

Indicators for Representative Response

Barry Schouten
Statistics Netherlands
Postbus 24500
2490HA Den Haag, The Netherlands
bstn@cbs.nl

Natalie Shlomo, Chris Skinner
University of Southampton
Highfield
Southampton, SO17 1BJ, UK
N.Shlomo@soton.ac.uk , C.Skinner@soton.ac.uk

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1. Introduction

The impact of nonresponse on survey quality is typically measured by the response rate. The response rate alone, however, is not sufficient as a quality indicator to capture the potential impact of nonresponse. The bias of estimates resulting from nonresponse also depends on the contrast between respondents and nonrespondents with respect to a target variable. The more they differ, the larger the bias will be. Good indicators that measure the degree to which the group of respondents of a survey still resembles the complete sample are currently lacking. Project RISQ (Representativeness Indicators for Survey Quality, www.risq-project.eu) was set up in order to fill the gap of indicators that measure the representativeness of the response to survey and register requests. We call these indicators Representativity indicators or R-indicators.

The indicators can be used in four different settings:

- To compare the response to different surveys that share the same target population, e.g. households or businesses
- To compare the response to a survey longitudinally, e.g. monthly, quarterly or annually
- To monitor the response to a survey during data collection, e.g. after various days, weeks or months of fieldwork
- To control the response to a survey by means of adaptive survey designs or responsive survey designs (e.g. Groves and Heeringa 2006, Mohl and Laflamme 2007, Wagner 2008)

In this paper we define representativeness and indicators for representative response. We illustrate the different types of indicators and we discuss the use of indicators in practical survey settings.

In section 2 we define representativeness and indicators. We illustrate the different types of indicators in section 3. Next, in section 4 we discuss the use of indicators in practical survey settings. RISQ is funded by the 7th EU Research Framework Programme.

2. Representativeness and indicators for representativeness

For the sake of brevity we only give condensed descriptions of representativeness and corresponding indicators. We refer to Schouten et al (2009), and the RISQ deliverables Shlomo et al (2009a) and Shlomo et al (2009b) for details.

2.1 Representativeness

Ideally we would like to define representativeness based on individual response probabilities. Their interpretation is not straightforward, however, and has been open to extensive debates in the literature; see e.g. various chapters in Madow and Olkin (1983). Moreover, it is impossible to estimate such probabilities based on a single response for each sample unit without making strong assumptions. For these reasons we restrict ourselves to response propensities. Let ρ_X denote the response propensity function for variable X , say age or gender, i.e. $\rho_X(x)$ is the probability that a population unit

carrying value $X = x$, say young people or females, will respond to the survey request. We suppose that X is a subset of a supervector \aleph of auxiliary variables that explains response behaviour and for which the response propensities ρ_{\aleph} can be viewed as individual response probabilities. This \aleph may be viewed as the whole of characteristics of a person or business that determines their response behaviour given a survey design.

We propose two definitions for representativeness of survey response; representative response and conditional representative response.

Definition: A response to a survey is representative with respect to X when response propensities are constant for X , i.e. when $\rho_X(x)$ is a constant function.

Definition: A response to a survey is conditional representative with respect to X given Z when conditional response propensities given Z are constant for X , i.e. when $\rho_{X,Z}(x, z) = \rho_Z(z)$ for all x .

The two definitions can be measured for any auxiliary vectors X and Z , e.g. age and gender or business size and type of business. In order to do that we need a distance function or metric, say $d(\rho_1, \rho_2)$, that measures distance between two vectors of response propensities ρ_1 and ρ_2 . For this purpose we use the Euclidean distance

$$d(\rho_1, \rho_2) = \sqrt{\frac{1}{N} \sum_U (\rho_{1,i} - \rho_{2,i})^2}, \quad (2.1)$$

where N is the population size, U the population units and i a label for a population unit.

This definition of representative response is proposed in Schouten et al (2009). The motivation for the definition is that it conforms to random samples, or in other words response leads to equal selection probabilities and can be considered as an additional phase in the sampling design. It's interpretation is straightforward as a result of that. It does not relate to a specific survey item, a specific estimator or a specific model for response behaviour other than that we assume that response propensities exist. The definition of conditional representative response is new.

