The University of Manchester
Manchester
Business School

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What are Bayesian Methods

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My background

- Took my first statistics course in 1970
  - No data; I thought Anova was a Russian mathematician!
- Introduced to Bayesian Statistics in 1971 by Adrian Smith and Mike Dempster
  - Morrie DeGroot (1970) Optimal Statistical Decisions
- Doctoral research on Bayesian Statistics in Protein Crystallography 1972-75
- Attended the first Valencia Conference in 1979
  - And have attended all since including the last in 2010
- Have worked on Bayesian *decision* analysis rather than *statistical* analysis ever since
  - But – hey! – its the same philosophy

The Bayesian paradigm

- explicitly models judgements and explores their implications
  - probabilities to represent beliefs and uncertainties
  - utilities to represent values
- is based upon a model of an idealised (consistent, rational) scientist
- focuses first on the individual scientist
- then by varying the scientist's beliefs enables the exploration of potential consensus.

For a Bayesian, knowledge is based on consensus

A philosophy ...

Bayesian analysis is an approach to inference and decision so it applies across all the interests of Methods@Manchester

- Visual/Sound
- Survey related
- E-Science/Data management
- Ethnographic methods
- Qualitative/interviewing
- Mixed methods
- Experimental methods
- Collaborative methods
- Quality assessment
- And probably any new ones that are introduced

Rev. Thomas Bayes

- 1701?-1761
- Main work published posthumously:
  T. Bayes (1763) An essay towards solving a problem in the doctrine of chances. *Phil Trans Roy. Soc.* **53** 370-418
- Bayes Theorem – inverse probability

