On the Limit Distribution of the F-Test Statistic for Fixed Effects

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October 16, 2004

Abstract

Virtually all standard econometric texts quote the F-test statistic for fixed effects, but offer no comment on the importance of the normality assumption when T (the number of time periods) is fixed, or small relative to N (the number of cross-sections). Wooldridge (2002) is an exception who notes that the asymptotic distribution (large N, fixed T) is unknown under non-normality. The result in this paper fills the gap in the literature by deriving the limit distribution of the appropriately normalised F-test statistic in the context of static panel data, under non-normality of the errors, when $N \rightarrow \infty$ and T fixed. Three results emerge: (i) the F test statistic asymptotically equivalent to the random effects test statistic; (ii) the nature of the limit distribution ensures that the commonly used F distribution still provides asymptotically valid inferences; and, (iii) the asymptotic theory appears to provide a good guide to sampling behaviour even in quite small samples.

1   Introduction

There is a huge statistics and econometrics literature on analysis of variance testing. However, it appears that the details of the asymptotic properties of the commonly used F-test for fixed effects remains incomplete. Virtually all standard econometric texts quote this test, but offer little or no comment on the finite sample importance of the normality assumption when T (the number of time periods) is fixed, or small relative to N (the number of cross-sections). However, Wooldridge (2002, p.274) does remark that the asymptotic distribution (large N, fixed T) is unknown under non-normality.

This paper derives the limit distribution of the appropriately normalised F-test statistic for fixed effects in the context of static panel data, under non-

1

normality of the errors, when $N \to \infty$ and T fixed. Three results emerge: (i) the F-test statistic is asymptotically equivalent to the random effects test statistic; (ii) the nature of the limit distribution ensures that the standard F-test procedure will still deliver asymptotically valid inferences; and, (iii) Monte Carlo evidence suggests that the procedure performs remarkably well even in quite small samples under non-normality.

The plan of this paper is as follows. Section 2 introduces the notation and the F-test statistic for fixed effects in static panel data models. Section 3 analyses the asymptotic behaviour of this test statistic. Section 4 illustrates the main findings by reporting the results of a small Monte Carlo study and Section 5 concludes.

2 The Notation, Model and Test Statistic

2.1 Notation and Model

Consider the following fixed effect model

$$y_{it} = \alpha_i + \mathbf{x}'_{it}\boldsymbol{\beta} + u_{it}, \ \ i = 1, ..., N, \quad t = 1, ..., T \tag{1}$$

where, for the purposes of the current analysis, $\alpha_N \equiv 0$, the innovations, u_{it}, are independently and identically distributed with zero mean and finite constant variance, $0 < \sigma^2 < \infty$, and $\mathbf{x}_{it} = (1, \mathbf{x}^{*\prime}_{it})'$ is a $((K+1) \times 1)$ vector of regressors. By stacking (1) for all t (for each i), we obtain

$$\mathbf{y}_i = \boldsymbol{\alpha}_i + \mathbf{X}_i\boldsymbol{\beta} + \mathbf{u}_i, \quad i = 1, ..., N \tag{2}$$

where $\mathbf{y}_i = (y_{i1}, ..., y_{iT})'$, $\mathbf{u}_i = (u_{i1}, ..., u_{iT})'$, $\boldsymbol{\alpha}_i = \alpha_i \boldsymbol{\iota}_T$, with $\boldsymbol{\iota}_T$ a $(T \times 1)$ vector of ones, and $\mathbf{X}_i = (\mathbf{x}_{i1}, ..., \mathbf{x}_{iT})'$, a $(T \times K')$ matrix with $K' = K + 1$. Stacking again, the model for all individuals becomes

$$\mathbf{y} = \mathbf{D}\boldsymbol{\alpha} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u}, \tag{3}$$

where $\mathbf{y} = (\mathbf{y}'_1, ..., \mathbf{y}'_N)'$ is a $(NT \times 1)$ vector, $\boldsymbol{\alpha} = (\alpha_1, ..., \alpha_{N-1})'$ is a $((N-1) \times 1)$ vector, $\mathbf{D} = \begin{bmatrix} \mathbf{I}_{N-1} \otimes \boldsymbol{\iota}_T \\ \mathbf{0} \end{bmatrix}$ is a $(NT \times (N-1))$ matrix, $\mathbf{X} = (\mathbf{X}'_1, ..., \mathbf{X}'_N)'$ is a $(NT \times K')$ matrix, and $\mathbf{u} = (\mathbf{u}'_1, ..., \mathbf{u}'_N)'$ is a $(NT \times 1)$ vector.

In general, define the projection matrices, $\mathbf{P_Z} = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$ and $\mathbf{M_Z} = \mathbf{I}_{NT} - \mathbf{P_Z}$, for any $(NT \times S)$ matrix $\mathbf{Z}$ of full column rank, with $\tilde{\mathbf{Z}} = \mathbf{M_D}\mathbf{Z}$ being the residual matrix from a multivariate least squares regression of $\mathbf{Z}$ on $\mathbf{D}$ which is, of course, the within transformation except for the Nth set of T cross sectional observations. For example, conformably with $\mathbf{X}$, $\tilde{\mathbf{X}} = (\tilde{\mathbf{X}}'_1, ..., \tilde{\mathbf{X}}'_{N-1}, \mathbf{X}'_N)'$, where $\tilde{\mathbf{X}}_i$ has rows $(\mathbf{x}_{it} - \bar{\mathbf{x}}_i)'$, $i = 1, ..., N-1$,

and $\bar{\mathbf{x}}_i = T^{-1} \sum_{t=1}^{T} \mathbf{x}_{it}$ and similarly for $\tilde{\mathbf{y}}$. [1] Then the fixed-effect (least squares dummy variable) estimator of $\boldsymbol{\beta}$ in (3) is given by

