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When the Going Gets Tough: Durable Consumption and the Equity Premium

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When the Going Gets Tough: Durable Consumption and the Equity Premium☆

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Abstract

I present an endowment economy where a representative agent has recursive preferences over the consumption of non-durable and durable goods, and uncertainty about the underlying endowments. Using parameter calibration consistent with real business cycle literature (risk aversion coefficient of 2.1 and elasticity of intertemporal substitution of 1.09), the model generates a high level of equity premium and a low and stable risk-free rate. The model is also able to explain up to 60% of the equity volatility. The volatile expenditure on durable consumption goods generates a high and volatile equity premium; endogenous time-varying uncertainty produces a counter-cyclical equity premium.

Keywords: asset pricing, equity premium, durable goods, uncertainty
JEL: E21, E32, E44, G12

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1. Introduction

Over the last hundred years the average difference between the returns to stocks and to US Treasury bills (the equity premium) was about 6% and has posed a challenge to the asset pricing literature ever since. The representative agent framework that separated durable from non-durable consumption provided one way of explaining the high equity premium observed in the data (see, for example, Dunn & Singleton, 1986; Eichenbaum & Hansen, 1990; Ogaki & Reinhart, 1998; Yogo, 2006; Gomes et al., 2009; Yang, 2011, among others). A different stream of literature approached this problem by studying the role aggregate macroeconomic uncertainty has on asset prices (Veronesi, 1999; Xia, 2001; Epstein & Schneider, 2008; Johannes, Lochstoer & Mou, 2016, and others). These two channels, while being complementary, were mostly studied on their own. The main goal of this paper is to analyse the properties of asset prices when both channels are present: when we take into account both the separation of durable and non-durable goods and there is aggregate uncertainty about the underlying state of the economy.

I show that the presence of these two channels, in conjunction with recursive preferences of Epstein & Zin (1989, 1991) plays a key role in matching the empirical facts of the asset pricing literature. While the model jointly incorporates both of these components, I am able to assess the relative contribution of each of the model's ingredients in explaining the mean and the volatility of the equity premium and the risk-free rate.

The role of durable goods consumption and the nonseparability of durable and nondurable consumption in the recursive framework of Epstein & Zin (1989, 1991) was neglected in most of the aforementioned consumption-based asset-pricing literature. The first paper to consider this framework is Yogo (2006), where he proposes a consumption-based explanation of the cross-sectional variation in the expected stock returns and countercyclical variation in the equity premium. The representative agent has a constant elasticity of substitution intraperiod utility function over the durable and nondurable consumption and also an Epstein & Zin (1989, 1991) recursive interperiod utility function. This model generalizes the previous durable consumption models (Dunn & Singleton, 1986; Eichenbaum & Hansen, 1990; Ogaki & Reinhart, 1998) by separating the risk-aversion coefficient from the elasticity of intertemporal substitution and is the first study to model durable consumption in a recursive framework.

Yang (2011) also studies a representative agent that has nonseparable utility over durable and nondurable consumption together with Epstein & Zin (1989, 1991) preferences. The economy is modeled in the spirit of a Bansal & Yaron (2004) type of long-run risk model and documents that there exists strong evidence of a highly persistent component in durable consumption growth. Furthermore, he shows that this long-run durable consumption risk helps explain the key features of asset markets and is an extension of Yogo's seminal work that allows the author to study the time-series properties of asset markets, rather than cross-sectional properties of returns as in Yogo (2006).

Building on the works of Yogo (2006) and Yang (2011), I propose a consumption-based explanation of aggregate market behaviour, where I model a representative agent that has a Cobb-Douglas intraperiod utility over the durable and nondurable consumption in conjunction with the recursive interperiod utility function of Epstein & Zin. I find that the presence of a durable component in the utility function is crucial for generating the value and the time properties of the risk-free rate and the equity premium. When utility is non separable between nondurable and durable consumption, and the elasticity of substitution between these two goods is higher than the elasticity of intertemporal substitution, the marginal utility of consumption is high when durable consumption is low. Since, empirically, the ratio of durable to nondurable consumption is highly procyclical, this ratio magnifies the countercyclical property of marginal utility, and thus of the equity premium.

A common feature of financial time series is their extraordinary change in behaviour during periods of financial crises or rapid growth (Cecchetti et al., 1993; Ang & Bekaert, 2002; Dai et al., 2007). To account for this feature and to allow for aggregate uncertainty in the economy, I model cash flows from non-durable consumption, durable consumption, and aggregate equity market as subject to a regime shift that is unobservable for the agent in a similar vein as Veronesi (1999). This way of modelling the economy complements the existing recent literature on asset pricing (for example, Brandt, Zeng & Zhang, 2004; Ju & Miao, 2012, among others, model the growth rates of consumption and dividends subject to a hidden regime, and the agent learns about the hidden state of the economy). The unobservability of the underlying state induces endogenously time-varying uncertainty due to inference problems. I fit the model to US consumption and dividend data from 1952:Q1 to 2014:Q4.

This paper studies the asset pricing implications of the consumption of durable goods and macroeconomic uncertainty. It is the first paper that combines the idea of modeling a broad consumption portfolio together with learning about hidden economic fundamentals (such as current state of the world). The cash flows from non-durable consumption, durable consumption, and aggregate equity markets are subject to a regime shift which is unobservable to the agent. I show that expenditure on durable consumption goods, and not the stock of the durable goods itself, determines the dynamics of asset prices in the economy. I also show that unobservability of the underlying fundamentals in the economy gives rise to another empirically observed fact about the equity premium - its variability over time. Another finding of this paper is the dependence of the results on the preference parameters. I find that the level and the volatility of the equity premium mostly depends on the share of the durable consumption of the agent. As long as the elasticity of substitution between nondurable and durable consumption is higher than the elasticity of intertemporal substitution, a high level of equity premium can be achieved with an empirically justified value of the risk aversion coefficient of around 2.

2. Model

2.1. *Preferences and Endowments*

Consider an infinitely lived representative household. In each period t, the household purchases C_t units of non-durable consumption goods and I_t units of durable consumption goods. Durable good provides a service flow for more than one period, while non-durable consumption good is non-storable and is entirely consumed in period t. The household accumulates the stock of durable goods K_t according to the law of motion

$$K_t = (1-\delta)K_{t-1} + I_t,$$

where $\delta \in (0,1)$ is the depreciation rate. Household's intertemporal utility is defined recursively as

$$U_t = \left\{(1-\beta)V_t^{\frac{1-\gamma}{\theta}} + \beta\left(\mathbb{E}\left[U_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \tag{2.1}$$

where V_t is given as the Cobb-Douglas function over K_t and C_t

$$V_t = C_t^{1-\alpha}K_t^{\alpha}$$

with $0 < \alpha < 1$. The parameters of the household's utility function are the subjective discount factor $\beta \in (0,1)$, the relative risk aversion coefficient $\gamma > 0$, and the elasticity of intratemporal substitution $\psi \geq 0$ with $\theta = (1-\gamma)/(1-\frac{1}{\psi})$.

2.2. *Assets and Dividends*

The household has an initial W_0 units of wealth. In every period t, the household splits his current wealth W_t between consumption and investment. Household invests $B_{i,t}$ units into one of the N available tradable assets in the economy. Each asset realizes the gross rate of return $R_{i,t+1}$ in period $t+1$. The household's budget constraint in period t is given by

$$W_t - C_t - P_tI_t = \sum_{i=1}^{N} B_{i,t},$$

where P_t is the relative price of consumer durable goods in terms of non-durable goods. The $t+1$ period wealth of the household is given by

$$W_{t+1} = \sum_{i=1}^{N} B_{i,t} R_{i,t+1}.$$

I consider two types of assets: equity, that provides stochastic amount of dividends in each period, and risk-less bonds, that pay zero coupons and act as purely discount bonds. Consider a stochastic endowment economy, where each period non-durable consumption C_t and durable consumption I_t arrives. In equilibrium, agents purchase C_t and I_t, such that markets clear and price of these goods is determined endogenously. I model the growth rates of endowment a (a non-durable good C_t, a durable good I_t, and equity) as a hidden Markov model in logs:

$$g_{t+1}^a = \mu_{S_t}^a + \sigma^a \varepsilon_{t+1}^a, \tag{2.2}$$

where (ε_t^a)'s are independent jointly standard normal error terms.

The model is an extension of Hamilton (1989) and Cecchetti et al. (1993). The predictable components $\mu_{S_t}^a$ are driven by the common Markov chain S_t with the state space

$$\mathcal{S} = \{1 = \text{expansion}, 0 = \text{recession}\}.$$

The unobservability of the underlying state induces time-varying uncertainty due to inference problems. All dividend parameters are estimated using a Maximum Likelihood Estimation from postwar U.S. consumption and dividend data.