Rev. T. Bayes

Bayes theorem

$$p(\theta | x) \propto p(x \mid \theta) \quad \times \quad p(\theta)$$

Posterior probability

$\propto$ likelihood $\times$ prior probability

Bayes theorem

$$p(\theta | x) \propto p(x | \theta) \times p(\theta)$$

Posterior probability

$\propto$ likelihood $\times$ prior probability

There is a constant, but 'easy' to find as probability adds (integrates) to one

Bayes theorem

$$p(\theta | x) \propto p(x | \theta) \times p(\theta)$$

Posterior probability

$\propto$ likelihood $\times$ prior probability

Probability distribution of parameters $p(\theta)$

Bayes theorem

$$p(\theta|\, x) \propto p(x \mid \theta) \quad \times \; p(\theta)$$

Posterior probability

$$\propto \text{likelihood} \times \text{prior probability}$$

likelihood of data
given
parameters $p(x|\theta)$

Bayes theorem

$$p(\theta | x) \propto p(x | \theta) \times p(\theta)$$

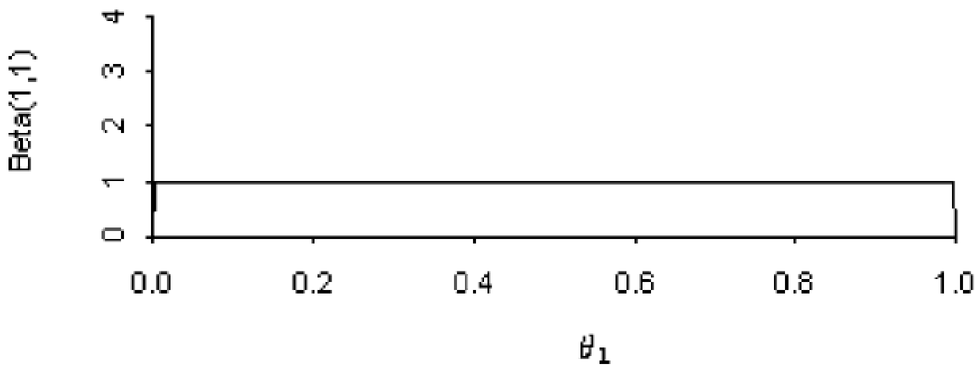
Posterior probability

$\propto$ likelihood $\times$ prior probability

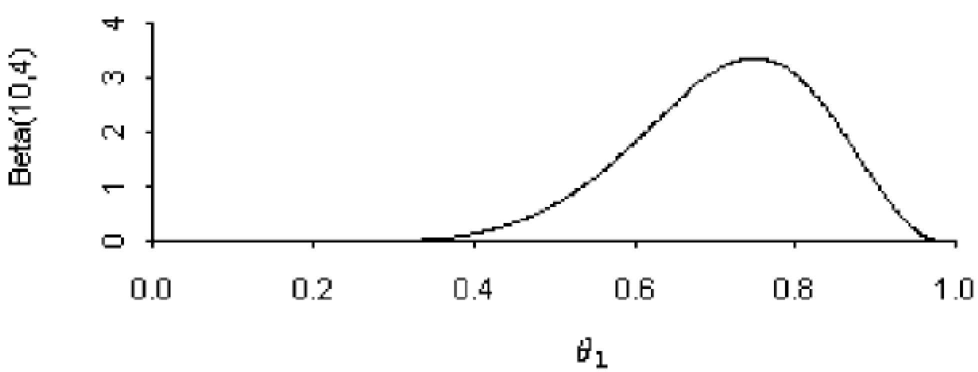
Probability distribution
of parameters
given data $p(\theta|x)$

