Economics

Discussion Paper

Series

EDP-2301

Measuring Vulnerability to Poverty: A Unified Framework

Indranil Dutta, Ajit Mishra

January 2023

Economics

School of Social Sciences

The University of Manchester

Manchester M13 9PL

Measuring Vulnerability to Poverty: A Unified Framework*

Indranil Dutta
University of Manchester, UK

Ajit Mishra
University of Bath, UK

ABSTRACT

In this paper, we provide an unified analytical framework to measure vulnerability to poverty. We conceptualise vulnerability as arising out of different shocks which makes it more intuitive and appealing. Our focus is on understanding impacts of shocks on income, both at the individual and societal level. We will mainly discuss conceptual issues around measuring vulnerability along with suggestions for empirical methods to estimate the level of vulnerability. We combine the literature on poverty measurement and decision making under uncertainty to provide a novel way of understanding and measuring vulnerability to poverty. Specifically, we employ a three step process where we first identify the vulnerable, second we calculate the vulnerability of each individual in the society and finally aggregate the vulnerability across all individuals to estimate the societal vulnerability. Our proposals also provide a broad theoretical framework to assess vulnerability under different information constraints.

Key Words: Poverty, Vulnerability, Uncertainty.
JEL Classification: D80, I32, O12

January, 2023

*We are very grateful to Ceasar Calvo, James Foster, Prasanta Pattanaik, Suman Seth, and Tony Shorrocks for long discussions over the years on issues related to vulnerability measurement which has improved our own understanding. We are also thankful to Jacques Silber and an anonymous referee for comments which has significantly improved the paper.

1 Introduction

Vulnerability means a lot of different things in different contexts – from the impact of climate crisis on countries, to financial stability of banks, to the impact of various shocks on individual lives.[1] In this paper, we examine a specific concept of vulnerability based on how shocks impact income, both at the individual and the societal level. Our interest here is in understanding vulnerability arising from shocks that push people into poverty. At the heart of the paper is a focus on conceptual issues around identifying individuals who are vulnerable to poverty, measuring their level of vulnerability and the overall societal vulnerability to poverty. Recent evidence of an increase in poverty from the Covid-19 shock demonstrates that, despite rapid strides in reducing poverty, a significant proportion of the population in many countries remains vulnerable to becoming poor (World Bank 2020). For instance, in Bangladesh, Sen et al. (2020) estimate that, around 25 million people would fall into poverty due to the pandemic shock.

Among the different approaches to conceptualize vulnerability is the macro-approach, such as the Economic Vulnerability Index (EVI), which considers the various dimensions over which an economy might be impacted by shocks. For example, a country's trade openness, export concentration and dependence on strategic imports are all important dimensions of the index (Bruguglio et al. 2009). A country's overall vulnerability would then be an aggregation of vulnerability over the different dimensions (Assa and Meddeb 2021). Methodologically, this approach is similar to the United Nations Development Programme Human Development Index (HDI), where a synthetic index is created by aggregating information across different dimensions (UNDP 2010).

In contrast, in this paper we will focus mainly on the micro-approach, whose foundations lie in welfare economics based on individuals. Instead of centring vulnerability on the different dimensions of the economy, we estimate societal vulnerability based on the vulnerability levels of individuals. The advantage of the micro-approach is that in addition to ranking different regions based on vulnerability,we can also rank various population subgroups (such as those based on gender, race, ethnicity, etc.) according to their level of vulnerability. The micro-approach makes the measurement of vulnerability centred on people. Several recent surveys on vulnerability discuss this approach (see Calvo 2018; Ceriani 2018; Gallardo 2018).

A defining characteristic of vulnerability that we examine is susceptibility to falling into

[1] For a rich discussion of factors impacting general vulnerability of a country see the report from the Commonwealth (2021) and Flanagan et al. (2011).

poverty in the future. Thus, vulnerability typically is an ex ante notion of poverty. This distinguishes vulnerability from other similar concepts, such as income poverty or chronic poverty. We are interested in how individuals and societies fare in terms of poverty in the future, which may be one or multiple time periodsahead. In estimating vulnerability to poverty, we will need to estimate future income distributions. Note that, although in this paper we focus on vulnerability to income poverty, conceptually we can formulate a broader notion of vulnerability based on a multi-dimensional concept of poverty. Vulnerability would then involve estimating the future joint distribution across the various indicators over which we are measuring multi-dimensional poverty.

In this paper we extend a framework proposed by Dutta et al. (2011) and Dutta et al.(2019) for measuring vulnerability which brings together two well-established but distinct literatures – measuring poverty (Sen 1976; Zheng 1997) and decision making under uncertainty (Gilboa 2009). Our framework adapts the steps used for measuring poverty to measure vulnerability. Specifically, we employ a three-step process: first, we identify the vulnerable; second, we calculate the vulnerability of each individual in the society; and, finally, we aggregate the vulnerability across all individuals to estimate the societal vulnerability. Based on the literature on decision making under uncertainty, in our framework we capture uncertainty over future income as a lottery. Thus, in the future there are associated probabilities with different possible incomes. This allows us to conceptualize vulnerability as arising out of different shocks. It makes the measure of vulnerability in our proposed framework intuitive and appealing. For instance, an individual can fall into poverty due to shocks which can be idiosyncratic (such as sudden illness or death, or loss of employment), systemic (such as natural shocks such as droughts or floods), or economy-wide shocks arising from trade policies. An increasing number of empirical papers – such as Heltberg and Lund (2009), Gloede et al. (2015), Haq (2015), Knippenberg and Hoddinott (2017), and Erman et al. (2018) – have taken this discrete shock-based approach to measure vulnerability.

