SKEWNESS AND THE RELATION BETWEEN RISK AND RETURN

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The authors would like to thank Roni Michaely, Denise R. Osborn, Neophytos Lambertides, Konstantinos Tolikas, the participants of the MFS conference (2013), the participants of the CFE conference (2012) and the participants of CREATES (Aarhus University) seminar series (2013). The authors are responsible for remaining errors.

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Abstract

The empirical literature examining the relation between expected returns and risk premium is voluminous and the results so far have been inconclusive. In this study we acknowledge that investors operate under a Mean-Variance framework where skewness and kurtosis are not relevant parameters in asset pricing models. However, in the estimation process skewness and kurtosis in the data contaminate this relation. Using a general framework that accounts for these effects, the results show a positive and statistically significant relation between risk and return. However, in case the effects of skewness are ignored in model specification, this relation usually turns insignificant with mixed signs.

JEL Classification: C18, C22, G15

1. Introduction

The tradeoff between risk and return has been one of the most important and extensively investigated issues in financial economics literature. The theoretical results predict a positive relation between the two.[1] That is, in equilibrium, under a Mean-Variance framework (which is the standard for popular asset pricing models) a larger expected return of an asset can only be justified by a larger standard deviation or variance of returns, the latter often used as measure of risk.

Many empirical investigations, based on parametric, semi- and non-parametric techniques, have been used in the determination of the risk-return tradeoffs. The findings however, so far have been inconclusive. Many well known scholars have found positive relationship, others a negative relationship and an equal number found no relationship.[2]

The main objective of this paper is to provide a framework that explains the contradictions in the aforementioned studies as well as determines the relation between risk and returns. As shown by the theoretical, simulations and empirical results the contradictions have their origins to the presence of skewness and kurtosis in the data used in empirical investigations.

In this study we acknowledge that the investors operate under the Mean-Variance (M-V) framework (such as the CAPM family models). In the M-V framework, skewness

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[1] There are however some studies that support the opposite (see for instance, Abel, 1998; Backus and Gregory, 1993; and Gennotte and Marsh, 1993).

[2] For instance, a positive relation is supported empirically by Bollerslev, Engle and Wooldridge (1988), Scruggs (1998), Harrison and Zhang (1999), Muradoglu, Berument and Metin (1999), Bansal and Lundblad (2002), Bali and Peng (2006), Lanne and Saikkonen (2007), Ludvigson and Ng (2007), Lundblad (2007), Pastór, Sinha and Swaminathan (2008) while evidence of no relation is supported by Poterba and Summers (1986), French, Schwert and Stambaugh (1987), Baillie and DeGennaro (1990), Cheung and Ng (1992), Chan, Karolyi and Stulz (1992) and Campbell and Hentschel (1992) Corhay and Tourani-Rad (1994) and Theodossiou and Lee (1995). In contrast to the above, a negative relation is documented by Fama and Schwert (1977), Campbell (1986), Breen, Glosten and Jagannathan (1989), LeBaron (1989), Pagan and Hong (1991), Nelson (1991), Glosten, Jagannathan, and Runkle (1993), Whitelaw (1994), Fraser and Power (1997), Balios (2008) and Lettau and Ludvigson (2009).

and kurtosis are not relevant parameters in the asset pricing models, however, in the estimation process, skewness and kurtosis in the data, affect the coefficients between expected returns and risk, which is usually interpreted as the price of risk.[3] This raises the question whether such coefficients are true measures of risk price, since they can be decomposed into two parts, the part of the real price of risk and the part that depends entirely on the skewness and indirectly on kurtosis.[4]

By decomposing these effects, our findings suggest a positive and statistically significant relation between risk and return once we account for the effects of skewness. However, in case the effects of skewness are ignored in model specification, this relation usually turns insignificant with mixed signs.

The rest of the paper is organized as follows. Section 2 presents the regression framework while Section 3 proceeds with Monte Carlo simulations. Section 4 provides an empirical illustration and reports the empirical results. Section 5 concludes.

2. Impact of Skewness on the Pricing of Risk

This section provides analytical derivations of the impact of skewness on the relation between risk and return. The analysis is carried out using Engle's autoregressive conditional heteroskedasticity in mean processes, under the skewed generalized t

[3] See analysis in section 2.
[4] Although many studies stressed out the importance of skewness in stock returns and volatility (see for instance, Arditti,1967; Scott and Horrath, 1980; Campbell and Hentschel, 1992; Hansen, 1994; Harvey and Siddique, 2000; Kim and White, 2004; Guedhami and Sy, 2005; Lanne and Saikkonen, 2007; Barberis and Huang, 2008; Grigoletto and Lisi, 2009; Mitton and Vorkink, 2010; Chang, Christoffersen and Jacobs, 2012; Conrad, Dittmar and Ghysels, 2013; among others) the current literature either omits the effects of skewness or treats skewness as an exogenous variable without accounting for its direct effects on the price of risk parameter.

distribution (SGT).[5] We could have used any other skewed type parametric density with the results however remaining qualitatively the same. Nevertheless, the SGT distribution is chosen because of its flexibility to capture stylized facts often observed in financial data. More specifically, it provides flexibility in modeling fat-tails, peakness and skewness, which are common characteristics in financial returns.[6] Furthermore, it encompasses several well known distributions often used in the finance literature, such as the GED, student t, Laplace and normal distributions. Therefore, the results are as general as they can be (see Theodossiou, 1998; Bali and Theodossiou, 2008; and Hansen, McDonald, Theodossiou and Larsen, 2010).

The link between conditional mean and risk is modeled using the GARCH in mean process, which has been the standard in the literature (e.g. Engle, Lilien, and Robins, 1987; Glosten, Jagannathan and Runkle, 1993). The stochastic behavior of a stock returns is modeled as

$$y_t = c\sigma_t + a + by_{t-1} + u_t \, , \tag{1}$$

where $\sigma_t^2 = \operatorname{var}\left(y_t \middle| I_{t-1}\right)$ is the conditional variance of returns based on some information set $I_{t\text{-}1}$ available prior to the realization of y_t, $y_{t\text{-}1}$ are past values of the series included in $I_{t\text{-}1}$, a and b are typical regression parameters and c is a parameter linking the first conditional (unconditional) moment of y_t to its conditional (unconditional) standard deviation. For practical purposes and without any loss of generality, we use a single lag value for y_t. The results however, with more lag values remain qualitatively similar.

Under the SGT framework, y_t is modeled as

[5] The latter specification allows for asymmetry in the response of conditional volatility to return innovations (a stylized fact which is particularly pronounce in financial data).
[6] Since its development in Theodossiou (1998), the SGT distribution, has been used widely by finance researchers (for instance in computing VaR measures, option pricing, estimating asset pricing models among others) and incorporated in econometric packages such as GAUSS.

$$f\left(y_t|\theta, I_{t-1}\right) = .5k\left(\frac{n+1}{k}\right)^{-\frac{1}{k}} B\left(\frac{n}{k},\frac{1}{k}\right)^{-1} \phi_t^{-1}\left(1+\frac{|u_t|^k}{\left((n+1)/k\right)\left(1+sign(u_t)\lambda\right)^k \phi_t^k}\right)^{-\frac{n+1}{k}} \qquad (2)$$

where

$$u_t = y_t - \left(c\sigma_t + a + by_{t-1}\right) = y_t - m_t \ , \qquad (3)$$

where $m_t \equiv c\sigma_t + a + by_{t-1}$ is the mode of y_t. The stochastic error, u_t, would have been the residuals in case of estimating the model under a symmetric pdf. In the case of skewed data, u_t, represents deviations from the mode rather that the mean of returns. ϕ_t is a time-varying scaling function related to σ_t, k and n are positive kurtosis parameters controlling the shape of the density around the mode and tails, λ is a skewness parameter constrained in the open interval (–1, 1), $sign(u_t)$ equals –1 for $u_t < 0$ and 1 for $u_t > 0$ and $B(x,y) = \Gamma(x)\Gamma(y)/\Gamma(x+y)$ is the beta function.[7]

The existence of the conditional variance constrains the kurtosis parameter n to values greater than two ($n > 2$). Values of k below two are associated with leptokurtic (peaked) densities relative to the normal distribution and smaller values of n result in fat tailed densities.

