Revisiting Francis Galton's Forecasting Competition

Kenneth F. Wallis

Department of Economics
University of Warwick
Coventry CV4 7AL, UK
[K.F.Wallis@warwick.ac.uk]

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Abstract This note re-examines the data from a weight-judging competition described in an article by Francis Galton published in 1907. Following the correction of some errors, it is shown that this forecasting competition is an interesting precursor of two more recent developments in the statistical forecasting literature. One is forecast combination, with the mean forecast here exactly coinciding with the outcome, and the second is the use of two-piece frequency and probability distributions to describe asymmetry.

Keywords: forecast competitions; forecast combinations; two-piece distributions; skewness; kurtosis; the wisdom of crowds

1.   Introduction: the weight-judging competition

In an article titled *Vox Populi* in the weekly journal of science, *Nature*, of March 7, 1907, Francis Galton, renowned anthropologist, biometrician and statistician, described a weight-judging competition conducted at the recent West of England Fat Stock and Poultry Exhibition. 'A fat ox having been selected, competitors bought stamped and numbered cards, for 6*d.* each, on which to inscribe their respective names, addresses, and estimates of what the ox would weigh after it had been slaughtered and "dressed." Those who guessed most successfully received prizes.' In accordance with his lifelong motto, reported by his biographer Karl Pearson as "Whenever you can, count" (1924, p.340), Galton was able to obtain the loan of all the entry cards for a short period. On studying the entries, he found that 'these afforded excellent material. The judgments were unbiassed by passion … . The sixpenny fee deterred practical joking, and the hope of a prize and the joy of competition prompted each competitor to do his best. The competitors included butchers and farmers, some of whom were highly expert in judging the weight of cattle.'

After weeding out 13 defective or illegible cards, there were 787 entries for analysis, and Galton began by constructing a ranked list of the estimates, converting from hundredweights, quarters and pounds to pounds. He then picked out the 5, 10, 15, … , 95 percentiles, which are tabulated as the 'distribution of the estimates' in his article, following his 'method of percentiles', developed over many years. His preferred measure of central tendency is the median although, having earlier introduced this term, in this article he retains the previous terminology: 'According to the democratic principle of "one vote one value," the middlemost estimate expresses the *vox populi*, every other estimate being condemned as too low or too high by a majority of the voters.' He reports that 'the middlemost estimate is 1207 lb., and the weight of the dressed ox proved to be 1198 lb.; so the *vox populi* was in this case

9 lb., or 0.8 per cent. of the whole weight too high.' Galton concludes that 'This result is, I think, more creditable to the trustworthiness of a democratic judgment than might have been expected.'

Such a competition provides an early example of the forecast competitions that have become familiar in the forecasting literature. In this note we take another look at Galton's data, and show that his article, despite some inaccuracy, is an interesting precursor of two more recent developments in the statistical forecasting literature. It has also attracted attention in the public choice literature: see, for example, Levy and Peart (2002), who reproduce the original article (Galton, 1907b), together with a Letter to the Editor of *Nature* published a week earlier, arguing for the median on democratic principles (Galton, 1907a). Also, more popularly, see Surowiecki's *The Wisdom of Crowds* (2004), which begins with the weight-judging competition and so brings it to the attention of a wider audience.

2.   A visit to the archives: some discrepancies

Galton's working papers, notes, some correspondence and a handwritten draft of his article are stored in the Galton Archive at University College, London. Study of this material reveals some slips that have a bearing on subsequent analysis and interpretation. Galton reached the age of 85 on February 16, 1907, and may have been in a hurry to attract 'immediate attention', for which *Nature* provided 'a ready means' of communication, as Pearson observed (1924, p.400).

There are small errors in all three figures appearing in the summary statement of results quoted above, although they are arithmetically consistent. First, with respect to the median entry, among 787 observations this is the 394th in the ranked list, which is 1208

pounds. Second, the outcome, that is, the dressed weight of the ox, was reported in a letter from the organiser of the competition as 10 cwt, 2 qt and 21 lbs, that is, 1197 pounds, and this figure appears in Galton's worksheets, being equal to the 353rd entry in the ranked list, as shown in the extract reproduced in Figure 1. So the true error in the middlemost estimate is 11 lb. Galton devoted a paragraph of a letter to his nephew, Edward Wheler Galton, dated February 4, 1907, to this subject. He says that he is 'just now at some statistics that might interest you', and concludes a brief account of the weight-judging data with the statement 'The average was 11 lbs. wrong' (Pearson, 1930, p.581).