The remainder of the paper is organised as follows. In the next section, we discuss in detail the three steps that can be used in measuring vulnerability of a society, and examine the rules that can be used to identify individuals. Then we proceed to explore the various ways in which individual vulnerability can be measured, and discuss some general rules of aggregation to arrive at the societal level of vulnerability. In the third section we provide a general discussion of our method along with a numerical example, while the last section concludes the paper.

2 Measuring Vulnerability

In measuring vulnerability based on the micro-approach, we combine two frameworks – choice under uncertainty and poverty measurement. As suggested by Sen (1981, Chapter 3), in measuring poverty, first one has to identify the individuals who are poor and then aggregate individual poverty to arrive at societal poverty. Without first establishing whether or not an individual is poor, measuring their level of poverty does not make much sense. In the same vein, we can argue that we need to first identify whether or not an individual is vulnerable before we measure their level of vulnerability. Thus, we will have three steps in measuring vulnerability: (i) identifying the vulnerable; (ii) measuring the level of vulnerability of those identified as vulnerable; and (iii) aggregating individual vulnerability to measure societal vulnerability. We discuss each of these steps in detail in this section, but first we present the basic notation.

2.1 Basic Notation

Each individual i, has a possibility of earning a finite set of income in the future $\{y_1^i, y_2^i, ..., y_m^i\}$ with associated probability $\{p_1^i, p_2^i, ..., p_m^i\}$. In the present we will consider each of these income to be net income – i.e. the income received after all taxes and transfers are made. These different income levels can be associated with m different states of nature, which arise from various shocks that individuals may face. We can bring both the income and probability together in a lottery $L^i = (p_1^i, y_1^i; p_2^i, y_2^i; ...; p_m^i, y_m^i)$, where $m \geq 2$ and $\forall s$, $y_s^i \geq 0$, $p_s^i \geq 0$, and ${}^{m}_{s=1} p_s^i = 1$.[2] An individual's future income then can be thought of as a lottery where different incomes can happen with the associated probabilities. We implicitly assume that no two states have the same income. Which income the individual finally earns in the future will be based on the state of nature revealed in the future period. We shall drop the superscript i, from the lotteries and the associated probabilities and deprivations when dealing with an individual. The probability vector associated with lottery L is represented as $\mathbf{p}^L = \{p_1, p_2, ..., p_m\} \in P$ where P is the set of all probability distributions over the m states. Let be the set of all such lotteries an individual may face such as $L = (p_1, y_1; p_2, y_2; ...; p_m, y_m)$, and $\widetilde{L} = (\widetilde{p}_1, y_1; \widetilde{p}_2, y_2; ...; \widetilde{p}_m, y_m)$. Let $z \in \mathbb{R}_{++}$ be the poverty line in the future, income below which individuals would be considered poor. Note that we have assumed the poverty line z to be fixed since we are interested in the shortfall of income in the future period.

[2] In our context, suppose an individual i earns y^{is} in state s. Then deprivation in that state is given by $d^{is} = (z - y^{is})/z$, if $y^{is} \leq z$, otherwise $d^{is} = 0$, where z is the future poverty line.

2.2 Identification of the vulnerable

Before measuring an individual's vulnerability we need to identify whether the individual is vulnerable or not. As in the case of poverty, identification is an important step. Even for extremely rich individuals (millionaires), the possibility exists, however small, that in the future their income could be very low. For instance, fall in income might be due to unemployment from economic shock (deep recession), closure of business due to a pandemic or loss of income from a health shock. Under any of these factors, suppose the rich individual of today has the possibility of becoming poor tomorrow. Should we include such individuals in our vulnerability calculations? In other words, should we identify individuals who are extremely rich but have a very low probability of becoming poor in the future as vulnerable? In such circumstances it may not be unreasonable to identify those rich individuals as not vulnerable to poverty. To identify the vulnerable, recent studies such as Dang and Lanjouw (2017), Chakravarty et al. (2016) and de la Fuente et al. (2015) have considered a vulnerability line, which is similar in concept to a poverty line. However, this approach operationalises the vulnerability line using ex post data, where we have to know the future income to set the vulnerability line, and thus ignores the ex ante nature of vulnerability.

In this section, we rely on Dutta et al.(2019) to formulate identification rules for vulnerability based on the level of information about a person's future income available in the current period. For $z \in \mathbb{R}_{++}$ and any $L \in$, an identification function $\rho : \mathbb{R}_{++} \times \longrightarrow [0, 1]$, an help identify whether an individual is vulnerable or not, subject to a threshold level $3b8 \in [0, 1]$, which is similar to a cut-off point. Thus, if $\rho(z, L) \geq \theta$, $\theta \in [0, 1]$, the individual is identified as vulnerable, otherwise the individual is not vulnerable. Similar to the literature on multi-dimensional poverty, we can have situations where everyone is considered vulnerable, or only those who are definitely If $\theta = 0$, every individual is deemed vulnerable. We refer to this as the *universal* rule. On the other hand, when $\theta = 1$, we have the *intersection* approach where an individual is deemed vulnerable if he is poor in all future states.