Under the SGT specification and for $n > 2$, the conditional expected value and variance of u_t (e.g., MacDonald, Michelfelder and Theodossiou 2009)[8] are equal to

$$E\left(u_t \mid I_{t-1}\right) = p\sigma_t \qquad (4)$$

and

$$\sigma_t^2 = \mathrm{var}\left(u_t \mid I_{t-1}\right) = \left(g - r^2\right)\phi_t^2 , \qquad (5)$$

where

[7] Setting $\lambda = 0$ in the SGT yields the generalized t (GT) of McDonald and Newey (1988). Similarly, setting k=2 yields the skewed t (ST) of Hansen (1994) which includes the student t distribution when $\lambda = 0$.
[8] See Appendix for the derivation of the moments of the SGT distribution.

$$p = \left(r \big/ \sqrt{g - r^2} \right), \tag{6}$$

$$r = 2\lambda \left(\frac{n+1}{k} \right)^{\frac{1}{k}} B\left(\frac{n-1}{k}, \frac{2}{k} \right) B\left(\frac{n}{k}, \frac{1}{k} \right)^{-1} \tag{7}$$

and

$$g = \left(1 + 3\lambda^2\right) \left(\frac{n+1}{k} \right)^{\frac{2}{k}} B\left(\frac{n-2}{k}, \frac{3}{k} \right) B\left(\frac{n}{k}, \frac{1}{k} \right)^{-1}. \tag{8}$$

In a sense, the parameter p represents the price of risk due to skewness. The size of parameter p is purely determined by the extent of skewness and kurtosis present in the financial return series (see equations 6 and 7). It has the same sign as the skewness parameter λ, estimated internally by the data. In the absence of skewness, its size is zero (i.e. $\lambda = 0$, symmetric SGT with $r = 0$ and $p = 0$). The larger the value of skewness, the larger its impact on the parameter.

Figure 1, shows the relation of the bias in the price of risk as measured by the parameter p, with the skewness parameter λ and kurtosis parameter k. The larger the λ the larger the bias of p for specific values of k. Similarly, for larger values of k, the bias of p increases.

It follows easily from the above that the conditional expected values and conditional variance of the regression variable y_t are

$$\mu_t = E\left(y_t \middle| I_{t-1} \right) = \left(c + p \right) \sigma_t + a + b y_{t-1} = \xi \sigma_t + a + b y_{t-1} \tag{9}$$

and

$$\sigma_t^2 = \mathrm{var}\left(y_t \mid I_{t-1} \right) = \mathrm{var}\left(u_t \mid I_{t-1} \right) = \left(g - r^2 \right) \phi_t^2 . \tag{10}$$

The regression equation in (1) can be written in the following equivalent form

$$y_t = \mu_t + \varepsilon_t = (c + p)\sigma_t + a + by_{t-1} + \varepsilon_t \tag{11}$$

Note that unlike u_t, the error term $\varepsilon_t = y_t - \mu_t = u_t - p\sigma_t$ has an expected value of zero.[9]

Equation (11), provides the basis for measuring the impact of skewness on the relation between risk and return.[10] The parameter $\xi = c + p$ (equation 9) is the risk-return parameter and measures the total impact of risk (standard deviation) on mean returns contaminated with the effects of skewness. Parameter c is the price of risk parameter and accounts for the impact of risk on mean returns net or the effects of skewness and kurtosis. On the other hand, the parameter p is purely determined by the sign of parameter λ and is directly linked to skewness. Therefore, depending on the sign of skewness the risk is either overpriced or underpriced (i.e. for the case where $\lambda>0$, $p>0$ and $\xi>c$ while for the case where $\lambda<0$, $p<0$ and $\xi<c$).

In general, the conditional variance σ_t^2 is specified as a function of the past values of the error ε_t or its standardized measure $z_t = \varepsilon_t / \sigma_t$. In this paper we consider the linear symmetric GARCH model of Bollerslev (1986)

$$\sigma_t^2 = v + \delta\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2, \tag{12a}$$

and the GJR-GARCH specification of Glosten, et al. (1993):

$$\sigma_t^2 = v + \delta\varepsilon_{t-1}^2 + \zeta N_{t-1}\varepsilon_{t-1}^2 + \gamma\sigma_{t-1}^2, \tag{12b}$$

where $N_{t-1} = 0$ for $\varepsilon_{t-1} \geq 0$ and $N_{t-1} = 1$ for $\varepsilon_{t-1} \leq 0$.

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[9] Note that $p = (\mu_t - m_t)/\sigma_t$ is the Pearson's skewness parameter.

[10] Under equation 11, there are two distinct aspects of skewness: one allowing for this in the distribution of u and the second (additional one) where this enters the mean equation.

In the above equations, long term persistence of volatility is mainly captured by the parameter γ and the short term persistence volatility by the parameter δ while parameter ζ captures the asymmetric effects in case of equation 12b.[11]

Let $\theta' = [c, a, b, \nu, \delta, \gamma, \lambda, k, n]$ (or $\theta' = [c, a, b, \nu, \delta, \zeta, \gamma, \lambda, k, n]$ in the case of GJR-GARCH model) be a vector of location parameters related to the above process. Estimates for θ can be obtained via the quasi maximum likelihood method by

$$\max_{\theta} L(\theta) = \sum_{t=1}^{T} \ln f\left(y_t \middle| \theta, I_{t-1}\right), \tag{13}$$

where f is a conditional probability density function for y_t. Estimation of the GARCH-M model without accounting for skewness will generally result in biased estimators for the in-mean effect. Moreover, computed error rates will not possess zero means and unit variance, thus the conditional variances will be inconsistent.

3. Monte Carlo Simulations

In this section we demonstrate that the mixed empirical evidence of risk-return relation documented throughout the literature may be attributed to the fact that the effects of skewness are ignored. Therefore, the effects of skewness on the risk-return relation within the GARCH-M framework are analysed using Monte Carlo (MC) simulations of one thousand samples of two-thousand and a hundred observations each, i.e., N= 1,000 and T= 2,100.

MC simulations proceed as follows:

[11] The usual constraints (in order the unconditional variance to be defined) apply. The results remain qualitatively similar for other forms of GARCH specification.

1. For each sample, a standardized random vector $z = [z_0, z_1, \cdots, z_T]$ of size T+1 is drawn randomly from the standardized SGT distribution with skewness parameter $\lambda = -0.25$ and kurtosis parameters k = 1.5 and n = 10, i.e.,

$$z \sim SGT(\mu = 0, \sigma = 1, \lambda = -0.25, k = 1.5, n = 10).$$

The skewness and kurtosis measures for these parameters (using equations A16 and A17 in the appendix) are equal to -0.186 and 2.444 respectively.[12]

2. The unconditional variance and unconditional mean of the regression variable y_t are used as starting values for the conditional mean and conditional variance of y_t, That is,

$$\sigma_0^2 = \frac{\nu}{1-\delta-\gamma} = \frac{0.05}{1-0.05-0.8} = 0.333$$

and

$$\mu_0 = \frac{(c+p)\sigma_0 + a}{1-b} = \frac{[0.3+(-0.344)]0.33+0.05}{1-0.1} = 0.039.$$

Note that $p = -0.344$ is computed using equation 6. Starting values for the regression error and variable are computed using $\varepsilon_0 = z_0\sigma_0$ and $y_0 = \mu_0 + \varepsilon_0$.

3. The values of the regression variable are generated using the following recursive equations:

$$\sigma_t^2 = 0.1 + 0.05\varepsilon_{t-1}^2 + 0.8\,\sigma_{t-1}^2$$

or

$$\sigma_t^2 = 0.1 + 0.05\,\varepsilon_{t-1}^2 + 0.05\,N_{t-1}\varepsilon_{t-1}^2 + 0.8\,\sigma_{t-1}^2$$

$$\varepsilon_t = z_t\sigma_t$$

and

[12] Values employed in the MC, are calibrated to estimates from actual data.

$$y_t = -0.044\sigma_t + 0.05 + 0.1y_{t-1} + \varepsilon_t ,$$

for $t = 1, 2, \ldots, T$. Note that $c + p = 0.3 - 0.344 = -0.044$.

For each artificially generated sample we estimate an AR(1)-GARCH(1,1)-M and an AR(1)-GJR-GARCH(1,1)-M model under three sample likelihood environments, based on 1) the normal distribution, 2) the symmetric generalized t (GT) distribution and 3) skewed generalized t (SGT) distribution. Note that the first two environments involve misspecified sample likelihood functions, thus cannot capture the impact of skewness in the regression variable y_t on the relationship between risk and return. To avoid problems arising from the starting values we use only the last two thousand observations of y_t.

Tables 1, 2 and 3 present the Monte Carlo statistics for the GARCH-M and GJR-GARCH-M models and all one thousand randomly generated samples. In each table, the first row of numbers presents the parameter values used in the simulation of data (true values). The remaining rows present the arithmetic means (*Avg.*), the standard deviations (*Std. Dev*.), the average standard deviations (*Avg. Std. Dev.*), and the percent of times, that the null hypothesis that the mean values of the parameters are statistically equal to their respective true values, is rejected (*% rejected*).

Tables 1 and 2 present the statistics for the Normal and GT distributions, respectively. The arithmetic means of all other parameters of the conditional means equations, except for that of *c* (GARCH-M effect), appear to be statistically similar to their expected values. The arithmetic mean of the parameter *c* is greatly influenced by the skewness due to the fact that it is not accounted in the estimation (misspecified likelihood function).[13] It seems pretty obvious from (9) that skewness affects the estimate of c. Since *p* is negative, it is anticipated that estimated parameter *c* will be downward biased. Interestingly, the parameters of the conditional variance equations are over-rejected under the Normal distribution while under the Symmetric GT these coefficients appear to be highly significant and statistically not different from their initial values.

