MANCHESTER
1824
The University of Manchester

Economics

Discussion Paper

Series

EDP-2104

The Long-Run Effects of Risk: An Equilibrium Approach

Christiaan van der Kwaak, Joao Madeira, Nuno Palma

February 2021

Updated January 2023

Economics

School of Social Sciences

The University of Manchester

Manchester M13 9PL

The long-run effects of risk: an equilibrium approach*

Christiaan van der Kwaak[†], João Madeira[‡], and Nuno Palma[§]

January 6, 2023

Forthcoming, **European Economic Review**

Abstract

Advanced economies tend to have large financial sectors which can be vulnerable to crises. We employ a DSGE model with banks featuring limited liability to investigate how risk shocks in the financial sector affect long-run macroeconomic outcomes. With full deposit insurance, banks expand balance sheets when risk increases, leading to higher investment and output. With no deposit insurance, we observe substantial drops in long-run credit provision, investment, and output. Reducing moral hazard by lowering the fraction of reimbursed deposits in case of bank default increases the probability of bank default in equilibrium. The long-run probability of bank default under a regime with no deposit insurance is more than 50% higher than under a regime with full deposit insurance for high levels of risk. These differences provide a novel argument in favor of deposit insurance. Our welfare analysis finds that increased risk always reduces welfare, except when there is full deposit insurance and deadweight costs from default are small.

Keywords: financial intermediation, risk, investment, regulation, endogenous leverage, limited liability, costly state verification, deposit insurance.

JEL Codes: E22, E44, G21, O16

*We thank Nicola Limodio, Manuel Mosquera Tarrio, Ryland Thomas and Francesco Zanetti for useful comments and suggestions. Alexandra Brown, Carlos Javier Charotti and Levi Kuiper provided research assistance. Nuno Palma acknowledges financial support from Fundação para a Ciência e a Tecnologia (CEECIND/04197/2017).

[†]Affiliation: University of Groningen

[‡]Affiliation: Department of Economics and BRU-Business Research Unit, ISCTE-University Institute of Lisbon

[§]Affiliation: University of Manchester; Instituto de Ciências Sociais, Universidade de Lisboa; CEPR

1 Introduction

Cross-country evidence reveals substantial heterogeneity across countries with respect to bank risk-taking attitudes. Developed countries tend to have large financial intermediation sectors which can be unstable, depending on a number of factors including the nature and extent of regulation (Bezemer et al., 2020; Calomiris and Haber, 2015; Dávila and Goldstein, 2021; De Roux and Limodio, 2023; Choudhary and Limodio, 2022). In this paper, we consider a quantitative equilibrium model with banks benefiting from limited liability which allows us to investigate how exogenous changes in risk affect credit provision by banks, and by implication long run macroeconomic outcomes.[1] By risk we mean the volatility or standard deviation of idiosyncratic shocks to the return on banks' assets (Christiano et al., 2014).[2] Therefore, our paper contrasts with the baseline Real Business Cycle or New Keynesian models where risk does not affect the steady state or balanced growth path.

Specifically, we develop a Dynamic Stochastic General Equilibrium (DSGE) model with financial intermediaries in which each generation of bankers operates for two periods, as in Clerc et al. (2015).[3] In our model, intermediate goods producers issue corporate securities that are purchased by banks.[4] Banks finance these securities through net worth and deposits.[5] Banks have limited liability and freely choose the degree of leverage in the first period by paying out part of their net worth in the form of dividends, subject to dividend adjustment costs (Jermann and Quadrini, 2012). However, the size of their balance sheet is limited by a minimum equity-deposits requirement, where equity is the amount of net worth after dividend payments.[6] Banks face the above-mentioned risk shocks which affect the performance of their securities portfolios in the second period of their existence, and therefore the profits they pay to their owners. Because bankers operate under limited liability, they only care about the distribution of their idiosyncratic shocks *conditional* on survival when making their balance sheet decisions in the first period (Diamond and Rajan, 2011). Therefore, changes in the distribution of the shock affect credit provision, investment and output. This contrasts with a standard DSGE model with unlimited liability, in which the distribution of idiosyncratic shocks faced by bankers does not

[1] Ranciere et al. (2008) find a link between higher risk in the financial sector and higher long run output in the form of a robust negative link between the skewness of credit growth and GDP growth in a large sample of countries. This is true irrespective of the fact that excessive credit growth can lead to financial crises (Schularick and Taylor, 2012; Reinhart and Rogoff, 2009).