With the definition of representative response we do not have estimation in mind but data collection. We do not consider a specific Y nor a specific parameter of any distribution. We question whether data collection succeeded in obtaining a balanced response for a set of pre-selected variables X that is available before and during data collection. Of course the selected variables may be of a general, wide interest when multiple surveys are compared, or may consist of relevant variables for a particular survey when that survey is compared to itself.

2.2 Measuring deviations from representative response

Given (2.1) we define a representativeness indicator or R-indicator, as the transformed distance between ρ_X , the response propensity function for X , and the constant vector $\rho_0 = (\rho, \rho, \dots, \rho)^T$, which equals the survey response rate ρ .

$$R(X) = 1 - 2d(\rho_X, \rho_0) = 1 - 2S(\rho_X) \quad (2.2)$$

It is easy to show that d is the standard deviation S of the response propensities for X . The transformation in (2.2) was made so that $R \in [0,1]$ and representative response is represented by a value of one (or 100%) for the indicator. A value of 0 indicates the largest possible deviation from representative response.

Note that $R(X_1) \geq R(X_2) \geq R(\mathfrak{N})$ when variable X_2 is nested in X_1 . The more refined the “resolution”, the more variation is observed. So one should not compare R-indicators based on different vectors of auxiliary variables.

In general X will be a vector of auxiliary variables like age, gender or urbanization for household surveys and business type and size for business surveys. If measuring representativeness is restricted to one auxiliary variable, say Z , then we call the indicator a partial representativeness indicator or partial R-indicator. At the variable level the partial R-indicator is defined as

$$P_u(Z) = d(\rho_Z, \rho) = S(\rho_Z), \quad (2.3)$$

the standard deviation of the response propensity function $\rho_Z(z)$ in the population.

The subscript u in (2.3) is given in order to distinguish partial R-indicators for unconditional representative response from those for conditional representative response that we will define in section 2.3. For any Z it holds that $P_u(Z) \in [0,1]$. Furthermore, $P_u(Z) \in [0, (1 - R(X))/2]$ when Z is an element of X .

Next, for categorical variables we define partial R-indicators for each category. Let Z be a categorical variable with categories $k = 1, 2, \dots, K$ and let Z_k be the 0-1 variable that indicates whether $Z = k$ or not. For example, Z represents age and Z_k is the indicator for being younger than 35 years of age. The partial R-indicator for a category k is defined as

$$P_u(Z, k) = \sqrt{\frac{N_k}{N}} (\rho_{Z_k} - \rho), \quad (2.4)$$

with $N_k = \sum_U Z_k$ the number of population units in category k . $P_u(Z, k)$ originates from dividing $P_u(Z)$ over the strata of Z while maintaining the signs between the stratum response propensity ρ_{Z_k} and the overall response rate ρ . Negative values indicate underrepresentation while positive values indicate overrepresentation. We have that $P_u(Z, k) \in [-1,1]$ and

$$P_u(Z) = \sqrt{\sum_{k=1}^K P_u^2(Z, k)}.$$

Note that (2.3), the partial R-indicator at the variable level, is in fact the square root of the “between” variance for variable Z . As such it is a component of the total variance of response propensities in (2.2), and, hence, always smaller than or equal to that variance.

2.3 Measuring deviations from conditional representative response

In measuring conditional representativeness we want to adjust the impact for one variable for the impact of other variables. Based on (2.1) we propose

$$P_c(Z | X) = d(\rho_{X,Z}, \rho_X) = \sqrt{\frac{1}{N-1} \sum_U (\rho_{X,Z}(x_i, z_i) - \rho_X(x_i))^2}, \quad (2.5)$$

the distance between propensities based on X and Z , and based on X alone. For example, X could be a vector containing household composition, household income and province of residence while Z equals the age of the head of the household.