$$
\begin{aligned}
\tilde{\boldsymbol{\beta}} &= (\mathbf{X}'\mathbf{M_D}\mathbf{X})^{-1}\mathbf{X}'\mathbf{M_D}\mathbf{y} \\
&= (\tilde{\mathbf{X}}'\tilde{\mathbf{X}})^{-1}\tilde{\mathbf{X}}'\tilde{\mathbf{y}}
\end{aligned}
\tag{4}
$$

and the corresponding estimator of $\boldsymbol{\alpha}$ is

$$
\tilde{\boldsymbol{\alpha}} = (\mathbf{D}'\mathbf{M_X}\mathbf{D})^{-1}\mathbf{D}'\mathbf{M_X}\mathbf{y}.
\tag{5}
$$

The null model of $H_0 : \boldsymbol{\alpha} = \mathbf{0}$, is the pooled regression model of

$$
\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}.
\tag{6}
$$

2.2 The F-statistic and Assumptions

Naturally, and as described by econometric text books, the joint test for $\boldsymbol{\alpha} = \mathbf{0}$ will be based on the statistic

$$
F_N = \frac{(RSS_R - RSS_U)/(N-1)}{RSS_U/(N(T-1)-K)},
\tag{7}
$$

where RSS_R is the restricted sum of squares (from the pooled regression (6)) and RSS_U is the unrestricted sum of squares (from the fixed effects regression (3)). This can also be expressed as

$$
F_N = \frac{\sqrt{N}\tilde{\boldsymbol{\alpha}}' \left(N^{-1}\mathbf{D}'\mathbf{M}_X\mathbf{D} \right) \sqrt{N}\tilde{\boldsymbol{\alpha}}}{(N-1)\tilde{\sigma}^2}
$$

where $\tilde{\sigma}^2 = RSS_U/(N(T-1)-K)$ is consistent for σ^2.

In the absence of normality, the asymptotically validity of the F-test procedure would usually rest on $W_N = (N-1)F_N$ having a limiting χ^2 distribution. However, this can not obtain with T fixed and $N \to \infty$, since neither the dimension of $\tilde{\boldsymbol{\alpha}}$ nor $N^{-1}\mathbf{D}'\mathbf{M}_X\mathbf{D}$ is bounded. Indeed, as Wooldridge (2002, p274) remarks:

> "Under the classical linear model assumptions (which require ... normality of the u_{it}), we can test the equality of the [fixed effects] using a standard F test for T of any size. ... Unfortunately, the properties of this test as $N \to \infty$ with T fixed are unknown without the normality assumption.".

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[1]To see this, note that

$$
\mathbf{P}_D = \begin{pmatrix} \mathbf{I}_{N-1} \otimes T^{-1}\boldsymbol{\iota}_T\boldsymbol{\iota}_T' & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{pmatrix}
$$

In the subsequent analysis this is addressed by developing the asymptotic analysis with $N \to \infty$ and T fixed, since it is for this case that the limit distribution of the F-test statistic has previously been unknown. To proceed, the following high-level assumptions are made:

Assumption 1:

$\{\mathbf{X}_i, \mathbf{u}_i\}$ is an independent sequence with $E(\mathbf{u}_i | \mathbf{X}_i) = \mathbf{0}$.

Assumption 2:

(i) $\mathbf{X}'\mathbf{X}/N - \mathbf{Q}_N = o_p(1)$, where $\mathbf{Q}_N = E(\mathbf{X}'\mathbf{X}/N)$ is $O(1)$ and uniformly positive definite;

(ii) $\tilde{\mathbf{X}}'\tilde{\mathbf{X}}/N - \tilde{\mathbf{Q}}_N = o_p(1)$, where $\tilde{\mathbf{Q}}_N = E(\tilde{\mathbf{X}}'\tilde{\mathbf{X}}/N)$ is $O(1)$ and uniformly positive definite.

Assumption 3:

(i) $\mathbf{Q}_N^{-1/2} \dfrac{1}{\sqrt{N}} \mathbf{X}'\mathbf{u} \overset{d}{\to} N(\mathbf{0}, \sigma^2 \mathbf{I})$;

(ii) $\tilde{\mathbf{Q}}_N^{-1/2} \dfrac{1}{\sqrt{N}} \tilde{\mathbf{X}}'\mathbf{u} \overset{d}{\to} N(\mathbf{0}, \sigma^2 \mathbf{I})$.

Assumption 1 imposes a strong exogeneity assumption on $\mathbf{X}_i$, ruling out (for example) lagged dependent variables, and it also implies that $E(\tilde{\mathbf{X}}_i'\mathbf{u}_i) = \mathbf{0}$. Together with *Assumption 2*, consistency of the pooled and fixed effects estimators for $\boldsymbol{\beta}$ is guaranteed. If weakened to $E(\mathbf{X}_i'\mathbf{u}_i) = \mathbf{0}$, or even $E(\mathbf{x}_{it}u_{it}) = \mathbf{0}$ (zero contemporaneous correlation), least squares estimation of (3) generally becomes inconsistent rendering the F-test statistic asymptotically invalid, anyway (see, for example, the discussion in Wooldridge, 2002, Sections 10.5 and 11.1). *Assumptions 3 (i)* and *(ii)* result from conditions discussed, for example, by White (2001, p.120), and justify asymptotic normality of the pooled and fixed effect estimators. (Note that *Assumptions 2* and *3* also ensure that both $\mathbf{X}'\mathbf{u}/N$ and $\tilde{\mathbf{X}}'\mathbf{u}/N$ are $o_p(1)$.)