[2] These idiosyncratic shocks can result from limits to diversification of bank assets (for example: regional/sectoral specialization) or unmodeled disturbances to banking costs and revenues Mendicino et al. (2018).

[3] See Bernanke et al. (1999); Gertler and Karadi (2011) for contributions on models with financial frictions.

[4] We refer to corporate securities rather than bank loans to emphasize their state-dependent nature following Gertler and Kiyotaki (2010); Gertler and Karadi (2011). Bank loans are typically not state-dependent since they have a fixed nominal principal (with exceptions for some indexed ones and in the case of default).

[5] Deposits within our model should be interpreted as any type of debt funding that banks might issue in reality, including bank bonds, wholesale funding, and so on. Therefore, we use the terms "deposits" and "bank debt" interchangeably.

[6] We introduce this minimum equity-deposit requirement in order to prevent a corner solution where banks attract deposits, and pay all funds as dividends to their owners without acquiring corporate securities. This minimum equity-deposit requirement can be mapped into a capital requirement that prescribes a minimum amount of equity as a fraction of assets. However, the minimum equity-deposits requirement prevents banks from paying all funds as dividends to its owners, something that is not ruled out by a capital requirement.

affect equilibrium investment or output, as long as the mean of the distribution is unchanged.

In our model, the government is responsible for the degree of deposit insurance, which amounts to the fraction of deposits that are reimbursed in case of bank default (Clerc et al., 2015). For simplicity, we do not explicitly model other government regulations that the banks have to abide by (except for the minimum equity-deposits requirement). However, we assume that the standard deviation of the idiosyncratic volatility shock can be indirectly influenced by government regulation, but we take the extent of regulation itself and its political origins and motivations as exogenous. In other words, we do not assume that the regulator has perfect control over the value of the standard deviation, but instead that it has the power to influence the standard deviation indirectly via regulation.[7] We interpret the standard deviation of shocks to bankers' idiosyncratic risk as an outcome of the degree to which banks are regulated, with a large standard deviation representing a regime with little regulation, as the financial sector will be able to invest in riskier projects that have both a higher payoff in the good state of the world, and a much lower payoff in the bad state.

Our analysis focuses on long-run outcomes, which we capture by studying the non-stochastic steady state of the model.[8] The combination of limited liability and (partial) deposit insurance introduces moral hazard into our model (Kareken and Wallace, 1978): the fact that banks' probability of default is not fully incorporated in the rate at which banks attract funding from depositors induces banks to increase leverage with respect to the case of no deposit insurance, everything else equal. As a result, an increase in the shock's standard deviation (which increases the probability of default) decreases the marginal cost of attracting an additional unit of deposits, thereby leading banks to hold more corporate securities (Diamond and Rajan, 2011). We analytically show this for the full deposit insurance case. Banks expanding credit in this way leads to an increase of steady state investment and output with respect to unlimited liability. Simultaneously, a higher standard deviation increases the probability of bank default (and therefore the fraction of banks that default ex post).[9] Finally, we show analytically that credit provision under full deposit insurance is always larger or equal to that under unlimited liability.

Moral hazard can be reduced by lowering the fraction of deposits that are reimbursed in case of bank default: the smaller the fraction the more the probability of default is priced in by banks'

[7]For example, in the 1970s, many states in the US still had usury laws in place. The interest rate ceiling on loans imposed by these laws became an important constraint with the environment of high inflation in the 1970s and limited growth in the use of credit cards (a risky type of debt). In 1978 the Marquette vs. First of Omaha ruling by the Supreme Court allowed banks to export the usury laws of their home state nationwide which set off a competitive wave of deregulation, resulting in the effective elimination of usury rate ceilings (Sherman, 2009). These interest rate ceilings on loans were constraining the distribution of overall risk of bank portfolios, because banks cannot be properly compensated (since they needed to charge higher interest rates to riskier borrowers, i.e. a risk premium). This meant that they would not lend to riskier borrowers.

[8]As we focus on the impact of risk, captured by the distribution of idiosyncratic shocks to the return on banks' assets, it suffices to use the non-stochastic steady state as a proxy for long-run macroeconomic outcomes: idiosyncratic shocks still arrive in the non-stochastic steady state for all bankers that are in the second period of their existence. Therefore, the distribution of these shocks affects the equilibrium even in the non-stochastic steady state.

