What is……Multilevel Modelling Vs Fixed Effects

Will Cook
Social Statistics

Intro

- Multilevel models are commonly employed in the social sciences with data that is hierarchically structured
- Estimated effects from multilevel models can be easily criticised for being driven by confounding variables
- Fixed effects models are an alternative to deal with this weakness and support causal conclusions
- There are however a number of drawbacks, including limiting the scope of the modelling undertaken

Intro

- Preference for the one of the two methods is partially driven by disciplinary norms
- Sociology & Education = Multilevel models
- Economics = Fixed effects models

Note on terminology:

- Multilevel models are also called ‘Random Effects’ models, sometimes denoted as MLM
- Confusingly it is common to refer to coefficient estimates on explanatory variables in multilevel models as ‘fixed effects’ !

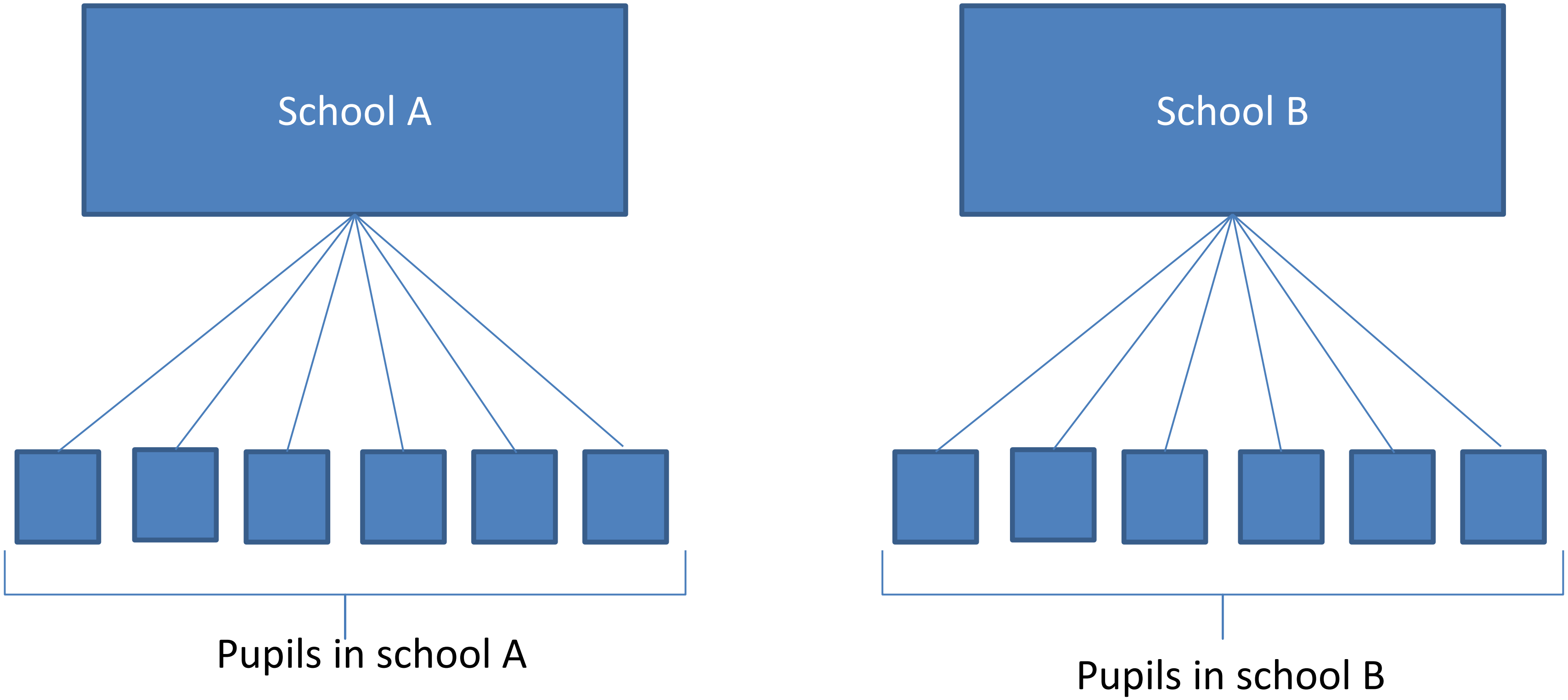
Examples used

- Cross sectional hierarchies
  - Peer effects in schools (contextual effects)
- Longitudinal Hierarchies
  - Effect of divorce on children

- Brief overview of multilevel models
- The causal inference problem
- Fixed effects models
- Problems with fixed effects models
- Which should I use?

