ_H leads therefore to an instantaneous jump in output. The fall in the debt-output ratio further accelerates in the following periods but slows down (as before) once diminishing returns set in. However, because of the relatively high productivity of health, MPSR performs better than MGR in terms of the debt-output ratio in the present case, while displaying the largest drop throughout the adjustment period. In addition, unlike what happens with an increase in maintenance spending, the debt-output ratio falls more under MGR and MPSR than under GR—even if additional spending in the former regimes is financed by borrowing. Although the BBR, GR and MGR rules give higher increases in the growth rate of output than MPSR initially, the MPSR again performs best in the longer run. Clearly, the indirect increases in spending on infrastructure and maintenance allowed by a higher tax base and government revenues are initially unable to compensate for the adverse crowding-out effect of private capital through interest payments in this case. This effect is even more significant because the jump in the rate of return on capital is also magnified by the productivity effect of health spending. Consequently, non-primary surplus rules give better results in in the initial phase of the adjustment process.

At the same time, the simulation results reveal that the dynamics of the debt-output ratio and the net liability position of the government are quite distinct from each other in the case at hand. Although health spending is highly productive and the debt-output ratio drops significantly, the effect of health spending on infrastructure capital operates through the increase in output (which enlarges the tax base) and subsequently higher investment on infrastructure. Thus, the MPSR exhibits a cyclical pattern with a worsening of the net liability position of the government in the long run, whereas under the MGR the transition is smoother and the long-run deviation from baseline is less pronounced. Under the PSR, the behavior of the net liability position mainly depends on the relative productivity of private capital and health—

as well as the upward movement in the interest rate due to the increase in the productivity of private capital—because infrastructure investment is proportional to the tax base, whereas under GR, the expansion of the tax base also triggers borrowing for infrastructure investment. This, in turn, limits the improvement in the net liability position of the government, relative to the PSR rule.

7.2 Reallocation of Unproductive Spending

We now consider the case where the government cuts the share of unproductive spending by 5 percentage points, that is, a reduction in υ_U from 0.15 to 0.1, and reallocates the additional resources to infrastructure investment, maintenance expenditure, and health spending. The results are shown in Figures 4 to 6.

Consider first the case where the resources created by the cut in unproductive spending are used to finance additional infrastructure investment. As shown in Figure 4, under every rule, the debt-output ratio behaves in a manner similar to the pattern observed in the previous experiment (increase in the share of spending on health). At first, the ratio drops quite significantly; but as the growth rate of output stabilizes, the magnitude of the drop falls markedly. However, the deviations are also strictly negative, because resources are now reallocated from unproductive to productive spending. With respect to the growth rate of output, the BBR, GR and MGR rules again deliver higher growth than the primary surplus rules initially, but the MPSR dominates in the longer run. The main reason for this is that the magnitude of the increase in the growth rate largely depends on the initial marginal product of infrastructure, which is highest in BBR due to lack of debt accumulation induced by higher public investment. However, as the fall in the interest rate stabilizes and interest payments begin to represent a larger part of the tax base, the government invests more in health and maintenance under the MPSR, and the increase in the growth rate of output in the long run tends to exceed the increase observed under the other rules.

Next, consider the case where the cut in unproductive spending is reallocated to maintenance expenditure. As illustrated in Figure 5, the dynamics of debt accumulation are affected in exactly the same way as above. However, this time around, the BBR, GR and the MGR perform better than the primary surplus rules with respect to growth throughout the adjustment period. The growth rate stabilizes at higher values than before, with a much lower dispersion across rules. Regarding the net liability position of the government, the allocation of resources toward infrastructure or maintenance leads to an improvement under all rules—and particularly so under MPSR. In both Figure 4 and Figure 5, the path of debt-public capital ratios follows closely the behavior of debt-output ratios.

Finally, Figure 6 illustrates the case where the resources generated by a cut in unproductive spending are used to increase spending on health. As in Figure 3, the increase in health spending yields a contemporaneous increase in labor productivity and output (and thus tax revenues), a drop in the debt-output ratio, and a jump in the growth rate. Debt dynamics in subsequent periods are, again, as described above, but volatility is larger due to the relatively high productivity of health. Except under MPSR, the increase in the growth rate converges smoothly under all rules. BBR and MGR deliver again the highest growth effects in the early phase of adjustment. Nevertheless, as in the case of a reallocation to infrastructure, the MPSR rule displays an increasing trend as the fall in the interest rate slows down, and the growth rate of output in the long run rises slightly more than the other rules.

At the same time, a shift from unproductive spending to health delivers mixed results with respect to the net liability position of the government because—unlike the shifts toward infrastructure and maintenance—output and the interest rate increase contemporenously as a result of the shock. Thus, the initial increase in the interest rate, and the additional borrowing for maintenance and health generated by the increase in the tax base, cause the net liability position to worsen under all rules initially. This is even more so under MPSR, because both interest payments and each category of (productive) government spending add to the debt stock. However, as the interest rate falls and infrastructure investment increases (as a result of higher output and tax revenues), the net liability position of the government improves in the longer run. Again, as before, the PSR and MPSR rules are more volatile than the GR and MGR rules, because of movements in interest payments.

8 Sensitivity Analysis

To examine how robust the above results are, we perform sensitivity analysis involving a lower value of the degree of efficiency of public investment outlays in infrastructure, a higher initial level of public debt, an endogenous risk premium (related to the debt-public capital ratio), and a higher degree of sensitivity of the depreciation rate of public capital to government spending on maintenance.

8.1 Low Efficiency of Infrastructure Investment

In order to assess the role of the degree of efficiency of public infrastructure investment, we considered the case where $\varphi = 0.5$ instead of 0.8. Figures 7 to 12 display the dynamics associated with an increase in υ_I, an increase in υ_M, an increase in υ_H, a shift from υ_U to υ_I, υ_M, and υ_H respectively, as in the previous section. Clearly, the impact of a lower φ is to reduce the productivity of all spending components due to gross complementarity. The debt-output ratio and the growth rate of output are therefore adversely affected. Whereas Figures 7 and 8 show that increases in the debt-output ratio in the long run are higher than the high-efficiency case following an increase in υ_I and υ_M, Figures 9-12 show that the drops in that ratio are lower after an increase in υ_H or a shift from unproductive to any category of productive spending. Moreover, a lower efficiency of public infrastructure investment favours PSR against the golden rules because the PSR rule does not allow the government to borrow for (relatively inefficient) infrastructure investment. The PSR rule now performs better than GR in terms of the debt-output ratio and yields higher drops in that ratio when there is an increase in health spending (Figure 9), or when there is a shift from υ_U to υ_I, υ_M, or υ_H (see Figures 10 to 12).

For the same reason, the net liability position of the govenment displays similar dynamics, and in the case of an increase in υ_H PSR yields the highest improvement (see Figure 9), whereas in the other figures the wedge between PSR and the other rules is the largest (lowest) where PSR initially performs better (worse). Similarly, falls in the long-run debt-output ratio are now very close to each other under MGR and PSR in case of a shift from υ_U to υ_M (see Figure 11), in contrast to the high efficiency case (see Figure 4).

At the same time, lower efficiency of public infrastructure investment reduces the growth performance of MPSR against the other rules. Whereas the wedges between MPSR and the other rules are smaller now, following an increase in υ_I and υ_M, it takes the MPSR rule longer to "catch up" with the BBR and the golden rules following an increase in υ_H (see Figures 7 to 9). This is also the case when there is a shift from unproductive spending toward infrastructure (see Figure 10) or toward health (see Figure 12), whereas in the case of a shift to maintenance spending, MPSR yields the lowest increase in the growth rate of output in the long run (see Figure 11).

8.2 High Initial Level of Public Debt

We also experimented with a high initial debt-output ratio, with an value of $B_0 = 700$ instead of 400. The results were very similar to those obtained earlier, and therefore we omit illustrative graphs to save space.[21] The main difference is the magnitude of the shifts in the debt-output ratio and the growth rate during the adjustment period.

Moreover, we found that the model displayed explosive behavior for the MPSR when $B_0 = 800$. Thus, as can be expected, if the initial value of debt is too high, the model displays unstable dynamics. Without an endogenous adjustment in the tax rate or a spending component, the rise in interest rate payments is such that it throws the model off its saddlepath.

8.3 Endogenous Risk Premium

It may be argued that the main reason primary surplus rules perform well (especially in terms of debt dynamics) in the foregoing experiments is that the interest rate on bonds is not linked to the outstanding level of debt—an assumption that may be viewed as questionable for most countries. In practice, the interest rate on bonds often includes a premium that increases with net liabilities due the higher (perceived) default risk. To account for this effect, we defined an endogenous risk premium, PR, as

$$PR = \frac{\kappa}{2}(\frac{B}{K_I})^2, \tag{58}$$

where $\kappa > 0$. Convexity implies that the faster the speed at which the net liability position deteriorates, the larger the increase in the risk premium demanded by markets for holding government debt.[22]

Using (58), the return on bonds, r_B, can be written as

$$r_B = r + PR.$$

[21] A complete set of figures is available upon request.

[22] Note that the premium could also be defined as a function not of net liabilities or assets (that is, stocks) but rather as a function of liquidity (flows), measured for instance by the ratio of interest payments at the "risk-free" rate, r, to tax revenues. However, the specification used in (58) is sufficient for the purpose at hand.

We chose MPSR as the most appropriate rule to perform this sensitivity exercise, given that it is the rule that is most responsive to changes in the interest rate. In the benchmark case considered earlier, $B/K_I = 400/600 =$ 67 percent, and the government's net liability position is negative. We set $\kappa = 0.0001$ and $\kappa = 0.0005$, and simulate the model for a 5 percentage-point increases in υ_I and υ_M. Figures 13 and 14 present the results. While a higher premium leads to a fall in deviations of the debt-output ratio from baseline and a lower growth rate when spending on infrastructure increases because the net liability position of the government improves, the reverse holds when υ_M is increased instead. Essentially, the effect of an increase in υ_I on the stock of infrastructure capital is higher than the effect on the debt stock. The government therefore benefits from a fall in the risk premium more as the sensitivity of the premium to the net liability position of the public sector increases.

However, the same result does not hold when maintenance expenditures increase, because the effect of maintenance spending on infrastructure capital (given our initial calibration) is not strong enough to offset the increase in the debt stock; as a result, following the shock, the net liability position of the government deteriorates. In this case, the government suffers from increasing interest payments as the premium rises. Additional productive government spending via a higher tax base brings higher growth than the baseline whereas the reverse is true for an increase in infrastructure investment. It must also be noted that for $\kappa \geq 0.001$, the rule displays explosive dynamics as excessive debt accumulation completely crowds out private investment.[23]

8.4 Maintenance and Depreciation of Public Capital

Next, we experimented with an alternative value of the coefficient that determines the effect of maintenance spending on the depreciation rate of infrastructure, ζ. We take GR, MGR and MPSR as above and simulate the model for $\zeta = 0.02$, instead of the baseline value of 0.01. The behavior of the growth rate of output in response to a 5 percentage-point increase in υ_M is presented in Figure 15 for each of these three rules. Unsurprisingly, the additional spending on maintenance contributes more to growth as ζ goes

[23]We also experimented, with an alternative specification of the premium, with B/K_P in (58); this relationship could be derived from a simple portfolio equation for the private sector. In that case, we were able to increase κ all the way up to 0.013. Beyond that value, however, the model also diverged.

up, and the shifts in transitional and long-run growth rates are very close to each other for the three regimes. The figures also suggest that although the magnitude of the wedge between the rules may be slightly affected in case of a higher productivity of maintenance spending, the ordering of the rules in Figures 3 and 9 is not altered.

8.5 Higher Health-Infrastructure Elasticity

Finally, we performed a sensitivity analysis with respect to the elasticity of health output to infrastructure capital, μ, under the GR, MGR, and MPSR rules and applying a 5 percentage-point increase to υ_I. The results are displayed in Figures 16 to 18 for $\mu = 0.1$, which corresponds to the baseline case, and $\mu = 0.2$. Clearly, the impact of a higher elasticity of health to infrastructure is to magnify the fall in the debt-output ratio, as well as the improvement in the net liability position of the government, under each rule. These effects are larger under MPSR, as could be expected. At the same time, the growth rate of output increases more significantly under GR because with $\mu = 0.2$ the government borrows to finance health expenditures that are less productive now (see equation (5)) under MGR and MPSR. Thus, the crowding-out effect of public debt accumulation on private capital formation is stronger, whereas the positive effect of productive government spending on output is weaker, than in the case where $\mu = 0.1$.

9 Concluding Remarks

The purpose of this paper was to examine the performance of alternative fiscal rules in an endogenous growth model with public capital and debt. The first part of the paper provided a brief review of the current debate on the golden rule, whereas the second part presented the analytical framework. In the model, in addition to investing in infrastructure, the government spends on maintenance (which improves the durability of public capital) and health (which raises labor productivity). Infrastructure affects the production of both commodities and health services.