S_t follows a two-state Markov chain with transition matrix $P = (p_{ij})$, where p_{ij} is the conditional probability $\mathbb{P}(S_{t+1} = j | S_t = i)$ of the process being in state j next period given it is in state i this period. I further assume that $\mu_1^a > \mu_0^a$, for every growth rate of endowment a, so that the growth rates in the expansion are higher than the growth rates in the recession .

Suppose we are in an economy with incomplete information, where the representative household knows the structure and the parameters of the model, but does not observe the state S_t. Let $\mathcal{F}_t$ be the information available to the household at time t, which consists of observed growth rates of endowment processes. We need to derive the evolution of the posterior state beliefs given $\mathcal{F}_t$. Let $\pi_t(i) = \mathbb{P}[S_t = i|\mathcal{F}_{t-1}]$ denote the posterior belief of state t being i, and suppose π_0 (stationary prior) is given. The agent uses Bayes' rule to update his belief about the hidden state:

$$\pi_{t+1}(i) \propto \sum_{j=0}^{1} \mathbb{P}(S_{t+1} = i|S_t = j) \cdot \mathbb{P}(S_t = j|\mathcal{F}_t),$$

where $\propto$ means that $\forall i,\ \pi_{t+1}(i)$ are multiplied by a constant term such that $\sum_i \pi_{t+1}(i) = 1$.

2.3. Consumption-Portfolio Choice

Define the total wealth of the household as

$$\widetilde{W}_t = W_t + (1-\delta)P_t K_{t-1}$$

that is, it includes the value of the stock of durable goods expressed in terms of relative price P_t. Let

$$B_{N+1,t} = P_t K_t,$$

where N is the total number of assets available to the household and in this case is equal to 4, and the return on the durable goods be equal to

$$R_{N+1,t+1} = (1-\delta)\frac{P_{t+1}}{P_t}.$$

This way we can rewrite the budget constraint to include the return not only from $N = 4$ available assets but also the return on the durable goods:

$$\widetilde{W}_{t+1} = (\widetilde{W}_t - C_t)\sum_{i=1}^{N+1} \omega_{it} R_{i,t+1} \tag{2.3}$$

with the condition that

$$\sum_{i=1}^{N+1} \omega_{it} = 1. \tag{2.4}$$

Given the household's current wealth level $\widetilde{W}_t$, the household chooses the level of non-durable consumption C_t and how much to invest into all of the available assets $\{\omega_{1,t}, \ldots, \omega_{N+1,t}\}$ to maximize utility (2.1) subject to (2.3) and (2.4). This leads to the Bellman equation for the problem of the form

$$J_t = \max_{C_t, \omega_{0,t}, \ldots, \omega_{N+1,t}} \left\{ (1-\beta) V_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathbb{E}_t \left[J_{t+1}^{1-\gamma} \right] \right)^{\frac{1}{\theta}} \right\}^{\frac{\theta}{1-\gamma}}.$$

I follow Yogo (2006) and conjecture that the value function J_t is a function of wealth $J_t = J_t(\widetilde{W}_t) = \phi_t \widetilde{W}_t$. It can be shown that (see Appendix A):

$$\phi_t = \left[(1-\beta)(1-\alpha) v \left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right)^{1-\frac{1}{\psi}} \right]^{\frac{1}{1-\frac{1}{\psi}}} \left(\frac{C_t}{\widetilde{W}_t} \right)^{\frac{1}{1-\psi}},$$

where

$$v \left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t} \right) = \left[\frac{\omega_{N+1,t}}{P_t} \left(\frac{\widetilde{W}_t}{C_t} - 1 \right) \right]^{\alpha}.$$

2.4. Asset Pricing

Let $R_{m,t+1}$ denote the return on wealth from an optimal portfolio, defined as

$$\widetilde{W}_{t+1} = \left(\widetilde{W}_t - C_t \right) R_{m,t+1},$$

and let

$$M_{t+1} = \beta^{\theta} \left(\frac{C_{t+1}}{C_t} \right)^{-\theta} \left(\frac{V_{t+1}}{V_t} \right)^{\theta - \frac{\theta}{\psi}} R_{m,t+1}^{\theta-1} \tag{2.5}$$

be the intertemporal marginal rate of substitution (IMRS) of the economy, the pricing kernel. Epstein & Zin (1989, 1991) show that the first-order condition for the consumption and the portfolio choice implies that the return on every tradable asset i in the economy satisfies the equation

$$\mathbb{E}_t[M_{t+1}R_{i,t+1}] = 1. \tag{2.6}$$

Moreover, the first-order condition with respect to the choice of durable goods is [1]

$$\mathbb{E}_t[M_{t+1}(R_{0,t+1} - R_{N+1,t+1})] = \frac{\frac{\partial V_t}{\partial K_t}}{P_t \frac{\partial V_t}{\partial C_t}},$$

where $V_t = C_t^{1-\alpha} K_t^{\alpha}$.

Equations (2.5) and (2.6) are used to derive the asset prices in the economy. I show in the Appendix B, that I conjecture that the wealth-consumption ratio $\frac{\widetilde{W}_{t+1}}{C_{t+1}}$ (and therefore the price-dividend ratio $\frac{P_{a,t}}{D_{a,t}}$ on any asset a) depend on a set of state variables $\boldsymbol{x}_t$. I take $\boldsymbol{x}_t$ to be a vector of two variables - current belief π_t and ratio of investment into durable goods over current stock of durable goods $\frac{I_t}{K_t}$. I then solve the model numerically (see Appendix B on how to apply the projection method of Judd (1992) to the model).

3. Data

3.1. *Source and Construction*

Personal consumption expenditure (PCE) data retrieved from the U.S. National Income and Product Accounts as provided by the Bureau of Economic Analysis. The measure of non-durable consumption includes personal consumption expenditure on non-durable goods (food and beverages purchased for off-premises consumption, clothing and footwear, and gasoline and other energy goods) and personal consumption expenditure on services (housing, health care, transportation and other services). The cor-

[1] If we equate the marginal utility of non-durable consumption per unit spent to marginal utility of durable consumption per unit spent, and taking the non-durable good as a numèraire we get:

$$\frac{\frac{\partial V_t}{\partial C_t}}{1} = \frac{\frac{\partial V_t}{\partial K_t}}{Q_t},$$

where Q_t can be thought of as a rental cost of service flow from the durable good. On the other hand, we can interpret Q_t as

$$Q_t = P_t - (1-\delta)\mathbb{E}_t[M_{t+1}P_{t+1}],$$

where we discount the future price P_{t+1} by M_{t+1}. By definition, $R_{N+1,t+1} = \frac{(1-\delta)P_{t+1}}{P_t}$, which gives us the first-order condition above.

responding seasonally adjusted quarterly quantity index for the sample period 1952:I–2014:IV is from lines 8 and line 13 of Table 2.3.3. (Real Personal Consumption Expenditures by Major Type of Product).

The measure of the stock of consumer durable goods includes motor vehicles, furnishings and durable household equipment, recreational goods and vehicles and other durable goods. The corresponding annual quantity index for the period 1952–2014 is from line 1 of Table 8.2 (Chain-Type Quantity Indexes for Net Stock of Consumer Durable Goods). The relative price of consumer durable goods is constructed as the ratio of the PCE price index for durable goods from line 3 over the PCE price index for non-durable goods from line 8 of Table 2.3.4 (Price Indexes for Personal Consumption Expenditures by Major Type of Product). The BEA reports only the annual series of the net stock of consumer durable goods, quarterly series are interpolated by assuming that the depreciation rate is constant within the year and by finding its implied value, which is consistent both with the annual stocks of net consumer durables at the beginning as well as the end of the year, and with quarterly series of PCE expenditures on durable goods.[2] The U.S. population measure used to calculate per-capita quantities covers the period 1952–2014 and may be retrieved from the Federal Reserve Bank of St. Louis.

The quarterly and annual returns on the common stock market as well as the short-term nominal interest rate for the sample period 1952:I–2014:IV are from the CRSP and provided by the University of Manchester. Because non-durable consumption is the numèraire in our analysis we deflate all asset returns with the PCE price index for non-durable goods to obtain real quantities.

3.2. Basic Description and Business Cycle Properties of Consumption Data

Table 1 reports descriptive statistics for nondurable and durable goods consumption growth and durable stock growth. Nondurable consumption and services growth has a mean 0.49% and standard deviation 0.46% per quarter. The expenditure to durable

[2]The law of motion of the consumer durable goods $K_{t+1} = (1-\delta_t) K_t + I_t$ yields after four iterations the equation $K_{t+4} = (1-\delta)^4 K_t + (1-\delta)^3 I_t + (1-\delta)^2 I_{t+1} + (1-\delta) I_{t+2} + I_{t+3}$ that implicitly defines the depreciation rate δ for the given year.

consumption has a mean 0.98% and standard deviation 3.21% per quarter. Durable goods stock growth has mean a 0.83% and standard deviation 1.01% per quarter. The first-order autocorrelations for the nondurable consumption growth, expenditure to durable goods growth and durable goods stock growth are equal to 0.45, -0.01, and 0.88, respectively.

