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Abstract

This paper studies the optimal allocation of government spending between health, education, and infrastructure in an endogenous growth framework. In the model, infrastructure affects not only the production of goods but also the supply of health and education services. The production of health (education) services depends also on the stock of educated labor (health spending). Transitional dynamics associated with budget-neutral shifts in the composition of expenditure are analyzed, and growth- and welfare-maximizing allocation rules are derived and compared. The discussion highlights the key role played by the parameters that characterize the health and education technologies.

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Contents

1 Introduction

Much of the literature on how health and nutrition affect economic growth has focused on labor productivity effects (see Strauss and Thomas (1998) and Hoddinott, Alderman, and Behrman (2005)). A common argument is that the chronically undernourished may be too weak to perform up to their physical potential; as a result, they do not get hired at any wage. Inadequate nutrition may thus engender poor health, low productivity, and continued low incomes and growth rates—in effect, preventing countries from escaping from persistent poverty (see, for instance, Mayer-Foulkes (2005)).

Other contributions have emphasized the indirect effects of health on growth. For instance, inadequate consumption of protein and energy, as well as deficiencies in key micronutrients (such as iodine, vitamin A, and iron), have been found to be key factors in the morbidity and mortality of children and adults.[1] Iron deficiency is also associated with malaria, intestinal parasitic infestations and chronic infections. By reducing life expectancy, malnutrition (or, more generally, poor health) may have an adverse, indirect effect on growth, by discouraging savings and investment. Conversely, healthy individuals both expect to live longer, which gives them an incentive to save, and more often than not do indeed end up living longer, which gives them more time to save and enjoy the fruits of their savings. In turn, higher savings rates tend to stimulate growth.[2]

Moreover, healthier children tend to do better in school—just like healthier workers perform their tasks better—thereby enhancing intellectual capacity and ultimately the quality of the labor force. Put differently, improvements in the health of individuals tend to increase also the effectiveness of

[1] The United Nations estimate that 55 percent of the nearly 12 million deaths each year among under five-year-old children in the developing world are associated with malnutrition; see Broca and Stamoulis (2003).

[2] See Chakraborty (2004) and Hashimoto and Tabata (2005) for an analysis—based on overlapping-generations models—of how health capital affects the probability of surviving across periods, and thus savings and growth.

education, as in the "food for thought" model of Galor and Meyer (2004). In addition, to the extent that spending on health increases an individual's lifespan, it may also raise the return (as measured by the discounted present value of wages) associated with greater expenditure on education. The increased incentive to accumulate human capital may spur economic growth. Conversely, poor health can have a significant adverse effect on educational attainment. When parents become ill for instance, children are often pulled out of school to care for them, take on other responsibilities (including menial tasks) in the household, or work to support their siblings. Thus, intra-family allocations regarding school and work time of children tend to be adjusted in the face of disease within the family (see Corrigan, Glomm, and Mendez (2005)) or when receiving foster children (see Deininger, Crommelynck, and Kempaka (2005)). In turn, these adjustments may influence the accumulation of both physical and human capital, and thus the growth rate.

At the same time, one line of research has shown that higher education levels can improve health. More educated mothers have greater awareness of health hazards and tend to take better care of their children. Another line of research has emphasized the positive impact that infrastructure (roads, electricity, clean water, telecommunications, and so on) may have on both health and education. Regarding the relationship between infrastructure and health, microeconomic studies have found that access to safe water and sanitation helps to improve health, particularly among children. By reducing the cost of boiling water, access to electricity helps to improve hygiene and health. Infrastructure may also have a sizable impact on educational outcomes; there is much evidence, for instance, of a direct linkage between education and access to roads. Electricity allows for more studying and access to computers, which may enhance the quality and depth of learning.

The foregoing discussion suggests that, at the microeconomic level, the relationship between health, education, and infrastructure services is largely complementary. At the macroeconomic level, however, potential trade-offs may emerge between the provision of various categories of services, which often falls under the responsibility of the state (at least in most low- and middle-income developing countries). With limited resources, governments must choose what services need to be provided in priority, whether it is to maximize the rate of economic growth or individual welfare.

The contribution of this paper is to examine the optimal allocation of government spending between health, education, and infrastructure, in a unified endogenous growth framework that accounts for both the complementarities emphasized by the microeconomic evidence and the aggregate budget constraint faced by policymakers.[3] To begin with, we assume that the economy is endowed only with "raw" labor, and raw labor must be educated to become productive. All public services are provided free of charge and are financed by a distortionary tax. Most importantly, and in line with the foregoing discussion, infrastructure services are assumed to affect simultaneously the production of goods, educated labor, and the provision of health services. In addition, the rate of human capital accumulation depends on the existing stock of educated labor (the number of "teachers") as well as the provision of health services, whereas the production of health services depends on the stock of educated labor (or "medical workers"). By imposing gross complementarity between production inputs, the model captures the positive externalities highlighted earlier between health, education, and infrastructure.

Our distinction between raw and educated labor dwells on the fact that, unlike Uzawa-Lucas type models, we assume that knowledge is (quite literally) embodied in individuals, as in Ehrlich and Lui (1991) and van Zon and Muysken (2005).[4] In addition, individuals can provide effective services from

[3]This paper draws on several earlier contributions by Agénor (2005*a*, 2005*b*, 2005*d*, 2005*e*), in which aspects of this issue were considered. But the present paper is (as far as we know) the first to provide a unified treatment of the links between education, health, and infrastructure and to compare systematically growth- and welfare-maximizing allocations.

[4]However, as indicated later, all of our results can be "recast" in the context of the

human capital only if they are healthy. Thus, by enhancing labor productivity, health influences growth directly, in addition to affecting individual welfare. It is "effective" labor (defined as a composite factor in educated labor and public health services) that is used in production. A lower flow of health services would therefore reduce the ability of each worker to produce. From that perspective, then, public spending on health and education are complementary. But from the point of view of the production of "effective" labor (through the schooling technology), the provision of health services is a substitute for the formation of educated labor, because it may reduce (everything else equal) government spending on education—as well as, possibly, spending on infrastructure services. At the same time, health services enter in the representative household's utility function and therefore affect welfare directly. Potential trade-offs imposed by the government budget constraint imply therefore that there is an optimal allocation of expenditure between education, health, and infrastructure, which in general depends on the technology for producing goods, human capital, and health services, as well as household preferences.

The remainder of the paper is organized as follows. Section II provides a brief overview of some of the recent empirical literature on the interactions between health, education, and infrastructure. Section III presents our framework. Section IV derives the balanced growth path (BGP) and discusses the dynamic properties of the model. Section V examines the short- and long-run effects of revenue-neutral increases in spending shares on infrastructure, health, and education. The issue that we address is whether (given that the production of educated labor and health services depends on infrastructure services) an increase in public spending on infrastructure is the most efficient way to stimulate long-run growth. As noted earlier, the provision of each category of services requires resources and this (given

Uzawa-Lucas framework of disembodied knowledge, by using the concept of "effective" human capital.

the overall constraint on tax revenues) creates trade-offs. The growth- and welfare-maximizing allocations of public expenditure are determined in Sections V and VI. We consider the optimal allocation of spending between any two categories of public services, assuming that the tax rate and the third spending category are arbitrarily set. The last section of the paper offers some concluding remarks.

2 Recent Evidence

This section provides a brief review of the recent evidence on the impact of health on economic growth, interactions between health and education outcomes, and the impact of infrastructure on health and education. In doing so, we dwell on both the micro and macro evidence.

2.1 Health and Economic Growth

Several recent studies have documented a sizable effect of nutrition and health outcomes on economic growth. Arcand (2001) and Wang and Taniguchi (2003) found that better nutrition enhances growth directly, through its impact on labor productivity, as well as indirectly, through improvements in life expectancy and possibly by speeding up the adoption of new production techniques.[5] Lorentzen, McMillan and Wacziarg (2005) found that countries with a high rate of adult mortality also tend to experience low rates of growth—possibly because when the risk of premature death is relatively high, incentives to save and invest in human capital are weakened.[6] More specifically, McCarthy, Wolf, and Wu (1999) found that malaria morbidity is

[5] Jamison, Lau and Wang (2004), however, conluded that differences in the impact of health on growth across countries were unlikely to be the result of differences in the effect of health on the rate of technical progress.

[6] They also found that the estimated effect of high adult mortality on growth is large enough to explain sub-Saharan Africa's poor economic performance between 1960 and 2000. Indeed, in the 40 countries with the highest adult mortality rates in their sample of 98 countries, all are in Sub-Saharan Africa, except three.

negatively correlated with the growth rate of output per capita across countries. Countries with a high incidence of malaria grew by 1.3 percent less per annum compared to unaffected countries during the period 1965-90, resulting in an income level 33 percent lower than that of countries without malaria. A 10% reduction in malaria was associated with a 0.3% increase in annual growth. In Sub-Saharan Africa alone, a one-percentage point increase in the morbidity rate associated with the disease tends to reduce the annual growth rate per capita by an average of 0.55 percent.

The direct impact of life expectancy (as an indicator of good health) on growth has been documented by Bloom, Canning, and Sevilla (2004) and Sala-i-Martin, Doppelhofer, and Miller (2004). The former study, based on a sample consisting of both developing and industrial countries, found that good health (proxied by life expectancy) has a sizable, positive effect on economic growth. A one-year improvement in the population's life expectancy contributes to an increase in the long-run growth rate of up to 4 percentage points.[7] Sala-i-Martin, Doppelhofer, and Miller (2004) also found that initial life expectancy has a positive effect on growth, whereas the prevalence of malaria, as well the fraction of tropical area (which may act as a proxy for exposure to tropical diseases) are both negatively correlated with growth. Using instead adult survival rates as an indicator of health, both Bhargava et al. (2001) and Weil (2005) found robust evidence that health has a strong effect on growth in low-income countries.

2.2 Interactions between Health and Education

Empirical studies have also found evidence of a strong impact of health on both the quantity and quality of human capital—and thus indirectly on growth. As noted earlier, healthier children tend to do better in school. In

[7] Using a production function approach, Bloom and Canning (2005) found that a one percentage point in adult survival rates raises labor productivity by 2.8 percent. Weil (2005), by contrast, found a calibrated value of 1.7 percent.

Tanzania, for instance, the use of insecticide-treated bednets reduced malaria increased attendance rates in schools (Bundy and others (2005, p. 2)). In Western Kenya, deworming treatment improved primary school participation by 9.3 percent, with an estimated 0.14 additional years of education per pupil treated (see Miguel and Kremer (2004)). McCarthy, Wolf, and Wu (1999) found that malaria morbidity (viewed as a proxy for the overall incidence of malaria among children) has a negative effect on secondary enrollment ratios. Bundy et al. (2005), in their overview of experience on the content and consequences of school health programs (which include for instance treatment for intestinal worm infections), have emphasized that these programs can raise productivity in adult life not only through higher levels of cognitive ability, but also through their effect on school participation and years of schooling attained. At the aggregate level, the cross-country regressions of Baldacci et al. (2004) show that health capital (as proxied by the under-5 child mortality rate) has a statistically significant effect on school enrollment rates. Finally, Bloom, Canning and Weston (2005) found that children vaccinated (against a range of diseases, including measles, polio and tuberculosis) as infants in the Philippines performed better in language and IQ scores at the age of ten than unvaccinated children—even within similar social groups. Thus, early vaccination may have a sizable effect on education outcomes (by enabling the accumulation of knowledge) and economic growth.

At the same time, several empirical studies have found that higher education levels can improve health.[8] Both micro and macro studies have found that where mothers are better educated infant mortality rates are lower, and attendance rates in school are higher (see Glewwe (1999) and the cross-country regressions of Baldacci et al. (2004) and Wagstaff and Claesson (2004)). Better-educated women tend, on average, to have more health knowledge and be more aware of the myriad of health risks that their chil-

[8] Glewwe (2002) provides a review of the evidence on the impact of schooling on adult and child health.

dren face. Paxson and Schady (2005), in a study of Ecuador, found that the cognitive development of children aged 3 to 6 years varies inversely with the level of education of their mother. More generally, during the period 1970–95, improvements in female secondary school enrollment rates are estimated to be responsible for 43 percent of the total 15.5 percent reduction in the child underweight rate of developing countries (Smith and Haddad (2001)). In sub-Saharan Africa alone, Summers (1994) estimated that five additional years of education for women could reduce infant mortality rates by up to 40 percent.

2.3 Infrastructure, Health and Education

A number of case studies (many of them summarized by Brenneman and Kerf (2002)) have found that infrastructure may have a very large impact on health and education outcomes. According to World Bank estimates, more than half of the population in the developing world still relies on traditional biomass fuels (such as wood and charcoal) for cooking and heating, which represent serious health hazards (see Saghir (2005)); improved and more efficient stoves would reduce indoor air pollution and harmful health effects. Access to clean energy for cooking and better transport (particularly in rural areas) may also contribute to better health. In another study, the World Bank (2005, p. 144) found that the dramatic drop in the maternal mortality ratio observed in recent years in Malaysia and Sri Lanka (from 2,136 in 1930 to 24 in 1996 in Sri Lanka, and from 1,085 in 1933 to 19 in 1997 in Malaysia) was due not only to a sharp increase in medical workers in rural and disadvantaged communities, but also to improved communication and transportation services—which helped to reduce geographic barriers. Transportation (in Malaysia) and transportation subsidies (in Sri Lanka) were provided for emergency visits to health care centers. Moreover, in Malaysia, health programs were part of integrated rural development efforts that included investment in clinics, rural roads, and rural schools. A similar approach was followed in Sri Lanka—better roads make it easier to get to rural health facilities. At a cross-country level, McCarthy, Wolf, and Wu (1999) found that access to clean water and sanitation has a significant effect on the incidence of malaria.