Again, we define partial R-indicators for classes of categorical variables by distributing (2.5) over the classes of Z .

$$P_c(Z, k | X) = \sqrt{\frac{1}{N-1} \sum_U Z_k (\rho_{X,Z}(x_i, z_i) - \rho_X(x_i))^2}. \quad (2.6)$$

Other than for the unconditional partial indicators, we cannot assign a positive or negative sign to the category level conditional partial indicators in (2.6). The reason is that the sign may be different for each subclass of X . In some subclasses a certain age of the head of the household may have a positive effect on response while in others it has a negative effect.

It can be shown that (2.5) is the square root of the “within” variance of the $\rho_{X,Z}$ propensities for a stratification of the population with X . In other words, it is the variation that is left within the cells defined by X . In our example, it represents the variation in response behaviour due to the age of the head of the household given its household composition and income and the province in which the household lives. As the within variance is again a component of the total variance, the conditional partial indicators too cannot exceed the total variance that makes up the R-indicator in (2.2). Furthermore, the conditional partial R-indicator for Z is always smaller than the unconditional partial R-indicator for that variable. This makes sense; the impact on response behaviour is to some extent removed by accounting for other characteristics of the population unit. In many survey settings, for instance, the impact of gender on response behaviour is completely or considerably removed by accounting for the age of the person.

2.4 Maximal absolute contrast and maximal absolute bias

In order to enable R-indicators to be interpreted in terms of the impact of nonresponse on survey estimation, we consider the standardized bias of the design-weighted response mean \hat{y}_r of an arbitrary survey item y .

$$\frac{|B(\hat{y}_r)|}{S(y)} = \frac{|Cov(y, \rho_Y)|}{\rho S(y)} = \frac{|Cov(y, \rho_N)|}{\rho S(y)} \leq \frac{S(\rho_N)}{\rho} = \frac{1 - R(N)}{2\rho}, \quad (2.7)$$

with ρ the average response propensity (or expected response rate). Clearly, we do not know ρ_N . Moreover, we want to have a measure that enables comparison of the representativeness of response in different surveys or the same survey over time. In such a setting we are interested in the general representativeness of a survey, i.e. not the representativeness with respect to single survey items. We use as an approximation for (2.7)

$$B_m(X) = \frac{1 - R(X)}{2\rho}. \quad (2.8)$$

B_m represents the maximal absolute standardized bias under the scenario that non-response correlates maximally to the selected auxiliary variables.

Additionally, we consider the maximal contrast between respondents and non-respondents. The contrast for a variable Y is the expected difference between the response mean and nonresponse mean of that variable. The bias of the response mean can be rewritten as the product of the non-response rate $1 - \rho$ and the contrast.

$$B(\hat{y}_r) = (1 - \rho)(E(\hat{y}_r) - E(\hat{y}_{nr})).$$

Hence, we may define the maximal absolute standardized contrast as the maximal absolute standardized bias divided by the non-response rate. We denote it by $C_m(X)$

$$C_m(X) = \frac{1 - R(X)}{2\rho(1 - \rho)}. \quad (2.9)$$

For convenience we will refer to B_m and C_m as the maximal bias and maximal contrast.

The R-indicator, the maximal bias and the maximal contrast provide means to evaluate the quality of response. Ideally, one would like to bound the R-indicator from below, i.e. to derive values of the R-indicator that are acceptable and values that are not. We construct three so-called response-representativity functions that can be used for deriving lower bounds for the R-indicator. They are a function of a threshold γ and the response rate ρ . The threshold γ represents a quality level. The response-representativity functions are defined as

$$\begin{aligned} RR_1(\gamma, \rho) &= 1 - \frac{2}{\xi_{1-0.5\alpha}} \gamma && \text{(maximal variation in response propensities)} \\ RR_2(\gamma, \rho) &= 1 - 2\rho\gamma && \text{(maximal bias)} \\ RR_3(\gamma, \rho) &= 1 - 2\rho(1 - \rho)\gamma, && \text{(maximal contrast)} \end{aligned}$$

with $\xi_{1-0.5\alpha}$ being the $1 - 0.5\alpha$ quantile of the standard normal distribution.