3 Asymptotic Distribution of F_N

The result is stated in the following Proposition:

Proposition 1

$$\sigma^2 \sqrt{N}\, (F_N - 1) = \frac{\sqrt{N}}{N(T-1)} \left(\sum_{i=1}^{N} v_i^2 - \sum_{i=1}^{N} \sum_{t=1}^{T} u_{it}^2 \right) + o_p(1),$$

where $v_i = \left(\sum_{t=1}^{T} u_{it} \right)$ are independent random variables, so that

$$\sqrt{N} \left(F_N - 1 \right) \xrightarrow{d} N \left(0, \frac{2T}{T-1} \right).$$

Proof. Define $F_N = \frac{R_N}{\tilde{\sigma}^2}$, where $R_N = (RSS_R - RSS_U)/(N-1)$, so that

$$\sigma^2 \sqrt{N}(F_N - 1) = \frac{\sigma^2}{\tilde{\sigma}^2} \sqrt{N} \left(R_N - \tilde{\sigma}^2 \right).$$

where $\tilde{\sigma}^2 - \sigma^2 = o_p(1)$. Consider $\sqrt{N} \left(R_N - \tilde{\sigma}^2 \right)$: By the Frisch–Waugh theorem, RRS_U is identical to that obtained from Ordinary Least Squares estimation of $\tilde{\mathbf{y}} = \tilde{\mathbf{X}} \boldsymbol{\beta} + \tilde{\mathbf{u}}$, so that

$$
\begin{aligned}
RSS_U &= \mathbf{u}' \mathbf{M_D} \mathbf{M_{\tilde{X}}} \mathbf{M_D} \mathbf{u} \\
&= \mathbf{u}' \left(\mathbf{M_{\tilde{X}}} - \mathbf{P_D} \right) \mathbf{u}
\end{aligned}
$$

since $\mathbf{M_D} \mathbf{M_{\tilde{X}}} \mathbf{M_D} = \mathbf{M_D} \left(\mathbf{M_{\tilde{X}}} - \mathbf{P_D} \right) = \mathbf{M_{\tilde{X}}} - \mathbf{P_D}$. Therefore, as $RSS_R = \mathbf{u}' \mathbf{M_X} \mathbf{u}$ and $\tilde{\sigma}^2 = \frac{1}{N(T-1)-K} \left[\mathbf{u}' \left(\mathbf{M_{\tilde{X}}} - \mathbf{P_D} \right) \mathbf{u} \right]$, $R_N - \tilde{\sigma}^2$ can be expressed as

$$
\begin{aligned}
R_N - \tilde{\sigma}^2 &= -\frac{\mathbf{u}' \mathbf{P_X} \mathbf{u}}{N-1} + \frac{\mathbf{u}' \mathbf{P_{\tilde{X}}} \mathbf{u}}{N-1} + \frac{\mathbf{u}' \mathbf{P_D} \mathbf{u}}{N-1} \\
&\quad - \frac{1}{(T-1) - K/N} \left[\frac{\mathbf{u}' \mathbf{u}}{N} - \frac{\mathbf{u}' \mathbf{P_{\tilde{X}}} \mathbf{u}}{N} - \frac{\mathbf{u}' \mathbf{P_D} \mathbf{u}}{N} \right] \\
&= \frac{\mathbf{u}' \mathbf{P_D} \mathbf{u}}{N} - \frac{1}{T-1} \frac{\mathbf{u}' \mathbf{u}}{N} + \frac{1}{T-1} \frac{\mathbf{u}' \mathbf{P_D} \mathbf{u}}{N} + O_p(N^{-1}) \\
&= \frac{1}{NT} \sum_{i=1}^{N} v_i^2 - \frac{1}{N(T-1)} \sum_{i=1}^{N} \sum_{t=1}^{T} u_{it}^2 + \frac{1}{NT(T-1)} \sum_{i=1}^{N} v_i^2 + O_p(N^{-1}) \\
&= \frac{1}{N(T-1)} \left(\sum_{i=1}^{N} v_i^2 - \sum_{i=1}^{N} \sum_{t=1}^{T} u_{it}^2 \right) + O_p(N^{-1}),
\end{aligned}
$$

by Assumptions 2 and 3 and noting that $\frac{N}{N(T-1)-K} = \frac{1}{(T-1)} + O(N^{-1})$, $\mathbf{u}' \mathbf{P_D} \mathbf{u} = \frac{1}{T} \sum_{i=1}^{N-1} \left(\sum_{t=1}^{T} u_{it} \right)^2$, so that

$$
\begin{aligned}
\frac{\mathbf{u}' \mathbf{P_D} \mathbf{u}}{N-1} &= \frac{1}{(N-1)T} \sum_{i=1}^{N-1} v_i^2 \\
&= \frac{1}{NT} \sum_{i=1}^{N} v_i^2 + O_p(N^{-1}).
\end{aligned}
$$

