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# A Simple Model of Herding and Contrarian Behaviour with Biased Informed Traders

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# A Simple Model of Herding and Contrarian Behaviour with Biased Informed Traders*

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**Abstract**

This paper unveils novel insights into the impact of trader bias on herding and contrarian behaviour in financial markets. We modify the behaviour of informed traders in a sequential trading microstructure model using cumulative prospect theory (CPT). By introducing CPT gain-loss asymmetry and loss-tolerant traders, we formulate a generalized CPT trader herding model. Our findings reveal that in markets with a substantial proportion of loss-tolerant agents, the elimination of gain-loss asymmetry can incur significant costs, emphasizing the necessity of employing the generalized model. Conversely, in markets dominated by loss-averse traders, such an assumption is less costly, allowing reliance on an extended model for closed-form results. We establish theoretical upper bounds on loss attitude, determining the threshold that triggers herding and contrarianism, thus facilitating regulatory monitoring. Contrary to previous models, our generalized approach suggests that a trader can engage in both herding and contrarian behaviour rather than a clear-cut preference, and they can occur at mild price deviations instead of only at extreme prices. We reconcile previous experimental evidence, addressing strong contrarian but weak herding tendencies under various market specifications.

**Keywords:** Herding and Contrarian, Social learning, Sequential Trading, Prospect Theory

**JEL Codes:** D82, D83, D84, G14, G41

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# 1 Introduction

Over the past two decades, the global economy has experienced significant uncertainties, often characterized by substantial price fluctuations in financial markets. In light of this, researchers in behavioural finance have been developing models that can capture the underlying drivers behind these crises, with a focus on studying emotions and psychological factors. One direction that has garnered considerable attention is herding and contrarian behaviour among investors – the tendency of individuals to trade in line with or against others' actions and against their private information. We investigate the impacts of biased informed traders on herding and contrarianism dynamics in the financial market. Our approach revolves around constructing a simple sequential trading market microstructure model, incorporating insights from cumulative prospect theory (CPT) by Tversky and Kahneman (1992) to account for investor decision-making biases.

CPT offers a suitable framework for our study as it accounts for various decision-making biases, such as loss attitude and probability distortions. It has two main ingredients, a value function indicates how payoffs are evaluated and perceived by the decision-maker. It contains loss attitude $\lambda$ and reference point. The associated exponent $\gamma$ shows the curvature of the value function, capturing value bias. A probability weighing function indicates how objective probabilities are perceived by decision-makers. The exponent $\delta$ shows the degree of probability distortion, capturing probability bias. Both elements feature different parameters in gain-loss regions; $\gamma_G$ and $\delta_G$ in the gain region, $\gamma_L$ and $\delta_L$ in the loss region, this captures gain-loss asymmetry.

The foundation of our research builds upon the seminal work of Avery and Zemsky (1998) (henceforth AZ), who examined herd behaviour in the financial market. Their model utilised the sequential trading market microstructure model by Glosten and Milgrom (1985). The models à la AZ all feature Bayesian updating informed traders who have an information advantage over the market maker. There are uninformed traders trading randomly due to exogenous reasons such as liquidity. There is also a market maker who sets prices in each trading round making zero profit subject to unmodelled competition. The market structure is common knowledge. There is one asset with two states. AZ demonstrated that herd behaviour could not occur when only uncertainty about the asset's value was present. Although this paper's assumptions might seem restrictive, it laid a solid theoretical foundation for exploring the influence of various factors on herding behaviour.

Subsequent research relaxed certain assumptions or introduced new elements. Cipriani and Guarino (2008) modelled heterogeneous informed traders. Park and Sabourian (2011) delved into the role of signal structure. Cipriani and Guarino (2014) built a structural model to empirically test herding. Cipriani, Guarino, and Uthemann (2022) extended this model by introducing price elastic noise traders to study the effects of financial transaction tax (FTT) on financial market outcomes. Kendall (2023) incorporated CPT traders to investigate the role of preferences. Another strand of literature dives into the effects of probability distribution ambiguity(J L Ford, Kelsey, and Pang 2005; Dong, Gu, and Han 2010; J. L. Ford, D. Kelsey, and W. Pang 2013; Boortz 2016).

However, early theoretical models struggled to match experimental observation on strong contrarian tendency and abstention from trade(Drehmann, Oechssler, and Roider 2005; Cipriani and Guarino 2005, 2009; Park and Sgroi 2016). Kendall (2023) demonstrated the significant role of CPT traders on herding and contrarianism. While his model offers an improved fit, it still encounters challenges in fully aligning with experimental evidence. Our model maintains a close connection to Kendall (2023)'s, yet introduces crucial differences.

Firstly, his model assumed no gain-loss asymmetry in CPT, setting $\gamma = \gamma_G = \gamma_L$ and $\delta = \delta_G = \delta_L$. This assumption generally do not hold in data. Secondly, Kendall only considered loss aversion ($\lambda > 1$), neglecting loss tolerance ($\lambda \leq 1$). More recent evidence supports the later too. Zeisberger, Vrecko, and Langer (2012) provided CPT estimates at individual level for 73 subjects, only 1 subject showed no gain-loss asymmetry in CPT and 25 displayed loss tolerance. Our proposed generalized CPT trader model avoids such assumptions, demonstrating the potential costs associated with imposing restrictions. Thirdly, we adhere to the definitions of herding and contrarianism proposed by AZ, commonly used in the literature. While Kendall follows the definition of Banerjee (1992) and Cipriani and Guarino (2009) for experimental reasons, leading to fundamental modelling differences. The former definition requires a clear history induced switching behaviour. Further elaboration on these differences is discussed in detail in Section 3.3.

The primary objective of our paper is not to demonstrate the mere occurrence of herding or contrarianism, as empirical and theoretical evidence supports their existence. Rather, we seek to uncover new insights by studying the dynamics resulting from biased informed traders in a generalised setting. Our findings consistent of 3 parts. In Part 1, we formally characterise a generalised CPT trader model and its dynamics. We show that they can engage in both herding and contrarian behaviour, contrary to a clear cut preference predicted by Kendall (2023)'s theory. This aligns the unexplained experimental observation by him that some subjects engaged in both behaviour. We also show that herding and contrarian behaviour can occur at mild price deviations, not just at extreme prices. Additionally, we demonstrate the occurrence of wrong information cascades, where the market belief diverges from the underlying state of the world. Finally, we show that for markets consisting of loss tolerant agents, shutting down gain-loss asymmetry can be very costly. While less so if proportion of loss-averse traders is high.

In Part 2, we present an extended CPT trader herding model without gain-loss asymmetry and explore various dynamics, with closed-form solutions for prices. Furthermore, by considering only lose averse traders, we obtain similar clear-cut preference as in Kendall (2023)'s work. In Part 3, we reconcile previous experimental evidence theoretically, addressing strong contrarian tendencies but weak herding tendencies under various market specifications. Under the same market structure as previous experiments, we match the levels of herding and contrarian behaviour. The rest of the paper is divided as follow. Section 2 reviews related literature on herd behaviour and prospect theory. Section 3 presents the model. Section 4 analyses dynamics within generalised CPT trader model. Section 5 discusses the dynamics within extended CPT trader model. Section 6 delves into experimental evidence reconciliation. Section 7 concludes.

# 2 Related Literature

## 2.1 Sequentially trading herd models

Banerjee (1992) was among the first to model herd behaviour. Agents follow a sequential decision process where earlier decisions can be observed. He showed situations where information revealed in the predecessor's action outweighs one's information so that own signal is disregarded. Similar sequential trading models have been used to show how herd behaviour can impede information coming into the market (Welch 1992; Chamley and Gale 1994; Bulow and Klemperer 1994).

However, those models are not suitable to analyse how herd behaviour can affect asset price. Price is inflexible in those models, but in the financial market earlier decisions are reflected in the subsequent price. AZ's paper was the first to address this issue by endogenous prices. They modified the sequential trading model by Glosten and Milgrom (1985). There is a single asset with two states; informed and noise traders; and a market maker providing bid and ask prices. Agents follow a Bayesian updating process when information arrives. Price mechanism prevents herd behaviour from happening when there is only uncertainty about the value of the asset. Herding is possible when the dimension of uncertainties increases. Our paper only focuses on value uncertainty.

Subsequent research was built on top of AZ and studied the role of different factors. Cipriani and Guarino (2008) incorporated heterogeneous informed traders with different views of fundamental values and showed that an information cascade can occur. They also showed that contagion can lead to informational cascades by adding another asset. Park and Sabourian (2011) modified the private signal structure by introducing a moderate state. Herding (contrarianism) can occur if private information satisfies U-shaped (hill-shaped) property, where investors place a higher(lower) weight on extreme states than on moderate ones. They argued that U-shaped signals induce herding instead of multidimensionality proposed by AZ. Kendall (2023) incorporated CPT to investigate the role of preferences, and showed that preference for future returns induce herding or contrarian.

The theoretical herd models are difficult to test empirically due to data availability on traders' private information. Cipriani and Guarino (2014) built a structural rational herding model that can be empirically estimated using financial data. They developed their framework using work by Easley and O'Hara (1987) to test parameters via maximum likelihood. Using data on an NYSE stock, Ashland Inc, they found 2% herd buy and 4% herd sell on average. Recently, Cipriani, Guarino, and Uthemann (2022) extended the model by introducing price elastic noise traders to study the effects of financial transaction tax (FTT) on welfare.

Another strand of literature focuses on the role of ambiguity on herding and contrarian behaviour. It describes a scenario when the agent does not know the precise distribution of an event. J L Ford, Kelsey, and Pang (2005) first demonstrated herding and contrarian behaviour can occur by introducing ambiguity. Dong, Gu, and Han (2010) allowed separation between ambiguity and ambiguity aversion. J. L. Ford, D. Kelsey, and W. Pang (2013) studied the impact of ambiguity using neo-additive capacities. Boortz (2016) built on top of this but relies on a more stringent definition of herding and varying ambiguity with respect to price. They found that herding is not possible if investors have fixed ambiguity preferences, while contrarianism is.

Various experiments tested the AZ predictions. Drehmann, Oechssler, and Roider (2005) conducted an internet experiment with more than 6,400 subjects including 267 consultants. Cipriani and Guarino (2005) tested this in a laboratory setting with 216 students, Park and Sgroi (2016) used student subjects with 1,350 trades [1]. These studies found a low level of herding, supporting AZ's no-herding prediction, but also observed contrarian behaviour and abstention from trade. Two other studies adopted a more relaxed herding definition. Cipriani and Guarino (2009) sampled 32 financial professionals, observed strong contrarian tendencies too. They compared their results with previous studies by adapting the same definition, noting reassuring similarity.

Kendall (2023)) used 46 student subjects in his study. In the main treatment, he directly provided subjects with the correct Bayesian posterior to control for Bayesian errors. This led to a significantly higher level of herding than contrarian behaviour. However, a main concern arises that providing subjects with the Bayesian posterior directly removes uncertainty in probabilities, as they are now given exogenously by the experimenter. This creates more herding-type traders than contrarian types. In the second treatment, where the correct Bayesian posterior is not provided directly, results are comparable to Cipriani and Guarino (2009). Amount of herding matches previous evidence but not contrarian behaviour. Across both treatments, he observed that a trader can engage in both herding and contrarian behaviour, a phenomenon not captured by his theory. Regardless, the author demonstrated that the CPT trader herding model fits the data better compared to previous ones. A detailed discussion on the experiments, their challenges, and how our generalized CPT trader model reconciles the evidence is provided in Section 6.

1. Park and Sgroi (2016) examined the theoretical framework proposed by Park and Sabourian (2011) with 3 types of signals. Notably, signals S1 and S3 exhibit a monotonic behaviour, akin to those discussed in the context of AZ. Our discussion focuses on this for comparison purposes.

## 2.2 Prospect Theory

Tversky and Kahneman (1992) (henceforth TK) provided cumulative prospect theory (CPT). The model comprises two primary components. Firstly, the value function captures the value assigned by the decision maker to an uncertain outcome. The functional form enables it to represent deviations from the reference point and loss attitude. The second element involves probability distortions. TK proposed a probability weighting function that transforms objective probabilities into subjective ones. This accounts for the phenomena of underweighting high-probability and overweighting low-probability events. Different parameters are allowed for the loss and gain domains.
(i) Value function $\Omega(.)$:

$$\Omega(\pi)\begin{cases}\pi^{\gamma_G} \text{ if } \pi \geq 0\\ -\lambda(-\pi)^{\gamma_L} \text{ if } \pi < 0\end{cases}$$

(ii) the probability weighting function $w(.)$ in gain and loss domains:

$$w^{+}(P)=\frac{P^{\delta_G}}{(P^{\delta_G}+(1-P)^{\delta_G})^{1/\delta_G}};w^{-}(P)=\frac{P^{\delta_L}}{(P^{\delta_L}+(1-P)^{\delta_L})^{1/\delta_L}}$$

where $\pi$ is the payoff relative to the reference point, $\gamma_G$ is the exponent of the value function in the gain region, and $\gamma_L$ is the exponent of the value function in the loss region. $\lambda$ indicates the attitude towards loss. It is typically assumed to be greater than 1, indicating loss aversion. If it's smaller than 1, it indicates loss tolerance. $\delta_G$ is the probability weighting function exponent in the gain domain. $\delta_L$ is the probability weighting function exponent in the loss domain. $P$ is the objective probability. In expected utility theory, a rational decision-maker computes the expected utility of a risky asset with two potential payoffs $\pi_1$ and $\pi_2$ according to the following: $E[u(\pi)] = Pu(\pi_1)+(1-P)u(\pi_2)$. In prospect theory, a decision-maker computes perceived expected utility according to the following: $E[\Omega(\pi)] = w(P)\Omega(\pi_1)+w(1-P)\Omega(\pi_2)$.