- Much social data is hierarchically structured……

Pupils in schools

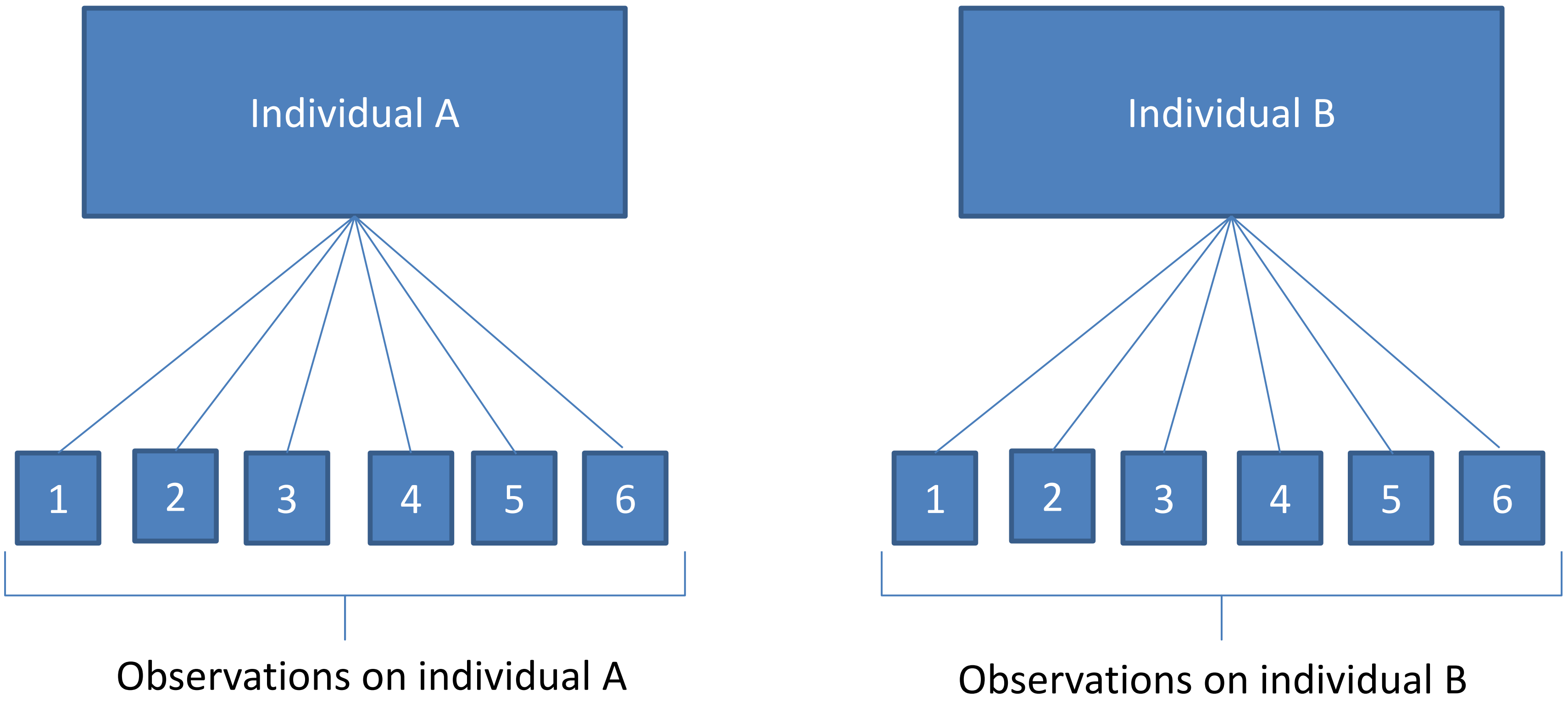


Other cross sectional hierarchical data

- Citizens within countries
- People within neighbourhoods
- Siblings within families
- Players within a sports team

Longitudinal hierarchical data

- Observations on an individual (or other unit of analysis) across time



More complex structures

- Individuals observed over time within a family, within a school
- Individuals observed over time within a family, within a school, accounting for changes in school attended
- Individuals observed over time within a family, within a school, accounting for family members attending different schools

- Much social data is hierarchically structured……

- Multilevel modelling allows us to recognise this in our model by assuming that the error term in a regression (i.e. everything that is not explained by the explanatory variables) is structured according to the known hierarchy.

E.g.

$$y_{ij} = b_0 + b_1x_{ij} + b_2p_j + e_{ij}$$

Becomes

$$y_{ij} = b_0 + b_1x_{ij} + b_2p_j + v_{ij} + u_j$$

- This means that standard errors on the coefficients (b) are not downwardly biased and therefore reduces risk of type I error.
- Also is a structure for estimating how relationships vary between contexts (random slopes and cross level interactions)

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Partitioned error, with assumptions the same as usual OLS (i.e. independence, normality, homoscedasticity)

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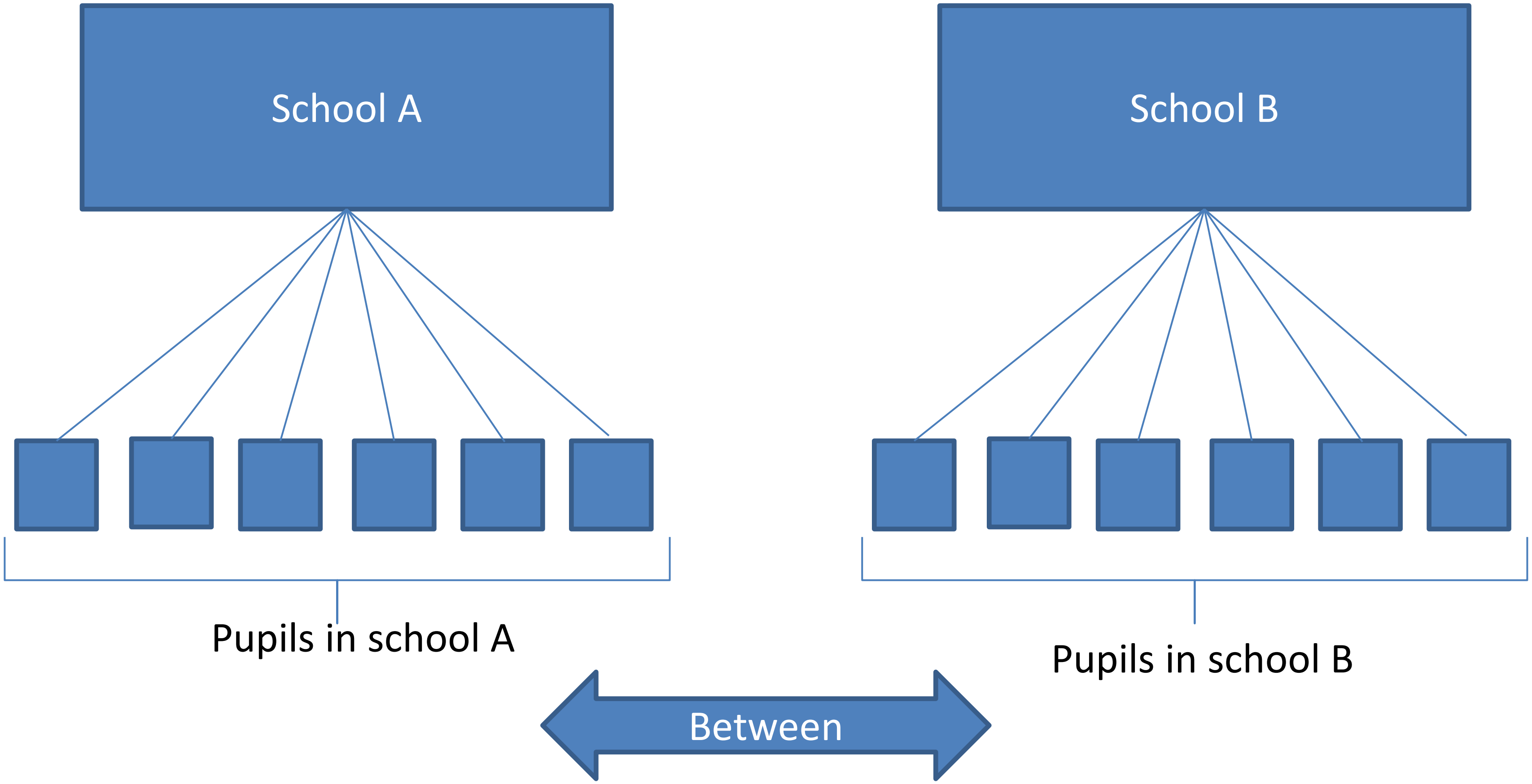
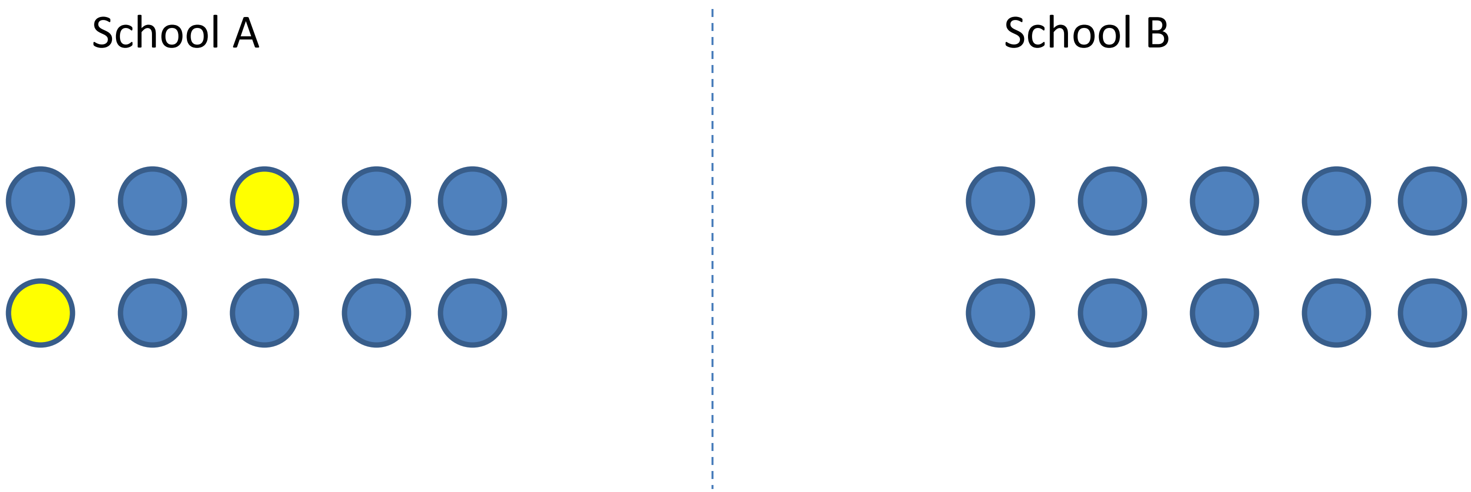
- Example I: Peer effects in schools