The third part discussed the properties of the model under a balanced budget rule. Because the model is analytically tractable under that rule, we fully characterized its stability and uniqueness properties. We then considered several alternative rules—a "standard" golden rule, a "standard" primary rule, and modified versions of both rules to exclude all productive spending (infrastructure investment, but also current outlays on maintenance and health). Because the resulting models are too complex to solve analytically (as a result of the higher dimension added by public debt accumulation), we resorted to numerical techniques for dynamic comparative analysis.

After discussing the calibration procedure, the performance of all rules was examined in response to two types of shocks—an increase in the shares of spending on infrastructure, maintenance, and health; and a reallocation of unproductive spending to alternative types of productive expenditure. Under a range of plausible parameter configurations (pertaining, in particular, to the degree of efficiency of public investment) and spending shares, and as long as the risk premium on government bonds is not too elastic to the debt-public capital ratio, our numerical simulations showed that in response to these shocks a primary surplus rule that excludes productive spending performs better than alternative rules—in the sense of yielding higher steady-state growth, stable debt-output ratios, and more rapid convergence. Thus, "narrow" rules, which do not account for the fact that some components of current spending may have a high pay-off in terms of growth without jeopardizing fiscal performance, can be described as tyrannical in nature.

The analysis presented in this paper could be extended in various directions. For instance, it could be worth examining the welfare effects of fiscal rules, in addition to their implications for growth and fiscal sustainability, as we have done here. Studies by Stockman (2001) and others, have shown indeed that welfare losses associated with (narrow) fiscal rules can be substantial. In that context, accounting for the effect of health services on utility in a non-separable manner (as, for instance, in Agénor (2005*d*)), or for a direct effect of health on the rate of time preference (as in Agénor (2006)) would be critical for ranking alternative rules—especially if the "modified" primary surplus rules are examined in more detail, by considering separately rules excluding and including expenditure on health only. Another useful extension would be to account, as in Agénor (2005*b*), for endogenous depreciation of *private* capital, in response to government spending on maintenance. As documented by Agénor and Moreno-Dodson (2006), such effects can be sizable, particularly in the area of transport.

However, although these extensions would be valuable in their own right, they are unlikely to alter some of the broad policy lessons that can be drawn from the present analysis. First, there are parameter configurations for which the primary surplus rules lead to unsustainable ratios of public debt (in the sense that the model does not converge, following a specific shock). Intuitively, if components of spending are productive but not very much so, excluding them from the primary surplus rule is bound to create instability. Although we focused our discussion on cases where the steady-state path is stable (in order to perform meaningful comparisons across rules), this is important to keep in mind. Similarly, financing all productive components of public spending through borrowing can be destabilizing if the initial level of debt is high to begin with and/or the risk premium is highly sensitive to the debt-public capital ratio, that is, the government's net liability position.

Second, to the extent that there is heterogeneity among parameters and starting values, imposing uniform fiscal rules "across the board" to a group of countries because they share a common monetary arrangement (as is presently the case in the Euro area or the Western African Economic and Monetary Union, for instance) makes little sense. In countries where stocks of public infrastructure assets are relatively low to begin with, greater latitude to finance investment outlays through borrowing makes sense economically—as long as the investment is sufficiently efficient (in the sense that it does indeed turn into capital) and productive. This may actually improve prospects for fiscal stability. By implication, imposing uniform debt-output ratios (or, more appropriately, debt-asset ratios) would also be sub-optimal in the sense that it would unduly constrain growth and actually hamper fiscal sustainability—unless, again, these ratios are high to begin with and have a large effect on the risk premium embedded in interest rates.

Third, our focus on growth (and transitional dynamics) should not lead us to lose sight of the fact that fiscal rules are also imposed in order to avoid procyclical government spending from exacerbating macroeconomic volatility. To the extent that such volatility is detrimental to growth (as shown in a number of recent studies), a potential trade-off may emerge. For instance, if a "standard" fiscal rule succeeds in lowering volatility significantly, constraining productive spending may be ultimately beneficial to growth.[24] The nature of this trade-off would normally depend on a number of institutional factors, in addition the structural characteristics of the economy. In countries where political polarization is high, or the national legislature is fragmented across a large number of political parties, the propensity to engage in procyclical spending may be quite strong and tight rules may be

[24]Moreover, volatility may have large welfare costs, as documented by Pallage and Robe (2003), independently of its impact on growth.

inevitable.

Fourth, the modified primary surplus rule in our analysis does not provide a clear guide as to where one should "draw the line" on current outlays. Indeed, we assumed the population to be constant in the model. But suppose that population is endogenous and that "raw" labor must be educated to become productive (as in Agénor (2005*a*, 2005*c*)). Suppose also that the education technology is such that it depends on government spending as well. The foregoing analysis suggests therefore that spending on education should also be excluded from the rule, as long as it is sufficiently productive. The reasoning can of course be pushed one step further; it could be argued that wages and salaries of public servants (a large share of public spending in most countries) are "productive" to some degree because they increase incentives to provide effort and facilitate private activity. Likewise, spending on defense and security, or the environment, could be viewed as being productive—feeling safer or breathing air of better quality may lead to higher productivity. Moreover, governments may have strong incentives to present various categories of spending as productive, even if the case for doing so is weak.[25] The implication therefore is that "drawing the line" becomes very difficult, making the practical implementation of the modified primary surplus rule problematic at best—given the objective of fiscal stability.

Our proposal is that, rather than focus solely on formal fiscal rules *per se*, governments should be required to announce and justify a multi-year, Core Productive Expenditure Program (CPEP) that is fully transparent. This would be a particularly important step for developing countries, where enhancing the growth effects of fiscal policy represent an acute policy issue. The CPEP should include only the "most productive" components of both current and public capital expenditures, as voted into law or adopted by Parliament. Moreover, it should be realistic, in the sense that it should be based on reasonable financing assumptions. In line with the foregoing analysis, therefore, the conventional distinction between current and capital spending would give way to a more economically meaningful distinction, at least from a growth perspective, between (highly) productive and (rel-

[25] In fact, the experience so far with fiscal policy rules has been mixed; although there have been some successful cases, a number of rules have been ineffective, suspended, or abandoned (see Kopits, 2001). In some cases, nontransparent, off-budget practices, or "creative accounting" have proliferated; see Bohn (2006) for the experience of some European countries under the Maastricht Criteria.

atively) unproductive outlays.[26] The explicit nature of the CPEP would force the government to explain its spending program, its financing (through both earmarked revenues and borrowing), and its implications for growth. A multi-year framework would also make expenditure management and resource allocation more responsive to national priorities, by providing a more predictable environment for program planning and implementation.

The CPEP would therefore be protected from the vagaries of fiscal stabilization. Indeed, budgets are often characterized by expenditure rigidities; a large fraction of outlays (pensions, social security, debt service, wages, transfers to sub-national governments) are not discretionary. Greater fiscal transparency would help the public to monitor what the government does, whether it is in the context of an explicit strategy for growth (as in low-income countries, for instance) or not. Moreover, imposing transparency would limit incentives to include components of spending for well-known political reasons (see Khemani (2006)), thereby alleviating the moral hazard and "create accounting" problems that are often raised in reference to the golden rule.

Here, it is worth making a parallel with the current state of information disclosure on monetary policy. Only 15 years ago, very few would have predicted that central banks (in developing and developed countries alike) would be as transparent as they are today in their operations. Our view is that such transparency with regard to a core program would only benefit fiscal policy.[27] Of course, for government-provided information to be credible may entail setting up an independent and non-partisan agency, such as a Fiscal Policy Committee (FPC) as proposed by Eichengreen, Hausman and von Hagen (1999) and Wyplosz (2002). In our proposal, in addition to having legal authority to impose explicit constraints on the overall size of public deficits and debt, the FPC would monitor progress (or lack thereof) toward achieving the CPEP and communicate this information to the public through regular publications. However, other institutional arrangements could be put in place. In some countries, for instance, this function could be delegated to a joint committee representing all the parties involved in the preparation

[26] To avoid the negative connotation associated with the term "unproductive," more neutral language could of course be used.

[27] Note that we are advocating greater transparency with respect only to a CPEP, not regarding all fiscal accounts (which would involve, for instance, explicit and accurate reporting of all contingent liabilities). Although the latter would be of course desirable, it has proved very difficult to achieve—for obvious political economy reasons.

of a strategy paper for poverty reduction, because of the inclusive nature of the process. In either case, however, an open question—which goes well beyond the scope of the present study—is how best to determinate the nature and extent of sanctions that should be imposed for non-compliance with the CPEP, that is, how to make governments accountable for poor performance.

Appendix
Stability with a Balanced Budget Rule

With a balanced budget, the dynamic system consists of equations (25) and (26), which are repeated here for convenience:

$$\frac{\dot{c}}{c} = (\sigma s - q)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c - \sigma\rho, \tag{A1}$$

$$\frac{\dot{k}_I}{k_I} = \tau^{1/\Omega}\chi {\upsilon_H}^{(1-\mu)\beta/\Omega} k_I^{-\eta/\Omega} - q(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} + c - 1. \tag{A2}$$

To investigate the dynamics in the vicinity of the steady state, this system can be linearized to give

$$\begin{bmatrix} \dot{c} \\ \dot{k}_I \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c - \tilde{c} \\ k_I - \tilde{k}_I \end{bmatrix}, \tag{A3}$$

with the a_{ij} given by

$$a_{11} = \tilde{c}, \qquad a_{12} = (\sigma s - q)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega}(\alpha + \mu\beta)/\Omega \tilde{k}_I^{-\eta/\Omega}\tilde{c}, \qquad a_{21} = \tilde{k}_I,$$

$$a_{22} = (-\eta/\Omega)\tau^{1/\Omega}\chi {\upsilon_H}^{(1-\mu)\beta/\Omega}\tilde{k}_I^{-\eta/\Omega} - q(\alpha+\mu\beta)/\Omega(\upsilon_H\tau)^{(1-\mu)\beta/\Omega}\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} < 0,$$

where $\tilde{x}$ denotes the stationary value of x. In principle, a_{12} can be either positive or negative; we assume in what follows that σ is sufficiently small to ensure that $a_{12} < 0$.[28]

The consumption-private capital ratio, c, is a jump variable, whereas the public-private capital ratio, k_I, is predetermined. Saddlepath stability requires one unstable (positive) root. To ensure that this condition holds, the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A3) must be negative, that is, $a_{11}a_{22} - a_{12}a_{21} < 0$, or equivalently, $-a_{12}/a_{11} < -a_{22}/a_{21}$. It can be verified that—regardless of the sign of a_{12}—this condition always holds in the present case.[29] Therefore, the system is saddlepath stable. The slope of the saddlepath SS, which is given by $\kappa = -a_{12}/(\tilde{c} - \nu)$, where ν is the negative root of the system, is positive, given that $a_{12} < 0$.

[28] See Agénor (2005*a*) for a discussion of the general case.

[29] To see this, note that from equations (21), (22), and (23) in the text, and using $(\alpha + \mu\beta)/\Omega = (1 - \eta/\Omega)$, we can write $a_{12}a_{21} = \left[\sigma\rho\tilde{c} - \tilde{c}^2\right](1 - \eta/\Omega)$, and $a_{22} = [(-\eta/\Omega)(1 - \tilde{c}) - (\gamma + \tilde{c}]$.

From (A1), setting $\dot{c} = 0$ yields

$$\tilde{c} = \sigma\rho + (q - \sigma s)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \tilde{k}_I^{(\alpha+\mu\beta)/\Omega}. \tag{A4}$$

Substituting (A4) in (A2) with $\dot{k}_I = 0$ yields the implicit function

$$F(\tilde{k}_I) \equiv \tau^{1/\Omega} \chi \upsilon_H{}^{(1-\mu)\beta/\Omega} \tilde{k}_I^{-\eta/\Omega} - \sigma s (\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \tilde{k}_I^{(\alpha+\mu\beta)/\Omega} + \sigma\rho - 1 = 0. \tag{A5}$$

To show that the BGP is unique, note first that from (A5),

$$F_{\tilde{k}_I} = (-\eta/\Omega)\tau^{1/\Omega} \chi \upsilon_{H}{}_I^{(1-\mu)\beta/\Omega 1} \tilde{k}_I^{-\eta/\Omega - 1} - (\alpha + \mu\beta)/\Omega \sigma s (\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \tilde{k}_I^{-\eta/\Omega},$$

which is negative along a BGP with $\tilde{k}_I > 0$. Thus, $F(\tilde{k}_I)$ cannot cross the horizontal axis from below. Now, note that

$$\lim_{\tilde{k}_I \to 0} F(\tilde{k}_I) = +\infty \text{ and } \lim_{\tilde{k}_I \to +\infty} F(\tilde{k}_I) = -\infty.$$

Given that $F(\tilde{k}_I)$ is a continuous, monotonically decreasing function of $\tilde{k}_I$, there is a unique positive value of $\tilde{k}_I$ that satisfies $F(\tilde{k}_I) = 0$. From (A4), there is also a unique positive value of $\tilde{c}$.