Regarding the relationship between infrastructure and education, there is also evidence of direct linkages between education, electricity, roads, and sanitation. As noted earlier, electricity allows for more studying and access to technology. Studies have shown that the quality of education tends to improve with better transportation networks in rural areas, whereas attendance rates for girls tend to increase with access to sanitation in schools. In the Philippines, for instance, after rural roads were built, school enrollment went up by 10 percent and drop-out rates fell by 55 percent. A similar project in Morocco raised girls' enrollments from 28 percent to 68 percent (see Levy (2004)). A study of Bangladesh shows also a correlation between access to water and sanitation facilities and increases in girls' attendance. Indeed, in most developing countries, the sanitary and hygienic conditions in schools are often appalling, characterized by the absence of proper functioning water supply, sanitation and hand washing facilities. Schools that lack access to basic water supply and sanitation services tend to have a higher incidence of major childhood illnesses among their students. In turn, as discussed earlier, poor health is an important underlying factor for low school enrollment, absenteeism (often the result of respiratory infections, as noted by Bundy et al. (2005)), poor classroom performance, and early school dropout. Inadequate nutrition, which often takes the form of deficiencies in micronutrients, also reduces the ability to learn and study. Thus, improving hygiene, sanitation, and access to food and safe water in schools can create an enabling learning environment that contributes to children's improved health and learning ability. In turn, these improvements may have a sizable impact on growth.

3 The Model

Our starting point is an economy with a single, infinitely-lived household who produces and consumes a single traded good. The good (whose price is fixed on world markets) can be used for consumption or investment. The economy's endowment consists of raw labor, which must be educated to be used in market activity. Raw labor is supplied inelastically. The government provides infrastructure services, as well as health and education services, all free of charge. It finances these expenditures by levying a flat tax on marketed output.

3.1 Market Production of Goods

Aggregate marketed output, Y, is produced with private physical capital, K_P, public infrastructure services, G_I, and effective labor. In turn, effective labor, Q, is defined as a composite input produced by combining the economy's flow supply of health services, H, and the share of educated workers in production, $\chi_P E$, under constant returns to scale:[9]

$$Q = H^{\varepsilon}(\chi_P E)^{1-\varepsilon}, \tag{1}$$

where E is the total number of educated workers in the economy, $\chi_P \in (0,1)$ the share of educated workers in production, and $\varepsilon \in (0,1)$ a share parameter.

Production exhibits constant returns to scale in all factors:

$$Y = G_I^{\alpha} Q^{\beta} K_P^{1-\alpha-\beta}, \tag{2}$$

where $\alpha, \beta \in (0,1)$. Substituting (1) in (2) yields

$$Y = A_P(\frac{G_I}{K_P})^{\alpha}[(\frac{H}{K_P})^{\varepsilon}(\frac{E}{K_P})^{1-\varepsilon}]^{\beta} K_P, \tag{3}$$

where $A_P \equiv \chi_P^{(1-\varepsilon)\beta} > 0$.

[9] Throughout the paper, the time subscript t is omitted whenever doing so does not result in confusion. A dot over a variable is used to denote its time derivative.

3.2 Household Preferences

The household maximizes the discounted stream of future utility

$$\max_C V = \int_0^\infty \frac{(CH^\kappa)^{1-1/\sigma}}{1-1/\sigma} \exp(-\rho t) dt, \tag{4}$$

where C is aggregate consumption and $\kappa > 0$ measures the contribution of health to utility and σ is the intertemporal elasticity of substitution. Thus, health services affect welfare directly and are included in the instantaneous utility function, together with consumption, in a non-separable manner. At first sight, specification (4) is similar to the one used by Corsetti and Roubini (1996) and Turnovsky (1996), among others.[10] In those papers, however, it is utility-enhancing public *spending* that enters directly in the utility function, whereas in the present case what matters is the supply of health services.

The household's resource constraint is

$$C + \dot{K}_P = (1-\tau)Y + (1-\chi_P)w_G E, \tag{5}$$

where $\tau \in (0,1)$ is the tax rate on income and $(1-\chi_P)w_G E$ represents salaries paid to teachers and doctors, with w_G a constant real wage (assumed identical for both categories of workers). For simplicity, the depreciation rate of private capital is assumed to be zero.

3.3 Schooling Technology

As noted earlier, the economy's raw labor endowment, which grows at a constant rate n, must be educated before it can be used in market production. Education is a public, nonmarket activity. Specifically, the production of educated labor requires the combination of government spending on education services (such as instructional materials), G_E , as well as infrastructure

[10] To ensure that the instantaneous utility function has the appropriate concavity properties, we impose the restrictions $\kappa(1-1/\sigma) < 1$ and $1 > (1-1/\sigma)(1+\kappa)$.

and health services, teachers (who are all on the government's payroll), and students:

$$\dot{E} = G_E^{\mu_1'} G_I^{\mu_2} H^{\mu_3} L^{\mu_4} (\chi_E E)^{1-\Sigma\mu_h}, \tag{6}$$

where $\dot{E}$ is the flow of newly-educated workers, χ_E the proportion of educated workers engaged in teaching, L the number of students, and $\mu_h \in (0,1)$, for $h = 1, ...4$. Thus, the education technology exhibits constant returns to scale in all inputs. This specification captures the view (discussed in the previous section) that healthier students learn better; consequently, the quality of education improves and this translates into a higher output of educated labor.[11] Infrastructure also matters—lack of access to electricity for instance, may prevent schools from functioning properly.[12] To ensure that the number of newly-educated workers does not exceed the number of students, we assume that $\dot{E} < L$.

Equation (6) can be rewritten as

$$\frac{\dot{E}}{E} = \chi_E^{1-\Sigma\mu_h} (\frac{G_E}{E})^{\mu_1'} (\frac{G_I}{E})^{\mu_2} (\frac{H}{E})^{\mu_3} (\frac{L}{E})^{\mu_4}. \tag{7}$$

In what follows, we ignore depreciation (or de-skilling) of educated labor. We also assume that the student-teacher ratio (an indicator of the quality of schooling) varies inversely with government spending on education per student:

$$\frac{L}{\chi_E E} = (\frac{G_E}{L})^{-a}, \tag{8}$$

where $a \in (0,1)$. This specification captures the idea that spending relatively more per student (on classroom equipment, for instance), tends to lower the student-teacher ratio. It also implies that, along the balanced growth path, where E and G_E grow at a constant rate γ (as shown below), the number

[11] The analysis can readily be extended to the case where healthier teachers provide better training as well.

[12] Note that the production of educated labor could also occur through informal job training, or as a product of experience (learning by doing). We abstract from these considerations and focus instead on knowledge accumulation through schooling.

of students must grow at the same rate. This, in turn, imposes a lower bound on the rate of growth of the *total* population from which students are drawn, $n \geq \gamma$. Individuals who remain illiterate (that is, those who are unable to get access to the schooling system) are assumed to turn to a subsistence (non-market) activity for survival—a plausible assumption for the low-income countries that we have in mind.[13]

Combining equations (7) and (8) yields

$$\frac{\dot{E}}{E} = A_E(\frac{G_E}{E})^{\mu_1}(\frac{G_I}{E})^{\mu_2}(\frac{H}{E})^{\mu_3}, \tag{9}$$

where $A_E \equiv \chi_E^{1-\mu_1-\mu_2-\mu_3+\mu_4\frac{a}{1-a}} > 0$ and $\mu_1 \equiv \mu_1' - \alpha\mu_4/(1-\alpha)$.[14]

3.4 Production of Health Services

Aggregate production of health services requires combining public spending on health and infrastructure, as well as doctors. Assuming that production takes place under constant returns to scale in all factors yields

$$H = (\chi_H E)^{\theta_1} G_I^{\theta_2} G_H^{1-\theta_1-\theta_2} = A_H(\frac{E}{G_H})^{\theta_1}(\frac{G_I}{G_H})^{\theta_2} G_H, \tag{10}$$

where $\chi_H \equiv 1 - \chi_P - \chi_E$, $\theta_h \in (0,1)$ and $A_H \equiv \chi_H^{\theta_1} > 0$.

An alternative (or, rather, complementary) explanation for introducing educated labor in the production function (10) is to assume that the education system is used for the delivery of health services. Indeed, as documented by Glewwe (2002) and Bundy et al. (2005), an increasingly common approach in developing countries has been to use teachers to deliver micronutrients in schools.

[13] See Agénor (2005*b*) for a more detailed discussion.

[14] Although we have developed our model in terms of embodied knowledge, it can be reinterpreted in the standard Uzawa-Lucas framework. With N denoting population, and Z the stock of knowledge, one would need to use the concept of "effective human capital" in the production function, defined as ZNH, with productivity assumed to be strictly proportional to the supply of health services (that is, $Q = H$). In that case, assuming a constant population, knowledge production would become $\dot{Z} = G_E^{\mu_1'} G_I^{\mu_2} H^{\mu_3} Z^{1-\Sigma\mu_h}$ instead of (6), and restriction (8) would not be needed.

3.5 Government

The government spends on education services, and invests in health and infrastructure. It levies a flat tax on marketed output at the rate τ. In addition, it cannot issue debt claims and therefore must keep a balanced budget at each moment in time. The government budget constraint is thus given by

$$G_E + G_H + G_I + S = \tau Y, \tag{11}$$

where $S = (1 - \chi_P)w_G E$ is the public sector wage bill. As noted earlier, the wage rate w_G is taken to be constant.[15]

All categories of spending on services are taken to be a constant fraction of tax revenues:

$$G_h = \upsilon_h \tau Y, \quad \text{for } h = E, H, I. \tag{12}$$

The government budget constraint can thus be rewritten as

$$S = \tau(1 - \Sigma \upsilon_h)Y, \tag{13}$$

or equivalently, assuming that wage payments are a fraction $\varphi \in (0, 1)$ of tax revenues,

$$\upsilon_E + \upsilon_H + \upsilon_I = 1 - \varphi. \tag{14}$$

4 The Decentralized Equilibrium

In the present setting, a decentralized equilibrium is a set of infinite sequences for the quantities $\{C, K_P, E\}_{t=0}^{\infty}$, such that $\{C, K_P\}_{t=0}^{\infty}$ maximizes equation (4) subject to (5), and the path $\{K_P, E\}_{t=0}^{\infty}$ satisfies equations (5), (9), and (10), for given values of the tax rate, τ, the ratio of government wages to

[15] We assume implicitly that educated workers seek employment in the public sector first, perhaps because of the rent-seeking opportunities (or, more generally, the non-pecuniary benefits) associated with government jobs. Given that these jobs are rationed (only a fraction $(1-\chi_P)E$ of the pool of educated workers can be employed as doctors or teachers), there is no arbitrage condition between ω_G and the prevailing wage in the private sector, $\beta Y/\chi_P E$.

revenue, φ, and the spending shares υ_h, with $h = E, H, I$, which must also satisfy the constraint (14).

This equilibrium can be characterized as follows. The household solves problem (4) subject to (5), taking the tax rate, τ, and health services, H, as given. Using (4), (5) and (3), the current-value Hamiltonian for this problem can be written as

$$\Lambda = \frac{(CH^{\kappa})^{1-1/\sigma}}{1-1/\sigma} + \lambda \left\{ (1-\tau)A_P(\frac{G_I}{K_P})^{\alpha}[(\frac{H}{K_P})^{\varepsilon}(\frac{E}{K_P})^{1-\varepsilon}]^{\beta} K_P + S - C \right\},$$

where λ is the co-state variable associated with constraint (5).

From the first-order condition $d\Lambda/dC = 0$ and the co-state condition $\dot{\lambda} = -d\Lambda/dK_P + \rho\lambda$, optimality conditions for this problem can be written, with $s \equiv (1-\tau)(1-\alpha-\beta) = (1-\tau)\eta$, as

$$H^{\kappa}(CH^{\kappa})^{-1/\sigma} = \lambda, \tag{15}$$

$$\dot{\lambda}/\lambda = \rho - sA_P(\frac{G_I}{K_P})^{\alpha}[(\frac{H}{K_P})^{\varepsilon}(\frac{E}{K_P})^{1-\varepsilon}]^{\beta}, \tag{16}$$

together with the budget constraint (5) and the transversality condition

$$\lim_{t\to\infty} \lambda K_P \exp(-\rho t) = 0. \tag{17}$$

Equation (15) can be rewritten as

$$C = \lambda^{-\sigma} H^{\sigma\kappa(1-1/\sigma)}.$$

Taking logs of this expression and differentiating with respect to time yields

$$\frac{\dot{C}}{C} = -\sigma(\frac{\dot{\lambda}}{\lambda}) + \nu(\frac{\dot{H}}{H}), \tag{18}$$

where $\nu \equiv \sigma\kappa(1-1/\sigma)$.