Therefore

$$\sqrt{N}\left(R_N - \tilde{\sigma}^2\right) = \frac{\sqrt{N}}{N(T-1)}\left(\sum_{i=1}^{N} v_i^2 - \sum_{i=1}^{N}\sum_{t=1}^{T} u_{it}^2\right) + o_p(1)$$

$$= \frac{1}{(T-1)\sqrt{N}}\sum_{i=1}^{N} W_{iT} + o_p(1)$$

where $\frac{1}{\sqrt{N}}\sum_{i=1}^{N} W_{iT}$ is asymptotically equivalent to the familiar test indicator for random effects; see, for example, Breusch and Pagan (1980), Chesher (1984) and Honda (1985). Since, by assumption, the u_{it} are independently and identically distributed so are the $W_{iT} = \left(\sum_{t=1}^{T} u_{it}\right)^2 - \sum_{t=1}^{T} u_{it}^2$. Then, writing $W_{iT} = \sum\sum_{t \neq s} u_{it}u_{is} = \mathbf{u}_i'\mathbf{A}\mathbf{u}_i$, where $\mathbf{A} = \iota_T\iota_T' - \mathbf{I}_T$, it is easy to determine that $E(W_{iT}) = 0$ and

$$var(W_{iT}) = 2\sigma^4 tr\left(\mathbf{A}^2\right)$$

$$= 2\sigma^4 T(T-1) < \infty$$

where $tr(.)$ signifies trace. *Thus, a standard Central Limit Theorem yields, $\sqrt{N}\left(R_N - \tilde{\sigma}^2\right) \xrightarrow{d} N\left(0, \frac{2\sigma^4 T}{T-1}\right)$, which is the familiar result for the standard random effects test indicator. Finally, since $\tilde{\sigma} - \sigma^2 = o_p(1)$, it follows that*

$$\sqrt{N}\left(F_N - 1\right) \xrightarrow{d} N\left(0, \frac{2T}{T-1}\right). \tag{8}$$

∎

3.1 Asymptotic validity of the F-test

Here it is shown that conventional testing procedure for fixed effects, based on F distribution critical values, will still provide asymptotically valid inferences, despite non-normality. Let $\xi_N \sim F(n_1, n_2)$, an F distribution with n_1 and n_2 degrees of freedom, respectively, where $n_1 = N - 1$ and $n_2 = N(T-1) - K$. Then

$$\sup_z \left| \Pr\left(\sqrt{\tfrac{N(T-1)}{2T}}(\xi_N - 1) \leq z\right) - \Pr\left(\sqrt{\tfrac{N(T-1)}{2T}}(F_N - 1) \leq z\right)\right| \to 0.$$

To see this, define $\Phi(z)$ to be the standard normal distribution function.

6

Then, by the triangle inequality,

$$\sup_z \left| \Pr\left(\sqrt{\tfrac{N(T-1)}{2T}}(\xi_N - 1) \leq z \right) - \Pr\left(\sqrt{\tfrac{N(T-1)}{2T}}(F_N - 1) \leq z \right) \right|$$

$$\leq \sup_z \left| \Pr\left(\sqrt{\tfrac{N(T-1)}{2T}}(\xi_N - 1) \leq z \right) - \Phi(z) \right|$$

$$+ \sup_z \left| \Pr\left(\sqrt{\tfrac{N(T-1)}{2T}}(F_N - 1) \leq z \right) - \Phi(z) \right|.$$

By Proposition 1, the second term of the right hand side is $o(1)$. The first term is also $o(1)$ from elementary distribution theory, as follows. Observe that ξ_N can be defined as $\xi_N = \dfrac{\chi^2_{n_1}/n_1}{\chi^2_{n_2}/n_2}$, where $\chi^2_{n_1}$ is chi-squared with n_1 degrees of freedom independent of $\chi^2_{n_2}$, and $n_1 = N-1$ and $n_2 = N(T-1) - K$. Now,

$$\sqrt{N}(\xi_N - 1) = \frac{\sqrt{N}\left(\chi^2_{n_1}/n_1 - \chi^2_{n_2}/n_2\right)}{\chi^2_{n_2}/n_2},$$

with $E\left[\sqrt{N}\left(\chi^2_{n_1}/n_1 - \chi^2_{n_2}/n_2\right)\right] = 0$ and $var\left[\sqrt{N}\left(\chi^2_{n_1}/n_1 - \chi^2_{n_2}/n_2\right)\right] = \dfrac{2N}{n_1} + \dfrac{2N}{n_2}$. Since, (i) $\chi^2_{n_1}/n_1 - \chi^2_{n_2}/n_2$ can be represented as sums of squares of (scaled) independent standard normal random variables, (ii) $0 < \dfrac{2N}{n_1} + \dfrac{2N}{n_2} \to \dfrac{2T}{T-1} = O(1)$, and, (iii) $\chi^2_{n_2}/n_2 \xrightarrow{p} 1$, a suitable central limit theorem yields

$$\sqrt{\tfrac{N(T-1)}{2T}}(\xi_N - 1) \xrightarrow{d} N(0,1)$$

and the result follows. Therefore, using critical values from the F distribution, in conjunction with the statistic F_N, becomes increasingly like using critical values from the $N\left(1, \dfrac{2T}{N(T-1)}\right)$ distribution, which is the asymptotic distribution of the F_N.

In the next section, the preceding analysis is supported by the results of a small Monte Carlo experiment which illustrates the asymptotic robustness of the F-test to non-normality.

4 Monte Carlo Simulation

In order to shed light on the relevance of the asymptotic analysis for finite sample behaviour, this section reports the results of a small Monte Carlo experiment which investigates the (null) sampling behaviour of F_N, under a

variety of error distributions using $N = 5, 20, 50$, $T = 5$, with 5000 replications. The null model used here is

$$y_{it} = \sum_{k=1}^{3} x_{it,k}\beta_k + u_{it}$$

where $x_{it,1} = 1$, $x_{it,2}$ is drawn from a uniform distribution on $(1, 31)$ independently for i and t, and $x_{it,3}$ is generated following Nerlove (1971), such that

$$x_{it,3} = 0.1t + 0.5x_{it-1,3} + v_{it},$$

where the value $x_{i0,3}$ is chosen as $5 + 10v_{i0}$, and v_{it} is drawn from the uniform distribution on $(-0.5, 0.5)$ independently for i and t, in order to avoid any normality in regressors. Also, observe that the regression design is not quadratically balanced.[2] Table 1 shows the value of the maximum leverage point and the number of leverage points[3], confirming that the regressors used are not quadratically balanced.