Table 3 presents the Monte Carlo statistics of the estimated parameters of the GARCH-M and the GJR-GARCH-M based on the SGT distribution. Note that the means of all parameters are statistically similar to their expected values. Furthermore, the rejection rates of the hypothesis that the parameters are equal to their initial values (*% rejection*) are close to the significance level of 5% except from the case of asymmetry parameter ζ, which is slightly over rejected. Unlike the previous cases, the risk-return relationship, measured by the mean value of the estimated parameter *c*, is statistically similar to the true value used in the simulations.

Overall the results suggest that skewness has an important role in the risk-return relation. In case the effects of skewness are ignored, the relation is negative and insignificant. This downward bias will be due to the negative skewness assumed; otherwise this relation is positive and highly significant in line with the theory.[14] In the next section, we introduce empirical evidence using real data from various international stock markets to examine how this relation behaves.

[13] Misspecified models estimate the risk-return relation contaminated with the effects of skewness, i.e. they estimate risk-return coefficient, ξ.

[14] Theory assumes a linear positive relationship between risk and return and is based on assumptions of rational behavior and economic utilitarianism (Ross, 1973).

4. Empirical Illustration

4.1 Data Description

In this section we use real data to examine the risk-return relation under the models developed in section 2 and we present the empirical results.

The financial series used (number of observations varies with the index and it was chosen according to the longest available period) are the S&P500 (US), FTSE100 (UK), Nikkei 225 (Japan), TSX 60 (Canada), CAC40 (France) and DAX30 (Germany) stock market indices. The series are transformed into continuously compounded daily percentage returns (logarithmic changes) using the formulae

$$y_t = 100 * \ln\left(P_t / P_{t-1}\right) \tag{14}$$

where ln is the natural logarithm and P_t is the level of each series at time t.[15, 16]

Preliminary statistics reported in Table 4 (Panel A) for the daily returns on the one hand show positive average returns for all indices except from Nikkei 225 at daily and monthly frequencies, with S&P500 possessing the highest value followed by DAX30. On the other hand, the Euro Area and Japanese markets have substantially higher (unconditional) volatility, compared to the US and the UK. The same picture is given for weekly and monthly returns (Panels B and C respectively).

The third and fourth rows give the statistics for skewness (b_1) and kurtosis (b_2). These are calculated using the formulae $b_1=m_3/m_2^{3/2}$ and for kurtosis is $b_2=m_4/m_2^{2}$, where m_j is the estimate for the j_{th} moment around the mean. Under the null hypothesis of normality, the two statistics are normally distributed with standard errors $se(b_1)=\sqrt{6/T}$

[15] Unlike the levels of the series, logarithmic changes are stationary processes (results are available upon request).

[16] Extreme values for each data series (i.e. values that are falling outside the range of plus-minus four standard deviations from the mean) are dropped from the sample.

and $se(b_2) = \sqrt{24 / T}$, where T is the sample size. Mean returns at all frequencies are negatively (and statistically significant) skewed for all of the markets. In addition, all returns are highly leptokurtic with respect to the normal distribution. The fact that all returns are negatively skewed implies that the returns distribution of indices have a probability of earnings greater than the mean (in other words the median return is greater than the mean).

The fifth row gives the Kolmogorov-Smirnov (KS) statistic for detecting departures of the data from normality. This is calculated by comparing the empirical and normal cumulative distributions for y_t, with the critical values being calculated using the formula $1.63/\sqrt{T}$ (see for further details L. H. Miller, 1956).[17] All KS values (at all frequencies) are greater than their respective critical values at the one-percent level of significance, rejecting the null hypothesis of normality for the data.

4.2 Empirical Findings

This section discusses the empirical findings using the *AR(ρ)-GJR-GARCH-M* model, where ρ denotes the lag order for the mean specification.[18] We opt to set the autoregressive lag equal to 1 (i.e. ρ=1) since we anticipate that stock markets will quickly respond to new information.[19]

The mean, volatility and SGT distribution estimates for the *AR(1)-GJR-GARCH-M* specification at daily, weekly and monthly frequency are presented in Tables 5, 6 and 7

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[17] The KS is estimated by: $KS = max[F_E(y_t) - F_G(y_t)]$, for $t = 1, 2, \ldots, T$, where $F_E(y_t)$ and $F_G(y_t)$ are the empirical and normal distributions for the indices, respectively.

[18] We adopt this specification because stock returns exhibit asymmetry in response of conditional volatility to positive and negative return shocks (Nelson, 1991 and Glosten et. al., 1993). Therefore, simple GARCH specification fails to account for asymmetric effect. Nevertheless, AR(p)-GARCH-M specification has also been estimated and the results (available upon request) are qualitatively the same.

[19] This is also supported by the Schwarz Information Criterion for the majority of the series.

respectively. Most of the series (at daily, weekly and monthly frequency) are characterized by statistically insignificant AR parameters with the exception of the S&P500 index (with positive and significant effects at daily frequency), CAC40 and TSX60 indices (with significant negative influence at weekly frequency) and TSX60 index (with positive and significant prediction at monthly frequency). As far as the volatility is concerned, in common with many previous studies, the results show volatility to be highly persistent (see parameter γ). Similarly, short term persistence (captured by parameter δ) is in line with the results found elsewhere in the literature while asymmetry parameter ζ points to leverage effects (i.e. negative shocks have greater impact on volatility than that of positive shocks of the same magnitude). Leverage effect disappears for the European markets as we move from daily to monthly frequency.

As for the parameters of the SGT distribution, each combination of the location parameters k and n indicates that the distribution of each series exhibits kurtosis beyond that permitted by the normal distribution.[20] The skewness parameter λ is negative and highly significant (at the 1% level) for all specifications at any frequency indicating that all series are negatively skewed, while parameter k for all series is around two indicating that all series are leptokurtic relative to the normal distribution. Furthermore, the skewness values (S_k) for the series calculated using equation A.16 in the appendix under both specifications range between -0.311 and -0.138 at daily frequency, -0.871 and -0.317 at weekly frequency, and -1.919 and -0.419 at monthly frequency. As for the kurtosis values (K_u) calculated using equation A.17 in the appendix, these range between 3.485 and 4.149 at daily frequency, 3.758 and 6.402 at weekly frequency, and 3.234 and 4.221 at monthly frequency.

[20] For the cases where estimated value of n explodes, we fix it to 100.

Because the normal distribution is nested within the skewed GT distribution, a log-likelihood ratio (LR) statistic could be used to test whether normal distribution is more appropriate.[21] The LR statistic is asymptotically distributed as a chi-square, $\chi^2(3)$, with three degrees of freedom. All LR values are greater than their critical value at the 5% level of significance (11.34), rejecting the null hypothesis (exceptions are only the cases of CAC40 - significant at 10% - and Nikkei 225 – insignificant – both at monthly frequency, indicating that as we move towards to lower frequency we approach the Normal distribution). Similar results are obtained for the case of the Generalized *t* (GT) distribution.[22]

Finally, we employ the ARCH test of Engle to examine the null hypothesis (H_0) of no remaining ARCH(1) effects. This test indicates that for most of the cases the chosen order is adequate. However there are some cases where the null is rejected. Therefore, we re-estimate the model using higher ARCH orders with the results remaining qualitatively the same.[23]

Turning to the results of the “in-mean-effects” we estimate the model twice. Firstly, we estimate the model without isolating the effects of skewness and the “in-mean-effect” is denoted by ξ (i.e. we estimate equation 9) and then we re-estimate the model removing the effects of skewness from the impact of risk to the expected return (i.e. we estimate equation 11, where c denotes the real price of risk and p the price of risk attributed to skewness).

In the first estimation, at daily frequency, parameters ξ for S&P500, FTSE100, CAC40 and DAX30 are positively correlated with the expected returns and they are

[21] The null hypothesis to be tested is H_0: λ=0, k=2 and n=∞.
[22] The null hypothesis is now H_0: λ=0 and the LR statistic is asymptotically distributed as $\chi^2(1)$.
[23] These estimations are available from the authors upon request.

statistically insignificant. For the case of TSX60, it is negative but still insignificant, while for the Nikkei225 is negative and significant. The range of point estimates varies from -0.062 (Nikkei225) to 0.037 (CAC40). At weekly frequency, most of the parameters are negative and insignificant (FTSE100, CAC40 and DAX30) with exception, only as far as the sign is concerned and not the significance the case of S&P500. Nikkei 225 is still negative and significant while TSX60 is negative and significant, with the point estimates at this frequency varying from -0.305 (TSX60) to 0.048 (S&P500). Finally, at monthly frequency all signs of the risk-return parameter ξ appear to be similar to weekly frequency (except CAC40, which is now positive), however only TSX60 is significant. The point estimates are now bigger in magnitude and vary from -0.416 (TSX60) to 0.157 (S&P500).

Overall, the above analysis reveals mostly insignificant relation between expected returns and volatility when the relation is contaminated with the effects of skewness (with some weak evidence of negative effects), supporting in that manner the insignificant/negative relation found in the related literature.

In the second estimation we modify the model to account for skewness and separate the "in-mean-effect" into two components: the real impact of risk to the expected return (parameter *c*) and the effect of skewness (parameter *p*).[24] Under this specification the results are completely different compared to the first estimation. In that case, at all frequencies, the real impact of risk is positive and significant (with minor exceptions being the cases of Canada at weekly frequency, France and Germany at monthly frequency and Japan at all frequencies; where the effect is still positive but insignificant).

