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The Cobweb, Borrowing and Financial Crises

by

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The Cobweb, Borrowing and Financial Crises

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Abstract

Studies of non-linear cobweb models have failed to address a fundamental issue: whether the complex dynamical behavior displayed by such models is consistent with the survival of producers. This paper indicates that where borrowing is unconstrained – as is implicitly assumed in standard cobweb models – borrowing results in financial crises. Incorporating constraints on borrowing is needed to salvage cobweb models and to bring them closer to reality. Industry performance – in terms both of profitability and of the incidence of bankruptcies – is shown to be highly sensitive to the nature of such credit restrictions.

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The Cobweb, Borrowing and Financial Crises

Studies of non-linear cobweb models have failed to address a fundamental issue: whether the complex dynamical behavior displayed by such models is consistent with the survival of producers. This paper indicates that where borrowing is unconstrained – as is implicitly assumed in standard cobweb models – borrowing results in financial crises. Incorporating constraints on borrowing is needed to salvage cobweb models and to bring them closer to reality. Industry performance – in terms both of profitability and of the incidence of bankruptcies – is shown to be highly sensitive to the nature of such credit restrictions.

1. Introduction

Since their introduction in the 1930s, cobweb models have played a pivotal role in developments in economic dynamics. They have traditionally been invoked to explain fluctuations in agricultural production and prices in terms of sequential production readjustments. In the standard model, firms in a competitive industry produce a single homogeneous product; there is a well-defined production period, with the producers' activities being synchronized; producers base decisions on price expectations; and the market-clearing price for the product is established instantaneously at the end of each period. With the sole link between periods being via price expectations, particular attention has been devoted to their formation. Indeed, it was in the context of cobweb models that adaptive expectations, rational expectations, expectations based on the mean of all past prices and heterogeneous expectations were first analyzed.

It is, however, curious that, in a context the essence of which is that production takes time, very little attention has been paid to how producers finance their production activities and to the possibility of their becoming bankrupt. As regards the financing of production, there typically appears to be an implicit assumption that producers in the industry concerned can borrow or lend as much as they wish at a given market rate of interest determined by the overall state of the economy. Certainly a 'perfect' financial capital market is a powerful simplification frequently invoked by economic theorists. In a cobweb model, it enables theorists to dispense with financial constraints on the behavior of producers and to concentrate on technological constraints. But it can be a very misleading simplification. Indeed, assuming that producers can borrow as much as they wish at a given rate of interest not only does not rule out the possibility of bankruptcy but makes it particularly likely. Nor is bankruptcy ruled out by assuming that producers pay for inputs at the end of the period in which they are used. To put the matter starkly, the possibility of bankruptcy is only necessarily eliminated if producers pay for inputs in advance and they rely exclusively on their own financial capital.

Section 2 sets out the assumptions of our model. Section 3 considers the case where producers can borrow or lend at a given rate of interest. Looking beyond the usual treatment of the dynamical behavior of price and quantity reveals a fundamental problem with non-linear cobweb models. However, this paper is not simply intended as a challenge to these models. In Section 4, following a brief examination of the case where firms rely exclusively on their own financial capital, we explore the implications of banks limiting what they are prepared to lend to producers on the basis of the latters' balance sheets. We examine the cases where the borrowing limits depend on the value of the durable assets available for use as collateral and where they depend on the producers' financial wealth levels. We are interested in the implications of the borrowing constraints for the profitability of the industry and for the incidence of bankruptcies.

2. Assumptions

There are N units of a homogeneous durable asset, denoted by L, that is specific to the industry and in perfectly inelastic supply (akin to Ricardian land). Since the ownership and use of one (and only one) unit of L is required for participation in the industry,[1] there are, in any period, N producers, where N is sufficiently large that each producer acts as if a price-taker for the product. Producers can acquire inputs of other factors of production but must pay for these at the outset of the well-defined production period using their own financial capital, possibly supplemented by borrowed funds. At the beginning of period t (before entering into any commitments for the ensuing period), the representative firm's total wealth is:

$$W_t = F_t + V_t \tag{1}$$

where F_t is its net financial wealth and $V_t \geq 0$ is the market value of its unit of L. The firm's output for the t^{th} period is:

$$q_t = q_f + q_{v,t} \tag{2}$$

[1] This asset could, for example, be land or a farm. With appropriate (re-)interpretations of what follows, it could be a transferable license or permit required for participating in the industry; or, more generally, an indivisible bundle of inputs.

where $q_f > 0$ is the output per period that would result from using solely its unit of L and where any extra output, $q_{v,t} \geq 0$, is achieved by the purchase of additional inputs. The specific cost function, which is invariant over time, is:

$$c\left(q_{v,t}\right) = q_{v,t}^{\alpha} \tag{3}$$

where $\alpha > 1$, so that marginal cost is increasing. The firm's *net* borrowing for period t is:

$$B_t = q_{v,t}^{\alpha} - F_t \tag{4}$$

where $B_t < 0$ implies having bank deposits on which interest is received. The rate of interest, r, on a loan for the duration of the production period is determined by the overall state of the economy and is invariant over time. The firms in the industry earn the same rate of interest r on any bank deposits, so that r constitutes the marginal opportunity cost of the use of own funds in financing the production process.[2]

Producers are motivated by the accumulation of wealth. At the beginning of period t, subject to any financial capital constraint, the representative firm maximizes its expected financial wealth at the beginning of period $(t+1)$ or – equivalently – maximizes its expected profit for period t. The representative firm's expected price for the output of period t, p_t^e, is based on adaptive expectations:

$$p_t^e = p_{t-1}^e + \gamma\left(p_{t-1} - p_{t-1}^e\right) \tag{5}$$

where $0 < \gamma \leq 1$ is the price expectations adjustment speed, with $\gamma = 1$ corresponding to naïve expectations. Expected profit for period t is then:

$$\pi_t^e = p_t^e q_t - \left(1 + r\right) q_{v,t}^{\alpha} \tag{6}$$

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[2] To assume that the interest rate charged by banks exceeds the rate paid on deposits would be more realistic but would complicate the model considerably.

This definition of expected profit allows for the opportunity cost of own funds used to finance production but does not take account of the cost of the funds tied up in ownership of *L*. Once the producer is committed to participation in the industry in the current period, the cost of ownership of *L* constitutes a sunk cost. From the perspective of the current period, expected profit, as defined here, could be interpreted as an expected quasi-rent accruing to the ownership of *L*.

Output is sold at the end of the period. For simplicity, we assume that the total expenditure, *E*, on the product of this industry is given and invariant over time, implying a unit elastic product demand curve. The market-clearing price, established instantaneously, is:

$$p_t = \frac{E}{Nq_t} \tag{7}$$

so that $0 < p_t \le \breve{p} \equiv E/\left(Nq_f\right)$. Since total revenue is invariant, the realized profit per firm is a strictly monotonically declining function of the output per firm:

$$\pi_t = p_t q_t - \left(1+r\right) q_{v,t}^{\alpha} = \breve{\pi} - \left(1+r\right) q_{v,t}^{\alpha} \tag{8}$$

where $\breve{\pi} \equiv E/N$ is the maximum profit per firm achieved when each produces the minimum output q_f. The firm's income for period *t* is:

$$y_t = rF_t + \pi_t \tag{9}$$

The firm's financial wealth at the end of the period is:

$$F_{t+1} = F_t + y_t = \left(1+r\right) F_t + \pi_t \tag{10}$$

The simplest assumption that captures the notion that the market value of a unit of *L* depends on the long-term profitability of its ownership is that it is given by the present value of the receipt in perpetuity of the mean of the representative producer's past profits:

$$V_{t+1} = \frac{1}{r\left(t+1\right)} \sum_{\tau=0}^{t} \pi_\tau \tag{11}$$

subject to $V_{t+1} \geq 0$. The representative firm's total wealth at the beginning of the next production cycle is then $W_{t+1} = F_{t+1} + V_{t+1}$.

3. Unconstrained Borrowing

Suppose initially that firms can borrow as much as they wish at the going market rate of interest. To maximize expected profit requires that marginal cost equal the expected price:

$$\left(1+r\right)\alpha q_{v,t}^{\alpha-1} = p_t^e \tag{12}$$

From (12):

$$q_t = q_f + q_{v,t} = q_f + \left(p_t^e\right)^{\frac{1}{\alpha-1}} \psi \tag{13}$$

where $\psi = \left[\left(1+r\right)\alpha\right]^{-\frac{1}{\alpha-1}}$. Using (5), (7) and (13) yields the map *f*:

$$p_t^e = f(p_{t-1}^e) = (1-\gamma)p_{t-1}^e + \frac{\gamma E}{N\left[q_f + \left(p_{t-1}^e\right)^{\frac{1}{\alpha-1}} \psi\right]} \tag{14}$$

Given an initial expected price p_0^e, the future time path of expected price is uniquely determined by (14). The time paths of q_t, p_t and π_t are determined uniquely from the time path of p_t^e; with an appropriate initial condition, the time path of V_t can be determined from that of π_t. The decomposition that results from unconstrained borrowing means that the time paths of p_t^e, q_t, p_t, π_t and V_t do *not* depend on the initial financial wealth, F_0. In contrast, the time paths for B_t, y_t, F_t, and W_t depend on F_0.