Bayes theorem

Toss a biased coin 12 times; obtain 9 heads



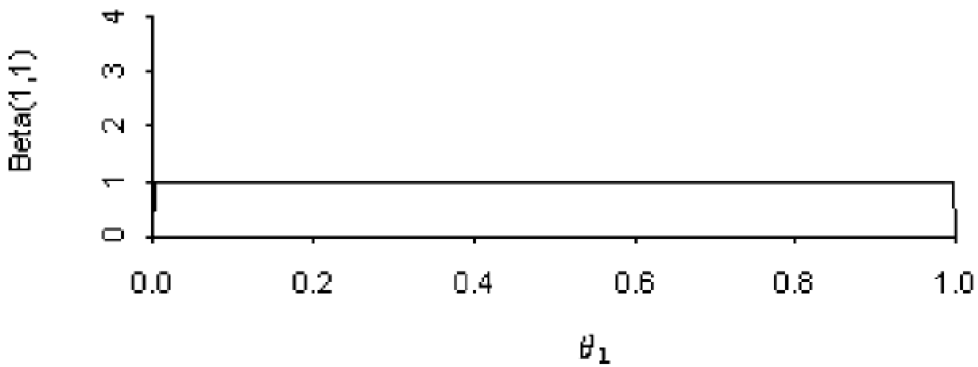
Prior



Posterior $\theta_1 | X_1 = 9$

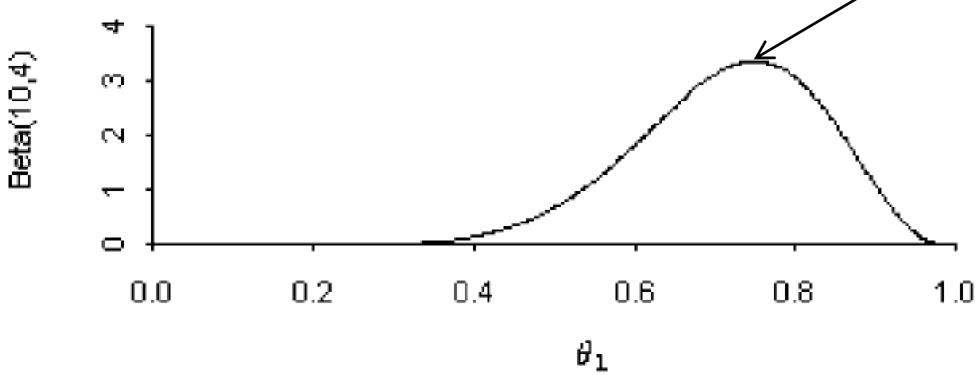
Bayesian Estimation

Toss a biased coin 12 times; obtain 9 heads



Prior

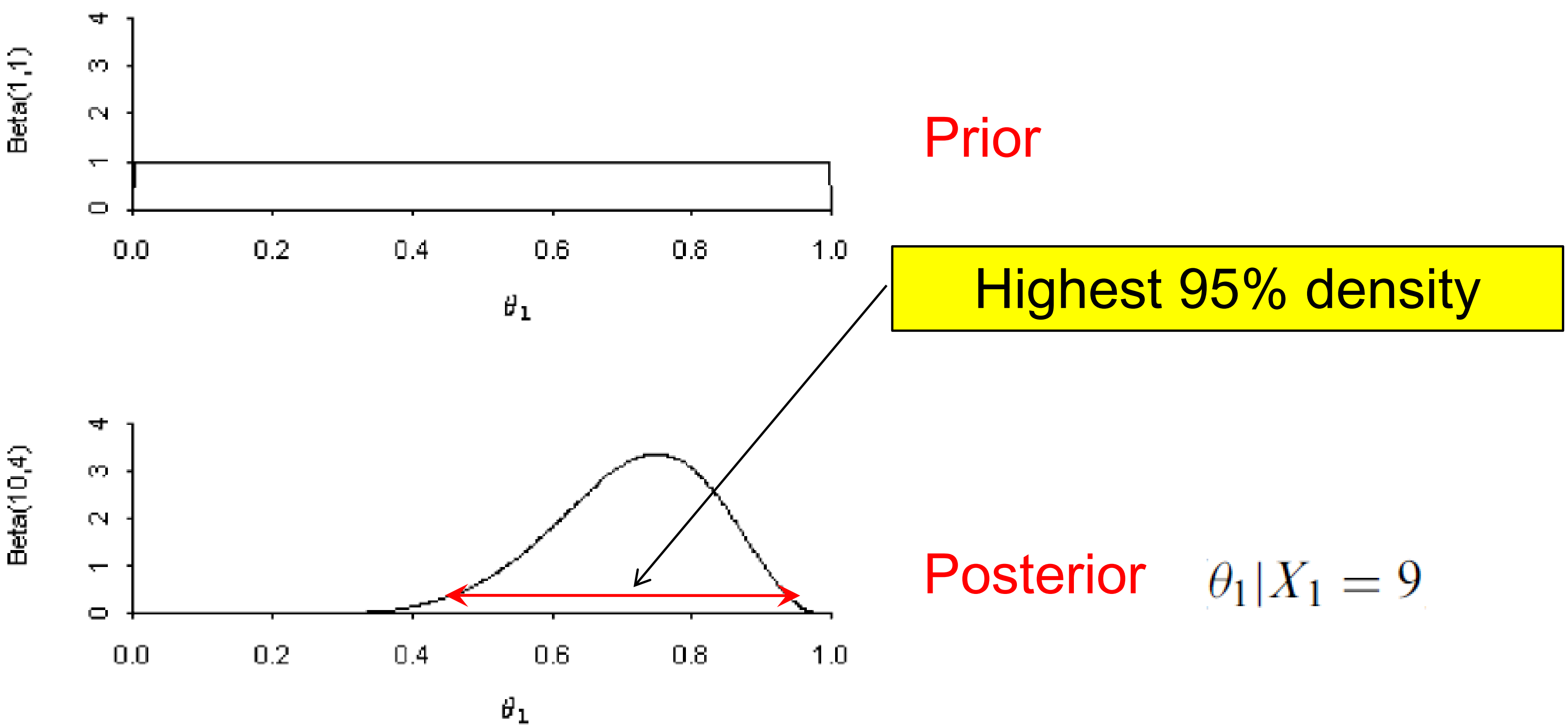
Take mean, median or mode