Using (10) and (16), and setting $\Omega \equiv 1-\alpha-\varepsilon\beta(1-\theta_1) > 0$, yields

$$\frac{\dot{C}}{C} = \sigma \left\{ sA_P(\frac{G_I}{K_P})^{\alpha}[(\frac{H}{K_P})^{\varepsilon}(\frac{E}{K_P})^{1-\varepsilon}]^{\beta} - \rho \right\} \tag{19}$$

$$+\nu \left\{ \frac{\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta}{\Omega}(\frac{\dot{E}}{E}) + \frac{\eta(1-\theta_1)}{\Omega}(\frac{\dot{K}_P}{K_P}) \right\},$$

where, as shown in Appendix A, equation (A3), the expression for $\dot{H}/H$ is obtained by combining (10), (3), and (12), using the fact that the latter implies $\dot{G}_I/G_I = \dot{G}_H/G_H = \dot{Y}/Y$. In addition, substituting (3) in (5) yields

$$\dot{K}_P = (1-\tau)A_P(\frac{G_I}{K_P})^{\alpha}[(\frac{H}{K_P})^{\varepsilon}(\frac{E}{K_P})^{1-\varepsilon}]^{\beta}K_P + S - C. \tag{20}$$

As shown in Appendix A, equations (9), (10), (13), (19), and (20) can be further manipulated to lead to a system of two nonlinear differential equations in $c = C/K_P$ and $e = E/K_P$ (see equations (A11) and (A13)). These equations, together with the initial condition $e_0 > 0$ and the transversality condition (17), determine the dynamics of the decentralized economy.

The balanced-growth path (BGP) is therefore a set of sequences $\{c, e\}_{t=0}^{\infty}$, spending shares and tax rate, such that for initial condition e_0 equations (9), (19), (20) and the transversality condition (17) are satisfied, and consumption, the stock of educated labor, and the stock of private capital, all grow at the same constant rate $\gamma^* = \dot{C}/C = \dot{E}/E = \dot{K}_P/K_P$.

From equations (A7) and (A9) (after substituting (A14) and (A7) in (A9)) of Appendix A, the steady-state growth rate γ^* is given by the equivalent forms[16]

$$\gamma^* = A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}\tilde{e}^{-M_3\eta}, \tag{21}$$

$$\gamma^* = \frac{\sigma}{1-\nu}\left\{ s(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}} \upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}} \upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}} \tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} \tilde{e}^{\frac{\beta[1-\varepsilon(1-\theta_1)]}{\Omega}} - \rho \right\}, \tag{22}$$

where $\delta \equiv 1-\theta_1-\theta_2$ and a tilde over a variable is used to denote its stationary value, and A, M_1, M_2, and M_3 (which are all positive terms) are defined in Appendix A.

From equation (22), the growth rate is positive if the rate of time preference is not too large, that is, if $\rho < s(\tilde{Y}/\tilde{K}_P)$, as well as $1-\nu > 0$. The second

[16] Alternatively, equation (22) can be obtained by substituting (A14) and (A7) into equation (A10).

condition can be rewritten as $\sigma < 1+1/\kappa$, which imposes an upper bound on the intertemporal elasticity of substitution. In turn, this condition—which can be derived from the convergence requirement $\gamma^* < \rho/(1-1/\sigma)$ combined with (22)—must hold for the transversality condition (17) to be satisfied along the BGP. Therefore, a steady-state solution exists as long as the rate of time preference and the growth rate are not too large.

As shown in Appendix A, along an equilibrium path with a strictly positive growth rate, the BGP is unique. In addition, there is only one stable path converging to the equilibrium. Thus, the model is locally determinate, and its dynamics are illustrated in Figure 1. Although the $\dot{e} = 0$ curve (denoted EE in the figure) has a concave shape, the $\dot{c} = 0$ curve (denoted CC) can be either upward- or downward-sloping, depending on the size of the elasticity of intertemporal substitution, σ. The upper (lower) panel corresponds to the case where σ is relatively high (low), in a sense made precise in Appendix A (see equation (A12)). Therefore, the slope of the saddlepath SS may be either positive or negative.

Following a jump in c (as a result, for instance, of a change in the tax rate or in one of the spending shares), c and e may or may not move in the same direction. The reason is that the transitional dynamics are driven by the ratio of educated labor to private capital, and as this ratio increases, the marginal productivity of capital increases, thereby raising the incentive to save and invest. Although the intertemporal substitution effect tends to reduce consumption on impact, the positive income effect (associated with the higher capital stock and output) tends to increase it. Given the relatively high (small) value of the elasticity of substitution, σ, in the upper (lower) panel, the former (latter) effect dominates and lowers (raises) the consumption-capital ratio. This is illustrated by a movement along SS from the left of point A towards A.

5 Transitional Dynamics

We now analyze the steady-state effects and transitional dynamics of the economy to an unanticipated, permanent revenue-neutral change between any two of the spending categories. We examine, in turn, a shift from health toward infrastructure spending, a shift from education toward infrastructure spending, and finally a shift from health expenditures toward education.[17] It is intuitively clear that all these experiments entail a trade-off with respect to their impact on economic growth and the levels of consumption, health, and education.

5.1 Shift in Spending toward Infrastructure

First, we examine the impact of an increase in υ_I when offset by a reduction in υ_H ($d\upsilon_I = -d\upsilon_H$), holding τ and φ constant. Appendix B establishes that such a shift in the government's expenditure composition has an ambiguous effect on the steady-state growth rate. In particular, if education and health services do not affect the education accumulation process ($\mu_1 = \mu_3 = 0$), the net effect on growth depends on whether the observed ratio υ_I/υ_H lies above or below its optimal value. In this case, the optimal ratio is a function of the elasticities of the goods and health technologies only. With $\mu_1 > 0$ and $\mu_3 > 0$, the net effect depends also on the education technology elasticities.

To understand the intuition behind these results, consider first the case where $\mu_1 = \mu_3 = 0$. Increasing the share of public expenditures on infrastructure has a positive impact on the marginal productivity of private capital and, therefore, growth (both directly through the goods technology and indirectly through the production of health and education services). At

[17] Although such resource shifting experiments from one type of *productive* government spending category to another have long been acknowledged as having important implications for growth (see, for instance, Glomm and Ravikumar (1997)), most of the literature has focused on shifts between productive and unproductive expenditures (see, for instance, Turnovsky and Fisher (1995)). In our model, this could be captured by setting $\theta_1+\theta_2 = 1$, which implies that G_H would become unproductive. Alternatively, we could set $\mu_1 = 0$.

the same time, however, this increase is accompanied by a lower provision of health services that reduce the supply of healthy workers, which tends to lower private production and reduce the growth rate. The net effect on growth, therefore, depends on whether the actual spending ratio υ_I/υ_H exceeds or falls short of an optimal value, which depends on the parameters characterizing the production of goods and health services (see Appendix B). If the observed ratio is lower than this optimal value, the growth effect will be positive, whereas the effect on the consumption-capital ratio will be negative.

With $\mu_1 > 0$ and $\mu_3 > 0$, the net growth effect is even more ambiguous. Now, it depends not only on the elasticities characterizing the production of goods and health services, but also on those determining the economy's ability to produce educated labor. Even if infrastructure services have a small impact on the production of goods (low α/β), a high relative importance of infrastructure in the production of educated labor (high μ_2/μ_3) and/or production of health services (high θ_2/δ) may suffice to lead to increases in $\tilde{e}$, $\tilde{c}$, and γ^*. In the particular case where $\theta_1+\theta_2 = 1$, that is, if health services do not affect the economy-wide level of health, $\delta = 0$ and the effect of an increase in υ_I on the steady-state rate of growth is unambiguously positive.

Figure 2 illustrates the possible effects on $\tilde{e}$ and $\tilde{c}$, in the presence of relatively small values of both μ_1, μ_3, and of the elasticity of intertemporal substitution, σ. As a result, both panels show an increase in the steady-state ratio of educated labor to capital. However, the consumption-capital ratio and the rate of growth may either increase or fall, depending on the ratio υ_I/υ_H. In both panels, a rise in υ_I shifts both curves CC and EE to the right. In the upper (lower) panel, where the ratio υ_I/υ_H is relatively small (large), CC shifts by more (less) than EE and the consumption-capital ratio falls (rises). In both cases, the economy converges monotonically to the new BGP, located at point A'.

A similar analysis examines the economic impact of an increase in infrastructure spending when compensated by an equivalent decrease in education expenditures ($d\upsilon_I = -d\upsilon_E$). As before, such a change in the composition of public spending creates a trade-off with respect to the rate of growth, as shown in Appendix B. Starting with the case where $\mu_2 = \mu_3 = 0$ so that infrastructure and health public services have no impact on the education technology, an increase in υ_I has growth-enhancing effects while the respective decrease in υ_E has growth-retarding effects. The positive growth effects take place through the output and health technologies, whereas the distorting effects are the result of the indirect influence of the education and health technologies on growth. As a result, the net growth impact depends on the relative importance of the two offsetting effects, as represented by the ratio $(\alpha + \varepsilon\beta\theta_2)/[1 - \varepsilon(1 - \theta_1)]\beta$. As established in Appendix B, if this ratio exceeds the elasticity of the steady-state value of the educated labor-capital ratio with respect to the share of spending in infrastructure, both growth and consumption increase.

These effects are illustrated in Figure 3 for relatively small values of μ_2, μ_3, and σ. In both panels the steady-state ratio of educated labor-capital declines, while the consumption-capital ratio and the rate of growth may either increase or fall. Both panels reveal that a rise in υ_I shifts both curves CC and EE to the left. In the upper (lower) panel, a low (high) elasticity of the steady-state value of the educated labor-capital ratio with respect to the share of spending in infrastructure causes the CC curve to shift by more (less) than EE so that the consumption-capital ratio rises (falls).

In the general case, where $\mu_2 > 0$ and $\mu_3 > 0$, the net effect on the steady-state ratio of educated labor to capital is also unclear. This in turn, implies that the effect on growth is even more obscure since it now also depends on the elasticities of the education technology with respect to spending on infrastructure and education. In this general case, a rise in υ_I may still lead to a higher $\tilde{e}$, $\tilde{c}$, and γ^* even if α/β is low, as long as θ_2/θ_1 and/or μ_2/μ_1 are sufficiently large (that is, as long as infrastructure is sufficiently productive in the education and health production technologies).[18]

5.2 Shift in Spending toward Education

The final experiment consists of a revenue-neutral shift in spending from health toward education ($d\upsilon_E = -d\upsilon_H$), keeping τ and φ constant. The intuition is similar to the above line of argument, which suggests two conflicting effects on growth. However, both effects now are *indirect* because they affect the goods production technology only through the human capital production techniques. Appendix B shows that in the simple case where $\mu_2 = \mu_3 = 0$, a rise in υ_E unambiguously raises the ratio of educated labor to physical capital; but in general, sufficiently high values of μ_2 and μ_3 may lead to a decrease in the steady-state value of e. The positive effect of an increase in spending on education will thus outweigh the negative effect of lower spending of health services on the stock of educated workers. The respective effect on the rate of growth (and the steady-state ratio of consumption to capital), depends on how far above or below υ_E/υ_H is, compared to its optimal ratio. For values above (below) the optimal value, both the rate of growth and $\tilde{c}$ will be positively (negatively) affected.

The steady-state effects and transitional dynamics of the increase in υ_E (again assuming low values for μ_2, μ_3, and σ) are also illustrated in Figure 2, where both the CC and EE curves shift to the right. At the new equilibrium, the education to private capital ratio is higher, while the consumption to capital ratio could be either lower (upper panel) or higher (lower panel). In both cases the adjustment path is reflected by the sequence ABA'.

[18] These results provide a generalization of those derived in Agénor (2005*b*), where the provision of health services is absent.

6 Growth-Maximizing Policies

Using the steady-state growth rate equations (21) and (22), we now examine the optimal allocation of public expenditures to infrastructure, education, and health in the decentralized equilibrium, treating the tax rate and one of the shares of spending as exogenously set (that is, $d\tau = 0$ and $d\upsilon_I = -d\upsilon_H$, $d\upsilon_I = -d\upsilon_E$, $d\upsilon_E = -d\upsilon_H$).

Following the same order of illustration as the section that dealt with transitional dynamics, we first examine a revenue-neutral shift in public spending from health to infrastructure. As a result of the budget constraint (14), and with φ constant, only one of these shares can be independently chosen.

Setting $d\gamma^*/d\upsilon_I = 0$ in equations (21) and (22), and assuming that the exogenously set values of υ_E and φ are zero, yields

$$\upsilon_I^*|_{d\upsilon_I=-d\upsilon_H} = \frac{Z_1 + Z_2}{\Theta_1 + Z_2} < 1, \tag{23}$$

where $Z_1 \equiv \beta\{(\mu_1\varepsilon+\mu_3)\theta_2+\mu_2(1-\varepsilon\delta)\} > 0$, $Z_2 \equiv \beta\{(\mu_1\varepsilon+\mu_3)(1-\theta_1)+\mu_2\} > 0$, and $\Theta_1 \equiv \alpha[\mu_1+\mu_2+\mu_3(1-\theta_1)] > 0$. Equation (23) shows that, in general, the optimal composition of spending depends on all the parameters characterizing the technologies for producing goods, health services, and educated labor.