A fixed point for the map *f* corresponds to a stationary state for the industry, where the representative producer is maximizing (expected) profit on the basis of a price expectation which is being realized. This requires (i) $\overline{p} = \overline{p}^e$, (ii) $\overline{q} = q_f + \overline{p}^{\frac{1}{\alpha-1}}\psi$, and (iii) $N\,\overline{q}\,\overline{p} = E$. The stationary (expected) price satisfies:

$$q_f\left(\breve{p} - \overline{p}\right) - \overline{p}^{\frac{\alpha}{\alpha-1}}\psi = 0 \tag{15}$$

where $\overline{p} < \breve{p}$. The stationary price, $\overline{p}$, and the corresponding industry output, $N\overline{q}$, are shown in Figure 1. The stationary profit, $\overline{\pi} = \breve{\pi} - \left(1+r\right)\overline{q}_v^{\alpha} > 0$, constitutes a return to the ownership of *L*. In a thorough-going stationary state, $\overline{V} = \overline{\pi}/r$ and *pure* profit – taking account of the opportunity cost of the wealth tied up in the ownership of *L* – is zero. In a stationary state, everything is stationary except for financial wealth, bank deposits and income.

The first derivative of the map *f* evaluated at the fixed point is:

$$f'(\overline{p}) = 1 - \frac{\gamma\left(\alpha - 1 + \overline{Z}\right)}{\alpha - 1} < 1 \tag{16}$$

where

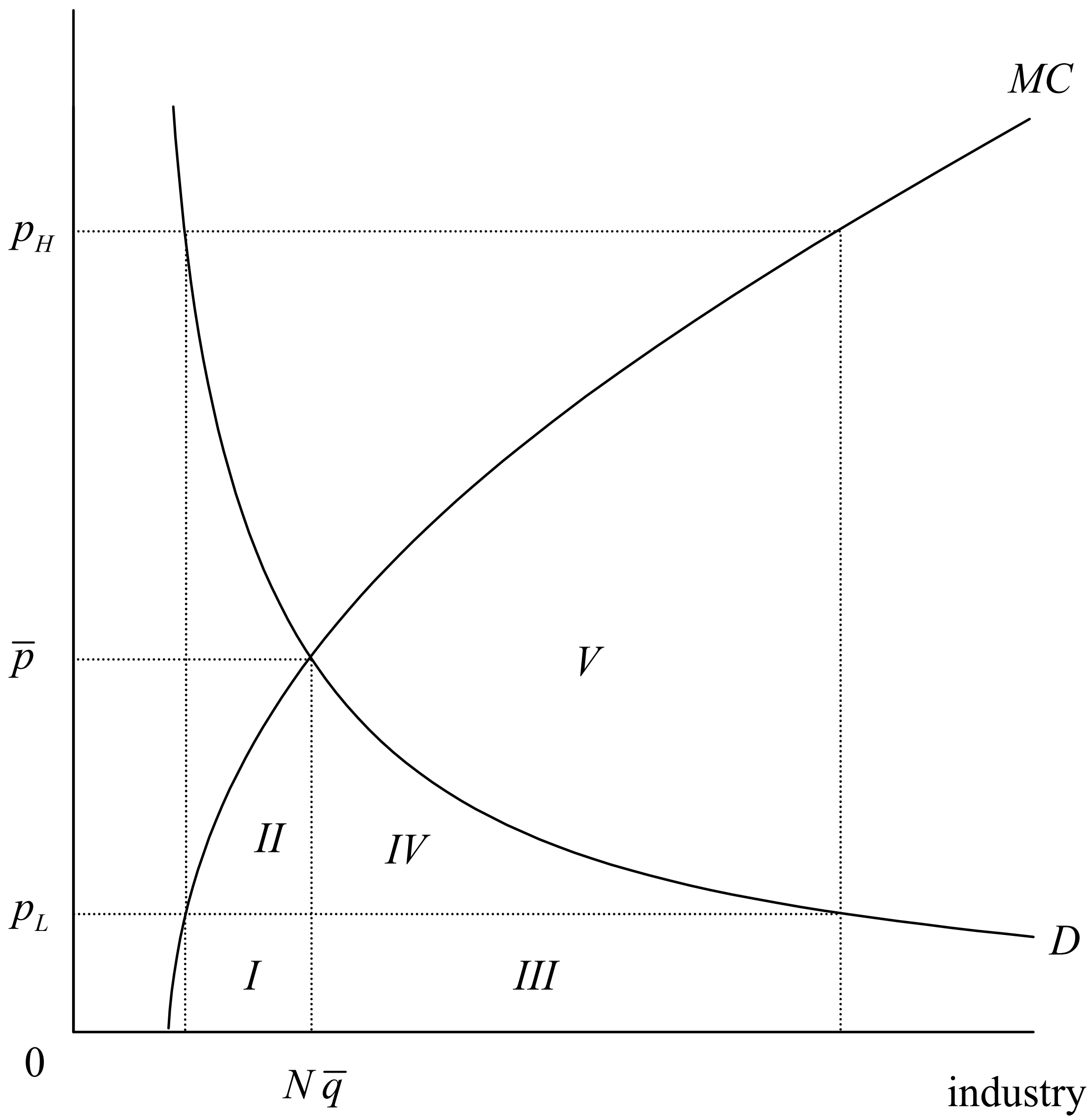
$$0 < \overline{Z} \equiv \frac{N\psi}{E}\overline{p}^{\frac{\alpha}{\alpha-1}} < 1 \tag{17}$$

The fixed point is stable if $f'(\overline{p}) > -1$, that is, if:

$$\frac{2-\gamma}{\gamma} - \frac{\overline{Z}}{\alpha - 1} > 0 \tag{18}$$

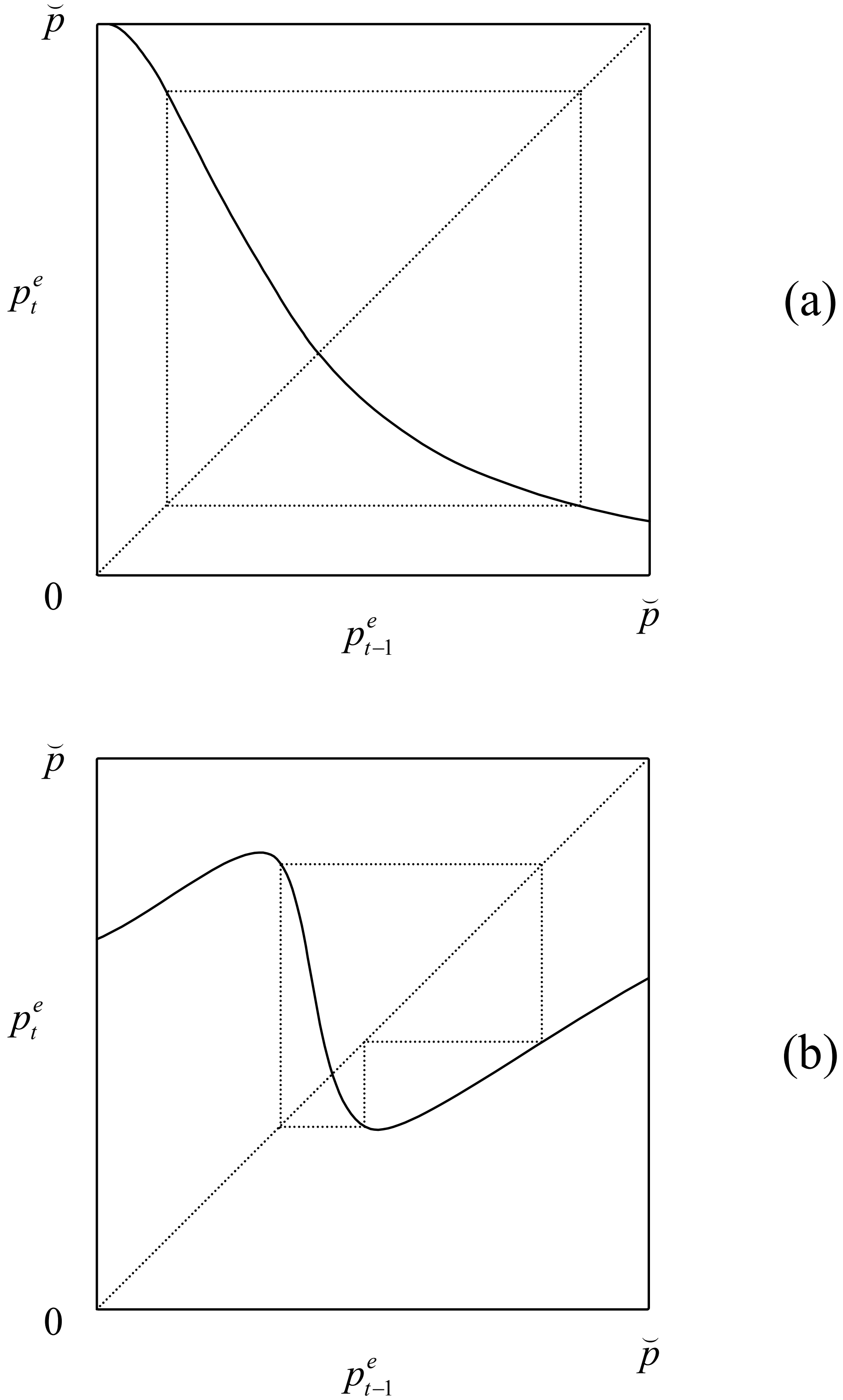
With naïve expectations, *f* is strictly monotonically decreasing: the higher is p_{t-1}^e, the higher is q_{t-1} and the lower is $p_{t-1} = p_t^e$. The system is attracted either to the fixed point (for $\alpha > 1 + \overline{Z}$) or to a period-two cycle (for $\alpha < 1 + \overline{Z}$). Figure 2(a), based on $\alpha = 1.5$ and $\gamma = 1$, shows the map *f* corresponding to Figure 1: the fixed point is repelling and the system is

