The University of Manchester

Economics

Discussion Paper

Series

EDP-2202

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April 2022

Economics

School of Social Sciences

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EVOLUTIONARY BEHAVIOURAL FINANCE: A MODEL WITH ENDOGENOUS ASSET PAYOFFS

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Abstract. The paper explores financial market dynamics from evolutionary and behavioural perspectives. Most of the studies on this topic deal with models in which asset payoffs are exogenous and depend only on the underlying stochastic process of states of the world. The present work proposes a model in which the payoffs of assets are endogenous: they depend on the share of total market wealth invested in the asset.

1 Introduction

Evolutionary Behavioural Finance (EBF) is a novel research area at the interface of Mathematical Economics and Mathematical Finance combining behavioural and evolutionary approaches to the modelling of financial markets. The classical theory (Radner [13, 14]) relies upon the hypothesis of full rationality of market players, who are assumed to maximize their utilities subject to budget constraints, i.e. solve well-defined and precisely stated constrained optimization problems. EBF models abandon this hypothesis and permit market players to have a whole variety of patterns of behaviour determined by their individual psychology, not necessarily describable in terms of utility maximization. Strategies may involve, for example, mimicking, satisficing, rules of thumb based on experience, etc. Objectives might be of an evolutionary nature: survival (especially in crisis environments), domination in a market segment, fastest capital growth, etc. They might be relative—taking into account the performance of the others.

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EBF deals with stochastic dynamic models of financial markets in which asset prices are determined endogenously by short-run equilibrium of supply and demand. Equilibrium is formed consecutively in each time period in the course of interaction of investment strategies of competing market participants. It is defined directly via the set of strategies of the market players specifying the patterns of their investment behaviour (behavioural equilibrium). An important feature of EBF models is that they employ only objectively observable market data and do not use unobservable individual agents' characteristics, such as their utilities and beliefs. Thereby they suggest a path towards creating an operational alternative to the traditional Dynamic Stochastic General Equilibrium analysis for financial markets.

The main focus of studies in the field is on questions of survival and extinction of investment strategies in the market selection process. A central goal is to identify those strategies that allow an investor to survive, i.e. to keep with probability one a strictly positive, bounded away from zero share of market wealth over an infinite time horizon, irrespective of the strategies used by the other players. Typical results show that under very general assumptions, survival strategies exist and are easily computable.

First models integrating evolutionary and behavioural approaches were proposed in Amir et al. [3, 4]. A survey describing the state of the art in EBF by 2016 and outlining a program for further research was given in Evstigneev et al. [8]. An elementary textbook treatment of the subject can be found in Evstigneev et al. [7], Ch. 20. For a most recent review of the development of studies related to this area see Holtfort [10]. General perspectives of a synthesis of behavioural and mainstream economics based on the evolutionary approach are discussed in a recent paper by Aumann [5].

The basic sources for EBF are behavioural economics and finance (Tversky and Kahneman [18], Shiller [16], Bachmann et al. [6]), evolutionary game theory (Weibull [19], Samuelson [15], Gintis [9], Kojima [11]) and games of survival (Milnor and Shapley [12], Shubik and Thompson [17])[3].

Most of the studies on EBF consider assets with exogenous asset payoffs or dividends. To the best of our knowledge the only exception is paper by Amir et al. [2] that introduced an EBF model with long-lived dividend-paying assets in which the dividends depend on the fraction of total market wealth invested in the asset. The present paper suggests an analogous model

[3]For a comprehensive discussion of game-theoretic aspects of EBF in a different but closely related model see Amir et al. [4], Sections 1 and 6.

with short-lived (one-period) assets.