We refer to Schouten and Bethlehem (2009) for background to the response-representativity functions. In section 4 we show how they can be used to plot traces of response rate and response representativeness over time or during data collection. As such they may be used to assess the number of days, weeks or months that is needed to get a response that satisfies a minimal quality level represented by the quality threshold.

2.5 Estimation of indicators and, contrast and bias

In the RISQ project we have proposed estimators for R , P_u , P_c , B_m and C_m . We refer to Shlomo et al (2009a and b) for the estimators and their details. The estimators replace population means by design-weighted sample and response means and response propensities by estimated propensities. Propensities are estimated by means of general linear models like linear regression, logistic regression or probit regression.

In the examples of this paper the auxiliary variable vector X is available at the sample level by means of direct linkage to frame data, registrations and administrative data. This is not feasible and realistic in many practical settings. Survey researchers may have access to population totals only. Within the RISQ project estimators based on population totals have been investigated. Shlomo et al (2009c) propose both sample-based and population-based estimators for response propensities and R-indicators. The population-based estimators employ population totals and no direct linkage is needed.

3. Examples

We illustrate the possible uses of the indicators with two household surveys, two business surveys and one business register. Sample size, response rate and modes are given in table 3.1. For the household surveys we have the following auxiliary variables at the sample level: gender, age, marital status, urbanization, average value of houses in a postal code area, job status (yes or no a paid job), type of household and ethnicity. For the business surveys we could dispose over business type, business size and VAT reported to Tax Office in previous year.

Table 3.1: Description of household and business surveys.

Survey	Consumer Sentiments Survey (CSS) 2005	Health Survey (HS) 2005	Short-Term Statistics (STS) retail 2007	Short-Term Statistics (STS) industry 2007
Sample size	17,908	15,411	93,799	64,413
Response rate	66,9%	67,3%	49,5% (15days) 78,0% (30days) 85,8% (45days) 88,2% (60days)	48,8% (15days) 78,7% (30days) 85,7% (45days) 88,3% (60days)
Mode	CATI ¹	CAPI ²	Web & paper	Web & paper

Use 1: comparing the representativeness of two household surveys

Table 3.2 contains R-indicators and their corresponding 95% confidence intervals for the two selected household surveys. Response propensities are estimated using logistic regression with main effects only for are gender, age x marital status, urbanization, house value, paid job, household type, and ethnic background.

Table 3.2: R-indicators for the two household surveys HS 2005 and CSS 2005.

HS 2005	CSS 2005
R = 0,808 95%CI = (0,794 – 0,823)	R = 0,821 95%CI = (0,807 – 0,834)

The Consumer Sentiments survey performs slightly better than the Health survey, but the difference in R-indicator is not significant at the 5% level. One can, therefore, conclude that there is no evidence that the response to the two surveys differs strongly in terms of representativeness.

Use 2: evaluating the representativeness of a business register in time

We computed the R-indicator for the business register of VAT reports for the months January, June and December. Businesses have to report their VAT to the Tax Board on a monthly, quarterly or annual basis depending on their size. Small companies report only once a year while big companies have to submit VAT reports every month. Statistics Netherlands uses the VAT records as input to statistics about business turnover. For monthly statistics the VAT reports need to be available between 25 and 30 days after the end of the reference month. After 25 days processing data begins and after 30 days the statistics are made public. Since the reporting frequency depends on the size of the company, the months January, June and December are very different. For January only monthly reports are available, while for June and December also, respectively, the quarterly and annual reports can be used. We view the completion of the register as response and R-indicators as measures of the representativeness of available reports.

The completion rates and R-indicators are given in table 3.3. For the estimation of the completion probabilities we used VAT reported one year earlier in the same month and the total wages of the reporting month. The total wages are also reported to the Tax Board and are available quickly after the end of the reporting month.

The completion rates are given after 25, 30 and 60 days. The completion rate for January is extremely low, only 20% of the businesses has submitted a tax report after 25 days. For June and December these rates are much higher. After 30 days more than 85% of the businesses has reported for December.