Galton's handwritten draft of the article, however, presents the results as follows. 'The weight of the dressed ox proved to be 1198 lbs. The estimates were scattered about their own middlemost value of 1208 lbs so the vox populi was in this case 11 lbs too high, or closely 1% of the weight.' Here the arithmetic is inconsistent, because the outcome has been incorrectly transcribed. But instead of correcting 1198 to 1197, so that the difference is 11, as correctly stated in the draft, in the published version 1208 has been changed to 1207, and the error is correspondingly reported as 9 lb. In attempting the correction, the wrong four-digit number has had its final 8 altered to 7. It is not clear at what point in the work this happened, since the wrong median also appears in the published table of the distribution of the estimates, although the first and third quartiles are correctly given. With 787 observations these require no interpolation, being the 197th and 591st observations in the ranked list, equal to 1162 and 1236 respectively.

Galton appears to have remained unaware that the results as presented in his article were inaccurate, since they reappear in his *Memories of My Life* (1908). A short account of his visit to the cattle exhibition 'a little more than a year ago' and his subsequent research is

given (pp.280-281); this refers to his 'memoir published in *Nature*' and repeats the incorrect numbers to be found there, respectively 1207 pounds and 1198 pounds.

3. Forecast combination

The idea that combining different forecasts of the same event might be worthwhile has gained wide acceptance since the seminal article of Bates and Granger (1969), some sixty years after Galton's *Vox Populi*. A substantial literature has subsequently appeared, mostly concerning point forecasts of the future realisation of a random variable, and although the median of a set of competing forecasts is sometimes a combined forecast of interest, simple averages and various weighted averages are more common, given that the statistical forecasting literature is largely founded on least squares principles.

The choice between the median and the mean was discussed in a letter to the editor of *Nature* published two weeks after Galton's article had appeared. The correspondent, Hooker (1907), wished that Galton had also reported the arithmetic mean of the 787 observations. He says that 'I have not the actual figures, but judging from the data in Mr. Galton's article, the mean would seem to be approximately 1196 lb., which is much closer to the ascertained weight (1198 lb.) than the median (1207 lb.)': he had calculated the mean of the percentiles in Galton's table. In his reply, one week later, Galton (1907c) reports the correct mean of all the figures as 1197 pounds, which for him shows that 'the compactness of a table of centiles is no hindrance to their wider use'. He does not remark on the fact that this is closer to the true value than the median, as Hooker had observed. Indeed, using the correct value of the outcome, the mean estimate has zero error. This early example of the gains from forecast combination would very likely have been cited by Bates and Granger (1969), had they been aware of it.

Surowiecki focusses on this mean estimate, reported in Galton's reply, as representative of 'the collective wisdom of the Plymouth crowd' (2004, p.xiii). He compares it to the published outcome of 1198 pounds and concludes that 'the crowd's judgment was essentially perfect'. Had he known the true outcome, his conclusion on the wisdom of crowds would no doubt have been more strongly expressed.

In this one-off exercise there is no possibility of using calculated weights to construct an alternative combined forecast. Other possibilities considered in the forecast combination literature include trimmed means: in this case symmetric trimming of the sample by increasing amounts simply moves the estimate from the mean of 1197 towards the median, which is the extreme example of a trimmed mean, and increases the error.

4.   Two-piece distributions

Galton discusses the dispersion and shape of the distribution of the individual forecasts of the dressed weight with reference to the normal distribution. As a measure of spread he uses the 'probable error', an archaic term for the semi-interquartile range: the probability of obtaining a value within one probable error of the centre of a symmetric distribution is 0.5. For normal variables the probable error is equal to $0.6745 \times (\text{standard deviation})$, hence Galton's probable error of $(1236 - 1162)/2 = 37$ is equivalent to a standard deviation of 54.9. To compare the empirical distribution with a normal distribution with mean (=median) 1207 and probable error 37, he plots each distribution's percentiles against the corresponding percentages (5, 10, … , 95); his 'ogive' curves are now called inverse cumulative distribution functions. His diagram shows that the normal distribution does not extend sufficiently far into the tails of the empirical distribution. To modern eyes, this is more readily apparent in

the comparison of the probability density function (PDF) of Galton's normal distribution and the sample histogram shown in Figure 2.

In the course of an extended review of Galton's statistical investigations, Pearson (1924, Ch. 13) revisits this dataset. In respect of Galton's methods, he observes (p.400) 'That Galton used median and quartiles so frequently … must, I think, be attributed to his great love of brief analysis. He found arithmetic in itself irksome;' For the present exercise, he notes (p.404) that 'the percentile method of tabulation does not permit of very ready determination of the mean and standard deviation and so of getting the best normal distribution', in other words, the one fitted by his method of moments! He finds, 'after some labour: mean 1197, standard deviation 61.895, … These give a far better fit than Galton's median and quartile values.' Pearson seems to have overlooked the mean value given by Galton (1907c). Figure 2 shows that his claim of 'a far better fit' is true away from the centre of the distribution, but less true around the peak.