Pupils in schools



Effect of SEN inclusion on pupil test scores (attainment)

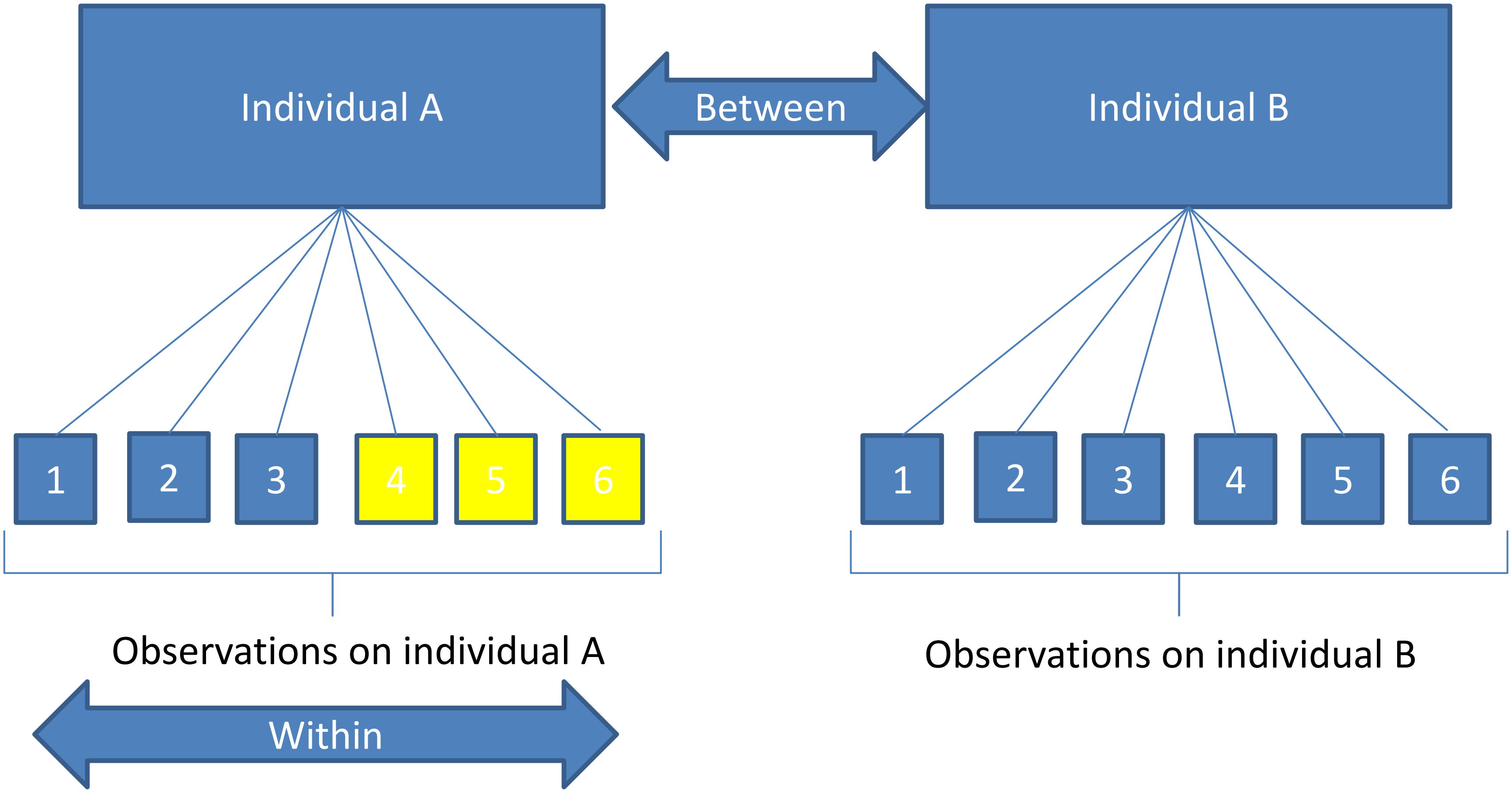


-Estimation of the effect of the SEN peers becomes a comparison between the outcomes in school A and school B

-Controlling for observed variables, the estimation of the effect simply is calculated as the mean of pupil attainment in school A minus the mean of pupil attainment in school B.

- Example II: Effect of divorce on children

Effect of divorce on children's behaviour



- Estimated effect is driven both by comparing the outcomes ‘between’ individuals and the change in outcomes ‘within’ individuals

- Multilevel modelling generates the correct standard errors and efficiently weights the between and within variation to generate the estimated effect based on the residual variances within and between individuals

- Brief overview of multilevel models
- The causal inference problem
- Fixed effects models
- Problems with fixed effects models
- Which should I use?