Table 1. Descriptive statistics[a]

Time series	Mean[b] (%)		SD[b] (%)		Autocorrelation	
	Est.	(S.E.)	Est.	(S.E.)	Est.	(S.E.)
Nondurable Goods	0.49	(0.06)	0.46	(0.03)	0.45	(0.08)
Durable Goods Expenditure	0.98	(0.15)	3.21	(0.25)	-0.01	(0.06)
Durable Goods Stock	0.83	(0.15)	1.01	(0.09)	0.88	(0.04)

[a] This table provides the descriptive statistics for the consumption data. Nondurable goods is the expenditure for nondurable consumption and services. Quarterly data for durable goods stock is obtained by assuming that the depreciation rate is constant within the year and by finding its implied value, which is consistent both with the annual stocks of net consumer durables at the beginning as well as the end of the year.

[b] All variables are in percentage. Standard errors obtained by performing a block bootstrap with each block having geometric distribution with length 32 quarters; 50,000 experiments performed. Sample period is 1952:I-2014:IV.

Figure 1 is a plot of the ratio of the stock of durable goods to nondurable consumption ($\frac{K_t}{C_t}$) and the relative price of durables to nondurables. The upward trend in $\frac{K_t}{C_t}$ is consistent with a downward trend of the relative price. $\frac{K_t}{C_t}$ increased by factor of almost 2.5 over the data span while relative price decreased by factor of almost 3.5. The ratio $\frac{K_t}{C_t}$ is pro-cyclical, it rises during booms and falls during recessions. The shaded regions are the recessions as defined by the NBER. Figure 2 plots the time series of expenditures to durable goods (solid line) and nondurable goods consumption (dashed line). Both time series exhibit an upward trend in the sample period and are strongly pro-cyclical.

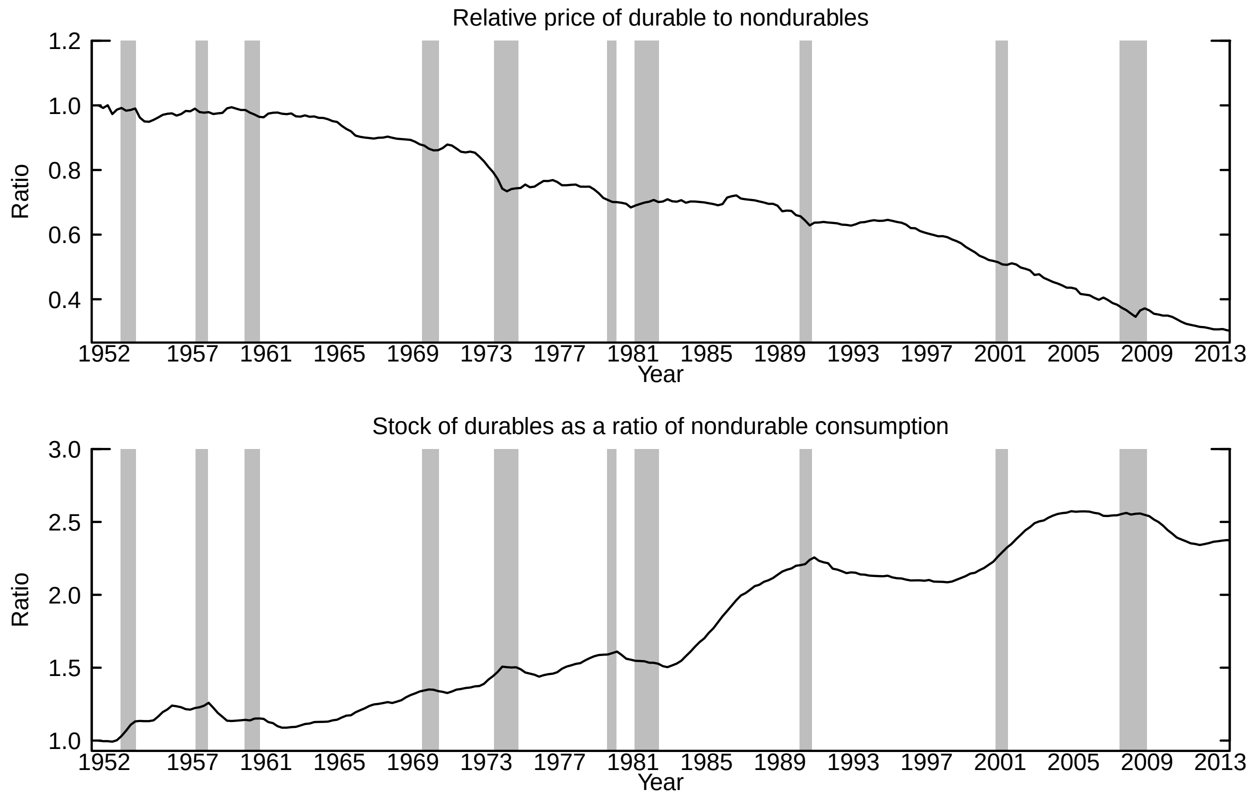

Figure 1. Price and Stock of Durables Relative to Nondurables. Time-series plot of the relative price of durables to nondurables (top panel) and the stock of durables as a ratio of nondurable consumption (bottom panel). The sample period is 1952:I - 2014:IV; the shaded regions indicate NBER recessions. For both series the 1952:I values are normalized to 1.

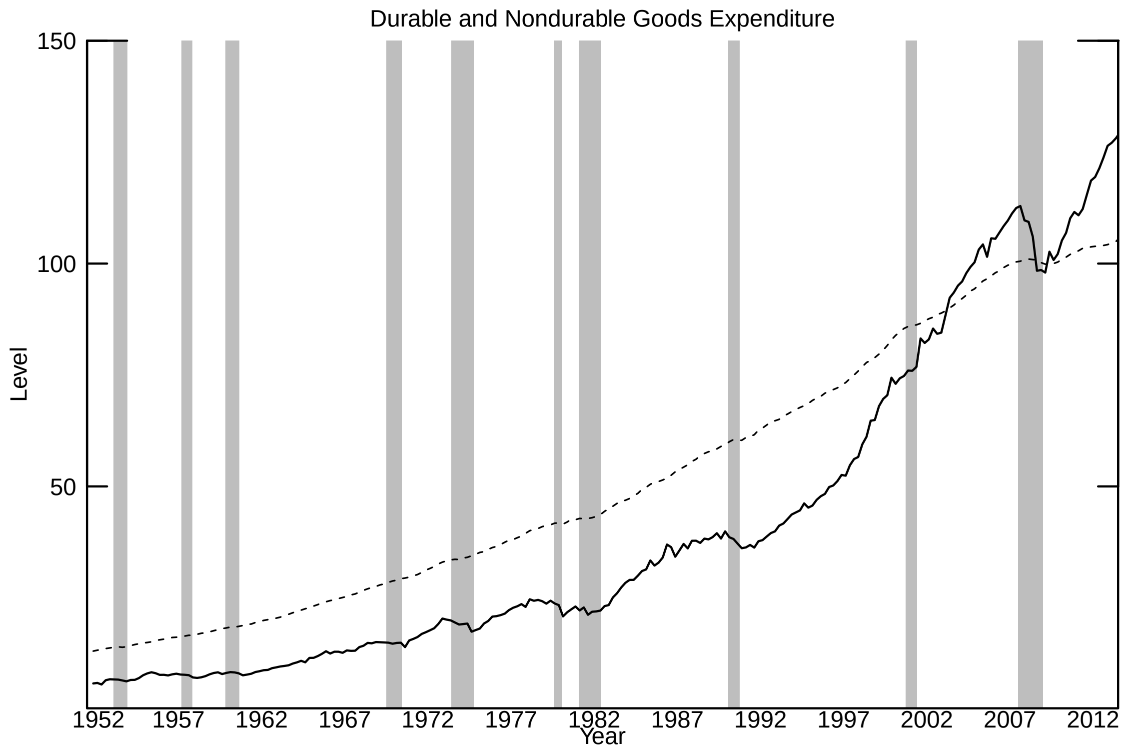

Figure 2. Durable and Nondurable Goods Expenditure. Time-series plot of the real durable goods expenditure (solid line) and the real nondurable goods consumption (dashed line), in levels. The sample period is 1952:I - 2014:IV; the shaded regions indicate NBER recessions.

Top panel of Figure 3 plots the corresponding growth rates of stock of durable goods and of nondurable consumption, respectively. Middle panel plots the growth rate of expenditure to durable goods, while the bottom panel plots the growth rate of relative price of durables. Growth rates of durable stock, expenditure to durable goods and nondurable consumption are strongly pro-cyclical, whereas the growth rate of relative price is strongly countercyclical. The growth rate of expenditure to durable goods is more pro-cyclical than nondurable consumption, and thus is a good business cycle indicator.