[24] Parameter *p* is predetermined and equal to $r\Big/\sqrt{g-r^2}$, see equation 6.

The point estimates of parameter c varies from 0.039 to 0.202, 0.095 to 0.361 and 0.045 to 0.776 at daily, weekly and monthly frequency respectively.

This finding is in line with MC simulations and highlights the importance of skewness in uncovering the true relation between risk and returns. The positive effect is in line with the seminal work on the dynamic risk return tradeoff equilibrium of Merton (1973) who supports that market participants will demand higher returns to hold riskier assets. The fact that (once the effects of skewness are taken into account) the relation turns significantly positive in higher frequencies (daily and weekly) may be attributed to the fact that skewness is more pronounced in higher frequencies while the insignificant effect for three cases at monthly frequency is linked to the findings of Lundblad (2007) who documents that small samples may lead to the insignificant relation between risk and return.

Parameter p is negative and relatively big in magnitude as far as the point estimates are concerned (ranging from -0.230 to -0.082, -0.518 to -0.210 and -1.192 to -0.350 at daily, weekly and monthly frequency respectively). The negative sign of the effects of skewness (p) is in line with the results of Boyer et. al. (2010) who propose that skewness-preferring investors are willing to pay a premium for riskier stocks in return for a chance at having greater returns. However, whether p significantly affects the relation is a matter of further investigation.

Since p is a function of other parameters (r and g), its standard error cannot be obtained directly; hence, we apply a parametric bootstrap procedure.[25] Using this technique we are able to obtain a clearer picture regarding the uncertainty associated with

[25] This procedure has been introduce by Rapach and Wohar (2009) in the context of intertemporal hedging demand under a VAR framework.

the point estimate of *p*. We assume that our series are generated by equations 11 and 12b, where the parameters of the model are set to their maximum likelihood estimates. In order to construct a pseudo sample for y_t, we make a T+100 independent draws from SGT distribution. Using the randomly drawn innovations, equations 11 and 12b with $\theta' = \hat{\theta}'$, we can build up a pseudo-sample of T+100 observations for y_t.[26] For the pseudo-sample, we estimate the model repeating this process 1000 times, obtaining in that manner an empirical distribution for each of the coefficients. We construct 90% confidence intervals for each of the coefficients from the empirical distributions using the percentile method described in Davidson and MacKinnon (1993, p. 766).[27]

For all cases we observe that the 90% confidence intervals for parameter *p* are relatively tight enough at all frequencies. This suggests that all estimates are significant and the null hypothesis that *p* is equal to zero is rejected according to the 90% confidence intervals. Overall, our findings suggest that the omission of the effects of skewness could bias the sign\magnitude of the impact of risk on market returns.

4.3 Time Varying behavior

An obvious concern regarding the above findings is whether the risk return relation changes over time. Our dataset includes different sample periods for the various returns and we expect that the characteristics of each series will vary with sample period and incidents occurred (such as crises). Therefore, assuming a time-invariant risk-return trade-off is rather a strong assumption. As indicated by Lundblad (2007) the risk-return

[26] First 100 observations are discarded in order to avoid initialization problems leaving us with a pseudo-sample of T observations for y_t, matching the original sample.
[27] We report results only for the parameter ρ. For the rest of the parameters (where standard error is directly obtained from the estimation process) results are available upon request.

trade-off is described by risk aversion, which may vary with economic conditions, evolution of financial markets, improved risk sharing, etc. As a consequence of the above, a snapshot of this relation may conceal substantial differences over time.

To capture the time varying behaviour of the relation, we rerun the model using rolling windows of 2500 observations for all indices.[28] A rolling window analysis of a time series is often used to assess parameter's stability over time. In this technique the estimate of risk-return relation (with and without accounting for the effects of skewness) is updated by removing the first observation and incorporating the next observation in the sample. The process is then repeated until the last available observation in the sample is used. We assess the stability by plotting the time varying behaviour of the coefficients of interest.

Figures 2a to 2f, show the time varying pattern of risk-return trade-off: (i) real impact of risk – parameter c (solid black line) and (ii) impact of risk contaminated with the effects of skewness – parameter ξ (grey line). In addition to the above, the time varying pattern of skewness - parameter λ - is depicted in dotted black line.

As it can be seen by the plots, the magnitude of all parameters is changing with time, suggesting that risk-return relation is time varying. Nevertheless, over the available data period the full record of risk-return trade-off is consistently positive once we account for the effects of skewness with some minor exceptions for some periods for the case of Nikkei225.[29] In contrast to the above, in case we do not account for the effects of skewness, the sign of risk-return tradeoff is mixed, supporting the often mixed findings reported in the literature.

[28] The results are robust using windows of 1500 and 2000 observations. These are available upon request.
[29] During these periods estimated skewness (λ) appears to be higher than parameter (ξ).

These findings, once again, suggest that the mixed results indicated in the literature may be attributed to the omission of important factors such as the effects of skewness.

5. Conclusions

This paper revisits the puzzle of the relation between risk and return and attempts to explain the contradictory empirical findings in the literature. The theoretical and empirical investigation of the relationship between risk and return in large portfolios is carried out using the analytical framework based on the popular SGT distribution, on Monte Carlo simulations, on bootstrapping and on rolling window regressions. Based on analytical derivation, the risk-return parameter is decomposed into two components: a price of risk component and an idiosyncratic component, that depends purely on the skewness and kurtosis. We claim that this idiosyncratic component, which disappears in lower frequency data, should not matter for pricing purposes. This claim is also consistent with the assumptions of M-V framework, which has been widely used in stock return models. Under this framework, skewness and kurtosis are not relevant parameters in asset pricing models, therefore should not be priced. However, they contaminate the estimation of the risk-return relation, and particularly the price of risk parameter through the estimation process (where we are able to decompose the effect into the real risk price effect and the effect that depends entirely on skewness).

By decomposing these effects, the main findings (using Monte Carlo simulations and real data at daily, weekly and monthly frequency) indicate that once we account for the effects of skewness, the risk-return relation becomes positive and highly significant (with the effect however becoming less influential as we move from daily to monthly data). The omission of the effects of skewness could bias the sign\magnitude of the impact of risk on market returns. These results are also supported when rolling window estimations are employed to uncover the time varying pattern of this relation.

Therefore, our study indicates that correctly specified parametric models that take into account the effects of skewness are able to deliver positive risk-return relations in line with the theory.

This finding has several important implications. Firstly, it helps to explain the often-mixed results in the literature because it demonstrates that the empirical relation between risk and returns is by the skewness in the data. Secondly, it is important for the estimation and measurement of required return in stock pricing models. Thirdly and more importantly, it has implications for international risk analysis and portfolio construction. For buy-and-hold investors, which includes the traditional institutional investors in a M-V world, skewness, for practical reasons, is an irrelevant issue (since skewness is not typically present in higher frequency data such as monthly, quarterly, etc). Being able to decompose the pricing of risk parameter into its real and skewness impact components, we are able to get the true measure of risk pricing irrespective of the frequency of data used. Since, as demonstrated in this article, the effects of skewness should not be omitted in assessing the risk–return relation in stock markets, investors should consider it when they construct their portfolio.

Finally, the above findings encourage for further research in the area of parametric models and specifically in the multivariate version of our specification, which has been used extensively so far, not only in finance but also in economic literature, however, without accounting for possible skewness effects in the data.

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APPENDIX 1.A - DERIVATIONS FOR THE SGT DISTRIBUTION

The r^{th} non-central moment of the random variable $u = y - m$ following the non-centered skewed generalized t (SGT) below

$$dF_u = C\left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1+sign(u)\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du\,, \tag{A.1}$$

for integer values of $r < n$, is

$$M_r = \int_{-\infty}^{\infty} u^r dF_u = C\int_{-\infty}^{\infty} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1+sign(u)\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du$$

$$= C\int_{-\infty}^{0} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1-\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du + C\int_{0}^{\infty} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1+\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du$$

$$= (-1)^r C\int_{0}^{\infty} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1-\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du + C\int_{0}^{\infty} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1+\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du\,. \tag{A.2}$$