Posterior $\theta_1 | X_1 = 9$

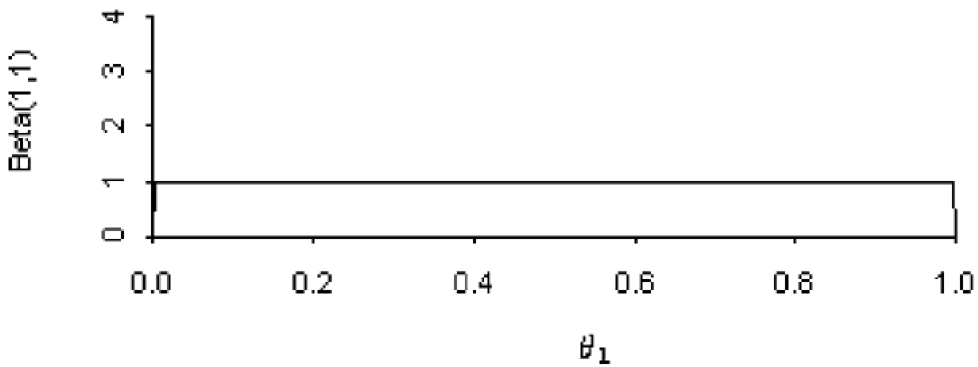
Bayesian confidence interval

Toss a biased coin 12 times; obtain 9 heads



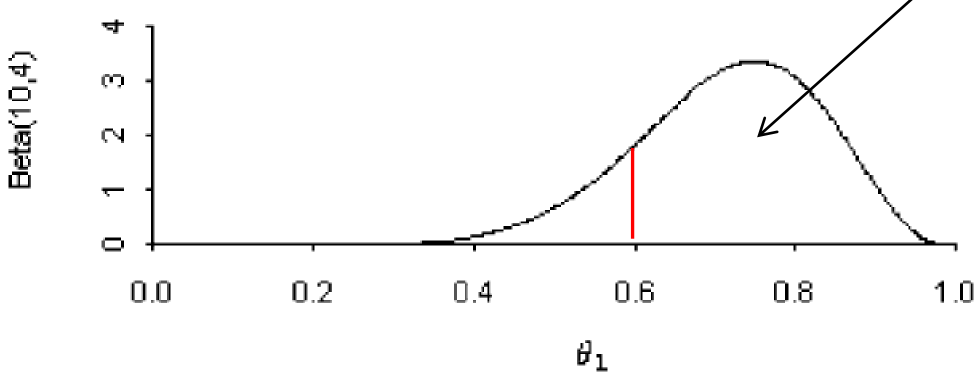
Bayesian hypothesis test

Toss a biased coin 12 times; obtain 9 heads



Prior

To test H_0: $\theta_1 > 0.6$
look at Prob(θ_1>0.6)