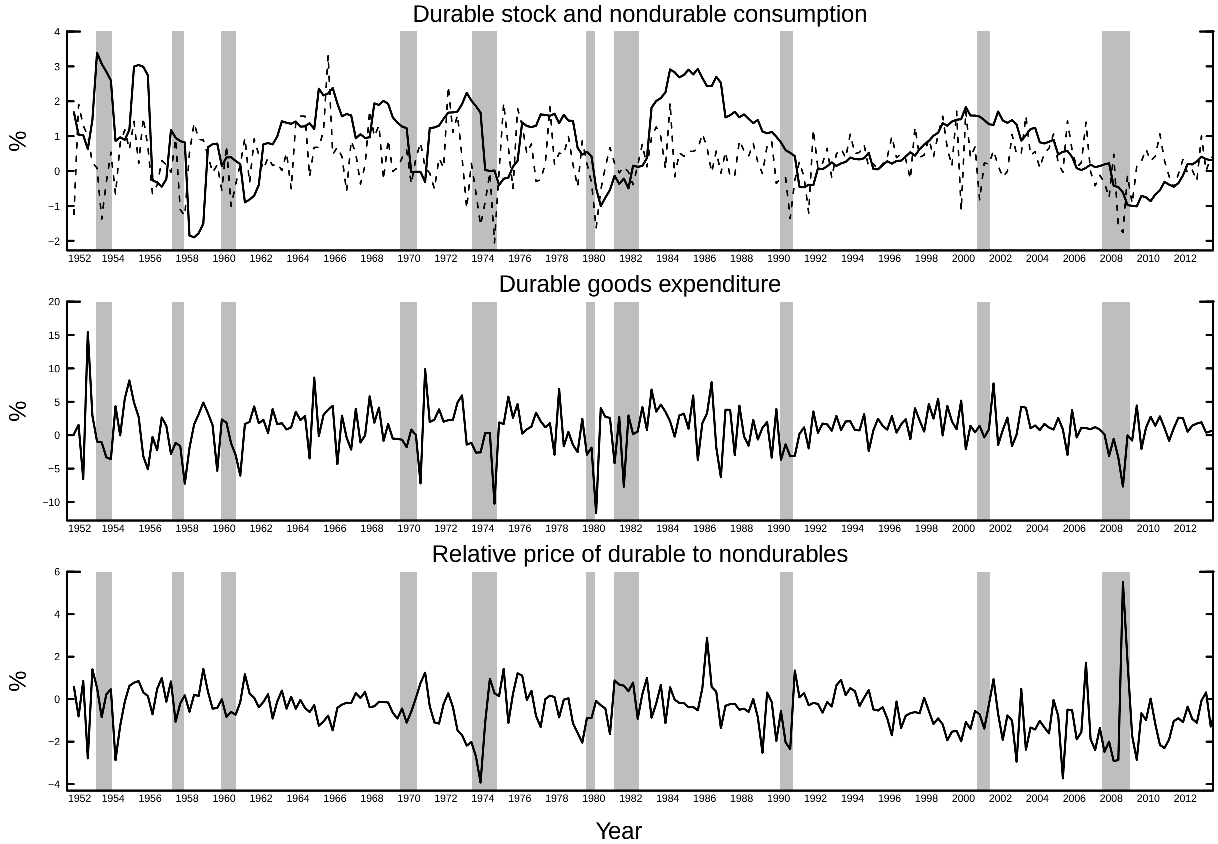

Figure 3. Growth Rates. Top panel is a time-series plot of the real growth rates of the stock of durables (thick solid line) and nondurable consumption (thin solid line), middle panel plots the real growth rate of durable goods expenditure, and bottom panel plots the growth rate of relative price of durables to nondurables. The sample period is 1952:I - 2014:IV; the shaded regions indicate NBER recessions.

4. Results

I calibrate the model at the quarterly frequency. The endowments coefficients are estimated using the US consumption and dividends data, and the preference parameters are set to be consistent with the previous literature. Since the model does not admit an analytical solution, I solve the model numerically and run simulations to compute the model implied moments. I then annualize these simulated moments to compare with the data. I also study the shape of the wealth-consumption and the price-dividend ratios, and the time-varying properties of the asset prices.

Coefficients of equations (2.2) are estimated using the Maximum Likelihood Estimation procedure from the post-war US data on consumption and dividends. Table 2 reports the estimation results. Non-durable consumption does not grow in the recession state and grows at the rate of more than 0.6% in the boom state. The volatility of non-durable consumption is estimated at almost 0.38%. The expenditure to durable consumption declines at the rate of 1.95% in the recession state and grows at the rate of almost 1.04% in the boom state. Durable consumption is more volatile than the non-durable consumption with the estimated volatility of approximately 3%. Finally, the growth rate of dividends in the recession is estimated at -1.83% whereas in the boom state at a 1.60%. The dividends are very volatile, with the estimated volatility of more that 5%. Transition probabilities for the regime-switching process are estimated as 0.95 and 0.75 for the boom and recession states, accordingly, implying the expected duration of recession equal to 4 quarters.

Table 2. Maximum Likelihood Estimation of a two-state Markov-switching model [a]

Parameter	Estimate		Parameter	Estimate	
	Value[b]	SE[b]		Value[b]	SE[b]
μ_0^c	0.00	0.07	p	0.95	NA
μ_0^d	-1.83	1.10	q	0.75	NA
μ_0^e	-1.95	0.54			
μ_1^c	0.63	0.03	$(\sigma^c)^2$	0.14	0.01
μ_1^d	1.04	0.41	$(\sigma^d)^2$	25.74	2.01
μ_1^e	1.60	0.23	$(\sigma^e)^2$	8.82	0.59

[a] This table provides the maximum likelihood estimation of the endowment process for nondurable and durable consumption expenditure and dividend series. p and q denotes the probability of expansion and recession, respectively. μ_i^c, μ_i^d and μ_i^e denote the estimated growth rates for nondurable consumption, dividends, and expenditure to durable consumption, respectively, in the state i, where $i = 0$ is recession and $i = 1$ is expansion. $(\sigma^c)^2$, $(\sigma^d)^2$, and $(\sigma^e)^2$ denotes the estimated variance of growth rates of nondurable consumption, dividends, and expenditure to durable consumption.

[b] All variables are in percentage. Data is quarterly. Sample period is 1952:I-2014:IV.

Figure 4 displays the plot of filtered (solid line) and smoothed (dashed line) probabilities of the boom state from the postwar US data over the NBER recessions.

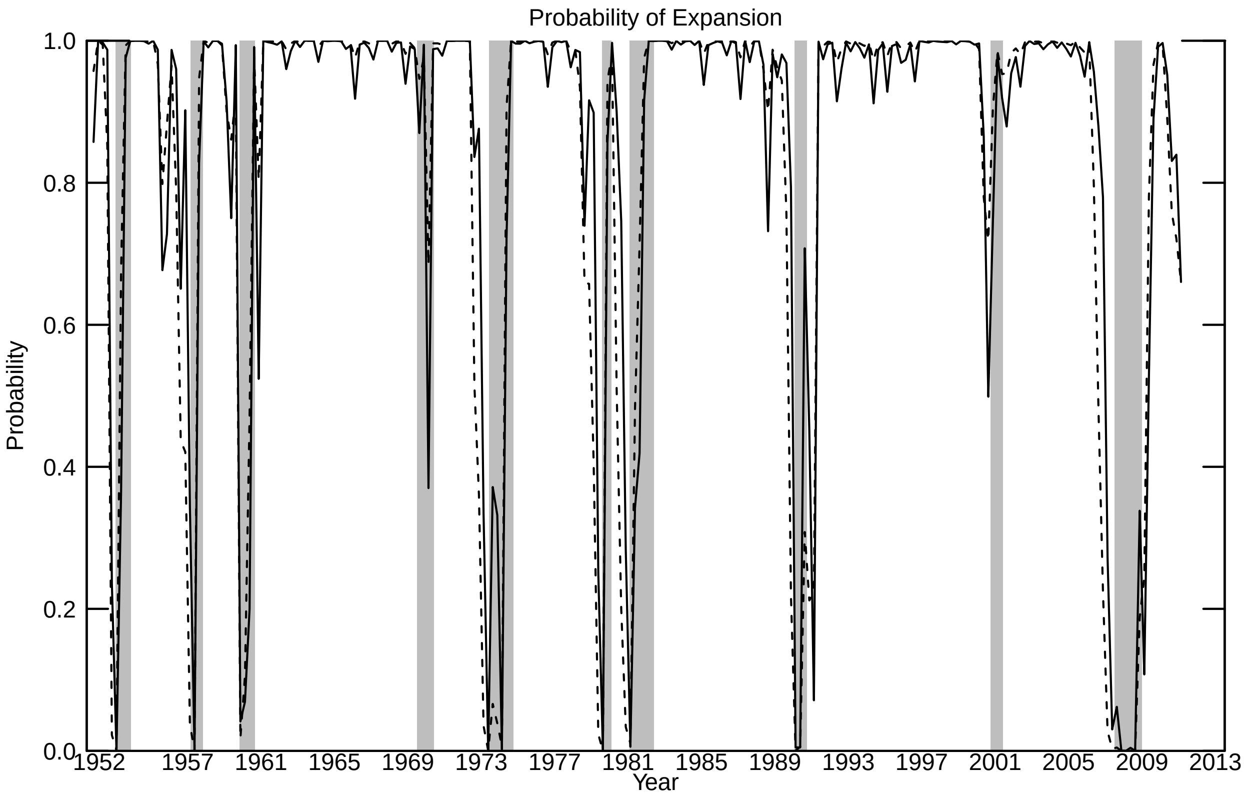

Figure 4. Probability of expansion. Figure displays the filtered (solid line) and smoothed (dashed line) probabilities of expansion. The sample period is 1952:I - 2014:IV; the shaded regions indicate NBER recessions.