[Table 1 about here]

Without loss of generality, the coefficients are set as $\beta_k = 1$ for $k = 1, 2, 3$ and the error terms, u_{it}, are all $iid(0, 1)$ in the experiments. They are drawn from the following distributions and standardised: (i) standard normal distribution (SN); (ii) Student's t distribution with 5 degrees of freedom ($t(5)$); (iii) uniform distribution over the unit interval (UN); (iv) mixture normal distribution from either $N(-1, 1)$ and $N(+1, 1)$ with and equal probability of 0.5 (MN); (v) log-normal distribution (LN); and, (vi) chi-square distribution with 2 degrees of freedom ($\chi^2(2)$).

The theoretical results of Section 3 imply that asymptotically valid inferences based on F_N can be made with reference to a $F(N-1, 4N-2)$ distribution. To illustrate this, Figures $1-6$ present Q-Q plots of F_N (obtained from the Monte Carlo experiments) against quantiles from the reference F distribution.

[Figures 1-6 about here]

In general, these confirm the asymptotic validity of using the $F(N-1, 4N-2)$ distribution, in this case, in conjunction with F_N (a procedure which is exact under normality). Agreement between the empirical and reference quantiles

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[2]The results of Ali and Sharma (1996) show that, with a quadratically balanced design, the effects of non-normality on the F-test for linear restrictions in the linear model is minimal. Hence the Monte Carlo design guards against that possibility.

[3]Denoting the diagonal elements of $\mathbf{P_X}$ as h_t, and the average of h_t as $\bar{h}$, following Belsley et al. (1980), we call h_t is a high leverage point if $h_t/\bar{h} > 2$.

is not so clear in the case of log-normal errors, which exhibits a very high coefficient of skewness (6.185), but appears adequate for $\chi^2(2)$ errors, whose coefficient of skewness is 2.

The Q-Q plots presented above, examine the quality of the reference distribution as a whole. Of perhaps more concern is the ability of the reference distribution to model empirical significance levels. Table 2 shows the estimated significance levels of the standard F-test procedure which uses the statistic F_N in conjunction with critical values from a $F(N-1, 4N-2)$ distribution designed to give nominal significance levels of 1%, 5% and 10%, respectively.

<div align="center">[Table 2 about here]</div>

This procedure will be exact under normality, since F_N is an exact pivot, and the results reflect this. Under non-normality such a procedure is only asymptotically valid, although it performs well (in terms of agreement between estimated and nominal significance levels) for all designs. However, and as noted above, with LN errors it appears somewhat less satisfactory with $N = 5, 20$.[4] This problem disappears when $N = 50$.

These results not only support the asymptotic analysis of Section 3, but also suggest that standard F-test procedure (as reported in standard econometric texts) remains robust to non-normality even in quite small samples, except in cases of extreme skewness of the error distribution. In the latter case, and since F_N is asymptotically pivotal, the results of Beran (1988) imply that improved finite sample performance may be obtained through the use of bootstrap critical values; see Yamagata (2004) who provides evidence which confirms this.

5   Conclusions

This paper has addressed an apparent gap in the econometrics literature - as identified, for example, by Wooldridge (2002) - by providing the limit distribution of the standard F-test statistic for fixed effects, in a static panel data model, under non-normality and fixed time periods. It has been shown, in fact, that the commonly cited test procedure remains asymptotically valid. Moreover, Monte Carlo evidence has been presented which suggests that the asymptotic results provide an excellent guide to finite sample behaviour even in quite small samples, under non-normality.

For the applied worker, then, the standard F-test procedure appears quite robust to non-normality even in small samples but (if required) further finite sample improvement may be afforded by the use of bootstrap critical values.

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[4]However, under highly skewed $\chi^2(2)$ errors, , there is still very good agreement between estimated and nominal significance levels.

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Table 1: Leverage Points

N	5	20	50
The value of maximum leverage point	4.6	5.4	5.7
The number of leverage points	1	5	16

Table 2: Estimated Significance Levels of F_N

	1%			5%			10%		
N	5	20	50	5	20	50	5	20	50
SN	0.88	1.14	0.96	4.70	4.44	4.68	9.82	9.40	9.76
$t(5)$	0.70	0.92	1.18	4.40	4.74	4.90	9.22	9.76	9.56
UN	0.96	1.08	0.84	5.60	5.46	4.86	10.28	10.20	9.78
MN	1.28	0.96	1.02	5.38	5.18	4.80	10.04	10.58	10.28
LN	0.70	1.02	1.68	<u>3.72</u>	4.46	4.96	<u>8.06</u>	<u>8.30</u>	9.44
$\chi^2(2)$	0.76	1.12	1.14	4.88	4.94	4.90	9.54	9.58	10.24

Notes:

Single (resp. double) underline denotes that the rejection frequency is not consistent
with the true significance level being between -1 (resp. -0.5)% and +1 (resp. +0.5) %
from its nominal level.