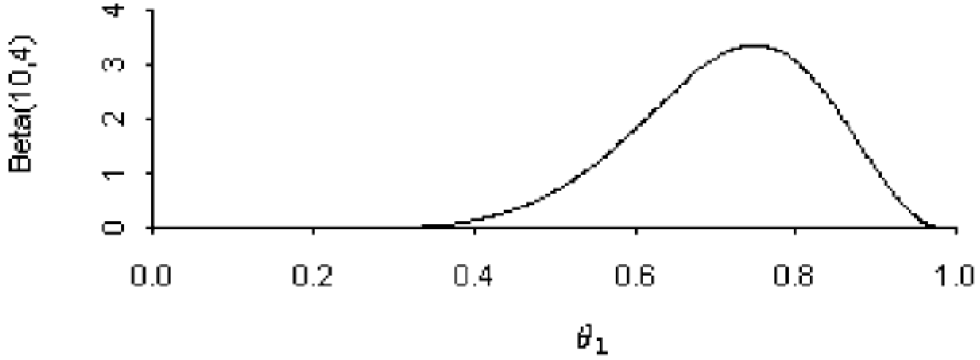


Posterior $\theta_1 | X_1 = 9$

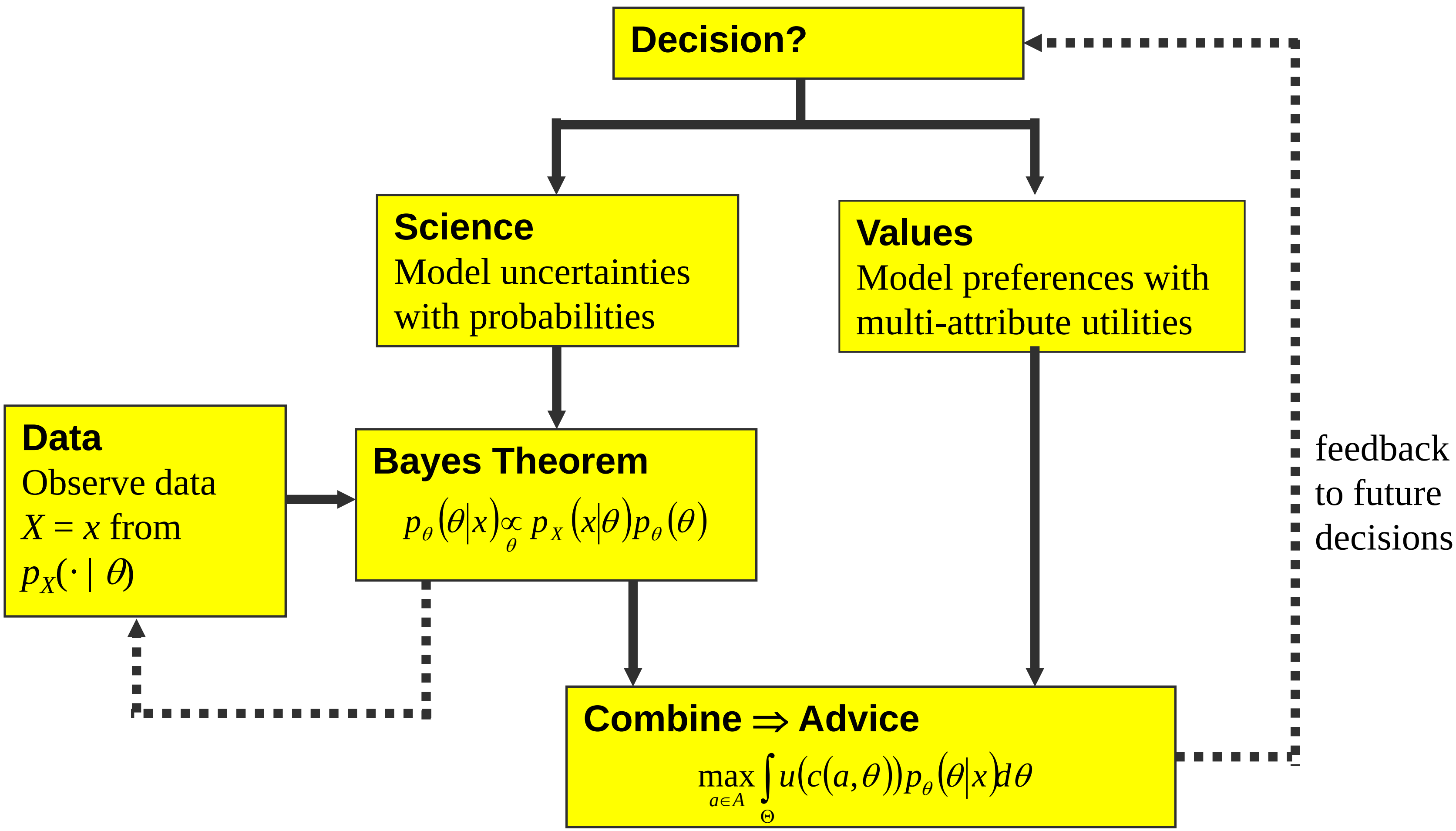
But why do any of these?

Just report the posterior.

It encodes all that is known about θ_1



Bayesian decision analysis



Evaluation of Bayes update and of expected utilities

- Analytic approaches
  - conjugate families of distributions
  - Kalman filters

- Numerical integration

- Markov Chain Monte Carlo (MCMC)
  - Gibbs Sampling
  - winBUGS
  - www.mrc-bsu.cam.ac.uk/bugs/welcome.shtml

Subjectivity vs Objectivity

- Bayesian statistics is explicitly subjective
- Science is (thought to be) objective

$\Rightarrow$ controversy!

- 1950s to 1980s

Controversial

"I have lamented that Bayesian statisticians do not stick closely enough to the pattern laid down by Bayes himself: if they would only do as he did and publish posthumously we should all be saved a lot of trouble."