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FIGURE 1

5 %Increase in vl

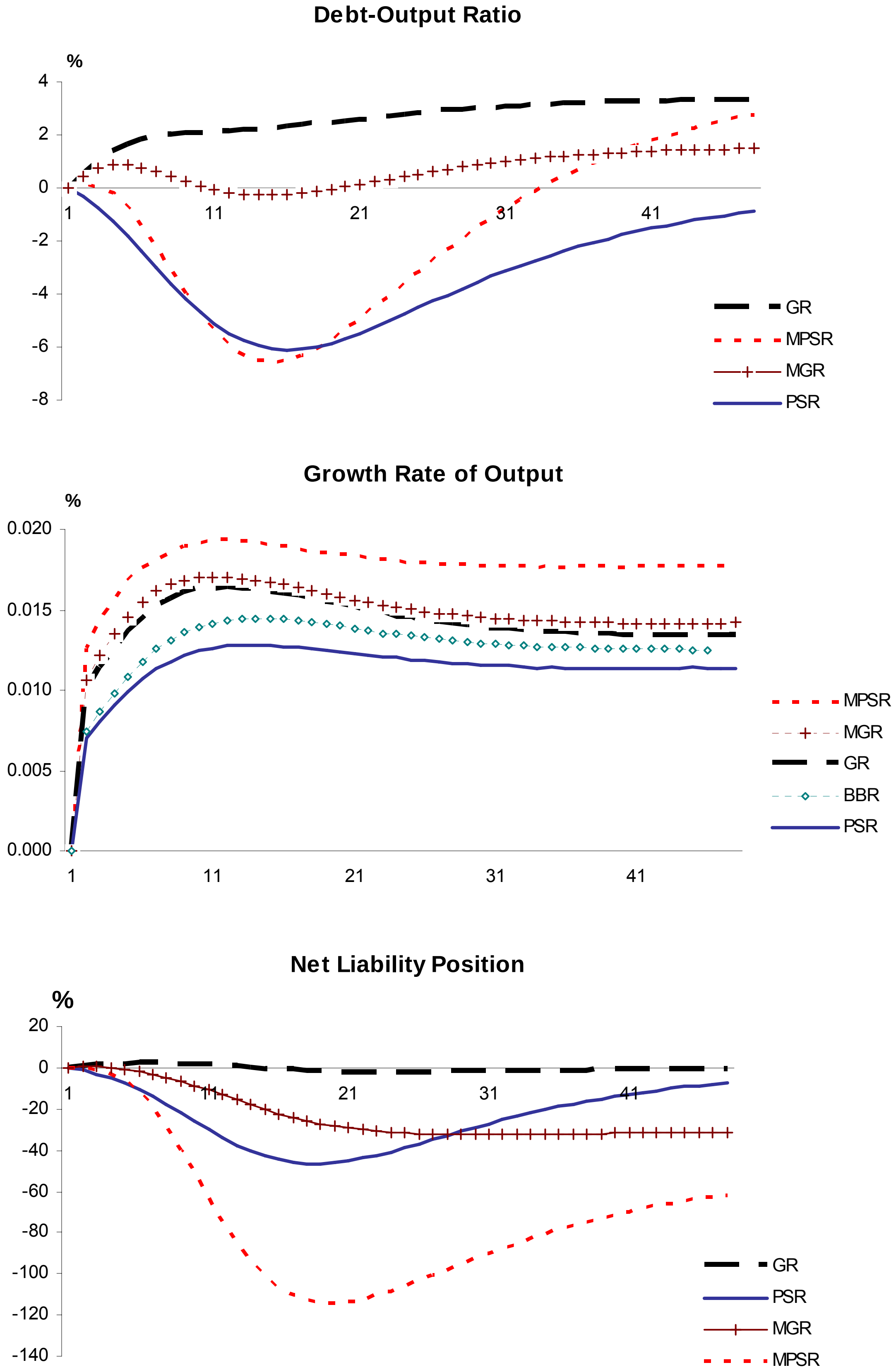


FIGURE 2

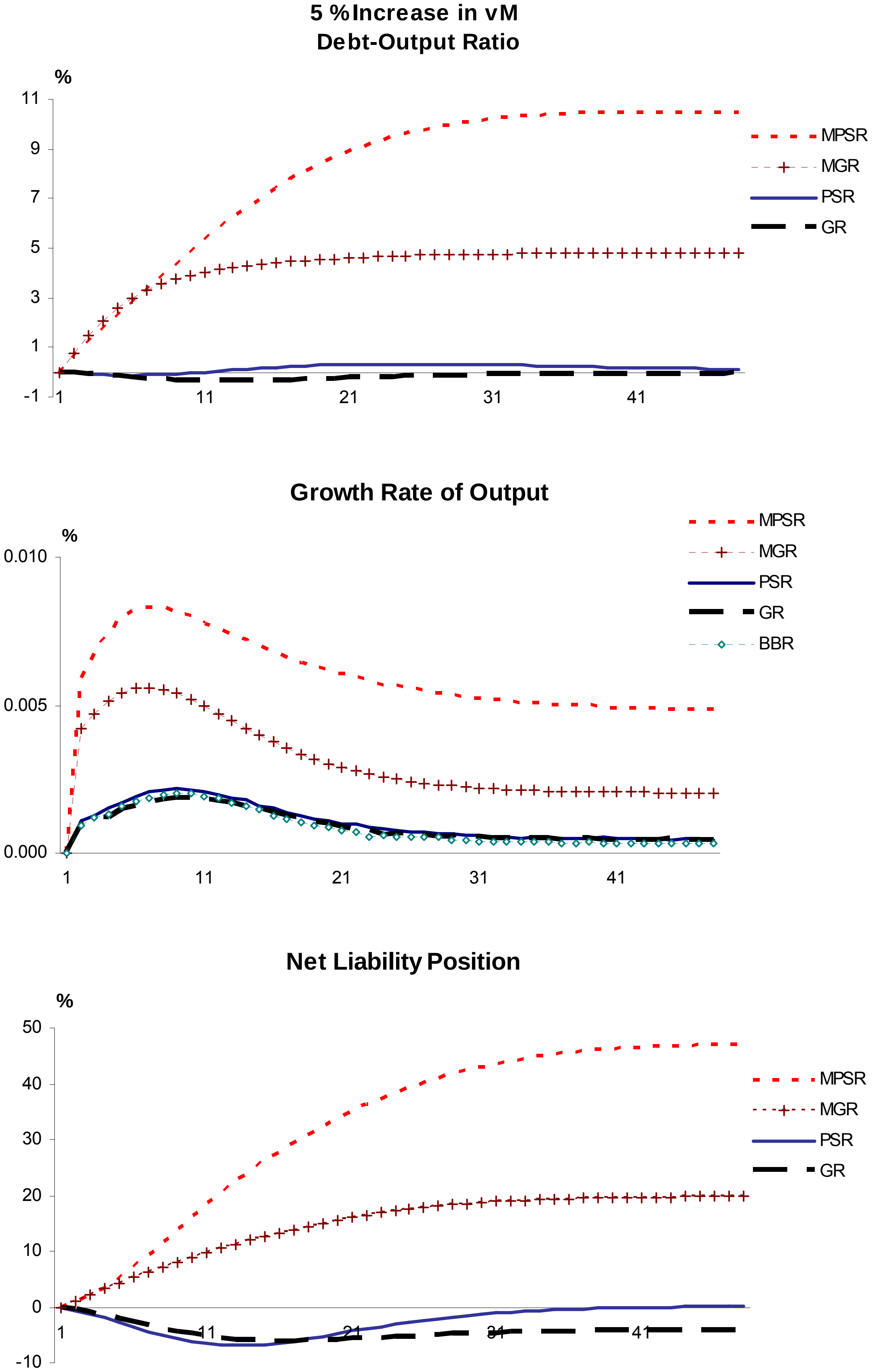
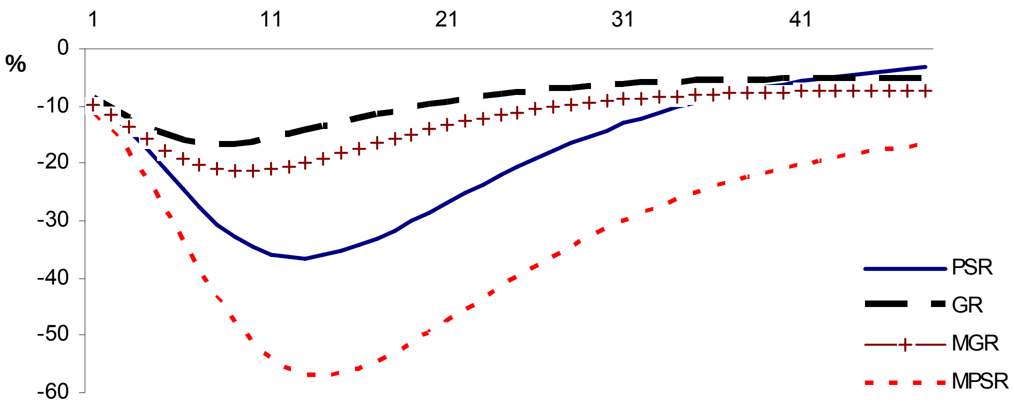

5 %Increase in vM
Debt-Output Ratio
%
11
9
7
5
3
1
-1
1
11
21
31
41
MPSR
MGR
PSR
GR
Growth Rate of Output
%
0.010
0.005
0.000
1
11
21
31
41
MPSR
MGR
PSR
GR
BBR
Net Liability Position
%
50
40
30
20
10
0
-10
1
11
21
31
41
MPSR
MGR
PSR
GR