In devising identification rules we must be mindful of the different information sets on which such rules can be based. For instance, we may just want to take into account: (a) the probability of falling into poverty in the future;(b) both the probability and the depth of poverty in the future; or (c) just the number of shocks in the future where the individual may fall into poverty. Each of these different information sets results in different identification rules. Associated with any lottery L we create a $(1 \times m)$ deprivation identification vector

$\mathbf{r}^L = \{r_1, r_2, ..., r_m\}$ based on the following rule

$$\forall s,\ r_s^L = \begin{cases} 1 & \text{if } y_s < z \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

for all $s = 1, ..., m$. The probability identification function (*P-rule)* is based on the total probability of falling into poverty in the future. We define the *P-rule* identification as:$\rho^P(z, L) = \mathbf{p}^L \bullet \mathbf{r}^L$. Thus $\rho^P(z, L) \geq \rho^P(z, L')$ if and only if $\mathbf{p}^L \bullet \mathbf{r}^L \geq \mathbf{p}^{L'} \bullet \mathbf{r}^{L'}$. Note that under this rule, whether a person falls a little or a lot below the poverty does not really matter. The *E-rule* identification function is based on both the probability and the depth of poverty in the future. The *E-rule* is defined as $\rho^P(z, L) = {}_{s=1}^{k} p_s(z - y_s/z)$, where $k \leq m$ is the number of states where $z \geq y_s$. On the other hand the counting rule (*C-rule)* identification can focus just on the number of states where the individual is going to be deprived. Hence, the *C-rule* is defined as $\rho^C(z, L) = \left\|\mathbf{r}^L\right\|$, which is the cardinality of the deprivation identification vector for lottery L. For the *C-rule*, we do not include any information of the probabilities or the depth of the shortfall, just the total number of future states in which the individual may fall below poverty.

These identification rules, which differentiates whether an individual is vulnerable or not, is very different conceptually from the next step which is about measuring an individual's level of vulnerability. The identification rule need not consider all the information that it available in the same way that we may do when measuring how vulnerable an individual is. For instance, in the *P-rule*, the depth of poverty is not being considered. It may be that for identification purposes, one would like to cast a wider net and therefore consider all those who are slightly below the poverty line along with those who are significantly below the poverty line. Hence, the depth of poverty in the different future states do no matter. In a similar manner various intuitions can be used to justify different information sets being used for identification purposes and for measuring individual vulnerability.

The difference in the information sets used can lead to inconsistencies between those who are identified as vulnerable and their level of vulnerability. For the *P-rule*, for instance, since we are not considering the depth of poverty, there is a possibility that those who are not identified as vulnerable can register a higher level of vulnerability when depth is taken into account, relative to those who are identified as vulnerable. One can argue that such inconsistencies arise due to the distinct nature of the two processes – identification and measuring individual vulnerability – and therefore is to be expected. On the other hand, if we do not want any inconsistencies to arise, then there should be no difference between the information sets and the functional forms used for identification and for measuring vulnerability. In such case, the difference between identification of who is vulnerable and what their level of vulnerability is, are not two distinct questions.

2.3 Individual Vulnerability

Once we identify the individual as vulnerable, i.e. those with $\rho(z, L) \geq \theta$, we then use the lottery the individual faces to come up with the level of vulnerability an individual faces. Thus for any individual i, vulnerability is measured by $V : [0, 1] \times \mathbb{R}_{++} \times \longrightarrow \mathbb{R}_{+}$, where $[0, 1]$ and $\mathbb{R}_{++}$ is the range of the identification function and the poverty line z, respectively. Finally, if $\rho(z, L) < \theta$, then $V(z, L) = 0$.[3]

2.3.1 The Expected Poverty Approach

A common method of measuring individual vulnerability is taking the expected deprivation for the lottery the individual faces. In particular, we can take the expected value of the poverty that individuals face in different future states. This is a natural extension of how lotteries are assessed under the von Neumann-Morgenstern framework of choice under uncertainty. Let the poverty in any future state s, of a lottery L, be represented as $d(z, y_s)$. The poverty function $d(z, y_s) \geq 0$ if $z \geq y_s$, otherwise $d(z, y_s) = 0$. Note that the poverty function d is state independent. The level of vulnerability represented by lottery L is given by

$$V(L) = \begin{cases} \sum_{s=1}^{m} p_s d(z, y_s) & \text{if } \rho(z, L) \geq \theta \\ 0 & \text{otherwise} \end{cases} . \tag{2}$$

The expected poverty is calculated only for those individuals who are identified as vulnerable. Given the functional form of the vulnerability measure, any restrictions on $V(L)$ will also impose similar restrictions on the poverty function $d(z, y_s)$. For instance, if $V(L) \in [0, 1]$ then (2) implies that for all s, $d(z, y_s) \in [0, 1]$.