Figure 1



Market demand and industry marginal cost, based on cost parameter $\alpha = 1.5$. The stationary market price and industry output are $\overline{p}$ and $N\overline{q}$. Naïve expectations give rise to a period-two cycle.

Figure 2



(a) Attracting period-two cycle for the map f for $\alpha = 1.5$ and $\gamma = 1$.

(b) Attracting period-three cycle for $\alpha = 1.075$ and $\gamma = 0.4$.

attracted to the depicted period-two cycle.[3] With adaptive price expectations, the possible long-term behaviors are considerably enriched. Figure 2(b), based on $\alpha = 1.075$ and $\gamma = 0.4$, shows the case of a period-three cycle, the hallmark of a system that exhibits complex dynamical behavior. Figure 3(a) is a bifurcation diagram, based on $\alpha = 1.1$, that shows the dependence of the long-term behavior of expected price on the expectations adjustment speed, γ. The fixed point is stable for sufficiently slow speeds, i.e., for:

$$\gamma < \gamma^{bif} \equiv 2\left(\frac{\alpha - 1}{\alpha + \bar{Z} - 1}\right) \cong 0.24 \tag{19}$$

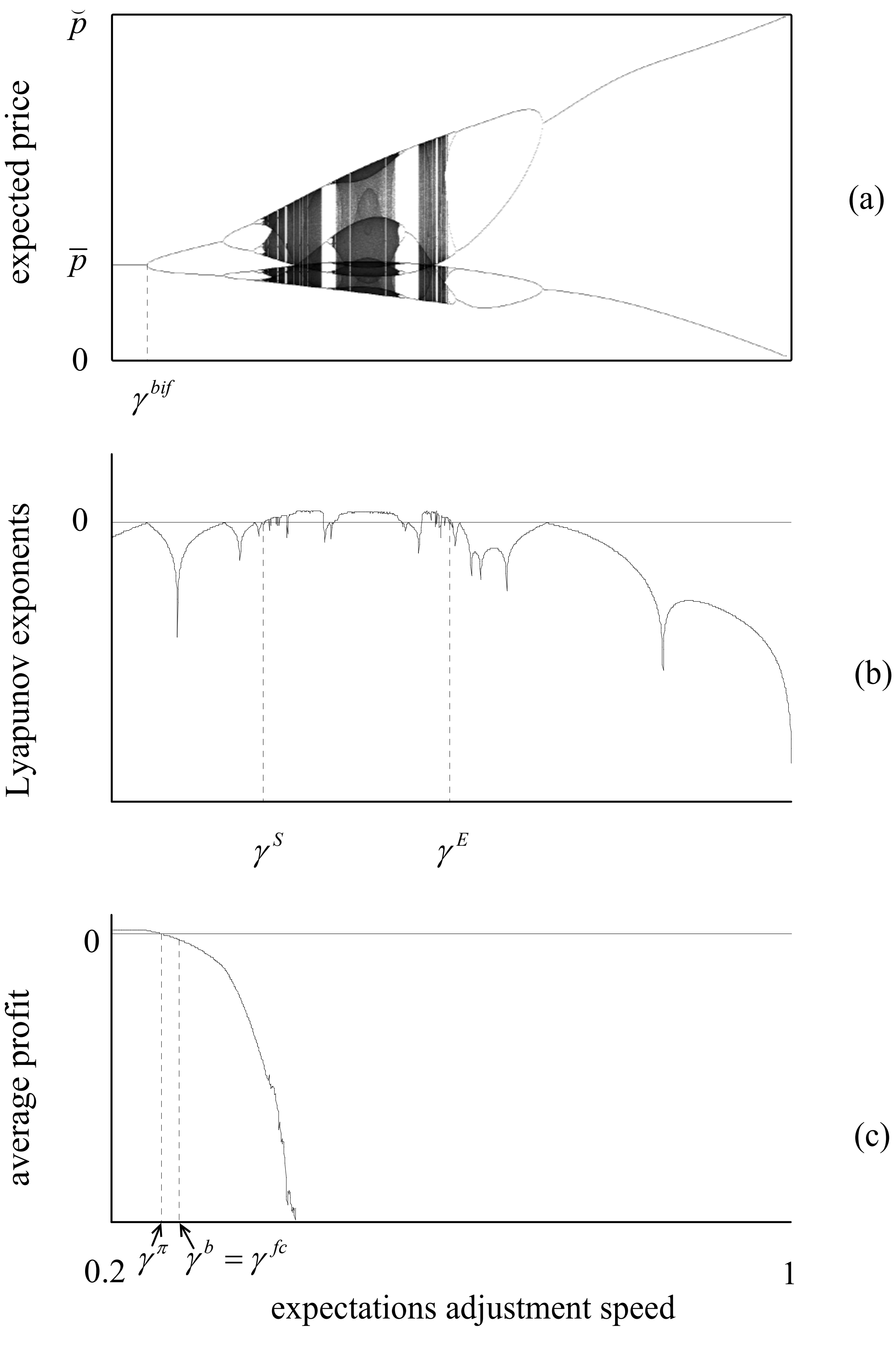
As γ increases through γ^{bif}, a sequence of period-doubling bifurcations occurs. For speeds between $\gamma^{S} \cong 0.381$ and $\gamma^{E} \cong 0.564$, intervals of chaos (a positive Lyapunov exponent in Figure 3(b)) and of order (a negative Lyapunov exponent) are intermingled. Increasing γ above γ^{E} gives rise to a sequence of period-halving (period-doubling reversed), until a stable period-two cycle is generated at $\gamma \cong 0.712$. As γ increases towards 1, the amplitude of the period-two cycle increases.

It should be stressed that our model involves normal assumptions about costs and demand and that, with the assumption of unconstrained borrowing, it constitutes a standard cobweb model. The map *f* belongs to the class of difference equations analyzed by Hommes (1994) involving adaptive expectations and non-linear but monotonic demand and supply.[4]

[3] All diagrams and all simulations assume $q_f = 1$, $r = 0.1$, $N = 1000$ and $E = 5000$. All simulations assume $p_0^e = 0.99\bar{p}$.

[4] This class of difference equations possesses two properties: 1) its is smooth, i.e., differentiable; and 2) the first derivative is less than 1. As a specific case, Hommes explores the properties of a map derived from an 'S-shaped' supply curve and a linear demand function. Hommes's map has similar properties to our map *f*, i.e, it has two critical points for an expectations adjustment speed between $0 < \gamma < 1$, and it is strictly decreasing at

Figure 3



(a) Bifurcation diagram, based on $\alpha = 1.1$, that shows the dependence of the long-term behavior of expected price on the expectations adjustment speed, for $0.2 \leq \gamma \leq 1$, with unconstrained borrowing. (b) The corresponding Lyapunov exponents. (c) The corresponding long-run average profit.