Table 3.3: The completion rate ρ , R-indicator and maximal bias for the VAT register of January, June and December after 25, 30 and 60 days of data collection.

¹ CATI = Computer Assisted Telephone Interviewing

² CAPI = Computer Assisted Personal Interviewing

	January			June			December		
	25 d	30 d	60 d	25 d	30 d	60 d	25 d	30 d	60 d
ρ	19,7%	26,1%	28,1%	64,1%	81,5%	83,2%	48,1%	84,1%	88,0%
$R(\rho)$	0,683	0,604	0,614	0,739	0,716	0,731	0,846	0,769	0,815
B	80,4%	75,8%	68,7%	20,4%	17,4%	16,2%	16,0%	13,8%	10,5%

From table 3.3 we can conclude that the representativeness is lowest for January and highest for December. As the completion rate follows the same pattern, the maximal bias is highest for January and lowest for December. However, for each of the three months it does not pay off to wait longer than 25 days when it comes to representativeness.

Use 3: evaluating the representativeness of response during data collection

Table 3.4 contains R-indicators for the two business surveys for all available auxiliary variables and a restricted set where VAT is omitted. The R-indicators are given for response after 15, 30, 45 and 60 days of fieldwork. STS surveys need to provide statistics 30 days after the end of the reference month. The R-indicators show that for retail representativeness does not improve over time and is especially affected by VAT. The representativeness for industry improves over time and is only mildly related to VAT of the previous year.

Table 3.4: R-indicators, maximal bias and maximal contrast using small and full sets of auxiliary variables. The R-indicators are computed after 15, 30, 45 and 60 days fieldwork. 95% confidence intervals are estimated for the R-indicators.

Survey		Small				Full			
		15d	30d	45d	60d	15d	30d	45d	60d
STS industry	R	0,921	0,933	0,940	0,942	0,905	0,918	0,931	0,933
	CI	0,913- 0,928	0,927- 0,940	0,935- 0,944	0,938- 0,946	0,897- 0,913	0,913- 0,922	0,926- 0,935	0,928- 0,938
	B	8,1%	4,2%	3,5%	3,3%	9,7%	5,2%	4,1%	3,8%
	C	15,8%	19,5%	24,6%	27,9%	19,0%	24,5%	28,2%	32,4%
STS retail	R	0,961	0,946	0,940	0,941	0,881	0,879	0,883	0,890
	CI	0,954- 0,967	0,940- 0,952	0,935- 0,945	0,936- 0,946	0,873- 0,888	0,873- 0,886	0,876- 0,889	0,883- 0,896
	B	3,9%	3,5%	3,5%	3,3%	12,0%	7,7%	6,8%	6,2%
	C	7,8%	15,7%	24,6%	28,3%	23,8%	36,0%	47,7%	53,2%

Figure 3.1 illustrates the response-representativity curves RR_1 , RR_2 and RR_3 for response from the 2007 STS for Industry and Retail business using the extended model with business type and business size x VAT. The response and R-indicator are taken from table 3.3 and are plotted for 15, 30, 45 and 60 days of fieldwork

For response in STS industry, the R-indicator is higher than the 10% RR_1 threshold after 15 days and is approaching the 5% RR_1 threshold after 60 days. For response in STS retail, the R-indicator reaches the 10% RR_1 only after 60 days. RR_2 presents a similar picture for both surveys. The R-indicators for both STS industry and retail exceed the 10% RR_2 threshold after 30 days. However, both surveys never reach the 1% threshold and the STS retail does not reach the 5% after 60 days. The picture from RR_3 is different as quality is decreasing with the number of fieldwork days. The maximal contrast increases after 15 days. For STS industry it is approaches the 20% RR_3 level, while for STS retail it is considerably lower than the 20% threshold.