[9]The ex ante probability of bank default and the fraction of banks that default ex post are equal in the non-stochastic steady state.

creditors.[10] However, we find that decreasing moral hazard in this way actually *increases* the probability of bank default in equilibrium because of a nonlinear feedback loop between banks' funding costs and the probability of default: the larger the fraction of deposits that are not reimbursed, the more creditors increase interest rates to price in the probability of bank default. This increase in funding costs, however, decreases banks' (expected) profitability, which then increases the probability of bank default.[11] This, in turn, further increases banks' funding costs, which has amplification effects. In equilibrium, the long-run probability of bank default under a regime with no deposit insurance is more than 50% higher than the case of full deposit insurance for high levels of risk. In addition, we prove analytically that the level of credit provision to the real economy under no insurance is always smaller or equal to the level of credit provision under unlimited liability, as well as that it is always decreasing in risk.

The impact on the real economy from a complete absence of deposit insurance is large: as a result of higher funding costs, credit provision can drop by approximately 90% for very large values of risk (with respect to the case of unlimited liability), causing long-run output and consumption to drop by more than 50%. This sharply contrasts with the case of full deposit insurance, where credit provision and output increase relative to unlimited liability. The negative impact on the macroeconomy from a complete absence of deposit insurance is mitigated when 50% of deposits are reimbursed, although credit provision, investment, and output still decrease substantially with respect to the unlimited liability case. Higher deadweight costs from default negatively affect welfare which always decreases with risk, except when there is full deposit insurance. In that case, consumption increases with risk, which positively affects welfare, everything else equal. However, the negative effects from deadweight costs trump the positive effects from higher consumption, except when the fraction of assets that cannot be recouped is small.

Therefore, our results provide an additional argument in favor of deposit insurance, in addition to the well-known Diamond and Dybvig (1983) argument about preventing bank runs (which are absent in our model): deposit insurance eliminates the above-mentioned feedback loop that would otherwise cause banks' funding costs to increase to such an extent that the probability of bank default increases far above that of the case of full deposit insurance. As such, financial instability (as defined by the fraction of banks that default) decreases with more deposit insurance, despite introducing moral hazard along the way.[12]

Our approach is related to that of Aghion et al. (2005), and Aghion et al. (2010), who focus on credit constraints being able to cause a shift towards long-term investment. These authors focus on how the tightness of credit constraints affects the composition of investment, whereas we focus on how the distribution of idiosyncratic risk shocks affects the level of investment. Our

[10] We follow Clerc et al. (2015) by assuming that bank creditors diversify their funds across banks in the economy, such that all banks borrow at the same interest rate.

[11] There is a counterpart in the sovereign default literature; for a review of this literature on debt-spirals see Obstfeld and Rogoff (1996).

[12] Bank runs are absent from our model. Our feedback loop is generated by the interaction between funding costs and the probability of default meaning that when funding costs get high, banks reduce lending, without any strategic considerations.

work is also related to other papers which study the effects of risk in the financial sector in DSGE models. Christiano et al. (2014) shows that fluctuations in risk are a key driver of business cycle volatility. Mendicino et al. (2018), Afanasyeva and Guntner (2020), and Elenev et al. (2021), among others, study the emergence of risk shifting in financial intermediaries, as we do; but these papers focus on the short run equilibrium. Bloom (2014) in turn, also focuses on the economic effects of uncertainty for short and medium-run horizons. Gertler et al. (2012) and de Groot (2014) investigate moral hazard and risk-taking within DSGE models but in their models banks are not subject to limited liability as in our model.

2 Model

We consider a closed economy which contains households, bankers, intermediate, retail and final goods producers, capital goods producers, and a government. Households provide labor services to intermediate goods producers, and save through deposits in banks. They are the ultimate owners of banks, producers, and capital goods producers. Profits and dividends of these firms and banks therefore accrue to households.

Intermediate goods producers issue corporate securities to finance capital purchases from capital goods producers. They use capital together with labor supplied by households to produce an intermediate good. Intermediate goods producers sell to retail goods producers who operate in an environment of monopolistic competition. Retail goods producers use their price-setting power to charge a markup over the marginal cost of production to perfectly competitive final good producers. However, they face quadratic adjustment costs when changing prices (Rotemberg, 1982).