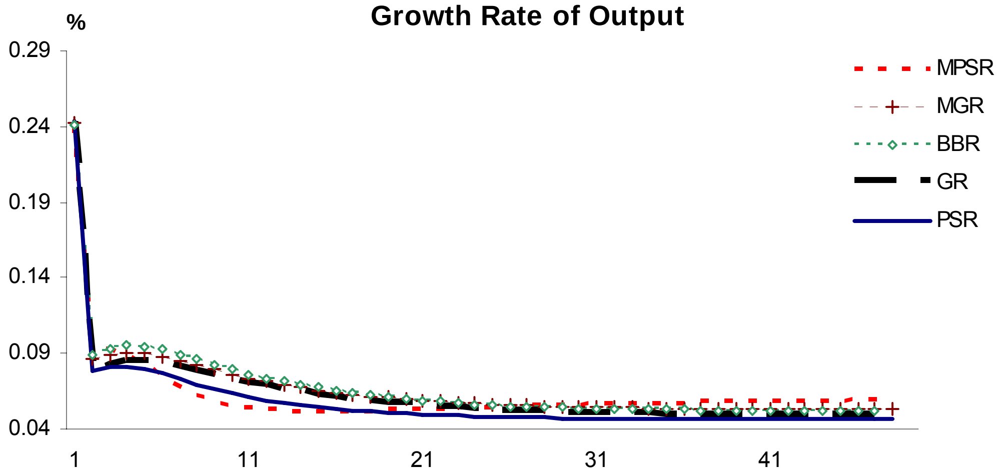
FIGURE 3

5 %Increase in vH

Debt-Output Ratio



Growth Rate of Output



Net Liability Position

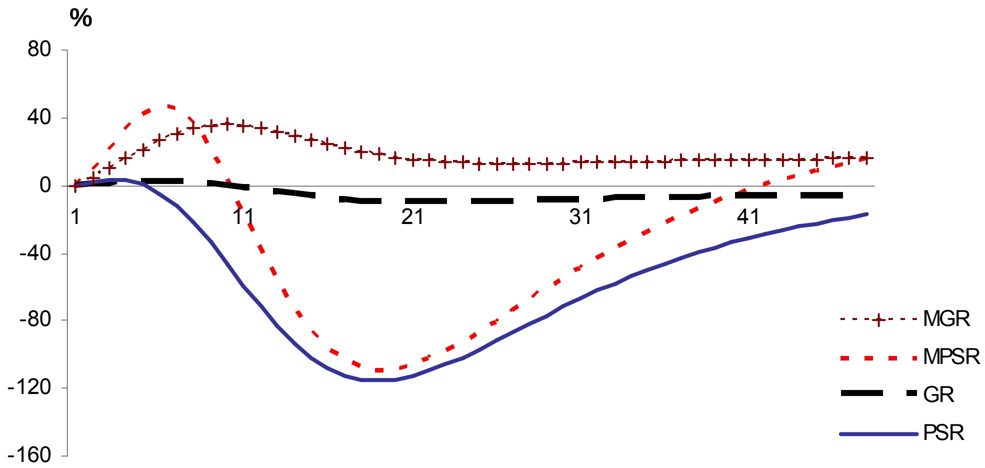


FIGURE 4

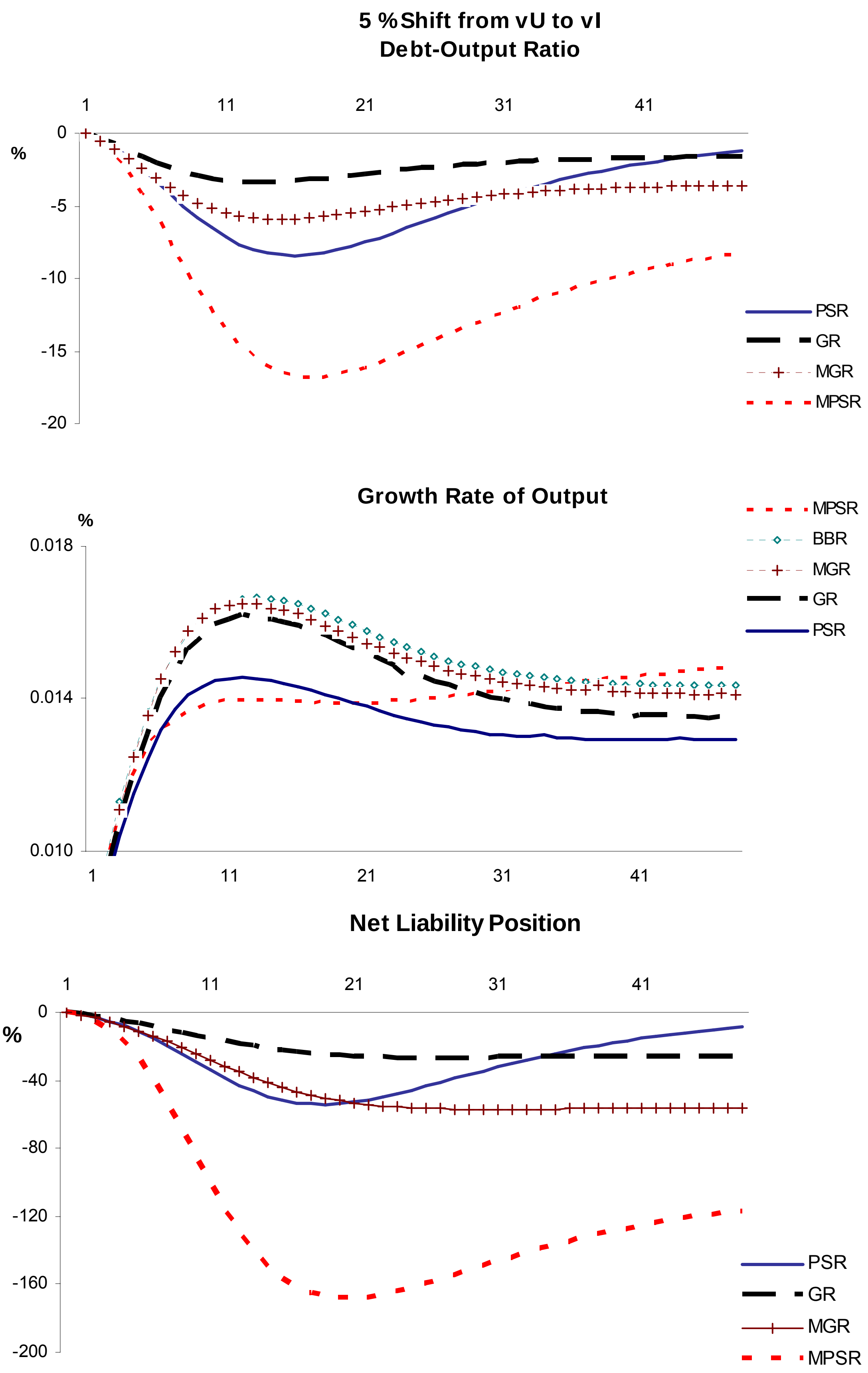
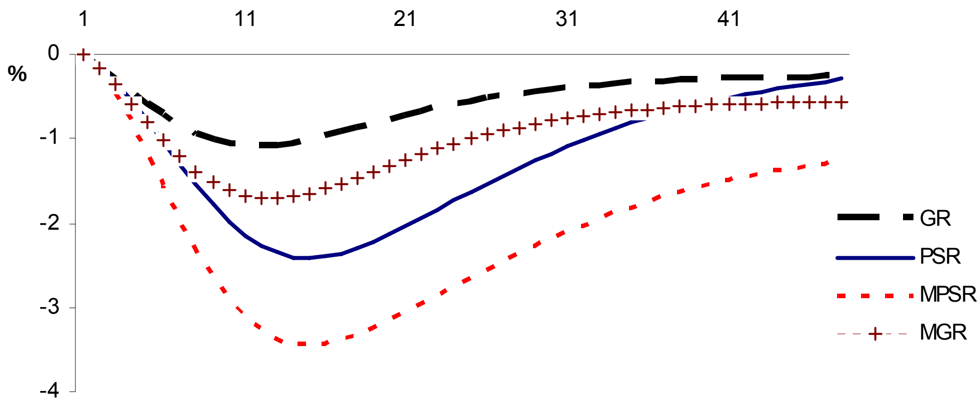
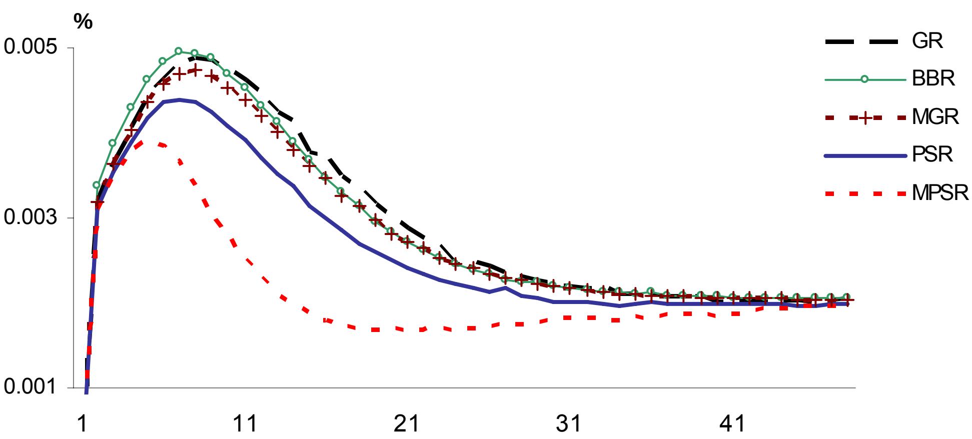

5 %Shift from vU to vI
Debt-Output Ratio
%
1
11
21
31
41
0
-5
-10
-15
-20
PSR
GR
MGR
MPSR
Growth Rate of Output
%
0.018
0.014
0.010
1
11
21
31
41
MPSR
BBR
MGR
GR
PSR
Net Liability Position
%
1
11
21
31
41
0
-40
-80
-120
-160
-200
PSR
GR
MGR
MPSR

FIGURE 5

5 % Shift from vU to vM
Debt-Output Ratio



Growth Rate of Output



Net Liability Position

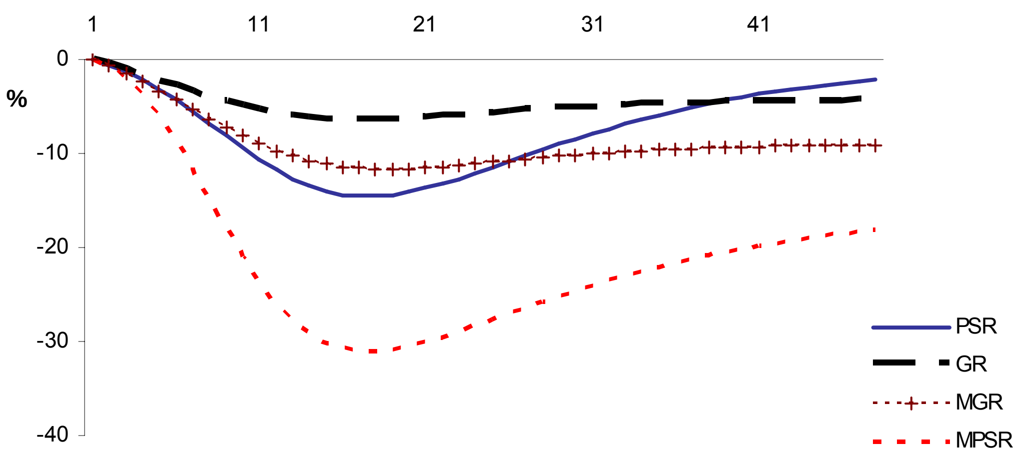
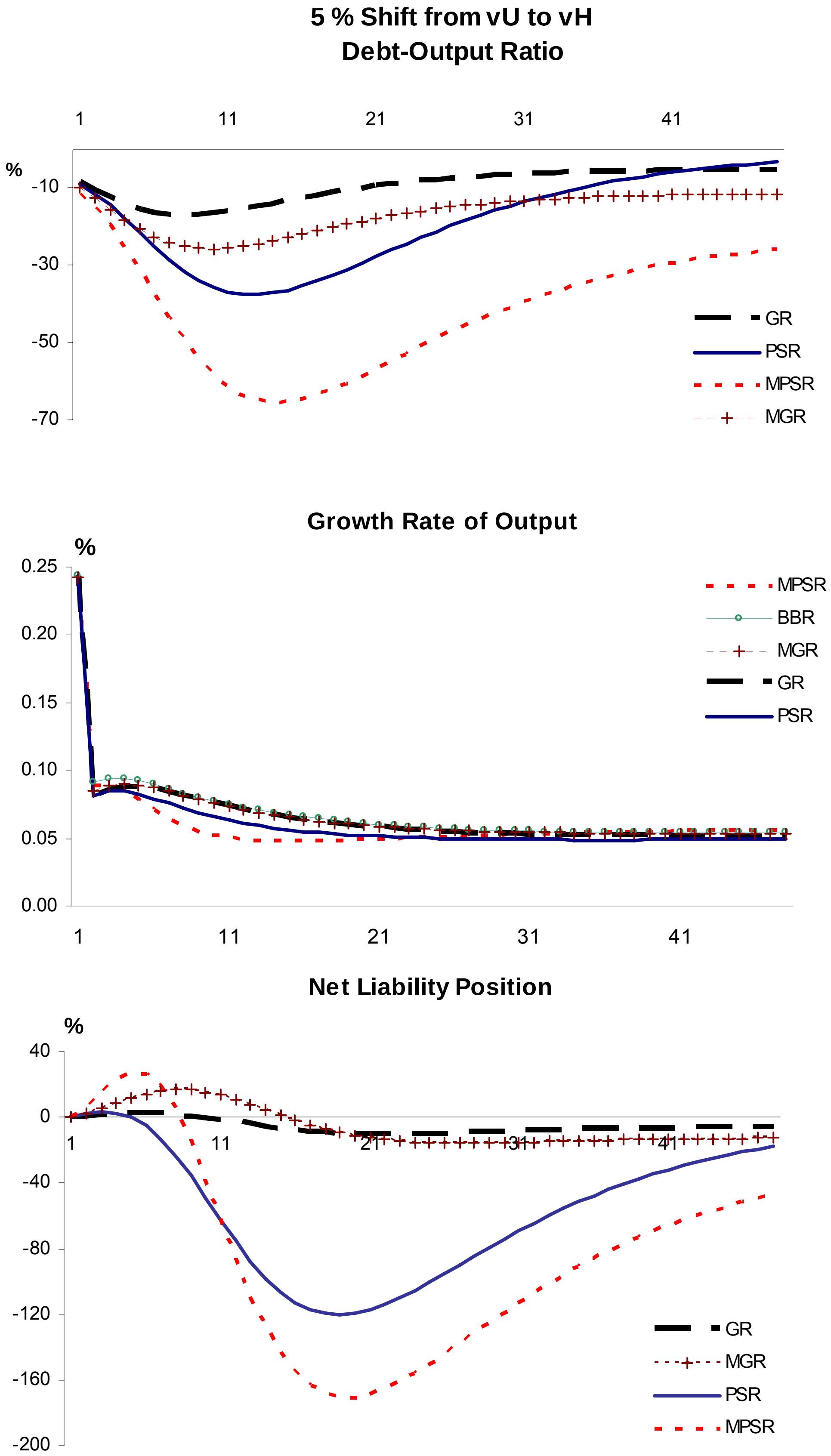