Depending upon the functional form of the poverty function $d(z, y_s)$, we can have different

[3]While we distinguish between the identification function $\rho(L)$ and the individual vulnerability measure $V(L)$, our framework allows for the possibility that $\rho(L)$ and $V(L)$ are the same. This is common in empirical applications.

measures of vulnerability. Many empirical studies have used the Foster, Greer and Thorbecke (1984) (FGT) functional form for $d(z, y_s) = (z - y_s/z)^2$. The general class of FGT vulnerability measure then would be given by

$$V(L) = \begin{cases} \sum\limits_{s=1}^{m} p_s \left(\frac{z-y_s}{z}\right)^{\alpha} & \text{if } \rho(z, L) \geq \theta,\ \alpha \geq 0 \\ 0 & \text{otherwise} \end{cases}. \tag{3}$$

If $\alpha = 0$, the vulnerability measure of the individual just considers the probability of falling into poverty in the future. It does not take into account the depth of poverty. For $\alpha = 1$, the vulnerability measure is the expected poverty gap and for $\alpha = 2$, it it the expected squared poverty gap.

Calvo and Dercon (2013) has considered the Chakravarty (1983) poverty measure for their vulnerability measure where $d(z, y_s) = 1 - (y_s/z)^{\beta}$. The Chakravarty vulnerability measure then becomes

$$V(L) = \begin{cases} \sum\limits_{s=1}^{m} p_s (1 - (\frac{y_s}{z})^{\beta}) & \text{if } \rho(z, L) \geq \theta,\ \beta \geq 1 \\ 0 & \text{otherwise} \end{cases}. \tag{4}$$

If $\beta = 1$, we are back to the case of the expected poverty gap.

We can similarly extend the vulnerability measures to the Watts poverty index (Zheng 1993)

$$V(L) = \begin{cases} \sum\limits_{s=1}^{m} p_s (\ln z - \ln y_s) & \text{if } \rho(z, L) \geq \theta \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

The expected poverty approach, subject to the identification, includes all who are definitely going to be poor in the future, along with those who have a possibility of fall into poverty in the future. Thus, the set of vulnerable individuals includes a broader set of people compared to just the poor.

2.3.2 Properties of Vulnerability Measures.

There are several properties of individual vulnerability measures that are quite similar to the axioms for standard poverty measures. In this part, we shall discuss some of the more common properties.

If an individual is not identified as vulnerable, then his/her level of vulnerability should not be positive. This is in a similar vein to the focus axiom of poverty measures where we are interested only in those individuals whose income is below the poverty line (Sen, 1976). For any individual with lottery L, and a given threshold θ, if the identification rule $\rho(L) < \theta$, the individual is not identified as vulnerable and hence $V(z, L) = 0$.

Axiom 1 *Focus (A1): Given z and $L = (p_1, y_1; p_2, y_2; ...; p_m, y_m)$ such that $\rho(z, L) < \theta$. Then $V(z, L) = 0$.*

Suppose an individual faces a lottery $L = (0.2, 10; 0.1, 30; 0.7, 100)$. Let $z = 50$. The deprivation vector based on (1) is $\mathbf{r}^L = \{1, 1, 0\}$. If we consider ρ^P (*P-rule*) as the identification rule, and $\theta = 0.4$ as the threshold, then given $\rho^P(z, L) = 0.3$, the individual is not identified as vulnerable. Hence, according to the focus axiom, the individual's vulnerability $V(z, L) = 0$. The value of θ is exogenously determined and may vary depending on the context. Note that this sets the lower bound for our vulnerability measure as zero.

The intuition for our next axiom has similarity with the monotonicity property under choice under uncertainty literature, where an increase in the probability of the 'good' states relative to the 'bad' states leads (Luce and Raiffa, 1957, p. 29). In the present context, it implies that vulnerability should increase when the probability of income which has higher shortfall increases.

Axiom 2 *Monotonicity (A2): Consider two lotteries $L = (p_1, y_1; ..., p_k, y_k; ...; p_{k'}, y_{k'}; ...; p_m, y_m)$ and $L' = (p_1, y_1; ...; p_k + \delta, y_k; ...; p_{k'} - \delta, y_{k'}; ...; p_m, y_m)$, $\delta > 0$ such that $p_k > 0$; $p_{k'} > \delta$, $z > y_{k'} \geq y_k \geq 0$. Then $V(z, L) \leq V(z, L')$.*

This is different from the traditional monotonicity axiom in the poverty literature in that we are not changing the future income profile, rather we are changing the probability distribution over the future income. We can, however, achieve very similar results by changing the probability distribution. Consider the following two lotteries: $L_1 = (0.2, 10; 0.1, 30; 0.7, 100)$ and $L_2 = (0.2, 50; 0.1, 30; 0.7, 100)$. In first glance, it seems that the probability distribution across the states (or the income profile) has remained the same but we are increasing the income in the first state from 10 to 50 as we move from L^1 to L^2. However, in our framework we accommodate this through a slightly different income profile of $(10, 50, 30, 100)$ with $L_1 = (0.2, 10; 0, 50; 0.1, 30; 0.7, 100)$ and $L_2 = (0, 10; 0.2, 50; 0.1, 30; 0.7, 100)$. Thus in L_1 the state with income 50 has zero probability and in L_2 the state with income 10 has zero probability.