The evidence for gain-loss asymmetry in CPT parameters is strong. Rieger, Wang, and Hens (2017) estimated the parameters using an international survey for 53 countries with a sample of 6912 university students. There is a clear heterogeneity and gain-loss asymmetry. Zeisberger, Vrecko, and Langer (2012) reported CPT at individual level, 72 out of 73 subjects displayed gain-loss asymmetry. Early studies supported loss aversion. TK's study reported 2.25. More recent evidence shows a much lower number. In Rieger, Wang, and Hens (2017)' study, $\lambda$ are 1.2 and 1.37 for subjects in the UK and USA. Barberis, Jin, and Wang (2021) used a value of 1.5 to explain stock market anomalies. Chapman et al. (2018) used a survey of the US population with a sample size of 2000. They found 50% of the sample showed loss tolerance. In Zeisberger, Vrecko, and Langer (2012)'s study, 34% of the subjects showed loss tolerance. Given the evidence, we build a generalised CPT trader herding model with gain-loss asymmetry and loss tolerance allowed.

# 3 The Model

We first present the AZ baseline market microstructure model in subsection 3.1. Then we set up our modified generalised CPT trader model in subsection 3.2. After that we define herding and contrarian behaviour formally in section 3.3. Finally, we simulate the models to compare dynamics between baseline and modified models.

## 3.1 Baseline model

The baseline model is as follow:

**The Asset:** There is a single risky asset with unknown fundamental value $V$. It is calculated based on the present value of future cash flows. It takes values $V_L$ and $V_H$ (where $V_H > V_L$). Without loss of generality, we can normalise $V_L = 0, V_H = 1$.

**The Market:** The traders' action is defined as: $x_t \in \{buy, sell, hold\}$. Trading takes place at a sequence of discrete time t ( t=1,2,3,...T), after T the value is revealed. For simplicity, one can define a sequence as a trading day. An informational event that affects asset value occurs before the day starts, and at the end of the day true value of the asset is revealed. Informed traders learn the value of the asset throughout the day with their private signal and observation of history. For a given sequence, $V$ follows a Bernoulli distribution with a single trial, with fixed probabilities for high and low states. Across sequences, for example, different trading days, $V$ can be assumed to be independently distributed, the same as in Cipriani and Guarino (2014). Though they are not necessarily identical. In this paper, we focus on the dynamics within a single sequence. A sequence could also be defined as a quarter, where quarterly earnings of an asset are typically revealed.

Each trader interacts with the market maker, exchanging one unit of an asset for cash or no trade (hold). The history is defined as $H_t = \{(x_1, p_1), ...(x_{t-1}, p_{t-1})\}$. Following the literature, the price of the asset is given by the public expectation of the asset's true value $p_t = E[V|H_t] = P(V_H|H_t)$. This is the price before trades take place in $t$ and reflect all public information. Price is a martingale with respect to the history of trades and prices. $E[p_{t+1}|H_t] = E[E[V|H_{t+1}]|H_t] = E[V|H_t] = p_t$. $H_1$ is the initial history before any trade occurs. The prior probability for the high state characterises the prior which we set exogenously to be 0.5; $p_1 = E[V|H_1] = E[V] = P(V_H) = 0.5$.

**The Market Maker:** The market maker makes 0 expected profit due to unmodelled competition. The market maker does not receive any private information. The information asymmetry between market makers and informed traders creates the bid-ask spread as shown by Glosten and Milgrom (1985). The market maker sets different prices at which they are willing to sell and buy, as they have to consider the possibility of information advantage of informed traders. The bid and ask prices are set by the market maker before traders in t arrive. $b_t = E[V|H_t, x_t = sell], a_t = E[V|H_t, x_t = buy]$

**The Traders:** The trading sequence is exogenously given, and the number of traders is finite. Each trader can only trade once at the time of arriving. There are two types of traders, informed and noise. Traders' type is private information and not known by the public or market maker. The probability of the informed trader arriving is exogenous $\mu$, and the noise trader is $1-\mu$.

Noise trader: they trade randomly for unmodelled reasons such as liquidity with exogenously given equal probability 1/3 for each action. The probability of a noise trader's action is $\theta = (1-\mu)/3$. AZ noted that their model is a special case of Glosten and Milgrom (1985) model. In their model, noise traders have inelastic demand and do not respond to prices. Thus, the market never collapses, and noise traders absorb any possible losses.

Informed trader: Informed traders receive exogenous private information $S \in \{S_L, S_H\}$ where $S$ can be high or low signals. They also observe the trading history and public information. Their expected value of the asset is $E[V|S,H_t] = P(V_H|S,H_t)$. They buy if the expected value is greater than the ask price, and sell if it is smaller than the bid price, otherwise is no trade. We denote as follow: (i)buy if $E[V|S,H_t] > a_t$. (ii)Sell if $E[V|S,H_t] < b_t$ (iii)No trade in other cases.

**The Signal:** The private signals received by informed traders are independent of history. The distribution is i.i.d and defined by $P(S|V)$, it is conditional on the state of the world. S is assumed to be symmetric binary signals with precision $1 > q > 0.5$. $P(S_L|V_L) = P(S_H|V_H) = q$. This means that the precision increases as q becomes larger, and the signals become more informative. The assumption that $q > 0.5$ implies the signals are informative.

**Updating process:** Public belief/price, bid and ask prices are updated from $t$ to $t+1$ when trading action $x_t$ is observed in $t+1$. Table 1 provides a summary of the process. Boortz (2016) has provided a good summary of the key formulas in AZ model, which I include in Appendix 8.1.

Table 1: Trading sequence summary

| | $t = 1 \Rightarrow$ | $t = 2 \Rightarrow$ The process repeats until T |
|---|---|---|
| History | $H_1$ the initial history before any trade occurs. It contains no useful information. | $H_2 = \{x_1, p_1\}$. Price and trading action are included in the history. |
| Market maker | The market maker does not know trader's action $x_1$ when updating $b_1$ and $a_1$, they update according to conditional expectation of the asset $b_1 = E[V|H_1, x_1 = sell]$, $a_1 = E[V|H_1, x_1 = buy]$. Since we assume the market maker knows the proportion of informed traders, the precision of private signal q and the probability of noise traders taking a certain action, they have sufficient information to compute this. | Similar to t=1, market maker sets $b_2 = E[V|H_2, x_2 = sell]$, $a_2 = E[V|H_2, x_2 = buy]$. However, $H_2$ now contains trading action $x_1$. Note that $x_2$ is not observed when $b_2$ and $a_2$ are set. Trading action $x_1$ could be due to an informed trader or noise trader, but public belief $p_2$ factors in this since the market structure is of common knowledge. |

| | | |
|---|---|---|
| Public expectation | $p_1 = E[V\|H_1] = 0.5$. We assume an initial prior of 0.5 for the high value state. | $p_2 = E[V\|H_2] = P(V = 1\|H_2)$.Reflects all public information up to t=1. |
| Traders | A trader arrives exogenously. It could be a noise or an informed trader. An informed trader trades based on private signals only. $E[V\|S] = E[V\|H_1, S]$, herding or contrarianism cannot occur this period. Trading action $x_1$ takes place. This stage is completed. | A trader arrives exogenously. Informed trader trade by comparing expected value based on private signal and history $E[V\|S, H_2]$ with $b_2$ and $a_2$. A trading action $x_2$ takes place. |

## 3.2 Modified model

We only modify the behaviour of informed traders, all other features of the market microstructure remain identical to the baseline model in section 3.1. We replace the expected utility theory with prospect theory.

**Informed traders with both public and private information**

They receive exogenous private information $S \in \{S_L, S_H\}$ where $S$ can be high or low signals. They also observe the trading history and public information $H_t$. Instead of computing the expected value of the asset and comparing it to bid and ask prices. Traders now compute the utility associated with the expected payoff of a certain trading action through prospect theory. If the action is expected to generate positive utility, they implement the action. Recall that $\Omega$ is the value function of CPT, $w$ is the probability weighting function, $a_t$ and $b_t$ are the bid and ask prices, $V_H$ and $V_L$ are the high and low-value states of the asset, $\pi$ is the payoff. We denote informed traders' decision-making with both private and public signals are as follow:

(i)Buy if $E\Omega[\pi|S, H_t] = \Omega(V_H - a_t) * w[P(V_H|S, H_t)] + \Omega(V_L - a_t) * w[P(V_L|S, H_t)] > 0$

(ii)Sell if $E\Omega[\pi|S, H_t] = \Omega(b_t - V_H) * w[P(V_H|S, H_t)] + \Omega(b_t - V_L) * w[P(V_L|S, H_t)] > 0$

(iii)No trade if neither (i) and (ii) are satisfied.

Condition (i) states that an informed trader buys if the expected utility of buying conditional on all available information is positive. The payoff is $V_H - a_t$ if the high-value state realises, since traders buy at price $a_t$. The value perceived by the trader of that payoff is $\Omega(V_H - a_t)$. The associated probability of the high-value state is $P(V_H|S, H_t)$. The probability perceived by the trader is $w[P(V_H|S, H_t)]$. A similar logic applies to the low-value state. Then we multiply the utility perceived $\Omega$ in each state by its associated probability $w$ and sum up. This gives us the expected utility of buying perceived by the informed trader. Bias in the decision-making process is captured by $\Omega$ and $w$ which we discussed in section 2.2. Condition (ii) follows the same idea for the sell side. Payoffs are $b_t - V_H$ and $b_t - V_L$ in high and low-value states now, since they can sell at price $b_t$. Condition (iii) is straightforward, giving us the no-trade condition.

**Informed traders with only private information**:

Our herding and contrarian definition in the next section require a hypothetical situation where informed traders only consider private signal. If we don't allow history in informed traders' decision making process, their decision-making are as follow:

(iv)Buy if $E\Omega[\pi|S] = \Omega(V_H - a_1) * P(V_H|S) + \Omega(V_L - a_1) * P(V_L|S) > 0$

(v)Sell if $E\Omega[\pi|S] = \Omega(b_1 - V_H) * P(V_H|S) + \Omega(b_1 - V_L) * P(V_L|S) > 0$

(vi)No trade if neither (iv) and (v) are satisfied.

Similar as above payoffs in the high and low-value states are transformed through prospect theory value function $\Omega(.)$. The probabilities without history are not transformed through the weighting function. The reasoning is that signal precision is exogenous given that is known to the informed traders, it is not affected by any biases. We can show that the probability of a state conditional on only the private signal equals the precision of the signal[2]. Put this formally, $P(V_H|S_H) = P(V_L|S_L) = q, P(V_L|S_H) = P(V_H|S_L) = 1 - q$. With history, the probabilities are first computed rationally using Bayesian formula as in the baseline model, then transformed through the weighting function $w(.)$ of prospect theory. The intuition is that informed traders face uncertainty about the value of the asset. They are affected by biases when they evaluate the probabilities generated using the Bayesian formula. Notice also we are computing the payoffs using bid and ask in first period when history is not allowed. As it contains only the initial period when there was no history of trades. This approach is standard and in line with Avery and Zemsky (1998) and Park and Sabourian (2011).

We return to the baseline scenario if we switch off all features of CPT, setting $\gamma_G = \gamma_L = \lambda = \delta_G = \delta_L = 1$.

## 3.3 Definition of herding and contrarian behaviour

There are several definitions of herding and contrarian behaviour in the literature. One notion requires actions to converge irrespective of private information, Lakonishok, Shleifer, and Vishny (1992) defined herding as the probability of fund managers taking the same trading decisions simultaneously. However, Park and Sabourian (2011) noted that such a case is not interesting in the context of an informationally efficient financial market. Since it is uninformative when informed traders act alike, prices remain unchanged.

Another strand defines herding and contrarian as trading independent of their own private signal. (Banerjee 1992; Bikhchandani, Hirshleifer, and Welch 1992; Cipriani and Guarino 2008, 2009; Kendall 2023). For instance, buy given high or low signal after price increase counts as herding. However, this means that a trader who trade in the same direction as private signal can be considered as herding or contrarian behaviour. It does not impede the flow of private information into the market, as order direction and private signal align. It could be that they are just following their private signal instead of trend following.