To provide a more intuitive interpretation, it is convenient to consider the particular case where infrastructure and health services do not affect directly the accumulation of educated labor (that is, $\mu_2 = \mu_3 = 0$), although similar intuition would follow if we instead set $\mu_1 = \mu_3 = 0$ or $\mu_1 = \mu_2 = 0$. In this way, we can write the growth-maximizing share of infrastructure as

$$\upsilon_I^*|_{d\upsilon_I=-d\upsilon_H} = \frac{\alpha + \varepsilon\beta\theta_2}{\alpha + \varepsilon\beta(1-\theta_1)}. \tag{24}$$

This expression is a generalization of the optimal allocation rule derived in Agénor (2005e), in a model where infrastructure enters also in the production of health services, but educated labor is absent (that is, $\theta_1 = 0$). It implies that if the production of health services depends on publicly-provided infrastructure, that is, $\theta_2 > 0$, then the optimal share of spending on infrastructure is higher than otherwise. Also note that this share is in general greater than α, implying that the strict Barro rule, which here corresponds to $\upsilon_I^* = \alpha + \beta$, is sub-optimal (see Barro (1990)). In the special case where educated labor and infrastructure expenditure do not affect the health production technology (that is, $\theta_1 = \theta_2 = 0$), the optimal share of spending on infrastructure is given by $\upsilon_I^* = \alpha/(\alpha + \varepsilon\beta)$, where $\varepsilon\beta$ can be viewed as the weighted elasticity of goods production with respect to effective labor. If $\varepsilon = 1$, then the optimal share of infrastructure is similar to the expression obtained in Agénor (2005a).

A more general presentation of the effects of all the related technology parameters on the optimal share is provided in the second column of Table 1, by using (23). The results are intuitively appealing; they show that an increase in the elasticities of the production of goods, educated labor, and health services, with respect to infrastructure outlays, α, μ_2, and θ_2, respectively, should be accommodated by an increase in the share of infrastructure spending. Conversely, governments should decrease υ_I^* (or increase υ_H^*) when the elasticity of production of goods with respect to effective labor, β, the responsiveness of productivity with respect to health, ε, and the elasticity of production of educated labor with respect to health, μ_3, improve. An increase in the elasticity of health with respect to education, θ_1, tends to increase υ_I^*. The reason is that the increase in θ_1 lowers the elasticity of health output with respect to spending on health, δ, while at the same time the shift toward infrastructure raises the supply of educated labor—which in turn raises output of health services and magnifies the initial effect on education. Finally, the effect that μ_1 (the responsiveness of the production of educated labor with respect to education spending) has on the share of infrastructure depends on the relative responsiveness of both the goods and education production technologies with respect to infrastructure compared to health spending, that is, on the ratios α/μ_2 and $\varepsilon\beta/\mu_3$. If the former (latter) dominates, then υ_I^* rises (falls).

A similar line of argument follows when, instead of financing an increase in the share of infrastructure by decreasing the share of health, there is a decrease in the share of education (that is, $d\upsilon_I = -d\upsilon_E$). In this case, with $\upsilon_H = \varphi = 0$, the optimal composition is

$$\upsilon_I^*|_{d\upsilon_I=-d\upsilon_E} = \frac{Z_1 + Z_2}{\Theta_2 + Z_2} < 1, \tag{25}$$

where $\Theta_2 \equiv \beta\{(\mu_1 + \mu_2)(1 - \varepsilon\delta) + \mu_3\theta_2\} > 0$.

For tractability of exposition, consider the case where $\mu_2 = \mu_3 = 0$. The optimal share of infrastructure is now

$$\upsilon_I^*|_{d\upsilon_I=-d\upsilon_E} = \frac{\alpha + \varepsilon\beta\theta_2}{\alpha + \beta(1 - \varepsilon\delta)}, \tag{26}$$

which, in contrast to the optimal share derived in equation (24) shows that a higher elasticity of the health technology with respect to education, θ_1, lowers rather than raises υ_I^*. Of course, this is an implication of the fact that now a higher share of infrastructure is financed by an equivalent reduction in the share of education spending, and as such, it diminishes the growth-enhancing effects of education. As before, in the special case where $\theta_1 = \theta_2 = 0$, so that $\delta = 1$, the optimal allocation of spending between infrastructure and education would depend only on the parameters characterizing the goods production technology, represented by the ratio $\alpha/\beta(1-\varepsilon)$. This would yield an optimal share of $\upsilon_I^* = \alpha/[\alpha + \beta(1 - \varepsilon)]$. If, in addition, $\varepsilon = 0$, the result is consistent with Agénor (2005a).

Table 1 (column 3) provides more information on the effects of the technology parameters on υ_I^*. As illustrated in the case where an increase in υ_I is offset by a decrease in υ_H, we get that higher values of α, μ_2, and θ_2 positively affect υ_I^*. But now, in addition, so does ε and μ_3, because a higher ε is associated with a lower responsiveness of the production of final goods (through productivity) with respect to educated labor, and because a higher μ_3 means that more spending on infrastructure, by raising output of health services, tends to mitigate the adverse effect of lower education spending on output of educated labor—in addition to its direct effect. By contrast, increases in β, μ_1, and θ_1 negatively affect the share of spending on infrastructure because they entail a higher degree of responsiveness of the production of goods, educated labor, and health, respectively, with respect to education spending (thus calling for higher υ_E^*). The fourth column of Table 1 presents the symmetrically opposite effects that changes in the technology elasticities have on the optimal share of education spending when financed with a cut in infrastructure expenditure.

The final optimal share of spending to determine is related to education being financed by an equivalent decrease in health expenditures (that is, $d\upsilon_E = -d\upsilon_H$). Setting $d\gamma^*/d\upsilon_E = 0$ and $\upsilon_I = \varphi = 0$ in equations (21) and (22) yields

$$\upsilon_E^*|_{d\upsilon_E=-d\upsilon_H} = \frac{\mu_1[1-\varepsilon(1-\theta_1)]}{\delta(\mu_2\varepsilon+\mu_3)+\mu_1(1-\varepsilon\theta_2)} < 1. \tag{27}$$

In the same vein as before, this revenue-neutral shift in spending from health to education creates two opposing effects on the marginal product of capital, and therefore growth. However, in contrast to the previous two cases examined, the growth effects now are only indirect, through the health and education production technologies. That is, there is no direct impact on the goods production technology (notice the absence of the parameters α and β). Equation (27) reveals that the more important is the responsiveness of the production of health services and educated labor with respect to education (health), as measured by θ_1 (δ) and μ_1 (μ_3) respectively, the higher is the optimal share of spending on education (health) services.

Table 1

Partial Effects of Technology Parameters
on Growth-Maximizing Spending Structure

	υ_I^*		υ_E^*	
Parameter	Offset: υ_H	Offset: υ_E	Offset: υ_I	Offset: υ_H
α	+	+	−	0
β	−	−	+	0
ε	−	+	−	−
μ_1	?	−	+	+
μ_2	+	+	−	−
μ_3	−	+	−	−
θ_1	+	−	+	+
θ_2	+	+	−	+

To get a more intuitive interpretation of υ_E^*, consider, as before, the case where $\mu_2 = \mu_3 = 0$. This gives rise to

$$\upsilon_E^*\big|_{d\upsilon_E=-d\upsilon_H} = \frac{1-\varepsilon(1-\theta_1)}{1-\varepsilon\theta_2}, \tag{28}$$

which implies that the growth-maximizing share of investment in education receives its maximum value ($\upsilon_E^* = 1$) if government spending on health services has no indirect effect on the production of output (so that $\varepsilon = 0$).

The final column of Table 1 illustrates in more detail the effects that changes in the technology parameters have on υ_E^*, by using the general rule spelt out in equation (27). As expected, an increase in the parameters that characterize the responsiveness, with respect to education spending, of output of goods, $1-\varepsilon$, educated workers, μ_1, and health, θ_1, has an enhancing effect on υ_E^*. In addition, an increase in θ_2 is also associated with a higher υ_E^*, as a consequence of a lower responsiveness of the production of health services with respect to health spending. Finally, a government would find beneficial (in terms of maximizing the rate of growth) a reduction in the share of public spending in education if μ_2 or μ_3 rise (because both are related with a lower impact of education spending on the educated stock of workers).

Having derived the *growth*-maximizing spending structure in a market economy, our next task is to examine the *welfare*-maximizing structure in a centrally planned economy and provide a comparison with the growth-maximizing policies that we have obtained.

7 Welfare-Maximizing Allocation

We now assume that an altruistic central planner maximizes the household's lifetime utility by organizing the production and allocation of resources in all the sectors of the economy. The planner, by having complete information, chooses all the quantities directly, taking into account both the welfare-enhancing effects of health and the process of human capital accumulation.[19]

To specify the planner's problem rewrite the output production function (3), by using (12) and (10), as

$$Y = (A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}} \upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}} \upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}} \tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} E^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} K_P^{\frac{\eta}{\Omega}}. \tag{29}$$

By using equations (C1), (C2), (C3), and (C4) derived in Appendix C, the social planner's problem is to maximize, with respect to $C, \upsilon_I, \upsilon_H, \tau, K_P$, and E,

$$\begin{aligned} \Lambda = & \frac{\{C[A_H E^{\theta_1} \upsilon_I^{\theta_2} \upsilon_H^{\delta} (\tau Y)^{1-\theta_1}]^{\kappa}\}^{1-1/\sigma}}{1-1/\sigma} \\ & +\zeta_K \{[1-(\upsilon_E+\upsilon_H+\upsilon_I)\tau]Y - C\} \\ & +\zeta_E \{B \upsilon_E^{\mu_1} \upsilon_I^{\mu_2+\mu_3\theta_2} \upsilon_H^{\mu_3\delta} (\tau Y)^{\mu_1+\mu_2+\mu_3(1-\theta_1)} E^{1-[\mu_1+\mu_2+\mu_3(1-\theta_1)]}\}, \end{aligned}$$

where ζ_K and ζ_E denote the co-state variables associated with equations (C3) and (C4) respectively.

The first-order optimality conditions are given by

[19] An alternative approach is to assume that the government solves optimally only for its fiscal policy instruments, taking as given the paths of consumption and capital accumulation determined by private maximization. See Park and Philippopoulos (2002) and Piras (2005) for a discussion.

$$H^{\kappa}(CH^{\kappa})^{-1/\sigma} = \zeta_K, \tag{30}$$

$$\kappa CH^{\kappa}(CH^{\kappa})^{-1/\sigma}\left\{\frac{\theta_2+\alpha\delta}{\upsilon_I} - \frac{(1-\alpha)\delta}{\upsilon_H}\right\} \tag{31}$$
$$+\zeta_K Y\left\{\psi(\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I} - \frac{\varepsilon\beta\delta}{\upsilon_H}) + \tau\Omega\right\}$$
$$= -\zeta_E A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}E^{1-M_3\eta}K_P^{-M_3\eta}\left\{\frac{M_1\Omega}{\upsilon_I} - \frac{\mu_1\Omega}{\upsilon_E} - \frac{M_2\Omega}{\upsilon_H}\right\},$$

$$\kappa CH^{\kappa}(CH^{\kappa})^{-1/\sigma}\left\{\frac{(1-\alpha)\delta}{\upsilon_H} - \frac{\theta_2+\alpha\delta}{\upsilon_I}\right\} \tag{32}$$
$$+\zeta_K Y\left\{\psi(\frac{\varepsilon\beta\delta}{\upsilon_H} - \frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I}) + \tau\Omega\right\}$$
$$= -\zeta_E A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}E^{1-M_3\eta}K_P^{-M_3\eta}\left\{\frac{M_2\Omega}{\upsilon_H} - \frac{\mu_1\Omega}{\upsilon_E} - \frac{M_1\Omega}{\upsilon_I}\right\},$$

$$\kappa CH^{\kappa}(CH^{\kappa})^{-1/\sigma}(1-\theta_1) + \zeta_K Y\left\{\psi[\alpha+\varepsilon\beta(1-\theta_1)] - \tau(\upsilon_I+\upsilon_H+\upsilon_E)\Omega\right\} \tag{33}$$
$$= -\zeta_E A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}E^{1-M_3\eta}K_P^{-M_3\eta}[\mu_1+\mu_2+\mu_3(1-\theta_1)],$$

$$\frac{\dot{\zeta}_K}{\zeta_K} = \rho - \frac{\eta}{\Omega}\psi\frac{Y}{K_P} - \frac{\zeta_E}{\zeta_K}\frac{[\mu_1+\mu_2+\mu_3(1-\theta_1)]\eta}{\Omega} \tag{34}$$
$$\times A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}E^{1-M_3\eta}K_P^{-M_3\eta}\frac{1}{K_P} - \frac{1}{\zeta_K}\kappa CH^{\kappa}(CH^{\kappa})^{-1/\sigma}\frac{(1-\theta_1)\eta}{\Omega}\frac{1}{K_P},$$

$$\frac{\dot{\zeta}_E}{\zeta_E} = \rho - \frac{\Omega-[\mu_1+\mu_2+\mu_3(1-\theta_1)]\eta}{\Omega}A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}E^{1-M_3\eta}K_P^{-M_3\eta}\frac{1}{E} \tag{35}$$
$$-\frac{\zeta_K}{\zeta_E}\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}\frac{Y}{E} - \frac{1}{\zeta_E}\kappa CH^{\kappa}(CH^{\kappa})^{-1/\sigma}\frac{\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta}{\Omega}\frac{1}{E},$$

and the transversality conditions

$$\lim_{t\to\infty}\zeta_K K_P\exp(-\rho t) = \lim_{t\to\infty}\zeta_E E\exp(-\rho t) = 0. \tag{36}$$

Rewriting (30), taking logs, and differentiating with respect to time yields (18),

$$\frac{\dot{C}}{C}=-\sigma(\frac{\dot{\zeta}_K}{\zeta_K})+\nu(\frac{\dot{H}}{H}), \tag{37}$$

which is repeated here for convenience.

