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MARGINAL TAX RATES AND INCOME IN THE LONG RUN: EVIDENCE FROM A STRUCTURAL ESTIMATION*

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Abstract

We estimate the long-run response of income to marginal tax rate changes within a life-cycle model of the US. We find statistically significant long-run tax elasticities of income of around 0.66 for all taxpayers. Estimated elasticities are largest for the richest 1% but are also positive for other income groups. In our economy, the most productive agents obtain higher returns on their wealth by choosing to be entrepreneurs. This crucial feature, in combination with earnings risk and tax progressivity, increases the incentives to save and invest for the richest, high-return entrepreneurs, thus amplifying their income responses to marginal tax changes.

JEL classification: E62, H21, H24

Keywords: marginal tax rate changes; elasticity of taxable income; life-cycle; entrepreneurs; structural estimation.

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1 Introduction

Are marginal tax rates important for long-run behavioral responses of income? Are incomes at the top of the distribution, who pay a large share of total fiscal revenues, responsive to marginal tax rate changes in the long run? Answering these questions is essential for understanding the transmission of tax policies and their role in dealing with economic growth and inequality. Nevertheless, the existing empirical literature studying US tax returns concentrates on short- to medium-run effects of marginal tax rate changes (e.g., [Feldstein, 1995](#), [Gruber and Saez, 2002](#), and [Mertens and Montiel Olea, 2018](#)). This is because estimates of the effects of marginal tax changes in the long run (i.e., exceeding a few years) are plagued by extremely difficult identification issues (e.g., [Saez et al., 2012](#)). For instance, the overall implications of a tax reform on households' investment and entrepreneurial decisions may not be reached for several years and hence they are very hard to trace back empirically to the original policy change. However, these long-run effects on economic outcomes are extremely important for policymaking and they represent a key motivation for legislated tax reforms in the US and other industrialized economies (e.g., [Romer and Romer, 2010](#) and [Cloyne, 2013](#)).

This paper tackles the identification problem by estimating the long-run effects of marginal tax changes within a structural life-cycle model of the US. We estimate that the long-run elasticity of total taxable income (ETI) with respect to net-of-tax rates – 1 minus the marginal tax rate – is substantial and statistically significant for all income groups, and is centered at 0.66. We also find that incomes in the top 1 percent of the distribution, who contribute to more than 40 percent of all fiscal revenues, are the most responsive to marginal tax rate changes, displaying a long-run ETI of 0.77.

Our economy features rich heterogeneity through overlapping generations, incomplete markets and progressive income taxation. Agents face persistent heterogeneity in their labor income (e.g., [Huggett, 1996](#) and [Conesa et al., 2009](#)) and, crucially, can obtain higher-than-average returns on their wealth by choosing to be entrepreneurs (e.g., [Cagetti and De Nardi, 2006](#) and [Guvenen et al., 2019](#)). We estimate the model via Simulated Method of Moments (SMM), in order to match cross-sectional variation in income and wealth at the household level, together with a broader set of distributional moments, as well as standard macroeconomic aggregates. In this way, we provide a suitable laboratory to evaluate the long-run effects of marginal tax changes on the economy as a whole and along the income distribution.

A key advantage of our structural approach is that it allows us to fully disentangle

how the various ingredients of our model (i.e., return heterogeneity, earnings risk and progressive taxation) affect the transmission of tax policies, through individual behavior and general equilibrium price changes. With our approach, we can also analyze heterogeneous responses to marginal tax rate changes along the income distribution, by income type (earnings and capital), by age (young, middle-aged and retirees) and by occupational choice (workers and entrepreneurs). Hence, our empirical methodology delivers precise insights and intuition about the agents' responses to tax changes, and therefore is particularly appealing for the question at hand about the long-run spillovers of tax policy.

In particular, we show that the combination of wealth return heterogeneity, earnings risk and progressive taxation is fundamental to match the salient distributional features of the US economy. We also find that these ingredients (return heterogeneity, earnings risk and progressive taxation) have both quantitatively and qualitatively important amplification effects for the transmission of marginal tax changes on all taxpayers, especially for the richest 1 percent, who are the most responsive to marginal tax rate changes. This is because in our economy, a cut in marginal tax rates increases the incentive to save and invest, but mainly for households in the top of the income distribution, who have high returns, significant amounts of labor income and high marginal tax rates. This reallocation of capital to high-productivity agents increases aggregate productivity and generates a larger equilibrium boost in wages, in turn benefiting also the bottom 90 percent, but nevertheless their response is smaller than at the top.

We put this transmission mechanism under scrutiny by running three external validity checks on untargeted statistics. First, we show that the return profiles at the top of the wealth distribution generated by our model replicate those found in the data (e.g., [Xavier, 2020](#)). This is an interesting finding that brings favorable evidence about our identification strategy and key parameter estimates. It also indicates that our microfoundation for the right tail of the wealth and income distributions is both quantitatively and qualitatively realistic.

Second, we show that our model matches the capital income shares at the top of the income distribution. This is important as various empirical studies have indicated that the distribution of capital income is pivotal for the understanding of income and wealth inequality in the US (e.g., [Piketty et al., 2018](#)). On this point, we also find that an alternative theory of the right tail of the distribution, the earnings superstate model (e.g., [Castañeda et al., 2003](#)), fails to capture the high concentration of capital income for the richest households and implies a much more muted long-run response to marginal tax changes. This result is interesting as it highlights in a transparent manner how the

modeling choice of the right tail of the income and wealth distributions is relevant for tax policy analysis in practice.

Finally, we show that our model produces short-run ETIs and multipliers that are consistent with a large chunk of the empirical evidence, both in public economics (e.g., [Kumar and Liang, 2020](#)) and in macroeconomics (e.g., [Barro and Redlick, 2011](#) and [Mertens and Ravn, 2013](#)). This latter result further increases the credibility of both the internal transmission mechanism at play in our model as well as of our estimates of long-run ETIs.

One possible drawback of our approach is that our estimates are conditional on the assumed data generation process. While our model contains several important ingredients and heterogeneity over various dimensions, it almost surely misses on some features that could have a sizeable impact on the transmission of marginal tax changes in the long run. For example, we do not include human capital accumulation nor tax avoidance. These factors are naturally expected to increase the distortionary effects marginal tax rates, particularly for households in the top end of income and wealth distributions. Thus, we expect that these extra ingredients could further magnify the quantitative significance of our results. However, we leave such an analysis for future research.

The remainder of the paper is the following. Section 2 explains how this paper relates with the literature. Section 3 presents our benchmark structural model. Section 4 describes the main estimation exercise. Section 5 presents the main policy experiment. Finally, Section 6 concludes.

2 Relation to the Literature

First of all, our paper relates to the public finance literature estimating the short-run tax elasticity of reported income, surveyed by [Saez et al. \(2012\)](#). This literature concentrates on estimating the short-run elasticity of taxable income (ETI), or policy elasticity in the sense of [Hendren \(2016\)](#), as a measure of the distortionary effects of taxation on the behavioral responses of labor supply, investment, as well as equilibrium changes in prices. The ETI is a popular measure in public finance because, under some regularity conditions, it is a sufficient statistic to evaluate the efficiency costs of tax policy reforms. Related to this literature, our main contribution consists of providing a structural identification scheme enabling us to estimate long-run ETIs. Moreover, our results about long-run ETIs along the income distribution contribute to the age-old debate on whether agents at the top or bottom of the distribution react the most to marginal

tax changes (Feldstein, 1995; Mertens and Montiel Olea, 2018; Zidar, 2019). On this, we provide a realistic general equilibrium mechanism based on incomplete markets and return heterogeneity and point out that in the long run, the richest households display the highest elasticities.

Second, our paper relates to the emerging literature on structural estimation of heterogeneous agents models that combines cross-sectional micro data with time-series macro data (e.g., Winberry, 2018, Liu and Plagborg-Møller, 2021, Auclert et al., 2021 and references within). The common approach of this literature is to use full-information econometric methods, which are generally implemented by approximating the structural model via perturbation techniques. For this reason, this identification scheme is local in nature and most suited for inference in analysis of the short-run, such as the cyclical properties of inequality (e.g., Bayer et al., 2020). Differently, our SMM technique belongs to the family of limited-information estimators and is global in nature. As such, it allows us to consider complex non-linearities on the model's long-run equilibrium. Therefore, it is most suited for the research question at hand about long-run spillovers of tax policy. From a technical point of view, we adopt the approach presented in Cocci and Plagborg-Møller (2021), which amounts to estimating the standard errors using the worst-case correlations for the unknown covariances between cross-sectional micro data and time-series macro data.

Our paper also relates to the literature that models uninsurable capital income risk within life-cycle heterogeneous-agent frameworks (e.g., Benhabib et al., 2011; Guvenen et al., 2019). This modeling feature has recently gained popularity for three reasons. First, the empirical evidence shows a substantial heterogeneity in capital income (Bach et al., 2020; Fagereng et al., 2020). Second, heterogeneity in capital income is a crucial ingredient in order to match the fat Pareto tail of the wealth distribution. Third, capital return heterogeneity has been shown to have important consequences for the setting of economic policies (e.g., Guvenen et al., 2019). Our contribution is to show that heterogeneity in capital income greatly modifies our understanding of the ETI along the income distribution, causing the top 1 percent to display the largest elasticities in the long run.

Another strand of literature links aggregate productivity, entrepreneurship and taxes. In a growth model with entrepreneurs, Jaimovich and Rebelo (2017) find higher capital income tax rates reduce incentives to be entrepreneurs and long-run growth. Guvenen et al. (2019) find that a wealth tax reduces misallocation and increases aggregate TFP by re-allocating capital toward more productive entrepreneurs. Differently, we focus on the effect of progressive income taxes on entrepreneurship and productivity when capital and labor are jointly taxed. Moreover, our paper offers support within a structural approach,

to a number of well established empirical results for short-run analysis, such as the relation between tax rates and entrepreneurial activity (e.g., [Djankov et al., 2010](#)), the effects of tax changes on aggregate productivity (e.g., [Cloyne, 2013](#)) and the relationship between tax progressivity and misallocation (e.g., [Fajgelbaum et al., 2019](#)).

Lastly, we relate to the literature on capital misallocation featuring financially constrained entrepreneurs with heterogeneous productivity (e.g., [Cagetti and De Nardi, 2006](#); [Moll, 2014](#); [Itskhoki and Moll, 2019](#)). On this aspect, the closest contribution is [Guvenen et al. \(2019\)](#), who analyze the effects of fiscal reforms, in particular a shift from capital income to wealth taxes, on capital misallocation. Instead, we focus on how marginal income tax changes affect aggregate productivity, via changes in the rate of entrepreneurship.

3 The Model

We present an incomplete-markets life-cycle model consisting of households, firms and a government who interact in competitive good and factor markets.

3.1 Households

The economy is populated by a continuum of households, who differ by age, labor productivity and entrepreneurial ability. Each period, a mass of new households is born, where the rate of population growth is exogenous and assumed to be n . During their life, households choose consumption, savings, and labor supply and whether or not to engage in entrepreneurial activity. Households also pay progressive taxes on total income and flat social security taxes on labor earnings (up to a cap). After retirement at age R , households receive social security benefits from the government.

Households also face a risk of early death. We denote by s_j the probability of surviving to age j , conditional on surviving to age $j - 1$, where $s_1 = 1$ and $s_{J+1} = 0$. The demographic patterns are stable, so that age- j agents make up a constant fraction μ_j of the total population.¹ Accidental bequests are redistributed to all living consumers as a lump-sum transfer, T_b .

¹The measure μ_j can be defined recursively, where $\mu_{j+1} = s_{j+1}\mu_j / (1 + n)$ for $j = 1, \dots, J - 1$ and μ_1 is set to normalize $\sum_{j=1}^J \mu_j = 1$.

Preferences All agents have identical preferences for consumption c_j and hours worked h_j over their lifetime:

$$E \left\{ \sum_{j=1}^J \beta^{j-1} \left(\prod_{k=1}^j s_k \right) u(c_j, h_j) \right\}, \quad (1)$$

where $\prod_{k=1}^j s_k$ is the unconditional probability an age-1 agent will survive to age j . As it is standard in the literature (e.g., [Conesa et al., 2009](#)), we assume that the period utility is of the form

$$u(c, h) = \frac{(c^\gamma(1-h)^{1-\gamma})^{1-\sigma}}{1-\sigma},$$

where γ is the consumption utility share and σ controls the household's risk aversion.²

Labor Productivity In each period before retirement, agents receive labor earnings equal to weh , where w is the real wage rate, e is the household's labor ability and h is hours worked. When households reach age R , they retire so that hours worked and total labor earnings become zero for ages $j \geq R$.

We assume ex-ante and ex-post heterogeneity in labor abilities as in, inter alia, [Kaplan and Violante \(2014\)](#) and [Guisen et al. \(2019\)](#). A household's labor ability $e_{i,j}(z_h)$ is given by

$$\log e_{i,j}(z_h) = \bar{e}_i + \alpha_0 + \alpha_1 j + \alpha_2 j^2 + \alpha_3 j^3 + \alpha_4 j^4 + \log z_h \quad (2)$$

A household's labor productivity depends on three factors. First, labor ability depends on a household-specific innate ability, \bar{e}_i . At birth, the household learns her type $i \in \{1, \dots, I\}$ which indexes its overall level of labor ability. We denote by π_i the probability a household will become type i . Second, labor ability explicitly depends on a fourth-order polynomial in age j . Third, labor ability is also affected by an idiosyncratic shock, z_h , which follows an AR(1) process:

$$\log z'_h = \rho_h \log z_h + \varepsilon_h, \quad \varepsilon_h \sim N(0, \sigma_{\varepsilon_h}^2), \quad (3)$$

where the initial $\log z_h$ is set to zero.

We assume that the household's innate ability, \bar{e}_i , is drawn from $N(0, \sigma_e^2)$. In our quantitative analysis, we will construct a discrete approximation for innate ability using I individual types. As a result, the innate abilities $\{\bar{e}_i\}_{i=1}^I$ and the type probabilities $\{\pi_i\}_{i=1}^I$ are all parameterized by one parameter, σ_e . See [Appendix B](#) for details.

²Given the assumption of a Cobb-Douglas utility function, the coefficient of relative risk aversion in consumption is $-cu_{cc}/u_c = 1 - \gamma(1 - \sigma)$.