Avery and Zemsky (1998) requires trader's private signal to be overwhelmed by the public signal that contain information about others' actions. This is the definition followed by majority of the literature, (Cipriani and Guarino 2005; Drehmann, Oechssler, and Roider 2005; Park and Sabourian 2011; Cipriani and Guarino 2014; Boortz 2016; Park and Sgroi 2016). We follow Park and Sabourian (2011)'s set-up precisely. For instance, given asset price has dropped, herd sell occurs if the trader sells based on private signal and history, but otherwise would have bought the asset if conditional on only the private signal. We define herding and contrarianism behaviour formally below:

2. $P(V_H|S_H) = \dfrac{P(S_H|V_H)P(V_H)}{P(S_H|V_H)P(V_H) + P(S_H|V_L)P(V_L)} = \dfrac{q * 0.5}{q * 0.5 + (1-q) * 0.5} = q.$

**Definition 1.** ***Herd and Contrarianism behaviour***

*1.A. Herd and Contrarianism buy*

*(B1)* $E\Omega[\pi|S] = \Omega(b_1 - V_H) * P(V_H|S) + \Omega(b_1 - V_L) * P(V_L|S) > 0$
*The expected utility of selling conditional on only private signal is positive*

*(B2)* $E\Omega[\pi|S,H_t] = \Omega(V_H - a_t) * w[P(V_H|S,H_t)] + \Omega(V_L - a_t) * w[P(V_L|S,H_t)] > 0$
*The expected utility of buying conditional on both private signal and history is positive*

*(B3)* *(i)*$E[V|H_t] > E[V]$. *Price has increased since first period,* $p_t > p_1$.
*(ii)*$E[V|H_t] < E[V]$. *Price has decreased since first period,* $p_t < p_1$.

*1.B. Herd and Contrarianism sell*

*(S1)* $E\Omega[\pi|S] = \Omega(V_H - a_1) * P(V_H|S) + \Omega(V_L - a_1) * P(V_L|S) > 0$
*The expected utility of buying conditional on only private signal is positive*

*(S2)* $E\Omega[\pi|S,H_t] = \Omega(b_t - V_H) * w[P(V_H|S,H_t)] + \Omega(b_t - V_L) * w[P(V_L|S,H_t)] > 0$
*The expected utility of selling conditional on both private signal and history is positive*

*(S3)* *(i)*$E[V|H_t] < E[V]$. *Price has decreased since first period,* $p_t < p_1$.
*(ii)*$E[V|H_t] > E[V]$. *Price has increased since first period,* $p_t > p_1$.

Herd (Contrarianism) buy occurs if and only if conditions B1, B2 and B3i (B1, B2, B3ii) are satisfied. Herd (Contrarianism) sell occurs if and only if conditions S1, S2 and S3i (S1, S2, S3ii) are satisfied. For herd and contrarianism buy, condition 1 checks whether informed traders' expected utility of selling is positive conditional on private signal only. If so they would have sold the asset. Condition 2 checks whether informed traders' expected utility of buying is positive conditional on private signal and history of trade. If so they buy. Condition B3 ensures that herding is in line with the movement of the crowd, and contrarianism is against the crowd. The opposite holds for herd and contrarianism sell. B1 and B2 $> 0$ or S1 and S2$> 0$ are necessary conditions for herd or contrarianism. They are not necessarily always greater than 0, if they are smaller than or equal to 0, herd or contrarianism cannot occur.

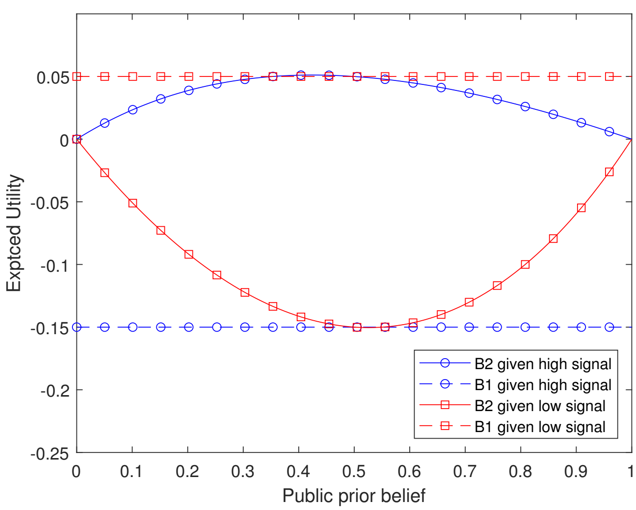
Conditions B2 and S2 are identical to the ones we discussed in the modified model. They check if the expected utility of a certain trading action conditional on both private signal and history is positive. Conditions B1 and S1 are "what if" situations, if informed traders only trade based on private signals, what would have happened? For instance, given both B1 and B2 are positive, the informed trader would have sold the asset if the decision was based on only their private signal. When the decision is based on both private signals and history, they buy. Either herding or contrarianism occurs, if the price has increased (B3i) then we have herd buy, if the price has dropped (B3ii) then we have contrarianism buy.

To compare to Kendall (2023)'s CPT trader model, I focus the discussion on buy side, sell side follows the same idea. Kendall's model only requires B2 to be satisfied given both high and low signals. For instance, if $B2 > 0$ given $S_H$ and $S_L$, either buy herding or contrarian behaviour occurs. B3 then pins down herding or contrarian behaviour. Thus, a buy order given high signal and price increase could be considered as herding. However, we don't know if the traders are following the trend or not. They could be just following this positive private signal, instead of buying because others are doing so. To truly pin down a trend following behaviour, one should check what happens if we shut down social leaning. That is removing history from their decision making, is there a switch in trading decision? If so then we have a clear trend following behaviour. Condition B1 allows us to do so, in line with the definition used in majority of the literature. Additional we introduce decision making bias in condition 1, as discussed in the section above. Our model imposes more stringent conditions for herd and contrarian behaviour.

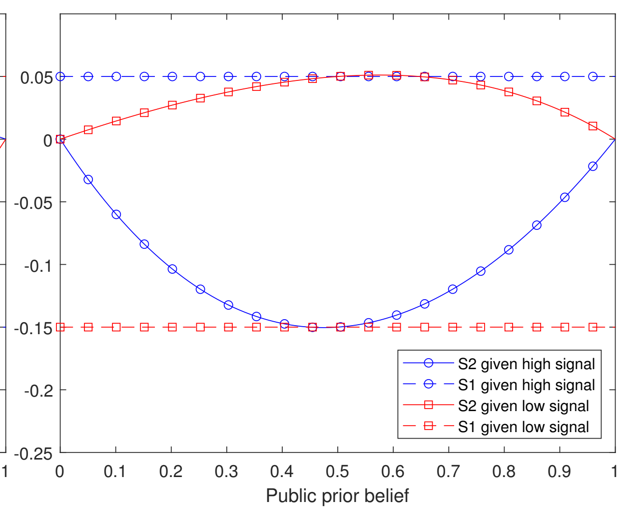
## 3.4 Model Simulation

In this section, we compare the dynamics in the baseline and modified model using simulations. We set the proportion of informed traders $\mu$ to be 0.4 and private signal precision $q$ to be 0.6. The choices of model parameters are not restrictive. For instance, we could have a market with a large proportion of informed traders, setting $\mu$ to 0.8, similar dynamics present. We set CPT values to $(\gamma_G, \gamma_L, \lambda, \delta_G, \delta_L, \text{reference point}) = (0.44, 0.49, 1.06, 0.47, 0.98, 0)$. This is the median CPT values for subjects in the UK reported in table 3 of Rieger, Wang, and Hens (2017). The CPT for the baseline model are $(1, 1, 1, 1, 1, 0)$, this switches off all CPT features and return to expected utility framework. We compute the utility when public prior belief $p_t$ is between 0 and 1. See appendix 8.2 for specific formulas of the model.

Figure 1 simulates the baseline model. 1(a) checks herd and contrarianism buy, 1(b) checks sell side. For a given signal, conditions 1 and 2 are never satisfied at the same time (lines of the same colour/shape to be above 0). Indeed herding and contrarianism cannot occur in the baseline model. Traders always make a trade in line with their private signal.




(a) Herd and Contrarianism buy check

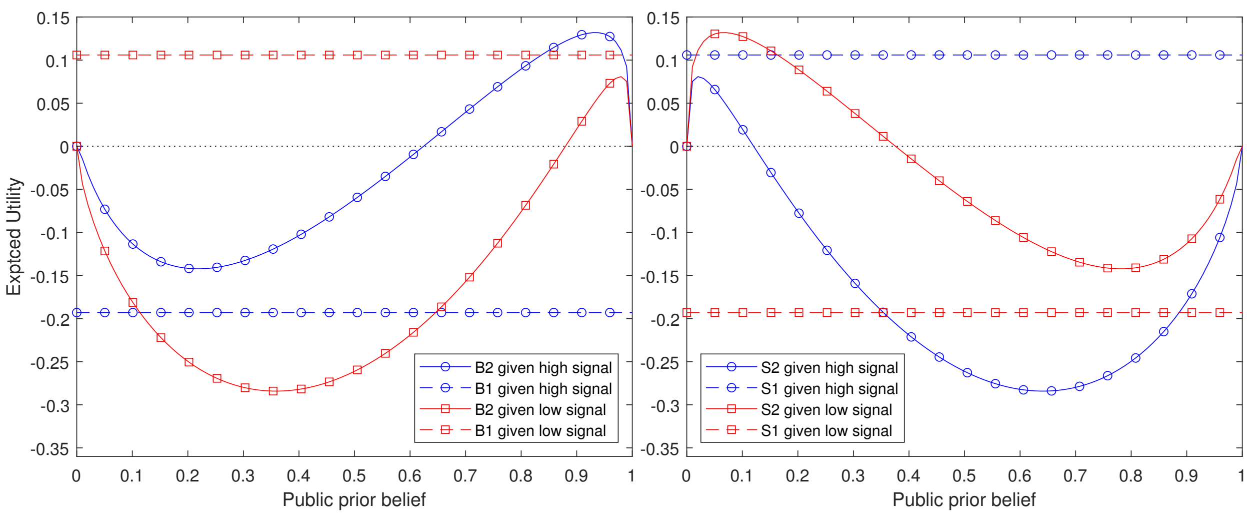



(b) Herd and Contrarianism sell check

Figure 1: Herd and Contrarianism check in baseline model

Figure 2 simulates the modified model. In Figure 2(a), we observe herd buy with a low signal when prior belief is between around 0.87 and 1. Since both B1 and B2 given low signal (red square lines) are positive, and B3i are satisfied too. In Figure 2(b), we observe a herd sell for informed traders with a high signal, when prior belief is between 0 and around 0.13. Since S1 S2 (blue circle lines) and S3i are satisfied. Also, there are situations when traders decide not to trade when B2 and S2 are both negative given a signal.

The figures allow us to see the impact of definitional difference between our and Kendall (2023)'s CPT trader herding model. In Kendall's model, B1 and S1 conditions are not needed. One needs to check for a given order direction, do both signals create positive utility. When prior belief is between around 0.87 and 1, informed traders would be considered to engage in herd buy. Since B2 given both high and low signals generate positive utility in that region. B2 given low signal is the driving condition in both our models, giving us the same price region. However, in Kendall's model informed traders with a high signal would be considered to engage in herd buy too. While in ours it does not qualify as herd buy, since B1 is negative. We don't see a clear switch behaviour in the trader's behaviour induced by the observing of other's action through prices. Considering such cases as herding or contrarian could lead to overestimation. This justifies our definition choice, consistent with majority of the literature.




(a) Herd and Contrarianism buy check

(b) Herd and Contrarianism sell check

Figure 2: Herd and Contrarianism check in modified Model

Before moving onto a generalised model without specific parameter values. Let's first consider the intuition behind the model mechanisms, and why it can generate herding and contrarian behaviour. In condition B1 the low value state produces the gain term that pulls informed traders towards selling. Since $b_t - V_L > 0$ and $b_t - V_H < 0$ by assumption. While in condition B2 the low value state produces the loss term that pulls informed traders away from buying. Attaching a higher probability to this state makes B1 easier to satisfy at the cost of B2, a low probability makes B2 easier to satisfy at the cost of B1. Intuitively, there should be optimal choices of parameters that allow both conditions to be satisfied.

# 4 Generalised CPT Trader Model Dynamics

There are many states for herding and contrarian behaviour. Buy/sell herding or contrarian behaviour given high and low signals. For each of the 8 states, there are 3 conditions to check (B1,B2,B3 or S1,S2,S3). For each of those conditions, there are 5 CPT parameters and 2 market structure parameters. This makes it challenging to track the model dynamics. We derive a generalised bias upper bound for sell(sGBUB) and buy orders(bGBUB). This bound is a function of model parameters that place upper bound on loss attitude, once satisfied either herding or contrarian behaviour occurs.