If we are interested in the expected poverty structure, then we need some decomposability between the different states of nature. For this property, we use the notion of convex combination of two lotteries which we define as follows:

Definition 1 *Suppose* $L = (p_1, y_1; p_2, y_2; ...; p_m, y_m)$ *and* $L^1 = (p'_1, y_1; p'_2, y_2; ...; p'_m, y_m)$. *Then a compound lottery* $\lambda L + (1-\lambda)L' = (\lambda p_1 + (1-\lambda)p'_1, y_1; \lambda p_2 + (1-\lambda)p'_2, y_2; ...; \lambda p_m + (1-\lambda)p'_m, y_m)$, *where* $0 < \lambda < 1$.

Our property of decomposability in the context of individual vulnerability is similar to Dutta, Foster and Mishra (2011). If an individual has lottery L with probability λ or lottery L' with probability $(1-\lambda)$, then the vulnerability from this compound lottery is the same as the convex combination of the vulnerability from two simple lotteries of L and L' . Hence, the vulnerability of a convex combination of lotteries should be the same as the convex combination of the vulnerability of each of the lotteries.[4]

Axiom 3 *Decomposability (A3): Consider any two deprivation lotteries* L *and* L' *such that* $V(z, L) > 0$ *and* $V(z, L') > 0$. *Then* $V(z, \lambda L + (1-\lambda)L') = \lambda V(z, L) + (1-\lambda)V(z, L')$.

The implication of this axiom would be to make the vulnerability measure linear in probabilities and allows us to decompose individual vulnerability into the vulnerability arising from each of the different shocks.[5]

Our final property is a normalization rule, which has become the norm in most of the poverty literature. We assume that if individuals are certain to have no income in the future, then that should reflect the highest vulnerability. Formally, the property is as follows

Axiom 4 *Normalization (A4): Consider lottery* $L = (p_1, y_1; ...; p_m, y_m)$, *such that for any* s, $y_s = 0$ *if* $p_s > 0$. *Then, for a given* z, $V(z, L) = 1$.

This provides an upper bound to our individual vulnerability measure. Along with Axiom 1, this property ensures that the vulnerability measure $V(z, L) \in [0, 1]$.

In all of these four properties we have discussed the impact on vulnerability measure $V(z, L)$ of changing lottery$L$, while holding z constant. We can also extend these properties for the case where we change z, holding L constant. For instance, we can claim for a given lottery, if we increase z, then vulnerability $V(z, L)$ should not decrease. This is akin to the monotonicity property for $V(z, L)$ that we discussed above but based on z.

[4]This axiom can be derived from more fundamental axioms (Gilboa, 2009, Chapter 8).

[5]The vulnerability from each shock is essentially the expected deprivation arising from each state of nature. This result is shown in Dutta, Foster and Mishra (2011, Lemma 3).

2.3.3 Extension: The Reference Dependent Approach

One way to distinguish the concept of vulnerability from being just a broader set of the poor is to measure the future shortfall from a hybrid poverty line (Dutta et al. 2011).[6] Under this approach, someone who is certain to be poor in the future is not necessarily vulnerable if their future income is higher than the hybrid poverty line. Thus, it creates a notion of vulnerability which is distinct from the idea of expected poverty. Empirical results such as Celidoni (2013), and Vo (2018) find that the vulnerability measure proposed in Dutta et al. (2011) is better at predicting who are going to fall into poverty in the future.

The hybrid poverty line under Dutta et al. (2011) is given by $z^h = z^{\alpha}c^{1-\alpha}$, $\alpha \geq 0$, where c is the current income and z is the standard poverty line. However, c need not be just the current income, it can be any parameter which reflects the individuals standard of living. For instance, it can be permanent income, based on multiple periods. Depending on the value of α, the hybrid poverty line takes different values. When $\alpha = 1$, the hybrid poverty line is the standard poverty line z, and when $\alpha = 0$, it is just the standard of living (or their current income) of individuals. Note that under $\alpha > 0$, each individual will have a different hybrid poverty line.

Reference dependent vulnerability measures in the FGT framework can be represented as:

$$V(L) = \begin{cases} \sum_{s=1}^{m} p_s \left(\frac{z^h - y_s}{z^h}\right)^{\alpha} & \text{if } \rho(z, L) \geq \theta,\ \alpha \geq 0 \\ 0 & \text{otherwise} \end{cases} . \tag{6}$$

The main difference with the expected poverty approach is that, instead of a fixed poverty line, we now have a reference dependent poverty line. Thus, the information requirement for reference dependent vulnerability measures is slightly more relative to the expected poverty approach. However, for most surveys the information is easily available. Note that, while we have left the identification function as earlier with a standard poverty line, a hybrid poverty line can also be used to identify the vulnerable. There is no compelling reason to decide on one way or the other since identification and measuring individual vulnerability levels are distinct processes. While we have presented a vulnerability measure based on the FGT framework, we can also have similar reference dependent vulnerability measure based on the Chakravarty or the Watts index. For a more general discussion of reference-dependent vulnerability measures that explicitly build on concepts of risk-aversion, see Günther and Maier (2014).