Asset Return Risk Through Entrepreneurship Following [Cagetti and De Nardi \(2006\)](#) and [Guvenen et al. \(2019\)](#), we introduce a role for entrepreneurship. All households can choose to be an entrepreneur, whereby they access a “backyard technology” that uses k units of capital to produce q units of an intermediate capital service. We assume a linear technology

$$q = z_r k \tag{4}$$

where z_r characterizes the household’s entrepreneurial productivity. We also assume that entrepreneurial productivity follows an AR(1) process of the type:

$$\log z'_r = \rho_r \log z_r + \varepsilon_r, \quad \varepsilon_r \sim N(0, \sigma_{\varepsilon_r}^2) \tag{5}$$

where the initial shock is drawn from the distribution $N(0, \sigma_{\varepsilon_r}^2 / (1 - \rho_r^2))$.

All households lend on the bond market their whole wealth at the riskless rate r . Those who also choose to be entrepreneurs borrow at rate r on the same market and use their own backyard technology to produce the intermediate capital service q . Entrepreneurs must also decide how much capital k to invest in their backyard technology. They are subject to a collateral constraint, i.e., $k \leq \lambda a$, where $\lambda \geq 1$ is exogenous and controls the leverage level, while a is the individual entrepreneur’s wealth (e.g., see [Moll, 2014](#), [Boar and Midrigan, 2019](#) and [Guvenen et al., 2019](#)). Entrepreneurs then maximize the following profit function,

$$\pi(a, z_r) = \max_{0 \leq k \leq \lambda a} \{p z_r k - (r + \delta)k\}, \tag{6}$$

where p is the price of the capital service, $r + \delta$ is the rental rate of capital, with δ representing the depreciation rate. The associated optimal capital demand is

$$k(a, z_r) = \begin{cases} \lambda a & \text{if } z_r \geq (r + \delta)/p \\ 0 & \text{if } z_r < (r + \delta)/p \end{cases} \tag{7}$$

Therefore, there exists an endogenous productivity threshold,

$$\bar{z}_r = (r + \delta)/p, \tag{8}$$

such that only households that are sufficiently productive will choose to be entrepreneurs, while the others will simply engage in lending activities. This feature derives from our assumption of constant returns to scale and it allows the model to match the entrepreneur-

ship rate observed in the data. Our framework also avoids the negative relationship between wealth and returns, which is counterfactual (e.g., see [Bach et al., 2020](#)).

To summarize, all households earn the interest rate r by lending their wealth on the bond market. Those households with sufficiently high entrepreneurial ability also choose to run a business, whereby they borrow at rate r , produce the intermediate good q and earn $\pi(a, z_r)$. Substituting the solution for $\pi(a, z_r)$, the household's total return on its wealth is given by

$$r_a(z_r) = r + \lambda \max(pz_r - (r + \delta), 0). \quad (9)$$

Therefore, there will be persistent idiosyncratic variation in returns across households, which is a crucial ingredient for the model's ability to match the fat tail of wealth and taxable income (e.g., see [Benhabib et al., 2011](#), [Benhabib et al., 2019](#) and [Güvener et al., 2019](#)). Furthermore, despite no explicit link between wealth and returns, high-wealth households will, on average, earn higher returns, consistent with the empirical evidence (e.g., see [Bach et al., 2020](#) and [Fagereng et al., 2020](#)).

3.2 Final Production Firm

The final good is produced according to a Cobb-Douglas production function:

$$Y = F(Q, L) = Q^\alpha L^{1-\alpha}$$

where L is aggregate labor and Q is the aggregate of the intermediate capital service produced by entrepreneurs.

It is straightforward to derive the following aggregate relationship:

$$Y = AK^\alpha L^{1-\alpha}$$

where K is aggregate capital and A is aggregate TFP. Aggregate TFP is $A = (Q/K)^\alpha$, where Q/K is the average productivity of entrepreneurs. Therefore, aggregate productivity depends crucially on the allocation of capital across entrepreneurs.

The market for the intermediate capital service and the market for labor are both perfectly competitive. Therefore, the representative firm takes as given the prices (w, p) and chooses Q and L to maximize profits, $\Pi = Q^\alpha L^{1-\alpha} - pQ - wL$.

3.3 Government

The government taxes income in order to finance a fixed and exogenous level of government spending, G , which provides agents no utility. The government operates a balanced budget and does not use debt, implying that G is just equal to aggregate income tax revenues. The government also runs a social security system with a dedicated budget.

Income Tax Labor and capital income are jointly taxable. This assumption derives from the difficulties in the data to precisely estimate separate tax functions for capital and labor, and it is standard in the literature (e.g., [Heathcote et al., 2014](#)).³ Households can also deduct part of the social security contribution (described below), up to an upper limit \bar{y} . The resulting household's taxable income is

$$y = we_{i,j}(z_h)h + r_a(z_r)a - \frac{1}{2}\tau_{ss} \min (we_{i,j}(z_h)h, \bar{y}). \quad (10)$$

We adopt a tax specification function belonging to a flexible three-parameter family, originally proposed by [Gouveia and Strauss \(1994\)](#) and popular in applied works (e.g., [Conesa et al., 2009](#) and [Guner et al., 2014](#)),

$$\mathcal{T}_y(y) = \tau_0 y \left(1 - (\tau_2 y^{\tau_1} + 1)^{-1/\tau_1}\right). \quad (11)$$

Roughly speaking, τ_0 governs the maximum tax rate, while τ_1 and τ_2 determine the progressivity of the tax schedule. For $\tau_1 \rightarrow 0$, the tax system reduces to a pure flat tax, while other values encompass a wide range of progressive and regressive tax functions. According to this specification, the marginal income tax rate converges to zero as taxable income converges to zero, while the marginal tax rate converges to the upper bound of τ_0 as taxable income grows large.

Social Security Scheme The government runs a pay-as-you-go social security scheme. Taxpayers pay a social security tax only out of their labor income (at the flat tax rate τ_{ss}), up to an upper bound \bar{y} . The government pays a type-specific social security benefit, $b_{i,j}$:

$$b_{i,j} = \begin{cases} 0 & \text{if } j < R \\ \bar{b}_i & \text{if } j \geq R. \end{cases}$$

³We relax this assumption in Appendix [E.1](#) and assume instead a progressive labor income tax alongside a flat capital income tax.

We assume that $\bar{b}_i = \chi w L_i$, where L_i is the average labor input of type- i agents and χ is the replacement rate.

Social security benefits are financed by a flat tax τ_{ss} on all labor earnings weh below \bar{y} . That is, a household with labor earnings weh will pay a social security tax of $\tau_{ss} \min(weh, \bar{y})$. Given the tax rate τ_{ss} and the cap \bar{y} , we internally set the replacement rate χ so that aggregate social security tax revenue equals aggregate social security benefits.

3.4 Value Function

Having presented the main features of our model economy, we can now describe the household's problem in recursive form. In each period, the household chooses consumption c , savings a' , and labor supply h given idiosyncratic risk, prices and the tax function. In retirement, households supply zero hours (i.e., $h = 0$), but they still choose consumption and savings. Let $V_{i,j}(a, z_h, z_r)$ denote the value of a type- i and age- j consumer with assets a and idiosyncratic shocks (z_h, z_r) . We can write the consumer's maximization problem as follows:

$$V_{i,j}(a, z_h, z_r) = \max_{c, h, a'} \{u(c, h) + \beta s_{j+1} E[V_{i,j+1}(a', z'_h, z'_r) | z_h, z_r]\} \quad (12)$$

subject to

$$\begin{aligned} y &= we_{i,j}(z_h)h + r_a(z_r)a - \frac{1}{2}\tau_{ss} \min(we_{i,j}(z_h)h, \bar{y}) \\ c + a' &= a(1 + r_a(z_r)) + we_{i,j}(z_h)h - \tau_{ss} \min(we_{i,j}(z_h)h, \bar{y}) - \mathcal{T}_y(y) + T_b + b_{i,j} \\ a' &\geq 0 \\ 0 &\leq h \leq \mathbb{1}\{j < R\}. \end{aligned}$$

3.5 Equilibrium

We focus on a stationary equilibrium, in which capital, labor, transfers and government consumption are all constant in per-capita terms. See Appendix A for a full definition.

4 Quantitative Analysis

In this section, we outline our estimation strategy, and then evaluate the model’s ability to account for a number of features in the data for the US. In our main exercise, we solve and estimate the model assuming the economy is in a steady state. One period corresponds to one year and we convert all nominal values into 2010 dollars. As the numerical strategy used to solve the model is completely standard, we relegate its description to Appendix B.

In the tradition of [Gourinchas and Parker \(2002\)](#), we adopt a two-step estimation procedure. This consists of splitting our parameters into two main groups: (i) a group of parameters that is externally set, either according to previous literature, via direct observation or through estimation; and (ii) a group of parameters that is internally set, estimated using a Simulated Method of Moments (SMM) estimator, in order to match relevant distributional moments in the Survey of Consumer Finances (SCF) for 2016 and other standard macroeconomic moments from national accounts.

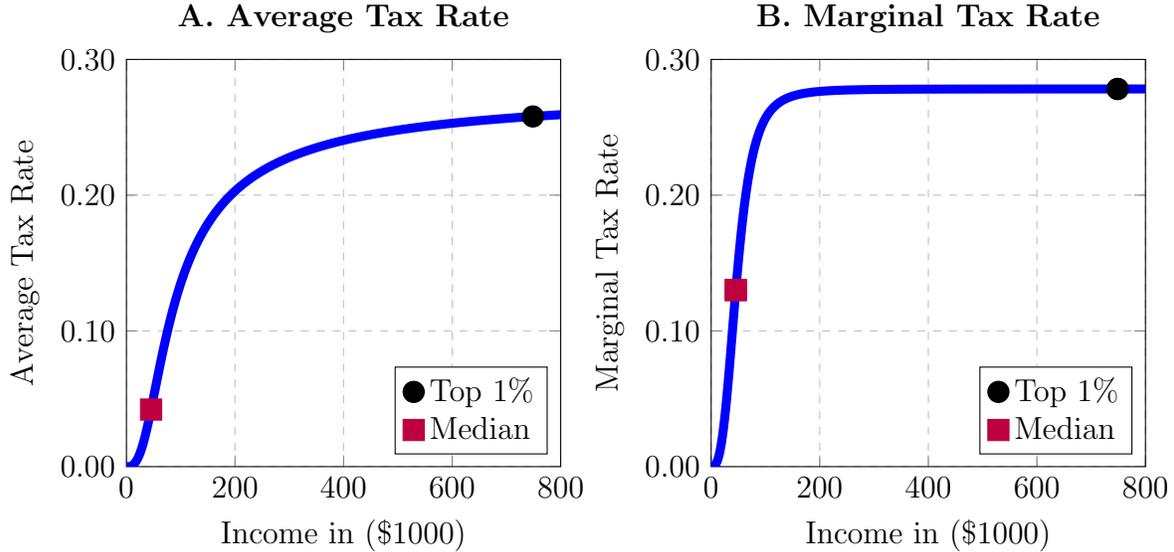
This methodological approach enables us to consider a rich equilibrium model that would otherwise be intractable to estimate. For this reason, the technique adopted in this paper is gaining popularity both in the quantitative macroeconomics literature (e.g., [Heathcote et al., 2014](#) and [Benhabib et al., 2019](#)) as well as in the corporate finance literature (e.g., [Bazdresch et al., 2018](#) and [Wang et al., 2020](#)).

4.1 Externally Set Parameters

Externally Fixed Parameters We fix two parameters consistently with the literature (see panel A, Table 1). The first of these is σ , which controls households’ risk aversion. We fix this parameter to 2, consistent with a large bulk of applied works in the life-cycle literature (e.g., [Nishiyama and Smetters, 2005](#) and [Benhabib et al., 2019](#)). Second, we fix the capital income share α , to 0.36, which is standard in the macroeconomics literature.

Then, we fix J , the maximum age in the model, to 85 and R , the retirement age, to 45. Assuming that age 1 in the model corresponds to age 21 in the real life, these choices for (J, R) correspond to ages 105 and 65 in real life/years. We set the population growth rate n to 0.7%, to be consistent with the U.S. population growth rate in the World Bank’s World Development Indicators. We obtained estimates of the survival probabilities s_j from the United States Mortality Database (see Appendix C.1 for details). Finally, we use data from the Internal Revenue Service (IRS) to set the linear social security tax, $\tau_{ss} = 12.4\%$, and the upper limit on the social security contribution, $\bar{y} = 107.7\text{k}$. Finally,

Figure 1 – Tax Function



Note: We estimate an income tax function using a measure of total income in the SCF and a measure of federal tax liabilities from NBER’s TAXSIM. Taxable income is expressed in 2010 dollars.

we internally set the parameter governing Social Security benefit (i.e., χ), to 0.305, to balance the government budget on Social Security contributions.

Externally Estimated Parameters Next, we focus on a set of parameters that we estimate outside the model (see panel B, Table 1). We estimate the parameters of the tax functions (τ_0, τ_1, τ_2) via a non-linear weighted least squares method (e.g., [Guner et al., 2014](#)). Using our SCF data, we construct a measure of income that includes all income flowing to households. We then calculate federal income tax liabilities using NBER’s TAXSIM program. See Appendix C.2 for details. Figure 1 illustrates the resulting tax function.

One important drawback of our analysis is that we consider a tax function for total income, where the tax authority does not discriminate between labor and capital income. This is mainly due to data limitation, as in microeconomic surveys like the SCF there is not a clear way to estimate distinct tax functions for labor and capital income. For this reason, our approach is canonical in the empirical public finance literature, where the elasticity of taxable income generally utilizes a broad definition of the tax base (e.g., [Saez et al., 2012](#), [Heathcote et al., 2014](#), and [Mertens and Montiel Olea, 2018](#)). However, in reality, the US tax authority taxes differently at least part of income based on its origin

(i.e., labor or capital). Our main results follow through in the alternative scenario where labor and capital incomes are taxed differently, with a progressive tax on earnings and a linear tax on capital income (see Appendix E.1 for details).

Next, we focus on the challenge of how to estimate the labor ability process in Equation (2). On the one hand, this fixed effect model would conform very well with the panel dimension of the Panel Study of Income Dynamics (PSID). The problem with this approach is that the labor earnings inequality and other inequality measures recorded in PSID are much lower than that observed on SCF – e.g., the earnings gini coefficient is more than 10 percentage points lower in PSID than in SCF. As such, by using this method, one would lose on a fundamental aspect of inequality, particularly for the top 1 percent of the distribution. Alternatively, one could only use SCF, which is more reliable for measuring earnings at the top of the distribution. The fundamental issue with this dataset is that it lacks a panel dimension, and so it would be very difficult to credibly estimate, for instance, dynamic features of the transitory idiosyncratic risk in (3).