As $q_f \to 0$, the map *f* tends to a form similar to that analyzed by Onozaki *et al*. (2000), who assume naïve expectations but cautious adjustment to the output that maximizes expected profit. But what these studies ignore is whether the long-term behavior implied by the models is consistent with the long-run viability of producers. When this issue is addressed, it becomes evident that there is a fundamental problem with standard cobweb models.

For our model, we may note at once that, for the period-two cycle depicted in Figures 1 and 2(a), average profit not only would be less than the stationary profit $\bar{\pi}$ [as is easily confirmed from the fact that area *III* + *IV* + *V* exceeds area *I* + *II* in Figure 1] but also would be negative. Figure 3(c) shows the dependence of long-run average profit on the expectations adjustment speed. For $\gamma > \gamma^{bif}$, long-run average profit declines monotonically as γ increases; for $\gamma > \gamma^{\pi} \cong 0.257$, average losses are incurred and they increase rapidly as γ increases towards the case of naïve expectations. Negative long-run average profits set off alarm bells; they suggest non-viability. However, the question of long-run viability cannot be settled conclusively by examining average profits: in general, negative average profits are neither necessary nor sufficient for financial crises to occur. To determine the parameters for which financial crises occur, we need to consider explicitly the behaviors of net borrowing and of financial wealth. We define γ^{b} as the speed below which producers never have

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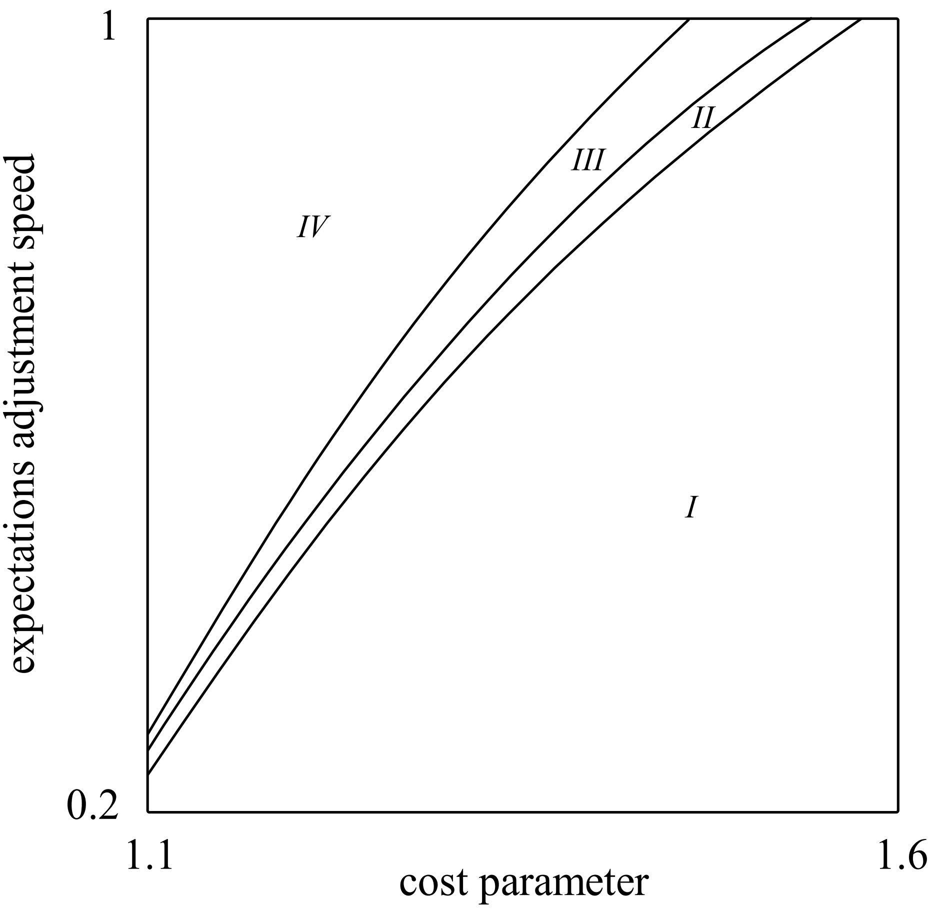
$\gamma = 1$. Moreover, the qualitative properties of the dynamics are substantially identical. A crucial difference between Hommes's analysis and our model is that Hommes, in order to focus on the mathematical properties of his model, normalizes prices by using the inflection point of his supply curve as the new origin. This type of normalization does not allow for an evaluation of profitability. The dynamic properties of unconstrained cobweb models with adaptive expectations and non-linear but monotonic demand and/or supply have been also explored by Chiarella (1988), Finkenstädt and Kuhbier (1992), Gallas and Nusse (1996) and Hommes (1991, 1998).

recourse to borrowing, i.e., they are always able to finance internally their desired input acquisitions. We define γ^{fc} as the speed below which financial crises do not occur, where, provisionally, we define a financial crisis as arising when the debt of the representative firm is increasing period after period. Since a financial crisis can only arise as a result of borrowing, $\gamma^{fc} \geq \gamma^{b}$. In fact, with unconstrained borrowing, borrowing results sooner or later in a financial crisis, i.e., $\gamma^{fc} = \gamma^{b}$. The latter value depends *inter alia* on the representative producer's initial financial wealth. Assuming that the initial financial wealth is sufficient to cover the cost of the initial expected profit maximizing output [i.e., $F_0 = \left(q_0 - q_f\right)^{\alpha}$ where $q_0 = q_f + \left(p_0^e\right)^{1/(\alpha-1)} \psi$], simulations show that $\gamma^{fc} = \gamma^{b} \cong 0.277$ for the parameters on which Figure 3 is based. Note that $\gamma^{fc} = \gamma^{b} > \gamma^{\pi}$, i.e., there is a range of speeds for which, even though average profit is negative, the representative firm does not borrow and cannot go bankrupt; the firm's production losses are being subsidized by the receipt of positive net interest. Figure 4 shows the impact on profitability and on the occurrence of financial crises of varying both the speed γ and the cost parameter α. Region *I* corresponds to parameter combinations that result in stationarity, i.e., that satisfy (18), and that give profit $\bar{\pi}$. In Region *II*, average profit is positive but below the stationary profit. In Region *III*, average profit is negative but firms do not borrow (and cannot go bankrupt). The boundary between Regions *III* and *IV* shows for any α the corresponding speed γ^{b} below which firms do not wish to borrow. In Region *IV*, firms do borrow and, sooner or later, they go bankrupt. From Figure 4, for most parameter combinations for which the model exhibits complex behavior, firms engage in borrowing and, with no constraints on that borrowing, they go bankrupt.

Before we consider the nature and implications of constraints on borrowing, some observations are appropriate. First, whereas the notion of negative average pure profits over

Figure 4



The impact on profitability and on the occurrence of financial crises of the expectations adjustment speed and the cost parameter with unconstrained borrowing. Region *I* involves stationarity. In region *II*, firms do not borrow and average profit is positive but below the stationary profit. In region *III*, firms do not borrow and average profit is negative. In region *IV*, firms borrow and go bankrupt.

the long run may be disquieting to those brought up on the standard theory of a perfectly competitive industry, it would not have been troublesome to Knight. In his classic work, *Risk, Uncertainty and Profit*, he advanced his strongly-held belief that "business as a whole suffers a loss" (1971, p. 365): he argued that entrepreneurs, motivated by the prospect of profits, actually realize negative pure profits on average and they sustain this essentially through foregoing some of the opportunity costs on those financial or physical resources which they themselves supply to their businesses. Second, the incidence of financial crises cannot simply be eliminated by a *ceteris paribus* increase in demand: increasing E (or reducing N) increases the intercept of the map f and is a destabilizing force. With cyclical or chaotic system behavior, average profit is less than the stationary profit; and fluctuations increase the likelihood of financial crises. Similarly, assuming a demand curve with a constant elasticity other than -1 does not alter our conclusions in any fundamental way. Finally, extending the model to incorporate the distribution of part of the representative firm's income to shareholders would complicate further the relationships between average profits, borrowing and the occurrence of bankruptcies. The greater the proportion of income that is distributed, the lower the critical speeds at which borrowing and financial crises occur. With such distribution, it is possible that there is a range of speeds for which firms borrow regularly without going bankrupt, i.e., $\gamma^{b} < \gamma^{fc}$ for a given α. Furthermore, it is possible that the distribution of earnings may be sufficiently high that bankruptcies occur even though long-run average profit is positive, i.e., $\gamma^{fc} < \gamma^{\pi}$ for a given α.