Figure 1: Q-Q plots of F_N to $F(N-1, 4N-2)$ distribution: SN Errors
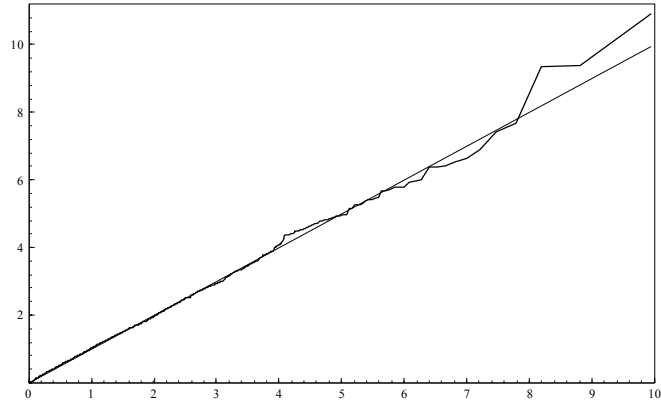
(a) $N = 5$
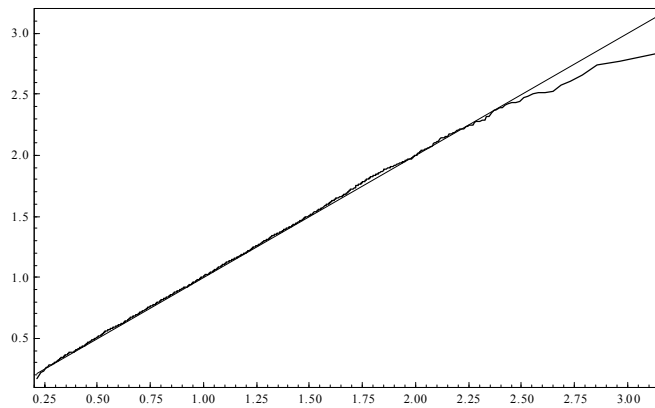
(b) $N = 20$
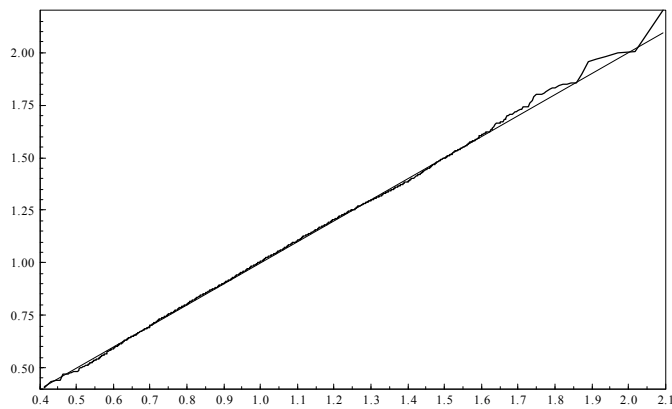
(c): $N = 50$

Figure 2: Q-Q plots of F_N to $F(N-1, 4N-2)$ distribution: $t(4)$ errors
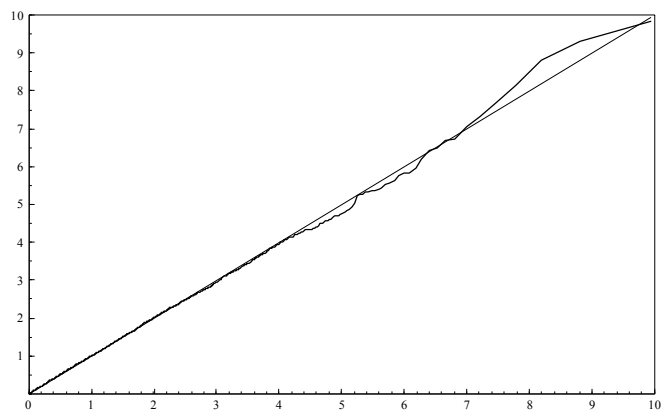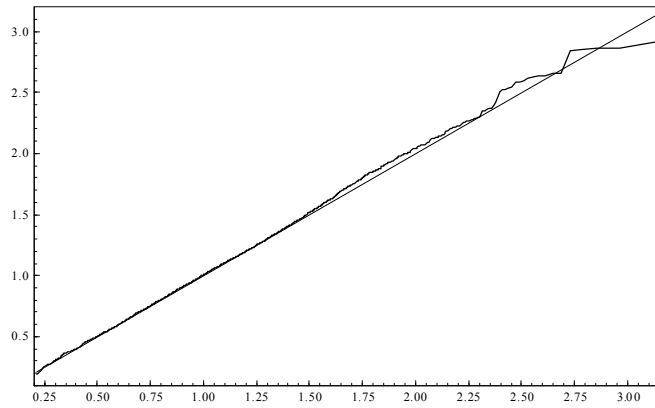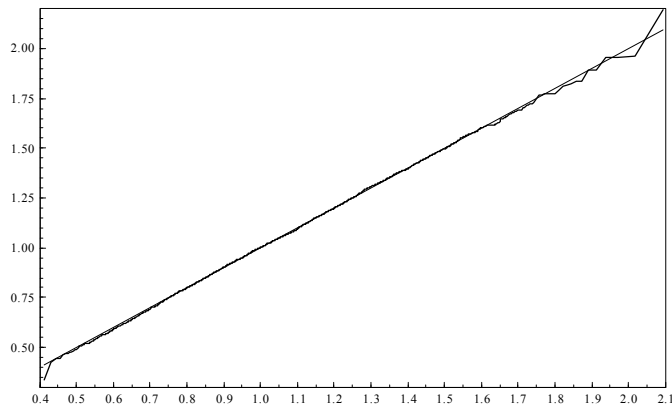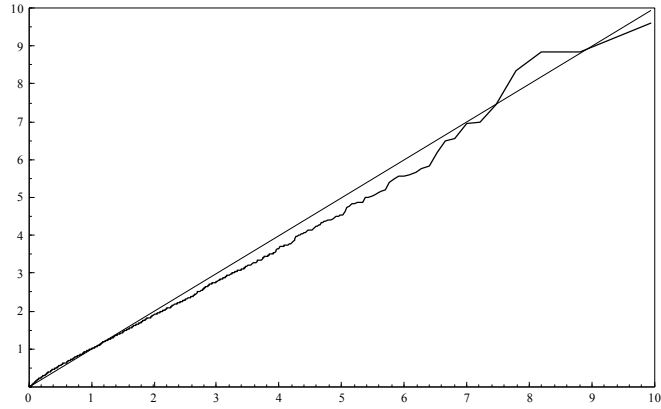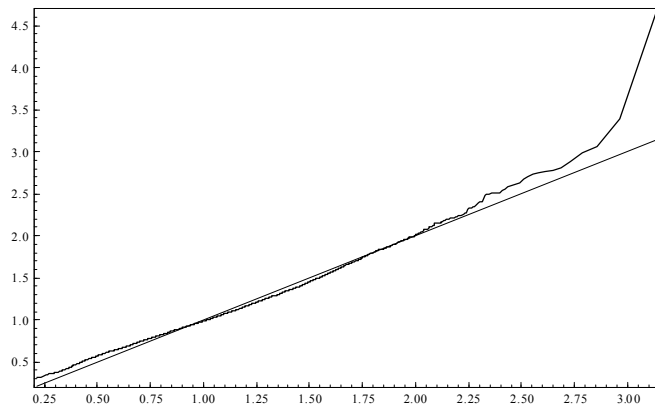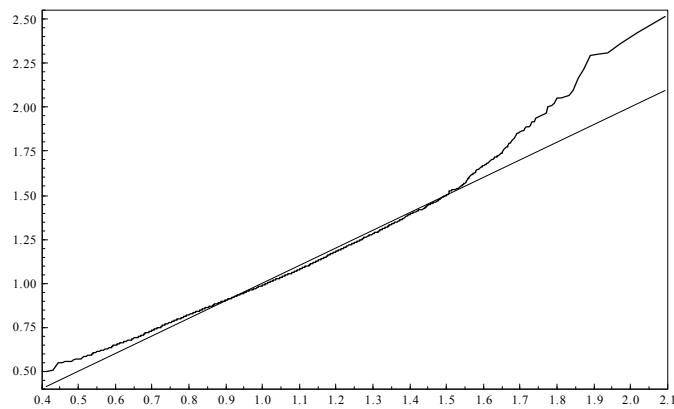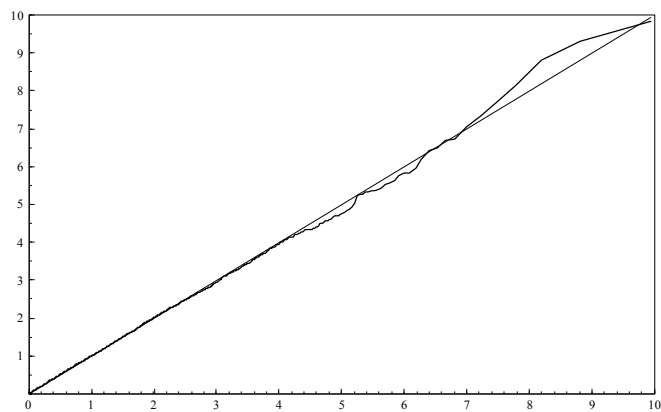