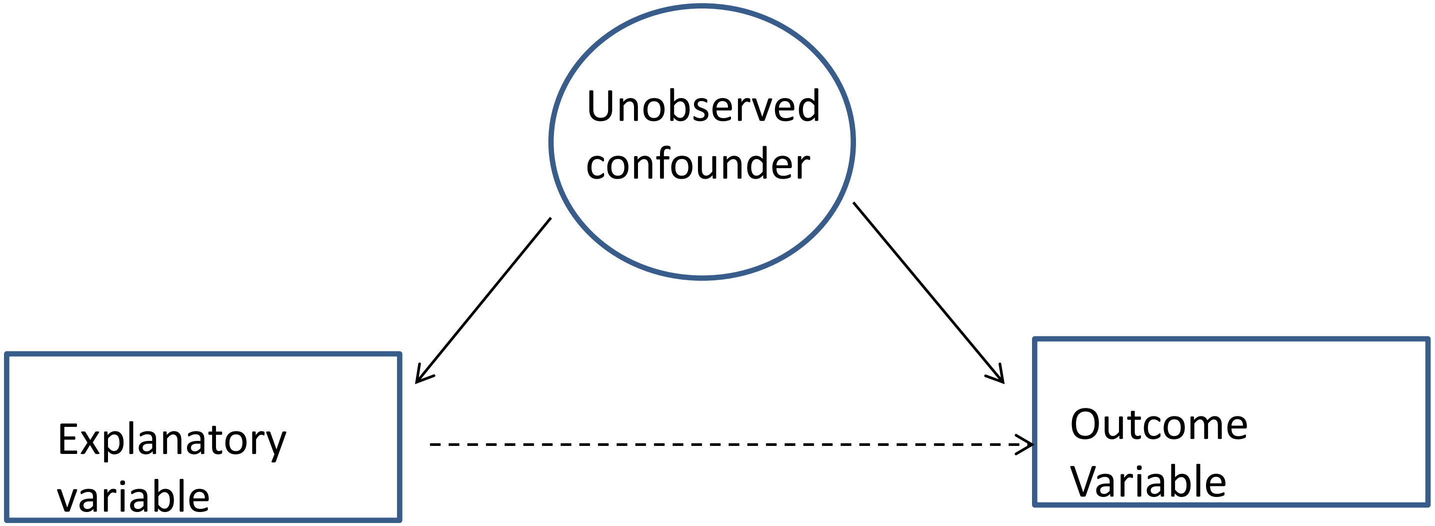
Is your data appropriate for MLM?

- Need to treat higher level units as if they were a sample:
    - Sufficient sample size (N>50?)
    - Random sampling
- Many MLM analyses do not conform to this (especially when the higher level unit is a country – see Mohring (2012))

Causal inference problem

- To generate unbiased estimates, in regression modelling we make the assumption that explanatory variables are uncorrelated with the error term
- Key problem in multilevel models is that variation between higher level units that generate your co-efficient estimates, might be related to unobserved confounding variables
- Problem in example 1: Cannot distinguish the *effect* of being in a group (e.g. school/neighbourhood etc) from the *reason* for being in a group (Hoxby, 2000)
- Problem in example 2: The propensity for individuals to experience change in the variable of interest is often determined by other pre-existing variables that vary between individuals and also affect outcomes.

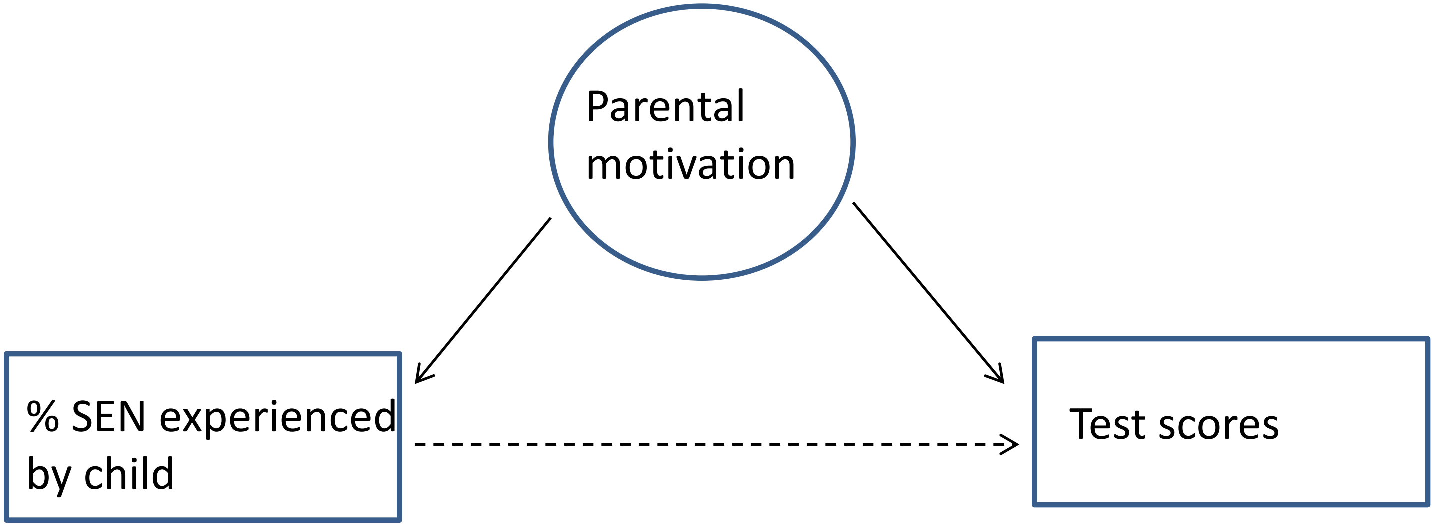
Problems with causal inference from observational studies



Example I: Sources of bias

- SEN pupils might go to certain types of school
- Pupils might be more likely to encounter a SEN peers based on unobserved pupil characteristics that are related to attainment.
- Parents may actively choose schools that do not have SEN peers; parental motivation is an unobserved pupil characteristic.