4. Constrained Borrowing

In analysing the impact of borrowing constraints, we denote the representative producer's financial capital fund at the beginning of period t by K_t, where this comprises both own financial wealth and the maximum that the producer could borrow. The financial capital constraint on output is:

$$q_{v,t}^{\alpha} \leq K_t \tag{20}$$

Maximizing expected profit subject to (20) requires:

$$q_t = q_f + q_{v,t} = q_f + \min\left\{\left(p_t^e\right)^{1/(\alpha-1)}\psi; K_t^{1/\alpha}\right\} \tag{21}$$

where $q_f + \left(p_t^e\right)^{1/(\alpha-1)}\psi$ is the output that would maximize expected profit in the absence of a financial constraint [see (13)] and $q_f + K_t^{1/\alpha}$ is the maximum output consistent with the financial constraint.

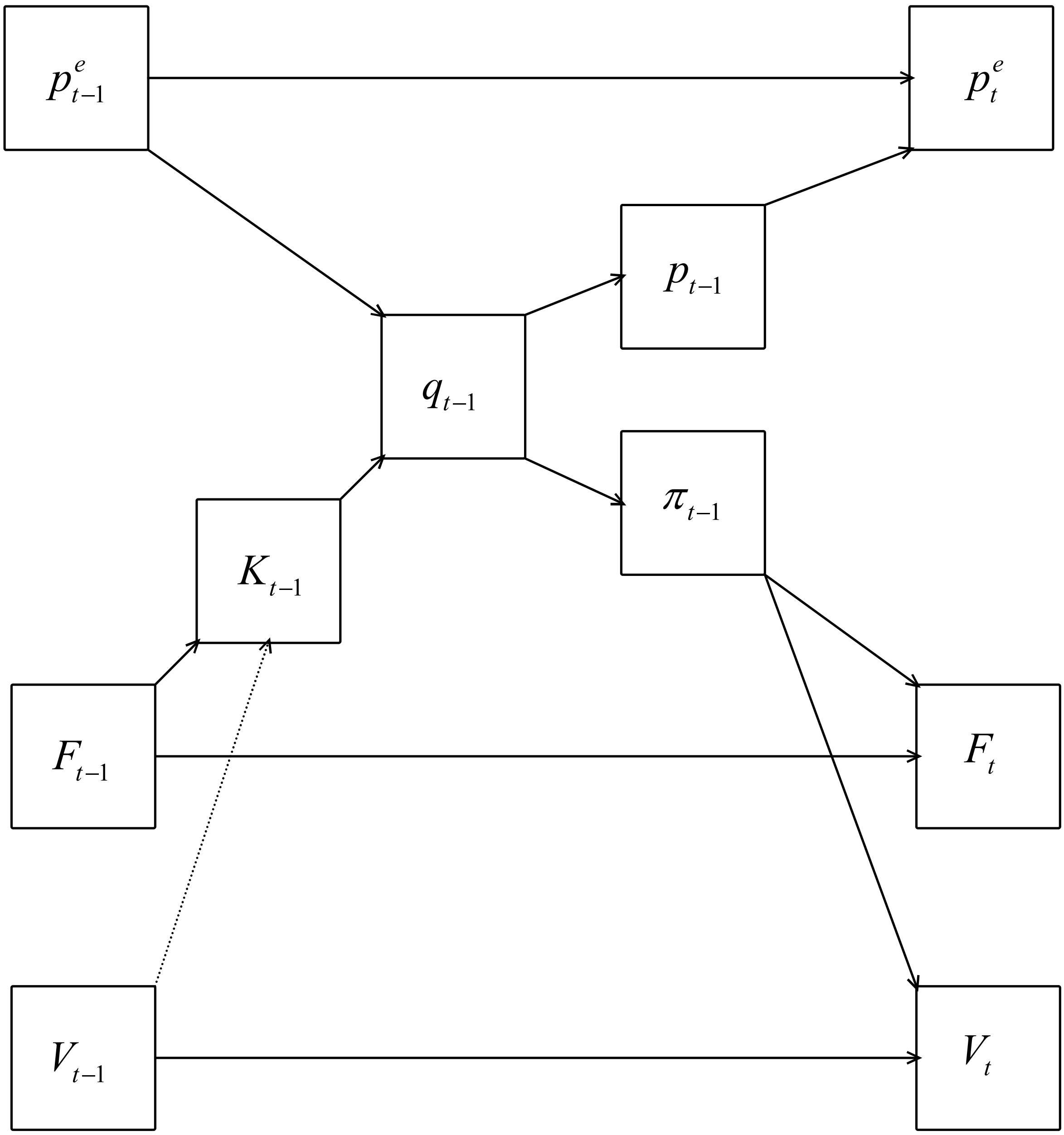
In order to accommodate the entry of a new cohort of producers to replace those that go bankrupt, it is necessary to specify precisely when firms are deemed bankrupt and what the financial position of entrants is. Our banks follow a simple rule: a firm is declared bankrupt if and only if it has a financial debt that is not diminishing. That is, the representative producer is deemed to be bankrupt at the beginning of period *t* iff:

$$F_t \leq F_{t-1} < 0 \tag{22}$$

We assume that, if bankruptcy occurs and the durable assets of bankrupt producers are sold to a new cohort of producers, the purchase exhausts the funds of the representative entrant, so that there is no financial capital left for acquiring inputs, i.e., $F_t = 0$ for a firm entering at the beginning of period *t*. The latter seems the least arbitrary assumption; and it implies at least that new firms get off to a good start, since each produces q_f and receives the maximum profit in its first period.

The dynamical system is depicted in Figure 5. The financial capital fund K_t will necessarily depend on own financial wealth F_t; K_t may or may not depend on the value of

Figure 5



Dynamical system with constrained borrowing the durable asset, V_t. Where K_t depends directly only on F_t, then p_t^e depends on p_{t-1}^e and on F_{t-1}; and F_t depends on p_{t-1}^e and on F_{t-1}. Given an initial expected price p_0^e and an initial financial wealth F_0, the future time paths of p_t^e, K_t, q_t, B_t, p_t, π_t, y_t and F_t are determined uniquely; with an appropriate initial condition, the time path of V_t can be determined from that of π_t. The decomposition that occurs with unconstrained borrowing breaks down because of the financial capital constraint (20); given that the latter is shifting over time as financial wealth changes, the system's dynamical behavior is considerably more complicated than that of the map *f* for unconstrained borrowing. Long-term behavior is only different when the constraint (20) impacts on the time path. Thus the stationary (fixed point) values $\overline{p}$, $\overline{q}$ and $\overline{\pi}$ are the same as for the map *f* for unconstrained borrowing; more generally, dynamical behavior is the same as for map *f* for parameter combinations in Regions *I*, *II* and *III* in Figure 4. The interesting (γ, α) combinations are those in Region *IV*.