We tackle these issues by adopting a hybrid approach between the two datasets. In particular, we start by estimating the parameters of the fourth order age-profile ($\alpha_1, \alpha_2, \alpha_3, \alpha_4$) directly using the SCF. Then, following the approach of Kaplan (2012), we recollect the process of transitory idiosyncratic risk by estimating a fixed effect model on PSID (see Appendix C.3 for details). This method relies on the assumption that the dynamics of idiosyncratic risk does not depend upon the mass in the right tail of the distribution. In this way, we estimate a persistence component, $\rho_h = 0.972$, and a standard deviation $\sigma_{\varepsilon_h} = 0.135$. Interestingly, these numbers are similar to the large body of literature estimating the process of the transitory component of labor abilities in linear Gaussian models (e.g., Guvenen et al., 2021). The remaining two parameters of the ability process, σ_e and α_0 will be internally estimated using data from the SCF (see the discussion in the next section).

4.2 Internally Estimated Parameters

We use SMM to estimate the remaining eight parameters, $(\gamma, \sigma_e, \alpha_0, \rho_r, \sigma_{\varepsilon_r}, \beta, \lambda, \delta)$. Intuitively, this consists of picking the parameters such that the moments computed from real data are as close as possible to those computed from data simulated from our model (e.g., Pakes and Pollard, 1989, Hennessy and Whited, 2007, Heathcote et al., 2014 and Benhabib et al., 2019). In particular, denoting the vector of parameters to be estimated

Table 1 – Externally Set Parameters

Parameters	Notation	Value	Std. Err.	Source
A: Fixed Parameters				
Risk Aversion	σ	2		Typical in lit.
Capital Share	α	0.36		Typical in lit.
Maximum Age	J	85		Corresp. to age 105
Retirement Age	R	45		Corresp. to age 65
Survival Prob.	s_j	Appendix C.1		USMD
Pop. Growth	n	0.007		World Bank
Soc. Sec. Tax	τ_{ss}	0.124		IRS
Soc. Sec. Cap	\bar{y}	107.7		IRS
Soc. Sec. Benefit	χ	0.311		Balanced budget
B: Estimated Parameters				
Maximal Tax Rate	τ_0	0.278	(0.003)	SCF/TAXSIM
Tax Progressivity 1	τ_1	2.85	(0.10)	SCF/TAXSIM
Tax Progressivity 2	τ_2	$1.14e^{-5}$	$(4.03e^{-6})$	SCF/TAXSIM
Ability Coef. 1	α_1	0.100	(0.014)	SCF
Ability Coef. 2	α_2	$-3.72e^{-3}$	$(1.19e^{-3})$	SCF
Ability Coef. 3	α_3	$6.37e^{-5}$	$(3.87e^{-5})$	SCF
Ability Coef. 4	α_4	$-4.20e^{-7}$	$(4.24e^{-7})$	SCF
Labor Ability Persist.	ρ_h	0.976	(0.005)	PSID
Labor Ability Std. Dev.	$\sigma_{\varepsilon h}$	0.135	(0.006)	PSID

Note: This table reports the externally set parameters. USMD stands for the United States Mortality Database. Standard errors are reported in parentheses.

by Θ , the SMM estimator solves the following minimum distance problem:

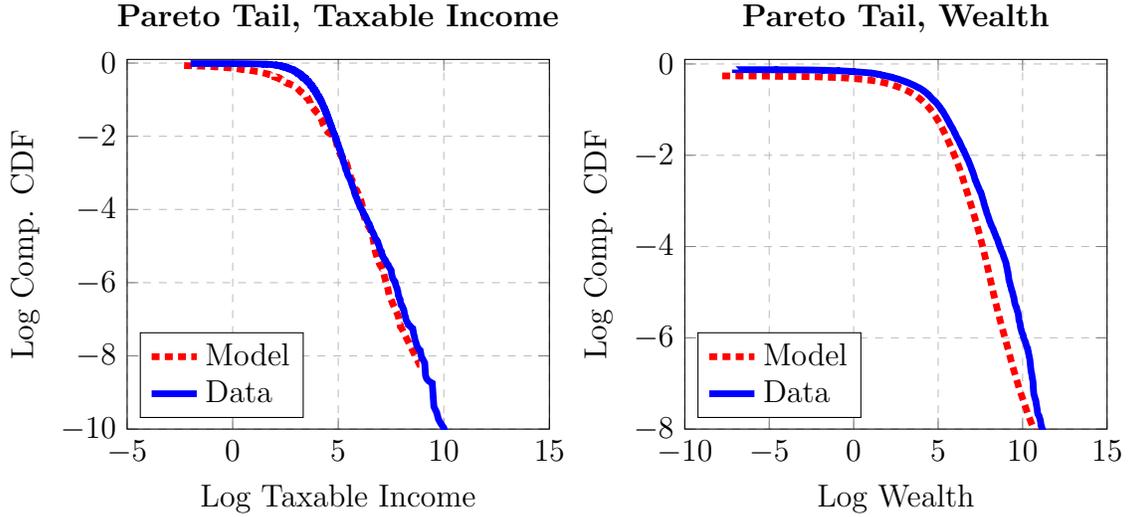
$$\hat{\Theta} = \arg \min_{\Theta} \left(\hat{M} - \hat{m}(\Theta) \right)' W \left(\hat{M} - \hat{m}(\Theta) \right), \quad (13)$$

where \hat{M} identifies the targeted cross-sectional moments from the 2016 SCF as well as macroeconomic moments from the NIPA tables and [Jordà et al. \(2019\)](#). The matrix $\hat{m}(\Theta)$ represents the moments implied by the model for a given set of parameters Θ , and W is a weighting matrix.⁴

In order to be transparent about our identification, we need to clarify how the param-

⁴We freely picked the weighting matrix W . In particular, we assumed the off-diagonal elements are all zero. For the diagonal elements, we assume $W_{ii} = 1/\hat{M}_i^2$, where \hat{M}_i is data moment i . This approach is common in the literature, in light of the Monte Carlo results presented by [Altonji and Segal \(1996\)](#), who argue that in standard applications there is a non-negligible small sample bias when using the optimal weighting matrix.

Figure 2 – Fat Tail: Model vs. Data



Note: This figure plots the complimentary cumulative distribution of taxable income and wealth, in the model and the data (SCF).

eters are relevant for individual moments. The preference parameter γ governs the utility weight of consumption. This parameter is useful for matching labor supply moments, such as average hours. The parameter σ_e is the standard deviation of permanent labor ability and it assists in matching the distribution of earnings, given the estimation of labor productivity risk. The parameter α_0 is the constant term in the ability profile and is useful for capturing average earnings. The parameters $(\rho_r, \sigma_{\epsilon_r})$ govern the capital income risk faced by individuals. These parameters are crucial for matching the right tails of wealth and taxable income, as well as the share of entrepreneurs. The parameter β is the discount factor and, as in the canonical macroeconomics literature, it assists in capturing the economy-wide capital-income-ratio. The parameter λ governs the collateral constraint on the leverage-ratio and it helps in matching the risk-free rate. Finally, δ is the capital depreciation rate and it is used to match the economy-wide investment-to-output ratio.

In order to compute the standard errors for our estimated parameters and thus to conduct inference, we need to obtain the variance-covariance matrix of the data moments. This represents a challenge, since the correlation between cross-sectional household level data and time-series data is generally unobservable. We get around this problem by adopting the approach presented in [Cocci and Plagborg-Møller \(2021\)](#), whereby the standard errors are computed using the worst-case correlations for the unknown covariances. See [Appendix D](#) for further details.

Table 2 – Estimated Parameters and Targeted Moments

Parameters	Notation	Value	Std. Err.
Utility Cons. Weight	γ	0.362	0.006
Labor Ability PC	σ_e	0.985	0.030
Labor Ability Constant	α_0	2.741	0.069
Return Persistence	ρ_r	0.968	0.008
Return Shock	$\sigma_{\varepsilon r}$	0.172	0.007
Discount Factor	β	0.989	0.005
Coll. Constraint	λ	3.037	0.104
Depreciation Rate	δ	0.050	0.002

Moments	Model	Data
<i>Cross-Sectional Moments</i>		
Average Hours (working age)	0.299	0.304
Entrepreneurship Rate	0.087	0.085
Wealth Gini	0.862	0.860
Wealth Share, Top 1%	0.406	0.386
Wealth Share, Top 5%	0.638	0.651
Wealth Share, Top 20%	0.888	0.883
Earnings Gini	0.735	0.680
Earnings Share, Top 1%	0.145	0.172
Earnings Share, Top 5%	0.363	0.327
Earnings Share, Top 20%	0.687	0.605
Average Earnings	54.83	55.30
Tax Revenue Share, Top 1%	0.419	0.424
Tax Revenue Share, Top 5%	0.702	0.659
Tax Revenue Share, Top 20%	0.959	0.881
Wealth-Income Slope, Top 20%	1.574	1.638
Wealth-Income Slope, Top 40%	0.915	0.959
Wealth-Income Slope, Top 60%	0.693	0.717
<i>Macroeconomic Moments</i>		
Capital-to-output Ratio	2.947	2.950
Investment-to-output Ratio	0.220	0.222
Borrowing Rate	0.019	0.019

Note: The top panel reports the estimated parameters with “worst case” standard errors (see [Cocci and Plagborg-Møller, 2021](#)), while the bottom panel reports the moments in the model and the data. The model parameters are estimated via Simulated Method of Moments (SMM). Cross-sectional moments are from the 2016 Survey of Consumer Finances (SCF), while macroeconomic moments are from national statistics and [Jordà et al. \(2019\)](#).

The estimated parameters are reported in the top panel of Table 2, while the moments are reported in bottom panel of the same table. All parameters are statistically different from zero and precisely estimated. This finding is not obvious and shows a tight link between the targeted moments and structural parameters. As parameter identification in SMM requires choosing moments whose predicted values are sensitive to the model’s underlying parameters, the results presented here indicate that we picked the right targets.

Furthermore, the model does very well in matching the moments from the SCF. It matches the wealth gini and the wealth shares of the wealthiest top 1, 5 and 20 percent, respectively. Similarly, our model matches the right tail in the distribution of taxable income and earnings. Interestingly, our model captures the wealth-income slope, average hours worked, the entrepreneurial rate as well as average earnings. Our model also matches almost perfectly the macroeconomic targets, such as the capital- and investment-to-GDP ratios and the market borrowing rate. The key ingredients of our model (wealth return risk, earnings risk, fixed labor productivity heterogeneity and progressive taxation) are crucial for matching the distributional and macroeconomic moments of interest presented in Table 2.

On this last point, Appendix E.3 shows how the model performs when each of its key ingredients is shut down in isolation. Briefly, return heterogeneity is important for both wealth and income concentrations. Earnings risk is important both for matching the concentration of labor income in the right tail of the distribution and as well to fine tuning the wealth concentration in the top 1 percent. Progressive taxation is crucial for matching the tax revenue shares along the income distribution and to avoid too much wealth accumulation in the right tail. Overall, it is interesting to see that these ingredients ensure our model successful in generating the (untargeted) fat Pareto tails of the taxable income and wealth distributions observed in the data (see Figure 2).

Most estimated parameters have values that are, broadly speaking, consistent with those found in the literature. This is the case for the discount factor β , the utility parameter γ , the collateral constraint λ and the depreciation rate δ . We have no good prior for the parameters governing the return profiles, although recent quantitative studies point to substantially persistent and moderately variable processes (e.g., Guvenen et al., 2019 and Xavier, 2020). As such, our estimates are consistent with these results.

Differently, the parameter governing the variability of the labor ability permanent component, σ_e , is larger than it is generally estimated in the literature using PSID. On the one hand, this may simply reflect that earnings inequality recorded in SCF is much greater than measured in PSID. Given we estimate the transitory component from PSID

and the permanent component from SCF, σ_e mechanically absorbs the residual variation in earnings. On the other hand, similar variability in the permanent component of labor abilities has been recently recorded in applied works using detailed administrative data, when estimating linear Gaussian models (e.g., [Guvenen et al., 2021](#)). This said, one might rightly wonder whether we are assigning too much variability to the permanent component of earnings and how this could affect/bias our results. It turns out that the variability in the permanent component of labor abilities dampen the sensitivity of income to marginal tax changes. This effect appears to be particularly strong at the top of the income distribution. In other words, amplifying the transitory stochastic component of the earnings process would increase ETI at the top of the distribution and for all taxpayers. As such, our results are conservative in this sense.⁵ See Appendix [E.2](#) for a detailed description of this case.

4.3 Model Fit with Untargeted Moments

In order to increase the credibility of our estimates and our transmission mechanism, we run three external validity exercises on untargeted statistics. First, we show that the return profiles at the top of the wealth distribution generated by our model are consistent with those found in the data. Second, we show that our model matches the capital income shares at the top of the income distribution. And third, we show that our model produces short-run ETIs and multipliers that are consistent with the empirical evidence.

Return Profiles The first aspect of our framework that we want to assess is to what extent the return heterogeneity necessary to capture distributional moments in income and wealth is reasonable and consistent with the empirical evidence on wealth returns. Comparing our model-based return profiles with the data is problematic. On the one hand, US-based data in the SCF lacks of a long-span panel dimension, and therefore one cannot fully measure the entirety of wealth returns such as unrealized capital gains. On the other hand, countries that provide panel data dimension on wealth returns (e.g., Norway) show that the portfolio composition of wealth (particularly at the top of the distribution) is quite different than in the US.⁶

⁵Intuitively, the more important the transitory stochastic part of labor earnings, the larger the precautionary saving effect of marginal tax changes at the top of the distribution.

⁶For example, according to [Fagereng et al. \(2020\)](#), in Norway, the share of housing in gross wealth held by the 95-99 percentiles (99-99.9 percentiles) is 0.73 (0.44). Meanwhile, it is only 0.33 (0.25) in the US (as reported in the 2016 SCF).

Table 3 – Return Profiles (Untargeted)

Wealth Percentile	Returns (Model)	Returns (Data)
[99-100]	0.071	0.074
[95-99)	0.039	0.066
[90-95)	0.033	0.059
[75-90)	0.029	0.053
[50-75)	0.025	0.049
[25-50)	0.021	0.040
[10-25)	0.019	0.021
[1-10)	0.021	0.028

Note: This table reports the resulting wealth returns by wealth percentile in the model and the data. For the data, we estimate the average return for each household’s portfolio in the SCF using estimates of the average returns of different asset types between 1990 and 2016, as reported by [Xavier \(2020\)](#).

To partly overcome these issues, we compute returns by wealth percentile in the SCF. Briefly, we estimate the return for each household’s portfolio in the SCF using outside estimates of the return on individual asset classes. We use the estimates of the average returns of different asset types between 1990 and 2016, as reported by [Xavier \(2020\)](#). [Table 3](#) presents the returns by wealth percentiles in the model and in the data. See [Appendix C.4](#) for further details.