Bankers live for two periods (Clerc et al., 2015), and receive a starting net worth from the previous generation of bankers. Then they decide how much net worth they pay out to households as dividends. We introduce this possibility for banks to choose the risk with which they operate. They face quadratic costs in the deviation of dividends from some target level of dividends. After paying dividends, they attract deposits, which they use to purchase securities issued by intermediate goods producers, who need these funds to purchase physical capital for production. Banks, however, are subject to a minimum equity-deposits ratio, where equity is defined as the amount of banks' net worth after dividend payments and adjustment costs. We model market power for banks in the market for deposits assuming a Dixit–Stiglitz framework, similar to Gerali et al. (2010), Güntner (2011), and Damjanovic et al. (2020). Bankers are profit maximizers and face limited liability. Depositors are (partially) protected by deposit insurance, and therefore do not fully price in the probability of bank default. As a result, bankers prefer deposit financing (Kareken and Wallace, 1978). When next period's revenues (which depend on an idiosyncratic shock to the return on assets) are not enough to cover repayment of deposits, the banker's assets are taken over by a government-owned deposit insurance company that partially reimburses depositors. However, as in Bernanke et al. (1999), this company faces costly state verification costs, as a result of which a fraction of revenues from the banker's assets cannot be recouped. We deviate from Bernanke et al. (1999) by having banks' creditors making an active savings decision. So in the optimization problem, bankers do not take a participation constraint into account, but incorporate in their decision the effect of their leverage choice on the probability of default. This in turn affects the interest rate at which they can borrow from depositors when there is no full deposit insurance.

Note that in our model, depositors do not have perfect foresight about outcomes in individual banks, therefore they cannot observe individual outcomes (defaults must be verified). However, they know the distribution of idiosyncratic shocks (which is the same for all banks), implying in equilibrium that every bank sets the same nominal interest rate and receives the same amount of deposits (there is monopolistic competition in the market for deposits). This means that households can correctly calculate expected returns.

2.1 Households

The economy is populated by a continuum of measure one of ex ante identical households. Households are risk averse and maximize expected utility given by:

$$\max_{\{c_{t+s},h_{t+s},a_{t+s},d_{t+s}\}_{s=0}^{\infty}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \left[\frac{\left(c_{t+s} - \upsilon c_{t-1+s}\right)^{1-\sigma_c} - 1}{1-\sigma_c} - \frac{\chi}{1+\varphi} \left(h_{t+s}\right)^{1+\varphi} \right] \right\} \tag{1}$$

where c_t denotes consumption and h_t households' labor supply. Households receive income from labor, remunerated at wage rate w_t, repayment (including interest) of previous period deposits d_{t-1} and profits Π_t^f from the firms and banks they own. Banks promise households in period $t-1$ a nominal interest rate $R_{t-1}^{n,d}$ in period t. Households cannot discriminate between banks because we will see in Section 2.3 that all banks make the same choices in equilibrium, and will therefore offer the same nominal interest $R_{t-1}^{n,d}$ in equilibrium. Therefore, households have their deposits perfectly diversified across banks. Arriving in period t, all banks will receive an idiosyncratic shock which is drawn from the same distribution for all banks. As a result of this shock, some banks will default and not be able to repay their depositors in full. A deposit insurance agency, however, reimburses a fraction of the deposits from those banks. If a substantial fraction of debt financing does not fall under deposit insurance scheme, investors in these types of debt will take losses. We capture this by assuming that a fraction γ of defaulting deposits is not reimbursed by the deposit insurance agency in our model. As household deposits are perfectly diversified across all banks, all households will receive the same effective rate of return $\tilde{R}_t^D = \left(1 - \gamma \Delta_t^b\right) \frac{R_{t-1}^{n,d}}{\pi_t}$ on their deposits in period t, where Δ_t^b denotes the fraction of defaulting banks, and $\pi_t \equiv P_t/P_{t-1}$ the gross inflation rate of the final goods. Observe that even though households are not capable of monitoring the behavior of individual banks, they are aware of the distribution from which the idiosyncratic shocks are drawn, and will therefore be able to correctly calculate the (expected) fraction of banks that will default.