The above two integrals can be re-written as

$$\int_{0}^{\infty} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)\left(1\pm\lambda\right)^k \phi^k}\right)^{-\frac{n+1}{k}} du = \int_{0}^{\infty} u^r \left(1+\frac{u^k}{q}\right)^{-\kappa} du \tag{A.3}$$

where $q \equiv \left((n+1)/k\right)\left(1\pm\lambda\right)^k \phi^k$ and $\kappa \equiv (n+1)/k$. Gradshteyn and Ryzhik (1994, p. 341) show that

$$\int_0^{\infty} u^r \left(1+\frac{u^k}{q}\right)^{-\kappa} du \;= k^{-1} q^{\frac{r+1}{k}} \Gamma\left(\frac{\kappa k-(r+1)}{k}\right)\Gamma\left(\frac{r+1}{k}\right)\Gamma(\kappa)^{-1}$$

$$= k^{-1} q^{\frac{r+1}{k}} B\left(\frac{\kappa k-(r+1)}{k},\frac{r+1}{k}\right), \qquad \text{(A.4)}$$

where $q \neq 0$, $n \neq 0$ and $0 < (r+1)/k < \kappa$ $(\text{or } 0 < r < n)$. It follows easily from the above that

$$\int_0^{\infty} u^r \left(1+\frac{|u|^k}{\left((n+1)/k\right)(1\pm\lambda)^k \phi^k}\right)^{-\frac{n+1}{k}} du = k^{-1}\left(\frac{n+1}{k}\right)^{\frac{r+1}{k}} (1\pm\lambda)^{r+1} \phi^{r+1} B\left(\frac{n-r}{k},\frac{r+1}{k}\right), \qquad \text{(A.5)}$$

and

$$M_r = (-1)^r Ck^{-1}\left(\frac{n+1}{k}\right)^{\frac{r+1}{k}} (1-\lambda)^{r+1}\phi^{r+1} B\left(\frac{n-r}{k},\frac{r+1}{k}\right) + Ck^{-1}\left(\frac{n+1}{k}\right)^{\frac{r+1}{k}} (1+\lambda)^{r+1}\phi^{r+1} B\left(\frac{n-r}{k},\frac{r+1}{k}\right)$$

$$= \left[(-1)^r (1-\lambda)^{r+1} + (1+\lambda)^{r+1}\right] Ck^{-1}\left(\frac{n+1}{k}\right)^{\frac{r+1}{k}} B\left(\frac{n-r}{k},\frac{r+1}{k}\right)\phi^{r+1} \qquad \text{(A.6)}$$

For dF to be proper probability density function,

$$M_0 = \int_{-\infty}^{\infty} dF_u = \left[(1-\lambda)+(1+\lambda)\right] Ck^{-1}\left(\frac{n+1}{k}\right)^{\frac{1}{k}} B\left(\frac{n}{k},\frac{1}{k}\right)\phi = 2Ck^{-1}\left(\frac{n+1}{k}\right)^{\frac{1}{k}} B\left(\frac{n}{k},\frac{1}{k}\right)\phi = 1,$$

thus

$$C = 0.5k\left(\frac{n+1}{k}\right)^{-\frac{1}{k}} B\left(\frac{n}{k},\frac{1}{k}\right)^{-1} \phi^{-1}. \qquad \text{(A.7)}$$

Substitution of C into the M_r equation gives,

$$M_r = 0.5\left[(-1)^r (1-\lambda)^{r+1} + (1+\lambda)^{r+1}\right]\left(\frac{n+1}{k}\right)^{\frac{r}{k}} B\left(\frac{n-r}{k},\frac{r+1}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1} \phi^r. \qquad \text{(A.8)}$$

The expected value of u, provided that n >1, is

$$E(u) = M_1 = 0.5\left[(-1)(1-\lambda)^2 + (1+\lambda)^2\right]\left(\frac{n+1}{k}\right)^{\frac{1}{k}} B\left(\frac{n-1}{k}, \frac{2}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} \phi$$

$$= 2\lambda\left(\frac{n+1}{k}\right)^{\frac{1}{k}} B\left(\frac{n-1}{k}, \frac{2}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} \phi = r\phi, \qquad \text{(A.9)}$$

where $r \equiv 2\lambda\left(\frac{n+1}{k}\right)^{\frac{1}{k}} B\left(\frac{n-1}{k}, \frac{2}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1}$.

The second non-centered moment of u, provided that n > 2, is

$$E(u^2) = M_2 = 0.5\left[(-1)^2(1-\lambda)^3 + (1+\lambda)^3\right]\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-2}{k}, \frac{3}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} \phi^2$$

$$= (1+3\lambda^2)\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-2}{k}, \frac{3}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} \phi^2 = g\phi^2 \qquad \text{(A.10)}$$

where $g \equiv (1+3\lambda^2)\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-2}{k}, \frac{3}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1}$.

In this case, the variance of u is

$$\sigma^2 = E(u^2) - E(u)^2 = (g - r^2)\phi^2, \qquad \text{(A.11)}$$

where $g - r^2 > 0$ (see below).

In the above expression the variance, expressed in terms of ϕ, exists for as long as n > 2, although the value of ϕ exists for any value of n > 0. Note that

$$g - r^2 = (1+3\lambda^2)\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-2}{k}, \frac{3}{k}\right) B\left(\frac{n}{k}, \frac{1}{k}\right)^{-1} - 4\lambda^2\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-1}{k}, \frac{2}{k}\right)^2 B\left(\frac{n}{k}, \frac{1}{k}\right)^{-2}$$

$$=\left[1+3\lambda^2-4\lambda^2 B\left(\frac{n-2}{k},\frac{3}{k}\right)^{-1} B\left(\frac{n-1}{k},\frac{2}{k}\right)^2 B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}\right]\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-2}{k},\frac{3}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}$$

$$=S(\lambda)\left(\frac{n+1}{k}\right)^{\frac{2}{k}} B\left(\frac{n-2}{k},\frac{3}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}>0$$

because $S(\lambda) > 0$ (the latter can be proven using the Stirling's approximation of the gamma function).

The third non-centered moment of u, provided that $n > 3$, is

$$E\left(u^3\right)=M_3=0.5\left[(-1)^3(1-\lambda)^4+(1+\lambda)^4\right]\left(\frac{n+1}{k}\right)^{\frac{3}{k}} B\left(\frac{n-3}{k},\frac{4}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}\phi^3$$

$$=4\lambda\left(1+\lambda^2\right)\left(\frac{n+1}{k}\right)^{\frac{3}{k}} B\left(\frac{n-3}{k},\frac{4}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}\phi^3=A_3\phi^3, \qquad \text{(A.12)}$$

where $A_3 \equiv 4\lambda\left(1+\lambda^2\right)\left(\frac{n+1}{k}\right)^{\frac{3}{k}} B\left(\frac{n-3}{k},\frac{4}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}\phi^3$

The third centered moment is,

$$m_3=E\left(u-M_1\right)^3=Eu^3-3M_1Eu^2+3M_1^2Eu-M_1^3$$

$$=A_3\phi^3-3gr\phi^3+2r^3\phi^3=\left(A_3-3gr+2r^3\right)\phi^3 \qquad \text{(A.13)}$$

The fourth non-centered moment of u, provided that $n > 4$, is

$$E\left(u^4\right)=M_3=0.5\left[(-1)^4(1-\lambda)^5+(1+\lambda)^5\right]\left(\frac{n+1}{k}\right)^{\frac{4}{k}} B\left(\frac{n-4}{k},\frac{5}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}\phi^4$$

$$=\left(1+10\lambda^2+5\lambda^4\right)\left(\frac{n+1}{k}\right)^{\frac{4}{k}} B\left(\frac{n-4}{k},\frac{5}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}\phi^4=A_4\phi^4 \qquad \text{(A.14)}$$

where $A_4 \equiv \left(1+10\lambda^2+5\lambda^4\right)\left(\frac{n+1}{k}\right)^{\frac{4}{k}} B\left(\frac{n-4}{k},\frac{5}{k}\right) B\left(\frac{n}{k},\frac{1}{k}\right)^{-1}$.

The fourth centered moment of u is

$$m_4 = E\left(u-M_1\right)^4 = Eu^4 - 4M_1Eu^3 + 6M_1^2Eu^2 - 4M_1^3Eu + M_1^4$$

$$= \left(A_4 - 4A_3r + 6gr^2 - 3r^4\right)\phi^4 \tag{A.15}$$

The skewness and kurtosis measures are

$$SK = \frac{m_3}{\sigma^3} = \frac{A_3 - 3gr + 2r^3}{\left(g-r^2\right)^{3/2}} \tag{A.16}$$

and

$$KU = \frac{m_3}{\sigma^4} = \frac{A_4 - 4A_3r + 6gr^2 - 3r^4}{\left(g-r^2\right)^2}. \tag{A.17}$$

Table 1: GARCH and GJR-GARCH under Normal distribution

Panel a: GARCH-M

	Conditional Mean			*Conditional Variance*			
	a	*b*	*c*	*v*	*δ*	*ζ*	*γ*
True Values	***0.05***	***0.10***	***0.30***	***0.10***	***0.05***	**-**	***0.80***
Avg	0.038	0.099	-0.063	0.140	0.061	-	0.763
St. Dev.	0.266	0.025	0.305	0.047	0.016	-	0.069
Avg St. Dev.	0.285	0.025	0.329	0.095	0.028	-	0.137
% rejected	0.029	0.053	0.346	0.229	0.237	-	0.256

Panel b: GJRGARCH-M

	Conditional Mean			*Conditional Variance*			
	a	*b*	*c*	*v*	*δ*	*ζ*	*γ*
True Values	***0.05***	***0.10***	***0.30***	***0.10***	***0.05***	***0.05***	***0.80***
Avg	0.045	0.100	-0.072	0.126	0.060	0.061	0.784
St. Dev.	0.151	0.026	0.157	0.029	0.024	0.030	0.039
Avg St. Dev.	0.157	0.025	0.165	0.045	0.035	0.046	0.059
% rejected	0.041	0.048	0.690	0.179	0.144	0.198	0.176

Notes: This table reports Monte Carlo simulation results for GJR and GARCH-in-mean specifications under Normal distribution without account for the effects of skewness, based on 1,000 randomly generated samples of 2000 observations each. *Avg.* refers to the average value of the estimated parameters, *St. Dev.* to the standard deviation of the estimates, *Avg. St. Dev.* to the average standard deviation of the estimated parameters, and *% rejected* to the test of whether the difference of the estimated parameter from its true value is significant or not.