Figure 3.1: RR_1 , RR_2 and RR_3 curves for STS industry and retail after 15, 30, 45 and 60 days for: $\gamma = 0.05$, $\gamma = 0.10$ (RR_1), $\gamma = 0.01$, $\gamma = 0.05$, $\gamma = 0.10$ (RR_2) and $\gamma = 0.05$, $\gamma = 0.10$, $\gamma = 0.20$ (RR_3)

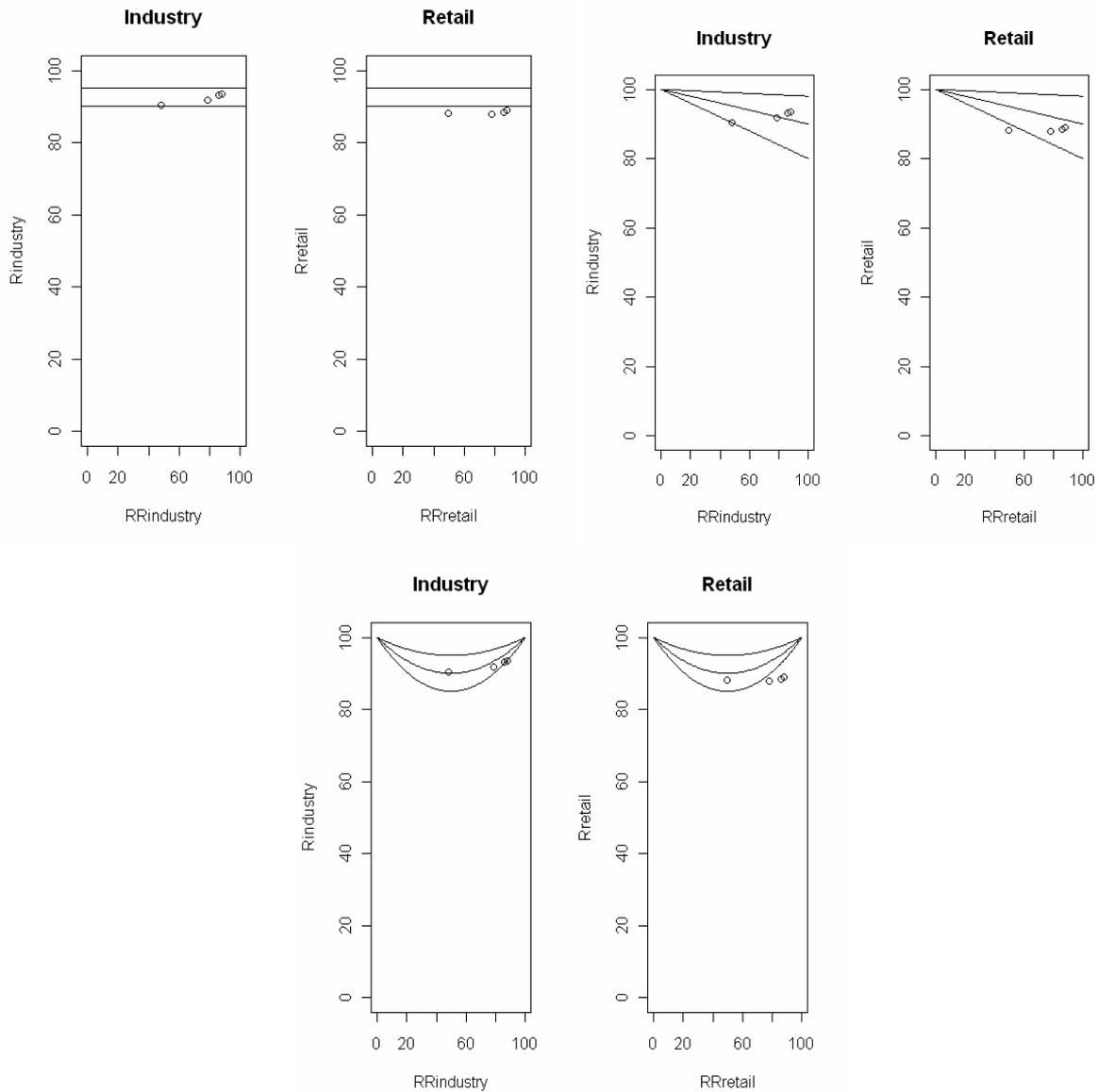
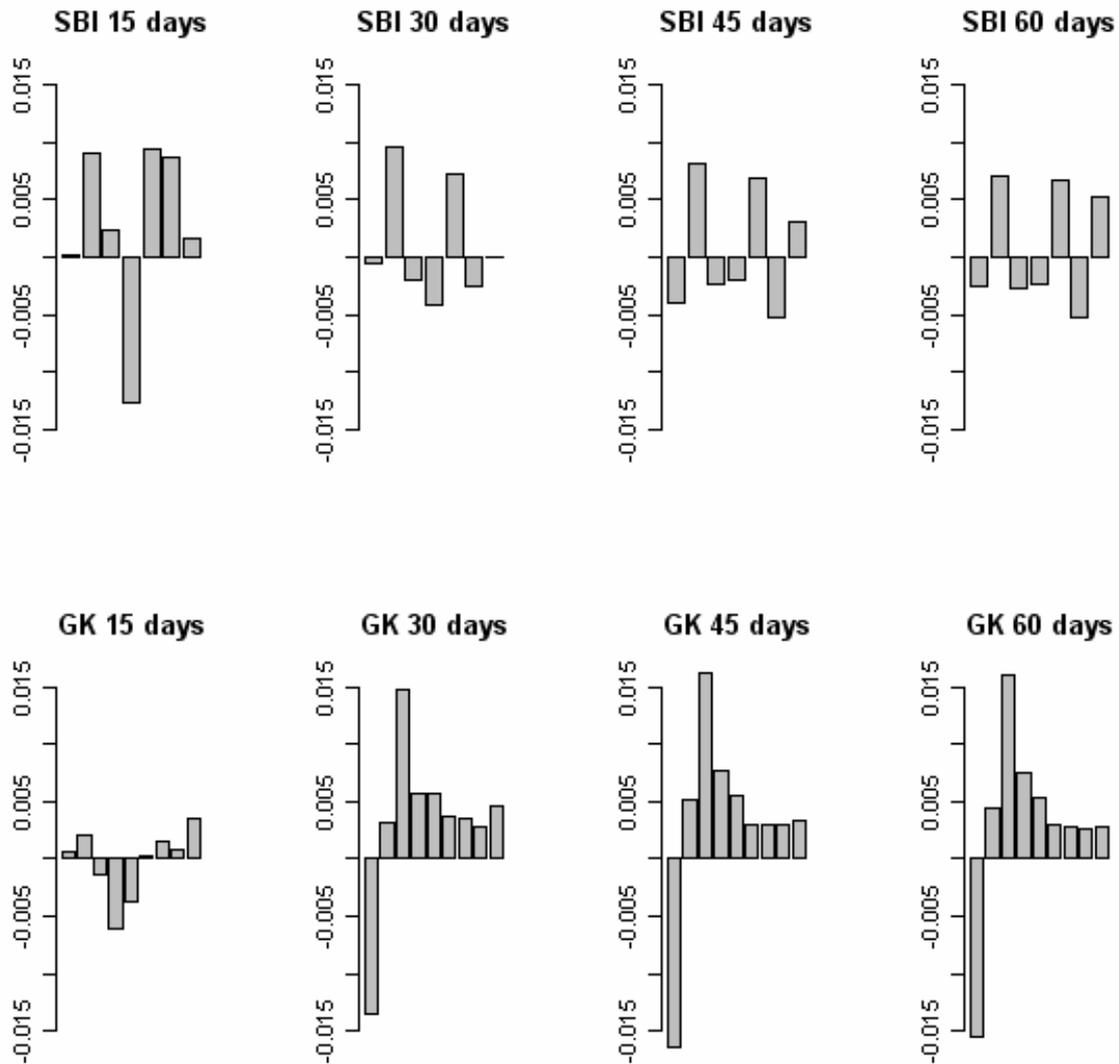