I choose three model parameters: the risk aversion coefficient γ, the elasticity of intertemporal substitution ψ and the share of durable consumption α to minimize the difference between the model implied and the data moments. For that, I solve and simulate the model for a large set of combinations of these two parameters and choose the ones that provide the best "fit". The implied value for risk aversion coefficient is 2.1, which lies well between the most commonly accepted values of 1 and 3 (some studies even suggesting values as low as 0.2 and as high as 10). The implied value of elasticity of intertemporal substitution is 1.09, which is also lies comfortably between the reported values of 0.2 and 2 (see, for example, Havránek, 2015, for recent survey). The share of durable consumption is set to 0.3 and is consistent with the data (durables make up of approximately 30% of consumers consumption basket). I set the other parameters of the model in the following way. The subjective discount factor β is set to match exactly the mean value of risk-free rate in the model. The implied value for β is 0.985. The depreciation rate of durables δ is set to 6%, which is the average sample observed value. These are summarized in Table 3 below:

Table 3. Benchmark values of the preference parameters

Parameter		Value
EIS	$\psi =$	1.09
Risk aversion	$\gamma =$	2.1
Subjective discount factor	$\beta =$	0.985
Share of durable consumption	$\alpha =$	0.3
Depreciation rate	$\delta =$	0.06

This tables lists the benchmark values for the preference parameters. The value of subjective discount factor is set to match the level of risk-free interest rate implied by the model. The value for the depreciation rate is set to match the average quarterly depreciation rate from the data for the sample period 1952:Q1-2014:Q4.

For the aforementioned set of parameters I also analyze two benchmark models in order to emphasize the importance of each of the ingredients of the model. Benchmark Model I analyses the endowment economy in which the agent has a recursive utility over consumption. The endowments are subject to the regime switch, but the state of the world is observable for the agent. Moreover, the agent has utility over nondurable consumption only. Benchmark Model II extends the previous benchmark model and analyses the case of recursive utility with durable consumption; again the underlying state of the economy is observed for the agent.

Table 4 summarizes the results. The first row of table 4 reports the annualized data moments. The average annual risk-free rate is equal to 1.2%, with the volatility of 0.99%. The mean equity premium is a bit more that 5.5% with large volatility of almost 19%. Second row of table 4 reports the results for the full model with recursive preferences over nondurable and durable consumption, and uncertainty about the endowments. We see, that both mean and volatility of the risk-free rate are matched perfectly. The model implied mean equity premium is slightly above the data value and is equal to more than 6%. The model implied volatility, on the other hand, is a bit less than 60% of the value observed in the data.

Table 4. Unconditional Moments of Returns [a]

	$\mathbb{E}(R_{f,t})$	$\mathsf{sd}(R_{f,t})$	$\mathbb{E}(R_{e,t+1})$	$\mathsf{sd}(R_{e,t+1})$
US Data [b]	1.20	0.99	5.57	18.94
Full Model [c]	1.20	0.99	6.60	10.38
Only Nondurable Consumption No Uncertainty [c]	1.20	0.30	1.88	11.14
Durable and Nondurable Consumption No uncertainty [c]	1.20	2.90	5.15	10.52

[a] This table reports the unconditional moments of returns from the data, the Full Model (with uncertainty and durables) and for Benchmark Model I (no uncertainty and no durables) and Benchmark Model II (no uncertainty, durables).

[b] Risk-free rate is the 3-month Treasury bill. Equity premium is defined as CRSP value-weighted return less the 3-month Treasury bill. All data is quarterly for the period 1952:Q1 - 2014:Q4, reported as annualized percentage values, and is available from CRSP.

[c] Model implied moments are obtained as a sample values from simulated 100,000 data points and annualized to match the corresponding data moments.

There is a substantial difference in the results between the full model and two benchmark cases. Implied volatility of the risk-free rate for Benchmark model I is only 0.30%, while implied mean equity premium is only at 1.88%. Benchmark model II, on the other hand, generates higher value of the equity premium, but fails to match the low volatility of the risk-free rate. Both benchmark cases generate about 60% of the volatility of the equity premium.

We see that with only three free parameters (the risk aversion, the elasticity of the intertemporal substitution and the share of durable consumption), the full model with recursive preferences over nondurable and durable consumption, and uncertainty about the endowments matches three moments of the returns observed in the data (mean and volatility of the risk-free rate, and mean of the equity premium), while also generating more than 50% of the equity premium volatility observed in the data. Finally, I further study the properties of the asset prices, such as the shape of the wealth-consumption and price-dividend ratios, and predictability of the price-dividend ratio.

4.1. *Properties of Asset Prices*

To further study the properties of the asset prices, such as the shape of the wealth-consumption and price-dividend ratios, time-varying properties of the asset prices, and predictability of the price-dividend ratio, I fix the values of the preferences parameters on the level that provides the best fit, and were reported in table 3. Appendix C summarizes the main properties of wealth-consumption and price-dividend ratios.

Predictive Regressions. One way to capture variation in the conditional equity risk is to run the predictive regressions. The most popular regressor in the literature is the dividend-price ratio (as in Campbell & Shiller, 1988; Fama & French, 1988, etc.). To see how the model captures return predictability by the dividend-price ratio, I compute predictive regressions for the simulated return series and compare those to the data. Table 5 reports the estimated coefficients, t-values and R^2 for regressions of (continuously compounded) one-, three-, five, and ten-year excess returns on the dividend-price ratio. The R^2 is high for the short-horizons (and close to data counterpart), and then decreases at the long-horizons. The slope coefficients also increase with the horizon (we can see the same effect when we run the regression for the data). Judging by the t-statistics, the slope coefficients are also statistically significant.

Table 5. Long-horizon Predictive Regressions [a]

	1 Year			3 Years			5 Years			10 Years		
	b	$t(b)$	R^2	b	$t(b)$	R^2	b	$t(b)$	R^2	b	$t(b)$	R^2
Data[b]	0.07	(2.69)	0.97%	0.21	(4.84)	3.16%	0.30	(5.72)	4.51%	0.55	7.64	8.41%
Model	0.32	(8.23)	0.67%	0.32	(4.71)	0.22%	0.34	(3.93)	0.15%	0.34	(2.80)	0.08%

[a] The regression equation is $r^e_{t\to t+k} = a + b \times dp_t + \varepsilon_{t+k}$, where $r^e_{t\to t+k}$ is the log excess return, continuously compounded over the horizon, and dp_t is the log dividend-price ratio. Return horizon is 1, 3, 5 and 10 years.

[b] Excess return is the CRSP value-weighted return less the 3-month Treasury bill. Data is quarterly from 1952:Q1 - 2014:Q4, available from the CRSP.

5. Conclusion

I consider an infinitely lived, representative-household, consumption-based, asset-pricing framework, where I (i) allow for a separation between durable and non-durable consumption, and (ii) model consumption and dividend growth as a hidden Markov regime-switching process and the household learns about the current state from past data. I show that the price-dividend ratios are dependent on posterior beliefs, which in turn drive the dynamics of the asset prices in the model. I follow existing literature on consumption-based explanations of asset market phenomena in the presence of consumption of durable goods (see, for example, Yogo, 2006; Yang, 2011) and contribute to the existing literature by proposing a hidden regime-switching, consumption-based, asset pricing model.

The findings suggest that the presence of a durable component in the utility has a considerable effect on the value and the time properties of risk-free rate and equity premium. When the utility is non separable between nondurable and durable consumption, and the elasticity of substitution between these two goods is higher than the EIS, marginal utility of consumption is high when durable consumption is low. Since the ratio of durable to nondurable consumption is highly pro-cyclical, it magnifies the countercyclical property of marginal utility, and thus of the equity premium.