2 The model

Asset structure. Trading on the asset market is possible at any of the moments of time (dates) $t = 0, 1, 2,...$ Random factors influencing the market are described in terms of a discrete-time stochastic process of states of the world $s_1, s_2, ...$with values in a measurable space S. The probability measure P on the space of paths $\omega = (s_1, s_2, ...)$ of this process is given exogenously. We denote by s^t the history of states of the world up to time $t \geq 1$:

$$s^t = (s_1, ..., s_t).$$

All functions we consider in this paper, in particular, functions of s^t $(t \geq 1)$ are supposed to be jointly measurable with respect to all its arguments.

There are $K \geq 2$ "short-lived" *assets* $k = 1, 2, ..., K$. They live one period but are identically reborn every next period. For each asset k and time $t \geq 0$ we are given a function $V_{t,k}(s^t) > 0$ describing exogenous *asset supply*—the total volume of "physical units" of asset k available in the market at time t. For $t = 0$ the supply $V_{0,k}$ of asset k is constant.

One unit of asset k issued at time $t \geq 0$ yields the *payoff*

$$D_{t+1,k}(s^{t+1}, W_{t,k}\ ) \geq 0$$

at time $t+1$, where s^{t+1} is the history of states of the world up to time $t+1$ and $W_{t,k}$ is the share of total wealth invested by all the market participants in asset k at time t (the precise definition of $W_{t,k}$ is provided below, see (4)). The given function $D_{t+1,k}(s^{t+1}, W_{t,k})$ of the history s^t and the number $W_{t,k} \geq 0$ is nonnegative and strictly positive if $W_{t,k} > 0$.

Investors and their portfolios. There are N *investors (traders)* $i = 1, ..., N$. Every investor i at each time $t = 0, 1, 2, ...$ selects a *portfolio*

$$x_t^i = (x_{t,1}^i, ..., x_{t,K}^i),$$

where $x_{t,k}^i$ is the number of units of asset k in the portfolio x_t^i. The procedure of portfolio selection based on investment proportions will be described later. The portfolio $x_t^i = x_t^i(s^t)$ for $t \geq 1$ depends, generally, on the history s^t of states of the world up to time t. To alleviate notation, we will often omit s^t, writing e.g. x_t^i in place of $x_t^i(s^t)$.

Equilibrium. In the model at hand, for each moment of time $t \geq 1$ and each random situation s^t, the market for every asset k is in equilibrium:

$$\sum_{i=1}^{N} x_{t,k}^i(s^t) = V_{t,k}(s^t). \tag{1}$$

This equality means that the total market demand of asset k is equal to the total supply of asset k, i.e., each asset is in the portfolio of some investor.

Wealth of an investor. The wealth w_t^i of investor i at time $t \geq 1$ can be computed as follows:

$$w_t^i = \sum_{k=1}^{K} D_{t,k} x_{t-1,k}^i, \tag{2}$$

where

$$D_{t,k} = D_{t,k}(s^t, W_{t-1}^k).$$

The sum in (2) represents the total payoff obtained at date t by trader i from her yesterday's portfolio $x_{t-1}^i = (x_{t-1,1}^i, ..., x_{t-1,K}^i)$. For $t = 0$, the initial wealth $w_0^i > 0$ of each investor i is a constant given in the model. Both the left-hand side and the right-hand side of (2) are functions of the history s^t (which is skipped for shortness).

Total market wealth. Total market wealth W_t at time $t \geq 1$ is computed as follows:

$$W_t = \sum_{i=1}^{N} w_t^i = \sum_{i=1}^{N} \sum_{k=1}^{K} D_{t,k} x_{t-1,k}^i = \sum_{k=1}^{K} D_{t,k} \sum_{i=1}^{N} x_{t-1,k}^i = \sum_{k=1}^{K} D_{t,k} V_{t-1,k}. \tag{3}$$

The last inequality holds by virtue of the equilibrium condition (1). For $t = 0$, we have

$$W_0 = \sum_{i=1}^{N} w_0^i \ ,$$

where w_0^i is the given initial wealth of investor i.