[6] The hybrid poverty line was first proposed by Foster (1998).

2.4 Societal Vulnerability

Our final step in measuring vulnerability is step 3, where we aggregate the individual vulnerability levels that we calculated in step 2, to form the over all societal vulnerability.

For a society with n individuals and m future states of the world, we have a $n \times m$ vulnerability matrix which we denote as $M^{n,m}$. Each cell of the matrix shows the probability and deprivation associated with an individual in that state. Thus each row of the matrix lists the probability and deprivation of one individual over the different states which is effectively the lottery that the individual faces. While a societal vulnerability measure based on the whole matrix $M^{n,m}$ is possible, in this paper we take the view that first we calculate the vulnerability measure for the individual and based on that we calculate the societal vulnerability. Thus, the societal vulnerability is a function $V^S : \mathbb{R}^n_+ \longrightarrow \mathbb{R}_+$.

A generalized societal vulnerability measure can be written as

$$V^S = \sum_{i=1}^{n} \omega_i V^i(z, L^i), \tag{7}$$

where ω_i is the weight society associates with the vulnerability of individual i and ${}^{n}_{i=1}\omega_i = 1$. In (7) the societal vulnerability is an weighted average of individual vulnerability measures. Thus, we have implicitly assumed that societal vulnerability measure is additively separable in individual vulnerability measures. This property can be derived from more fundamental axioms similar to subgroup consistency in poverty measurement literature (see Foster and Shorrocks, 1991). It allow us to decompose the societal vulnerability into vulnerability of subgroups, which may be based on gender, or region. Note that in this general case, we allow the vulnerability function of each individual to differ. With additional restrictions, such as anonymity, we can easily show that societal vulnerability measure is a simple average of individual measures.

3 Discussion and Example

The dominant method in estimating vulnerability empirically assumes that an individual's income follow a log normal distribution (see for instance, Ward 2016, Imai et al. 2011, Jha and Dang 2010, Zhang and Wan 2009, Christiaensen and Subbarao 2005, Chaudhuri 2003). Based on a set of characteristics which impact income, the mean and the variance are estimated and from there, given a poverty line z, the expected shortfall is calibrated for each individual. Those with vulnerability above a minimum threshold (typically 0.5) are identified as vulnerable. The identification rule (or function) under this method is the same as the individual vulnerability function. The societal vulnerability is a simple average of the vulnerability of all those who are identified as vulnerable.

In this chapter, we suggest several different improvements to the dominant method. Applications should follow a path similar to the existing literature on poverty measurement, where identification is an important part and is typically carried out before measuring individual vulnerability. Once the individuals who are vulnerable are identified, we then measure their level of vulnerability. It is important to note that the identification function and the individual vulnerability function could be different. The identification method can be generalized and different cut-off values should be used. Finally we also suggest that applications could think of measuring vulnerability based on different shocks, rather than assuming a continuous distribution of income.

We provide a simple application of the methods that we discussed in this chapter. Consider the following vulnerability matrix with $n = 3$ and $m = 3$:

$$M^{3,3} = \begin{pmatrix} \text{State 1} & \text{State 2} & \text{State 3} \\ 0.1{,}90 & 0.8{,}200 & 0.1{,}80 \\ 0.2{,}400 & 0.1{,}150 & 0.7,\ 20 \\ 0.5{,}60 & 0.25{,}40 & 0.25{,}20 \end{pmatrix}$$

where in each cell the first number is the probability and the second is the income earned by an individual. Let the poverty line $z = 100$. Thus the lotteries faced by each of the individuals are, $L^1 = (0.1, 90; 0.8, 200; 0.1, 80)$, $L^2 = (0.2, 400; 0.1, 150; 0.7, 20)$, and $L^3 = (0.5, 60; 0.25, 40; 0.25, 20)$. Thus $\mathbf{r}^1 = \{1, 0, 1\}$, $\mathbf{r}^2 = \{0, 0, 1\}$, and $\mathbf{r}^3 = \{1, 1, 1\}$.

First, let us begin with Step 1. Let the cut-off value for identification be $\theta = 0.5$. $\mathbf{p}^{L^1} = \{0.1, 0.8, 0.1\}$ and then using *P-rule* identification strategy we find $\mathbf{p}^{L^1} \bullet \mathbf{r}^1 = 0.2 < 0.5$. Hence individual 1 is not considered vulnerable. On the other hand $\mathbf{p}^{L^2} = \{0.2, 0.1, 0.7\}$ and $\mathbf{p}^{L^2} \bullet \mathbf{r}^2 = 0.7 > 0.5$ which implies that individual 2 is vulnerable. Similarly for the third individual, $\mathbf{p}^{L^3} = \{0.5, 0.25, 0.25\}$ and $\mathbf{p}^{L^3} \bullet \mathbf{r}^3 = 1 > 0.5$ which indicates that individual 3 is also vulnerable.