M.G. Kendall (1968)

Tea, Haydn and a right ‘Tosser’

Three experiments:

- Tea tasting
- Music score recognition
- Tossing a coin

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The data and first analysis

Data: ‘10 out of 10’

H_0 the expert is right no more than would be expected by chance.

H_1 the expert performs better than chance.

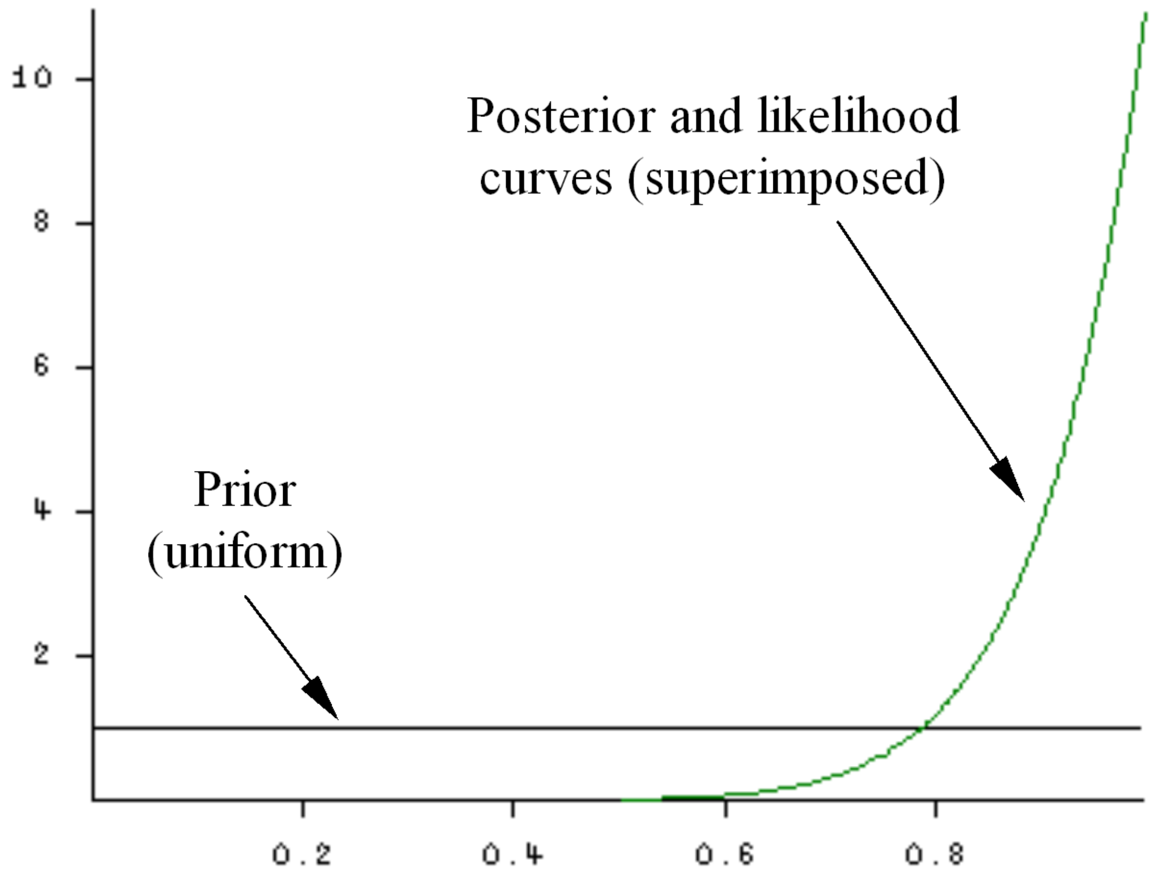
Significant evidence against H_0 in each case

Same evidence, same conclusion?