Using (34) and (A3), equation (37) becomes

$$\frac{\dot{C}}{C}=\sigma\{\frac{\eta}{\Omega}\psi\frac{Y}{K_P}+\frac{\zeta_E}{\zeta_K}\frac{[\mu_1+\mu_2+\mu_3(1-\theta_1)]\eta}{\Omega} \tag{38}$$

$$\times A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}E^{1-M_3\eta}K_P^{-M_3\eta}\frac{1}{K_P}+\frac{1}{\zeta_K}\kappa CH^{\kappa}(CH^{\kappa})^{-1/\sigma}\frac{(1-\theta_1)\eta}{\Omega}\frac{1}{K_P}-\rho\}$$

$$+\nu\left\{\frac{\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta}{\Omega}(\frac{\dot{E}}{E})+\frac{\eta(1-\theta_1)}{\Omega}(\frac{\dot{K}_P}{K_P})\right\}.$$

In addition, from Appendix C, equation (C3) implies

$$\dot{K}_P=\psi(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}E^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}}K_P^{\frac{\eta}{\Omega}}-C. \tag{39}$$

Appendix C illustrates how equations (38) and (39) can be further manipulated to produce two nonlinear differential equations in c and e, which together with the initial condition $e_0>0$ and the transversality condition for private capital (36) characterize the dynamics of the centrally-planned economy. The BGP is, again, a set of functions $\{c,e\}_{t=0}^{\infty}$, such that equations (38), (39), (C4) and the transversality condition (36) are satisfied, and consumption and the stocks of educated labor and private capital, all grow at the same constant rate γ^{**}.

The steady-state growth rate γ^{**} is given by equation (21) and its equivalent form[20]

$$\gamma^{**}=\frac{\sigma}{\Omega(1-\nu)}\{\left[\psi+[\mu_1+\mu_2+\mu_3(1-\theta_1)]\frac{\upsilon_E\tau}{\mu_1}+(1-\theta_1)\kappa(\frac{\tilde{C}}{\tilde{Y}})\right] \tag{40}$$

[20] Equation (40) can be obtained in two ways. First, by substituting (C10) into (A9). Second, by substituting (C10) and (A7) into (C6).

$$\times\eta(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}\tilde{e}^{\frac{\beta[1-\varepsilon(1-\theta_1)]}{\Omega}}-\Omega\rho\}.$$

It is straightforward to verify that this equilibrium is consistent with the transversality condition (36). Finally, Appendix C illustrates the uniqueness and stability of the BGP, where its dynamics, being qualitatively similar to those derived for the decentralized equilibrium, are illustrated in Figure 1.

As for the market economy, we now study the optimal allocation of public spending to the three categories (infrastructure, education, and health) in a command environment. With the use of equations (21) and (40), we examine the welfare-maximizing composition of these expenditure shares in the case of revenue-neutral shifts from health to infrastructure, education to infrastructure, and health to education. The respective optimal shares of spending that emerge are

$$\upsilon_I^{**}|_{d\upsilon_I=-d\upsilon_H}=\frac{Z_1+Z_2+T_1}{\Theta_1+Z_2+T_2}<1, \tag{41}$$

where Z_1, Z_2, and Θ_1 are as defined earlier, $T_1\equiv(1-\theta_1)\kappa(\tilde{C}/\tilde{Y})[(\mu_1+\mu_3)\theta_2+\mu_2(\theta_1+\theta_2)]>0$, and $T_2\equiv(1-\theta_1)\kappa(\tilde{C}/\tilde{Y})[(\mu_1+\mu_3)(1-\theta_1)+\mu_2]>0$.

$$\upsilon_I^{**}|_{d\upsilon_I=-d\upsilon_E}=\frac{Z_1+Z_2+T_1}{\Theta_2+Z_2+T_3}<1, \tag{42}$$

where $T_3\equiv(1-\theta_1)\kappa(\tilde{C}/\tilde{Y})[(\mu_1+\mu_2)(\theta_1+\theta_2)+\mu_2\theta_2]>0$, and

$$\upsilon_E^{**}|_{d\upsilon_E=-d\upsilon_H}=\frac{\beta\mu_1[1-\varepsilon(1-\theta_1)]+T_4}{\beta\delta(\mu_2\varepsilon+\mu_3)+\beta\mu_1(1-\varepsilon\theta_2)+T_5}<1, \tag{43}$$

where $T_4\equiv(1-\theta_1)\kappa(\tilde{C}/\tilde{Y})\mu_1\theta_1>0$ and $T_5\equiv(1-\theta_1)\kappa(\tilde{C}/\tilde{Y})[\mu_1(1-\theta_2)+\delta(\mu_2+\mu_3)]>0$.

In the particular case where $\kappa=0$, so that the supply of health services does not affect utility, $T_h=0$ for $h=1,..,5$, and formulas (41), (42), and (43) are identical to (23), (25), and (27), respectively. In general, however, this is not the case. As Table 2 illustrates, when increases in the shares of infrastructure and education are offset by a decline in health spending, the utility-maximizing shares of infrastructure and education fall below their respective growth-maximizing share (see the first and last columns, respectively). In both cases, the magnitude of the wedge depends on κ, implying that the greater the role of health services in utility, the larger the difference between the two optimizing rules.

Intuitively, spending on health services is now more important to the social planner, given its complementarity with consumption. Choosing shares of spending on infrastructure and education that are lower than their growth-maximizing rates reduces the growth rate but also leads to a reallocation of government outlays toward health services. If δ, the elasticity of the health production technology with respect to health expenditure is not too low, and neither is μ_3, the elasticity of the education technology with respect to spending on health, this reallocation leads to higher output of health services and the supply of educated labor, and thus higher productivity, which tends to mitigate the adverse productivity effect induced by a decline in public outlays in infrastructure or education. In turn, with $\kappa > 0$, the increase in output of health services translates into a higher level of consumption and a subsequent increase in welfare. In each case, this positive welfare effect dominates the negative effect of a lower growth rate. The higher θ_2 is, with respect to the trade-off between infrastructure and health spending, and the higher θ_1 and μ_1 are, with respect to the trade-off between education and health spending, the smaller the difference between the two optimizing solutions. In the limit case where $\theta_2 = 1$, then $\upsilon_I^{**}|_{d\upsilon_I=-d\upsilon_H} = \upsilon_I^{*}|_{d\upsilon_I=-d\upsilon_H} = 1$. Hence, both the growth- and welfare-maximizing solutions indicate that all government revenues should be allocated to infrastructure. In the same vein, when $\theta_1 = \mu_1 = 1$, then $\upsilon_E^{**}|_{d\upsilon_E=-d\upsilon_H} = \upsilon_E^{*}|_{d\upsilon_E=-d\upsilon_H} = 1$, and public resources should be directed toward education.

Table 2

Comparison of the Growth- and Welfare-Maximizing Spending Structure

υ_I^* and υ_I^{**}		υ_E^* and υ_E^{**}	
Offset: υ_H	Offset: υ_E	Offset: υ_I	Offset: υ_H
$\upsilon_I^* > \upsilon_I^{**}$	$\upsilon_I^* \lesseqgtr \upsilon_I^{**}$	$\upsilon_E^* \gtreqless \upsilon_E^{**}$	$\upsilon_E^* > \upsilon_E^{**}$

Table 2 also compares the two optimizing rules when the government finances more infrastructure with a cutback in education expenditures (or vice-versa), and therefore there is no direct change in health spending. As expected, in such event, it is not clear which of the two maximizing solutions dominates because there is no direct effect on welfare through health. The ultimate outcome depends critically on the responsiveness of the production of goods and health services with respect to infrastructure compared to education spending. That is, if infrastructure is relatively more productive than education, $\alpha/\theta_2 > \beta(1-\varepsilon)/\theta_1$, then the growth-maximizing share of infrastructure exceeds the welfare-maximizing share, and the intuition is similar to the one outlined for the trade-off between infrastructure and health spending. However, the parameter of interest in this case is μ_2, so that in the limit case where $\mu_2 = 1$, then $\upsilon_I^{**}|_{d\upsilon_I=-d\upsilon_E} = \upsilon_I^*|_{d\upsilon_I=-d\upsilon_E} = 1$.

Finally, we have also constructed a table, in the spirit of Table 1, that describes the partial effects of the various technology parameters on the welfare-maximizing spending allocation. The table turns out to be the same as Table 1, revealing that these parameters have equivalent effects on both the growth- and welfare-optimizing rules.[21]

[21] The only exception pertains to the tradeoff between infrastructure and health spending (see equation (41)) with respect to the impact of effective labor services (β). Although the effect of β on the growth-maximizing share of infrastructure was shown to be negative (see Table 1), its effect on the welfare-maximizing solution is ambiguous. As long as infrastructure is sufficiently productive in the production of goods compared to the production of educated labor (that is, if α/μ_2 is large enough), however, the negative effect will continue to hold.

8 Concluding Remarks

This paper studied the optimal allocation of government spending between health, education, and infrastructure in an endogenous growth framework. In the model, infrastructure affects not only the production of goods but also the supply of health and education services. Moreover, we also account for the fact that good health contributes not only to labor productivity but also to the quality of education, by improving the ability to attend school and learn. Thus, in contrast to the literature that followed from the seminal work of Lucas (1988), our model accounts for the fact that both knowledge and health are embodied in individuals.

The first part of the paper provided a brief overview of the recent evidence, at both the micro and macro levels, on the impact of health on growth, interactions between health and education, and the impact of infrastructure on health and learning outcomes. We noted, in particular, that there is significant evidence suggesting that better education of mothers tends to reduce the incidence of disease in children, that healthier children tend to do better in class, and that access to roads and electricity tends to improve the ability to attend school and visit health clinics, while at the same time enhancing the quality of education and health services.

The second part presented the model and the third described the derivation of the balanced growth path in the decentralized equilibrium. The fourth part analyzed the properties of the model by considering a series of revenue-neutral shifts in spending—a shift from education or health spending toward infrastructure outlays, and a shift in spending from health to education. This analysis allowed us to highlight the nature of the trade-offs that are embedded in the model, as a result of a binding budget constraint, despite the micro complementarities.

The last two parts of the paper provided a full characterization of both the growth- and welfare-maximizing structures of expenditure. They also compared the results from these two optimizing allocations. Although there are several cases where the comparison is ambiguous, there are also several instances where the optimal solutions are different. Our analysis showed that the degree of complementarity between health and consumption in utility, as well as the parameters characterizing the health and education technology, play a key role. In particular, if the elasticity of the health production technology with respect to infrastructure, and the elasticities of the health and education technologies with respect to educated labor and government spending on education are not too high, choosing shares of spending on infrastructure and education that are lower than their growth-maximizing rates is optimal from a welfare point of view. The reason is that although this allocation has a direct negative effect on the growth rate, it also leads to a reallocation of government outlays toward health services. In turn, this reallocation leads to a higher output of health services, and thus higher labor productivity, which tends to mitigate the drop in public outlays on infrastructure and education, respectively. In addition, the increase in the supply of health services translates into a higher level of consumption and a subsequent increase in welfare.

The model could be extended in various directions. One direction would be to account for congestion costs in the use of infrastructure and health services, as for instance in Eicher and Turnovsky (2000) and Piras (2005), and for a possible inverse relationship between the rate of depreciation of human capital (educated labor) and health expenditure. Another would be to account for the intertemporal welfare effect associated with the impact of health on longevity, as for instance in von Zon and Muysken (2005). Neither do we account for the fact that the probability of surviving between periods may be a function of health capital, which is augmented through public investment—as, for instance, in the overlapping-generations models of Chakraborty (2004) and Hashimoto and Tabata (2005). If, as noted in the introduction, an increase in life expectancy raises the incentive to save, the impact of an increase in public spending on infrastructure on growth would be magnified. This issue is further discussed in Agénor (2006), in a model where the rate of time preference is directly related to consumption of health services.

A more complex issue would be to explore how infrastructure may accelerate the demographic transition. By improving health, it reduces the need to have too many children, as postulated in the "old age" hypothesis; fertility rates would therefore drop. Parents may substitute quality for quantity and therefore invest more in the education of their (fewer) number of children, which would in turn enhance not only schooling but also health outcomes. However, to explore these issues an overlapping-generations model would be more appropriate than the single agent, infinite horizon framework developed in this paper.