Table 2: GARCH and GJR-GARCH under Symmetric GT distribution

Panel a: GARCH-M

	Conditional Mean			*Conditional Variance*				*SGT parameters*		
	a	*b*	*c*	*v*	*δ*	*ζ*	*γ*	*n*	*k*	*λ*
True Values	**0.05**	**0.10**	**0.30**	**0.10**	**0.05**	**-**	**0.80**	**10**	**1.5**	**0**
Avg.	0.043	0.098	0.011	0.132	0.058	-	0.780	5.807	1.853	-
St. Dev.	0.270	0.023	0.305	0.072	0.022	-	0.103	3.238	0.224	-
Avg. St. Dev.	0.264	0.022	0.300	0.061	0.021	-	0.087	2.061	0.252	-
% rejected	0.035	0.052	0.299	0.080	0.064	-	0.105	0.714	0.167	-

Panel b: GJRGARCH-M

	Conditional Mean			*Conditional Variance*				*SGT parameters*		
	a	*b*	*c*	*v*	*δ*	*ζ*	*γ*	*n*	*k*	*λ*
True Values	**0.05**	**0.10**	**0.30**	**0.10**	**0.05**	**0.05**	**0.80**	**10**	**1.5**	**0**
Avg.	0.040	0.098	0.014	0.123	0.057	0.070	0.803	5.925	1.857	-
St. Dev.	0.131	0.024	0.127	0.036	0.033	0.039	0.043	4.507	0.231	-
Avg. St. Dev.	0.125	0.023	0.128	0.034	0.031	0.037	0.041	3.238	0.252	-
% rejected	0.058	0.064	0.639	0.045	0.064	0.091	0.077	0.714	0.192	-

Notes: This table reports Monte Carlo simulation results for GJR and GARCH-in-mean specifications under SGT distribution taking into account the effects of skewness, based on 1,000 randomly generated samples of 2000 observations each. *Avg.* refers to the average value of the estimated parameters, *St. Dev.* to the standard deviation of the estimates, *Avg. St. Dev.* to the average standard deviation of the estimated parameters, and *% rejected* to the test of whether the difference of the estimated parameter from its true value is significant or not. To calculate *% rejected* for the case of parameter *p* we use *St. Dev.*

Table 3: GARCH and GJR-GARCH under SGT distribution (taking into account skewness)

Panel a: GARCH-M

	Conditional Mean			*Conditional Variance*				*SGT parameters*			*Skewness effect*
	a	*b*	*c*	*v*	*δ*	*ζ*	*γ*	*n*	*k*	*λ*	*P*
True Values	**0.05**	**0.10**	**0.30**	**0.10**	**0.05**	**-**	**0.80**	**10**	**1.5**	**-0.25**	**-0.344**
Avg.	0.012	0.099	0.325	0.130	0.058	-	0.779	15.67	1.495	-0.251	-0.360
St. Dev.	0.240	0.021	0.127	0.058	0.020	-	0.084	22.58	0.169	0.027	0.037
Avg. St. Dev.	0.293	0.021	0.133	0.075	0.022	-	0.107	14.34	0.158	0.027	-
% rejected	0.046	0.065	0.056	0.053	0.052	-	0.107	0.090	0.061	0.055	0.068

Panel b: GJRGARCH-M

	Conditional Mean			*Conditional Variance*				*SGT parameters*			*Skewness effect*
	a	*b*	*c*	*v*	*δ*	*ζ*	*γ*	*n*	*k*	*λ*	*P*
True Values	**0.05**	**0.10**	**0.30**	**0.10**	**0.05**	**0.05**	**0.80**	**10**	**1.5**	**-0.25**	**-0.344**
Avg.	0.046	0.099	0.285	0.120	0.051	0.084	0.803	16.52	1.489	-0.249	-0.358
St. Dev.	0.120	0.022	0.121	0.031	0.027	0.036	0.038	21.99	0.165	0.027	0.038
Avg. St. Dev.	0.118	0.021	0.120	0.033	0.027	0.037	0.041	15.62	0.155	0.027	-
% rejected	0.044	0.049	0.052	0.053	0.037	0.156	0.078	0.081	0.073	0.050	0.066

Notes: This table reports Monte Carlo simulation results for GJR and GARCH-in-mean specifications under SGT distribution taking into account the effects of skewness, based on 1,000 randomly generated samples of 2000 observations each. *Avg.* refers to the average value of the estimated parameters, *St. Dev.* to the standard deviation of the estimates, *Avg. St. Dev.* to the average standard deviation of the estimated parameters, and *% rejected* to the test of whether the difference of the estimated parameter from its true value is significant or not. To calculate *% rejected* for the case of parameter *p* we use *St. Dev.*

Table 4. Preliminary Statistics

	S&P500 (US)	FTSE100 (UK)	NIKKEI225 (JAPAN)	TSX60 (CANADA)	CAC40 (FRANCE)	DAX30 (GERMANY)
Period	03/01/1950-05/06/2012	02/04/1984-06/06/2012	04/01/1984-06/06/2012	03/01/1971-06/06/2012	01/03/1990-06/06/2012	01/03/1990-06/06/2012
Panel a: Daily frequency						
Mean	0.031	0.024	0.000	0.016	0.007	0.025
Variance	0.839	1.169	1.950	1.586	1.961	2.074
Skewness	**-0.090*** **	**-0.203*** **	**-0.130*** **	**-0.269*** **	**-0.176*** **	**-0.230*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
Kurtosis	**6.013*** **	**5.603*** **	**5.437*** **	**6.175*** **	**5.336*** **	**5.707*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
KS	**0.089*** **	**0.042*** **	**0.050*** **	**0.051*** **	**0.047*** **	**0.049*** **
	(0.000)	**(0.001)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
Panel b: Weekly frequency						
Mean	0.137	0.123	0.003	0.053	0.055	0.141
Variance	4.135	5.215	7.961	7.148	8.684	9.642
Skewness	**-0.312*** **	**-0.295*** **	**-0.279*** **	**-0.515*** **	**-0.253*** **	**-0.296*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
Kurtosis	**5.035*** **	**5.051*** **	**4.755*** **	**6.122*** **	**4.401*** **	**5.145*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
KS	**0.167*** **	**0.188*** **	**0.221*** **	**0.198*** **	**0.251*** **	**0.253*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
Panel c: Monthly frequency						
Mean	0.589	0.497	-0.045	0.182	0.164	0.575
Variance	17.411	20.365	37.890	22.059	32.668	39.341
Skewness	**-0.461*** **	**-0.601*** **	**-0.480** **	**-0.947*** **	**-0.486*** **	**-0.724*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.001)**	**(0.000)**
Kurtosis	**4.157*** **	**3.995*** **	**3.872*** **	**4.729*** **	**3.244*** **	**4.904*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**
KS	**0.361*** **	**0.399*** **	**0.367*** **	**0.382*** **	**0.389*** **	**0.423*** **
	(0.000)	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**	**(0.000)**

Notes: The statistics for skewness is $b_1=m_3/m_2^{3/2}$ and for kurtosis is $b_2=m_4/m_2^2$, where m_j is the estimate for the j_{th} moment around the mean. The Bera-Jarque statistic for testing normality is $BJ=T(b_1^2/6 + b_2^2/24)$. This is asymptotically distributed as $\chi^2(2)$ with 2 degrees of freedom. Its critical value at the one-percent level is 9.21. KS is the Kolmogorov-Smirnov statistic for testing the null hypothesis of normality. p-values in the brackets.