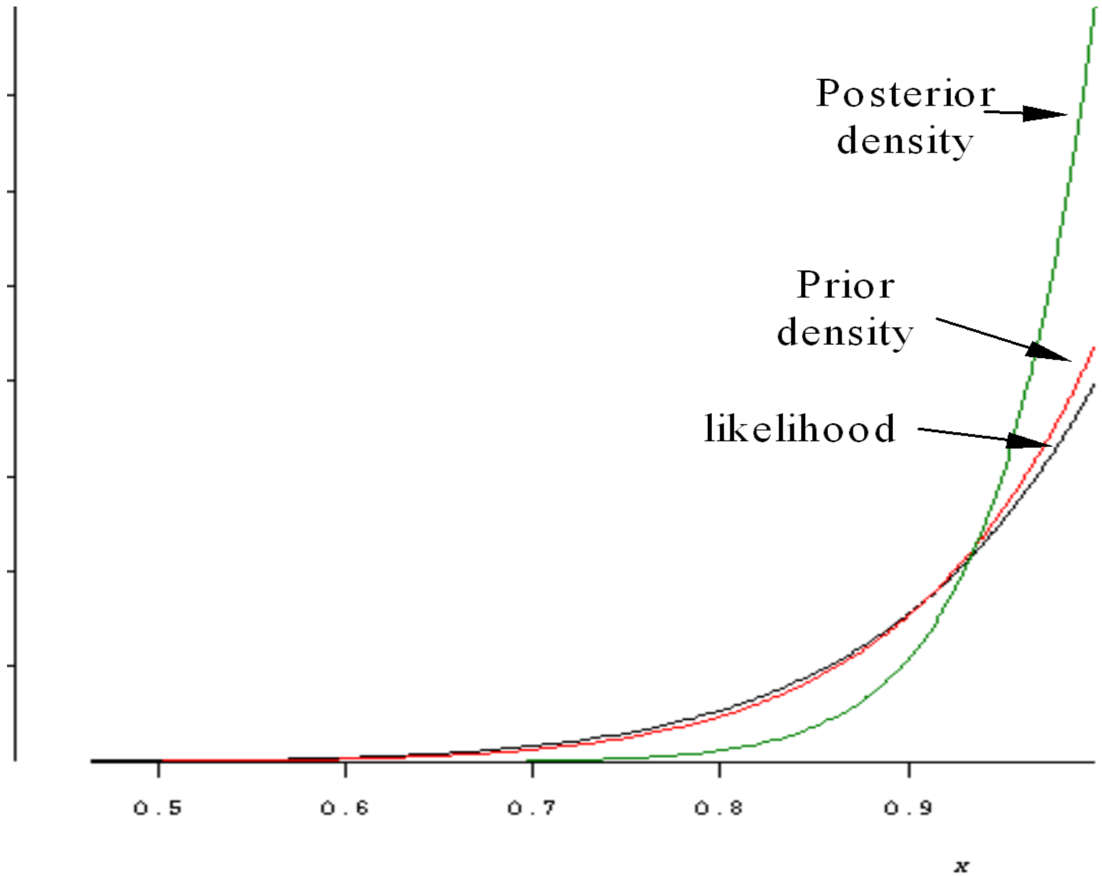
But a Bayesian analysis recognises differences