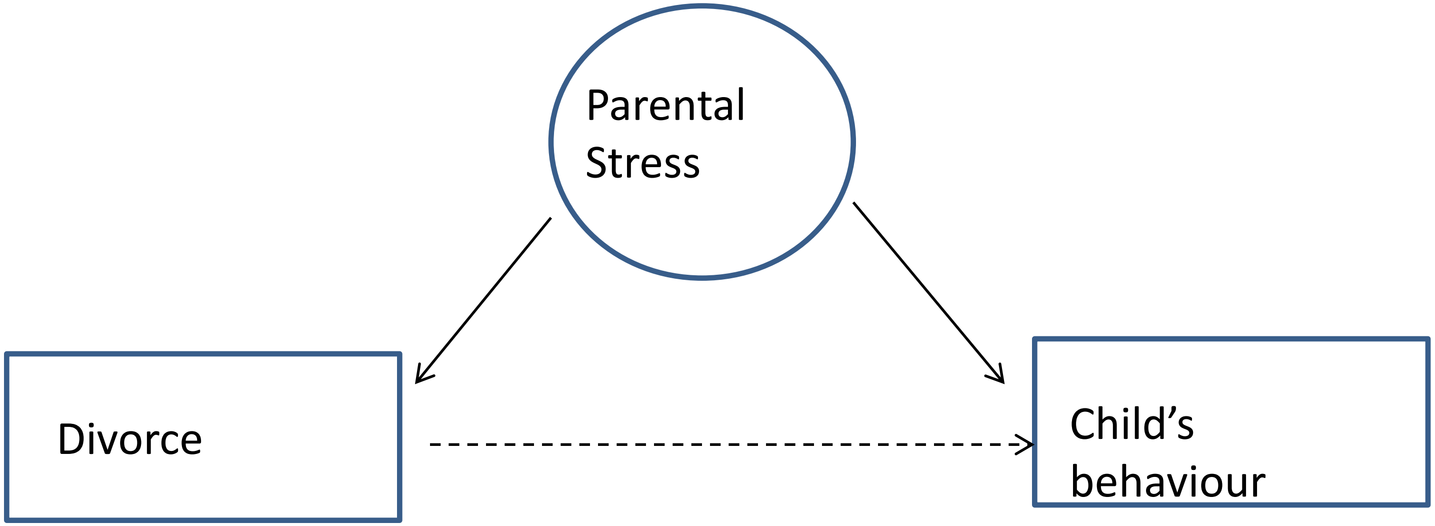
Problems with causal inference in a peer effect study



Example II: Sources of bias

- Socio-economic deprivation
- Parenting style (linked to many other factors)
- Number and gender of siblings

Problems with causal inference in a longitudinal study



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Fixed effects - overview

- Fixed effects models eliminate any variation in higher level units in coefficient estimation; they rule out ‘between’ variation.

- This means that (fixed) unobserved differences between higher level units (e.g. individuals, schools, neighbourhoods etc) no longer bias our estimates.
- Estimation is straightforward (OLS), in STATA it is implemented using the *xtreg,fe* command

Fixed effects – dummy variable model

- In fixed effects models, the higher level effect is no longer part of the residual
- Instead, fixed effects models can be thought of as including a dummy variable for each higher level unit.
- These dummy variables control for all variation (observed and unobserved) at the higher level

Within transformation

- Another way of thinking about fixed effects models is that they transform the variables into the deviation from the higher level unit mean.

- The actual research question being analysed by the data is whether the deviation of the outcome variable around its (group or individual level) mean is related to the corresponding deviation in the variables of interest from its mean.

- This is called the within transformation and is what is usually estimated in statistical packages.

- The overall effect is the same: all between variation is removed.

- Example I : Group fixed effects

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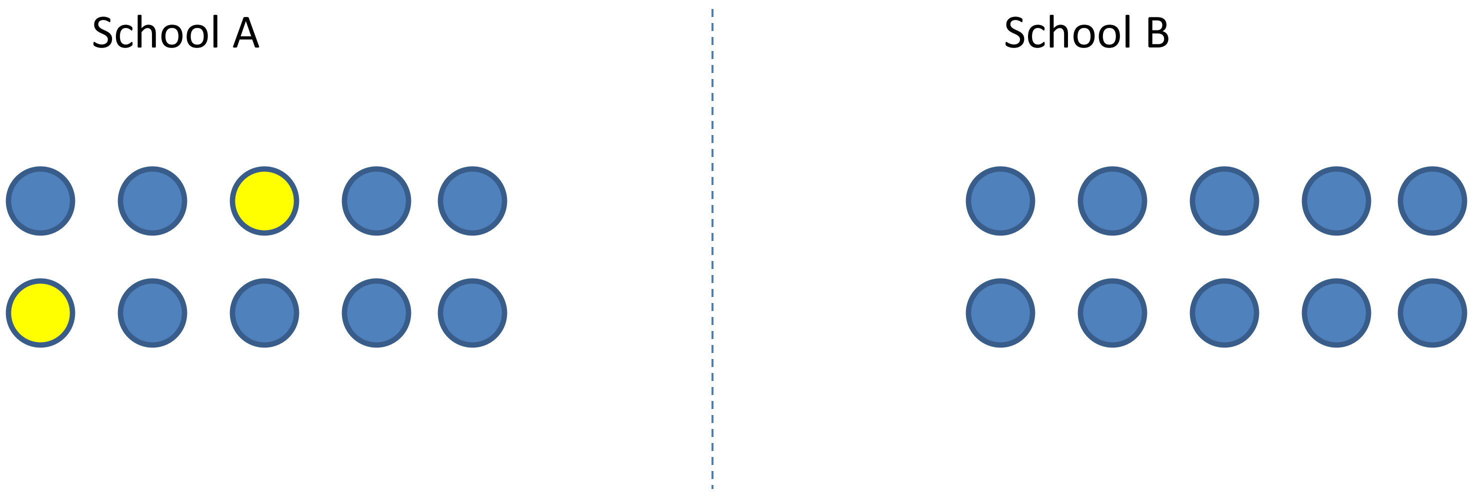