The model is able to explain the wide variety of asset market puzzles. The consumption-based model with incomplete information generates the average equity risk premium observed in the data and matches the mean and the volatility of the risk-free rate. The model is also able to generate more than 60% of the observed volatility of the equity premium. Last but not least, the model generates the pro-cyclical variation in wealth-consumption and price-dividend ratios and countercyclical variation in equity premium.

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Appendix A. Numerical solution

In this subsection, we present the numerical solution to the model. I start by guessing a solution of the form $U_t = \phi_t \widetilde{W}_t$, where $\widetilde{W}_t$ denotes the total wealth

$$\widetilde{W}_{t+1} = W_{t+1} + (1-\delta)P_{t+1}K_t = \sum_{i=1}^{N} \omega_{i,t} R_{i,t+1} + (1-\delta)P_{t+1}K_t = \sum_{i=1}^{N+1} \omega_{i,t} R_{i,t+1},$$

meaning I treat the durable good as an asset. We can further rewrite the budget constraint as

$$\widetilde{W}_{t+1} = (\widetilde{W}_t - C_t) \sum_{i=1}^{N+1} \omega_{i,t} R_{i,t+1} = (\widetilde{W}_t - C_t) \cdot \omega_t' R_{t+1},$$

where $\omega_t = (\omega_{1,t}, \ldots, \omega_{N+1,t})'$ is the vector of weights and $R_{t+1} = (R_{1,t+1}, \ldots, R_{N+1,t+1})'$ is the vector of returns. For some x and y lets define function

$$v(x,y) = \left[1 - \alpha + \alpha \left(\frac{y}{P_t}\left(\frac{1}{x} - 1\right)\right)^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}}, \quad \text{when } \rho \neq 1,$$

and

$$v(x,y) = \left[\frac{y}{P_t}\left(\frac{1}{x} - 1\right)\right]^{\alpha}, \quad \text{when } \rho = 1,$$

Note that

$$V_t = C_t \left[1 - \alpha + \alpha \left(\frac{\omega_{N+1,t}}{P_t}\left(\frac{\widetilde{W}_t}{C_t} - 1\right)\right)^{1-\frac{1}{\rho}}\right]^{\frac{1}{1-\frac{1}{\rho}}} = C_t\, v\left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t}\right).$$

Using the equations above, Bellman equation

$$J_t = \left\{(1-\beta) V_t^{\frac{1-\gamma}{\theta}} + \beta \left(\mathbb{E}_t\left[J_{t+1}^{1-\gamma}\right]\right)^{\frac{1}{\theta}}\right\}^{\frac{\theta}{1-\gamma}} \tag{A.1}$$

simplifies to

$$\left(\phi_t \widetilde{W}_t\right)^{1-\frac{1}{\psi}} = (1-\beta)\left(C_t \cdot v\left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t}\right)\right)^{1-\frac{1}{\psi}} + \beta \left(\widetilde{W}_t - C_t\right)^{1-\frac{1}{\psi}} y_t \tag{A.2}$$

with

$$y_t = \mathbb{E}_t\left[(\phi_{t+1}R_{m,t+1})^{1-\gamma}\right]^{\frac{1}{\theta}}.$$

We are interested in deriving the expression for ϕ_t as a function of $\frac{C_t}{\widetilde{W}_t}$ and $\omega_{N+1,t}$. Using a similar approach as Epstein & Zin (1989, 1991), the FOC of the equation (A.2) with respect to C_t is

$$0 = (1-\beta)\frac{\partial\left(\left(C_t \cdot v\left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t}\right)\right)^{1-\frac{1}{\psi}}\right)}{\partial C_t} - \left(1-\frac{1}{\psi}\right)\beta\left(\widetilde{W}_t - C_t\right)^{-\frac{1}{\psi}} y_t,$$

$$\frac{\partial\left((C_t \cdot v(\cdot,\cdot))^{1-\frac{1}{\psi}}\right)}{\partial C_t} = \left(1-\frac{1}{\psi}\right)(C_t \cdot v(\cdot,\cdot))^{-\frac{1}{\psi}}\left[v(\cdot,\cdot) + C_t\frac{\partial v(\cdot,\cdot)}{\partial C_t}\right] =$$
$$\left(1-\frac{1}{\psi}\right)(C_t \cdot v(\cdot,\cdot))^{-\frac{1}{\psi}}\left[v(\cdot,\cdot) + \left(v(\cdot,\cdot) + (\alpha-1)v(\cdot,\cdot)^{\frac{1}{\rho}}\right)\frac{-\widetilde{W}_t}{\widetilde{W}_t - C_t}\right],$$

where

$$v(\cdot,\cdot) = v\left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t}\right).$$

The FOC can be rewritten as

$$\beta\left(\widetilde{W}_t - C_t\right)^{1-\frac{1}{\psi}} y_t = (1-\beta)(1-\alpha)v^{\frac{1}{\rho}}\widetilde{W}_t\,(C_t \cdot v)^{-\frac{1}{\psi}} - (1-\beta)\,(C_t \cdot v)^{1-\frac{1}{\psi}}.$$

Substituting expression above to the Bellman equation (A.2) we get:

$$\left(\phi_t\widetilde{W}_t\right)^{1-\frac{1}{\psi}} = (1-\beta)(1-\alpha)v^{\frac{1}{\rho}}\widetilde{W}_t\,(C_t \cdot v(\cdot,\cdot))^{-\frac{1}{\psi}}.$$

Hence,

$$\phi_t = \left[(1-\beta)(1-\alpha)v\left(\frac{C_t}{\widetilde{W}_t}, \omega_{N+1,t}\right)^{\frac{1}{\rho}-\frac{1}{\psi}}\right]^{\frac{1}{1-\frac{1}{\psi}}}\left(\frac{C_t}{\widetilde{W}_t}\right)^{\frac{1}{1-\psi}}.$$

Following Yogo (2006), the Euler equation is of the form:

$$\mathbb{E}_t\left[\beta^{\theta}\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{\theta}{\rho}}\left(\frac{V_{t+1}}{V_t}\right)^{\frac{\theta}{\rho}-\frac{\theta}{\psi}}R_{m,t+1}^{\theta}\right]=1, \tag{A.3}$$

where $R_{m,t+1}$ denotes the return on wealth from an optimal portfolio, defined as

$$\widetilde{W}_{t+1}=\left(\widetilde{W}_t-C_t\right)R_{m,t+1}.$$

Using the functional form for V_t, we can further eliminate $\frac{V_{t+1}}{V_t}$ from the Euler equation (A.3). We have

$$\frac{V_{t+1}}{V_t}=\left\{\frac{(1-\alpha)\left(\frac{C_{t+1}}{K_{t+1}}\right)^{1-\frac{1}{\rho}}\left(\frac{K_{t+1}}{K_t}\right)^{1-\frac{1}{\rho}}+\alpha\left(\frac{K_{t+1}}{K_t}\right)^{1-\frac{1}{\rho}}}{(1-\alpha)\left(\frac{C_t}{K_t}\right)^{1-\frac{1}{\rho}}+\alpha}\right\}^{\frac{1}{1-\frac{1}{\rho}}}.$$

Consider the case when $\rho=1$. Since $\rho\neq 1$ introduces a new state variable into the model, we will consider the Cobb-Douglas specification of the intratemporal utility

$$V_t=C_t^{1-\alpha}K_t^{\alpha}.$$

Following Yang (2011), it can be shown that

$$\frac{V_{t+1}}{V_t}=\left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t}\right)^{\alpha}\frac{C_{t+1}}{C_t}.$$

From the budget constraint we get

$$\widetilde{W}_{t+1}=\left(\widetilde{W}_t-C_t\right)R_{m,t+1},$$

which can be rewritten as

$$R_{m,t+1}=\frac{\frac{\widetilde{W}_{t+1}}{C_{t+1}}}{\frac{\widetilde{W}_t}{C_t}-1}\frac{C_{t+1}}{C_t}.$$

Let $\xi_t = \frac{\widetilde{W}_t}{C_t}$ denote the wealth-consumption ratio. Then,

$$R_{m,t+1} = \frac{\xi_{t+1}}{\xi_t - 1} \frac{C_{t+1}}{C_n}.$$

The Euler equation then becomes

$$\mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{\theta\left(1-\frac{1}{\psi}\right)} \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t} \right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] = (\xi_t - 1)^\theta.$$

Appendix B explains how to solve for ξ_t. Taking the FOC of the Bellman equation with respect to $\omega_{i,t}$ results in another Euler equation of the form:

$$\mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{1-\theta} \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t} \right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} R_{m,t+1}^{\theta-1} R_{i,t+1} \right] = 1.$$

The return $R_{i,t+1}$ can be written as

$$R_{i,t+1} = \frac{P_{R,t+1} + D_{R,t+1}}{P_{R,t}} = \frac{\frac{P_{R,t+1}}{D_{R,t+1}} + 1}{\frac{P_{R,t}}{D_{R,t}}} \frac{D_{R,t+1}}{D_{R,t}} = \frac{\lambda_{t+1} + 1}{\lambda_t} \frac{D_{R,t+1}}{D_{R,t}}.$$

Therefore, the Euler equation can be written as

$$\mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{\theta\left(1-\frac{1}{\psi}\right)-1} \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t} \right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} \left(\frac{\xi_{t+1}}{\xi_t - 1} \right)^{\theta-1} \frac{D_{R,t+1}}{D_{R,t}} \left(\lambda_{t+1} + 1\right) \right] = \lambda_t.$$

Appendix B explains how to solve for λ_t.