In Step 2, we assess individual vulnerability using the expected FGT poverty index (3) where $\alpha = 1$; $V(L^1) = 0$, $V(L^2) = 0.56$ and $V(L^3) = 0.55$.

In Step 3, for the overall societal vulnerability we take a simple average of individual vulnerability levels. Thus, $V^S(M^{3,3}) = 1/3(V(L^1) + V(L^2) + V(L^3)) = 0.37$.

In this example, although individual 2 is poor only in one state compared to individual 1 (who is poor in two states), we identify individual 2 as vulnerable instead of individual 1. This is because the probability associated with the state in which individual 2 is poor is far higher than the combined probability of the states in which individual 1 is poor. Thus individual 2 is more likely to be in poverty in the future than individual 1. While we have used the *P-rule* for this application, one can use other identification rules to identify vulnerable individuals.

4 Conclusion

In this paper, we suggested various improvements to the dominant method for empirical estimations of vulnerability. Applications should follow a path similar to the existing literature on poverty measurement, where identification is an important part and is typically carried out before measuring individual vulnerability. Once the individuals who are vulnerable are identified, we then measure their level of vulnerability. It is important to note that the identification function and the individual vulnerability function could be different. The identification method can be generalized and different cut-off values should be used. Finally, we also suggested that applications could consider measuring vulnerability based on different shocks, rather than assuming a continuous distribution of income.

It should not be too daunting to undertake a shock-based approach to measuring vulnerability. Many survey data sets now ask respondents about the different shocks they faced in the last 3–5 years. Included with that information is the typical loss of income associated with the shocks and the frequency of the shocks. All this information can be used to identify vulnerable individuals and measure their level of vulnerability to poverty. One approach would be to use a frequency-based method to calculate the probabilities of the shocks and use the average income losses associated with each shock. However, this information is based on past events. Individuals' exposure to the shocks and their adaptive capability to the shocks might have changed in the interim. In this case, how best to estimate the probabilities of the various shocks and the associated losses remain open areas of research.

This paper has mainly focused on tying together the different parts of the vulnerability measurement literature. We suggested a framework based on the existing literature on poverty.

In particular, we suggest that vulnerability measures should follow three steps: identify who the vulnerable are; measure their level of vulnerability; and aggregate individual vulnerability to get societal vulnerability. Each of these steps is distinct but interrelated. Hence, there is no requirement for the functional forms in the different steps to be the same. Based on the decision making under uncertainty literature we have framed future uncertainty in income in terms of a lottery. One aspect of such a framing is to think of the different future incomes as arising from various shocks to income. Conceptualising measurement of vulnerability as arising out of shocks to future income also appeals to our intuitive sense of what vulnerability means, and is thus useful for policy (O'Brien et al. 2018).

The identification functions employed here use different data sets, which can be quite handy, particularly when it comes to targeting the vulnerable since information at the individual or household level may be difficult to collect and not easily accessible. In many situations where information is limited, there will still be some method of identifying the vulnerable. For instance, absent information on exact income levels under various shocks in the future, we could sapply the P-rule to identify the vulnerable, if we have information on whether the individual is in poverty or not for each of the future states and the probabilities associated with those states. This is different from existing empirical approaches where full information on both probability and associated income levels is required before individuals can be identified as vulnerable. We have also suggested how different poverty measures can be adapted to measure vulnerability, and discussed how hybrid poverty lines can be incorporated in the measurement of vulnerability. This paper provides a framework on how these different measures can be linked in an integrated approach to measuring vulnerability. It is only through such systematic measurement that effective policies to reduce vulnerability can be evaluated.

References

Assa, J., Meddeb, R. (2021): Towards a Multidimensional Vulnerability Index, Discussion Paper, UNDP.

Bruguglio, L., Cordina, G., Farrugia, N., Vella, S. (2009): Economic Vulnerability and Resilience: Concepts and Measurements, Oxford Development Studies, 37:3, 229-247,

Calvo, C. (2018): Vulnerability to Poverty: Theoretical Approaches, in D'Ambrosio, C., (Ed). Handbook of Research on Economic and Social Wellbeing, Edward Elgar, Chapter 11.

Calvo, C., Dercon, S. (2013): Vulnerability to Individual and Aggregate Poverty, Social Choice and Welfare, 41, 721-740

Celidoni, M. (2013): Vulnerability to Poverty: An Empirical Comparison of Alternative Measures, Applied Economics, 45, 1493-1506.

Chakravarty, S. (1983): A New Index of Poverty, Mathematical Social Sciences, 6, 307-313

Chakravarty, S. (2016): Measuring the Impact of Vulnerability on the Number of Poor: A New Methodology with Empirical Illustrations, Asian Development Bank Institute Working Paper Series, 612

Chaudhuri, S.(2003): Assessing Vulnerability to Poverty: Concepts, Empirical Methods and Illustrative Examples, Working Paper, Columbia University

Christiaensen, L., Subbarao, K. (2005): Towards an Understanding of Household Vulnerability in Rural Kenya, J. of African Economies, 14, 520-558.

Commonwealth (2021): The Commonwealth Universal Vulnerability Index, Report by the Commonwealth Secretariat.