Table 5. AR(1)–GJRGARCH-M Skewed GT Estimates – Daily Returns						
	S&P500 (US)	**FTSE100 (UK)**	**NIKKEI225 (JAPAN)**	**TSX 60 (CANADA)**	**CAC40 (FRANCE)**	**DAX30 (GERMANY)**
Panel a: Maximum Likelihood Estimates						
a	0.013	0.003	**0.094*** **	0.040	-0.027	0.030
	[0.015]	[0.034]	**[0.031]**	[0.057]	[0.050]	[0.039]
b	**0.096*** **	0.016	-0.003	-0.028	0.001	-0.008
	[0.008]	[0.012]	[0.012]	[0.017]	[0.013]	[0.013]
c	**0.106*** **	**0.200*** **	0.039	**0.202*** **	**0.180*** **	**0.161*** **
	[0.027]	**[0.052]**	[0.039]	**[0.076]**	**[0.055]**	**[0.049]**
ξ	0.024	0.030	**-0.062** **	-0.029	0.037	0.006
	[0.022]	[0.041]	**[0.030]**	[0.050]	[0.045]	[0.037]
v	**0.006*** **	**0.015*** **	**0.018*** **	**0.015*** **	**0.026*** **	**0.019*** **
	[0.001]	**[0.003]**	**[0.004]**	**[0.004]**	**[0.006]**	**[0.005]**
δ	**0.022*** **	**0.028*** **	**0.032*** **	0.007	0.008	**0.022*** **
	[0.004]	**[0.006]**	**[0.006]**	[0.009]	[0.008]	**[0.006]**
ζ	**0.092*** **	**0.080*** **	**0.111*** **	**0.109*** **	**0.106*** **	**0.103*** **
	[0.009]	**[0.010]**	**[0.014]**	**[0.018]**	**[0.013]**	**[0.015]**
γ	**0.925*** **	**0.916*** **	**0.903*** **	**0.926*** **	**0.924*** **	**0.916*** **
	[0.006]	**[0.008]**	**[0.008]**	**[0.011]**	**[0.009]**	**[0.009]**
n	**11.622*** **	**11.307*** **	**10.551*** **	24.908	**14.712*** **	**13.135*** **
	[2.178]	**[2.741]**	**[2.250]**	[30.785]	**[5.565]**	**[4.619]**
k	**1.824*** **	**2.271*** **	**1.916*** **	**1.818*** **	**2.057*** **	**1.912*** **
	[0.072]	**[0.143]**	**[0.109]**	**[0.177]**	**[0.145]**	**[0.132]**
λ	**-0.054*** **	**-0.108*** **	**-0.066*** **	**-0.150*** **	**-0.092*** **	**-0.100*** **
	[0.011]	**[0.018]**	**[0.016]**	**[0.025]**	**[0.020]**	**[0.019]**
Panel b: Confidence Intervals for parameter *p* using Rapach and Wohar (2009) bootstrapping procedure						
p	**-0.082**	**-0.170**	**-0.101**	**-0.231**	**-0.143**	**-0.155**
	{-0.114 -0.056}	**{-0.219 - 0.126}**	**{-0.142 -0.065}**	**{-0.298 -0.170}**	**{-0.193 -0.092}**	**{-0.207 -0.111}**
logL	-18169.7	-9577.2	-11154.5	-4515.0	-9100.7	-8707.3
LR	**469.56*** **	**83.03*** **	**190.7*** **	**65.53*** **	**73.31*** **	**131.75*** **
Sk	-0.138	-0.207	-0.165	-0.311	-0.184	-0.229
Ku	4.139	3.485	4.100	3.609	3.510	3.842
ARCH(1)	**21.999*** **	0.317	0.401	**3.396* **	**2.949* **	**5.661** **
p-value	**(0.004)**	(0.574)	(0.527)	**(0.065)**	**(0.086)**	**(0.017)**

Notes. Series are expressed as continuously compounded daily returns (logarithmic changes). Standard errors for the estimators are included in squared brackets. Bold letters denote significance at 10% (*), 5% (**) and 1% (***) level respectively. LR is the log-likelihood ratio for testing the null hypothesis that the series are distributed as normal against the alternative hypothesis that the series are distributed as skewed GT. The LR follows $\chi^2(3)$. Its critical value at the one-percent level of significance is 11.34. S_k and K_u, are the skewness and kurtosis measures calculated using equations (A16) and (A17) in the appendix. ARCH denotes the LM test statistic for remaining ARCH effects in the series. p-values in the brackets.

Table 6. AR(1)–GJRGARCH-M Skewed GT Estimates –Weekly Returns						
	S&P500 (US)	**FTSE100 (UK)**	**NIKKEI225 (JAPAN)**	**TSX 60 (CANADA)**	**CAC40 (FRANCE)**	**DAX30 (GERMANY)**
Panel a: Maximum Likelihood Estimates						
a	0.071	0.189	**0.457****	**0.714*****	0.099	0.201
	[0.107]	[0.207]	**[0.236]**	**[0.186]**	[0.249]	[0.270]
b	-0.017	-0.028	0.000	**-0.115*****	**-0.065*****	-0.051
	[0.019]	[0.027]	[0.028]	**[0.041]**	**[0.030]**	[0.032]
c	**0.336*****	**0.180****	0.095	0.183	**0.261****	**0.361*****
	[0.076]	**[0.091]**	[0.120]	[0.177]	**[0.136]**	**[0.133]**
ξ	0.048	-0.030	**-0.160***	**-0.340*****	-0.013	-0.010
	[0.062]	[0.107]	**[0.095]**	**[0.087]**	[0.098]	[0.097]
v	**0.123*****	**0.225*****	**0.0317****	0.072	**0.330*****	**0.441*****
	[0.037]	**[0.075]**	**[0.144]**	[0.048]	**[0.129]**	**[0.187]**
δ	**0.037*****	0.014	0.029	-0.011	**0.029***	**0.037***
	[0.011]	[0.017]	[0.019]	[0.039]	**[0.017]**	**[0.022]**
ζ	**0.0122*****	**0.156*****	**0.162*****	**0.184*****	**0.158*****	**0.182*****
	[0.029]	**[0.034]**	**[0.057]**	**[0.052]**	**[0.041]**	**[0.062]**
γ	**0.869*****	**0.863*****	**0.851*****	**0.918*****	**0.852*****	**0.822*****
	[0.023]	**[0.027]**	**[0.037]**	**[0.031]**	**[0.026]**	**[0.046]**
n	**10.006*****	**7.979*****	**5.088*****	100	**6.790****	**8.269*****
	[3.033]	**[2.984]**	**[0.995]**	[.]	**[1.864]**	**[3.739]**
k	**2.201*****	**2.254*****	**2.806*****	**1.423*****	**3.023*****	**2.381*****
	[0.217]	**[0.332]**	**[0.436]**	**[0.398]**	**[0.527]**	**[0.395]**
λ	**-0.186*****	**-0.135*****	**-0.166*****	**-0.360*****	**-0.173*****	**-0.240*****
	[0.027]	**[0.041]**	**[0.043]**	**[0.055]**	**[0.050]**	**[0.049]**
Panel b: Confidence Intervals for parameter *p* using Rapach and Wohar (2009) bootstrapping procedure						
p	**-0.288**	**-0.210**	**-0.255**	**-0.522**	**-0.274**	**-0.371**
	{-0.359 -0.230}	**{-0.313 - 0.108}**	**{-0.352 -0.137}**	**{-0.990 -0.457}**	**{-0.428 -0.156}**	**{-0.483 -0.266}**
logL	-6550.0	-3155.7	-3482.9	-1433.3	-2784.9	-2699.6
LR	**94.99*****	**38.22*****	**71.64*****	**67.49*****	**27.55*****	**42.55*****
Sk	-0.390	-0.317	-0.485	-0.883	-0.323	-0.501
Ku	3.834	4.144	6.402	4.662	3.758	4.058
ARCH(1)	1.937	0.010	0.751	0.171	0.097	0.424
p-value	(0.164)	(0.919)	(0.386)	(0.676)	(0.755)	(0.515)

Notes. Series are expressed as continuously compounded daily returns (logarithmic changes). Standard errors for the estimators are included in squared brackets. Bold letters denote significance at 10% (*), 5% (**) and 1% (***) level respectively. LR is the log-likelihood ratio for testing the null hypothesis that the series are distributed as normal against the alternative hypothesis that the series are distributed as skewed GT. The LR follows $\chi^2(3)$. Its critical value at the one-percent level of significance is 11.34. S_k and K_u, are the skewness and kurtosis measures calculated using equations (A16) and (A17) in the appendix. ARCH denotes the LM test statistic for remaining ARCH effects in the series. p-values in the brackets.