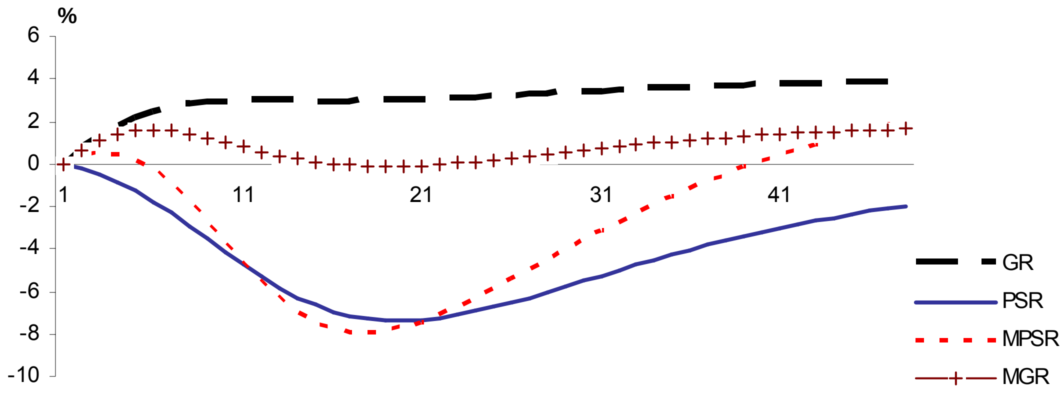
FIGURE 6



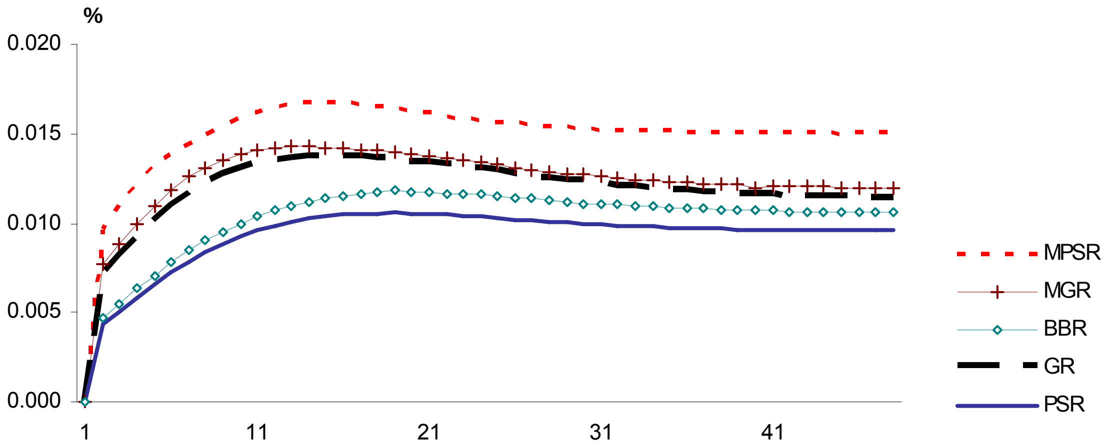
5 % Shift from vU to vH
Debt-Output Ratio
%
1
11
21
31
41
-10
-30
-50
-70
GR
PSR
MPSR
MGR
Growth Rate of Output
%
0.25
0.20
0.15
0.10
0.05
0.00
1
11
21
31
41
MPSR
BBR
MGR
GR
PSR
Net Liability Position
%
40
0
-40
-80
-120
-160
-200
1
11
21
31
41
GR
MGR
PSR
MPSR

FIGURE 7
φ=0.5

5 %Increase in vl
Debt-Output Ratio



Growth Rate of Output



Net Liability Position

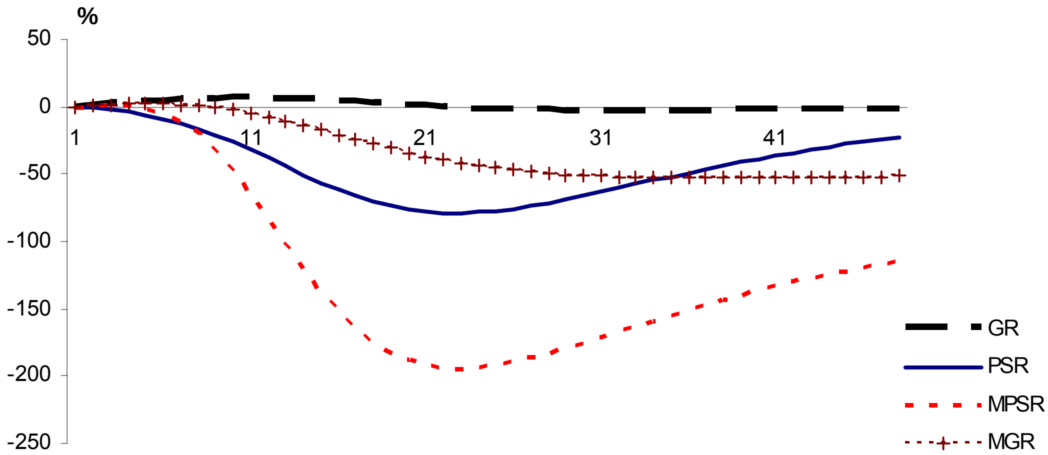
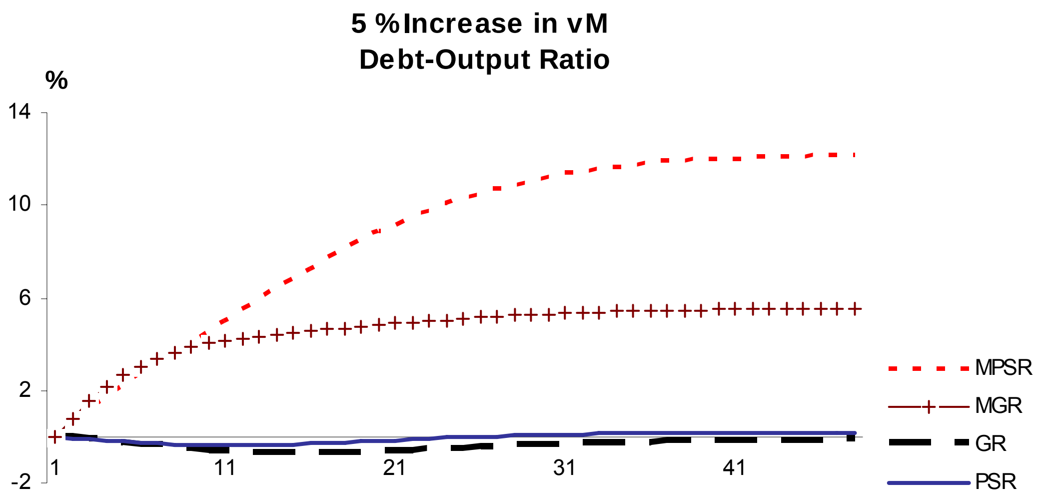
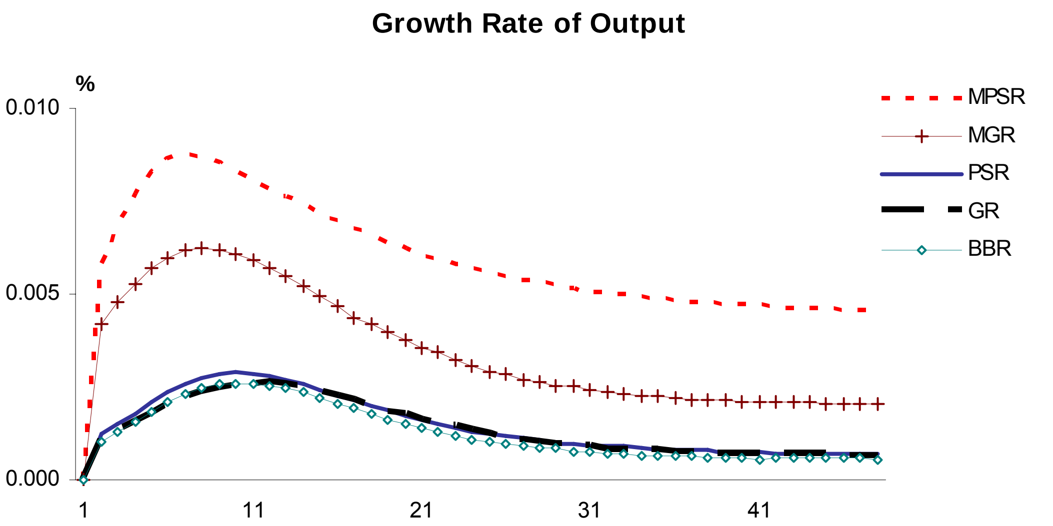


FIGURE 8
φ=0.5


5 %Increase in vM
Debt-Output Ratio
%
14
10
6
2
-2
1
11
21
31
41
MPSR
MGR
GR
PSR


Growth Rate of Output
%
0.010
0.005
0.000
1
11
21
31
41
MPSR
MGR
PSR
GR
BBR

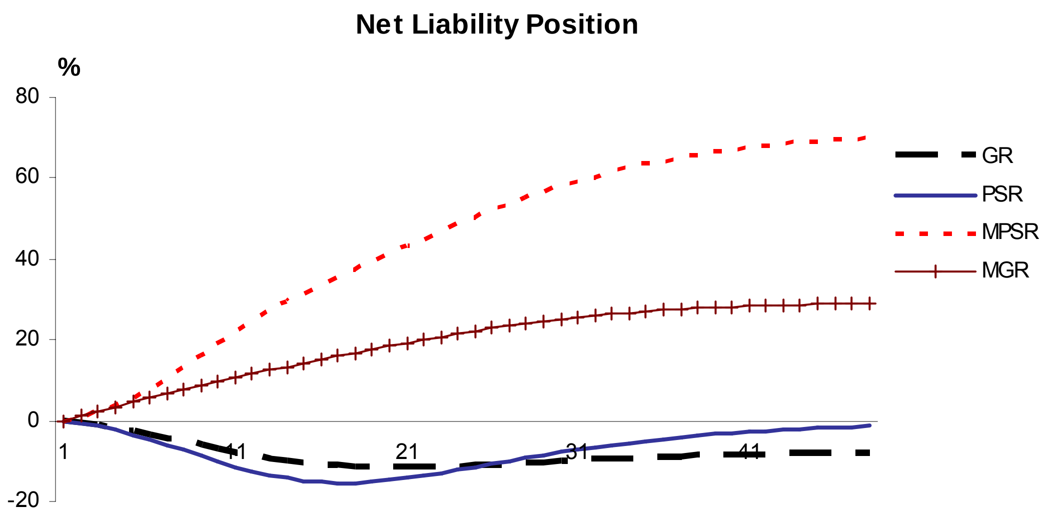
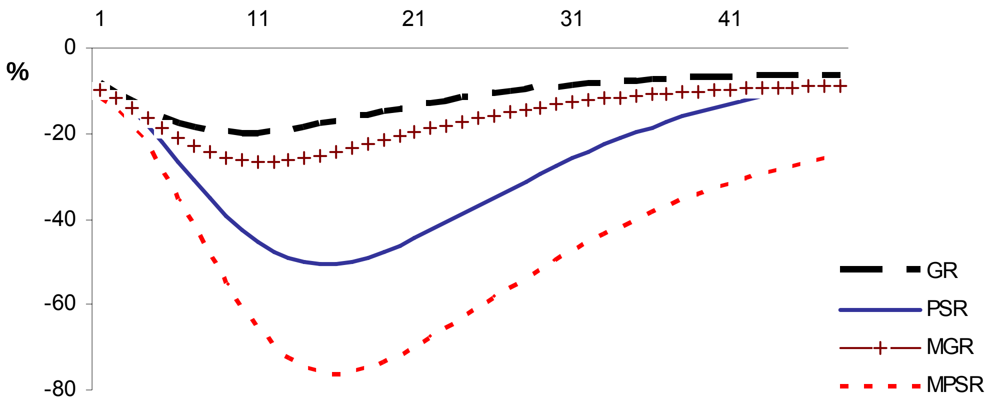
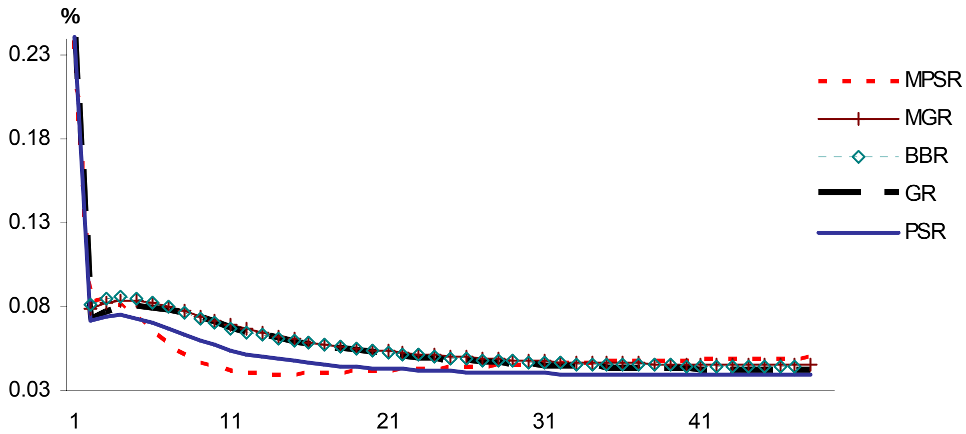