In this section, we allow loss tolerance ($\lambda < 1$) and gain-loss asymmetry in CPT($\delta_G, \delta_L, \gamma_G, \gamma_L$). We refer this model as generalised CPT trader herding model. In section 4.1, we derive the sGBUB and bGBUB to characteristic the necessary and sufficient conditions for herding and contrarian behaviour. We show how our model can capture unexplained observations in Kendall (2023) study. In section 4.2, we show the occurrence of information cascades by simulating a market for 1000 periods with heterogeneous CPT traders. In section 4.3, through calibration of CPT for 73 subjects at individual level using Zeisberger, Vrecko, and Langer (2012)'s study. We argue that for markets consisting of high proportion of loss averse traders, shutting down gain-loss asymmetry is not costly. However, if a fair amount of loss tolerant subjects presents, shutting down gain-loss asymmetry in CPT is extremely costly, leading to inaccurate herding and contrarian behaviour predictions.

## 4.1 Generalised Herding and Contrarianism Conditions

**Lemma 1.** *Conditions 1 and 2 of buy herding and contrarianism for the generalised model can be expressed with buy generalised bias upper bound (bGBUB); sell herding and contrarianism can be expressed with sell generalised bias upper bound (sGBUB):*

*(i)Buy:* $\lambda < bGBUB$. $bGBUB = \dfrac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \dfrac{K}{1-K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \dfrac{(1-K)^{1+\delta_G}}{K^{1+\delta_L}} \dfrac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B\}$

*(ii)Sell:* $\lambda < sGBUB$. $sGBUB = \dfrac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \dfrac{1-K}{K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \dfrac{(K)^{1+\delta_G}}{(1-K)^{1+\delta_L}} \dfrac{p_t^{\gamma_G-\delta_L}}{(1-p_t)^{\gamma_L-\delta_G}} V_S\}$

*where* $V_B = C[(\mu q+\theta)p_t + (\mu(1-q)+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$, $V_S = C[(\mu(1-q)+\theta)p_t + (\mu q+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$

$C = [(1-K)p_t + K(1-p_t)]^{\delta_L-\delta_G} \dfrac{\{[(1-K)p_t]^{\delta_L} + [K(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-K)p_t]^{\delta_G} + [K(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$

*K is a dummy variable takes value q if we have low signal,* $1-q$ *if we have high signal*

Recall that in definition 2 of herding and contrarianism, there are 3 conditions (B1,B2,B3) for the buy side, and 3 conditions for the sell side (S1,S2,S3). Additionally, the private signals that informed traders receive have two states. To reduce the difficulty of the problems, lemma 1 rewrites condition 1 and 2 as upper bound on loss attitude. The use of dummy variable K allows us to capture the symmetric property of the conditions. Lemma 1(i) condenses B1 given high signal, B2 given high signal, B1 given low signal, B2 given low signal into one inequality. This gives us an upper bound on loss attitude, which we refer to as buy generalised bias upper bound (bGBUB). This bound is a function of model parameters. Same idea applies to sell side in lemmma 1(ii), where we have sell generalised bias upper bound (sGBUB). The proof is fairly technical, we include it in appendix 8.3.

**Theorem 1.** *Necessary and sufficient conditions for herding and contrarianism for the generalised model. (i)If an informed trader engages in buy herding (contrarianism), then $\lambda$ is smaller than bGBUB and $p_t > 0.5$ ($p_t < 0.5$). (ii)If an informed trader engages in sell herding (contrarianism), then $\lambda$ is smaller than sGBUB and $p_t < 0.5$ ($p_t > 0.5$).*

Theorem 1 follows naturally from lemma 1. It characterises the necessary and sufficient conditions for herding and contrarianism by placing an upper bound on loss attitude. Recall in definition 1, conditions 1 (B1,S1)and 2(B2,S2) are the key ones. They indicate a switch in trading direction by an informed trader if public history $H_t$ were removed from decision-making process. Conditions 3 (B3,S3) are almost satisfied at all time, since it only requires price movement. Hence, if an informed trader herds or act as a contrarian, conditions 1 and 2 have to be satisfied, captured by bGBUB and sGBUB. The exact price movements pins down the type, herding or contrarian behaviour.

Given the proportion of informed traders $\mu$, private signal precision $q$, and CPT parameters. One can check if the loss attitude is within bGBUB or sGBUB to induce herding or contrarianism. For instance, in figure 2(a), we observed herd buy given low signal. Using $K = q$ and model parameters, the highest bGBUB is around 1.4 for $p_t > 0.5$. In the simulation we assumed $\lambda = 1.06$, so the necessary and sufficient condition is satisfied. When $p_t < 0.5$, bBUP is smaller than 1.06. So we do not observe contrarianism buy.

The implications of the theorem is as follow: firstly, regulators and practitioners could utilise these predictions. One can monitor the occurrence of herding or contrarian behaviour based on the range of loss attitudes shown by traders of certain asset classes, markets or stocks. Secondly, the necessary condition places an upper bound on loss attitude, a smaller loss attitude parameter allows a higher chance of herding or contrarianism. This suggests traders who engage in herding or contrarianism are likely to be less loss-averse than the ones who don't. This can cause large price deviation.

**Proposition 1.** *Generalised CPT Trader can engage in both herding and contrarian behaviour.*

Kendall (2023)'s theory predicts that a CPT trader makes either herding or contrarian decision, but not both. If $\gamma > \delta$, the trader can engage in only contrarian behaviour. If $\gamma < \delta$, the trader can engage in only herding. He showed that 79% of experimental subjects across both his treatments have a clear preference, consistent with the theory. However, a 10% decision deviation was allowed in this fitness calculation. For instance, if a herd type ($\gamma < \delta$) also engaged in less than 10% of contrarian decisions. This behaviour is classified as consistent with the theory. Removing the deviation allowance, only around 22% of the subjects show a clear-cut preference for either herding or contrarian. It is clear that a fair amount of the subjects are not predicted by his theory, even if 10% decision deviation were allowed. Our generalised model addresses this. Proposition 1 states that a generalised CPT trader can engage in both herding and contrarian behaviour.

Since we allowed gain-loss asymmetry and loss tolerance, it is challenging to derive the proof explicitly. We prove this using a set of plausible CPT values reported by Zeisberger, Vrecko, and Langer (2012).

We use subject 27 in table 5 pooled session results. This subject has $(\gamma_G, \gamma_L, \lambda, \delta_G, \delta_L, \text{reference point}) = (0.87, 1.03, 0.97, 1.14, 0.61, 0)$. We plot bGBUB given low signal. If loss attitude is smaller than a GBUB with price movements, then the necessary and sufficient condition of our theorem 1 is satisfied. In figure 3 we observe contrarian buy when price is smaller than 0.5, since $\lambda$ is smaller than bGBUB under some price regions. There is herd buy when price is greater than 0.5 at all region (other than boundary point when $p_t = 1$). A generalised CPT trader can engage in both herding and contrarian behaviour instead of a clear-cut preference for a certain type of behaviour. Loss tolerant traders have a higher tendency for such behaviour.

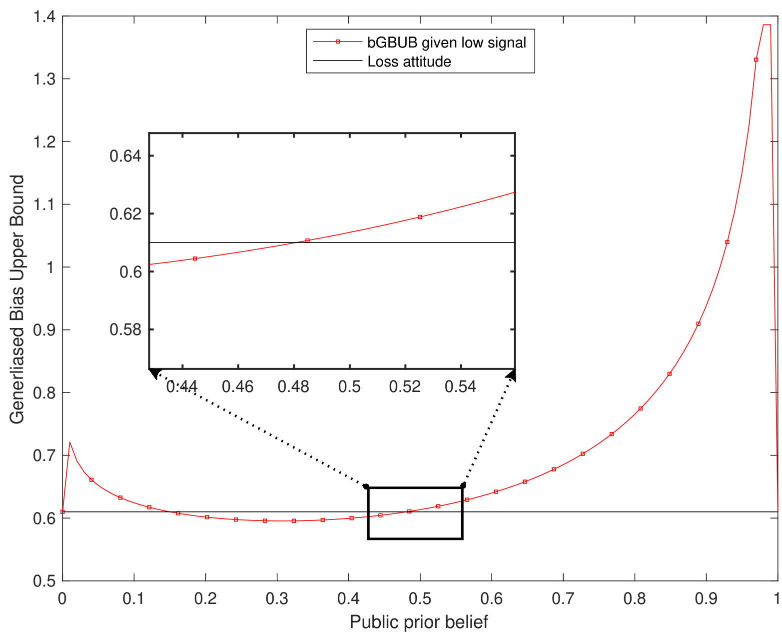



Figure 3: bGBUB given low signal

**Proposition 2.** *Herd and contrarian behaviour do not necessarily occur at extreme prices, they could also occur at mild price deviation.*

Without gain-loss asymmetry in CPT and loss tolerance. Herding and contrarian behaviour should only occur at extreme prices, as noted by Kendall (2023). This is not necessarily true in our generalised model. Proposition 2 suggests that under mild price deviations, such behaviour could also occur. The price threshold is not necessarily unique anymore that once crossed herding or contrarian occurs for certain. As figure 3 indicates, it occurs when price is around 0 to 0.15, but also 0.48 to 0.5. The later has a very mild deviation from original price 0.5. Same idea apply to sell side given high signal. There are no closed-form solutions for the price thresholds, but one can solve it numerically using root-finding algorithm.

## 4.2 Information Cascades

So far we have examined the impacts for a given trader. What are the dynamics if we consider a sequence of traders? To answer this, we simulate the price path for 1000 periods, and show how information cascades can occur. It occurs when public fails to aggregate information by observing the history of trade according to AZ, this leads to inefficiency. It is herding in extreme.

In each period a trader arrives and makes a trading decision buy sell or hold. We keep market structure the same as in previous simulations. The proportion of informed traders $\mu$ is 0.4, and the signal precision $q$ is 0.6. The sequence of traders is set exogenously and randomly. We assume the underlying state of the world is $V_L = 0$. There are 400 informed traders, of which 240 obtain a low signal, and the other 160 obtain a high signal. The 600 uninformed traders buy sell hold with equal probabilities, in expectation 200 of them buy, 200 of them sell, and 200 of them hold. The first-period price is 0.5. For the informed traders, we draw CPT values from the 73 subjects in Zeisberger, Vrecko, and Langer (2012)'s study with equal probabilities.

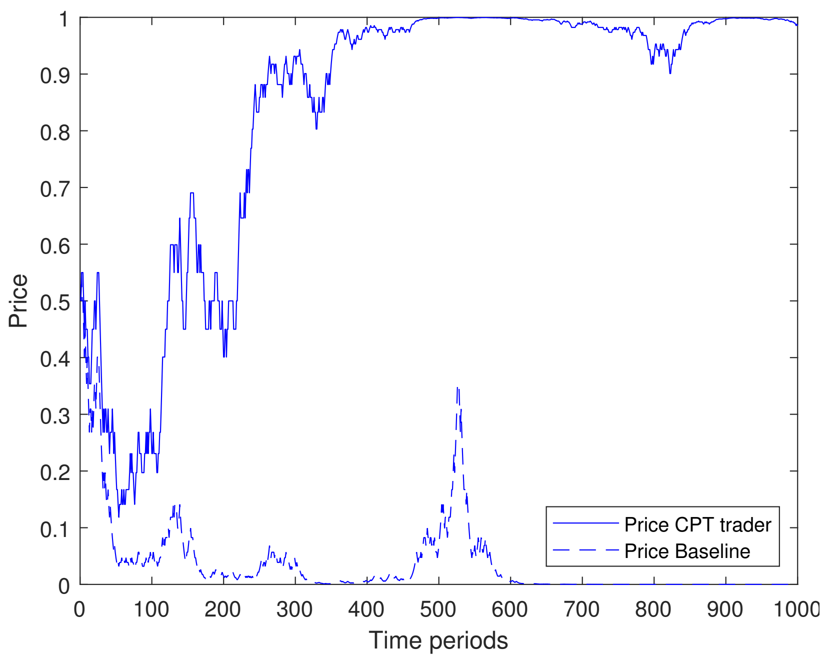



Figure 4: Information cascades

Figure 4 shows the price path in the baseline model and generalised CPT trader model. Even though the market is poorly informed and noisy with only 40% of informed traders, we observe a downward trend approaching the underlying state $V_L$. Since herding and contrarian behaviour do not occur and private information is efficiently aggregated by the market. However, in the generalised CPT trader model, we see the formation of bubbles and wrong information cascades. Asset price approaches value 1, the opposite state of the world. The price from period 500 onwards hardly moves, in line with our information cascades definition. From period 750 to 800, multiple informed sell pushes price closer to low value state temporarily. Though it is eventually dominated by multiple herd buy orders again. The formation of wrong cascades is due to the presence of multiple herd buy orders, as private signal is not revealed by the traders actions.