A major problem, as Galton observes, is that the distribution of forecasts is asymmetric, the extent of underestimation of the outcome being the greater. His final suggestion is that the lower half of the distribution might agree well with a normal distribution with probable error 45, and the upper half with one having probable error 29. He had made a similar suggestion on a previous occasion (Galton, 1896), only to have it immediately pointed out that placing two half-normal distributions together in this way would result in a discontinuity at the join point (Yule, 1896). The two-piece normal distribution, introduced by Fechner (1897) and subsequently rediscovered several times (Wallis, 2013), appropriately scales each half-normal distribution and is continuous at the mode. This

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distribution has found widespread use as a representation of asymmetric risks in density forecasting.

A second major problem, however, is that the sample distribution is leptokurtic, that is, more peaked and with fatter tails than the normal distribution: the conventional β_2 measure of kurtosis is 6.01. The corresponding lack of 'shoulders' in the distribution explains why Galton's estimate of the standard deviation based on the interquartile range is smaller than Pearson's second moment estimate. In the two-piece normal distribution we have $3 \leq \beta_2 \leq 3.87$, but this apparent kurtosis is simply a consequence of its asymmetry, with each half-normal distribution being rescaled, not reshaped. Attention then turns to two-piece versions of more kurtotic distributions, such as the Student-*t* distribution with a relatively small degrees of freedom. However, incorporating skewness in this way mainly accommodates skewness in the central part of the distribution, and does not allow the two tails of the distribution to have different rates of decay. Accordingly, Zhu and Galbraith (2010) develop a class of generalized asymmetric Student-*t* (AST) distributions which has one skewness parameter and two tail parameters, which offer the possibility of improved fit in the tail regions.

Fitting the (CDF of the) AST distribution to Galton's data by maximum likelihood gives the result shown (as the PDF) in Figure 3. It is seen that the main features of the data are well-represented by this five-parameter form, compared to the two-parameter forms shown in Figure 2. Basing model selection on the Akaike information criterion leads to a clear preference for the five-parameter form over any restricted version of the distribution. The two "degrees of freedom" parameter estimates are 4.97 and 2.73 respectively in the lower and upper pieces of the distribution. The skewness measure of Arnold and Groeneveld

(1995), which is equal to the difference between the areas under the PDF above and below the mode, is -0.32. It is hoped that this successful example will encourage further applications of the AST distribution, not only in financial econometrics, for which it was designed, but also in other fields.

5. Epilogue

Nothing in this account of Galton's inaccuracies should be taken as detrimental to his standing as a major figure in the development of modern statistics. Stigler (1986, Ch.8) assesses his contributions, describing him (p.266) as 'a romantic figure in the history of statistics, perhaps the last of the gentleman scientists'. More recently (Stigler, 2010), he dates the beginning of the half-century that he calls the Statistical Enlightenment from Galton's address to the 1885 meeting of the British Association for the Advancement of Science. By 1907, however, Galton's major accomplishments were behind him, and the analysis of the weight-judging data was the last piece of statistical work that he published. He was aged 85, and in poor health, which was the reason for his presence in the West of England. He explained in a letter to his nephew in October 1906 that 'London in November would help to, or quite, kill me', but whereas it had been his custom to spend winters in Southern Europe, 'I funk now foreign travel' (Pearson, 1930, p.579). He had not lost his mental powers, however, and, once in Plymouth, could not resist the opportunity that access to these data offered him. Galton died on January 17, 1911, and is buried at Claverdon, a few miles from Warwick.

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Note: facsimiles of the Galton and Pearson items are available at http://www.galton.org/.

Figure 1

Extract from Galton's worksheets

Note: Galton recorded the rank order(s) of the competitor(s) who chose each
possible weight, recorded in pounds. In this relatively dense part of the distribution
of estimated weights, it is remarkable that there was only a single winner, and no
immediately adjacent runner-up.