Appendix B. Application of the projection method

This section describes the application of the projection method of Judd (1992) to our model. We start with equilibrium wealth-consumption ratio. Using the budget constraint, we express $R_{m,t+1}$ in terms of a wealth-consumption ratio as:

$$R_{m,t+1} = \frac{\frac{\widetilde{W}_{t+1}}{C_{t+1}}}{\frac{\widetilde{W}_t}{C_t} - 1} \frac{C_{t+1}}{C_t}.$$

We make a conjecture that the wealth-consumption ratio is a function ξ_t of state variables $\boldsymbol{x}_t = (\pi_t, \frac{I_t}{K_t})$ and thus

$$R_{m,t+1} = \frac{\xi_{t+1}(\boldsymbol{x}_{t+1})}{\xi_t(\boldsymbol{x}_t) - 1} \frac{C_{t+1}}{C_n},$$

and we use the Euler equation and apply the projection method to obtain the functional form of ξ_t. By the Euler equation

$$\mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{\theta\left(1-\frac{1}{\psi}\right)} \left(\frac{K_{t+1}/C_{t+1}}{K_t/C_t} \right)^{\alpha\theta\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] - (\xi_t - 1)^\theta = 0.$$

We can further rewrite the Euler equation as:

$$\mathbb{E}_t \left[\beta^\theta \left(\frac{C_{t+1}}{C_t} \right)^{\theta\alpha\left(\frac{1}{\psi}-1\right)} \left(\frac{K_{t+1}}{K_t} \right)^{\theta\alpha\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] = (\xi_t - 1)^\theta.$$

Let x_t denote $\frac{I_t}{K_t}$. Then, using $K_{t+1} = (1-\delta)K_t + I_t$ we get

$$\beta^\theta \mathbb{E}_t \left[e^{\theta\alpha\left(\frac{1}{\psi}-1\right)\Delta c_{t+1}} (1-\delta+x_t)^{\theta\alpha\left(1-\frac{1}{\psi}\right)} \xi_{t+1}^\theta \right] - (\xi_t - 1)^\theta = 0.$$

where $\Delta c_{t+1} = \log\left(\frac{C_{t+1}}{C_t}\right)$. Let $\kappa = \theta\alpha\left(\frac{1}{\psi}-1\right)$ and $\zeta = -\kappa = \theta\alpha\left(1-\frac{1}{\psi}\right)$. We conjecture that ξ_t is a function of x_t and π_t, where $x_t = \frac{I_t}{K_t}$ and π_t is a posterior state belief. Then, equation becomes:

$$
\begin{aligned}
\beta^{\theta}\mathbb{E}_t\left[e^{\kappa\Delta c_{t+1}}\cdot(1-\delta+x_t)^{\zeta}\cdot\xi_{t+1}^{\theta}(x_{t+1},\pi_{t+1})\right]& \\
-(\xi_t(x_t,\pi_t)-1)^{\theta} &= 0.
\end{aligned}
$$

We approximate function ξ_t as

$$\hat{\xi}_t(x_t,\pi_t)=\sum_{i,j=0}^{n}\phi_{i,j}\psi_i(x_t)\psi_j(\pi_t),$$

where $\{\psi_i(\cdot)\}_{i=1}^{n}$, $\{\psi_j(\cdot)\}_{j=1}^{n}$ is a basis of complete Chebyshev polymonials of order n, and $\phi_{i,j}$ are the coefficients of the polynomials. We next define the residual equation

$$
\begin{aligned}
R(x_t,\pi_t;\phi) &= \beta^{\theta}\mathbb{E}_t\left[e^{\kappa\Delta c_{t+1}}\cdot(1-\delta+x_t)^{\zeta}\cdot\xi_{t+1}^{\theta}(x_{t+1},\pi_{t+1})\right] \\
&- (\xi_t(x_t,\pi_t)-1)^{\theta}.
\end{aligned}
$$

Substituting for $\hat{\xi}_t$ we get

$$
\begin{aligned}
\hat{R}(x_t,\pi_t;\phi) &= \beta^{\theta}\mathbb{E}_t\left[e^{\kappa\Delta c_{t+1}}\cdot(1-\delta+x_t)^{\zeta}\cdot\hat{\xi}_{t+1}^{\theta}(x_{t+1},\pi_{t+1})\right] \\
&- (\hat{\xi}_t(x_t,\pi_t)-1)^{\theta}.
\end{aligned}
$$

Denote by $f(S_t,\varepsilon_{t+1})$ the integrand in the equation above. Then,

$$\mathbb{E}_t\left[f(S_t,\varepsilon_{t+1})\right]=\int\left[\pi_t f(1,\varepsilon_{t+1})+(1-\pi_t)f(0,\varepsilon_{t+1})\right]d\Phi(\varepsilon),$$

where $\Phi(\varepsilon)$ is a cdf of the standard normal distribution, and

$$f(S_t, \varepsilon_{t+1}) = e^{\kappa(\mu^c_{S_t} + \sigma^c \varepsilon_{t+1})}(1 - \delta + x_t)^\zeta \, \hat{\xi}^\theta_{t+1}(x_{t+1}(S_t), \pi_{t+1}(S_t)), \quad S_t \in \{0, 1\}$$

The residual function becomes

$$\hat{R}(x_t, \pi_t; \phi) = -(\hat{\xi}_t(x_t, \pi_t) - 1)^\theta + \beta^\theta \int [\pi_t f(1, \varepsilon_{t+1}) + (1 - \pi_t) f(0, \varepsilon_{t+1})] \, d\Phi(\varepsilon).$$

We choose ϕ so that $\hat{R}$ is exactly zero at n collocation points. These points are chosen as a roots of nth order Chebyshev polynomial. The integral $\int [\pi_t f(1, \varepsilon_{t+1}) + (1 - \pi_t) f(0, \varepsilon_{t+1})] \, d\Phi(\varepsilon)$ is evaluated using the Gauss-Hermite quadrature.

Further, let $P_{a,t}$ denote the date t price of dividend claim a. We conjecture that the price-dividend ratio $\frac{P_{a,t}}{D_{a,t}}$ is a function λ_t of state variables $\boldsymbol{x}_t$, which are as above. By definition,

$$R_{a,t+1} = \frac{P_{a,t+1} + D_{a,t+1}}{P_{a,t}} = \frac{D_{a,t+1}}{D_{a,t}} \frac{1 + \lambda(\pi_{t+1}, x_{t+1})}{\lambda(\pi_t, x_t)}.$$

Using the same procedure as before, we can solve for the price-dividend ratio.

Appendix C. Properties of Wealth-Consumption and Price-Dividend Ratios

To understand what explains the value and the dynamics of asset returns in the model it is worth looking at the properties of wealth-consumption and price-dividend ratios. Figures C.5 and C.6 present the wealth-consumption ratio as a function of two state variables in the model: the posterior probabilities π_t (blue solid line) and ratio of durable goods expenditure to stock of durables x_t (blue dashed line) for the baseline values of parameters from section 4. As we see from figure C.5, wealth-consumption ratio is increasing and convex function of π_t. The intuition for this fact is similar to Veronesi (1999) in the case of time-additive expected exponential utility. When π_t is close to 1 (meaning that the times are good), bad news decreases π_t and hence decreases the future consumption growth. At the same time, the agent's uncertainty about the consumption growth is increased, since π_t is now closer to 0.5. The agent wants to be compensated for being exposed to more risk, and thus he will require an extra discount on consumption claim. Therefore, this reduction of the price because of the bad news is higher than the reduction in expected future consumption. On contrary, if π_t is close to zero, and the times are bad, the good peace of news increases the expected future consumption growth, but also increases agent's uncertainty (π_t is closer to 0.5). Thus, the wealth-consumption ratio is increasing and convex. Figure C.6 depicts the wealth-consumption ratio as a function of x_t. In this case, the wealth-consumption ratio is increasing and concave function of x_t.