## 4.3 Simplicity or Predictive Power

In our generalised CPT trader herding model, it is challenging to track the driving forces. Shutting down gain-loss asymmetry in CPT can give us clean predictions, as shown in Kendall (2023). There is a trade-off between predictive power and closed-form predictions. We show that for markets dominated by loss-averse traders, shutting down gain-loss asymmetry is not too costly. One can rely on an extended CPT trader herding model without gain-loss asymmetry. However, for markets consisting of substantial proportion of loss tolerant traders, such restriction is very costly. To test this, we use table 3 pooled session results in Zeisberger, Vrecko, and Langer (2012), where they reported individual CPT values for 73 subjects.

We generate predictions for those subjects using the generalised CPT trader herding model and extended CPT trader herding model without gain-loss asymmetry. This gives us proportion of herding and contrarian behaviour. Then we mark those with absolute differences smaller than 1% as consistent, then dividing by total number of predictions, giving us the fitness rate. For the extended model without gain-loss asymmetry, we set value function exponents to be the same in loss and gain region $\gamma = (\gamma_G + \gamma_L)/2$; the probability weighting exponents in loss and gain region to be the same too $\delta = (\delta_G + \delta_L)/2$.

We do so for various market specifications, $\mu = 0.2, 0.4, 0.6, 0.8, 1$, $q = 0.6, 0.7, 0.8, 0.9$, and absolute trade imbalances of 1,2,3,4. Cipriani and Guarino (2005) defined trade imbalances as the number of buy orders at time $t$ minus number of sell orders at time t-1. For instance, if at $t = 1$ informed trader bought the asset, then we have imbalance of 1. If at $t = 2$, another trader bought the asset, then we have imbalance of 2. We call associated price IB. IB1=$[P(buy|V_H)P(V_H)]/[P(buy|V_H)P(V_H) + P(buy|V_L)P(V_L)]$. IB2=$[P(buy|V_H)IB1]/[P(buy|V_H)IB1 + P(buy|V_L)(1-IB1)]$. Same logic applies to imbalance of 3 and 4, it changes according to Bayesian updating. Assuming $\mu = 1, q = 0.7$, the price at trade imbalances 1,2,3,4 are 0.7,0.84,0.93,0.97 respectively. Absolute trade imbalances captures additional the symmetric side too, trade imbalance of -1,-2,-3,-4 follow price sequence of 0.3,0.16,0.07,0.03. The price sequence for each trade imbalances changes with market composition $q$ and $\mu$. Therefore, we consider fixed sets of trade imbalances instead of points of prices. The specifications give us a total of 160 predictions, computed by number of $\mu$*number of q*number of trade imbalances=5*4*(4*2).

Table 2 shows the cost of shutting down gain-loss asymmetry in CPT when only loss averse subjects are considered. Out of the 73 subjects, 48 are loss averse, we use those subjects. For instance, 1% under contrarian when $\mu$ is 0.2, q is 0.6 and absolute trade imbalance is 1. It indicates that when shutting down gain-loss asymmetry, we obtain 1% higher or lower contrarian behaviour. Deviations smaller than 1% are marked as consistent, the model fitness is 87%. Therefore, turning off gain-loss asymmetry is generally not too costly if the subject group consists of high proportion of loss averse subjects.

Table 3 reports the same, but considers all subjects. The overall model fitness is only 51%. Under certain market specification, it can be extremely costly. When $\mu$ is 0.4, q is 0.6 at 1 trade imbalance, the difference in proportion of contrarian behaviour predicted differs by 8%. Therefore, one has to rely on the generalised CPT trader herding model when there is a substantial number of loss tolerant subjects. In this exercise, around 34% subjects are loss tolerant.

Table 2: Cost of shutting down gain-loss asymmetry:loss averse only

| | | $\mu$=0.2 | | $\mu$=0.4 | | $\mu$=0.6 | | $\mu$=0.8 | | $\mu$=1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | IB | H | C | H | C | H | C | H | C | H | C |
| $q=0.6$ | 1 | 0% | 1% | 0% | 1% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 2 | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 0% | 0% | 0% |
| | 3 | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 0% | 0% | 0% |
| | 4 | 0% | 1% | 0% | 1% | 0% | 0% | 0% | 0% | 0% | 0% |
| $q=0.7$ | 1 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 2 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 3 | 0% | 0% | 0% | 1% | 0% | 1% | 0% | 0% | 0% | 0% |
| | 4 | 0% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| $q=0.8$ | 1 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 2 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 3 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 1% | 0% | 1% |
| | 4 | 0% | 0% | 0% | 1% | 0% | 0% | 0% | 0% | 0% | 0% |
| $q=0.9$ | 1 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 2 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 3 | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 1% | 0% | 1% |
| | 4 | 0% | 0% | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 0% |

Absolute difference in percentage with and without gain-loss asymmetry. H indicate herding, C indicates contrarian, IB indicates absolute trade imbalances. Lose averse subjects only.

Table 3: Cost of shutting down gain-loss asymmetry:all types

| | | $\mu$=0.2 | | $\mu$=0.4 | | $\mu$=0.6 | | $\mu$=0.8 | | $\mu$=1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | IB | H | C | H | C | H | C | H | C | H | C |
| $q=0.6$ | 1 | 5% | 5% | 7% | 8% | 2% | 3% | 2% | 4% | 3% | 3% |
| | 2 | 5% | 5% | 7% | 6% | 1% | 3% | 2% | 5% | 3% | 5% |
| | 3 | 6% | 5% | 7% | 6% | 1% | 4% | 3% | 5% | 5% | 5% |
| | 4 | 5% | 5% | 7% | 6% | 2% | 2% | 4% | 3% | 5% | 5% |
| $q=0.7$ | 1 | 1% | 0% | 2% | 1% | 3% | 1% | 3% | 1% | 1% | 1% |
| | 2 | 0% | 0% | 3% | 2% | 3% | 1% | 2% | 1% | 1% | 1% |
| | 3 | 1% | 0% | 3% | 2% | 2% | 2% | 1% | 2% | 1% | 2% |
| | 4 | 1% | 1% | 3% | 3% | 2% | 3% | 1% | 2% | 0% | 2% |
| $q=0.8$ | 1 | 2% | 1% | 1% | 0% | 0% | 1% | 0% | 1% | 0% | 1% |
| | 2 | 2% | 0% | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 1% |
| | 3 | 2% | 0% | 0% | 1% | 0% | 1% | 0% | 2% | 0% | 2% |
| | 4 | 1% | 0% | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 1% |
| $q=0.9$ | 1 | 0% | 1% | 0% | 0% | 0% | 0% | 0% | 0% | 0% | 0% |
| | 2 | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 1% |
| | 3 | 0% | 0% | 0% | 1% | 0% | 1% | 0% | 1% | 0% | 1% |
| | 4 | 0% | 1% | 0% | 1% | 0% | 2% | 0% | 2% | 0% | 1% |

Absolute difference in percentage with and without gain-loss asymmetry. H indicate herding, C indicates contrarian, IB indicates absolute trade imbalances. All subjects.

# 5 Extended CPT Trader Model Dynamics

In last section, we have shown that for markets consist of high proportion of loss averse informed CPT traders, one can shut down gain-loss asymmetry of CPT. In section 5.1, we first present the extended CPT trade model without gain-loss asymmetry. Then we obtain closed form solutions for prices under herding, contrarian behaviour and abstention from trade. We don't place restrictions on informed traders being loss averse. Even though the proportion of loss tolerant traders has to be small to use the extended model, it is no 0. We still need theory to characterise those traders despite it being imperfect, otherwise the predictive power would be reduced more. In section 5.2, we additionally impose restrictions by allowing only loss averse traders. This framework is very similar to Kendall (2023)'s model, the difference comes from definition of herding and contrarian behaviour. Regardless, we show that our model can create similar predictions as in Kendall (2023)'s model, giving us additional assurance on our modelling. All proofs are in appendix 8.3.

## 5.1 No Gain-Loss Asymmetry In CPT

**Lemma 2.** *Conditions 1 and 2 of buy herding and contrarianism for the extended model can be expressed with buy extended bias upper bound (bEBUB); sell herding and contrarianism can be expressed with sell extended bias upper bound (sEBUB):*

*(i)Buy:* $\lambda < bEBUB$. $bEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}\frac{K}{1-K}min\{1,(\frac{1-K}{K})^{1+\delta}(\frac{p_t}{1-p_t})^{\delta-\gamma}\}$

*(ii)Sell:* $\lambda < sEBUB$. $sEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}\frac{1-K}{K}min\{1,(\frac{K}{1-K})^{1+\delta}(\frac{p_t}{1-p_t})^{\gamma-\delta}\}$

*where K is a dummy variable takes value q if we have low signal,* $1-q$ *if we have high signal*

Lemma 2 builds on top of lemma 1 directly. Shutting down gain-loss asymmetry gives us a much more clean bias upper bound on loss attitude. $V_B$ and $V_S$ variables in lemma 1 essentially become 1. We call the new bias upper bound as buy/sell extended bias upper bound (bEBUB,sEBUB).

**Theorem 2.** *Necessary and sufficient conditions for herding and contrarianism for the extended model.*
*(i)If an informed trader engages in buy herds (contrarianism), then* $\lambda$ *is smaller than bEBUB and* $p_t > 0.5$ *(*$p_t < 0.5$*). (ii)If an informed trader engages in sell herds (contrarianism), then* $\lambda$ *is smaller than sEBUB and* $p_t < 0.5$ *(*$p_t > 0.5$*).*

Theorem 2 follows the same idea of 1, but expressed in terms of extended bias upper bounds. Having laid out the extended model, now we can investigate the effects of contrarianism and herding on prices. We denote $\Delta p_t$ as the price deviation from the initial period.

**Proposition 3.** *The region of prices and deviation from initial price under contrarianism and herding are:*

*Scenario 1: $\gamma > \delta$*

*(i) Contrarianism buy:* $p_t \in (0, \frac{1}{1+L_1})$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

*(ii) Contrarianism sell:* $p_t \in (\frac{1}{1+L_2}, 1)$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

*(iii) Herd buy:* $p_t \in (0.5, \frac{1}{1+L_1})$; $|\Delta p_t| \in (0\%, |\frac{1-L_1}{1+L_1}|)$

*(iv) Herd sell:* $p_t \in (\frac{1}{1+L_2}, 0.5)$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

*Scenario 2: $\gamma < \delta$*

*(i) Contrarianism buy:* $p_t \in (\frac{1}{1+L_1}, 0.5)$; $|\Delta p_t| \in (0\%, \frac{1-L_1}{1+L_1})$

*(ii) Contrarianism sell:* $p_t \in (0.5, \frac{1}{1+L_2})$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

*(iii) Herd buy:* $p_t \in (\frac{1}{1+L_1}, 1)$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

*(iv) Herd sell:* $p_t \in (0, \frac{1}{1+L_2})$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

*where* $L_1 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-K}{K}]^{\delta}\}^{1/(\delta-\gamma)}$, $L_2 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{K}{1-K}]^{\delta}\}^{1/(\gamma-\delta)}$

Given that $\gamma > \delta$. The effects on prices under contrarianism are much more significant than under herding. Under contrarianism, given buy order, the price deviation is at least $|(1-L_1)/(1+L_1)|$ and up to 100%. While under herding, price deviation is at most $|(1-L_1)/(1+L_1)|$ and as low as 0%. Same idea applies the sell side but we have $L_2$ instead. The results are opposite when $\gamma < \delta$, where effects on price under herding are more significant than under contrarianism. Under contrarianism, given buy order, the price deviation is at most $|(1-L_1)/(1+L_1)|$. While under herding, price deviation is at least $|(1-L_1)/(1+L_1)|$ and up to 100%. Same idea applies to the sell side but we have $L_2$ instead.

The intuition is as follow. Recall in our modified model, the actions of informed traders are: (i)Buy if $E\Omega[\pi|S,H_t] = \Omega(V_H - a_t) * w[P(V_H|S,H_t)] + \Omega(V_L - a_t) * w[P(V_L|S,H_t)] > 0$. (ii)Sell if $E\Omega[\pi|S,H_t] = \Omega(b_t - V_H) * w[P(V_H|S,H_t)] + \Omega(b_t - V_L) * w[P(V_L|S,H_t)] > 0$.

The expected utility of a trading action has two parts. Firstly the value function $\Omega$ gives the perceived utility by the trader for potential payoffs at low and high-value states. Secondly the perceived probability associated with each state. Under scenario 1, the probability bias $\delta$ is not as strong as the value bias $\gamma$. As such, the first part perceived utility for potential payoffs are dominating the traders' expectation. Loosely speaking, contrarians buy when price is low and sell when price is high. While herd traders buy when price is high and sell when price is low. The price needs to deviate more from the initial price for contrarians' action to be profitable. The larger the deviation, the larger the potential payoff and utility for contrarians. Thus, given $\gamma > \delta$, price deviation under contrarianism is more significant than herding.