Investment strategies (portfolio rules). Investment strategies will be characterized in terms of vectors $\lambda_t^i = (\lambda_{t,1}^i, ..., \lambda_{t,K}^i)$ of *investment proportions*

$$\lambda_{t,k}^i \geq 0, \ \sum_{k=1}^{K} \lambda_{t,k}^i = 1,$$

according to which traders $i = 1, ..., N$ allocate wealth across assets. In game-theoretic terms, λ_t^i may be regarded as an *action* that is undertaken by player i at time t in the asset market game at hand. An *investment strategy* (*portfolio rule*) Λ^i of player i is described in terms of a vector function $\Lambda_t^i(s^t, H_{t-1})$ specifying what vector of investment proportions

$$\lambda_t^i = \Lambda_t^i(s^t, H_{t-1})$$

to select at each time $t \geq 1$ depending on the history $s^t = (s_1, ..., s_t)$ of states of the world and the history of play

$$H_{t-1} = \{\lambda_s^i : 0 \leq s \leq t-1,\ i = 1, ..., N\}.$$

The latter contains information about the actions λ_s^i of all traders $i = 1, ..., N$ at all moments of time $s = 0, 1, ..., t-1$. Additionally, the strategy Λ^i indicates the vector λ_0^i of investment proportions to be chosen at time 0.

Among all strategies $\Lambda_t^i(s^t, H_{t-1})$ we will distinguish those ones for which Λ_t^i depends only on s^t and not on H_{t-1}. Such strategies are called *basic*. A basic portfolio rule Λ^i of investor i is specified by a sequence of vectors of investment proportions

$$\Lambda^i = (\lambda_0^i, \lambda_1^i(s^1), \lambda_2^i(s^2), ...)$$

describing the investment behaviour of player i at each time t and in every random situation s^t.

Wealth $w_{t,k}^i$ of investor i invested in asset k. If $\lambda_t^i = (\lambda_{t,1}^i, ..., \lambda_{t,K}^i)$ is the vector of investment proportions selected by investor i at time t, then her wealth $w_{t,k}^i$ invested into asset k at this time is

$$w_{t,k}^i = \lambda_{t,k}^i w_t^i,$$

where w_t^i is the investor i's wealth (budget) at time t.

The share $W_{t,k}$ of the total market wealth invested in asset k. The total wealth invested in asset k can be expressed as

$$\sum_{i=1}^{N} w_{t,k}^i = \sum_{i=1}^{N} \lambda_{t,k}^i w_t^i \ .$$

The share $W_{t,k}$ of the total market wealth W_t (see (3)) invested in asset k is equal to

$$W_{t,k} = \frac{\sum_{i=1}^{N} w_{t,k}^i}{W_t} = \frac{\sum_{i=1}^{N} \lambda_{t,k}^i w_t^i}{W_t}. \tag{4}$$

Equilibrium asset prices. Given the vectors of investment proportions

$$\lambda_t^i = (\lambda_{t,1}^i, ..., \lambda_{t,K}^i), \ i = 1, ..., N,$$

of investors $i = 1, ..., N$ at time t the equation

$$p_{t,k} V_{t,k} = \sum_{j=1}^{N} \lambda_{t,k}^j w_t^j \tag{5}$$

determines for each k and s^t the *equilibrium* (*market clearing*) *price*

$$p_{t,k} = p_{t,k}(s^t)$$

of asset k. Equation (5) is the equilibrium condition expressed in financial terms. The product $p_{t,k} V_{t,k}$ on the left-hand side is the value of the total mass of asset k available in the market. The sum on the right-hand side, $\sum_{j=1}^{N} \lambda_{t,k}^j w_t^j$, expresses the total amount invested in asset k by all the market players. From equation (5) we obtain the following formula for the equilibrium price of asset k at time t:

$$p_{t,k} = \frac{1}{V_{t,k}} \sum_{j=1}^{N} \lambda_{t,k}^j w_t^j. \tag{6}$$

Investors' portfolios expressed via strategies. If investor i's wealth is w_t^i, the proportion of wealth invested in asset k is $\lambda_{t,k}^i$, and the price of asset k is $p_{t,k}$, then the number $x_{t,k}^i$ of units of asset k in the portfolio of investor i at time t is equal to

$$x_{t,k}^i = \frac{\lambda_{t,k}^i w_t^i}{p_{t,k}}. \tag{7}$$

In view of (6) this yields

$$x_{t,k}^i = V_{t,k} \frac{\lambda_{t,k}^i w_t^i}{\sum_{j=1}^{N} \lambda_{t,k}^j w_t^j}. \tag{8}$$

By summing up these equations for $i = 1, ..., N$, we get (1), as it should be according to the equilibrium hypothesis.

Dynamics of wealth of the investors. If investor i's portfolio at time $t \geq 0$ is $x_t^i = (x_{t,1}^i, ..., x_{t,K}^i)$, then her wealth at time $t+1$ is

$$w_{t+1}^i = \sum_{k=1}^{K} D_{t+1,k}(s^{t+1}, W_{t,k}\ )x_{t,k}^i,$$

where $W_{t,k}$, we recall, is the share of the total market wealth invested in asset k at time t. By using (8), we get

$$w_{t+1}^i = \sum_{k=1}^{K} D_{t+1,k}(s^{t+1}, W_{t,k}\ )V_{t,k}\frac{\lambda_{t,k}^i w_t^i}{\sum_{j=1}^{N}\lambda_{t,k}^j w_t^j}. \tag{9}$$

Admissible strategy profiles. Note that formula (7) makes sense only if $p_{t,k} > 0$. In this connection we introduce the following definition. A strategy profile $(\Lambda^1, ..., \Lambda^N)$ of all the investors $i = 1, ..., N$ is called *admissible* if the vectors of investment proportions $\lambda_t^i = (\lambda_{t,1}^i, ..., \lambda_{t,K}^i)$, $i = 1, ..., N$, generated by this strategy profile satisfy (for all k, t and s^t) the following condition:

$$\sum_{i=1}^{N} \lambda_{t,k}^i > 0, \tag{10}$$

which means that for each asset k there is at least one investor allocating a strictly positive share of her wealth in it. Clearly this condition holds if at least one investor i uses a fully diversified strategy guaranteeing that all the investment proportions $\lambda_{t,k}^i$ are strictly positive. In what follows *we will assume that all strategy profiles under consideration are admissible.* The assumption of admissibility implies that $w_t^i > 0$ and $p_{t,k} > 0$ for each t, i and k. This is proved by induction (step by step, passing from t to $t+1$) by using (9), (6) and the assumption that if $W > 0$ then $D_{t,k}(s^t, W) > 0$.

3 A central result

Market dynamics: An evolutionary perspective. The main focus of our analysis will be on the dynamics of the *market shares* of the investors

$$r_t^i = \frac{w_t^i}{W_t}\ (i = 1, ..., N),$$

where w_t^i is the investor i's wealth and $W_t = \sum_{i=1}^N w_t^i$ is the total market wealth at time t. We will use in this analysis equations (9) that describe the dynamics of the wealth w_t^i of investors $i = 1, 2, ..., N$ whose investment strategies $\Lambda^1, ..., \Lambda^N$ generate the vectors of investment proportions $\lambda_t^i = (\lambda_{t,1}^i, ..., \lambda_{t,K}^i)$.

Suppose that the market players $i = 1, 2, ..., N$ use strategies $\Lambda^1, ..., \Lambda^N$, respectively. Investor i, or her portfolio rule Λ^i, is said to *survive* in the market selection process (within the strategy profile $(\Lambda^1, ..., \Lambda^N)$) if the market share r_t^i is strictly positive and bounded away from zero almost surely:

$$\inf_{t \geq 0} r_t^i > 0 \text{ (a.s.)}.$$

Here, "almost surely" is understood with respect to the given probability P on the space of paths $\omega = (s_1, s_2, ...)$ of the process of states of the world.