Figure 2

Fitted normal curves
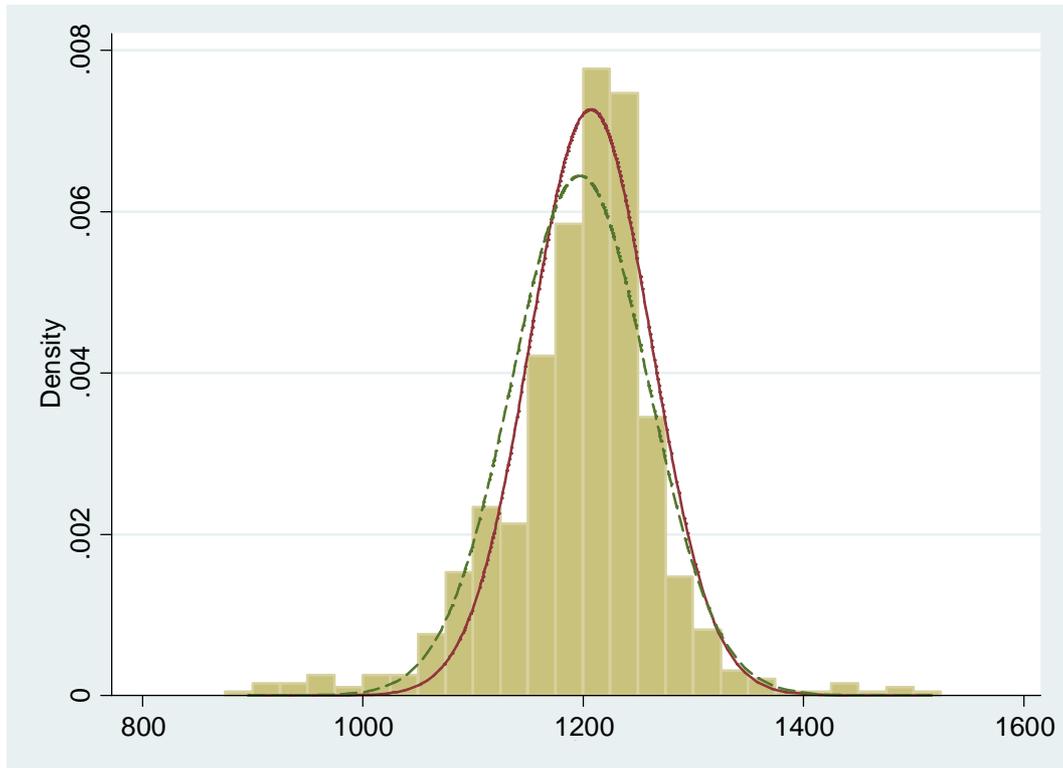
Histogram: original data

Solid line: PDF of the *N*(1207, 54.9) distribution (Galton)

Dashed line: PDF of the *N*(1197, 61.9) distribution (Pearson)

Figure 3

The asymmetric Student-*t* distribution
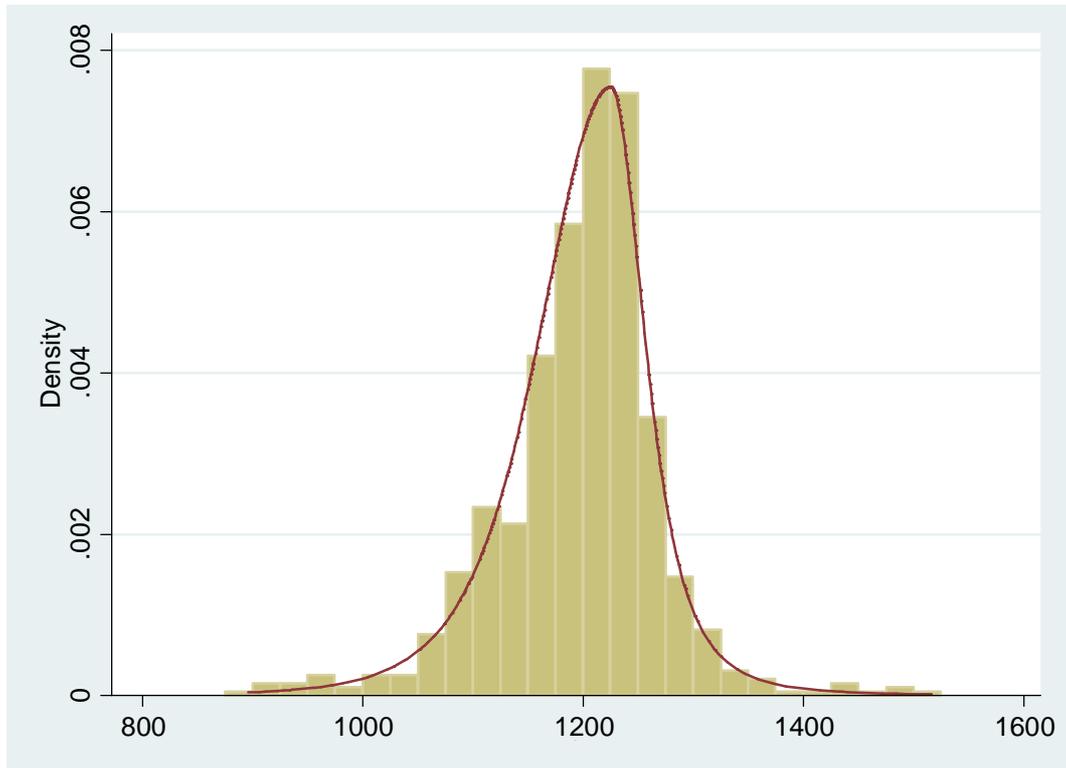
Histogram: original data

Solid line: PDF of the fitted AST distribution (Zhu and Galbraith, 2010)