A few considerations are in order. First, qualitatively, our model captures the positive correlation between wealth and wealth returns. The mechanism behind this effect is clear. Agents (entrepreneurs) enjoying high productivity in capital income have an incentive to accumulate larger wealth. This is important as the correlation between returns and wealth size is a robust feature of recent empirical studies (e.g., [Benhabib et al., 2019](#); [Fagereng et al., 2020](#); [Bach et al., 2020](#)). Second, the consistency of our model-implied returns at the bottom 25 percent of the wealth distribution and, crucially, in the top 1 percent is striking, since these moments were not targeted in the estimation exercise. We also find that the imputed returns from the SCF seem to be higher than those implied by the model in the middle of the wealth distribution. This is mostly due to returns related to housing for which the model abstracts. All in all, the main take home from this exercise is that the return profile generated by our model is generally consistent with the empirical evidence, particularly at the top of the distribution.

Capital Income Shares A second potential concern about our quantitative exercise is whether our model captures the capital income shares along the income distribution

Table 4 – Capital Income Share Along Income Distribution for 2016 (Untargeted)

	Model	Data
All Taxpayers	0.24	0.28
Income Top 1%	0.60	0.61
Income Top 5%	0.39	0.46
Income Top 10%	0.32	0.41
Income Bottom 90%	0.10	0.18

Note: Updated data series from [Piketty et al. \(2018\)](#), available on Gabriel Zucman’s website at <https://gabriel-zucman.eu/usdina/>. Aggregate income is GDP minus depreciation of capital. Capital income includes: i) income from equity; ii) net interest payments; iii) income from housing rents; iv) capital component of mixed income; v) property income paid to pensions.

found in the data. This is important, as recent empirical studies estimate that in the US, the dynamics in income concentration over the past three decades are mainly driven by a boom in capital income at the top (e.g., [Piketty et al., 2018](#)). Table 4 reports the capital income shares of the model with the ones in the data. Reassuringly, the model implies shares of capital income in aggregate and along the distribution that are broadly consistent with those found in the data. This is particularly striking for the richest 1 percent. Overall, this confirms the ability of our model to provide a realistic explanation of the distributional features of the US economy and as such to provide a suitable laboratory for sound policy analysis.

Short-Run Elasticities Our main policy experiment in Section 5 will evaluate the long-run effects of a permanent tax change. But first, to give credibility to our estimates for the long run, we evaluate the implications of our estimated model for the short-run effects of a temporary change in marginal taxes, where we can more readily compare our model’s implications to existing empirical evidence. Evaluating our model in such a dimension is challenging, as our framework has been primarily developed for long-run analysis. As such it misses some obvious ingredients and frictions, that have been shown to be pivotal for analyzing tax policies in the short-run. For example, our model abstracts from investment adjustment costs and, perhaps more crucially, from price and wage rigidities (e.g., [McKay and Reis, 2016](#) and [Bayer et al., 2020](#)).

In order to partly overcome these issues, we simulate an unexpected and temporary change in the marginal tax rate (via a change in τ_0), assuming that such a tax change lasts for five years. Any fiscal need is satisfied via a lump-sum. Then we calculate the transition response of our model under two alternative assumptions about price adjustments. First,

Table 5 – Effects of Marginal Tax Changes in the Short-Run

	General Equilibrium	Partial Equilibrium
A. Short-Run Multiplier (GDP)		
Impact	1.15	1.56
Following Year	1.40	1.81
After Two Years	1.28	1.52
B. Short-Run ETI		
Impact	0.55	0.69
Following Year	0.65	0.77
After Two Years	0.56	0.61

Note: GDP Multiplier is calculated as GDP dollar change over the dollar change in fiscal revenues.

we calculate the equilibrium transition path under the assumption that prices fully adjust in an internally consistent, general equilibrium fashion. Second, we compute the same policy experiment under partial equilibrium, where prices do not adjust along the transition. Interestingly, this latter scenario is commonly used in the public finance literature to compute short-run elasticities in life-cycle economies (e.g., [Attanasio et al., 2018](#) and [Kindermann and Krueger, 2020](#)). In our interpretation, it can represent a reduced-form version of a model with nominal rigidities.

For each scenario, we compute two standard statistics for short-run evaluation of fiscal policy. The first statistic is the tax multiplier, commonly used in the macroeconomics literature (e.g., [Romer and Romer, 2010](#) and [Cloyne, 2013](#)). This measure captures the GDP increase (in dollars) for each dollar decrease in fiscal revenues that the marginal tax cut brings about. The second measure we compute is the elasticity of taxable income (ETI) along the transition path with respect to net-of-tax rates (1 minus the average marginal tax rate) on impact and is commonly adopted by the public finance literature (e.g., [Saez et al., 2012](#)). We will discuss this measure in more detail in [Section 5](#). Results from this exercise are reported in [Table 5](#).

The first result worth noting is that in all cases the effects of a marginal tax change are substantially larger in partial equilibrium. This result stems from the fact that in general equilibrium, factor price adjustments due to the increase in labor supply and savings offset part of the expansionary effects of the tax policy.

Second, we find that our structural model produces short-run tax multipliers, under both assumptions about prices, that are consistent with the empirical evidence of the US economy, that generally estimate multipliers centered between 1 and 2, one year after

the tax reform. For instance, [Barro and Redlick \(2011\)](#) report marginal tax multipliers between 0.8 and 1.4, while [Mertens and Ravn \(2013\)](#) estimate somewhat higher marginal tax multipliers between 1 and 2.5.

Third, differently from the tax multiplier, the public finance literature has not reached a firm consensus about short-run ETIs. Estimates vary from as little as close to zero (e.g., [Saez et al., 2012](#) and [Romer and Romer, 2014](#)), to large values above one (e.g., [Feldstein, 1995](#) and [Mertens and Montiel Olea, 2018](#)). Our short-run ETIs are between 0.55 and 0.65 in general equilibrium and between 0.61 and 0.77 in partial equilibrium. Thus, they lie in the middle of the spectrum and are similar, for example, to the ETIs of around 0.57 estimated by [Kumar and Liang \(2020\)](#). At the same time, they are also consistent with the somewhat wide estimates of [Mertens and Montiel Olea \(2018\)](#) who report ETIs one year after the tax change in the range of 0.5-2.5. Overall, the results from this exercise show that the transmission mechanism of our framework in the short-run is reasonable and resembles a wide range of available estimates in the applied literature both in public finance and in macroeconomics.

5 The Tax Policy Exercise

We use our structural model to analyze the long-run distortionary effects of a change in marginal income tax rates. In particular, the main policy experiment we consider consists of measuring the steady-state effects of a permanent change in τ_0 , assuming that government spending adjusts accordingly. This type of tax changes mimics various policy reforms implemented by the US federal government and affects the marginal tax rates of all income groups. In this sense, our policy experiment differs from [Guner et al. \(2016\)](#), [Kindermann and Krueger \(2020\)](#) and [Badel et al. \(2020\)](#), who instead focus on the effects of changing taxes for the top 1 percent of the income distribution.⁷

ETI and Policy Elasticity We analyze the effects of our tax change by computing the elasticity of taxable income (y) with respect to net-of-tax rates (1 minus the average marginal tax rate) in the long run:

$$ETI = \frac{d \ln y}{d \ln(1 - AMTR)}.$$

⁷In Appendix F, we analyze the effect of a reform which increases marginal tax rates only for the top 1 percent. In this scenario, we obtain similar results, but marginal tax changes are more distortionary. In this sense, the exercise presented here is more conservative from a quantitative point of view.

This measure can also be interpreted as a *policy elasticity* in the sense of [Hendren \(2016\)](#). As described in [Saez et al. \(2012\)](#), in settings without fiscal externalities and income shifting, like the one under consideration here, the ETI represents a sufficient statistic to evaluate the efficiency effects of a tax change. As such, it is a fundamental measure for policy analysis that can be used in wide range of applications.⁸

We proceed by explaining our results as follows. First, we study how the ETI varies for different income sources (i.e., capital and labor), and how it varies along the age, income distribution and whether or not a household decides to run a business. Second, we study the aggregate implications of marginal tax rate policies, particularly for prices, macroeconomic aggregates, productivity and misallocation. Third, in order to isolate the role of various ingredients of our model, we compare our benchmark economy with alternative scenarios where we separately shut down the key ingredients of the model (e.g., return heterogeneity, earnings risk and tax progressivity). Finally, we analyze the effects of a marginal tax rate change in an alternative life-cycle model calibrated to match the same moments as the benchmark (see [Table 2](#)) through a superstar earnings state (e.g., [Castañeda et al., 2003](#)).

5.1 Benchmark Estimates

We start by reporting the benchmark estimates and confidence bounds for the long-run effects of marginal tax rate changes. [Table 6](#) presents the ETIs along the income distribution ([Panel A](#)), by type of income ([Panels B and C](#)), by age ([Panel D](#)) and by occupation ([Panel E](#)). A few results are worth stressing. First, all measures of interest (e.g., ETIs) are statistically different from zero and precisely estimated. This is reassuring and brings further evidence in favor of our identification strategy. This said, our estimates also show heterogeneous degrees of uncertainty. For example, estimates about elasticities in the top 1 percent of the distribution come with wider confidence bounds than those in the bottom 99 percent. Similarly, we find that the ETI for entrepreneurs is less precisely estimated than those for the rest of population. This finding reflects how the uncertainty in the measurement of the data (e.g., the entrepreneurial rate) translates into the distribution of our statistics of interest (e.g., ETIs).

The second important result is that while the ETIs are substantial and positive for all taxpayers, they are largest for households in the top 1 percent of the income distribution.

⁸In [Appendix G](#), we show how the ETI can be related to the amount of revenue the government loses to behavioral responses.

This is because the combination of return heterogeneity with earnings risk and progressive taxation increases the incentive to save and invest after a cut in marginal tax rates, but mainly for high-return agents in the top of the income distribution, whose capital income share is 60 percent (see Table 4). As a consequence of this, the elasticity of capital income is almost three times higher in the top income group (0.95) than for the average taxpayer (0.36) and the ETI of entrepreneurs is around 37 percent higher than the rest of the population (0.78 vs. 0.57, see Panel E of Table 6).

Table 7 shows that marginal tax changes also affect macroeconomic aggregates, such as aggregate quantities, prices and TFP. For example, a 1 percent increase in net-of-marginal tax rates increases aggregate productivity by 0.16 percent and entrepreneurial productivity by 0.45 percent. Moreover, the supply of aggregate capital increases (+1.13), leading to a decrease in both the price of capital (-0.86) and the borrowing rate (-3.08). For this reason, agents in bottom of the distribution whose capital income mainly consists of lending at the riskless rate, have a negative elasticity of capital income (e.g., -4.02 for the bottom 90%). Furthermore, the increase in aggregate productivity endogenously increases the response of real wages (+0.48). This effect boosts the elasticity of earnings, but mostly for agents in the bottom of the distribution whose income is mainly derived from labor (see Table 4). Consistently, we find that the elasticity of earnings decreases monotonically with income. Agents in the top 1 percent display an elasticity of earnings (0.51) that is less than half of those in the bottom 90 percent (1.26).

The estimated elasticities by age display a U-shape pattern. ETIs appear larger for younger households and for older ones. This is mostly driven by income composition by age and earnings life-cycle profiles. Young agents are generally wealth poor (agents enter the economy with no wealth). Furthermore, given their age-dependent productivity profiles, young agents have relatively low labor income. These two combined effects make young agents more likely to be in the bottom of the income distribution, where earnings elasticities are high. As households age, their labor productivity increases, making them richer. This effect tends to decrease ETIs as households age, via lower earnings elasticities. At the same time, agents start accumulating wealth throughout their life cycle. In isolation, this latter effect pushes up households' capital income elasticities. The tension of these two effects implies decreasing ETIs for age groups where labor income is predominant (i.e., ages 21-50), while it starts increasing in age groups where capital income become mores important (i.e., ages 51-64 and retirement).

Table 6 – Elasticities of an Income Tax Change

	Point Estimates	95% Bands
A. ETI		
All Taxpayers	0.66	[0.64 0.68]
Income Top 1%	0.77	[0.67 0.87]
Income Top 5%	0.62	[0.57 0.67]
Income Top 10%	0.60	[0.55 0.65]
Income Bottom 99%	0.55	[0.49 0.61]
Income Bottom 90%	0.70	[0.54 0.86]
B. Elasticity of Capital Income		
All Taxpayers	0.36	[0.30 0.42]
Income Top 1%	0.95	[0.77 1.12]
Income Top 5%	0.74	[0.61 0.88]
Income Top 10%	0.64	[0.50 0.77]
Income Bottom 99%	-1.76	[-2.49 -1.03]
Income Bottom 90%	-4.02	[-5.74 -2.31]
C. Elasticity of Earnings		
All Taxpayers	0.72	[0.70 0.75]
Income Top 1%	0.51	[0.48 0.54]
Income Top 5%	0.53	[0.52 0.55]
Income Top 10%	0.57	[0.55 0.59]
Income Bottom 99%	0.82	[0.79 0.86]
Income Bottom 90%	1.26	[1.14 1.38]
D. ETI by Age (Years)		
Ages 21-30	0.93	[0.87 0.98]
Ages 31-40	0.56	[0.45 0.66]
Ages 41-50	0.53	[0.47 0.59]
Ages 51-64	0.66	[0.63 0.70]
Ages 65+	0.73	[0.59 0.87]
E. ETI by Occupation		
Entrepreneurs	0.78	[0.59 0.97]
Rest of Population	0.57	[0.47 0.67]

Note: We report the elasticities with respect to the average net of marginal tax rate. The elasticity for variable X is defined as $d \ln X / d \ln(1 - AMTR)$. Earnings is total labor income. Income is total taxable income.

Table 7 – Effects of an Income Tax Change

Variables	Point Estimates	95% Bands
<u>A. Prices</u>		
Real Wage, w	0.48	[0.47 0.50]
Price of Capital, p	-0.86	[-0.88 -0.83]
Borrowing Rate, r	-3.08	[-4.30 -1.86]
<u>B. Aggregate Quantities</u>		
Output, Y	0.72	[0.70 0.75]
Capital, K	1.13	[0.98 1.28]
Labor, L	0.24	[0.21 0.27]
<u>C. Productivity</u>		
Aggregate TFP	0.16	[0.10 0.22]
Entrepreneurial Productivity	0.45	[0.29 0.61]
Entrepreneurial Rate	-1.87	[-2.72 -1.02]

Note: We report the elasticities with respect to the average net of marginal tax rate. The elasticity for variable X is defined as $d \ln X / d \ln(1 - AMTR)$.