A portfolio rule is termed a *survival strategy* if it guarantees unconditional survival for the investor using it irrespective of what strategies are used by the other market participants.

In order to survive you have to win! One might think that the focus on survival substantially restricts the scope of the analysis, since "one should care about survival only if things go wrong". It turns out, however, that the class of survival strategies in most of the EBF models coincides with the class of *unbeatable strategies* performing in the long run not worse in terms of wealth accumulation than any other strategies competing in the market. To demonstrate this let us reformulate the notion of a survival strategy in terms of the wealth processes $(w_t^i)_{t=0}^\infty$ of the market players $i = 1, 2, ..., N$. Survival of a portfolio rule Λ^1 used by player 1 means that $w_t^1 \geq c \sum_{i=1}^N w_t^i$, where c is a strictly positive random variable. The last inequality holds if and only if

$$w_t^i \leq C w_t^1, \ i = 1, ..., N, \tag{11}$$

where C is some random variable. Property (11) expresses the fact that the wealth of any player i using any strategy Λ^i cannot grow asymptotically faster than the wealth of player 1 who uses the strategy Λ^1. If this is the case, the portfolio rule Λ^1 is called *unbeatable*. Thus survival strategies are those and only those that are unbeatable, and so "in order to survive, you have to win!"

Key assumption. A central result of this note is a construction of a survival strategy in the model at hand. To this end we will need the following assumption:

(**D**) For each k, t and s^t, the payoff function $D_{t+1,k}(s^{t+1}, W_{t,k})$ of asset k is linear and strictly increasing in $W_{t,k}$:

$$D_{t+1,k}(s^{t+1}, W_{t,k}) = D_{t+1,k}(s^{t+1})W_{t,k}, \text{ where } D_{t+1,k}(s^{t+1}) > 0.$$

The Kelly portfolio rule. Denote by Δ^K the unit simplex consisting of all non-negative vectors $\lambda = (\lambda_1, ..., \lambda_K)$ such that $\sum_{k=1}^K \lambda_k = 1$. Define the *relative payoffs* of assets $k = 1, 2, ..., K$ by

$$R_{t+1,k}(s^{t+1}) = \frac{D_{t+1,k}(s^{t+1})V_{t,k}(s^t)}{\sum_{m=1}^K D_{t+1,m}(s^{t+1})V_{t,m}(s^t)} \tag{12}$$

and put

$$R_{t+1}(s^{t+1}) = (R_{t+1,1}(s^{t+1}), ..., R_{t+1,K}(s^{t+1})). \tag{13}$$

Clearly the vector $R_{t+1}(s^{t+1})$ belongs to Δ^K for each s^{t+1}. It will be convenient to use the following scalar product notation:

$$\langle R_{t+1}(s^{t+1}), \lambda\rangle = \sum_{k=1}^K R_{t+1,k}(s^{t+1})\lambda_k \tag{14}$$

where $\lambda = (\lambda_1, ..., \lambda_K) \in \Delta^K$.

The basic investment strategy

$$\Lambda^* = (\lambda_0^*, \lambda_1^*(s^1), \lambda_2^*(s^2), ...)$$

is called the *Kelly* (*log-optimal*) *portfolio rule* if for each $t \geq 0$ and any function $\lambda(s^t)$ with values in Δ^K we have

$$E \ln\langle R_{t+1}(s^{t+1}), \lambda_t^*(s^t)\rangle \geq E \ln\langle R_{t+1}(s^{t+1}), \lambda(s^t)\rangle.$$

Here E stands for the expectation with respect to the probability measure P on the space of paths $\omega = (s_1, s_2, ...)$ of the process of states of the world exogenously given in the model. It is known (see Algoet and Cover 1988, p. 877) that the Kelly investment strategy always exists and under some assumptions of non-degeneracy of the structure of relative payoffs is unique up to a.s. equivalence.