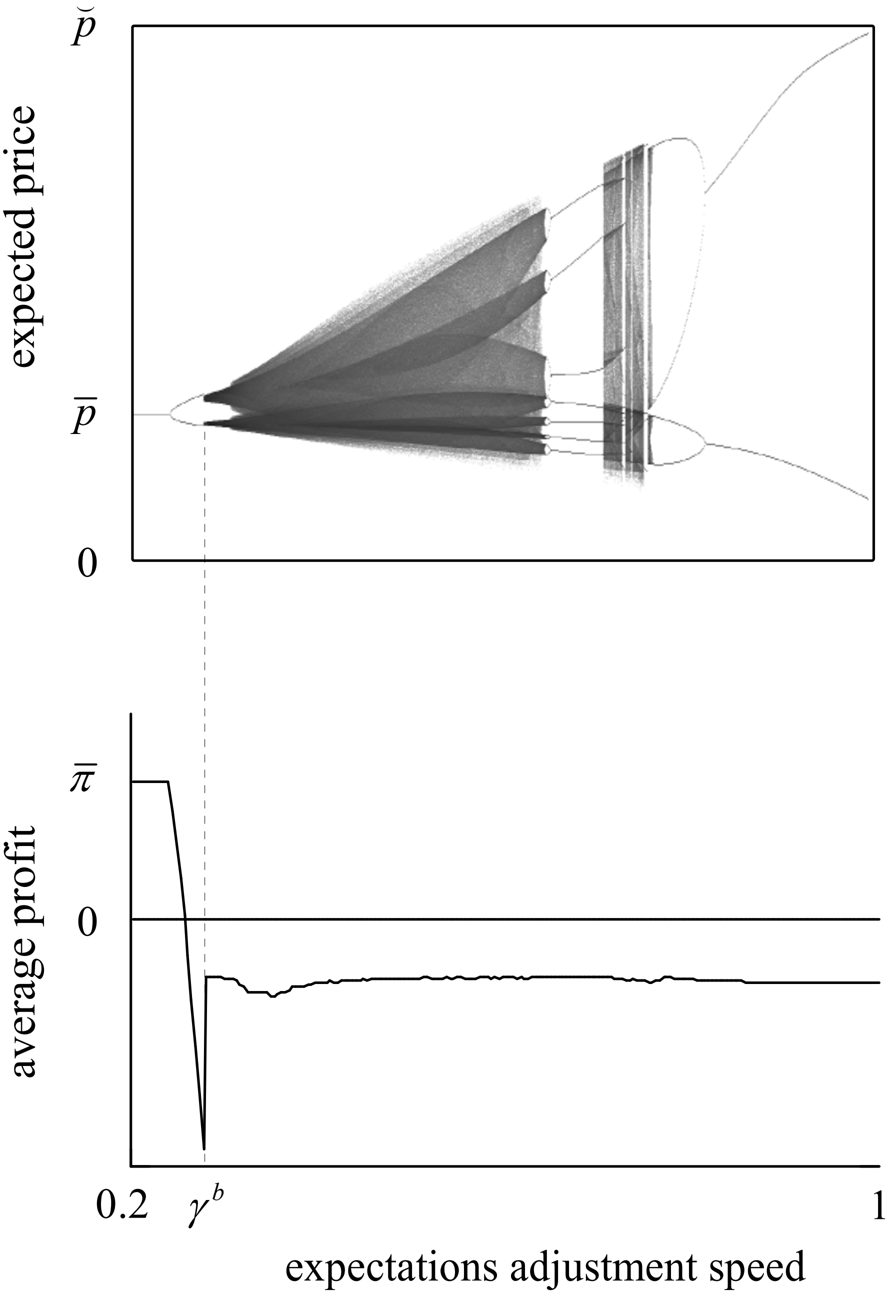
4.1 Pure Internal Finance

Suppose initially that producers must rely exclusively on their own financial capital. With pure internal finance:

$$K_t = F_t \geq 0 \tag{23}$$

Pure internal finance excludes any possibility of bankruptcy: a firm that cannot borrow never falls into debt. Figure 6, based on $\alpha = 1.1$ to permit comparisons with Figure 3, shows the dependence of the long-run behavior of expected price and of long-run average profit on the expectations adjustment speed. Comparing pure internal finance with unconstrained borrowing for $\gamma > \gamma^b$, the long-run behavior of expected price is not overtly very different. Over the chaotic region, the behavior of expected price appears rather more 'noisy' in Figure

Figure 6



Bifurcation diagram and average profit, based on $\alpha = 1.1$, for pure internal finance.

6 but the ranges of variation at any speed are similar. However, the crucial difference is not evident from the bifurcation diagrams. Whereas recurrent financial crises are inevitable with unconstrained borrowing for $\gamma > \gamma^b$, bankruptcies cannot occur with pure internal finance, notwithstanding the negative average profits. For example, for naïve expectations, the period-two cycle with unconstrained borrowing would effectively imply a firm lifetime of just two periods;[5] in contrast, the period-two cycle with pure internal finance is consistent with the continuing survival of firms. For *all* (γ, α) combinations in Region *IV* in Figure 4, pure internal finance involves survival with negative average profits.

4.2 Credit Rationing

Typically, firms are able to borrow but their ability to do so is constrained. Banks, facing the risk that a borrower may fail to repay the interest and the principal, ration credit. Lending to producers in a wide variety of industries and facing asymmetric information, our banks follow behavioral rules that discriminate between prospective borrowers according to their balance sheets.[6]

A natural case to consider first is where the producers' durable input *L* provides collateral for loans. Specifically, suppose that a bank is prepared to lend a producer up to a limit of $V_t/(1+r)$; provided that the value of *L* does not fall, the proceeds from its sale

[5] An entrant reliant on borrowed funds would be in debt at the end of its first period of operation; it would have an even higher debt at the end of its second period, despite making the maximum profit in that period; according to (22), it would be declared bankrupt at the end of its second period.

[6] Bernanke and Gertler (1989), in their macro-analysis of business cycles, examine the significance of the creditworthiness of borrowers being dependent on their net worth. They postulate an inverse relationship between a borrower's net worth and agency costs.

would cover both the principal and the interest, protecting the bank against default.[7] However, in our model, this credit constraint results in the same long-run dynamical industry behavior as for pure internal finance. The explanation is that, for those parameter combinations for which the firms' own financial capital is insufficient to finance desired input acquisition [i.e., for Region *IV* in Figure 4], long-run average profits are negative; according to (11), the durable asset is effectively worthless [i.e., $V_t \cong 0$] and it cannot be used as collateral for a loan.[8]

A more interesting possibility is that banks discriminate between producers according to their financial wealth levels. This would be equivalent to basing the limit on the own capital that producers risk in production – for example, where banks are prepared to 'match' the own funds invested by borrowers. Following Day (1967, 1994) and Day *et al*. (1974), suppose that a sufficient number of banks are willing to lend up to a multiple θ of the producers' own financial capital, where $\theta > 0$ reflects the degree of cautiousness of the banking community.[9] The representative producer's financial capital fund, including borrowed funds, is then:

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[7] In their analysis of credit cycles, Kiyotaki and Moore (1997, p. 218) invoke a similar borrowing constraint. In their rational expectations model, agents have perfect foresight of future durable asset prices.

[8] A different specification of the determination of the value of L could modify this conclusion. For example, at the opposite extreme from (11) would be where, subject to a non-negativity constraint, the market value of L is based solely on the naïve expectation that it will yield in perpetuity the profit of the previous period. In this case, the collateral constraint would have an impact on dynamical behavior: positive profits in one period would permit borrowing in the next period.

[9] Fixing limits to loans is a crucial component of banks' portfolio management. Cohen and Hammer (1972) and Walker (1997) present recursive programming models in which banks follow asset management rules which involve constraints on bank loans based *inter alia* on the availability of financial resources.

$$K_t = \begin{cases} (1+\theta)F_t & for \quad F_t \geq 0 \\ 0 & for \quad F_t < 0 \end{cases} \tag{24}$$

It is worth noting that, since a firm in debt cannot borrow, the simple bankruptcy rule (22) – plausible for an individual bank which lacks information about the industry – turns out to be a sensible one for the banking community as a whole. To see this, suppose that, at the outset of period $(t-1)$, the representative firm was in financial debt. Unable to borrow funds for acquiring inputs, it produced q_f. With each firm supplying q_f to the market, each received the maximum profit $\breve{\pi}$. A failure to make any positive contribution to paying off its debt, i.e., $F_t \leq F_{t-1} < 0$, is equivalent to $\breve{\pi} \leq r|F_{t-1}|$. Since the receipt of $\breve{\pi}$ in the previous period made no contribution to paying off the representative firm's debt, its financial position is irretrievable: if it continued in production, its debt would inexorably deteriorate period after period if $F_t < F_{t-1} < 0$ and would remain the same in the (fluke) case in which $F_t = F_{t-1} < 0$.[10] Thus, by deeming firms to be bankrupt if they have made no contribution to paying off their debts, banks are rationally cutting their losses. In contrast, if firms did make some contribution over the previous period to paying off their debts, i.e., $\breve{\pi} > r|F_{t-1}|$, it would

[10] If (22) were amended so that a firm is deemed bankrupt at date *t* iff its debt has *strictly* deteriorated over the previous period, it would be possible to have an idiosyncratic fixed point in which the representative firm has a debt that remains the same period after period: each firm would produce q_f; price would be at its maximum $\breve{p}$; each firm would receive the maximum profit $\breve{\pi}$; and the latter would precisely cover the interest on the firm's debt.