Appendix A
Stability Conditions in the Market Economy

To obtain the expression for $\dot{H}/H$ in equation (19), first use (10) and $\dot{G}_I/G_I = \dot{G}_H/G_H = \dot{Y}/Y$ from (12) to get

$$\frac{\dot{H}}{H} = \theta_1(\frac{\dot{E}}{E}) + (1-\theta_1)(\frac{\dot{Y}}{Y}), \tag{A1}$$

and then, from (3)

$$\frac{\dot{Y}}{Y} = \frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}(\frac{\dot{E}}{E}) + \frac{\eta}{\Omega}(\frac{\dot{K}_P}{K_P}), \tag{A2}$$

where $\eta \equiv 1-\alpha-\beta$.

Combining these two expressions yields

$$\frac{\dot{H}}{H} = \frac{\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta}{\Omega}(\frac{\dot{E}}{E}) + \frac{\eta(1-\theta_1)}{\Omega}(\frac{\dot{K}_P}{K_P}). \tag{A3}$$

The next step is to eliminate $\dot{E}/E$ in equation (A3). From (9) and (10),

$$\frac{\dot{E}}{E} = B(\frac{G_E}{Y}\frac{Y}{K_P}\frac{K_P}{E})^{\mu_1}(\frac{G_I}{Y}\frac{Y}{K_P}\frac{K_P}{E})^{\mu_2+\mu_3\theta_2}(\frac{G_H}{Y}\frac{Y}{K_P}\frac{K_P}{E})^{\mu_3\delta}, \tag{A4}$$

where $B \equiv A_E A_H^{\mu_3}$. Using (12), this expression simplifies to

$$\begin{aligned}\frac{\dot{E}}{E} &= B\upsilon_E^{\mu_1}\upsilon_I^{\mu_2+\mu_3\theta_2}\upsilon_H^{\mu_3\delta}\tau^{\mu_1+\mu_2+\mu_3(1-\theta_1)} \\ &\quad \times e^{-[\mu_1+\mu_2+\mu_3(1-\theta_1)]}(\frac{Y}{K_P})^{\mu_1+\mu_2+\mu_3(1-\theta_1)}.\end{aligned} \tag{A5}$$

From (3), (10), and (12)

$$\frac{Y}{K_P} = (A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}}. \tag{A6}$$

Combining this result with (A5) yields

$$\frac{\dot{E}}{E} = A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta}, \tag{A7}$$

where

$$A \equiv B(A_P A_H^{\varepsilon\beta})^{\frac{\mu_1+\mu_2+\mu_3(1-\theta_1)}{\Omega}} > 0,$$

$$M_1 \equiv \frac{\mu_1(\alpha+\varepsilon\beta\theta_2)+\mu_2(1-\varepsilon\beta\delta)+\mu_3(\theta_2+\alpha\delta)}{\Omega} > 0,$$

$$M_2 \equiv \frac{\delta[\mu_3(1-\alpha)+\varepsilon\beta(\mu_1+\mu_2)]}{\Omega} > 0,$$

$$M_3 \equiv \frac{\mu_1+\mu_2+\mu_3(1-\theta_1)}{\Omega} > 0.$$

Equation (A7) can be substituted in (A3) to give

$$\begin{aligned}\frac{\dot{H}}{H} &= \frac{1}{\Omega}\{[\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta]A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta} \\ &\quad .+\eta(1-\theta_1)(\frac{\dot{K}_P}{K_P})\}.\end{aligned} \tag{A8}$$

Now, combining equations (13) and (20), and setting $\psi \equiv 1-(\upsilon_E+\upsilon_H+\upsilon_I)\tau > 0$, yields

$$\frac{\dot{K}_P}{K_P} = \psi\frac{Y}{K_P} - c,$$

which, by the use of (A6), simplifies to

$$\frac{\dot{K}_P}{K_P} = \psi(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} - c. \tag{A9}$$

Substituting this result in (A8) and then the resulting expression, along with (A7), in (19) yields

$$\frac{\dot{C}}{C} = \Pi(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} \tag{A10}$$

$$+\frac{\nu(\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta)}{\Omega}A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta} - \frac{\nu\eta(1-\theta_1)}{\Omega}c - \sigma\rho,$$

where

$$\Pi \equiv \frac{1}{\Omega}\{\sigma s\Omega + \nu\eta(1-\theta_1)\psi\} > 0.$$

Subtracting (A9) from (A10) yields

$$\frac{\dot{c}}{c} = \Phi(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} \tag{A11}$$

$$+\frac{\nu(\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta)}{\Omega}A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta}-\frac{\nu\eta(1-\theta_1)-\Omega}{\Omega}c-\sigma\rho,$$

where $\Phi \equiv \Pi - \psi$. The sign of Φ depends on the size of σ as follows:

$$\text{sg}(\Phi)=\text{sg}(\sigma-\frac{\psi[\Omega+\kappa\eta(1-\theta_1)]}{s\Omega+\kappa\eta(1-\theta_1)\psi}). \tag{A12}$$

In the particular case where $\varphi = 0$, this condition boils down to

$$\text{sg}(\Phi)=\text{sg}(\sigma-\frac{1}{\eta}).$$

Given that $\eta \equiv 1-\alpha-\beta$, a sufficient (although not necessary) condition for $\Phi < 0$ is $\sigma < 1$. This condition on σ also ensures that $\sigma < 1+1/\kappa$, which, as shown in the text, is necessary for the transversality condition (17) to hold.

Similarly, subtracting (A9) from (A7) yields

$$\frac{\dot{e}}{e}=A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta} \tag{A13}$$

$$-\psi(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}}+c.$$

Equations (A11) and (A13) represent a system of two nonlinear differential equations in $c = C/K_P$, and $e = E/K_P$.

To examine the uniqueness of the BGP, first set $\dot{c} = 0$ in (A11) to get

$$\tilde{c}=\frac{1}{\nu\eta(1-\theta_1)-\Omega}\{\Phi\Omega(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} \tag{A14}$$

$$\times\tilde{e}^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}}+\nu(\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta)A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}\tilde{e}^{-M_3\eta}-\Omega\sigma\rho\},$$

and then substitute (A14) in (A13) with $\dot{e} = 0$ to yield the implicit form

$$F(\tilde{e})=\frac{1}{\Omega-\nu\eta(1-\theta_1)}\{\Omega(1-\nu)A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}\tilde{e}^{-M_3\eta} \tag{A15}$$

$$+\Omega\sigma\rho-\Omega\sigma s(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}\tilde{e}^{\frac{\beta[1-\varepsilon(1-\theta_1)]}{\Omega}}\}.$$

where $\tilde{c}$ and $\tilde{e}$ denote the stationary values of c and e.

To show that the BGP is unique, note first that from (A15), and using (21) and (22),

$$F_{\tilde{e}} = -\frac{M_3\eta\Omega(1-\nu)\gamma^* + [1-\varepsilon(1-\theta_1)]\beta[\gamma^*(1-\nu)+\sigma\rho]}{[\Omega-\nu\eta(1-\theta_1)]\tilde{e}}, \tag{A16}$$

which can be established to be negative along a BGP with a strictly positive γ^*, for values of $\sigma < 1+1/\kappa$. Thus, $F(\tilde{e})$ cannot cross the horizontal axis from below. Now, we also have $F(0) = \Omega\sigma\rho/[\Omega-\nu\eta(1-\theta_1)] > 0$. Given that $F(\tilde{e})$ is a continuous, monotonically decreasing function of $\tilde{e}$, there is a unique positive value of $\tilde{e}$ that satisfies $F(\tilde{e}) = 0$. From (A14), there is also a unique positive value of $\tilde{c}$. Therefore, the BGP is unique.

To investigate the dynamics in the vicinity of the unique steady-state equilibrium, the system of equations (A11) and (A13) can be linearized to give

$$\begin{bmatrix} \dot{c} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c-\tilde{c} \\ e-\tilde{e} \end{bmatrix} \tag{A17}$$

where the a_{ij} are given by

$$a_{11} = \frac{\Omega-\nu\eta(1-\theta_1)}{\Omega}\tilde{c} > 0,$$

$$a_{12} = \frac{\tilde{c}}{\tilde{e}}\frac{1}{\Omega}\left\{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\psi}(\gamma^*+\tilde{c})\Phi\right.$$

$$\left.-\frac{\mu_1+\mu_2+\mu_3(1-\theta_1)}{\Omega^2}\nu\eta(\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta)\gamma^*\right\},$$

$$a_{21} = \tilde{e} > 0,$$

$$a_{22} = -\frac{[\mu_1+\mu_2+\mu_3(1-\theta_1)]\eta}{\Omega}\gamma^* - \frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}(\gamma^*+\tilde{c}) < 0,$$

where the sign of a_{12} is determined to be negative if $\Phi < 0$, that is, for a sufficiently low σ as shown in (A12).

From (A17), the slopes of CC and EE in Figure 1 are given by

$$\frac{d\tilde{c}}{d\tilde{e}}\Big|_{\dot{c}=0} = -\frac{a_{12}}{a_{11}}, \qquad \frac{d\tilde{c}}{d\tilde{e}}\Big|_{\dot{e}=0} = -\frac{a_{22}}{a_{21}},$$

where although EE always has a positive slope, the slope of CC is upward (downward) sloping if Φ is negative (positive and sufficiently large).

c is a jump variable, whereas e is predetermined over time. Saddlepath stability requires one unstable (positive) root. To ensure that this condition holds, the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A17), Δ, must be negative, that is, $\Delta = a_{11}a_{22} - a_{12}a_{21} < 0$. In the present environment, this condition is always satisfied. In the case where the slope of CC is upward-sloping ($a_{12} < 0$), EE has to be steeper than CC, as shown in the lower panel of Figure 1. The slope of the saddlepath SS is given by $-a_{12}/(\tilde{c} - \chi)$, where χ is the negative root of the system, and is thus positive (negative) if $a_{12} < 0$ ($a_{12} > 0$).

Appendix B
Transitional Dynamics of Spending Shifts

Equations (A14) and (A15) can be used to examine the impact of a revenue-neutral shift in the composition of the spending shares on the steady-state levels of c and e. In particular, using the implicit function theorem, it can be established that when an increase in infrastructure spending is financed with a reduction in the share of health ($d\upsilon_I = -d\upsilon_H$ holding τ and φ constant), $\partial\tilde{e}/\partial\upsilon_I \mid_{d\upsilon_I=-d\upsilon_H} = -F_{\upsilon_I}/F_{\tilde{e}}$ is in general ambiguous. Given that, from (A16) $F_{\tilde{e}} < 0$, we have sg$(\partial\tilde{e}/\partial\upsilon_I)$ =sg(F_{υ_I}). In turn, F_{υ_I} can be shown to be equal to

$$F_{\upsilon_I} \quad | \quad_{d\upsilon_I=-d\upsilon_H} = \frac{1}{\Omega - \nu\eta(1-\theta_1)}\{\Omega(1-\nu)\gamma^*\left[\frac{M_1}{\upsilon_I} - \frac{M_2}{\upsilon_H}\right] \tag{B1}$$

$$-\sigma s(\frac{\tilde{Y}}{\tilde{K}_P})\left[\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I} - \frac{\varepsilon\beta\delta}{\upsilon_H}\right]\},$$

from where it can be established that $F_{\upsilon_I} > 0$ if $\mu_1 = \mu_3 = 0$. This, in turn, implies that $\partial\tilde{e}/\partial\upsilon_I > 0$.

In general, from (A14),

$$\frac{\partial\tilde{c}}{\partial\upsilon_I} \mid_{d\upsilon_I=-d\upsilon_H} = \frac{1}{\nu\eta(1-\theta_1)-\Omega}\{\Phi(\frac{\tilde{Y}}{\tilde{K}_P}) \tag{B2}$$

$$\times\left[\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I} - \frac{\varepsilon\beta\delta}{\upsilon_H} + \frac{[1-\varepsilon(1-\theta_1)]\beta}{\tilde{e}}(\frac{\partial\tilde{e}}{\partial\upsilon_I})\right]$$

$$+\nu[\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta]\gamma^*\left[\frac{M_1}{\upsilon_I} - \frac{M_2}{\upsilon_H} - \frac{M_3\eta}{\tilde{e}}(\frac{\partial\tilde{e}}{\partial\upsilon_I})\right]\}.$$

Similarly, from (22),

$$\frac{\partial\gamma^*}{\partial\upsilon_I} \mid_{d\upsilon_I=-d\upsilon_H} = \frac{\sigma s}{1-\nu}(\frac{\tilde{Y}}{\tilde{K}_P})\frac{1}{\Omega}\left[\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I} - \frac{\varepsilon\beta\delta}{\upsilon_H} + \frac{[1-\varepsilon(1-\theta_1)]\beta}{\tilde{e}}(\frac{\partial\tilde{e}}{\partial\upsilon_I})\right].$$

Denoting $\varepsilon_{\tilde{e}/\upsilon_I} = (\partial\tilde{e}/\partial\upsilon_I)(\upsilon_I/\tilde{e})$, noting that from (23) $\varepsilon^*_{\tilde{e}/\upsilon_I} = 1$, and given that $\Phi < 0$ in Figure 2, we therefore have

$$\text{sg}(\frac{\partial\gamma^*}{\partial\upsilon_I}) = -\text{sg}(\frac{\partial\tilde{c}}{\partial\upsilon_I}) = \text{sg}(\frac{\alpha+\beta-\varepsilon\beta\delta}{\varepsilon\beta\delta} - \frac{\upsilon_I}{\upsilon_H}).$$

If υ_I/υ_H is lower (greater) than the critical ratio $(\alpha+\beta-\varepsilon\beta\delta)/\varepsilon\beta\delta$, which depicts the optimal ratio of υ_I/υ_H, an increase in υ_I has a positive (negative) effect on growth. Graphically, it can be verified from (A14) and (A15) that a rise in υ_I leads to a rightward shift in both CC and EE.