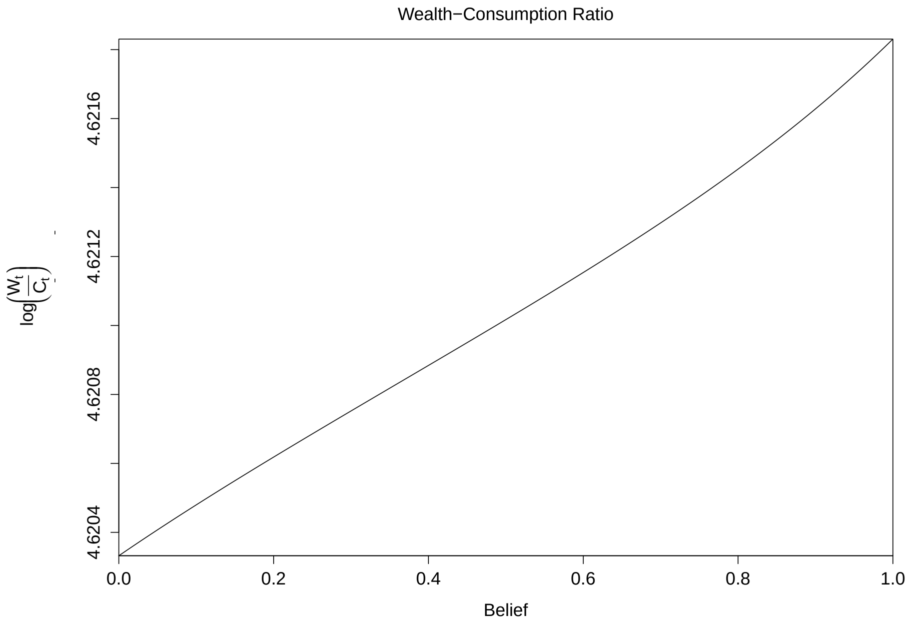

Figure C.5. The plot of wealth-consumption ratio as a function of posterior belief π_t, keeping the other state variable x_t fixed. The wealth-consumption ratio is calculated for the benchmark values of the preferences parameters.

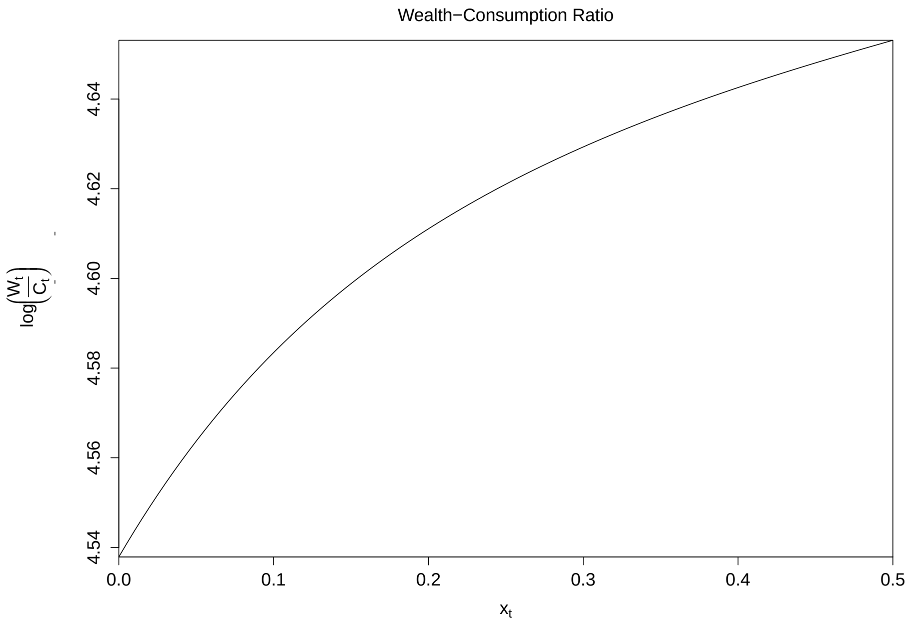

Figure C.6. The plot of wealth-consumption ratio as a function of the ratio of durable goods expenditure to stock of durables x_t, keeping the other state variable π_t fixed. The wealth-consumption ratio is calculated for the benchmark values of the preferences parameters.

Similarly, figures C.7 and C.8 present the price-dividend ratio as a function of π_t (black solid line) and x_t (black dashed line). We find that price-dividend ratio shares the similar features as wealth-consumption ratio (see figures C.5 and C.6)

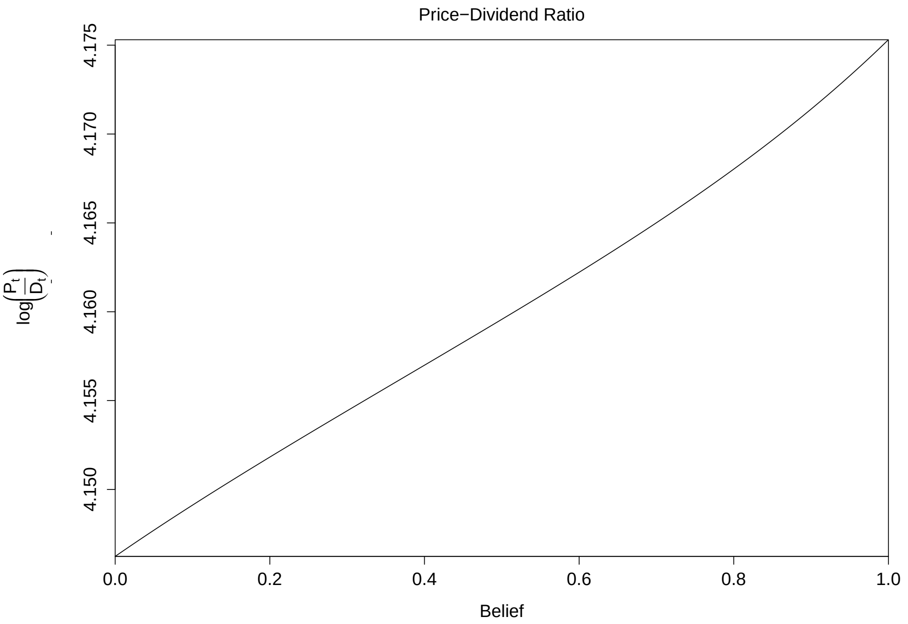

Figure C.7. The plot of price-divided ratio as a function of posterior belief π_t, keeping the other state variable x_t fixed. The price-dividend ratio is calculated for the benchmark values of the preferences parameters.

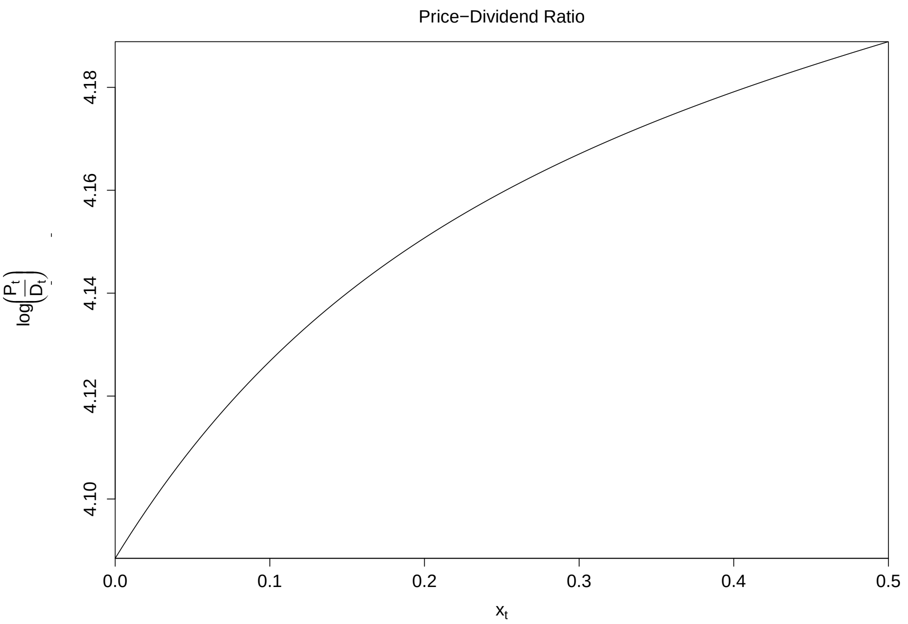

Figure C.8. The plot of price-dividend ratio as a function of the ratio of durable goods expenditure to stock of durables x_t, keeping the other state variable π_t fixed. The price-dividend ratio is calculated for the benchmark values of the preferences parameters.

Notice, that both the wealth-consumption and price-dividend ratios are concave and increasing functions of the state variable x_t. Moreover, the variation of the wealth-consumption and price-dividend ratios with respect to x_t is much higher than with respect to π_t, giving another evidence in favor of using the durable consumption in the asset-pricing models. The fact that these ratios are increasing in x_t is straightforward due to our ordering of states. Intuitively, the concavity property means that the wealth-consumption and price-dividend ratios dampen the low realizations of x_t (when the state of the world is bad) but exaggerate the high realizations (when the state of the world is good).