For scenario 2, the probability bias $\delta$ is stronger than the value bias $\gamma$. As such, the second part, associated probabilities for each state dominates the traders' expectation. Holding everything else constant, a higher $\delta$ implies that high-value state has a lower perceived probability, and low-value state has a higher perceived probability. Since now probability distortion is more server. The buy low sell high strategy by contrarians gives a lower expected perceived utility now. For contrarians to buy, price needs to be higher so that $P(V_H|S,H_t)$ becomes higher to offset the larger underweighting by $w[.]$ due to higher $\delta$. This means price is closer to the initial price now. For contrarians to sell, lower price is needed so that a lower $P(V_L|S,H_t)$ is required to offset the larger overweighting by $w[.]$ due to higher $\delta$. This means price is closer to the initial price too. A similar idea applies to herd traders, the buy high sell low strategy now gives a higher expected perceived utility. Even though, a larger price deviation squeezes out potential payoffs. It provides a strong enough signal to bring up probability for the winning state so that changes in probability bias are offset. Thus, given $\gamma < \delta$, price deviation under herding is more significant than contrarianism.

**Proposition 4.** *Given the signal. If $\delta > \gamma$, the no trade price region is $p_t \in [\frac{1}{1+L_2}, \frac{1}{1+L_1}]$. If $\delta < \gamma$, the no trade price region is $p_t \in [\frac{1}{1+L_1}, \frac{1}{1+L_2}]$.*

Experiments on AZ theory typically found high proportion of no trade, inconsistent with the theory. This impedes the flow of information into the market and affects the price discovery process negatively. With CPT traders, we are able to generate similar dynamics. This follows naturally from proportion 3, where we defined variables $L_1$ and $L_2$ as functions of model parameters. Recall that, an informed trader do no trade if neither buy or sell generates positive utility for her. This is also equivalent in saying if given a signal, B2 and S2 conditions of our definition 1 for herding and contrarian behaviour are violated. Then there is no trade. Those are encoded in the second part of the min term in our bEBUB and SEBUB.

## 5.2 Loss Averse Traders Only

How do our model compare to Kendall (2023)'s CPT trader herding model? To answer this, we make the same assumptions that there are no gain-loss asymmetry and only loss averse traders. We use the extended model and only allow loss averse traders. Proposition 5 highlights the difference in our models, coming from how we defined herding and contrarian behaviour. Our definition leads to lower herding and contrarian behaviour. Proposition 6 shows the similarity of our models, both can predict herding and contrarian dynamics based on relation between $\gamma$ and $\delta$.

**Proposition 5.** *Given loss-averse informed traders ($\lambda > 1$), buy herding and contrarianism cannot occur given a high signal, and sell herding and contrarianism cannot occur given a low signal.*

This proposition can be derived directly using theorem 2. bEBUB is smaller than 1 for buy herding or contrarianism given a high signal, sEBUB is smaller than 1 for sell herding or contrarianism given a low signal. Loss-averse informed traders with $\lambda > 1$ violate the condition. The intuition is that a loss-averse informed trader would never obtain a positive expected utility by trading against their private signal when considering on only the private signal. Trading against private signals is costly and generates losses, loss averse traders are not willing to do so. Essentially, condition 1 of definition 1 for herding and contrarianism is violated. In other words, informed traders with a high signal would never have sold conditional on only private signal, this prevents buy herding or contrarianism since condition B1 is violated. Informed traders with a low signal would never have bought conditional on only private signal, this prevents sell herding or contrarianism since condition S1 is violated. This proposition does not hold in Kendall (2023)'s model, since condition 1 of our herding and contrarianism definition is the driving force here, which is absent in his model. This reduces potential overestimation.

**Proposition 6.** *Given loss aversion, when $\gamma > \delta$ only contrarian behaviour is possible, when $\delta > \gamma$ only herding is possible.*

Proposition 6 tells us that if informed traders are loss averse and probability bias $\delta$ is smaller than value bias $\gamma$, only contrarianism can occur (buy contrarianism given a low signal, sell contrarianism given a high signal to be precise). When $\delta > \gamma$, only herding can occur (buy herding given low signal, sell herding given high signal). This proposition share similarity to Kendall (2023)'s predictions, but we require a clear switch in trading decision. The similarity to his results gives us additional assurance on our modelling.

Loosely speaking, contrarian buys when asset value is low and sells when it's high; herd traders buy high and sell low. If both types have bought(sold) the asset, followed by asset value drop(increases), the potential loss is bigger for herd traders than contrarian since they bought(sold) at relatively higher(lower) prices. When $\gamma > \delta$, traders care more about the payoff in each state instead of it's associated probabilities. The loss aversion is enough to dominate herd trader's perceived expected utility so that they don't engage in herding. While contrarians face higher payoffs once realised, so loss aversion is not enough to dominate their expected utility. When $\gamma < \delta$, traders care more about the associated probabilities in each state instead of payoffs. Contrarians prefer to buy when price is low, so they have big gain once high value state realises. Though a low price indicates a small probability for high state. Since they care more about probabilities now, contrarian's expected utility is negative. While for herd traders they buy at high price. Even though the potential payoff is small, a high price indicates a high probability for such state, so herding is possible.

# 6 Experimental Evidence Reconciliation

Multiple experiments tested AZ theory directly. Both Cipriani and Guarino (2005) and Drehmann, Oechssler, and Roider (2005) showed that informed traders rarely herd, in line with AZ theory. However, both also observed high proportion of contrarian behaviour, something not captured by the original theory. Cipriani and Guarino (2009) followed a more relaxed definition of herding and contrarian behaviour and observed higher proportion of both behaviour. In section 5 of their paper, they controlled for such definition difference and noted that results are consistent with previous experiments. Cipriani and Guarino (2005) found 12% of herding and 19% of contrarian behaviour, Cipriani and Guarino (2009) found 5% of herding ad 28% of contrarian. Authors have noted that such similarity was reassuring.

Evidence in Kendall (2023) is mixed. He followed the definition of Cipriani and Guarino (2009). In his main treatment, he provided subjects directly the correct Bayesian posterior to control for Bayesian errors. He found significantly higher level of herding and contrarian behaviour, with the former being 34% and later being 10.6%. However, a main concern is that by providing subjects Bayesian posterior directly, they take away the uncertainty in probabilities since it is now given exogenously by the experimenter. This would force the probability weighting function exponent of CPT to be 1, creating much more herding type traders than contrarian types. Since his underlying CPT trader theory and our proposition 6 suggest that when $\delta > \gamma$, only herding is possible. The author noted that there are almost 90% subjects showed $\delta > \gamma$. In their second treatment, they don't provide the correct Bayesian posterior directly. Thus, results are comparable with Cipriani and Guarino (2009). Herding is now much lower in the second treatment, dropping to only 13.9%, while contrarian increased to 12.8%. Suggesting both behaviour have similar occurrence rate. This matches herding level in Cipriani and Guarino (2009), but not the level of contrarian.

Overall, majority of the experimental evidences suggest that traders have a strong tendency to trade against the market and act as contrarian, ranging from 19% to 28% using AZ definition. While traders rarely engage in herding behaviour, ranging from 5% to 12% of the decisions. To reconcile this, Kendall (2023)'s CPT trader model provides us a good starting point. However, it is difficulty to explain those evidence giving the definitional difference in the underlying model. Even if one adjust for definition difference, such CPT trader model still rely on gain-loss asymmetry in CPT and only able to predict behaviour of loss averse traders. Both assumptions can be restrictive as we showed in previous sections.

We use Zeisberger, Vrecko, and Langer (2012)'s estimates of CPT for 73 subjects to generate theoretical predictions using our generalised CPT trader model. To keep it comparable with previous experiments, we only check when trade imbalance is greater than 2. We also restrict it to be smaller than 12, since in Cipriani and Guarino (2005)'s study each round had 12 subjects with maximum 12 trade imbalances. Table 4 reports our theoretical predictions under various signal precision $q$ and proportion of informed traders $\mu$. Under all market specification, we observe high proportion of contrarian trading and small proportion of herd trading. When $q = 0.7, \mu = 1$, the market structure align with previous experiments, where all traders are informed and signal is fairly precise. We found 4% of herding, in line with previous findings of 12% and 5%. We also observe 21% of contrarian behaviour, consistent with previous findings of 19% and 28%.

Table 4: Proportion of Herding and Contrarian Using AZ definition:Aggregate

| | $\mu$=0.2 | | $\mu$=0.4 | | $\mu$=0.6 | | $\mu$=0.8 | | $\mu$=1 | |
|---|---|---|---|---|---|---|---|---|---|---|
| | H | C | H | C | H | C | H | C | H | C |
| $q = 0.6$ | 19% | 26% | 18% | 26% | 14% | 25% | 12% | 22% | 12% | 22% |
| $q = 0.7$ | 7% | 23% | 8% | 23% | 7% | 25% | 4% | 22% | **4%** | **21%** |
| $q = 0.8$ | 3% | 14% | 1% | 22% | 1% | 25% | 1% | 23% | 3% | 22% |
| $q = 0.9$ | 0% | 11% | 0% | 16% | 1% | 25% | 1% | 25% | 1% | 21% |

H indicates herding, C indicates contrarian. This table shows the proportion of herding and contrarian behaviour when absolute value of trader imbalance is greater than 2 and smaller than 12.

# 7 Conclusion

We have presented a generalised model of herding and contrarian behaviour in financial markets by incorporating biased informed traders through cumulative prospect theory by Tversky and Kahneman (1992). Our study builds on the sequential trading market microstructure herding framework established by Avery and Zemsky (1998) and extends it beyond the confines of rational expected utility theory. By integrating CPT, we consider psychological factors such as loss attitude and probability distortions, crucial in understanding traders' decision-making. This complements Kendall (2023)'s work on CPT trader herding model. Our model is more general in the sense that we allow gain-loss asymmetry in CPT and loss tolerant traders, both supported by recent literature. Our model adopts a stringent definition, requiring a clear switch in trading decisions induced by history, aligning with the majority of literature started with Avery and Zemsky (1998).

Contrary to Kendall (2023)'s predictions, we found that generalised CPT trader can engage in both herding and contrarian behaviour, instead of a clear cut preference. This aligns the unexplained experimental observation by him. Additionally, we found herding and contrarian behaviour could occur at mild price deviations too instead of just extreme prices. Through simulations, we showed how wrong information cascades can arise with heterogeneous CPT trader, leading to market inefficiency. More importantly, Our findings reveal that in markets with a substantial proportion of loss-tolerant agents, the elimination of gain-loss asymmetry can incur significant costs, emphasizing the necessity of employing the generalized model. Conversely, in markets dominated by loss-averse traders, such an assumption is less costly, allowing reliance on an extended model without gain-loss asymmetry for closed-form results. Then we show that the extended model can generate consistent results with Kendall (2023)'s model, providing us extreme assurance on our modelling. Finally, we reconciled previous experimental evidence using Zeisberger, Vrecko, and Langer (2012)'s CPT estimates. We show strong contrarian tendencies under various market specifications. Under the same market structure as previous experiments, we match the levels of herding and contrarian behaviour.

Our findings have important implications for regulators and practitioners. Our theorem 1 can generate market-specific predictions. This allows regulators to monitor the occurrence of herding or contrarianism. For instance, the bias upper bound fluctuates with prices and market structure. Once this bound becomes larger than informed traders' loss attitude, herding or contrarianism occurs.

Our research opens avenues for future exploration. Firstly, by aligning our model with Cipriani and Guarino (2014)'s and carefully calibrate CPT values for that market, empirical testing of our model becomes feasible. This could potentially generate interesting market and country specific predictions on herding and contrarian behaviour. Secondly, future experiments testing our model could yield interesting findings. Specifically, researchers should examine trader behaviour when only private signals are permitted in decision-making(condition 1 of our definition). This involves identifying such decisions in the first period before any trading take place. This would allow one to observe a clear switch in trading decisions induced by history. Additionally, one should control for Bayesian updating errors in the fashion as Kendall (2023)'s main treatment. However, the Bayesian posterior should not be provided directly. As it may accidentally eliminate probability distortion of CPT, creating more herd type traders. One can perhaps provide the Bayesian updating formula instead so that uncertainty and probability distortion still present.

# 8 Appendix

## 8.1 Key formulas for the AZ model

Here I present the key formulas of AZ framework, taken from Boortz (2016).