Dang, H., Lanjouw, P. (2017): Welfare Dynamics Measurement: Two Definitions of a Vulnerability Line and Their Empirical Application. Rev. of Income and Wealth, 63, 633-660

De La Fuente, A., Ortiz-Juárez, E., Rodríguez-Castelán, C. (2015): Living on the Edge: Vulnerability to Poverty and Public Transfers in Mexico, World Bank Policy Research Working Paper 7165.

Dutta, I., Foster, J., Mishra, A. (2011): On Measuring Vulnerability to Poverty, Social Choice and Welfare, 37, 729-741.

Dutta, I., J. Foster, A. Mishra and Shasi Nandeibam, 2019. 'Identifying the Vulnerable: Concepts and Measurement', Economics Discussion Paper Series 1915, Economics, The University of Manchester, revised Apr 2022.

Erman, A., Motte, E., Goyal, R., Asare, A., Takamatsu, S., Chen, X., Malgioglio, S., Skinner, A., Yoshida, N., Hallegatte, S.(2018): The Road to Recovery The Role of Poverty in the Exposure, Vulnerability and Resilience to Floods in Accra, Policy Research Working Paper, World Bank, WPS 8469,

Flanagan, B. E.; Gregory, E. W.; Hallisey, E. J.; Heitgerd, J. L., Lewis, B. (2011): A Social Vulnerability Index for Disaster Management, Journal of Homeland Security and Emergency Management: Vol. 8: Iss. 1, Article 3.

Foster, J., Greer, J., E. Thorbecke, E. (1984): A Class of Decomposable Poverty Measures, Econometrica, 52, 761-766.

Foster, J., Shorrocks, A. (1991): Subgroup Consistent Poverty Indices, Econometrica, 59, 687-709

Foster, J. (1998): Absolute versus Relative Poverty, American Economic Review, 1998, 88, 335-341.

Gallardo, M.(2017): Identifying Vulnerability to Poverty: A Critical Survey, Journal of Economic Surveys, https://doi.org/10.1111/joes.12216,

Gilboa, I.(2009): Theory of Decision under Uncertainty, Econometric Society Monographs, Cambridge University Press.

Gloede, O., Menkoff, L., Waibel, H. (2015): Shocks, Individual Risk Attitude, and Vulnerability to Poverty Among Rural Households in Thailand and Vietnam, World Development, 71, 54-78;

Haq, R. (2015): Quantifying Vulnerability to Poverty in a Developing Economy, Pakistan Dev. Rev., 54, 915-929.

Heltberg, R., Lund, N (2009): Shocks, Coping, and Outcomes for Pakistan's Poor: Health Risks Predominate, J. of Dev. Stud., 45, 889-910.

Hohberg, M., Landau, K., Kneib, T., Klasen, S., Zucchini, W.: (2018) Vulnerability to Poverty Revisited: Flexible Modeling and Better Predictive Performance, Journal of Economic Inequality, https://doi.org/10.1007/s10888-017-9374-6.

Imai, K., Gaiha, R., Kang, W (2011).: Vulnerability and Poverty Dynamics in Vietnam, Applied Econ., 43, 3603-3618.

Jha, R., Wang, T. (2010): Vulnerability to Poverty in Papua New Guinea in 1996, Asian Econ. J., 24, 235-251

Knippenberg, E., Hoddinott, J. (2017): Shocks, Social Protection and Resilience: Evidence from Ethiopia, ESSP Working Paper, International Food Policy Research Institute (IF-PRI)

Luce, D., Raiffa, H. (1957): Games and Decisions: Introduction and Critical Survey, New York, Wiley.

O'Brien, C., Holmes R., Scott, Z., Barca, V. (2018) Shock-Responsive Social Protection Systems Toolkit: Appraising the Use of Social Protection in Addressing Largescale Shocks, Oxford Policy Management, Oxford, UK.

Sen, A.(1976): Poverty: An Ordinal Approach to Measurement, Econometrica, 44, 219-231, (1976).

Sen, A.(1981): Poverty and Famines: An Essay on Entitlement and Deprivation, Clarendon Press, Oxford.

Sen, B., Ali, Z., Murshed, M. (2020): Poverty in the Time of Corona: Trends, Drivers, Vulnerability and Policy Responses in Bangladesh, Bangladesh Institute of Development Studies.

Ward, P.(2016): Transient Poverty, Poverty Dynamics, and Vulnerability to Poverty: An Empirical Analysis using a Balanced Panel from Rural China, World Development, 78, 541-553.

World Bank. 2020. Poverty and Shared Prosperity 2020: Reversals of Fortune. Washington, DC: World Bank. Doi: 10.1596/978-1-4648-1602-4.

Vo, T. T. (2018): Household Vulnerability as Expected Poverty in Viet Nam, World Dev. Perspectives, 10-12, 1-14.

Zhang, Y., Wan, G. (2009): Can we Predict Vulnerability to Poverty? Oxford Dev. Stud., 37, 277-287

Zheng, B. (1993): An Axiomatic Characterization of the Watts Poverty Index, Economics Letters, 42, 81-86,

Zheng, B. (1997), Aggregate Poverty Measures. Journal of Economic Surveys, 11: 123-162.