Household income is used for consumption, new deposits, and lump sum taxes τ_t. Finally, we introduce a (nominally) risk-free asset a_t that is in zero net supply, on which the central bank sets the nominal interest rate R_t^n. Hence households face the following budget constraint:

$$c_t + a_t + d_t + \tau_t = w_t h_t + \left(\frac{R_{t-1}^n}{\pi_t}\right) a_{t-1} + \tilde{R}_t^D d_{t-1} + \Pi_t^f, \tag{2}$$

This results in the following first order conditions for savings through the risk-free asset and bank deposits:

$$a_t \quad : \quad E_t \left[\beta \Lambda_{t,t+1} \frac{R_t^n}{\pi_{t+1}}\right] = 1, \tag{3}$$

$$d_t \quad : \quad E_t \left[\beta \Lambda_{t,t+1} \left(1 - \gamma \Delta_{t+1}^b\right) \frac{R_t^{n,d}}{\pi_{t+1}}\right] = 1, \tag{4}$$

where $\beta^s \Lambda_{t,t+s} = \beta^s \lambda_{t+s}/\lambda_t$ denotes households' stochastic discount factor to discount cash flows from period $t+s$ into utility of period t, while λ_{t+s} denotes households' marginal utility from consumption in period $t+s$.

2.2 Producers

2.2.1 Final goods producers

Final goods producers operate in a perfectly competitive market. They purchase $y_{j,t}$ units of goods from retail firm $j \in [0,1]$ for a price $P_{j,t}$ and sell the final good y_t at price P_t. Final goods producers employ the following technology to produce final goods from intermediate goods:

$$y_t = \left(\int_0^1 (y_{j,t})^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}, \tag{5}$$

where ϵ denotes the elasticity of substitution between two different intermediate goods. They aim at maximizing period-by-period profits. Hence their optimization problem is static, and given by:

$$\max_{\{y_{j,t}\}} \left(P_t y_t - \int_0^1 P_{j,t} y_{j,t} dj\right), \tag{6}$$

subject to (5). This leads to the following first order condition, which describes the demand function for intermediate good j produced by intermediate firm $j \in [0,1]$:

$$y_{j,t} = \left(\frac{P_{j,t}}{P_t}\right)^{-\epsilon} y_t. \tag{7}$$

Substitution of this first order condition into the production technology of final goods producers leads to the following aggregate price level:

$$P_t = \left[\int_0^1 (P_{j,t})^{1-\epsilon} \, dj \right]^{1/(1-\epsilon)}. \tag{8}$$

2.2.2 Intermediate goods producers

Intermediate goods producers employ a constant returns to scale technology using capital $k_{j,t-1}$ and labor $h_{j,t}$ as inputs to produce $y_{j,t}$:

$$y_{j,t} = z_t k_{j,t-1}^{\alpha} h_{j,t}^{1-\alpha}, \tag{9}$$

where z_t denotes aggregate productivity, which is given by a log AR(1) process:

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{z,t}, \tag{10}$$

where $\rho_z \in [0, 1)$ and $\varepsilon_{z,t} \sim N(0, \sigma^2)$. At the end of period $t-1$, intermediate goods producer j issues securities $s_{j,t-1}^k$ at a price q_{t-1}^k to bankers in exchange for pledging next period's after-wage profits to bankers. We assume there are no financial frictions between bankers and intermediate goods producers, and also no monitoring costs. Therefore intermediate goods producers are capable of credibly pledging next period's profits (Gertler and Kiyotaki, 2010). They use the funds to buy physical capital $k_{j,t-1}$. Hence in equilibrium we will have that $s_{j,t-1}^k = k_{j,t-1}$. Exogenous shocks arrive at the beginning of period t, after which production takes place. Afterwards, intermediate goods producers pay wages to their workers, and sell the depreciated capital stock $(1-\delta) k_{,t-1}$ to capital goods producers at a price q_t^k. Finally, they pay a gross real return R_t^k to the owners of the corporate securities. Therefore, profits $\Pi_{j,t}^i$ of intermediate goods producer j are given by:

$$\Pi_{j,t}^i = mc_t y_{j,t} + q_t^k (1-\delta) k_{,t-1} - R_t^k q_{t-1}^k k_{j,t-1} - w_t h_{j,t}, \tag{11}$$

subject to equation (9), and with mc_t the price at which intermediate goods are sold to retail goods producers, and with w_t denoting the real wage rate. Intermediate goods producers operate in a perfectly competitive market, and aim to maximize after-wage profits. Labor is hired in a perfectly competitive labor market. Therefore, the optimal labor demand condition is given by:

$$w_t = (1-\alpha) mc_t z_t k_{j,t-1}^{\alpha} h_{j,t}^{-\alpha}. \tag{12}$$

We substitute this condition into the expression for profits (11), after which we set profits to zero since all after-wage revenues are paid to the owners of corporate securities. Doing so allows us to solve for the ex post return R_t^k on corporate securities:

$$R_t^k = \frac{r_t^k + q_t^k(1-\delta)}{q_{t-1}^k}, \tag{13}$$

where r_t^k is given by:

$$r_t^k = \alpha mc_t z_t k_{j,t-1}^{\alpha-1} h_{j,t}^{1-\alpha}. \tag{14}$$

Expression (13) is the same as in Gertler and Kiyotaki (2010); Gertler and Karadi (2011, 2013). They explain that these securities are state-contingent (on the realization of the exogenous shocks) and therefore more equity-like. Therefore, we refer to these assets as corporate securities rather than bank loans.