Theorem 1. *The Kelly portfolio rule* Λ *is a survival strategy.*

Proof of Theorem 1. Under assumption (**D**), the dynamics of wealth of investor $i = 1, ..., N$ employing investment proportions $\lambda_t^i = (\lambda_{t,1}^i, ..., \lambda_{t,K}^i)$ is governed by the equations

$$w_{t+1}^i = \sum_{k=1}^K D_{t+1,k} W_{t,k} V_{t,k} \frac{\lambda_{t,k}^i w_t^i}{\sum_{n=1}^N \lambda_{t,k}^n w_t^n}$$

following from (9). Consequently, for any two investors i and j, we have

$$\frac{w_{t+1}^i}{w_{t+1}^j} = \frac{\sum_{k=1}^K D_{t+1,k} W_{t,k} V_{t,k} \dfrac{\lambda_{t,k}^i w_t^i}{\sum_{n=1}^N \lambda_{t,k}^n w_t^n / W_t}}{\sum_{k=1}^K D_{t+1,k} W_{t,k} V_{t,k} \dfrac{\lambda_{t,k}^j w_t^j}{\sum_{n=1}^N \lambda_{t,k}^n w_t^n / W_t}}$$

$$= \frac{w_t^i}{w_t^j} \frac{\sum_{k=1}^K D_{t+1,k} V_{t,k} \lambda_{t,k}^i}{\sum_{k=1}^K D_{t+1,k} V_{t,k} \lambda_{t,k}^j} \tag{15}$$

(see (4)). By using the notation (12) – (14), we get from (15)

$$\frac{w_{t+1}^i}{w_{t+1}^j} = \frac{w_t^i}{w_t^j} \frac{\sum_{k=1}^K D_{t+1,k} V_{t,k} \lambda_{t,k}^i / \sum_{m=1}^K D_{t+1,m} V_{t,m}}{\sum_{k=1}^K D_{t+1,k} V_{t,k} \lambda_{t,k}^j / \sum_{m=1}^K D_{t+1,m} V_{t,m}}$$

$$= \frac{w_t^i}{w_t^j} \frac{\sum_{k=1}^K R_{t+1,k} \lambda_{t,k}^i}{\sum_{k=1}^K R_{t+1,k} \lambda_{t,k}^j} = \frac{w_t^i}{w_t^j} \frac{\langle R_{t+1}, \lambda_t^i \rangle}{\langle R_{t+1}, \lambda_t^j \rangle}. \tag{16}$$

Suppose that investor j uses the Kelly strategy Λ^* and investor i employs any strategy Λ^i so that the strategy profile at hand leads to the sequence of vectors λ_t^i of i's investment proportions. Then formula (16) yields

$$\frac{w_{t+1}^i}{w_{t+1}^*} = \frac{\Pi_{t+1}}{\Pi_{t+1}^*},$$

where $(w_t^*)_{t \geq 0}$ is the wealth process of the Kelly investor and

$$\Pi_{t+1} = w_0^i \langle R_1, \lambda_0^i \rangle ... \langle R_{t+1}, \lambda_t^i \rangle, \ \Pi_{t+1}^* = w_0^* \langle R_1, \lambda_0^* \rangle ... \langle R_{t+1}, \lambda_t^* \rangle.$$

By virtue of the Algoet-Cover (1988, p. 877) theorem, the random sequence Π_{t+1}/Π_{t+1}^* is a non-negative supermartingale. Therefore it converges a.s. to a finite limit, which implies that the random sequence w_{t+1}^i / w_{t+1}^* is bounded above by some random variable $C_i > 0$. Consequently, $w_{t+1}^i \leq C_i w_{t+1}^*$ for all i and t (a.s.), and hence the strategy Λ^* is unbeatable. As has been noted above, this is so if and only if Λ^* is a survival strategy.

The proof is complete.

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