5.2 Analyzing the Mechanism

As discussed before, one of the main advantages of our structural identification is that it permits us to be as transparent as possible about how the different features of our general equilibrium model affect the transmission of tax policies on the economy. In order to do this, we now measure the effects of a marginal tax rate change by selectively removing individual ingredients from the benchmark framework. Results from this exercise are reported in Tables 8, 9 and 10.

From these exercises, we will show that three key elements (return heterogeneity, earnings risk and progressive taxation) are crucial for explaining the size of the overall ETI, and also why they differ by income and age. In particular, we will show that these three ingredients explain why elasticities are so large for top incomes. From this, one of the main policy lessons of our study is that the combination of capital income, earnings risk and progressive taxation is crucial for precise quantitative assessment of fiscal policy, particularly for the distributional impact of marginal tax policies.

The Role of Return Heterogeneity In this section, we present the results from an alternative model, in all parts isomorphic to the benchmark, but with no return hetero-

geneity. In this alternative economy, all agents have the same entrepreneurial productivity. As a result, everyone is indifferent between being an entrepreneur or not, and thus all households earn the same return on their wealth, r . Thus, this setting is equivalent to the standard case with a single final-good production sector. Comparing this model without return heterogeneity to our benchmark economy will isolate the effect of return heterogeneity on the ETI.

The main result from this exercise is that return heterogeneity substantially affects the transmission mechanism of marginal tax policies. From a quantitative point of view, return heterogeneity increases the ETI for all taxpayers from 0.49 to 0.66 (+34%). This effect is strongest for the top 1 percent of the income distribution (+111%), but it is also high for the bottom 90 percent (+18%), with the smallest increase accruing to the bottom 99 percent (+5%). In the model without return heterogeneity, the ETI decreases monotonically with income. In contrast, our benchmark economy exhibits a U-shaped relationship between the ETI and income, with the top 1 percent of the income distribution displaying the highest ETI.

Comparing the two models, it can be seen that return heterogeneity amplifies the ETI for the top 1 percent because it increases the response of their capital income to marginal tax changes. It also shifts the composition of their income towards capital, where the elasticity is relatively higher. Intuitively, a cut in marginal taxes will generate an accumulation of wealth, most concentrated among high-return individuals at the top of the income distribution. In the benchmark economy, earnings are more elastic at the top as well because agents with high entrepreneurial ability react to a tax cut by working relatively more in order to accumulate more assets, relax their financial constraint and expand their backyard production.

To a smaller degree, return heterogeneity also amplifies the ETI at the bottom of the income distribution, partly because it boosts the general equilibrium response of wages. With return heterogeneity, a cut in marginal tax rates leads to an increase in TFP, and this leads to a larger general equilibrium increase in wages (see Table 9). Furthermore, for the bottom 90 percent, the composition of income shifts more towards labor income (where the elasticity is relatively higher), which further increases the ETI at the bottom.

Lastly, we turn to the effect of return heterogeneity on the ETI by age (see panel D, Table 8). Overall, there is a U-shaped relationship between the ETI and age, a pattern which is amplified in the benchmark model with investment risk. Specifically, return heterogeneity increases the ETI from 0.59 to 0.93 for young individuals (age 21-30). For those near retirement (age 51-64), return heterogeneity increases the ETI from 0.53 to

0.66, and generates an even larger response for retired individuals (age 65+). With return heterogeneity, marginal tax changes generate larger responses in labor earnings, particularly for the young and those nearing retirement. For retired individuals (age 65+), return heterogeneity generates a larger response of capital income to tax changes. Therefore, return heterogeneity is essential for explaining the sharp U-shaped relationship of the ETI by age.

The Role of Earnings Risk In this section, we analyze the effects of earnings risk for the transmission mechanism of marginal tax changes. Specifically, we consider an alternative economy where agents face no idiosyncratic risk in labor earnings. All other features (e.g., return heterogeneity, tax progressivity, etc.) are kept the same as in the benchmark economy. Agents populating this alternative economy are still heterogeneous in their permanent labor productivity, and in their deterministic life-cycle labor ability profiles.⁹ Comparing this model without earnings risk to our benchmark economy will isolate the effect of earnings risk on the ETI.

The main result from this exercise is that earnings risk has very little effect on the ETI for all taxpayers, but has big effects along the income distribution (see Table 8). In particular, relative to the setting with no earnings risk, the ETIs in the benchmark economy increase for all the top income groups, while they decrease in the bottom of the income distribution. This highlights the important role for earnings risk, in addition to return heterogeneity, in explaining the high elasticities at the top. In fact, absent earnings risk, the top 1 percent would display the smallest elasticities.

Interestingly, earnings risk has little effect on earnings elasticities, but a big distributional impact on the elasticities of capital income. Specifically, earnings risk doubles the elasticity of capital income for the top 1 percent, while decreasing the elasticity of capital income at the bottom (i.e., making it more negative). Mechanically, in our benchmark economy, top incomes have a larger overall ETI because their capital income elasticities are higher. This effect operates partly through the larger response of entrepreneurs in the top 1 percent, whose ETI is 73 percent higher in the benchmark economy.

Intuitively, earnings risk strengthens the response of precautionary savings following a marginal tax change, particularly for high-return entrepreneurs with top incomes. For example, a cut in marginal taxes will reduce the insurance benefit provided by progressive income taxes. In the benchmark economy, households in the top 1 percent will have

⁹For sake of brevity, we analyze the alternative setting with no fixed-effect in labor productivity in Appendix E.2.

a stronger precautionary savings motive, not only because they face earnings risk but also because they have a higher labor income share (see Table 10). As a result, these households react to a cut in marginal taxes by increasing their precautionary savings in order to better insure themselves from a negative earnings shock. Entrepreneurs are particularly responsive, as a negative earnings shock will reduce the resources available to invest in their own business.

In the same fashion, the role of earnings risk on the ETI along the life cycle is now clear. It increases the response to marginal tax changes for those income groups relatively more wealth rich and more dependent on capital income (age 65+). At the same time, it decreases the response to marginal tax changes for those age groups relying on labor income (age 21-50). Interestingly, earnings risk increases the elasticities for retired individuals, despite the fact that retired individuals do not face any earnings risk in either model. The important factor here is the different general equilibrium response in the two models (e.g., entrepreneurs see a larger decrease in their borrowing rate in the model with earnings risk).

The Role of Progressive Taxation In this section, we discuss the importance of the progressivity of the income tax schedule. In order to do so, we repeat the benchmark policy experiment, but assume that all agents face the same marginal tax rate (i.e., we impose linear taxes on income). We impose that the new linear tax raises the same fiscal revenues as in the benchmark case. The rest of the model remains unchanged. Comparing this model with flat taxes to our benchmark economy will isolate the effect of progressive taxation on the ETI.

The main result from this exercise is that progressive taxes increase the response to a tax change, both at the top and bottom of the income distribution. As a result, the economy-wide ETI is 60 percent higher in the benchmark economy (0.66 vs. 0.41). The higher ETIs at the top are driven mainly by higher capital income elasticities, while the higher ETIs at the bottom are mainly driven by higher earnings elasticities. Quantitatively, the effect of progressive taxes is strongest for the bottom 90 percent (+88%), but is also high for the top 1 percent (+72%), with the smallest increase accruing to incomes in the top 10 percent (+45%). Thus, progressive taxes are crucial for understanding why ETIs display a U-shaped pattern with income.

Intuitively, progressive taxes increase the distortionary effects of taxation. As a result, agents react more to tax changes, as households can control their marginal tax rate by adjusting their income. This is reflected on the elasticity of earnings (mainly for low-

Table 8 – Effects of an Marginal Income Tax Change

Variable	Bench.	No Ret. Het.	No Earnings Risk	Linear Taxes	Earnings Superstate
<u>A. ETI</u>					
All Taxpayers	0.66	0.49	0.67	0.41	0.43
Income Top 1%	0.77	0.37	0.49	0.45	0.29
Income Top 5%	0.62	0.40	0.54	0.42	0.35
Income Top 10%	0.60	0.43	0.53	0.41	0.38
Income Bottom 99%	0.55	0.53	0.70	0.38	0.50
Income Bottom 90%	0.70	0.59	1.05	0.37	0.56
<u>B. Elasticity of Capital Income</u>					
All Taxpayers	0.36	0.01	0.39	-0.01	-0.16
Income Top 1%	0.95	0.78	0.47	0.38	0.29
Income Top 5%	0.74	0.76	0.49	0.26	0.35
Income Top 10%	0.64	0.70	0.42	0.19	0.35
Income Bottom 99%	-1.76	-0.13	-1.02	-1.20	-0.45
Income Bottom 90%	-4.02	-0.96	-2.06	-1.56	-1.44
<u>C. Elasticity of Earnings</u>					
All Taxpayers	0.72	0.62	0.74	0.52	0.57
Income Top 1%	0.51	0.29	0.51	0.55	0.29
Income Top 5%	0.53	0.33	0.57	0.53	0.34
Income Top 10%	0.57	0.37	0.58	0.51	0.38
Income Bottom 99%	0.82	0.71	0.86	0.52	0.71
Income Bottom 90%	1.26	1.18	1.36	0.53	1.14
<u>D. ETI by Age (Years)</u>					
Ages 21-30	0.93	0.59	0.96	0.48	0.52
Ages 31-40	0.56	0.45	0.62	0.34	0.41
Ages 41-50	0.53	0.47	0.63	0.38	0.42
Ages 51-64	0.66	0.53	0.67	0.53	0.48
Ages 65+	0.73	0.18	0.54	0.12	-0.16
<u>E. ETI by Occupation</u>					
Entrepreneurs	0.78	-	0.45	0.46	-
Rest of Population	0.57	-	0.75	0.38	-

Note: We report the elasticities with respect to the average net of marginal tax rate. The elasticity for variable X is defined as $d \ln X / d \ln(1 - AMTR)$.

Table 9 – Macroeconomic Effects of a Marginal Income Tax Change

Variable	Bench.	No Ret. Het.	No Earnings Risk	Linear Taxes	Earnings Superstate
A. Prices					
Real Wage, w	0.48	0.31	0.47	0.47	0.32
Price of Capital, p	-0.86	-0.55	-0.83	-0.83	-0.57
Borrowing Rate, r	-3.08	-1.16	-1.51	-3.61	-1.29
B. Quantities					
Output, Y	0.72	0.62	0.75	0.52	0.57
Capital, K	1.13	1.17	1.26	1.18	1.14
Labor, L	0.24	0.31	0.28	0.06	0.24
C. Productivity					
Aggregate TFP	0.16	0.00	0.12	0.06	0.00
Entrep. Productivity	0.45	-	0.32	0.16	-
Entrep. Rate	-1.87	-	-2.84	-0.91	-

Note: We report the elasticities with respect to the average net of marginal tax rate. The elasticity for variable X is defined as $d \ln X / d \ln(1 - AMTR)$.

income households), as well as on that of capital.

Looking at how ETIs vary with age, interestingly, we see that progressive taxes help explain the strong U-shaped relationship of ETIs and age. For young households, who are more reliant on labor income and more likely to have lower incomes, progressive taxes increase earnings elasticities. For older households, who are more likely to be reliant on capital income (especially retirees), progressive taxes increase capital income elasticities.

And finally, progressive taxes amplify the macroeconomic effects of a marginal tax change (see Table 9). Relative to the economy with flat taxes, the response of aggregate output in the benchmark model is 38 percent higher. This result is mainly driven by the much larger effect on aggregate labor supply. Moreover, the progressivity in the tax schedule implies a larger reallocation of capital towards high-productivity agents. As a result, the increase in TFP that a tax cut brings about is much larger in the benchmark.

5.3 The Earnings Superstate Model

In this section, we repeat the main policy exercise – namely, a cut in aggregate marginal tax rates, within a different model of the right tail of the wealth and income distributions. This alternative setting features the same life-cycle structure of the benchmark case

Table 10 – Capital Income Shares Along the Taxable Income Distribution

Income Percentile	Bench.	No Ret. Het.	No Earnings Risk	Linear Taxes	Earnings Superstate
All Taxpayers	0.24	0.21	0.26	0.24	0.20
Income Top 1%	0.60	0.16	0.71	0.62	0.24
Income Top 5%	0.39	0.16	0.45	0.40	0.19
Income Top 10%	0.32	0.16	0.37	0.33	0.18
Income Bottom 99%	0.11	0.22	0.09	0.08	0.18
Income Bottom 90%	0.10	0.27	0.08	0.08	0.22

Note: We report the capital income shares along the income distribution. Aggregate income is GDP minus depreciation of capital.

(incomplete markets and progressive taxation) but all households face the same return on capital. However, workers, with a low probability, can move to a transitory earnings superstate that increases their labor earnings by several times the median (see [Castañeda et al., 2003](#)). As shown by a large literature (e.g., [Kindermann and Krueger, 2020](#)), this feature, combined with the ingredients of the benchmark economy, allows the model to match the high income concentration and the even higher wealth concentration at the top of the distribution.

In particular, in our numerical solution, we augment our benchmark specification with an earnings superstate. That is, the gridpoints for the transitory labor ability shocks z_h are $\mathcal{Z}_h = \{z_1, \dots, z_{n-1}, z_n\}$, with the earnings “superstate” $\log z_n = \log z_{n-1} + \bar{e}$, with $\bar{e} > 0$. We then set $\{z_1, \dots, z_{n-1}\}$ and the transition probabilities $\pi(z'_h|z_h)$ as an approximation of the AR(1) process given by (3) using the Rouwenhorst method of [Kopecky and Suen \(2010\)](#). We assume a constant probability p_s of transitioning into superstate z_n from the “normal” states $\{z_1, \dots, z_{n-1}\}$ and a constant probability q_s of transitioning out of the superstate z_n . When a household leaves the superstate, she transitions to $z_{n/2}$ (assuming n is even). We can then write the probability transition matrix as:

$$P_n = \left[\begin{array}{cccc|c} \pi(z_1|z_1)(1-p_s) & \cdots & \cdots & \cdots & \pi(z_{n-1}|z_1)(1-p_s) & p_s \\ \vdots & & & & \vdots & \vdots \\ \pi(z_1|z_{n-1})(1-p_s) & \cdots & \cdots & \cdots & \pi(z_{n-1}|z_{n-1})(1-p_s) & p_s \\ \hline 0 & \cdots & q_s & \cdots & 0 & 1-q_s \end{array} \right].$$

In the quantitative evaluation of this model, we need to discipline the three parameters (\bar{e} , p_s and q_s) which govern the superstate status. We proceed as follows. First, we shut

down return heterogeneity, as we did in Section 5.2. This means we no longer set the parameters $(\rho_r, \sigma_{er}, \lambda)$, as these are only relevant in the model with return heterogeneity. Second, given all parameters in the benchmark model, we calibrate the three earnings-state parameters – i.e., $(\bar{e}, p_s$ and $q_s)$ – in order to match the same moments as in the benchmark (except the entrepreneurship rate).