Figure 3.2 shows unconditional partial R-indicators for the retail businesses after 15, 30, 45 and 60 days. The unconditional indicators are computed for business type and business size. The representativeness with respect to business type does not show a fixed pattern until 45 days. After 45 days the unconditional partial R-indicators are stable. Throughout the data collection business type 2 enterprises are overrepresented. However, business type 4 starts with a strong underrepresentation but catches up after 30 days. The unconditional partial R-indicators for business size are stable after 30 days and show that small businesses (GK equals 1) are strongly underrepresented.

The other three papers in the Q2010 special topic session *ESSNet on Representativity Indicators for Survey Quality* present detailed examples of the use of R-indicators and partial R-indicators in fieldwork monitoring, adaptive survey design and responsive survey design.

Figure 3.2: For $t=15, 30, 45, 60$ unconditional partial indicators for STS retail for $Z = \text{business type (SBI)}$ and $Z = \text{business size (GK)}$.



4. Discussion

From the examples in section 3 it becomes clear that R-indicators and partial R-indicators may be useful tools in assessing, evaluating and monitoring response quality. They also need to be evaluated carefully. First, the variables that are selected for the prediction of response play an important role. Second, the sample size reduces the strength of conclusions. Furthermore, we have to restrict the assessment of representative response to available auxiliary variables. Only if we would dispose of a “super”vector \mathfrak{N} , containing all relevant variables for explaining response behaviour, we would be able to apply the variation in response propensities $S(\rho_{\mathfrak{N}})$, the maximal absolute bias $B_m(\rho_{\mathfrak{N}})$ and the maximal absolute contrast $C_m(\rho_{\mathfrak{N}})$ to all possible survey items. For a specific choice of X , it implies that $B_m(\rho_X)$ and $C_m(\rho_X)$ may underestimate the true maximal bias and maximal contrast.

The interpretation of the R-indicator and the partial R-indicators is, however, straightforward. They are based on response propensities which have a clear interpretation. It measures the (transformed) standard deviation of those propensities which is a measure that is commonly used in many statistical

settings and its components, the between and within variance. The more diverse the nonresponse is, the larger the standard deviation.

The objective of RISQ is the development of measures that can be used irrespective of the set of survey variables, the population parameters or statistics that one is interested in, and the models that are used to explain response behaviour. The R-indicator corresponds to that goal. It does not depend on survey items, estimators or models. The partial R-indicators that are derived from the R-indicator correspond to variations in response propensities within and between subpopulations. In that sense they appeal directly to the practice in data collection departments as data collection strategies are usually improved based on groups that have a relatively low response propensity.

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References

- Groves, R., Heeringa, S. (2006), Responsive design for household surveys: tools for actively controlling survey errors and costs, *Journal of the Royal Statistical Society A*, 169, 439-457.
- Madow, W.G., Olkin, I. (1983), *Incomplete data in sample surveys*, Proceedings of a Symposium, Academic Press, New York, USA.
- Mohl, C., Laflamme, F (2007), Research and responsive design options for survey data collection at Statistics Canada, Proceedings of ASA Joint Statistical Meeting, Section 293, July 29 – August 2, Salt Lake City, USA.
- Schouten, B., Bethlehem, J. (2009), Representativeness indicators for measuring and enhancing the composition of survey response, RISQ deliverable, www.risq-project.eu.
- Schouten, B., Cobben, F., Bethlehem, J. (2009), Indicators for the representativeness of survey response, *Survey Methodology*, 35 (1), 101 – 113.
- Shlomo, N., Skinner, C., Schouten, B., Bethlehem, J., Zhang, L.C. (2009a), Statistical properties of representativity indicators, RISQ deliverable, www.risq-project.eu.
- Shlomo, N., Skinner, N., Schouten, B., Carolina, N., Morren, M. (2009b), Partial indicators for representative response, RISQ deliverable, available at www.risq-project.eu.
- Shlomo, N., Skinner, C., Schouten, B., Heij, V. de, Bethlehem, J., Ouwehand, P. (2009c), Indicators for representative response based on population totals, RISQ deliverable, available at www.risq-project.eu.
- Wagner, J. (2008), Adaptive survey design to reduce nonresponse bias, PhD thesis, University of Michigan, USA..