*(i)Conditional buy and sell probabilities.*

$$P(x_t = buy|V_L) = P(x_t = sell|V_H) = \mu(1-q)+\theta \quad (1)$$

$$P(x_t = sell|V_L) = P(x_t = buy|V_H) = \mu q+\theta \quad (2)$$

*(ii)Ask and bid prices*

$$a_t = E[V|H_t, x_t = buy] = \frac{(\mu q+\theta)p_t}{(\mu q+\theta)p_t + [\mu(1-q)+\theta](1-p_t)} \quad (3)$$

$$b_t = E[V|H_t, x_t = sell] = \frac{[\mu(1-q)+\theta]p_t}{[\mu(1-q)+\theta]p_t + (\mu q+\theta)(1-p_t)} \quad (4)$$

*(iii)Expected value of assets given low and high signal*

$$E[V|S_L, H_t] = \frac{(1-q)p_t}{(1-q)p_t + q(1-p_t)} \quad (5)$$

$$E[V|S_H, H_t] = \frac{qp_t}{qp_t + (1-q)(1-p_t)} \quad (6)$$

## 8.2 Key formulas for the modified model

**(i)Conditional asset value probabilities.** $P(S_H|V_H) = P(S_L|V_L) = q$; $P(S_H|V_L) = P(S_L|V_H) = 1-q$

$$P(V_H|S_H, H_t) = \frac{P(S_H|V_H)P(V_H|H_t)}{P(S_H|V_H)P(V_H|H_t) + P(S_H|V_L)P(V_L|H_t)} = \frac{qp_t}{qp_t + (1-q)(1-p_t)}$$

$$P(V_H|S_L, H_t) = \frac{P(S_L|V_H)P(V_H|H_t)}{P(S_L|V_H)P(V_H|H_t) + P(S_L|V_L)P(V_L|H_t)} = \frac{(1-q)p_t}{(1-q)p_t + q(1-p_t)}$$

$$P(V_L|S_H, H_t) = 1 - P(V_H|S_H, H_t) = \frac{(1-q)(1-p_t)}{qp_t + (1-q)(1-p_t)}$$

$$P(V_L|S_L, H_t) = 1 - P(V_H|S_L, H_t) = \frac{q(1-p_t)}{(1-q)p_t + q(1-p_t)}$$

**(ii) Herd and Contrarianism buy**

*High Signal case*

$$\text{(B1)} \quad \Omega[\pi|S_H] = -\lambda[-(b_1 - V_H)]^{\gamma_L} q + (b_1 - V_L)^{\gamma_G}(1-q)$$

$$\text{(B2)} \quad \Omega[\pi|S_H, H_t] = (V_H - a_t)^{\gamma_G} \frac{P(V_H|S_H, H_t)^{\delta_G}}{(P(V_H|S_H, H_t)^{\delta_G} + [1 - P(V_H|S_H, H_t)]^{\delta_G})^{1/\delta_G}}$$
$$-\lambda[-(V_L - a_t)]^{\gamma_L} \frac{P(V_L|S_H, H_t)^{\delta_L}}{(P(V_L|S_H, H_t)^{\delta_L} + [1 - P(V_L|S_H, H_t)]^{\delta_L})^{1/\delta_L}}$$

*Low Signal case*

(B1) $\Omega[\pi|S_L] = -\lambda[-(b_1 - V_H)]^{\gamma_L}(1-q) + (b_1 - V_L)^{\gamma_G} q$

(B2) $\Omega[\pi|S_L,H_t] = (V_H - a_t)^{\gamma_G} \dfrac{P(V_H|S_L,H_t)^{\delta_G}}{(P(V_H|S_L,H_t)^{\delta_G} + [1-P(V_H|S_L,H_t)]^{\delta_G})^{1/\delta_G}}$
$-\lambda[-(V_L - a_t)]^{\gamma_L} \dfrac{P(V_L|S_L,H_t)^{\delta_L}}{(P(V_L|S_L,H_t)^{\delta_L} + [1-P(V_L|S_L,H_t)]^{\delta_L})^{1/\delta_L}}$

**(iii) Herd and Contrarianism sell**

*High Signal case*

(B1) $\Omega[\pi|S_H] = (V_H - a_1)^{\gamma_G} q - \lambda[-(V_L - a_1)]^{\gamma_L}(1-q)$

(B2) $\Omega[\pi|S_H,H_t] = -\lambda[-(b_t - V_H)]^{\gamma_L} \dfrac{P(V_H|S_H,H_t)^{\delta_L}}{(P(V_H|S_H,H_t)^{\delta_L} + [1-P(V_H|S_H,H_t)]^{\delta_L})^{1/\delta_L}}$
$+(b_t - V_L)^{\gamma_G} \dfrac{P(V_L|S_H,H_t)^{\delta_G}}{(P(V_L|S_H,H_t)^{\delta_G} + [1-P(V_L|S_H,H_t)]^{\delta_G})^{1/\delta_G}}$

*Low Signal case*

(B1) $\Omega[\pi|S_L] = (V_H - a_1)^{\gamma_G}(1-q) - \lambda[-(V_H - a_1)]^{\gamma_L} q$

(B2) $\Omega[\pi|S_L,H_t] = -\lambda[-(b_t - V_H)]^{\gamma_L} \dfrac{P(V_H|S_L,H_t)^{\delta_L}}{(P(V_H|S_L,H_t)^{\delta_L} + [1-P(V_H|S_L,H_t)]^{\delta_L})^{1/\delta_L}}$
$+(b_t - V_L)^{\gamma_G} \dfrac{P(V_L|S_L,H_t)^{\delta_G}}{(P(V_L|S_L,H_t)^{\delta_G} + [1-P(V_L|S_L,H_t)]^{\delta_G})^{1/\delta_G}}$

## 8.3 Proofs

**Lemma 1**: *Conditions 1 and 2 of buy herding and contrarianism for the generalised model can be expressed with buy generalised bias upper bound (bGBUB); sell herding and contrarianism can be expressed with sell generalised bias upper bound (sGBUB):*

*(i)Buy:* $\lambda < bGBUB$. $bGBUB = \dfrac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \dfrac{K}{1-K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \dfrac{(1-K)^{1+\delta_G}}{K^{1+\delta_L}} \dfrac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B\}$

*(ii)Sell:* $\lambda < sGBUB$. $sGBUB = \dfrac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \dfrac{1-K}{K} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \dfrac{(K)^{1+\delta_G}}{(1-K)^{1+\delta_L}} \dfrac{p_t^{\gamma_G-\delta_L}}{(1-p_t)^{\gamma_L-\delta_G}} V_S\}$

*where* $V_B = C[(\mu q+\theta)p_t + (\mu(1-q)+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$, $V_S = C[(\mu(1-q)+\theta)p_t + (\mu q+\theta)(1-p_t)]^{\gamma_L-\gamma_G}$
$C = [(1-K)p_t + K(1-p_t)]^{\delta_L-\delta_G} \dfrac{\{[(1-K)p_t]^{\delta_L} + [K(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-K)p_t]^{\delta_G} + [K(1-p_t)]^{\delta_G}\}^{1/\delta_G}}$
*K is a dummy variable takes value q if we have low signal,* $1-q$ *if we have high signal*

*Proof.* I prove for buy herding and contrarianism given low signal first.
(B1)We prove this using definition 1.A:

$$0 < -\lambda(V_H - b_1)^{\gamma_L} P(V_H|S_L) + (b_1 - V_L)^{\gamma_G} P(V_L|S_L) \Rightarrow \lambda < \frac{P(V_L|S_L)}{P(V_H|S_L)}[\frac{(b_1 - V_L)^{\gamma_G}}{(V_H - b_1)^{\gamma_L}}] \Rightarrow \lambda < \frac{q}{1-q}[\frac{(b_1)^{\gamma_G}}{(1-b_1)^{\gamma_L}}]$$

Given that prior $p_1 = 0.5$, using the formula on bid price in Appendix 8.1 we can derive that:

$$
\begin{aligned}
b_1 &= \frac{\mu(1-q)+\theta}{\mu-\mu q+\theta+\mu q+\theta} = \frac{\mu(1-q)+\theta}{\mu+2\theta} \Rightarrow 1-b_1 = \frac{\theta+\mu q}{\mu+2\theta} \\
\frac{(b_1)^{\gamma_G}}{(1-b_1)^{\gamma_L}} &= (\frac{\mu(1-q)+\theta}{\mu+2\theta})^{\gamma_G}(\frac{\mu+2\theta}{\theta+\mu q})^{\gamma_L} = \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}}(\mu+2\theta)^{\gamma_L-\gamma_G}
\end{aligned}
$$

Substitute this into the inequality.

$$
\lambda < \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}}(\mu+2\theta)^{\gamma_L-\gamma_G}(\frac{q}{1-q})
$$

(B2)Now we derive condition (B2):

$$
\begin{aligned}
0 &< (V_H-a_t)^{\gamma_G} w[P(V_H|S_L,H_t)] - \lambda(a_t-V_L)^{\gamma_L} w[P(V_L|S_L,H_t)] \\
\lambda &< \frac{(V_H-a_t)^{\gamma_G}}{(a_t-V_L)^{\gamma_L}} \frac{w[P(V_H|S_L,H_t)]}{w[P(V_L|S_L,H_t)]} \Rightarrow \lambda < \frac{(1-a_t)^{\gamma_G}}{(a_t)^{\gamma_L}} \frac{w[P(V_H|S_L,H_t)]}{w[P(V_L|S_L,H_t)]}
\end{aligned}
$$

For the first term, using formula for $a_t$ in Appendix 8.1, we derive:

$$
\begin{aligned}
&(1-a_t)^{\gamma_G} = (\frac{[\mu(1-q)+\theta](1-p_t)}{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)})^{\gamma_G}, (\frac{1}{a_t})^{\gamma_L} = (\frac{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)}{(\mu q+\theta)p_t})^{\gamma_L} \\
&\frac{(1-a_t)^{\gamma_G}}{(a_t)^{\gamma_L}} = \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\mu q+\theta)^{\gamma_L}} \frac{(1-p_t)^{\gamma_G}}{p_t^{\gamma_L}} \{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)\}^{\gamma_L-\gamma_G}
\end{aligned}
$$

For the second term, using the formula for probabilities in Appendix 8.2, we derive:

$$
\begin{aligned}
\frac{w[P(V_H|S_L,H_t)]}{w[P(V_L|S_L,H_t)]} &= \frac{P(V_H|S_L,H_t)^{\delta_G}}{(P(V_H|S_L,H_t)^{\delta_G}+[1-P(V_H|S_L,H_t)]^{\delta_G})^{1/\delta_G}} \frac{(P(V_L|S_L,H_t)^{\delta_L}+[1-P(V_L|S_L,H_t)]^{\delta_L})^{1/\delta_L}}{P(V_L|S_L,H_t)^{\delta_L}} \\
&= \frac{P(V_H|S_L,H_t)^{\delta_G}}{(P(V_H|S_L,H_t)^{\delta_G}+P(V_L|S_L,H_t)^{\delta_G})^{1/\delta_G}} \frac{(P(V_L|S_L,H_t)^{\delta_L}+P(V_H|S_L,H_t)^{\delta_L})^{1/\delta_L}}{P(V_L|S_L,H_t)^{\delta_L}} \\
\text{Friction 1} &= [\frac{(1-q)p_t}{(1-q)p_t+q(1-p_t)}]^{\delta_G} \frac{(1-q)p_t+q(1-p_t)}{\{[(1-q)p_t]^{\delta_G}+[q(1-p_t)]^{\delta_G}\}^{1/\delta_G}} \\
\text{Friction 2} &= [\frac{(1-q)p_t+q(1-p_t)}{q(1-p_t)}]^{\delta_L} \frac{\{[q(1-p_t)]^{\delta_L}+[(1-q)p_t]^{\delta_L}\}^{1/\delta_L}}{(1-q)p_t+q(1-p_t)} \\
\text{Combined} &= \frac{[(1-q)p_t]^{\delta_G}}{[q(1-p_t)]^{\delta_L}}[(1-q)p_t+q(1-p_t)]^{\delta_L-\delta_G} \frac{\{[(1-q)p_t]^{\delta_L}+[q(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-q)p_t]^{\delta_G}+[q(1-p_t)]^{\delta_G}\}^{1/\delta_G}}
\end{aligned}
$$

Define below:

$$
\begin{aligned}
V_B &= C\{(\mu q+\theta)p_t+[\mu(1-q)+\theta](1-p_t)\}^{\gamma_L-\gamma_G} \\
C &= [(1-q)p_t+q(1-p_t)]^{\delta_L-\delta_G} \frac{\{[(1-q)p_t]^{\delta_L}+[q(1-p_t)]^{\delta_L}\}^{1/\delta_L}}{\{[(1-q)p_t]^{\delta_G}+[q(1-p_t)]^{\delta_G}\}^{1/\delta_G}}
\end{aligned}
$$

Combine terms give us: $\lambda < \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\mu q+\theta)^{\gamma_L}} \frac{(1-q)^{\delta_G}}{q^{\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B$

(Necessary condition):

Now we use B1 and B2 to derive the necessary condition. Both B1 and B2 provide an upper bound on $\lambda$, we only need the more restrictive one to be satisfied. B2 can be rewritten as:

$$\lambda < \frac{(\mu(1-q)+\theta)^{\gamma_G}}{(\mu q+\theta)^{\gamma_L}} \frac{q}{1-q} \frac{(1-q)^{1+\delta_G}}{q^{1+\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B$$

We can rewrite B1 and B2 as:

$$\lambda < \frac{[\mu(1-q)+\theta]^{\gamma_G}}{(\theta+\mu q)^{\gamma_L}} \frac{q}{1-q} min\{(2\theta+\mu)^{\gamma_L-\gamma_G}, \frac{(1-q)^{1+\delta_G}}{q^{1+\delta_L}} \frac{p_t^{\delta_G-\gamma_L}}{(1-p_t)^{\delta_L-\gamma_G}} V_B\}$$

The rest follow the same idea and are symmetric, once derived, we apply dummy variable D to capture this.