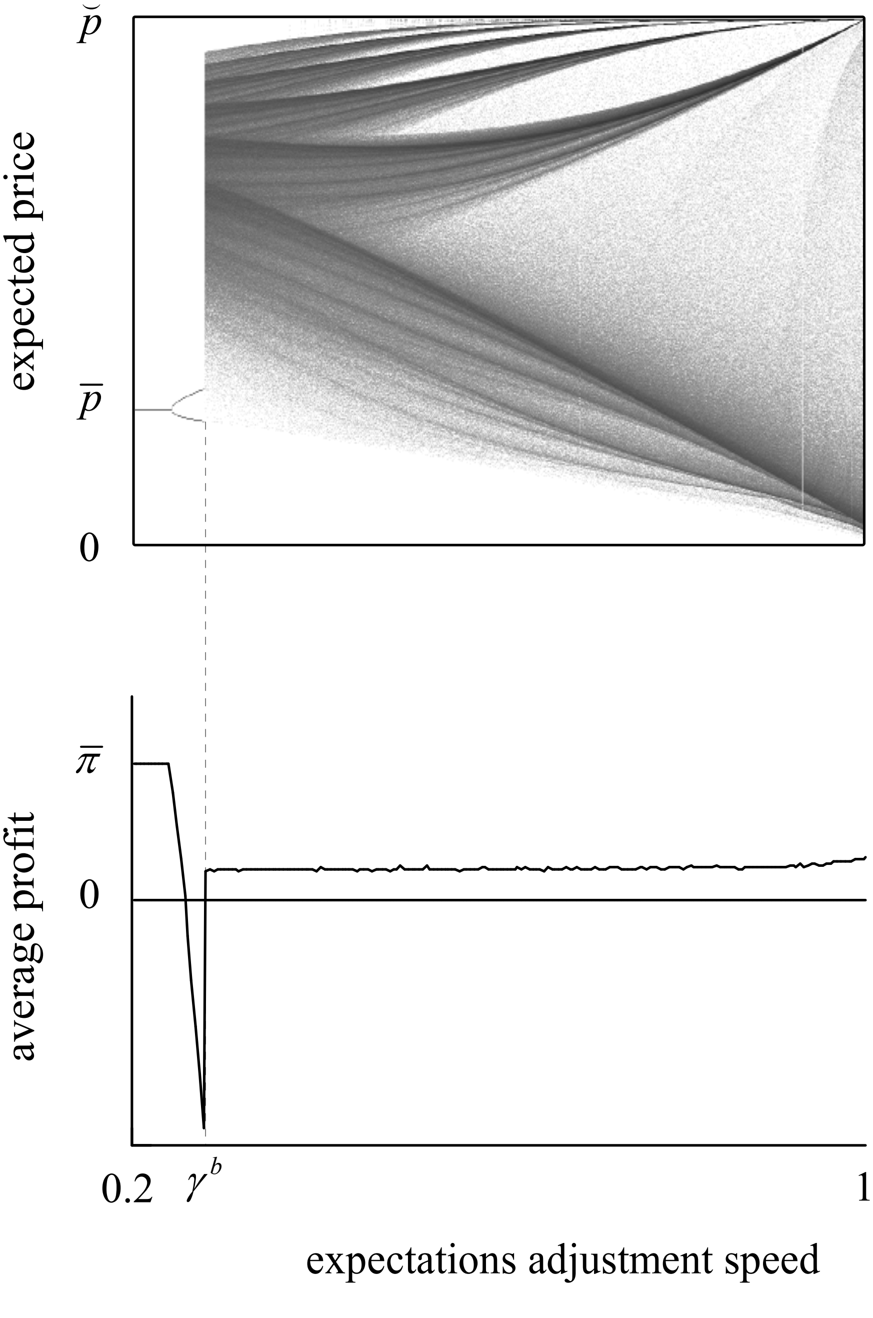
not pay banks to deem them to be bankrupt. Firms would continue to reduce the debts period by period until they are cleared.[11]

Compared with pure internal finance (which would correspond to $\theta = 0$), the ability to borrow results in much greater system volatility for $\gamma > \gamma^b$. Figure 7, based on $\alpha = 1.1$, shows the dependence of the long-run behavior of expected price and of long-run average profit on the expectations adjustment speed for $\theta = 4$. A crude story would be as follows. Starting with positive financial wealth, a low output in period *t* results both in a high price and in a high profit. In turn, the high price results in a high expected price; and the high profit enhances the producers' ability to borrow. The resulting high output in period $(t+1)$ gives rise to a loss.[12] If this loss results in the producer being in debt, output in the next period is at its minimum level, with price and profit at their highest levels. This continues until the debt is cleared. That the ability to borrow results in sustained periods of debt and of low outputs lies behind another striking feature of Figure 7, namely, that, for $\theta = 4$, long-run average profits are positive. This particular increase in average profits (compared to pure internal finance and *a fortiori* to the case of unlimited borrowing) is acquired without much

[11] Note that, since $\bar{\pi}/r$ is the maximum conceivable value for *L*, our bankruptcy condition implies that a bankrupt firm's total wealth cannot be positive. In other words, our bankruptcy condition is equivalent to postulating that the representative firm is deemed bankrupt if its (hypothetically) receiving the discounted present value of the future infinite stream of the maximum possible profits would not leave it with a positive total wealth.

[12] For their model, Bernanke and Gertler (1989, p. 27) claim: "In good times, when profits are high and balance sheets are healthy, it is easier for firms to obtain outside funds. This stimulates investments and propagates the good times. Conversely, poor financial health in bad times reduces investment and reinforces the decline of output". In contrast, in our model, with a unit-elastic product market demand curve, the immediate effect of greater access to outside funds is a fall in profits.

Figure 7



Bifurcation diagram and average profit, based on $\alpha = 1.1$, for constrained borrowing with credit rationing parameter $\theta = 4$.

risk to banks, since, for $\theta = 4$ and for the assumed cost structure [i.e., for $\alpha = 1.1$], financial crises occur only for a very few isolated speeds.

Figure 8 shows for different values of the credit rationing parameter θ the impact of γ and of α on long-run profitability, borrowing and bankruptcy.[13] The interpretations of the colors are shown in Table 1. Regions *I*, *II* and *III* in Figure 4 necessarily appear in Figure 8 as white, light blue and dark blue, respectively. The ability to borrow only impacts on Region *IV* (which would be red with unconstrained borrowing and dark blue for pure internal finance). In the case where banks are just prepared to match the producer's own funds, i.e., $\theta = 1$, firms take advantage of the opportunity to borrow, long-run average profits are still negative but bankruptcies do not occur. For $\theta = 4$ and $\theta = 7$, the beneficial impact of constrained borrowing is manifested in the light green areas, indicating positive average profits; the hazards are reflected in the incidence of bankruptcies signified by the purple and red areas. For $\theta = 10$, the incidences of bankruptcy are more common, though the yellow areas signify that, for some (γ, α) combinations, firms not only survive but also earn average profits above the stationary profit.[14] For more lax credit limits, borrowing almost invariably results in bankruptcy. Figure 9 focuses on the case of naïve expectations; it shows

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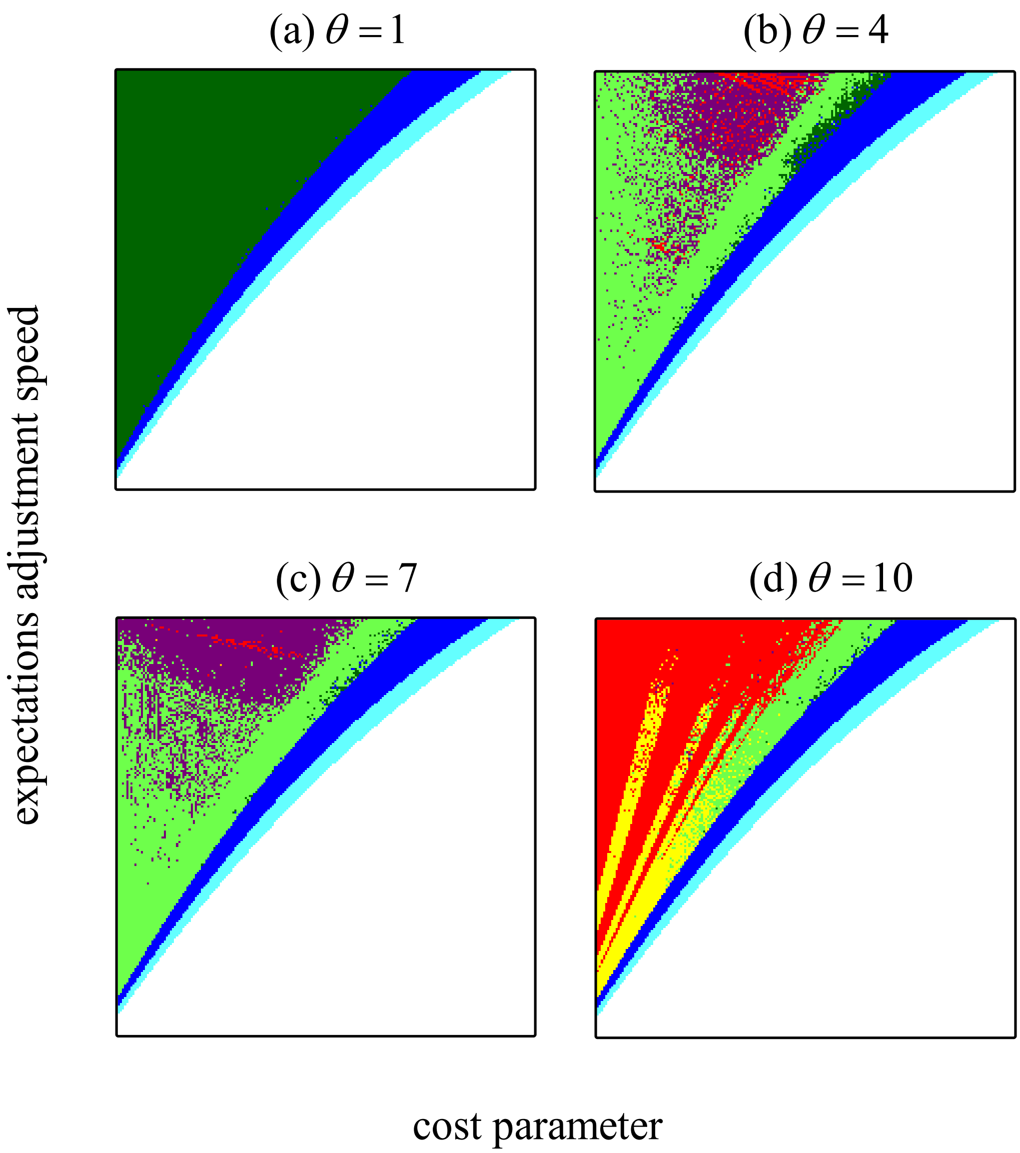
[13] For each parameter combination, we ran the simulations for 2000 periods. Since we are interested in *long-term* behavior, we disregarded bankruptcies that occurred in the first 500 periods and only calculated average profits over the range $501 \le t \le 2000$.