(a) $N = 5$
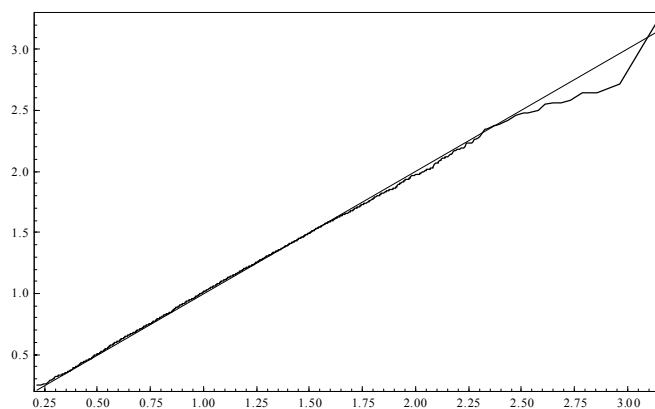
(b) $N = 20$
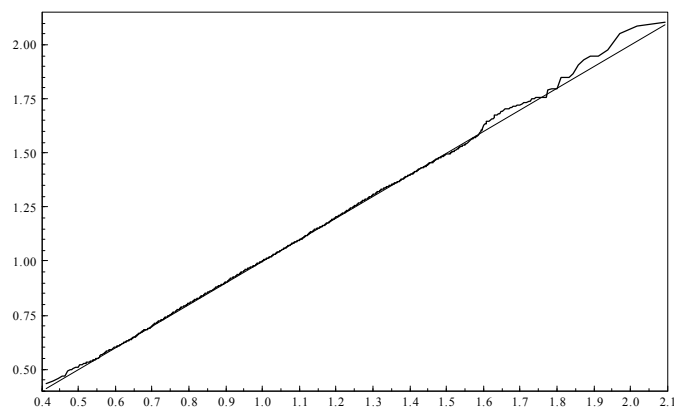
(c) $N = 50$