Net Liability Position
%
80
60
40
20
0
-20
1
11
21
31
41
GR
PSR
MPSR
MGR

FIGURE 9
φ=0.5

5 %Increase in vH
Debt Output Ratio



Growth Rate of Output



Net Liability Position

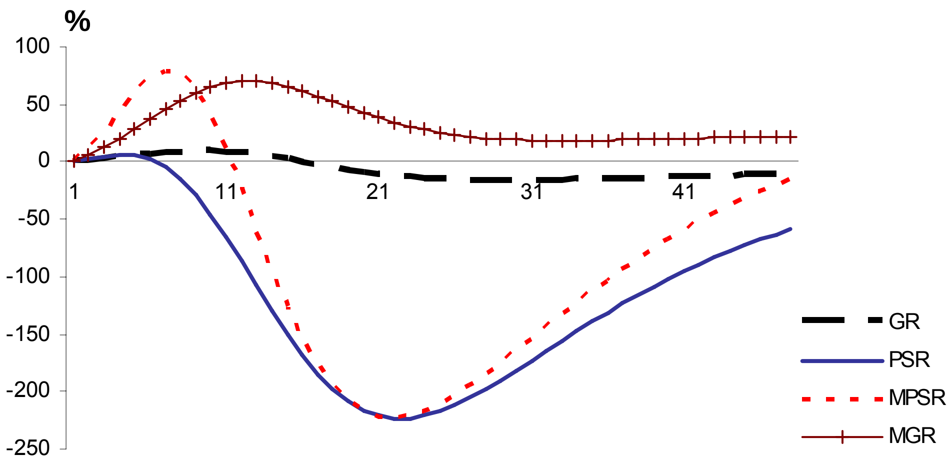
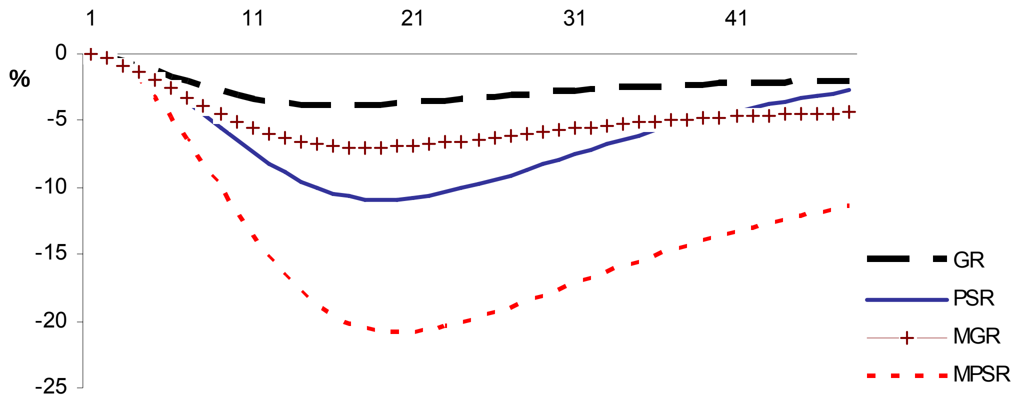
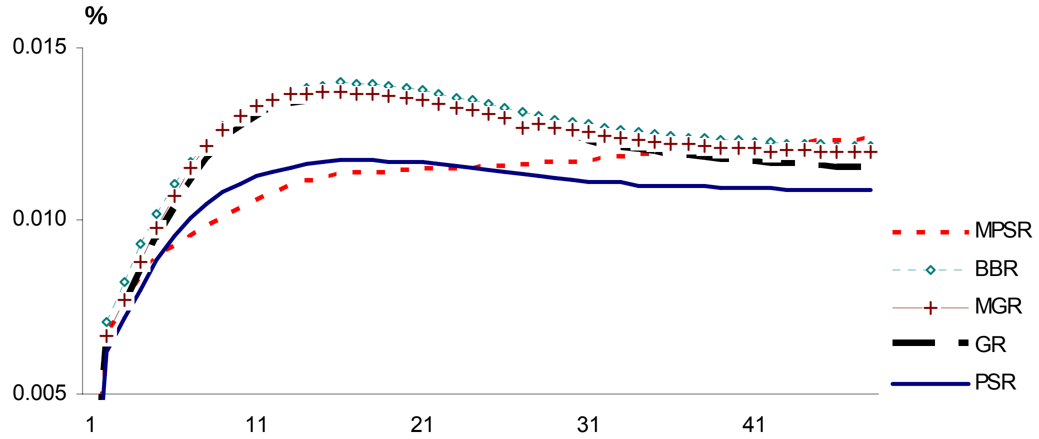


FIGURE 10
φ=0.5

5 % Shift from vU to vI
Debt-Output Ratio



Growth Rate of Output



Net Liability Position

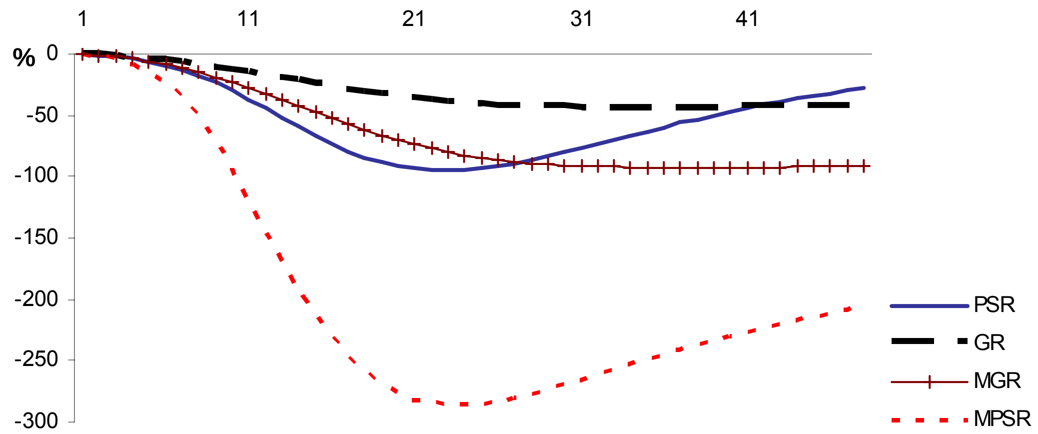
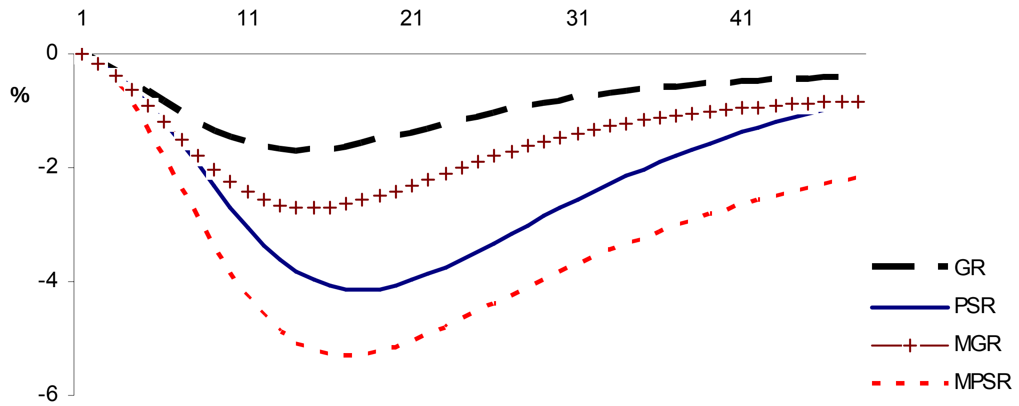
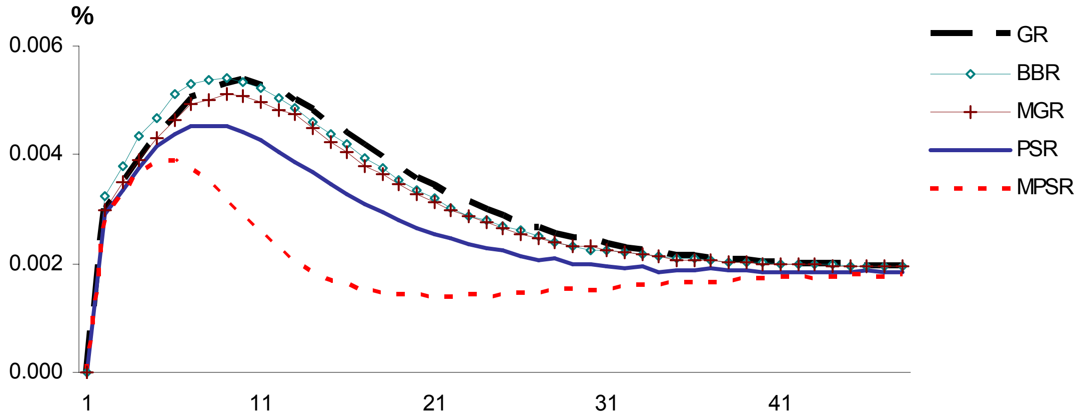


FIGURE 11
φ=0.5

5 % Shift from vU to vM
Debt-Output Ratio



Growth Rate of Output



Net Liability Position

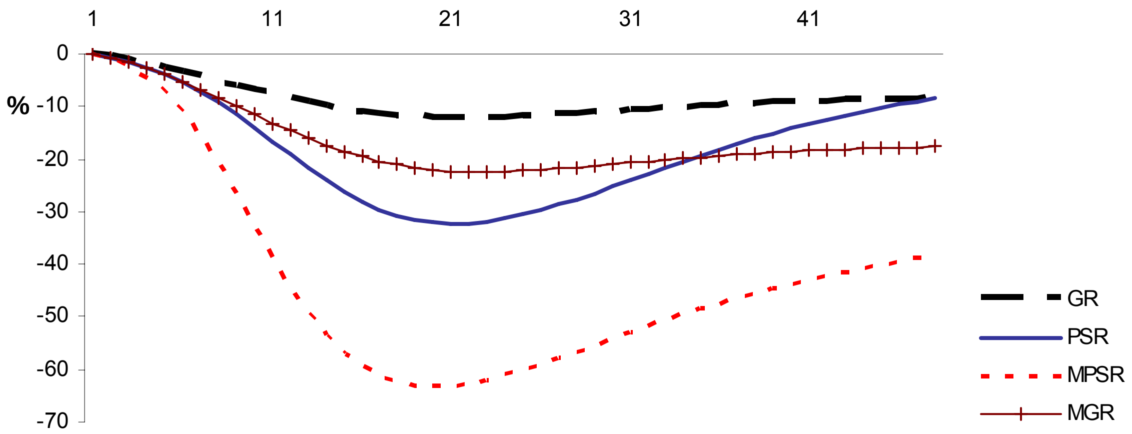
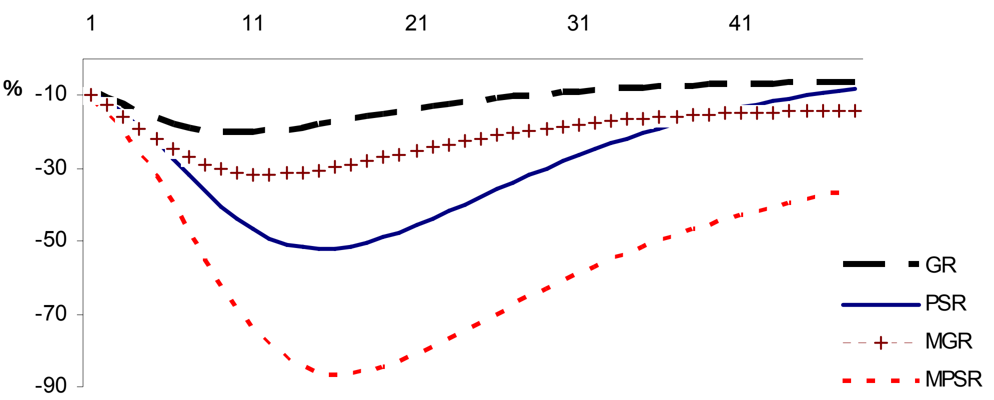
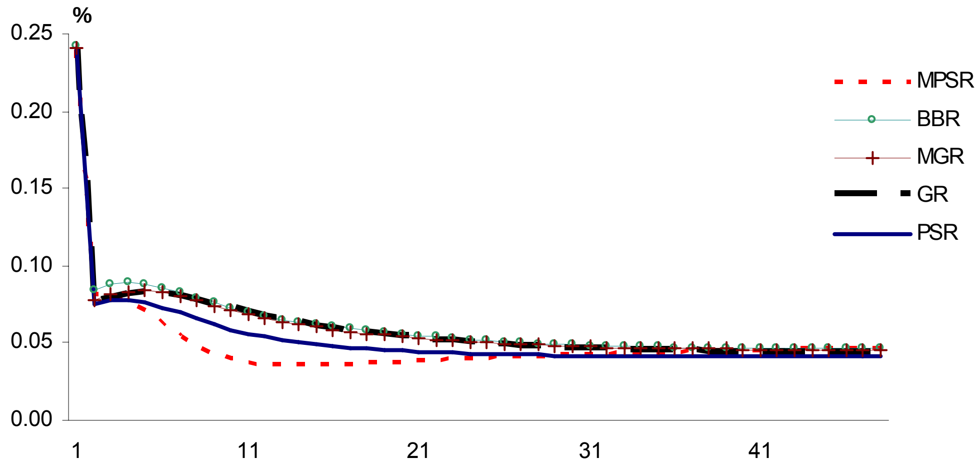