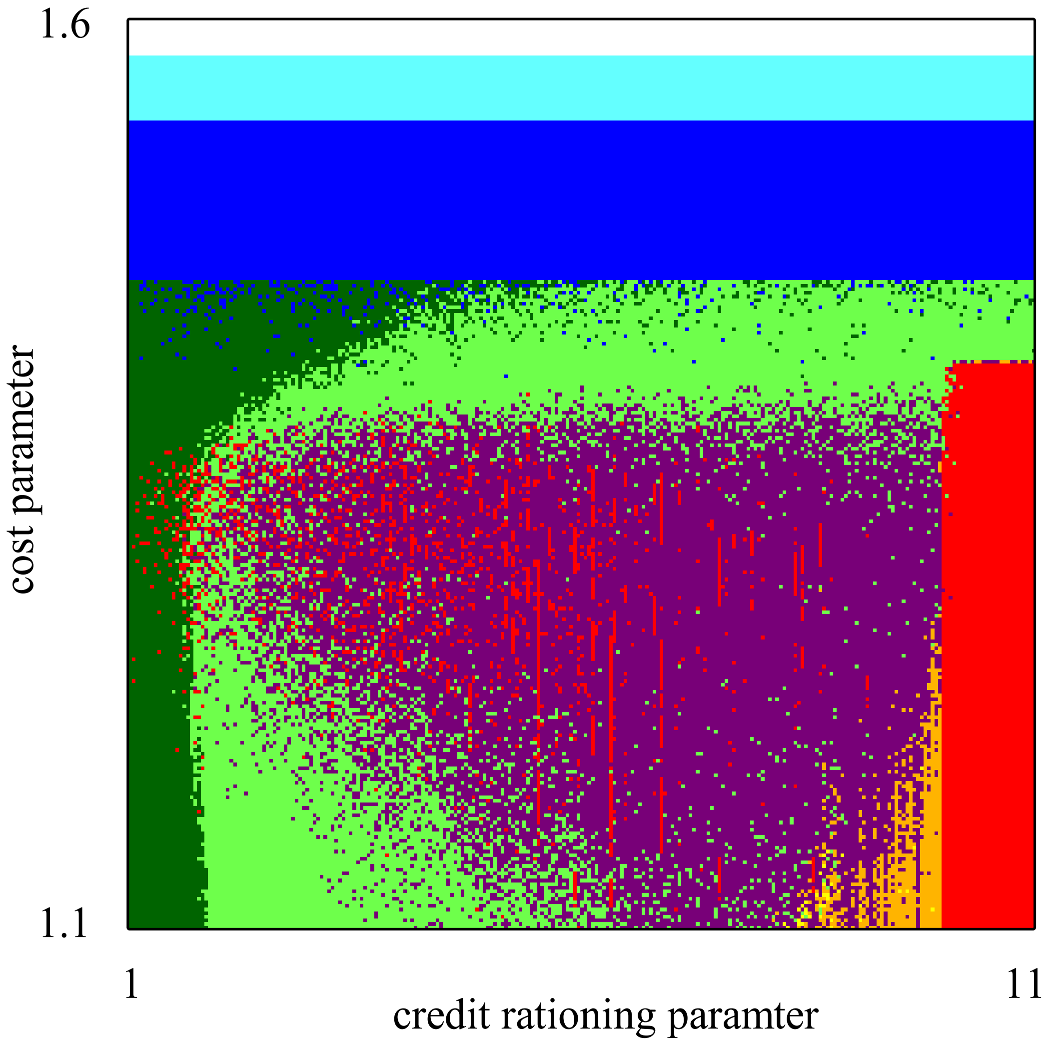
[14] This is consistent with Huang (1995), who shows that, under certain circumstances, 'cautious' responses by firms to fluctuating prices may result in long-run average profit above the stationary profit. Such responses involve upper bounds on the growth rates of output, which Huang suggests might be attributable to "capacity constraints, financial constraints and cautious response to price uncertainty by firms" (p. 261).

Figure 8



The impact of the expectations adjustment speed, $0.2 \leq \gamma \leq 1$, and of the cost parameter, $1.1 \leq \alpha \leq 1.6$, on long-run profitability, borrowing and bankruptcy for different values of the credit rationing parameter θ. White signifies stationarity; light blue signifies no borrowing and a positive average profit below the stationary profit; dark blue signifies no borrowing and a negative average profit; yellow signifies borrowing (but no bankruptcies) with an average profit above the stationary profit; light green signifies borrowing (but no bankruptcies) with a positive average profit below the stationary profit; dark green signifies borrowing (but no bankruptcies) with a negative average profit; orange signifies an average profit above the stationary profit but with bankruptcies; purple signifies a positive average profit below the stationary profit but with bankruptcies; red signifies a negative average profit with bankruptcies.

Figure 9



The impact of the cost parameter, $1.1 \le \alpha \le 1.6$, and of the credit rationing parameter, $1 \le \theta \le 11$, on long-run profitability, borrowing and bankruptcy for the case of naïve expectations.

the complex impact of (θ, α) combinations on profitability and on the incidence of financial crises.

We may conclude, broadly, that increases in the credit rationing parameter θ can increase average profitability but at a cost of a greater risk of financial crises. Where bankruptcies are avoided, the increased average profits are accompanied by increased variability of profits and possibly with the representative producer being frequently in debt. Above all, however, Figures 8 and 9 confirm the sensitivity of behavior to credit restrictions and they caution against overly-simple comparative dynamic propositions, particularly claims about which credit parameters would be 'preferable'.[15]

5. Some Concluding Comments

In reality, producers are constrained in their ability to borrow. In reality, producers go bankrupt. Our borrowing constraints and our bankruptcy condition presuppose that the banking community follows very simple behavioral rules. This paper is only a first step towards analyzing the dynamical implications of constraints on borrowing for the behavior of a competitive industry. The model could be extended by allowing credit limits to depend on the history of repayment defaults in this industry; by assuming that the rate of interest depends on the amount borrowed; or by introducing heterogeneity in the financial wealth levels of producers. Such amendments would surely reinforce our central conclusion:

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[15] The borrowing limit could depend on the producer's total wealth, $W_t = F_t + V_t$. To the extent that the constrained borrowing results in a positive average profit, so that $V_t > 0$, the dynamical behaviour would differ from the case where the limit depends purely on financial wealth. The difference would be manifested at higher credit rationing parameters in higher incidences of bankruptcy than where the borrowing limit depends only on financial wealth.

industry performance – in terms both of profitability and of the incidence of bankruptcies – is highly sensitive to the nature and degree of credit restrictions.

However simple the behavioral rules of our banks, they are certainly more plausible than the assumption – implicit in standard cobweb models – that banks will lend any amount to a firm, even to one that is falling further and further into debt. The conclusion from our model – which involves standard assumptions about costs and demand – is stark: unconstrained borrowing results in bankruptcies. This suggests that those analyzing non-linear cobweb models need, at the very least, to address the issue of whether the complex dynamical behavior displayed by such models is consistent with the survival of producers.

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	Average Profit ($av.$)	**Borrowing**	**Bankruptcy**
white	$av. = \overline{\pi}$	✗	✗
light blue	$0 \leq av. < \overline{\pi}$	✗	✗
dark blue	$av. < 0$	✗	✗
yellow	$av. \geq \overline{\pi}$	✓	✗
light green	$0 \leq av. < \overline{\pi}$	✓	✗
dark green	$av. < 0$	✓	✗
orange	$av. \geq \overline{\pi}$	✓	✓
purple	$0 \leq av. < \overline{\pi}$	✓	✓
red	$av. < 0$	✓	✓

Table 1