Figure 3: Q-Q plots of F_N to $F(N-1, 4N-2)$ distribution: UN errors
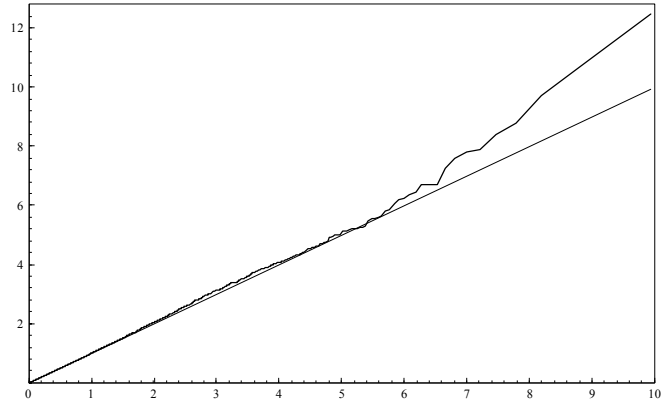
(a) $N = 5$
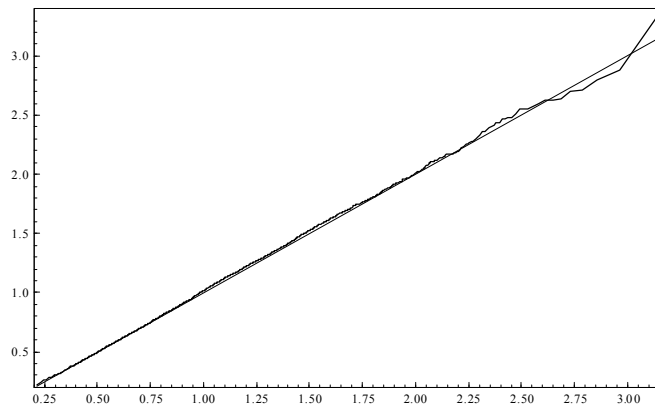
(b) $N = 20$
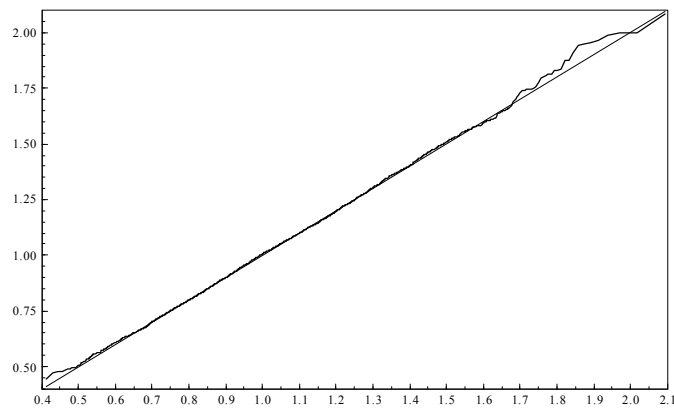
(c) $N = 50$

Figure 4: Q-Q plots of F_N to $F(N-1, 4N-2)$ distribution: MN errors
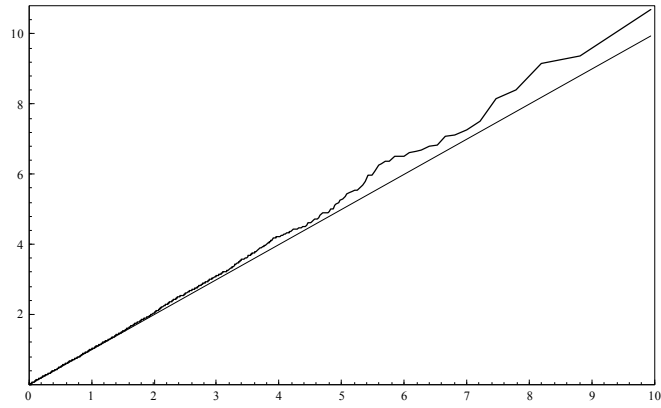
(a) $N = 5$

(b) $N = 20$

(c) $N = 50$

Figure 5: Q-Q plots of F_N to $F(N-1, 4N-2)$ distribution: LN errors
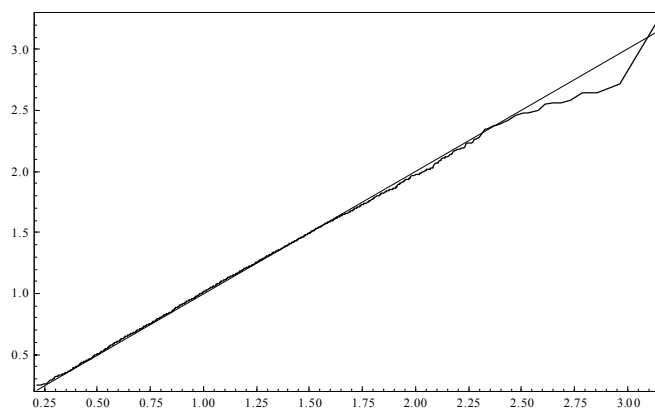

(a) $N = 5$

(b) $N = 20$
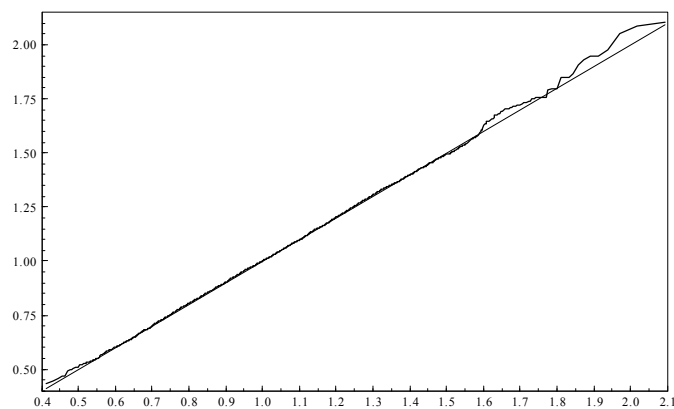

(c) $N = 50$

Figure 6: Q-Q plots of F_N to $F(N-1, 4N-2)$ distribution: $\chi^2(2)$ errors

(a) $N = 5$

(b) $N = 20$

(c) $N = 50$