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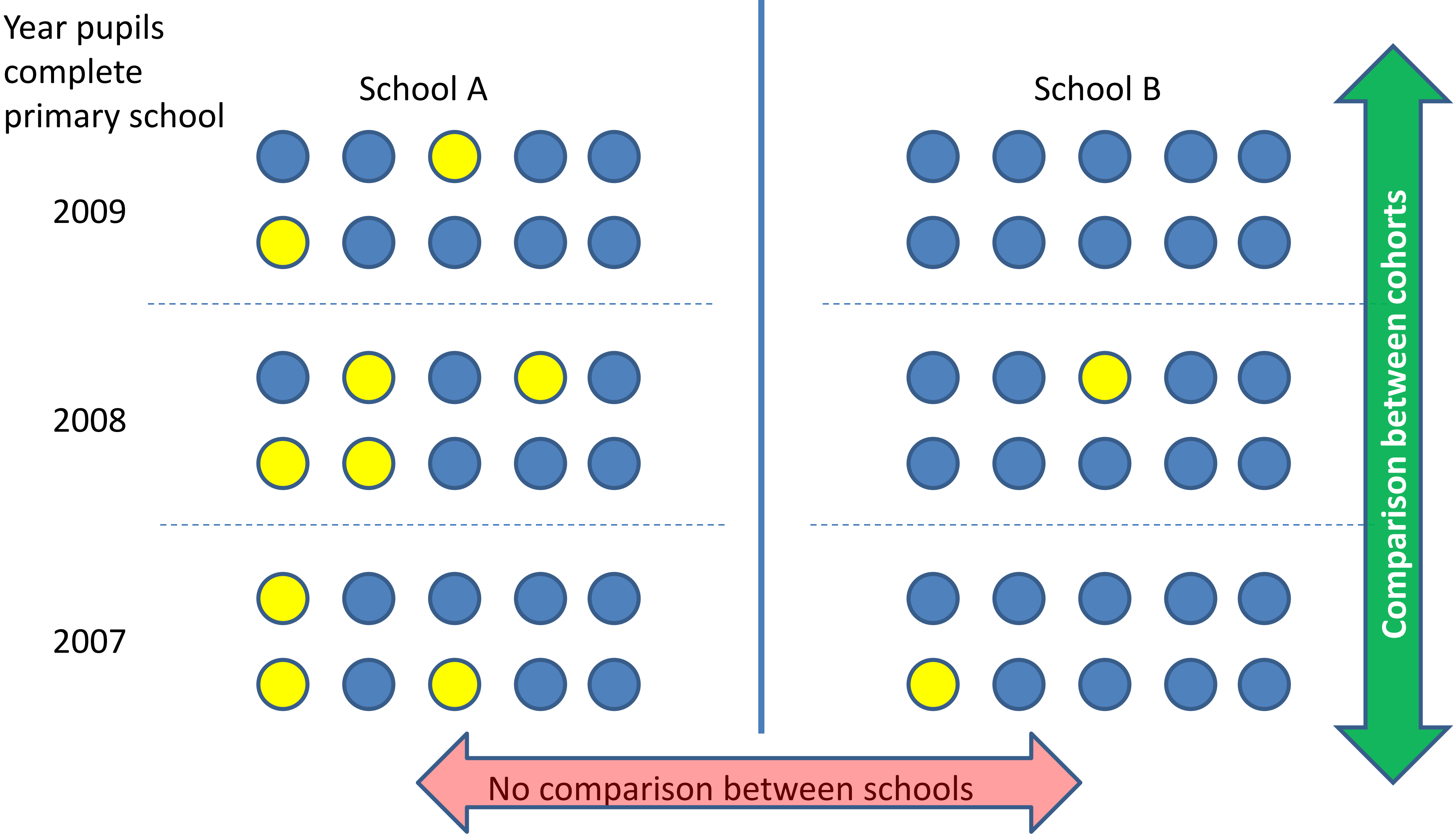
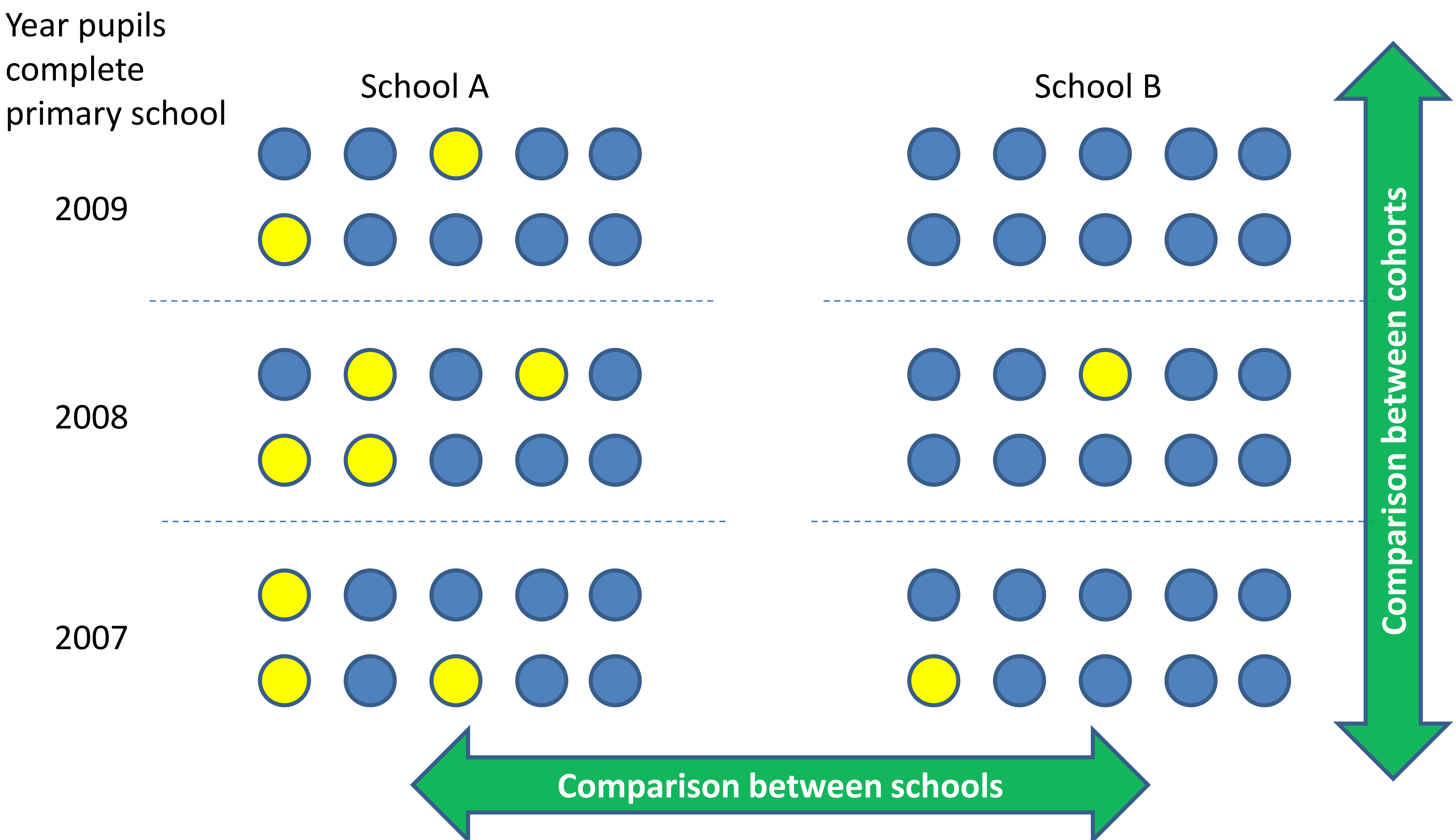
Sources of bias

- SEN pupils might go to certain types of school
- Pupils might be more likely to encounter a SEN peers based on unobserved pupil characteristics that are related to attainment.
- Parents may actively choose schools that do not have SEN peers; parental motivation is an unobserved pupil characteristic.
- Therefore comparing pupils between schools may produce biased results