The impact of a rise in υ_I on the consumption-private capital ratio, given that $\partial e_0/\partial \upsilon_I = 0$, is

$$\frac{\partial c_0}{\partial \upsilon_I} = \frac{\partial \tilde{c}}{\partial \upsilon_I} - \frac{a_{12}}{\tilde{c}-\chi}(\frac{\partial \tilde{e}}{\partial \upsilon_I}), \tag{B3}$$

which is also ambiguous in general, given that $\partial \tilde{c}/\partial \upsilon_I$ is ambiguous. With $\partial \tilde{e}/\partial \upsilon_I > 0$, given that $a_{12} < 0$, then $\partial c_0/\partial \upsilon_I > 0$ if $\partial \tilde{c}/\partial \upsilon_I > 0$, as shown in the lower panel of Figure 2.

Within a similar framework, the effects of an increase in infrastructure expenditures financed by a decrease in education spending ($d\upsilon_I = -d\upsilon_E$ with $d\tau = d\varphi = 0$), can be illustrated as follows. The implicit function theorem implies, as before, the ambiguity of $\partial \tilde{e}/\partial \upsilon_I \mid_{d\upsilon_I=-d\upsilon_E} = -F_{\upsilon_I}/F_{\tilde{e}}$, where again sg$(\partial \tilde{e}/\partial \upsilon_I)$ =sg(F_{υ_I}). However, now F_{υ_I} is shown to be

$$F_{\upsilon_I} \mid_{d\upsilon_I=-d\upsilon_E} = \frac{1}{\Omega-\nu\eta(1-\theta_1)}\{\Omega(1-\nu)\gamma^*\left[\frac{M_1}{\upsilon_I} - \frac{\mu_1}{\upsilon_E}\right] \tag{B4}$$

$$-\sigma s(\frac{\tilde{Y}}{\tilde{K}_P})(\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I})\},$$

which is negative ($F_{\upsilon_I} < 0$) if $\mu_2 = \mu_3 = 0$. As a consequence, $\partial \tilde{e}/\partial \upsilon_I < 0$.

Equations (A14) and (22) imply respectively that,

$$\frac{\partial \tilde{c}}{\partial \upsilon_I} \mid_{d\upsilon_I=-d\upsilon_E} = \frac{1}{\nu\eta(1-\theta_1)-\Omega}\{\Phi(\frac{\tilde{Y}}{\tilde{K}_P}) \tag{B5}$$

$$\times\left[\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I} + \frac{[1-\varepsilon(1-\theta_1)]\beta}{\tilde{e}}(\frac{\partial \tilde{e}}{\partial \upsilon_I})\right]$$

$$+\nu[\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta]\gamma^*\left[\frac{M_1}{\upsilon_I} - \frac{\mu_1}{\upsilon_E} - \frac{M_3\eta}{\tilde{e}}(\frac{\partial \tilde{e}}{\partial \upsilon_I})\right]\},$$

and

$$\frac{\partial \gamma^*}{\partial \upsilon_I} \mid_{d\upsilon_I=-d\upsilon_E} = \frac{\sigma s}{1-\nu}(\frac{\tilde{Y}}{\tilde{K}_P})\frac{1}{\Omega}\left[\frac{\alpha+\varepsilon\beta\theta_2}{\upsilon_I} + \frac{[1-\varepsilon(1-\theta_1)]\beta}{\tilde{e}}(\frac{\partial \tilde{e}}{\partial \upsilon_I})\right].$$

With $\mu_2 = \mu_3 = 0$ and $\Phi < 0$ in Figure 3, we have

$$\mathrm{sg}(\frac{\partial \gamma^*}{\partial \upsilon_I}) = \mathrm{sg}(\frac{\partial \tilde{c}}{\partial \upsilon_I}) = \mathrm{sg}(\frac{\alpha + \varepsilon\beta\theta_2}{[1-\varepsilon(1-\theta_1)]\beta} + \varepsilon_{\tilde{e}/\upsilon_I}).$$

If $\varepsilon_{\tilde{e}/\upsilon_I} < 0$ (which is always the case if μ_2 and μ_3 are both zero or very small), the effect on growth is positive if

$$\frac{\alpha + \varepsilon\beta\theta_2}{[1-\varepsilon(1-\theta_1)]\beta} > -\varepsilon_{\tilde{e}/\upsilon_I}.$$

If $\varepsilon_{\tilde{e}/\upsilon_I} < 0$, the growth effect is always positive. Figure 3 depicts that an increase in υ_I leads both CC and EE to shift to the left. As before the instantaneous effect on c, is shown by (B3), which is in general ambiguous. With $\partial\tilde{e}/\partial\upsilon_I < 0$, given that $a_{12} < 0$, then $\partial c_0/\partial\upsilon_I < 0$ if $\partial\tilde{c}/\partial\upsilon_I < 0$, as shown in the lower panel of Figure 3.

Finally, we examine the steady-state effects and transitional dynamics of the government's decision to substitute health spending with additional education expenditures ($d\upsilon_E = -d\upsilon_H$ holding τ and φ constant). Using the implicit function theorem, we obtain $\partial\tilde{e}/\partial\upsilon_E \mid_{d\upsilon_E=-d\upsilon_H} = -F_{\upsilon_E}/F_{\tilde{e}}$, so that sg$(\partial\tilde{e}/\partial\upsilon_E)$ =sg(F_{υ_E}). Using equation (A15),

$$F_{\upsilon_E} \mid_{d\upsilon_E=-d\upsilon_H} = \frac{1}{\Omega - \nu\eta(1-\theta_1)}\{\Omega(1-\nu)\gamma^*\left[\frac{\mu_1}{\upsilon_E} - \frac{M_2}{\upsilon_H}\right] \tag{B6a}$$

$$+\sigma s(\frac{\tilde{Y}}{\tilde{K}_P})\left(\frac{\varepsilon\beta\delta}{\upsilon_H}\right)\},$$

which yields $F_{\upsilon_E} > 0$ for $\mu_2 = \mu_3 = 0$. This, in turn, implies that $\partial\tilde{e}/\partial\upsilon_E > 0$.

Next, we can show that

$$\frac{\partial\tilde{c}}{\partial\upsilon_E} \mid_{d\upsilon_E=-d\upsilon_H} = \frac{1}{\nu\eta(1-\theta_1) - \Omega}\{\Phi(\frac{\tilde{Y}}{\tilde{K}_P}) \tag{B7}$$

$$\times\left[-\frac{\varepsilon\beta\delta}{\upsilon_H} + \frac{[1-\varepsilon(1-\theta_1)]\beta}{\tilde{e}}(\frac{\partial\tilde{e}}{\partial\upsilon_I})\right]$$

$$+\nu[\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta]\gamma^*\left[\frac{\mu_1}{\upsilon_E} - \frac{M_2}{\upsilon_H} - \frac{M_3\eta}{\tilde{e}}(\frac{\partial\tilde{e}}{\partial\upsilon_I})\right]\},$$

and

$$\frac{\partial \gamma^*}{\partial \upsilon_E} \mid_{d\upsilon_E = -d\upsilon_H} = \frac{\sigma s}{1-\nu} (\frac{\tilde{Y}}{\tilde{K}_P}) \frac{1}{\Omega} \left[-\frac{\varepsilon \beta \delta}{\upsilon_H} + \frac{[1-\varepsilon(1-\theta_1)]\beta}{\tilde{e}} (\frac{\partial \tilde{e}}{\partial \upsilon_I}) \right].$$

Assuming that $\mu_2 = \mu_3 = 0$ and $\Phi < 0$ in Figure 2, we have

$$\mathrm{sg}(\frac{\partial \gamma^*}{\partial \upsilon_E}) = \mathrm{sg}(\frac{\partial \tilde{c}}{\partial \upsilon_E}) = \mathrm{sg}[\frac{\upsilon_E}{\upsilon_H} - \frac{1-\varepsilon(1-\theta_1)}{\varepsilon \delta}].$$

If υ_E / υ_H is greater (lower) than the critical ratio $[1 - \varepsilon(1-\theta_1)]/\varepsilon\delta$, an increase in υ_E has a positive (negative) effect on growth. Graphically, it can be verified from (A14) and (A15) that a rise in υ_E leads to a rightward shift in both CC and EE. The impact of a rise in υ_E on the consumption-private capital ratio, is given by

$$\frac{\partial c_0}{\partial \upsilon_E} = \frac{\partial \tilde{c}}{\partial \upsilon_E} - \frac{a_{12}}{\tilde{c} - \chi} (\frac{\partial \tilde{e}}{\partial \upsilon_E}), \tag{B8}$$

where with $\partial \tilde{e} / \partial \upsilon_E > 0$, given that $a_{12} < 0$, then $\partial c_0 / \partial \upsilon_E > 0$ if $\partial \tilde{c} / \partial \upsilon_E > 0$, as shown in the lower panel of Figure 2.

Appendix C
Stability Conditions in the Command Economy

To express the social planners' problem, replace (12) and (29) into (10) to obtain an expression for the production of health services

$$H = (A_P^{1-\theta_1} A_H^{1-\alpha})^{\frac{1}{\Omega}} \upsilon_I^{\frac{\theta_2+\alpha\delta}{\Omega}} \upsilon_H^{\frac{(1-\alpha)\delta}{\Omega}} \tau^{\frac{1-\theta_1}{\Omega}} E^{\frac{\theta_1\eta+[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} K_P^{\frac{\eta(1-\theta_1)}{\Omega}}. \tag{C1}$$

From (5) and (11), the economy's consolidated budget constraint can be written as

$$Y = C + \dot{K}_P + G_E + G_H + G_I, \tag{C2}$$

that is, using (12),

$$\dot{K}_P = \psi(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}} \upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}} \upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}} \tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} E^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} K_P^{\frac{\eta}{\Omega}} - C. \tag{C3}$$

Finally, the education accumulation equation (6) with the use of (12), (10), and (29) becomes

$$\dot{E} = A\upsilon_E^{\mu_1} \upsilon_I^{M_1} \upsilon_H^{M_2} \tau^{M_3} E^{1-M_3\eta} K_P^{-M_3\eta}. \tag{C4}$$

where A, M_1, M_2, and M_3 are as defined in Appendix A.

To get an expression for ζ_E/ζ_K, use (31) and (32) to obtain

$$q \equiv \frac{\zeta_E}{\zeta_K} = \frac{\upsilon_E \tau}{\mu_1} \frac{1}{\gamma^{**} e}(\frac{Y}{K_P}), \tag{C5}$$

where γ^{**} is described by (21) and (40). Equation (C5) implies that $\dot{q} = 0$. Thereafter, use (C5) into (38) to obtain

$$\frac{\dot{C}}{C} = \Xi(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}} \upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}} \upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}} \tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} \tag{C6}$$

$$+\frac{\nu(\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta)}{\Omega} A\upsilon_E^{\mu_1} \upsilon_I^{M_1} \upsilon_H^{M_2} \tau^{M_3} e^{-M_3\eta} - \frac{\nu\eta(1-\theta_1)}{\Omega} c - \sigma\rho,$$

where

$$J \equiv \mu_1 + \mu_2 + \mu_3(1-\theta_1),$$

$$\Xi \equiv \frac{1}{\Omega}\left\{\sigma\eta J\frac{\upsilon_E\tau}{\mu_1}+\sigma\eta(1-\theta_1)\kappa(\frac{C}{Y})+\psi\eta[\sigma+\nu(1-\theta_1)]\right\}>0.$$

Divide (C3) by K_P to obtain (A9), and subtract (A9) from (C6) to get

$$\frac{\dot{c}}{c}=\Upsilon(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}} \tag{C7}$$

$$+\frac{\nu(\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta)}{\Omega}A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta}-\frac{\nu\eta(1-\theta_1)-\Omega}{\Omega}c-\sigma\rho,$$

where $\Upsilon\equiv\Xi-\psi$. The sign of this expression depends on the size of σ as follows:

$$\mathrm{sg}(\Upsilon)=\mathrm{sg}(\sigma-\frac{\psi[\Omega+\kappa\eta(1-\theta_1)]}{\eta\{\psi[1+\kappa(1-\theta_1)]+(1-\theta_1)\kappa(\frac{C}{Y})+\eta J\upsilon_E\tau/\mu_1\}}). \tag{C8}$$

Next divide (C4) by E to obtain (A7), and subtract (A9) from (A7) to obtain

$$\frac{\dot{e}}{e}=A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}e^{-M_3\eta} \tag{C9}$$

$$-\psi(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}}e^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}}+c.$$

which is the same as (A13). Equations (C7) and (C9) represent a system of two nonlinear differential equations in $c=C/K_P$ and $e=E/K_P$.