The three key parameters of the earnings superstate are $p_s = 4.94e^{-5}$, $q_s = 0.48$ and $\bar{e} = 4.75$. This implies that an agent has an approximately 0.005 percent chance to transition to the earnings superstate. Once there, in each period, a household has 48 percent chance to lose the superstate status and transition back to $z_{n/2}$. In the earnings superstate, a household earns around 650 times the median earnings. These values are, broadly speaking, consistent with the literature (e.g. Kindermann and Krueger, 2020). Results from the benchmark tax experiment in the earnings superstate model are reported in Tables 8, 9 and 10. In Appendix E.3, we report the targeted moments.

The superstate earnings model, like the benchmark model, does a good job in matching income and wealth inequality and their concentration in the tail of the distributions, particularly in the top 1 percent (see Appendix E.3). Intuitively, income inequality and concentration is matched by the large boost in productivity and therefore earnings that lucky households get when they enjoy the productivity superstate. At the same time, the risk of leaving the superstate is so high that when in this state, households massively increase their savings. In this way, they can efficiently smooth their consumption along the life-cycle. This mechanism allows the model to match the high concentration of wealth at the top, as well as the wealth-income slope. In this respect, the microfoundation at play in the earnings superstate model is fundamentally different from the one of the benchmark model, where the high concentration of income and wealth at the top is mainly driven by investment opportunities of higher wealth returns. Not surprisingly, the two models imply drastically different capital income shares along the income distribution, with the earnings superstate model counter-factually overstating the importance of labor earnings at the top of income distribution (see Table 10).

The earnings superstate model also implies a radically different transmission mechanism of marginal tax policies. The long-run ETIs generated by this model are much smaller than in the benchmark. The ETI for all taxpayers is 54 percent higher in the benchmark model (0.66 vs. 0.43), and the difference is most dramatic for the richest 1 percent, where the ETI in the benchmark model is more than double what it is in the earnings superstate framework (0.77 vs. 0.29). In turn, in the earnings superstate framework, the richest 1 percent become the least elastic to marginal tax changes. This is a

starkly different result relative to our benchmark estimates.

The intuition behind this result is the following. First, the transitory and extraordinary nature of the earnings superstate implies that when a household enjoys that state, it becomes extremely inelastic. Indeed, no matter the marginal tax rate one faces, it is always optimal to exploit the earnings possibility that the superstate offers, work as much as possible and accumulate savings to be used for consumption purposes once one transitions back to the normal earnings state. This is reflected in the low elasticities of earnings and capital for the richest households (those enjoying the superstate status), the smaller reaction of real wages, and the lower long-run ETIs for all age groups. Second, the general equilibrium effect on the borrowing rate implies a negative elasticity of capital income in the aggregate.

These results point out in a transparent manner that the modeling choice of the right tail of the wealth distribution has far-reaching consequences for policy analysis in practice. In particular, we show that models based on the transitory nature of an extremely lucky income state (e.g., [Castañeda et al., 2003](#)) imply much smaller response to marginal tax changes relative to models where households can obtain higher wealth returns by becoming entrepreneurs. Our results show that these differences are particularly dramatic for the richest 1 percent of the distribution, responsible for around 40 percent of all income tax revenues. At the same time, we also indicate a potential pitfall of the earnings superstate model, as it overstates the importance of labor income at the top of the distribution, and in turn a negative elasticity of capital income. This appears to be counterfactual and could potentially be inconsistent with the observed upsurge in capital income since the early 1990s, related directly and indirectly to entrepreneurial activities, at the top end of the distribution (see [Piketty et al., 2018](#)).

6 Conclusion

This paper adopts a structural approach in order to estimate the effects of marginal tax changes in the long run. We estimate that long-run elasticity of total taxable income (ETI) with respect to net-of-tax rates – 1 minus the marginal tax rate – is substantial and statistically significant for all income groups and is centred at 0.66. We also find that incomes in the top 1 percent of the distribution, who contribute to more than 40 percent of all fiscal revenues, are the most responsive to marginal tax rate changes.

Our results are interesting for several reasons. First of all, they quantify the long-run efficacy of marginal tax reforms. These long-run effects are crucial for policymakers

as they represent a key motivation behind discretionary tax reforms in the US and other industrialized economies (e.g., [Romer and Romer, 2010](#) and [Cloyne, 2013](#)). In this respect, our paper naturally complements, not only the canonical public finance estimates on short-run ETIs, but also the studies that use narrative datasets to estimate the effects of marginal tax changes aimed to improve long-run economic performance (e.g., [Mertens and Montiel Olea, 2018](#); [Zidar, 2019](#)).

Second, our results address one of the primary focuses in public economics, namely whether or not agents at the top end of the distribution are responsive to marginal income tax changes (e.g., [Feldstein, 1995](#); [Saez et al., 2012](#); [Mertens and Montiel Olea, 2018](#); [Zidar, 2019](#)). This is a fundamental issue as agents in the top 1, 5 and 20 percent of the income distribution pay respectively the 42, 66 and 88 percent of total tax revenues. We contribute to this by providing a realistic general equilibrium mechanism based on incomplete markets and return heterogeneity that enables us to explain why ETIs, in the long run, are highest for the richest households. Our structural approach enables us to precisely quantify how each model ingredient contributes to the empirical properties of the model and, in turn, to the main results. In doing so, we can clarify the transmission mechanism by income type (labor and capital), age structure (young, middle age and retirees) and occupational choice (workers and entrepreneurs).

Our framework could be extended in several ways. For example, one could analyze how the long-run effects of tax changes are affected by the presence of an informal sector. When agents have the possibility of avoiding paying taxes, the effects of a tax cut should sensibly increase. Second, it would be interesting to have a model with human capital accumulation. In the long run, lower taxes might push more people into acquiring education, which in turn might have beneficial effects on entrepreneurial productivity, and income and wealth inequality.

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Appendix

A Definition of Equilibrium

We focus on a stationary equilibrium, in which capital, labor, transfers, and government consumption are all constant in per-capita terms. Let $\psi_{i,j}(a, z_h, z_r)$ denote the distribution of agents with type i and age j , over assets a and idiosyncratic shocks (z_h, z_r) .

Definition 1. *The stationary recursive equilibrium consists of*

- (i) *the value function, $V_{i,j}(a, z_h, z_r)$;*
- (ii) *the policy functions, $c_{i,j}(a, z_h, z_r)$, $a'_{i,j}(a, z_h, z_r)$, $h_{i,j}(a, z_h, z_r)$;*
- (iii) *the entrepreneurial profit function $\pi(a, z_r)$ and associated capital demand $k(a, z_r)$;*
- (iv) *the prices (w, p, r) ;*
- (v) *the per-capita stocks of capital K , intermediate good Q , labor L , lump-sum transfers T_b , government spending G ;*
- (vi) *the per-capita benefit levels \bar{b}_i and labor L_i for types $i = 1, \dots, I$; and*
- (vii) *distributions (μ_1, \dots, μ_J) , $(\psi_{i,1}, \dots, \psi_{i,J})$ for $i = 1, \dots, I$*

such that the following conditions hold.

1. *The value function $V_{i,j}(a, z_h, z_r)$ solves the Bellman equation in (12) and $c_{i,j}(a, z_h, z_r)$, $a'_{i,j}(a, z_h, z_r)$, $h_{i,j}(a, z_h, z_r)$ are the associated policy functions.*
2. *Household profits $\pi(a, z_r)$ solve (6) and capital demand $k(a, z_r)$ is given by (7).*
3. *The final goods producer maximizes its profits, requiring that $F_Q(Q, L) = p$ and $F_L(Q, L) = w$.*

4. Markets clear:

$$\begin{aligned}
\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int [c_{i,j}(a, z_h, z_r) + a'_{i,j}(a, z_h, z_r)] d\psi_{i,j} + G &= F(Q, L) + (1 - \delta)K \\
\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int z_r k(a, z_r) d\psi_{i,j} &= Q \\
\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int a d\psi_{i,j} &= K \\
\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int [k(a, z_r) - a] d\psi_{i,j} &= 0 \\
\sum_{j=1}^J \mu_j \int e_{i,j}(z_h) h_{i,j}(a, z_h, z_r) d\psi_{i,j} &= L_i \\
\sum_{i=1}^I \pi_i L_i &= L.
\end{aligned}$$

5. The distribution of agents across age groups, μ_1, \dots, μ_J , satisfies

$$\mu_{j+1} = \frac{s_{j+1} \mu_j}{1 + n} \text{ for } j = 1, \dots, J - 1$$

where μ_1 is normalized so that $\sum_{t=1}^J \mu_j = 1$.

6. The distributions of agents within each age group j and type i , $\psi_{i,1}, \dots, \psi_{i,J}$, for $i = 1, \dots, I$, are consistent with individual behavior. That is, the law of motion for $\psi_{i,j}$ is

$$\psi_{i,j+1}(a', z'_h, z'_r) = \int f(z'_h | z_h) f(z'_r | z_r) \mathbf{1} \{a'_{i,j}(a, z_h, z_r) = a'\} d\psi_{i,j}(a, z_h, z_r)$$

where $f(z'_h | z_h)$ and $f(z'_r | z_r)$ are the conditional probability densities for the household transitioning to z'_h and z'_r given that its current shocks are z_h and z_r , respectively.

Furthermore, in the initial distribution $\psi_{i,1}(a, z_h, z_r)$ for each type $i \in \{1, \dots, I\}$, all age-1 agents are born with no assets (i.e., $a = 0$), the initial labor productivity shock, $\log z_h$, is zero and the initial $\log z_r$ is drawn from $N(0, \sigma_{\varepsilon_r}^2 / (1 - \rho_r^2))$.

7. The government budget constraint is satisfied

$$G = T_y \equiv \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \mathcal{T}_y(y_{i,j}(a, z_h, z_r)) d\psi_{i,j}$$

where taxable income is

$$y_{i,j}(a, z_h, z_r) = we_{i,j}(z_h)h_{i,j}(a, z_h, z_r) + r_a(z_r)a - \frac{1}{2}\tau_{ss} \min (we_{i,j}(z_h)h_{i,j}(a, z_h, z_r), \bar{y}).$$

8. Social security benefits equal social security taxes:

$$\tau_{ss} \sum_{i=1}^I \pi_i \sum_{j=1}^J \int \min (we_{i,j}(z_h)h_{i,j}(a, z_h, z_r), \bar{y}) d\psi_{i,j} = \sum_{i=1}^I \pi_i \bar{b}_i \left(\sum_{j=R}^J \mu_j \right).$$

9. The the type-specific benefit levels are $\bar{b}_i = \chi w L_i$.

10. Lump-sum transfers T_b are consistent with individual behavior,

$$T_b = \frac{1}{1+n} \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j (1 - s_{j+1}) \int E [a'_{i,j}(a, z_h, z_r)(1 + r_a(z'_r)) | z_r] d\psi_{i,j}.$$

B Numerical Solution Technique

The numerical solution technique is standard. First, we describe the discrete approximations we make for the idiosyncratic shocks and the fixed levels of innate ability. Second, we describe how we solve for the stationary equilibrium.

Discrete Approximations First, we discretize the AR(1) processes for the idiosyncratic shocks (z_h, z_r) using the Rouwenhorst method (see [Kopecky and Suen, 2010](#)). Second, we discretize the fixed levels of labor ability. That is, given the standard deviation of innate ability σ_e , we set $\{\bar{e}_i\}_{i=1}^I$ as I individual points, linearly spaced between $-3\sigma_e$ and $+3\sigma_e$. Second, assuming innate labor ability is normally distributed with mean zero and variance σ_e^2 , we construct the individual type probabilities $\{\pi_i\}_{i=1}^I$ using the approximation method of [Tauchen \(1986\)](#).

Solving for the Stationary Equilibrium To solve for the stationary equilibrium, we use a multi-dimensional root-finding algorithm to solve for the equilibrium. Specifically, we solve for the vector $(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$ such that $f_k = 0$ for $k = 1, \dots, 4 + I$, where the vector function $f_k(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$ is defined below. Given $(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$, f_k for $k = 1, \dots, 4 + I$ is computed as follows.

1. Given $\{L_i\}_{i=1}^I$, compute aggregate labor $L = \sum_{i=1}^I \pi_i L_i$.
2. Given Q and L , determine prices $p = F_Q(Q, L)$ and $w = F_L(Q, L)$.
3. Given χ, w and L_i , determine the social security benefit $b_{i,j} = \chi w L_i \times \mathbb{1}\{j \geq R\}$.
4. Given $w, r, p, b_{i,j}, T_b$, solve for the policy functions $a'_{i,j}(a, z_h, z_r), h_{i,j}(a, z_h, z_r)$ for $i = 1, \dots, I, j = 1, \dots, J$ by iterating on the Bellman equation defined in (12). We use Schumaker interpolation to interpolate the value function over assets. To determine the optimal choices of hours and savings, we use grid search followed by the BOBYQA local minimization algorithm. Consumption is then determined by the household's budget constraint.
5. Calculate the distributions $\psi_{i,j}$ for $i = 1, \dots, I$ and $j = 1, \dots, J$ using Monte Carlo simulation.
6. Given $\psi_{i,j}$ and $(r, \chi, Q, T_b, \{L_i\}_{i=1}^I)$, compute f_k for $k = 1, \dots, 4 + I$, where f_k is defined as follows:

$$\begin{aligned}
f_1 &= \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int (k(a, z_r) - a) d\psi_{i,j} \\
f_2 &= \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int \tau_{ss} \min(w e_{i,j}(z_h) h_{i,j}(a, z_h, z_r), \bar{y}) d\psi_{i,j} - \chi w L \left(\sum_{j=R}^J \mu_j \right) \\
f_3 &= Q - \sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j \int z_r k(a, z_r) d\psi_{i,j} \\
f_4 &= T_b - \frac{1}{1+n} \left[\sum_{i=1}^I \pi_i \sum_{j=1}^J \mu_j (1 - s_{j+1}) \int E [a'_{i,j}(a, z_h, z_r) (1 + r_a(z'_r)) | z_r] d\psi_{i,j} \right] \\
f_{4+i} &= L_i - \sum_{j=1}^J \mu_j \int e_{i,j}(z_h) h_{i,j}(a, z_h, z_r) d\psi_{i,j} \quad \text{for } i = 1, \dots, I.
\end{aligned}$$

C Additional Estimates

C.1 Survival Probabilities

The survival probabilities were obtained from the 2016 Period Life Tables from United States Mortality Database (see Table C.1). We utilized survival probabilities for both genders across the entire United States. Since the maximum age is $J = 85$ in the model (which corresponds to age 105 in real life), we impose that $s_{J+1} = 0$.