QED

**Lemma 2**: *Conditions 1 and 2 of buy herding and contrarianism for the extended model can be expressed with buy extended bias upper bound (bEBUB); sell herding and contrarianism can be expressed with sell extended bias upper bound (sEBUB):*

*(i)Buy:* $\lambda < bEBUB$. $bEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma} \frac{K}{1-K} min\{1, (\frac{1-K}{K})^{1+\delta} (\frac{p_t}{1-p_t})^{\delta-\gamma}\}$

*(ii)Sell:* $\lambda < sEBUB$. $sEBUB = [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma} \frac{1-K}{K} min\{1, (\frac{K}{1-K})^{1+\delta} (\frac{p_t}{1-p_t})^{\gamma-\delta}\}$

*where K is a dummy variable takes value q if we have low signal,* $1-q$ *if we have high signal*

*Proof.* Set $\gamma$ and $\delta$ in both gain and loss region in lemma 1, we have

$C = [(1-K)p_t + K(1-p_t)]^{\delta-\delta} \frac{\{[(1-K)p_t]^{\delta} + [K(1-p_t)]^{\delta}\}^{1/\delta}}{\{[(1-K)p_t]^{\delta} + [K(1-p_t)]^{\delta}\}^{1/\delta}} = 1$

$V_B = 1[(\mu q+\theta)p_t + (\mu(1-q)+\theta)(1-p_t)]^{\gamma-\gamma} = 1$, $V_S = 1[(\mu(1-q)+\theta)p_t + (\mu q+\theta)(1-p_t)]^{\gamma-\gamma} = 1$

Rename bEBUB and sEBUB to bEBUB and sEBUB to reflect the restricted model, we have:

$bEBUB = \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}} \frac{K}{1-K} min\{(2\theta+\mu)^{\gamma-\gamma}, \frac{(1-K)^{1+\delta}}{K^{1+\delta}} \frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}} 1\}$

$sEBUB = \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}} \frac{1-K}{K} min\{(2\theta+\mu)^{\gamma-\gamma}, \frac{(K)^{1+\delta}}{(1-K)^{1+\delta}} \frac{p_t^{\gamma-\delta}}{(1-p_t)^{\gamma-\delta}} 1\}$

Then, using that $(2\theta+\mu)^{\gamma-\gamma} = 1$, we obtain bEBUB and sEBUB QED

**Proposition 3**: *The region of prices and deviation from initial price under contrarianism and herding are:*

*Scenario 1: $\gamma > \delta$*

*(i) Contrarianism buy:* $p_t \in (0, \frac{1}{1+L_1})$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

*(ii) Contrarianism sell:* $p_t \in (\frac{1}{1+L_2}, 1)$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

*(iii) Herd buy:* $p_t \in (0.5, \frac{1}{1+L_1})$; $|\Delta p_t| \in (0\%, |\frac{1-L_1}{1+L_1}|)$

*(iv) Herd sell:* $p_t \in (\frac{1}{1+L_2}, 0.5)$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

*Scenario 2: $\gamma < \delta$*

*(i) Contrarianism buy:* $p_t \in (\frac{1}{1+L_1}, 0.5)$; $|\Delta p_t| \in (0\%, \frac{1-L_1}{1+L_1})$

*(ii) Contrarianism sell:* $p_t \in (0.5, \frac{1}{1+L_2})$; $|\Delta p_t| \in (0\%, |\frac{1-L_2}{1+L_2}|)$

*(iii) Herd buy:* $p_t \in (\frac{1}{1+L_1}, 1)$; $|\Delta p_t| \in (|\frac{1-L_1}{1+L_1}|, 100\%)$

*(iv) Herd sell:* $p_t \in (0, \frac{1}{1+L_2})$; $|\Delta p_t| \in (|\frac{1-L_2}{1+L_2}|, 100\%)$

*where* $L_1 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-K}{K}]^{\delta}\}^{1/(\delta-\gamma)}$, $L_2 = \{\frac{1}{\lambda}[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{K}{1-K}]^{\delta}\}^{1/(\gamma-\delta)}$

*Proof.* We utilise Lemma 2(i) and prove buy side scenario 2 here. Proof for the rest follows the same idea. From lemma 2(i) we know that $bEBUB = \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}min\{1, \frac{(1-K)^{1+\delta}}{K^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}}\}$ $=min\{\frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}, \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}\frac{(1-K)^{1+\delta}}{K^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}}\}$. Given that herding or contrarain behaviour occurs, second part of the min term gives us the solution for price. It is essentially condition 2 (B2) of our herding and contrarian behaviour definition. Assuming $\delta > \gamma$, We have:

$$\lambda < \frac{[\mu(1-q)+\theta]^{\gamma}}{(\theta+\mu q)^{\gamma}}\frac{K}{1-K}\frac{(1-K)^{1+\delta}}{K^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}} \to \lambda < (\frac{[\mu(1-q)+\theta]}{(\theta+\mu q)})^{\gamma}(\frac{1-K}{K})^{\delta}(\frac{p_t}{1-p_t})^{\delta-\gamma}$$
$$\to (\frac{1-p_t}{p_t})^{\delta-\gamma} < \frac{1}{\lambda}(\frac{[\mu(1-q)+\theta]}{(\theta+\mu q)})^{\gamma}(\frac{1-K}{K})^{\delta} \to \frac{1-p_t}{p_t} < [\frac{1}{\lambda}(\frac{[\mu(1-q)+\theta]}{(\theta+\mu q)})^{\gamma}(\frac{1-K}{K})^{\delta}]^{1/(\delta-\gamma)}$$
$$\to \frac{1-p_t}{p_t} < L_1 \to p_t > \frac{1}{1+L_1}$$

This is the lower bound on price, the upper bound price for contrarianism buy is 0.5 and for herd buy is 1 by definition. The price deviation for lower bound is $|\Delta p_t| = |\frac{\frac{1}{1+L_1}-0.5}{0.5}| = |\frac{2}{1+L_1}-1| = |\frac{1-L_1}{1+L_1}|$. The price deviation for upper bound of contrarianism buy is $\Delta p_t = \frac{0.5-0.5}{0.5} = 0$; for herd buy is $\Delta p_t = \frac{1-0.5}{0.5} = 100\%$.

QED

**Proposition 4**: *Given the signal. If $\delta > \gamma$, the no trade price region is $p_t \in [\frac{1}{1+L_2}, \frac{1}{1+L_1}]$. If $\delta < \gamma$, the no trade price region is $p_t \in [\frac{1}{1+L_1}, \frac{1}{1+L_2}]$.*

*Proof.* To prove this, we show that neither buy or sell creates positive utility for a given signal. Using proposition 3:
If $\gamma > \delta$, for a buy order to occur, $p_t$ has to be smaller than $\frac{1}{1+L_1}$. Thus, it is violated if $p_t \geq \frac{1}{1+L_1}$. For a sell order to occur, $p_t$ has to be greater than $\frac{1}{1+L_2}$. Thus, it is violated if $p_t \leq \frac{1}{1+L_2}$. As such, there is no trade if $p_t \in [\frac{1}{1+L_1}, \frac{1}{1+L_2}]$. If $\gamma < \delta$, for a buy order to occur, $p_t$ has to be greater than $\frac{1}{1+L_1}$. Thus, it is violated if $p_t \leq \frac{1}{1+L_1}$. For a sell order to occur, $p_t$ has to be smaller than $\frac{1}{1+L_2}$. Thus, it is violated if $p_t \geq \frac{1}{1+L_2}$. As such, there is no trade if $p_t \in [\frac{1}{1+L_2}, \frac{1}{1+L_1}]$. QED

**Proposition 5**: *Given loss-averse informed traders ($\lambda > 1$), buy herding and contrarianism cannot occur given a high signal, and sell herding and contrarianism cannot occur given a low signal.*

*Proof.* For buy herding and contrarianism given high signal, $K = 1-q$ by definition. For sell herding and contrarianism given low signal, $K = q$ by definition. Following from lemma 2 we have:
$bEBUB = \frac{[\mu(1-q)+\theta]^\gamma}{(\theta+\mu q)^\gamma}\frac{1-q}{q}min\{1, \frac{(q)^{1+\delta}}{(1-q)^{1+\delta}}\frac{p_t^{\delta-\gamma}}{(1-p_t)^{\delta-\gamma}}\}$
$sEBUB = \frac{[\mu(1-q)+\theta]^\gamma}{(\theta+\mu q)^\gamma}\frac{1-q}{q}min\{1, \frac{(q)^{1+\delta}}{(1-q)^{1+\delta}}\frac{p_t^{\gamma-\delta}}{(1-p_t)^{\gamma-\delta}}\}$
Given the private signal is informative ($q > 0.5 \Rightarrow 1-q < q$). $\theta, \mu, \gamma > 0$ by assumption. We can show (i)$\frac{1-q}{q} < 1$ (ii)$[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^\gamma < 1$. The proof for (ii) is as follow:

$$(1-q) < q \Rightarrow \mu(1-q) < \mu q \Rightarrow \mu(1-q)+\theta < \theta+\mu q \;\Rightarrow \frac{\mu(1-q)+\theta}{\theta+\mu q} < 1 \Rightarrow [\frac{\mu(1-q)+\theta}{\theta+\mu q}]^\gamma < 1$$

Therefore: $(\frac{\mu(1-q)+\theta}{\theta+\mu q})^\gamma(\frac{1-q}{q}) < 1$. $bEBUB = (term < 1) * min[1, ...] < 1$, $sEBUB = (term < 1) * min[1, ...] < 1$. Both bEBUB and sRBUp are smaller than 1, a loss averse trader would violate the conditions.
QED

**Proposition 6**: *Given loss aversion, when $\gamma > \delta$ only contrarian behaviour is possible, when $\delta > \gamma$ only herding is possible.*

*Proof.* For proposition 2, we need to prove that given loss-averse traders. When $\gamma > \delta$ buy herding cannot occur given a low signal and sell herding cannot occur given a high signal. hen $\gamma < \delta$ buy contrarian cannot occur given a low signal and sell contrarian cannot occur given a high signal.We compute the relevant relaxed bias upper bound using theorem 2 and lemma 2.
From proposition 1 proof, $\lambda < [\frac{1-q}{q}]^\delta[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^\gamma < 1$.
Given a low signal, $K = q$. For herd buy $p_t > 0.5$, $\frac{p_t}{1-p_t} > 1$, $(\frac{p_t}{1-p_t})^{(\delta-\gamma)} < 1$ if $\gamma > \delta$. For contrarian

buy $p_t < 0.5$, $\frac{p_t}{1-p_t} < 1$, $(\frac{p_t}{1-p_t})^{(\delta-\gamma)} < 1$ if $\delta > \gamma$. $bEBUB$ are:

$$\begin{aligned} bEBUB &= (\frac{\mu(1-q)+\theta}{\theta+\mu q})^{\gamma}(\frac{q}{1-q})min[1,(\frac{1-q}{q})^{(\delta+1)}(\frac{p_t}{1-p_t})^{(\delta-\gamma)}] \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}],[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-q}{q}]^{\delta}(\frac{p_t}{1-p_t})^{(\delta-\gamma)}\} \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}], term < 1\} < 1 \end{aligned}$$

Given a high signal, $K = 1-q$. For herd sell $p_t < 0.5$, $\frac{p_t}{1-p_t} < 1$, $(\frac{p_t}{1-p_t})^{(\gamma-\delta)} < 1$ if $\gamma > \delta$. For contrarian sell $p_t > 0.5$, $\frac{p_t}{1-p_t} > 1$, $(\frac{p_t}{1-p_t})^{(\gamma-\delta)} < 1$ if $\delta > \gamma$. $sEBUB$ are:

$$\begin{aligned} sEBUB &= (\frac{\mu(1-q)+\theta}{\theta+\mu q})^{\gamma}(\frac{q}{1-q})min[1,(\frac{1-q}{q})^{(\delta+1)}(\frac{p_t}{1-p_t})^{(\gamma-\delta)}] \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}],[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{1-q}{q}]^{\delta}[\frac{p_t}{1-p_t}]^{(\gamma-\delta)}\} \\ &= min\{[\frac{\mu(1-q)+\theta}{\theta+\mu q}]^{\gamma}[\frac{q}{1-q}], term < 1\} < 1 \end{aligned}$$

Therefore, bEBUB sEBUB are not satisfied under those scenarios.

QED

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