To examine the uniqueness of the BGP, first set $\dot{c}=0$ in (C7) to get

$$\tilde{c}=\frac{1}{\nu\eta(1-\theta_1)-\Omega}\{\Upsilon\Omega(A_PA_H^{\varepsilon\beta})^{\frac{1}{\Omega}}\upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}}\upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}}\tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} \tag{C10}$$

$$\times\tilde{e}^{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}}+\nu(\eta\theta_1+[1-\varepsilon(1-\theta_1)]\beta)A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}\tilde{e}^{-M_3\eta}-\Omega\sigma\rho\},$$

and then substitute (C10) in (C9) with $\dot{e}=0$ to yield the implicit form

$$F(\tilde{e})=\frac{1}{\Omega-\nu\eta(1-\theta_1)}\{\Omega(1-\nu)A\upsilon_E^{\mu_1}\upsilon_I^{M_1}\upsilon_H^{M_2}\tau^{M_3}\tilde{e}^{-M_3\eta} \tag{C11}$$

$$+\Omega\sigma\rho-\sigma\eta[\psi+(1-\theta_1)\kappa(\frac{\tilde{C}}{\tilde{Y}})+\frac{J\upsilon_E\tau}{\mu_1}]$$

$$\times(A_P A_H^{\varepsilon\beta})^{\frac{1}{\Omega}} \upsilon_I^{\frac{\alpha+\varepsilon\beta\theta_2}{\Omega}} \upsilon_H^{\frac{\varepsilon\beta\delta}{\Omega}} \tau^{\frac{\alpha+\varepsilon\beta(1-\theta_1)}{\Omega}} \tilde{e}^{\frac{\beta[1-\varepsilon(1-\theta_1)]}{\Omega}} \}.$$

To show that the BGP is unique, note that from (C11), and using (21) and (40),

$$F_{\tilde{e}} = -\frac{1}{[\Omega - \nu\eta(1-\theta_1)]\tilde{e}} \{M_3\eta\Omega(1-\nu)\gamma^{**} + [1-\varepsilon(1-\theta_1)]\beta[\gamma^{**}(1-\nu)+\sigma\rho] \tag{C12}$$
$$+\sigma\eta(1-\theta_1)\kappa\tilde{c}\frac{\eta\theta_1}{\Omega}\},$$

which, for values of $\sigma < 1 + 1/\kappa$, is negative along a BGP with a strictly positive γ^{**}. With $F(0) = \Omega\sigma\rho/[\Omega - \nu\eta(1-\theta_1)] > 0$, $F(\tilde{e})$ cannot cross the horizontal axis from below. Given that $F(\tilde{e})$ is a continuous, monotonically decreasing function of $\tilde{e}$, there is a unique positive value of $\tilde{e}$ that satisfies $F(\tilde{e}) = 0$. This, in turn, implies a well-defined unique steady-state where both $\tilde{c} > 0$ and $\tilde{q} > 0$.

To investigate the dynamics in the vicinity of the unique steady-state equilibrium, the system of equations (C7) and (C9) can be linearized to give

$$\begin{bmatrix} \dot{c} \\ \dot{e} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} c-\tilde{c} \\ e-\tilde{e} \end{bmatrix} \tag{C13}$$

where the a_{ij} are now given by

$$a_{11} = \frac{\Omega - \nu\eta(1-\theta_1)}{\Omega}\tilde{c} > 0,$$

$$a_{12} = \frac{\tilde{c}}{\tilde{e}}\frac{1}{\Omega}\left\{\frac{[1-\varepsilon(1-\theta_1)]\beta}{\psi}(\gamma^{**}+\tilde{c})\Upsilon + \sigma\eta(1-\theta_1)\kappa\tilde{c}\frac{\eta\theta_1}{\Omega}\right.$$
$$\left. -\frac{J}{\Omega^2}\nu\eta(\eta\theta_1 + [1-\varepsilon(1-\theta_1)]\beta)\gamma^{**}\right\},$$

$$a_{21} = \tilde{e} > 0,$$

$$a_{22} = -\frac{J\eta}{\Omega}\gamma^{**} - \frac{[1-\varepsilon(1-\theta_1)]\beta}{\Omega}(\gamma^{**}+\tilde{c}) < 0,$$

where the sign of a_{12} could be negative if $\Upsilon < 0$, that is, for a sufficiently low σ as shown in (C8).

The linearized system (C13) has one unstable positive root, implying saddlepath stability. As before, c is free to jump, whereas e is constrained to continuous adjustments. Figure 1 equally represents the phase diagram of the centrally-planned economy.

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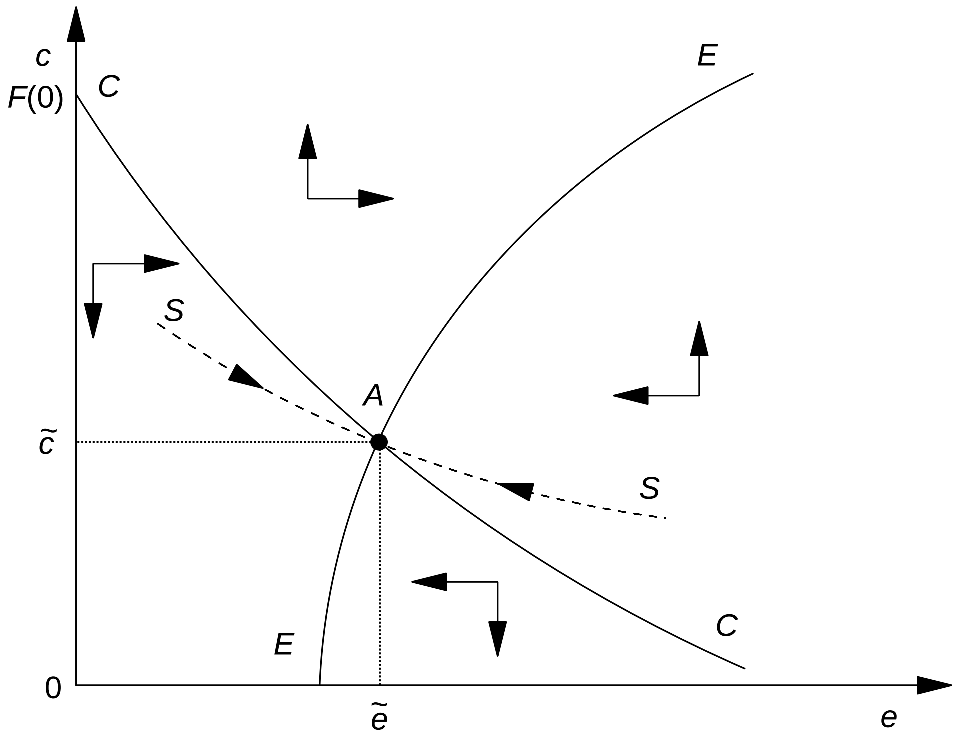
Figure 1
The Balanced Growth Equilibrium



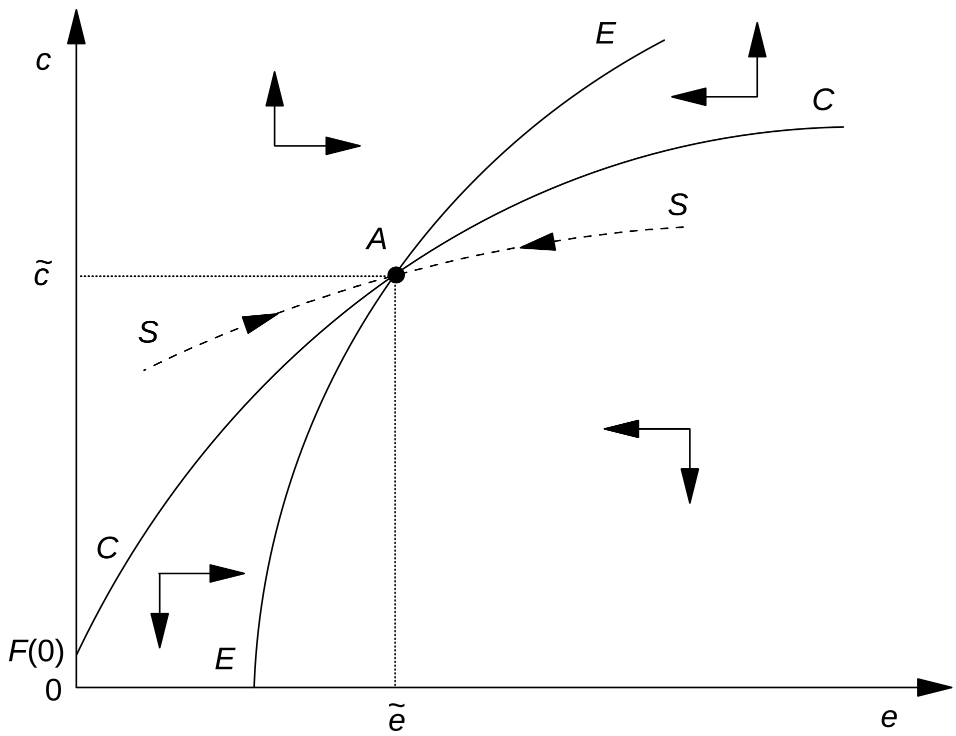


Figure 2

Shift in the Composition of Government Spending
from Health to Infrastructure

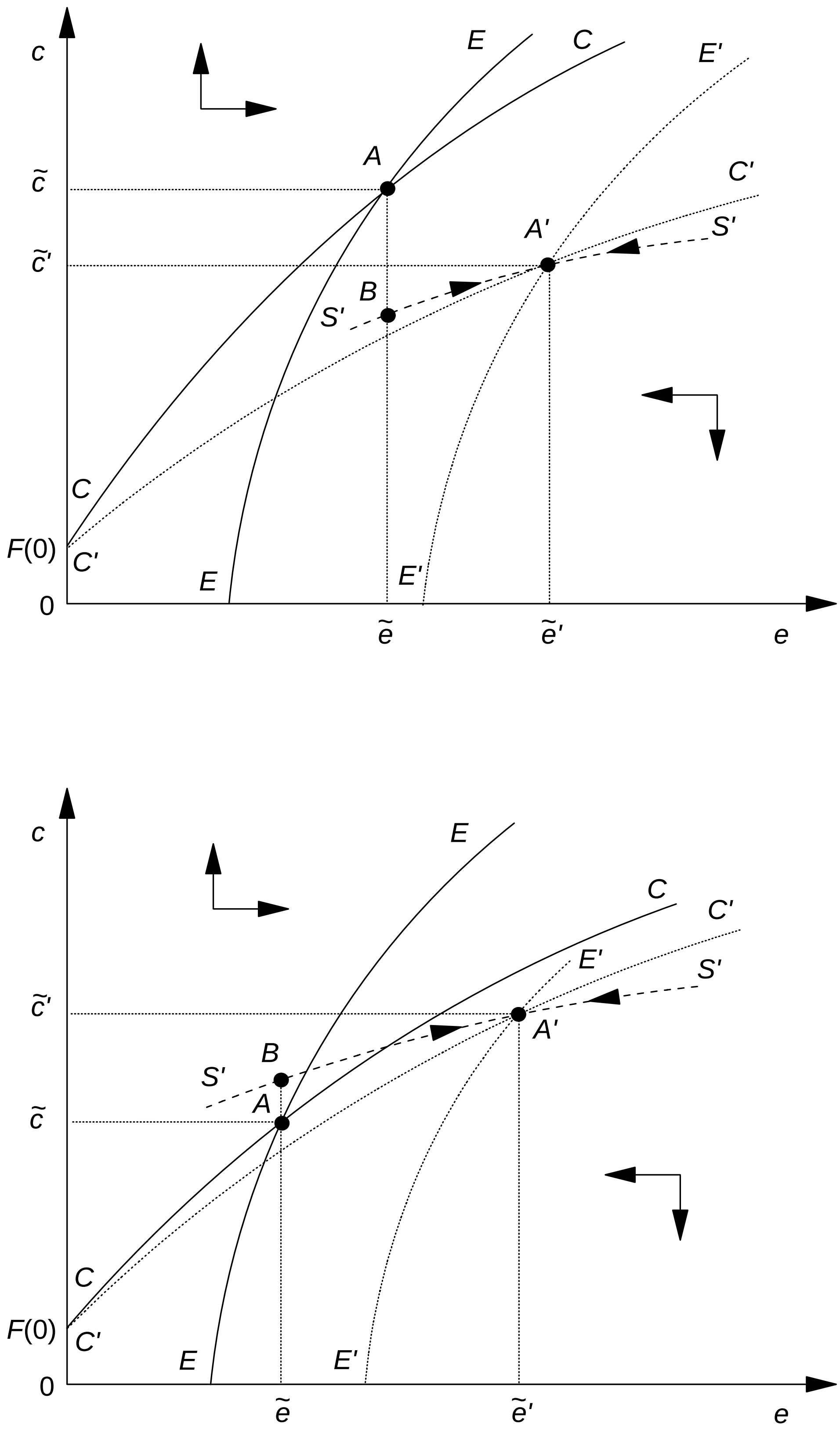


Figure 3

Shift in the Composition of Government Spending
from Education to Infrastructure