FIGURE 12
φ=0.5

5 %Shift from vU to vH
Debt-Output Ratio



Growth Rate of Output



Net Liability Position

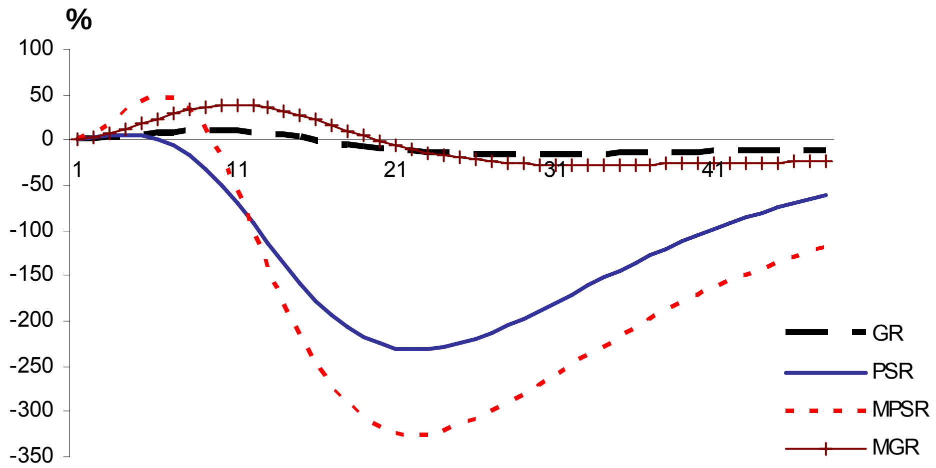
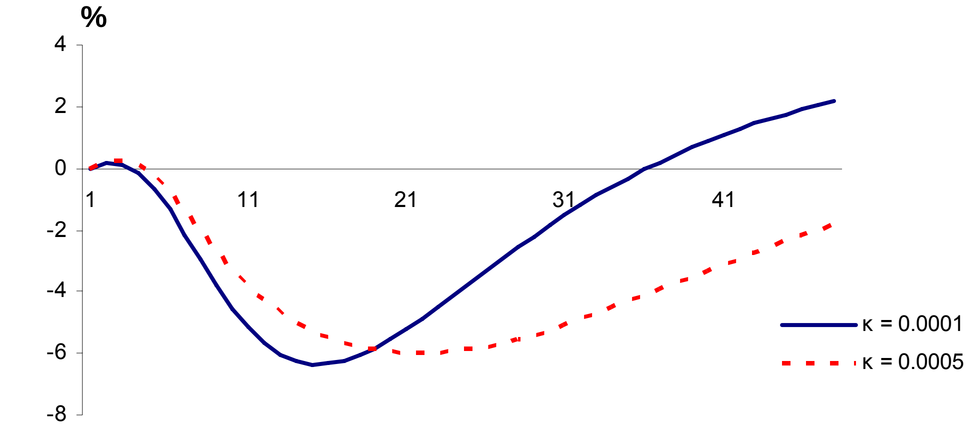
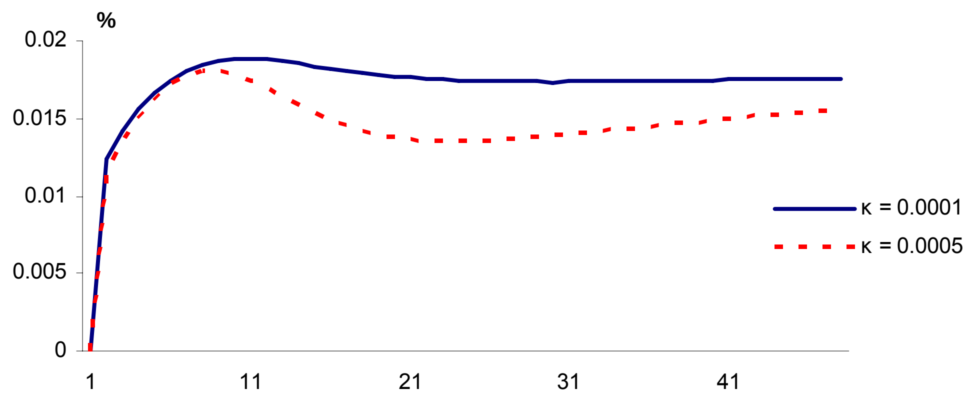


FIGURE 13

5 %Increase in vl
Debt-Output Ratio (MPSR)



5 %Increase in vl
Growth Rate (MPSR)



5 %Increase in vl
Net Liability Position (MPSR)

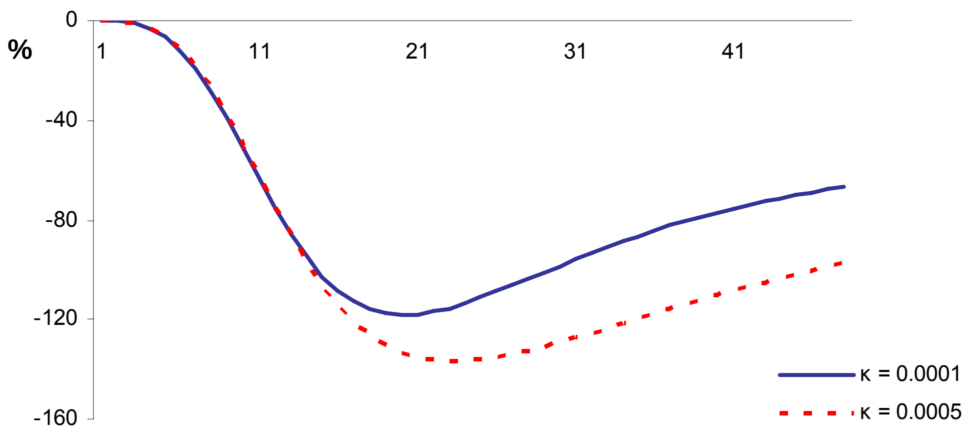
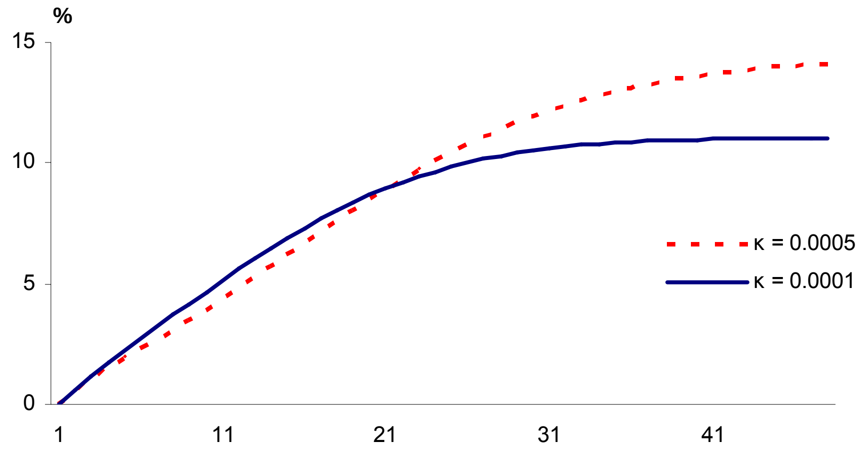
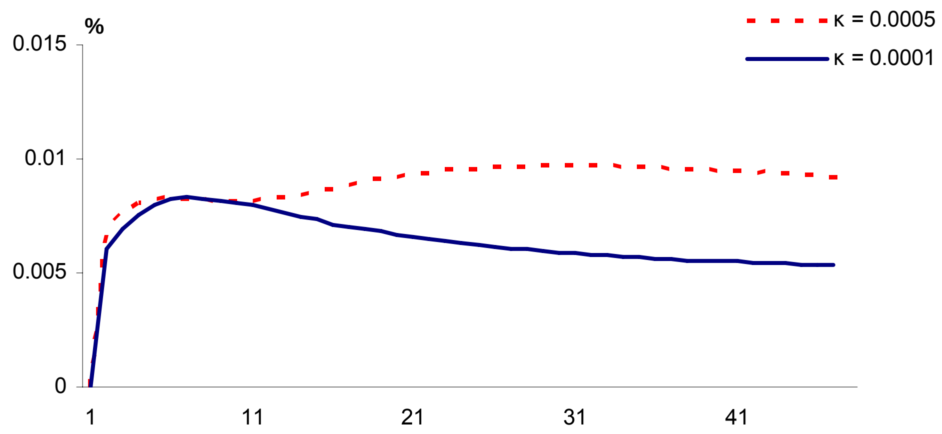


FIGURE 14

5 %Increase in vM
Debt-Output Ratio (MPSR)



5 %Increase in vM
Growth Rate (MPSR)



5 %Increase in vM
Net Liability Position (MPSR)

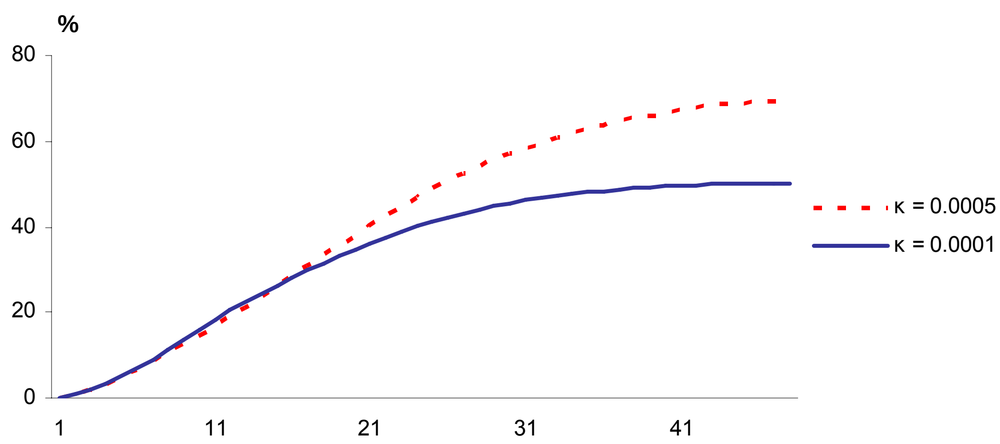
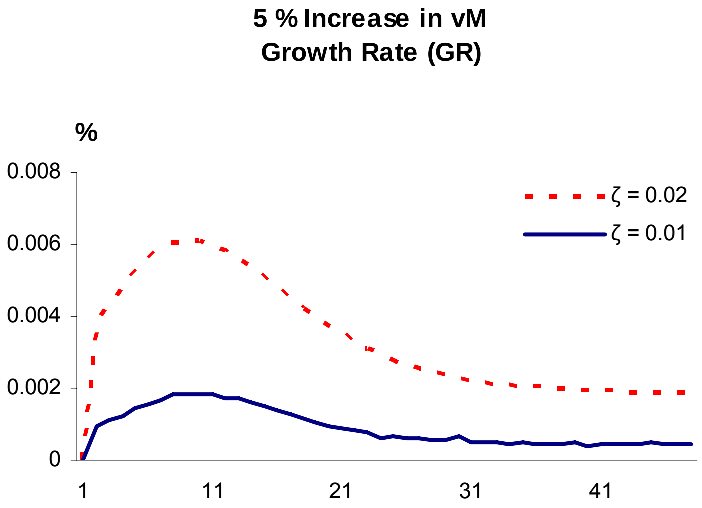
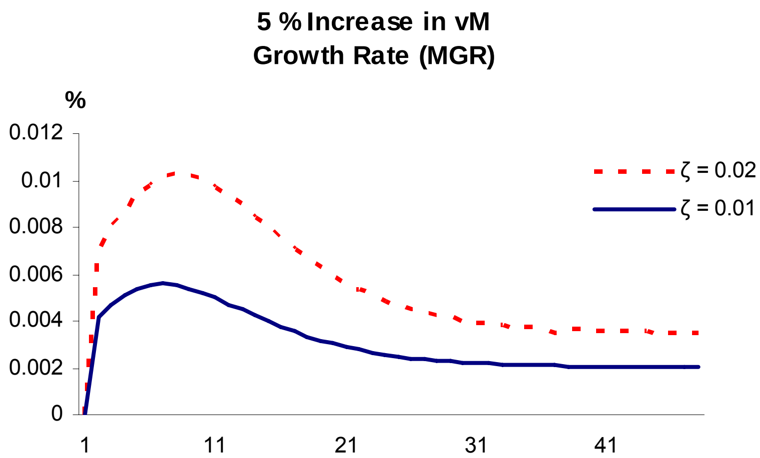


FIGURE 15


5 % Increase in vM
Growth Rate (GR)
%
0.008
0.006
0.004
0.002
0
1
11
21
31
41
ζ = 0.02
ζ = 0.01


5 % Increase in vM
Growth Rate (MGR)
%
0.012
0.01
0.008
0.006
0.004
0.002
0
1
11
21
31
41
ζ = 0.02
ζ = 0.01