Table 7. AR(1)–GJRGARCH-M Skewed GT Estimates –Monthly Returns						
	S&P500 (US)	**FTSE100 (UK)**	**NIKKEI225 (JAPAN)**	**TSX 60 (CANADA)**	**CAC40 (FRANCE)**	**DAX30 (GERMANY)**
Panel a: Maximum Likelihood Estimates						
a	-0.020	1.171	1.976	**1.610****	0.060	2.462
	[0.852]	[0.733]	[2.534]	**[0.692]**	[0.397]	[1.726]
b	-0.003	-0.028	0.031	**0.221****	0.078	0.005
	[0.044]	[0.057]	[0.085]	**[0.108]**	[0.085]	[0.059]
c	**0.507****	**0.271***	0.045	**0.776*****	0.462	0.105
	[0.258]	**[0.165]**	[0.383]	**[0.220]**	[0.370]	[0.298]
ξ	0.157	-0.138	-0.341	**-0.416*****	0.028	-0.295
	[0.220]	[0.174]	[0.440]	**[0.141]**	[0.101]	[0.288]
v	**1.803*****	1.020	**8.392****	1.051	5.753	**2.775****
	[0.694]	[1.450]	**[4.216]**	[0.900]	[5.231]	**[1.332]**
δ	0.021	0.146	**-0.089***	-0.094	0.054	0.054
	[0.036]	[0.090]	**[0.047]**	[0.133]	[0.095]	[0.072]
ζ	**0.151*****	0.037	**0.312****	**0.399****	0.227	0.103
	[0.054]	[0.190]	**[0.101]**	**[0.216]**	[0.182]	[0.086]
γ	**0.793*****	**0.800*****	**0.695*****	**0.885*****	**0.642*****	**0.830*****
	[0.055]	**[0.124]**	**[0.124]**	**[0.090]**	**[0.187]**	**[0.041]**
n	8.627	29.344	57.494	100	100	**3.600*****
	[5.467]	[120.69]	[95.952]	[.]	[.]	**[0.967]**
k	**2.201*****	**1.631*****	**1.960*****	2.238	**1.972*****	**3.250*****
	[0.453]	**[0.562]**	**[0.343]**	[1.706]	**[0.376]**	**[1.282]**
λ	**-0.228*****	**-0.272*****	**-0.248*****	**-0.838*****	**-0.278*****	**-0.278*****
	[0.056]	**[0.064]**	**[0.086]**	**[0.091]**	**[0.077]**	**[0.108]**
Panel b: Confidence Intervals for parameter *p* using Rapach and Wohar (2009) bootstrapping procedure						
p	**-0.350**	**-0.409**	**-0.386**	**-1.192**	**-0.434**	**-0.400**
	{-0.506 -0.206}	**{-0.626 - 0.209}**	**{-0.625 -0.218}**	**{-1.300 -0.263}**	**{-0.689 -0.142}**	**{-0.679 -0.105}**
logL	-2083.4	-966.7	-1081.4	-416.4	-825.6	-816.1
LR	**30.84*****	**19.97*****	0.474	**22.14*****	**9.427***	**27.54*****
Sk	-0.512	-0.624	-0.419	-0.858	-0.450	-1.919
Ku	4.221	4.132	3.290	3.503	3.244	3.648
ARCH(1)	0.459	**7.171*****	0.152	0.991	0.979	0.121
p-value	(0.498)	(0.007)	(0.696)	(0.319)	(0.323)	(0.728)

Notes. Series are expressed as continuously compounded daily returns (logarithmic changes). Standard errors for the estimators are included in squared brackets. Bold letters denote significance at 10% (*), 5% (**) and 1% (***) level respectively. LR is the log-likelihood ratio for testing the null hypothesis that the series are distributed as normal against the alternative hypothesis that the series are distributed as skewed GT. The LR follows $\chi^2(3)$. Its critical value at the one-percent level of significance is 11.34. S_k and K_u, are the skewness and kurtosis measures calculated using equations (A16) and (A17) in the appendix. ARCH denotes the LM test statistic for remaining ARCH effects in the series. p-values in the brackets.

Figure 1. The Bias in the Price of Risk due to Skewness and Kurtosis

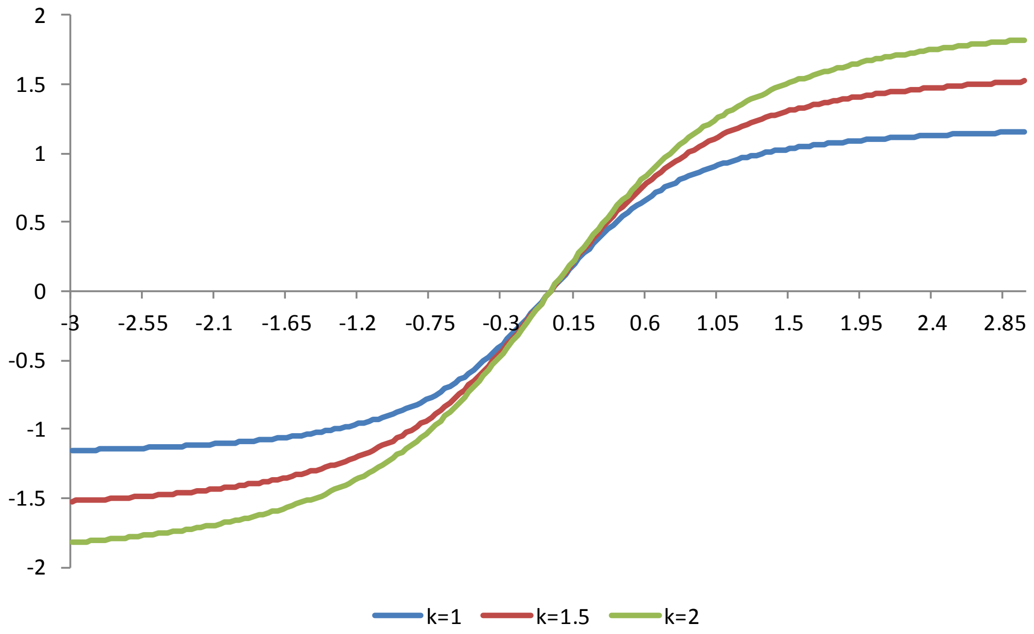
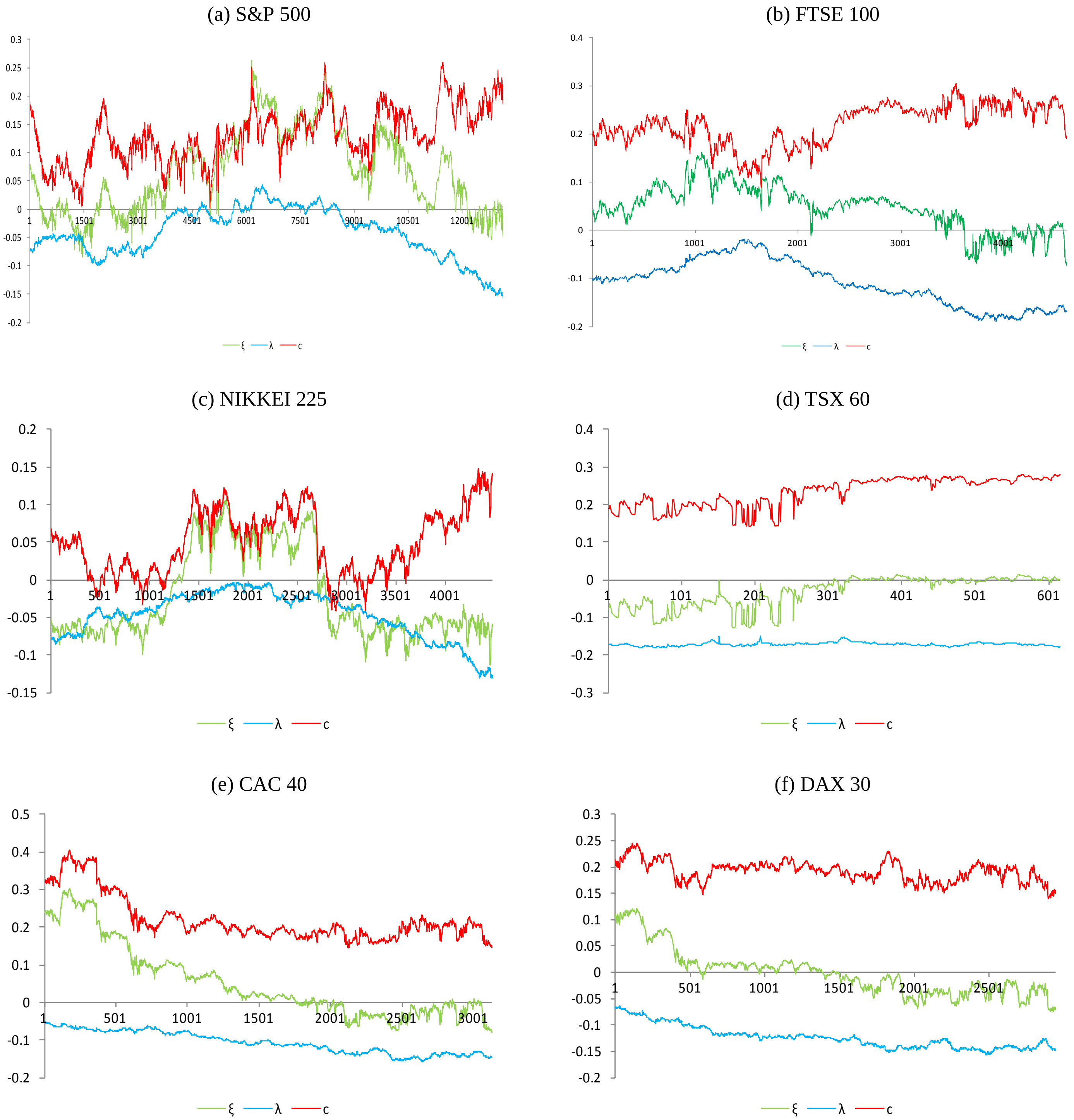


Figure 2. Time Varying Relation Between Risk and Returns



Notes. This figure shows the time varying pattern of risk-return tradeoff using rolling windows estimations. The real impact of risk – parameter c - is in red, the impact of risk contaminated with the effects of skewness – parameter ξ - is in green and the In addition, the time varying pattern of skewness - parameter λ - is in blue.