Recognises different prior knowledge

The Tea Taster



The Haydn Expert



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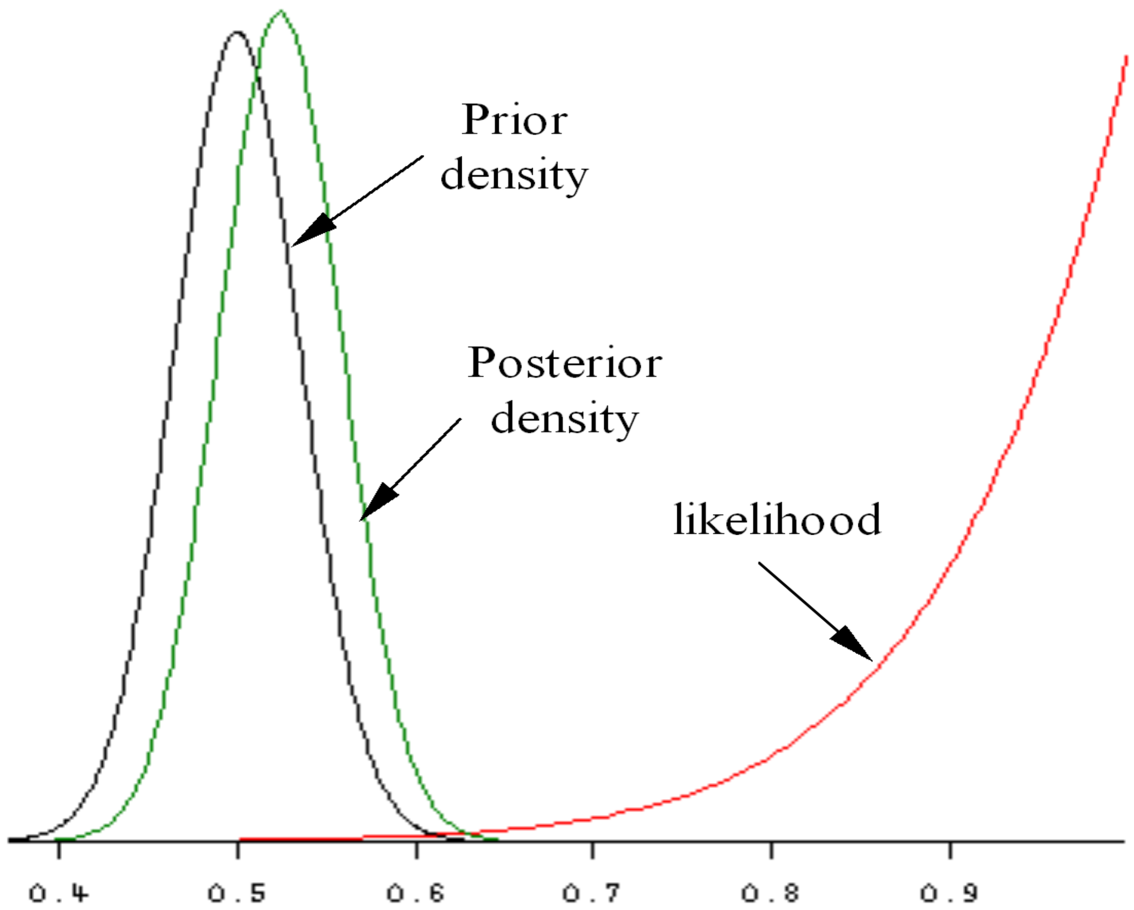
The Drunk



Importance of prior

- Different priors lead to different conclusions
  - ⇒ subjective
  - ⇒ not *scientific*?
- Can use:
  - ignorant (vague, non-informative) prior to 'let data speak for themselves'
  - precise prior to capture agreed common knowledge
  - Sensitivity analysis to explore the importance of the priors
- Indeed can use sensitivity analysis to explore agreements and disagreements on many aspects of the model not just the prior

BUGS Software

- **B**ayesian inference **U**sing **G**ibbs **S**ampling
- Lunn, D.J., Thomas, A., Best, N., and Spiegelhalter, D. (2000) WinBUGS -- a Bayesian modelling framework: concepts, structure, and extensibility. *Statistics and Computing*, **10**:325—337
- http://www.mrc-bsu.cam.ac.uk/bugs/

ISBA

- **I**nternational **S**ociety for **B**ayesian **A**nalysis
- www.bayesian.org
- Many resources and guide to software, literature, etc.
- Newsletter
- Open journal: ***Bayesian Analysis***

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Reading

W.M. Bolstad (2004). *Introduction to Bayesian Statistics*. Hoboken, NJ, John Wiley and Sons.

R. Christensen, W. Johnson, A. Branscum and T.E. Hanson (2011) *Bayesian Ideas and Data Analysis*. Boca Raton, CRC/Chapman and Hall

P. Congdon (2001) *Bayesian Statistical Modelling*. Chichester, John Wiley and Sons

S. French and D. Rios Insua (2000). *Statistical Decision Theory*. London, Arnold.

A. O'Hagan and J. Forester (2004). *Bayesian Statistics*. London, Edward Arnold.

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