School fixed effects models using multiple cohorts attempt to avoid these biases



Multilevel models using both between and within school comparisons

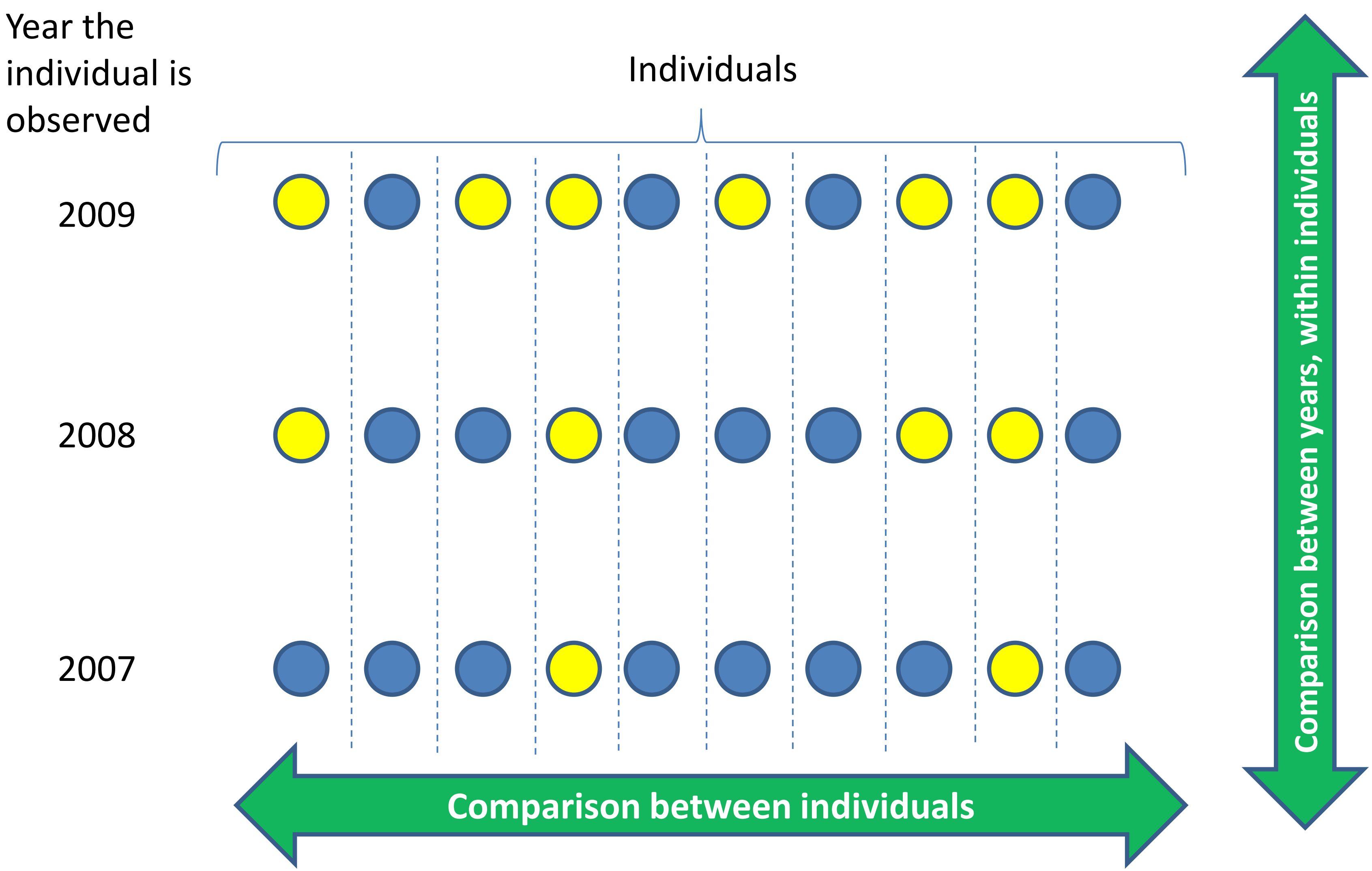


- Fixed effects models of peer effects tend to find smaller effects than multilevel models

- Note that the fixed effects model is estimating the peer effect by comparing pupils within the same school but in different cohorts
- Assumes that cohort to cohort variation in peer groups within a school is random

- Example II : Individual fixed effects

Longitudinal multilevel models use all source of variation in estimation



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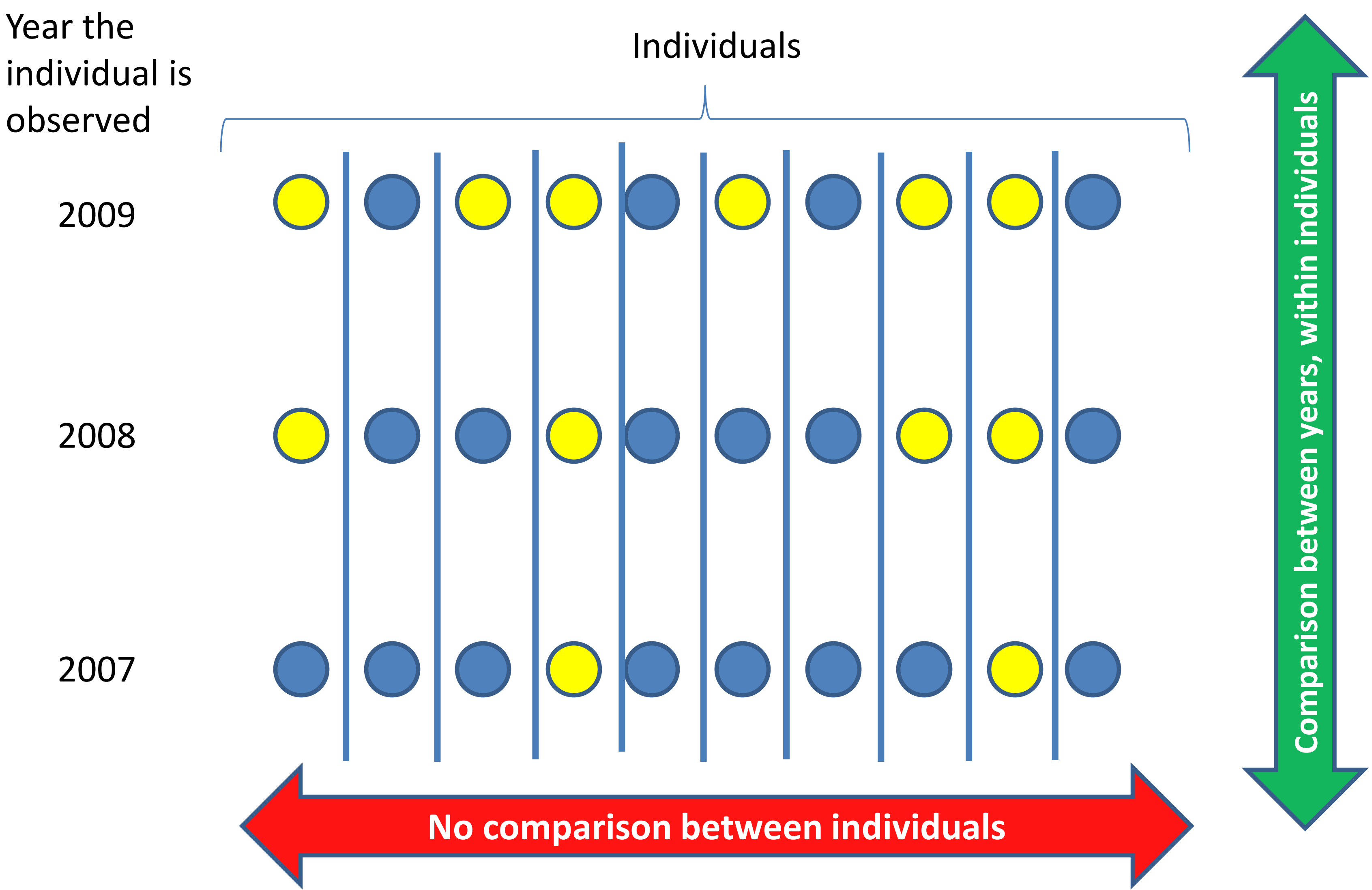
Individual fixed effects eliminate all comparisons between individuals