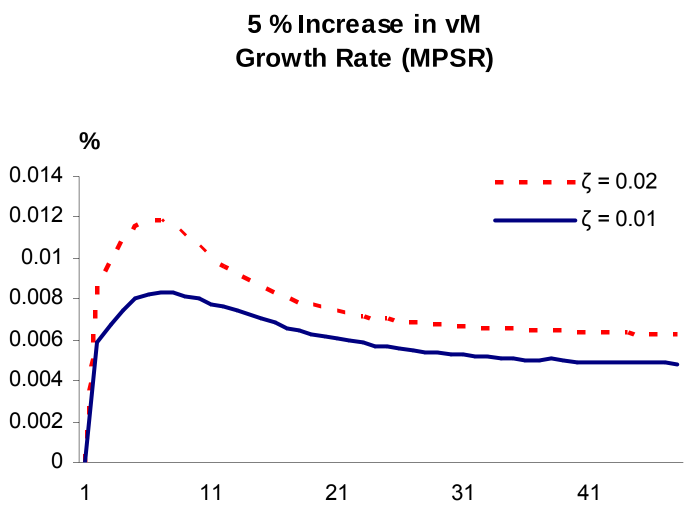
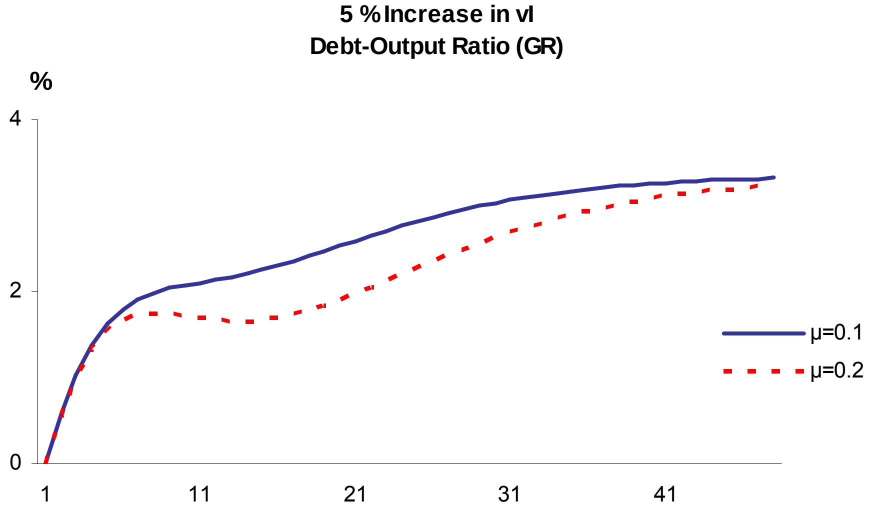
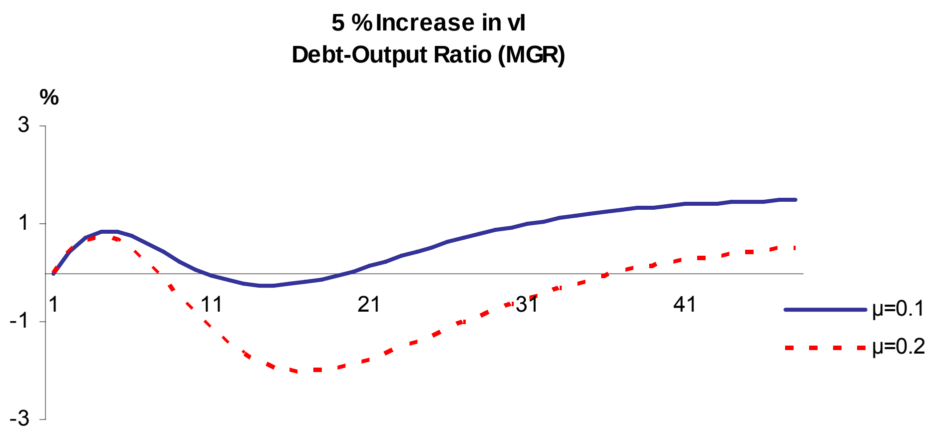

5 % Increase in vM
Growth Rate (MPSR)
%
0.014
0.012
0.01
0.008
0.006
0.004
0.002
0
1
11
21
31
41
ζ = 0.02
ζ = 0.01

FIGURE 16


5 % Increase in vl
Debt-Output Ratio (GR)
%
4
2
0
1
11
21
31
41
μ=0.1
μ=0.2


5 % Increase in vl
Debt-Output Ratio (MGR)
%
3
1
-1
-3
1
11
21
31
41
μ=0.1
μ=0.2

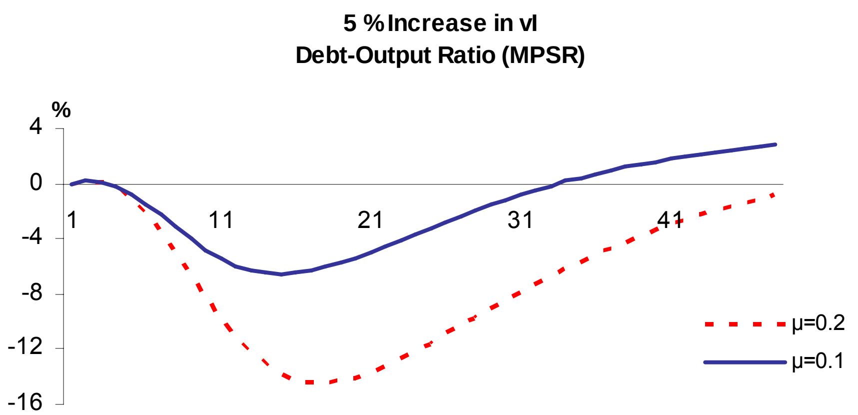
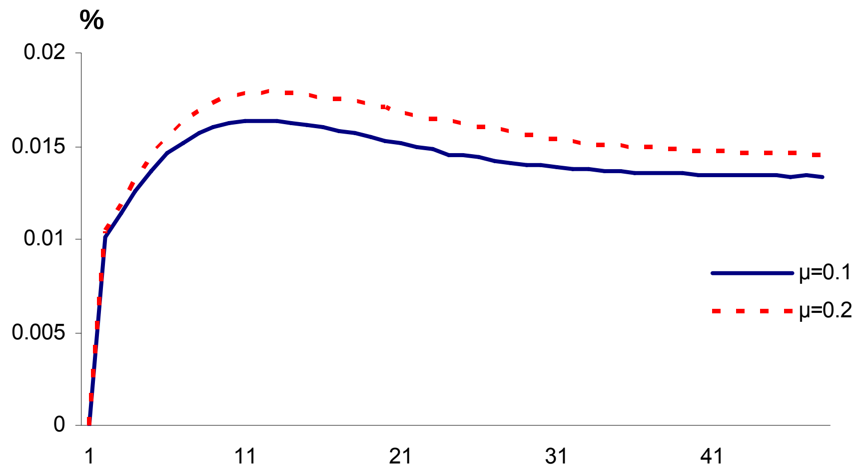
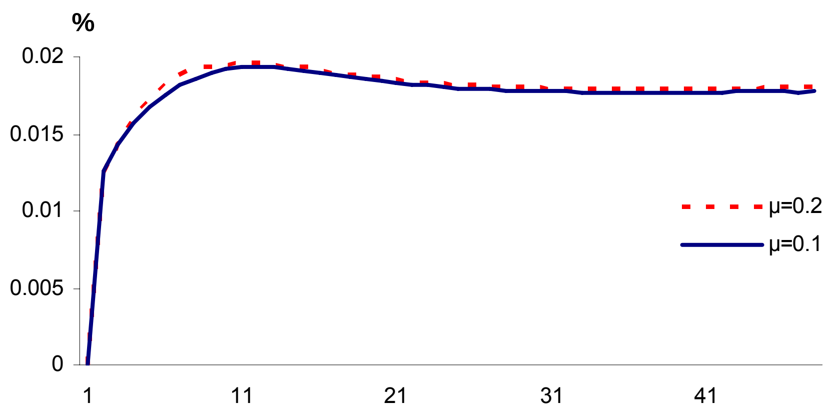

5 % Increase in vl
Debt-Output Ratio (MPSR)
%
4
0
-4
-8
-12
-16
1
11
21
31
41
μ=0.2
μ=0.1

FIGURE 17

5 % Increase in vl
Growth Rate (GR)



5 % Increase in vl
Growth Rate (MGR)



5 % Increase in vl
Growth Rate (MPSR)

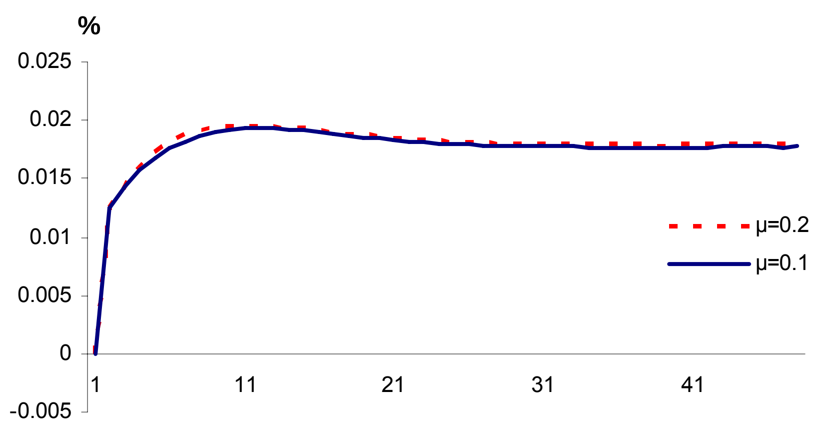
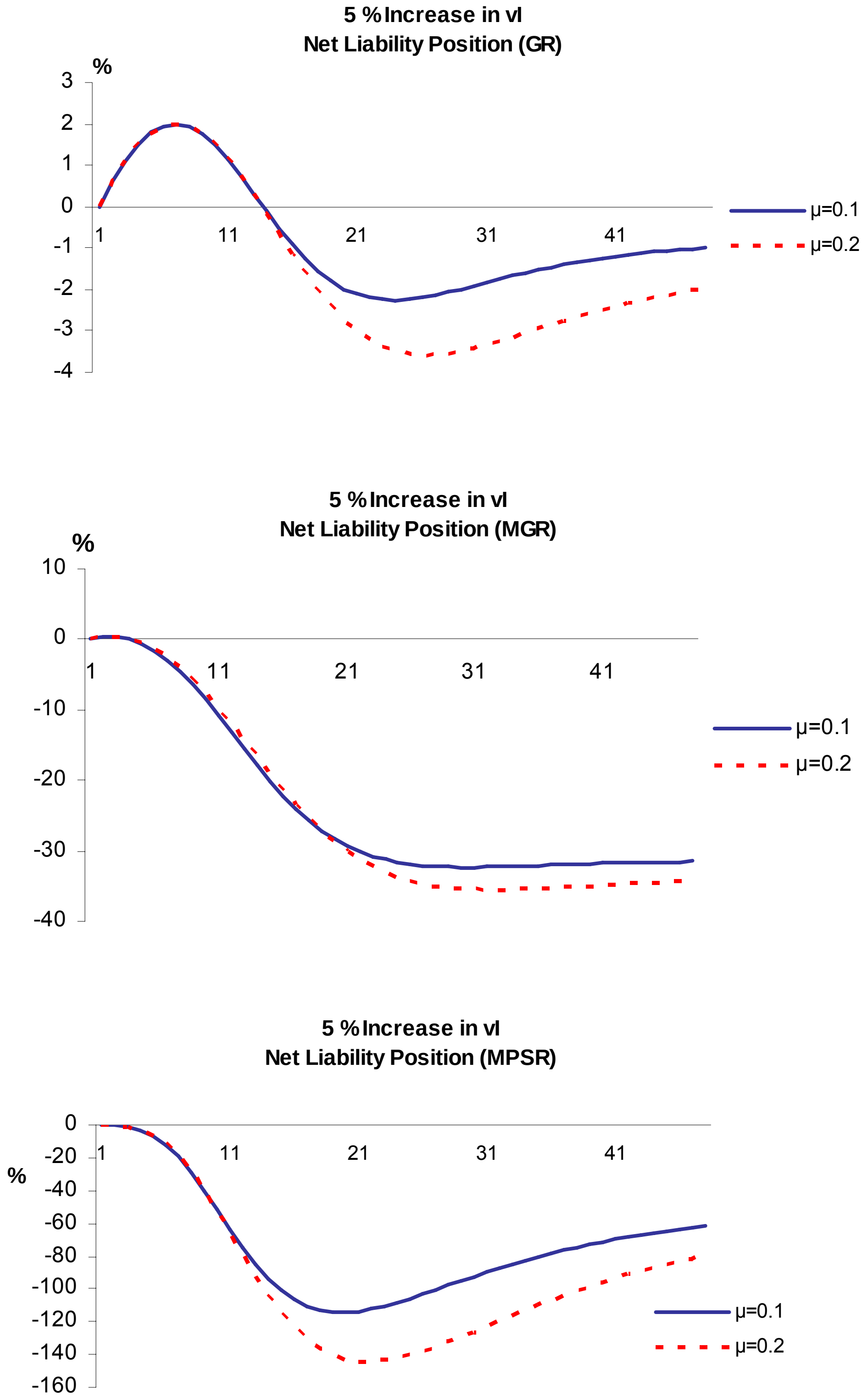


FIGURE 18


5 % Increase in vl
Net Liability Position (GR)
%
3
2
1
0
-1
-2
-3
-4
1
11
21
31
41
μ=0.1
μ=0.2
5 % Increase in vl
Net Liability Position (MGR)
%
10
0
-10
-20
-30
-40
1
11
21
31
41
μ=0.1
μ=0.2
5 % Increase in vl
Net Liability Position (MPSR)
%
0
-20
-40
-60
-80
-100
-120
-140
-160
1
11
21
31
41
μ=0.1
μ=0.2