Table C.1 – Survival Probabilities

Age (j)	s_{j+1}	Age (j)	s_{j+1}	Age (j)	s_{j+1}
1	0.9990	31	0.9956	61	0.9483
2	0.9990	32	0.9951	62	0.9437
3	0.9990	33	0.9946	63	0.9373
4	0.9989	34	0.9942	64	0.9313
5	0.9989	35	0.9937	65	0.9251
6	0.9989	36	0.9933	66	0.9161
7	0.9988	37	0.9926	67	0.9065
8	0.9987	38	0.9921	68	0.8954
9	0.9988	39	0.9914	69	0.8826
10	0.9987	40	0.9910	70	0.8694
11	0.9986	41	0.9903	71	0.8543
12	0.9986	42	0.9895	72	0.8394
13	0.9986	43	0.9889	73	0.8226
14	0.9985	44	0.9880	74	0.8051
15	0.9984	45	0.9873	75	0.7920
16	0.9984	46	0.9867	76	0.7736
17	0.9983	47	0.9855	77	0.7543
18	0.9982	48	0.9845	78	0.7344
19	0.9981	49	0.9821	79	0.7139
20	0.9981	50	0.9826	80	0.6932
21	0.9980	51	0.9799	81	0.6720
22	0.9979	52	0.9780	82	0.6506
23	0.9977	53	0.9751	83	0.6299
24	0.9975	54	0.9738	84	0.6096
25	0.9973	55	0.9710	85	0
26	0.9971	56	0.9688		
27	0.9970	57	0.9654		
28	0.9966	58	0.9620		
29	0.9963	59	0.9582		
30	0.9959	60	0.9534		

C.2 Tax Function Estimation

We use the 2016 wave of Survey of Consumer Finances (SCF) to estimate the parameters of tax function, given in Equation (11). Our measure of total household income follows closely Gouveia and Strauss (1994) and includes all income flows accruing to households. In particular, it includes salaries and wages, both taxable and non-taxable income, dividend and interest income, capital gains, total pensions and annuities received (including taxable IRA distributions), unemployment compensation and social security benefits, and alimony received. To calculate federal income tax liabilities, we use NBER’s TAXSIM program. Our notion of tax liability includes capital gains rates, surtaxes, AMT as well as refundable and non-refundable credits. In our estimation, we restrict the sample to those households whose income is strictly positive and whose measured average tax rate is less than 100%.

C.3 Estimation of Ability Process from PSID

To estimate the parameters $(\rho_h, \sigma_{\varepsilon_h})$ of the labor ability process, we follow the approach of Kaplan (2012). Specifically, we utilize the the Panel Study of Income Dynamics (PSID), using data from the 1968-2017 waves. We use selection criteria similar to Kaplan (2012), where we (i) retain only the core Survey Research Center (SRC) subsample, (ii) keep only males, (iii) drop observations with missing data on years of education, (iv) keep only individuals aged between 21 and 64, (v) drop households with a second earner who earned at least half the amount earned by the male head, (vi) keep only individuals who worked between 520 and 5200 hours during the calendar year, (vii) drop observations where the nominal wage is less than 1 dollar, (viii) drop observations where real income is below \$1,500 (in 2010 dollars, deflated using the CPI). The final sample contains 62,683 individual/year observations and 7,510 distinct individuals.

We measure the household’s real earnings as the head of household’s labor income, deflated by the CPI. We divide real earnings by the head of household’s yearly hours to obtain a measure of “ability” (i.e., real wages). First, we regress log ability on a full set of year and race dummies. Let $\log e_{i,j}$ be the residual from this regression for individual i at age j . The benchmark specification for the statistical process governing $\log e_{i,j}$ is

$$\begin{aligned}\log e_{i,j} &= \kappa_j + \bar{e}_i + z_{h,j} + \varepsilon_{e,j} \\ z_{h,j} &= \rho_h z_{h,j-1} + \varepsilon_{h,j} \\ z_{h,1} &= 0\end{aligned}$$

Table C.2 – Parameter Estimates of Ability Process from PSID

Parameter	Value	Std. Err.
ρ_h	0.976	(0.005)
$\sigma_{\varepsilon h}$	0.135	(0.006)
σ_e	0.279	(0.015)

Note: This table reports the parameter estimates for the ability process from PSID, which are obtained using GMM. Standard errors, reported in parentheses, are obtained by bootstrap with 250 repetitions.

where $E[\varepsilon_{h,j}] = E[\bar{e}_i] = 0$, $\text{Var}(\varepsilon_{h,j}) = \sigma_{\varepsilon h}^2$, $\text{Var}(\bar{e}_i) = \sigma_e^2$ and $\text{Var}(\varepsilon_{e,j}) = \sigma_{\varepsilon e}^2$. This specification matches our benchmark specification for labor ability given by Equation (2) and (3) in the text, with the exception that we allow for i.i.d. measurement error, $\varepsilon_{e,j}$. For notational convenience, we have slightly altered the notation here, where i indexes individual i and κ_j corresponds to the non-stochastic age profile of log ability. It can be shown that $\text{Var}(z_{h,j}) = \sigma_{\varepsilon h}^2 \left[1 - \rho_h^{2(j-1)} \right] / [1 - \rho_h^2]$ and $\text{Cov}(z_{h,j}, z_{h,j+s}) = \rho_h^s \text{Var}(z_{h,j})$ for $s > 0$. Denote an element of the autocovariance function of log $e_{i,j}$ as $\sigma_{j,j+s} \equiv \text{Cov}(\log e_{i,j}, \log e_{i,j+s})$. The autocovariance moments for this process is then given by

$$\begin{aligned} \sigma_{j,j} &= \sigma_e^2 + \frac{1 - \rho_h^{2(j-1)}}{1 - \rho_h^2} \sigma_{\varepsilon h}^2 + \sigma_{\varepsilon e}^2, \\ \sigma_{j,j+s} &= \sigma_e^2 + \rho_h^s \frac{1 - \rho_h^{2(j-1)}}{1 - \rho_h^2} \sigma_{\varepsilon h}^2 \quad \text{for } s > 0. \end{aligned}$$

Note that these moments are independent of the age profile, κ_j . We construct estimates of these autocovariances by age and year. We use a maximum of 25 lags and retain moments that were constructed with at least 30 observations. We assume that the variance of the i.i.d. measurement error is $\sigma_{\varepsilon e}^2 = 0.03$, using the estimates of measurement error of earnings and hours in Kaplan (2012).

We use generalized method of moments (GMM) to estimate the parameters of this process. Denote the parameters to be estimated by $\Theta = (\rho_h, \sigma_{\varepsilon h}, \sigma_e)$. The GMM estimator solves the following minimization problem

$$\hat{\Theta} = \arg \min_{\Theta} \left(\hat{M} - \hat{m}(\Theta) \right)' W \left(\hat{M} - \hat{m}(\Theta) \right)$$

where \hat{M} is the vector targeted moments in PSID, and $\hat{m}(\Theta)$ are the corresponding model moments, and W is a weighting matrix. We assumed the off-diagonal elements of the weighting matrix W were zero. For the diagonal elements, we assumed $W_{ii} = \sqrt{n_i}$, where

Table C.3 – Return on Wealth by Wealth Percentiles, Benchmark Model vs. Data

Wealth Percentile	Returns (Model)	Returns (Data)
[99-100]	0.071	0.074
[95-99)	0.039	0.066
[90-95)	0.033	0.059
[75-90)	0.029	0.053
[50-75)	0.025	0.049
[25-50)	0.021	0.040
[10-25)	0.019	0.021
[1-10)	0.021	0.028

Note: This table reports the resulting wealth returns by wealth percentile in the model and the data. For the data, we estimate the average return for each household’s portfolio in the SCF using estimates of the average returns of different asset types between 1990 and 2016, as reported by [Xavier \(2020\)](#).

n_i is the number of observations used in the construction of moment i . To minimize the GMM criterion, we used a scatter-search algorithm which generates random start points for the interior-point minimization algorithm. Standard errors are obtained by bootstrap with 250 repetitions. The resulting parameter estimates are reported in [Table C.2](#).

C.4 Calculating Return Profiles in SCF

Here we describe the technique used to estimate the return on wealth at household level in the SCF. We define r_j and ζ_j to be the return of asset type j and its share in each household’s portfolio, respectively.¹⁰ Consistently, the aggregate return on wealth can be computed as

$$r_w = \sum_j \zeta_j r_j.$$

The return r_j of each asset type is composed of its yield (income generated by the asset) and its capital gain (price changes in the asset). For each asset type, we obtain the average return over the period 1990-2016 from [Xavier \(2020\)](#). Finally, we compute the household’s portfolio shares ζ_j directly in the 2016 SCF.

In [Table C.3](#), we report the resulting returns by wealth percentile in the 2016 SCF. The data display the same qualitative patterns as that of our model: households with higher wealth obtain higher than average returns. Moreover, our model implied-returns at the bottom 25 percent of the wealth distribution and the top 1 percent are quite close to the

¹⁰The j categories are: interest earning assets, stocks, private businesses, real estate, other financial and non financial assets and debt (with a negative sign).

imputed returns in the SCF. In the middle of the distribution, imputed returns from the SCF seem to be higher than those implied by the model. Overall these results (particularly for the consistency at the top of the wealth distribution) confirm the ability of our model to provide a realistic explanation of the fat tail in wealth and income distributions.

D Calculation of Standard Errors

With the *Worst-Case* estimator of [Cocci and Plagborg-Møller \(2021\)](#), standard errors are computed as follows. Let p be the number of parameters and m be the number of moments. Suppose V is a $m \times m$ sample variance-covariance matrix of the data moments. In principle, we do not know the full matrix V , but suppose we are able to estimate the block diagonal of V :

$$V = \begin{bmatrix} V_{(1)} & ? & ? & \cdots & ? \\ ? & V_2 & ? & \cdots & ? \\ \vdots & & \ddots & & \vdots \\ \vdots & & & \ddots & \vdots \\ ? & ? & \cdots & ? & V_{(K)} \end{bmatrix}$$

where $V_{(k)}$ are known square symmetric matrices (possibly of different dimensions), for $k = 1, \dots, K$. In practice, think of $V_{(1)}$ as the variance-covariance matrix as estimated from the SCF moments, $V_{(2)}$, $V_{(3)}$, and $V_{(4)}$ correspond to the macroeconomic moments. Then define the $m \times 1$ vector $x_i = WG(G'WG)^{-1}\lambda_i$, where λ_i is a $p \times 1$ vector in which all elements are zero, except for the i -th element, which is one (corresponding to parameter θ_i). The matrix G is the $m \times p$ gradient matrix of the moments, and W is the $m \times m$ weighting matrix. As with V , we can partition the vector x_i into $x_i = (x_{i,(1)}; \dots; x_{i,(K)})$. Given parameters $\Theta = (\theta_1; \dots; \theta_p)$, the standard error of parameter θ_i is estimated to be

$$se(\theta_i) = \sum_{k=1}^K \sqrt{x'_{i,(k)} V_{(k)} x_{i,(k)}}.$$

E Alternative Models

E.1 Model with Separate Capital and Labor Taxes

In our benchmark economy, both capital and labor income are taxed jointly, which is consistent with the US tax code and the empirical literature. In this section, we consider the implications for our results if labor income and capital income are taxed separately.

Specifically, we now assume that taxable labor income is given by

$$y = we_{i,j}(z_h)h - \frac{1}{2} \min(we_{i,j}(z_h)h, \bar{y})$$

The total labor income tax is then given by $\mathcal{T}_y(y)$, given by Equation (11) in the text. We assume this function has the same parameters as our benchmark economy. Households separately pay a flat tax on capital income, given by $\tau_k r_a(z_r)a$. We set τ_k so that total tax revenue is identical to the revenue in our benchmark economy. This requires $\tau_k = 0.226$.

Table E.1 – Elasticities with Separate Capital and Labor Taxation

Variable	Benchmark	Separate Taxation		
	Income Tax Change	Labor Tax Change	Capital Tax Change	Cap. & Lab. Tax Change
<u>A. Capital Income</u>				
Income Top 1%	0.94		0.72	0.65
Income Top 5%	0.74		0.50	0.50
Income Top 10%	0.64		0.41	0.41
Income Bottom 99%	-1.76		-1.49	-1.08
Income Bottom 90%	-4.02		-1.82	-1.41
All	0.36		0.12	0.18
<u>B. Labor Income</u>				
Income Top 1%	0.51	0.15		0.52
Income Top 5%	0.53	0.19		0.55
Income Top 10%	0.57	0.24		0.58
Income Bottom 99%	0.82	0.39		0.84
Income Bottom 90%	1.26	0.63		1.30
All	0.72	0.34		0.77

Note: This table reports the elasticities of taxable labor income and capital income in a model with separate capital and labor taxation. For both labor and capital income, we group individuals based on their position in the total income distribution.

Since capital and labor are now taxed at separate rates, we cannot compute an elasticity of taxable income for total income. Therefore, we separately define an elasticity of labor income and an elasticity of capital income, where each elasticity is defined relative to the relevant net-of-marginal tax rate. We then consider the effects of a change in the labor income tax rate (i.e., a change in τ_0), a change in the capital income tax rate (τ_k) and a change in both tax rates.

E.2 Model with No Fixed-Heterogeneity in Labor Abilities

Here we repeat the main policy exercise, namely an across the board cut in marginal income tax rate, in the benchmark model where we shut down the fixed permanent heterogeneity in labor productivity. In this alternative scenario, productivity differences (at a given age) are purely driven by transitory idiosyncratic shocks. Results from this case are reported in Tables E.2, E.3 and E.4.

By eliminating the permanent component in earnings, we artificially increase the relative importance of risk. This increases the precautionary effects (both in savings and labor supply) of a marginal tax change cut, which in turn boosts the long-run ETI for all taxpayers and along the income distribution. This mechanism does not change the *qualitative* effects of the marginal tax change: the richest 1 percent displays the highest ETIs. Similarly the same amplification effect implies, relative to the benchmark, higher elasticities for income sources (labor and capital), for each group age (young, middle-aged and retirees) and occupation type (entrepreneurs and workers).

These results are interesting as they show that the permanent component in earning differentials has a dampening effect on the ETIs. As such we can show in a transparent manner that if anything, our estimates of σ_e work against the main mechanism of the model and reduce, rather than amplify, the main results presented in Section 5.1.