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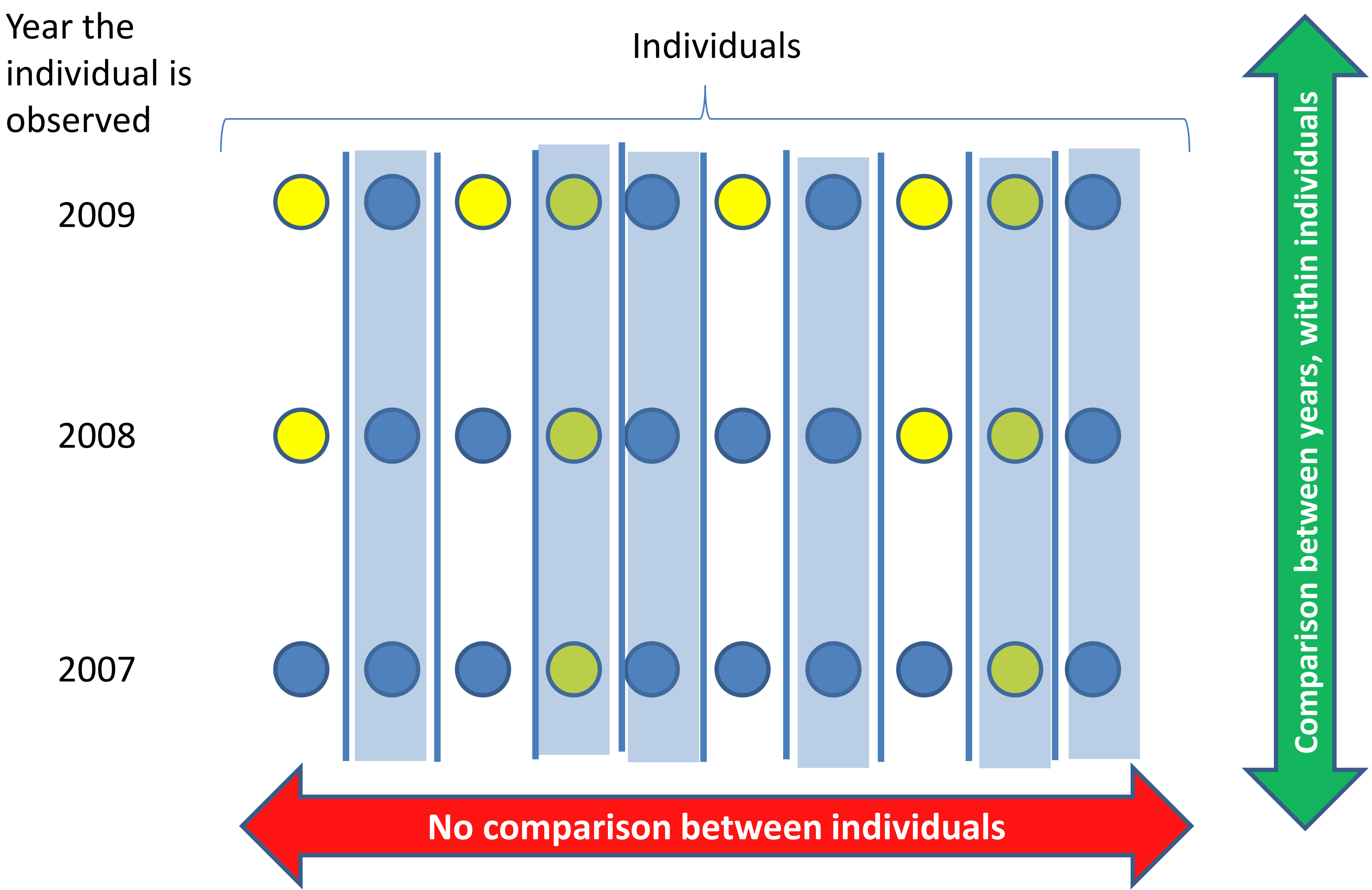

- Fixed effects models of divorce on childhood outcomes (e.g. behaviour) tend to find smaller effects than multilevel models and in some cases zero effects

- Note that the fixed effects model here is estimating the effect of divorce by comparing outcomes for a particular individual before and after parental divorce; it excludes cases where there is no change in state over period of analysis

- Assumes the *timing* of the divorce is random

- Brief overview of multilevel models
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Problem 1: Less variation

- Lack of variation
  - Measurement error
  - Non-linearities
  - Low power

Problem 2: Elimination of variation of interest

- Excludes ‘useful’ variation, though note that cross level interactions are still possible
- Equivalent of random slopes models is unwieldy
- In the case of education research, removing the school level from the analysis can be problematic
- Does not control for time-varying heterogeneity even at the higher level

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Hausman test

- Formally you can test for whether to use a multilevel model using the Hausman test

- Tests whether the coefficient estimates from the multilevel model are statistically significantly different from the fixed effects estimates (assumed to be unbiased)
- Assumes a well specified model.

Using Judgement

- Clarke et al (2010) recommend:

  “when the selection mechanism is fairly well understood and the researcher has access to rich data, the random effects model should naturally be preferred because it can produce policy-relevant estimates while allowing a wider range of research questions to be addressed.”

- This would seem to suggest that fixed effects models should be preferred in most cases.

Use both?

- Multilevel models have powerful descriptive uses
- If the data allows, multilevel estimates should be checked against those from fixed effects models
- Discrepancies and similarities between multilevel and fixed effects estimates are informative in themselves
- Adding contextual level variables to a multilevel model should eliminate *some* bias (e.g. means of level one variables) but may be problematic if small number of higher level units
- Straightforward to test both approaches (e.g. *xtreg,re* vs. *xtreg,fe* in STATA)

Summary

- In choosing between the two methods you should consider:
  - Are you interested in estimating a causal relationship?
  - Do you have concerns that unobserved higher level variables may affect the estimation of this relationship?
- If both answers are yes then fixed effects models should be preferred.
- No harm in using both methods and where possible, both should be considered.

References

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- Goldstein, H. (2010) Multilevel Statistical Models, 4th Edition, Wiley.
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