E.3 Targeted Moments in Various Models

In Table E.5, we report how the targeted moments change under various models we consider in the main text, including (i) no return heterogeneity, (ii) no earnings risk, (iii) linear taxes and (iv) the earnings superstate model. For convenience, we also include the data and the moments for the benchmark model.

F Alternative Policy Reform

Here we consider an alternative policy reform where we increase income taxes only for individuals in the top 1 percent. Specifically, we assume households face a modified income tax function, $\hat{\mathcal{T}}_y(y)$, given by:

$$\hat{\mathcal{T}}_y(y) = \begin{cases} \mathcal{T}_y(y) & \text{if } y < \hat{y} \\ \mathcal{T}_y(\hat{y}) + \hat{\tau}(y - \hat{y}) & \text{if } y \geq \hat{y} \end{cases}$$

Table E.2 – Effects of a Marginal Income Tax Change

Variable	Bench.	No Fixed Earnings Heterogeneity
A. ETI		
All Taxpayers	0.67	1.06
Income Top 1%	0.77	1.17
Income Top 5%	0.62	0.97
Income Top 10%	0.60	0.96
Income Bottom 99%	0.55	0.82
Income Bottom 90%	0.70	0.94
B. Elasticity of Capital Income		
All Taxpayers	0.36	1.00
Income Top 1%	0.94	1.22
Income Top 5%	0.74	0.97
Income Top 10%	0.64	0.99
Income Bottom 99%	-1.76	-4.01
Income Bottom 90%	-4.02	-8.54
C. Elasticity of Earnings		
All Taxpayers	0.72	1.03
Income Top 1%	0.51	0.80
Income Top 5%	0.53	0.94
Income Top 10%	0.57	0.91
Income Bottom 99%	0.82	1.29
Income Bottom 90%	1.26	1.47
D. ETI by Age (Years)		
Ages 21-30	0.93	2.02
Ages 31-40	0.56	0.83
Ages 41-50	0.53	0.73
Ages 51-64	0.66	0.91
Ages 65+	0.73	1.42
E. ETI by Occupation		
Entrepreneurs	0.78	1.35
Rest of Population	0.57	0.82

Note: We report the elasticities with respect to the average net of marginal tax rate. The elasticity for variable X is defined as $d \ln X / d \ln(1 - AMTR)$.

For income below a threshold \hat{y} , the household's income tax is given by $\mathcal{T}_y(y)$, as in our benchmark economy. For income above \hat{y} , households face a flat marginal tax, $\hat{\tau}$. We

Table E.3 – Macroeconomic Effects of a Marginal Income Tax Change

Variable	Bench.	No Fixed Earnings Heterogeneity
<u>A. Prices</u>		
Real Wage, w	0.48	0.65
Price of Capital, p	-0.86	-1.16
Borrowing Rate, r	-3.08	-4.26
<u>B. Quantities</u>		
Output, Y	0.72	1.03
Capital, K	1.13	0.96
Labor, L	0.24	0.38
<u>C. Productivity</u>		
Aggregate TFP	0.16	0.44
Entrep. Productivity	0.45	1.23
Entrep. Rate	-1.87	-6.96

Note: We report the effects of a tax cut via τ_0 as elasticities with respect to the average net of marginal tax rate. The elasticity for variable X is defined as $d \ln X / d \ln(1 - AMTR)$.

Table E.4 – Capital Income Shares Along the Taxable Income Distribution

Income Percentile	Bench.	No Fixed Earnings Heterogeneity
All Taxpayers	0.26	0.24
Income Top 1%	0.60	0.85
Income Top 5%	0.39	0.57
Income Top 10%	0.33	0.47
Income Bottom 99%	0.11	0.09
Income Bottom 90%	0.10	0.06

Note: We report the capital income shares along the income distribution. Aggregate income is GDP minus depreciation of capital.

choose the threshold \hat{y} so that the marginal tax rate $\hat{\tau}$ applies to the top 1 percent of households and set $\hat{\tau} = \mathcal{T}'_y(\hat{y})$.

We consider a policy reform in which the top marginal tax rate $\hat{\tau}$ is increased by 1 percentage point. Compared to our main policy experiment in which income taxes were changed for all taxpayers, we see that this policy reform is more distortionary. The elasticity of taxable income, for the top 1 percent and overall, is higher in this economy relative to our main policy experiment (see panel A, Table F.1). Similarly, the

Table E.5 – Moments from the Data and in Various Models

Moment	Data	Bench.	No Return Het.	No Earnings Risk	Linear Taxes	Earnings Superstate
Average Hours	0.30	0.30	0.30	0.31	0.29	0.29
Share of Entrepreneurs	0.09	0.09	-	0.04	0.07	-
Wealth Gini	0.86	0.86	0.71	0.84	0.90	0.80
Wealth Share, Top 1%	0.39	0.41	0.17	0.46	0.48	0.40
Wealth Share, Top 5%	0.65	0.64	0.41	0.64	0.71	0.58
Wealth Share, Top 20%	0.88	0.89	0.73	0.85	0.94	0.82
Earnings Gini	0.68	0.73	0.74	0.68	0.74	0.76
Earnings Share, Top 1%	0.17	0.14	0.14	0.11	0.14	0.23
Earnings Share, Top 5%	0.33	0.36	0.36	0.30	0.36	0.43
Earnings Share, Top 20%	0.61	0.69	0.69	0.60	0.69	0.72
Average Earnings	55.30	54.83	63.06	46.42	59.22	72.75
Tax Revenue Share, Top 1%	0.42	0.42	0.25	0.45	0.30	0.39
Tax Revenue Share, Top 5%	0.66	0.70	0.57	0.71	0.51	0.66
Tax Revenue Share, Top 20%	0.88	0.96	0.91	0.94	0.80	0.94
Wealth-Inc. Slope, Top 20%	1.64	1.57	1.67	0.88	1.43	1.65
Wealth-Inc. Slope, Top 40%	0.96	0.92	1.12	0.74	0.90	1.07
Wealth-Inc. Slope, Top 60%	0.72	0.69	0.84	0.58	0.77	0.80
Capital-to-output Ratio	2.95	2.95	3.81	2.51	2.96	4.03
Investment-to-output Ratio	0.22	0.22	0.27	0.19	0.22	0.28
Borrowing Rate	0.02	0.02	0.04	0.02	0.01	0.04

Table F.1 – Effects of Income Tax Increase for Top 1%

Variable	Alternative Experiment
A. Elasticity of Taxable Income	
Income Top 1%	0.83
All	1.00
B. Revenue Losses	
Income Top 1%	51.0
All	53.9

Note: This table reports the results of the policy experiment where the top marginal tax rate is increased only for individuals in the top 1 percent of the income distribution.

behavioral revenue losses are also higher, where the government would lose a little more than half of the additional tax revenue to the behavioral response (see panel B, Table F.1). Furthermore, as in our main policy experiment, these results are amplified by the presence of return heterogeneity.

G ETI and Behavioral Revenue Loss

In our framework, there is a link between the elasticity of taxable income and the amount of additional revenue the government loses to behavioral responses (e.g., from a tax increase). Specifically, to fix concepts, suppose the government were to raise marginal taxes by a small amount (i.e., $d\tau_0 > 0$) and that the increase in total tax revenue is given by dT_y . Thus, there will be two effects on tax revenues. First, there is a “mechanical” increase in tax revenues due to the fact that all taxpayers will face a higher marginal tax rate. We denote the increase in revenues from the mechanical effect by dM . Second, the higher tax rate will trigger a behavioral response, whereby households reduce their average taxable income (by altering their labor supply or savings/investment decisions). Intuitively, when the ETI is higher, the government will lose more revenue to the behavioral response.

In Table G.1, we compute the revenue loss due to the behavioral response as the percentage difference between the mechanical response and the actual revenue change (i.e., $(dM - dT_y)/dM$). Overall, in our benchmark economy, the government loses 28.5 percent of additional tax revenue because of behavioral responses. Higher losses are observed both for the top 1 percent (30.9 percent) and the bottom 90 percent (41.0 percent) of the income distribution. These patterns are further amplified by the presence of return heterogeneity (as with the elasticity of taxable income). This is especially true for individuals in the

Table G.1 – Revenue Losses from an Income Tax Change

Group	Benchmark
<u>A. Total Revenue Losses</u>	
Income Top 1%	30.9
Income Top 5%	26.1
Income Top 10%	26.3
Income Bottom 99%	26.9
Income Bottom 90%	41.0
All	28.5
<u>B. Share of Revenue Losses</u>	
Income Top 1%	45.0
Income Top 5%	64.1
Income Top 10%	78.0
Income Bottom 99%	55.0
Income Bottom 90%	22.0
All	100.0

Note: Panel A reports the percent of tax revenue lost to behavioral responses, given uniform change in the income tax via τ_0 , by income group. Panel B reports the share of the total revenue loss which is attributable to each income group.

top 1 percent, where return heterogeneity more than doubles the behavioral losses from tax changes. Furthermore, almost 80 percent of the total revenue losses come from the behavioral responses of individuals in the top 10 percent. Nevertheless, a sizeable fraction of losses (20 percent) come from the bottom 90 percent.

Given our policy exercise, we can derive an approximate link between the ETI and the amount of tax revenue the government loses to behavioral responses. The marginal change in fiscal revenues, dT_y , for a marginal change in τ_0 , is approximately given by

$$dT_y \approx dM \left(1 - \underbrace{\frac{AMTR}{1 - AMTR} \frac{AMTR}{ATR}}_{\text{behavioral response}} \times ETI \right). \quad (\text{G.1})$$

See below for a derivation. The expression in (G.1) is a sufficient statistic to estimate the overall revenue change from the type of tax policy reforms studied in this paper. This indicates that the behavioral revenue loss depends on the elasticity of taxable income (ETI), as well as the average marginal tax rate (AMTR) and the average tax rate (ATR).

Naturally, higher elasticities will be associated with higher revenue losses. Furthermore, due to the progressivity of the tax schedule, $AMTR > ATR$, and this will tend to increase the behavioral losses for a given elasticity. This suggests why revenue losses are relatively high for the bottom 90 percent. While households in the bottom 90 percent tend to have a relatively high ETI in our benchmark economy (0.70), they do also have a lower AMTR, which would lower the revenue losses. At the same time, the progressivity of the tax code generates a larger gap between the AMTR and the ATR for the bottom 90 percent, pushing up their behavioral losses.

Derivation of Relationship between ETI and Behavioral Revenue Losses In this section, we derive an approximate relationship between the elasticity of taxable income and the behavioral revenue losses. Specifically, denote the tax function $\mathcal{T}_y(y) = \tau_0 \bar{\mathcal{T}}_y(y)$, where $\bar{\mathcal{T}}_y(y) = y(1 - (\tau_2 y^{\tau_1} + 1)^{-1/\tau_1})$. Let $\Gamma(y)$ be the equilibrium distribution of agents over taxable income y . Γ can refer to the entire distribution, or to subsets of the entire distribution (e.g., top 1 percent, top 5 percent, etc.). Let T_y denote the aggregate amount of tax revenue:

$$T_y = \tau_0 \int \bar{\mathcal{T}}_y(y) d\Gamma(y) \quad (\text{G.2})$$

Differentiate Equation (G.2) with respect to τ_0 :

$$\frac{dT_y}{d\tau_0} = \underbrace{\int \bar{\mathcal{T}}_y(y) d\Gamma(y)}_{\text{mechanical effect}} + \tau_0 \underbrace{\frac{d \int \bar{\mathcal{T}}_y(y) d\Gamma(y)}{d\tau_0}}_{\text{behavioral effect}}. \quad (\text{G.3})$$

The first term is the mechanical effect, or the marginal increase in tax revenue absent any changes in taxable income. The second term is the behavioral response, which captures the effect of a marginal change in income on aggregate tax revenue. Let $\bar{T}_y = \int \bar{\mathcal{T}}_y(y) d\Gamma(y)$ be normalized tax revenue and $Y = \int y d\Gamma(y)$ denote aggregate taxable income. To simplify Equation (G.3), we assume the following approximation:

$$\frac{d\bar{T}_y}{d\tau_0} \approx \overline{AMTR} \frac{dY}{d\tau_0} \quad (\text{G.4})$$

where $\overline{AMTR} = \int \frac{\bar{\mathcal{T}}'_y(y)y}{Y} d\Gamma(y)$ is the normalized average marginal tax rate. We also assume that \overline{AMTR} is roughly constant for a small change in τ_0 . Essentially, we are assuming that the marginal change in aggregate normalized tax revenue, \bar{T}_y , is approximately equal to the marginal change in aggregate income Y times the normalized average

marginal tax rate, \overline{AMTR} . This will approximately be true when most of the higher-income individuals within the distribution Γ face a similar marginal tax rate. This would also be true if the marginal tax rate function, $\mathcal{T}'_y(y)$, could be approximated by an increasing sequence of constant marginal tax rates (e.g., consistent with the US tax code).

Substituting Equation (G.4) into Equation (G.3), we get:

$$\frac{dT_y}{d\tau_0} \approx \bar{T}_y + \tau_0 \overline{AMTR} \frac{dY}{d\tau_0} \quad (\text{G.5})$$

Let $AMTR = \tau_0 \overline{AMTR} = \int \frac{\mathcal{T}'_y(y)y}{Y} d\Gamma(y)$ denote the average marginal tax rate. Then:

$$\frac{dY}{d\tau_0} = - \frac{dY}{d(1 - AMTR)} \overline{AMTR}. \quad (\text{G.6})$$

Substituting Equation (G.6) into (G.5) and re-arranging, we obtain:

$$\frac{dT_y}{d\tau_0} \approx \bar{T}_y \left[1 - \frac{\tau_0 \overline{AMTR}}{1 - AMTR} \frac{\overline{AMTR}}{\bar{T}_y/Y} \frac{dY}{d(1 - AMTR)} \frac{1 - AMTR}{Y} \right] \quad (\text{G.7})$$

Using that the elasticity of taxable income is $ETI = \frac{dY}{d(1 - AMTR)} \frac{1 - AMTR}{Y}$, $AMTR = \tau_0 \overline{AMTR}$ and the average tax rate is $ATR = T_y/Y$, (G.7) becomes:

$$\frac{dT_y}{d\tau_0} \approx \bar{T}_y \left[1 - \frac{AMTR}{1 - AMTR} \frac{AMTR}{ATR} \times ETI \right]$$

For a small tax change $d\tau_0$, the mechanical effect is $dM = \bar{T}_y d\tau_0$ and the marginal change in tax revenue is

$$dT_y \approx dM \left[1 - \frac{AMTR}{1 - AMTR} \frac{AMTR}{ATR} \times ETI \right].$$

This is Equation (G.1).