2.2.3 Retail goods producers

Retail goods producers buy from intermediate goods producers, and convert this one-for-one into a unique retail good. Since final goods producers buy from all retail goods producers using a CES production technology (5), retail good producers face monopolistic competition. Therefore, they take into account the demand function (7) when setting the price $P_{j,t}$ of retail good j. We follow Rotemberg (1982) by assuming that retail firms face quadratic adjustment costs when changing prices. Hence their optimization problem is given by:

$$\max_{P_{j,t}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left[\left(\frac{P_{j,t+s}}{P_{t+s}} \right) y_{j,t+s} - mc_{t+s} y_{j,t+s} - \frac{\kappa_P}{2} \left(\frac{P_{j,t+s}}{P_{j,t-1+s}} - \pi_{t-1+s}^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right)^2 y_{t+s} \right] \right\}, \tag{15}$$

subject to (7), where $\Lambda_{t,t+s} = \lambda_{t+s}/\lambda_t$ is the ratio of future to present marginal utility of consumption, and $\pi_t = P_t/P_{t-1}$ is the inflation rate of the price level of the final good. Taking the first order condition, and imposing a symmetric equilibrium, we find:

$$\kappa_p \left(\pi_t - \pi_{t-1}^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right) \pi_t y_t = (1-\epsilon)\, y_t + \epsilon mc_t y_t + E_t \left[\beta \Lambda_{t,t+1} \kappa_p \left(\pi_{t+1} - \pi_t^{\gamma_P} \bar{\pi}^{1-\gamma_P} \right) \pi_{t+1} y_{t+1} \right]. \tag{16}$$

2.2.4 Capital producers

After intermediate goods producers have employed the capital stock k_{t-1} for production in period t, they sell the depreciated capital stock $(1-\delta)\, k_{t-1}$ to capital goods producers at a price q_t^k. Capital goods producers transform the old (depreciated) capital stock one-for-one into new capital. They also purchase final goods i_t for conversion into new capital. However, converting i_t final goods into new capital is subject to adjustment costs, which are quadratic in the change in investment i_t/i_{t-1} with respect to the previous period. The new capital stock at the end of period t is then given by:

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \frac{\kappa_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 \right] i_t, \tag{17}$$

Capital goods producers' profits in period t are the difference between the revenue from selling the newly produced capital k_t at a price q_t^k and the costs from purchasing the old capital stock $q_t^k (1-\delta) k_{t-1}$ and the final goods for investment i_t. Capital goods producers maximize the expected sum of discounted future profits. They are owned by households, and therefore discount future profits with the households' stochastic discount factor. Hence their optimization problem is the following:

$$\max_{\{i_{t+s}\}_{s=0}^{\infty}} E_t \left\{ \sum_{s=0}^{\infty} \beta^s \Lambda_{t,t+s} \left[q_{t+s}^k k_{t+s} - q_{t+s}^k (1-\delta) k_{t-1+s} - i_{t+s} \right] \right\}. \tag{18}$$

After substituting equation (17), we differentiate with respect to i_t to get the following first order condition:

$$\begin{aligned} \frac{1}{q_t^k} &= 1 - \frac{\kappa_k}{2} \left(\frac{i_t}{i_{t-1}} - 1 \right)^2 - \kappa_k \frac{i_t}{i_{t-1}} \left(\frac{i_t}{i_{t-1}} - 1 \right) \\ &+ E_t \left[\beta \frac{\lambda_{t+1}}{\lambda_t} \frac{q_{t+1}^k}{q_t^k} \left(\frac{i_{t+1}}{i_t} \right)^2 \kappa_k \left(\frac{i_{t+1}}{i_t} - 1 \right) \right] \end{aligned} \tag{19}$$

2.3 Bankers

We assume an overlapping generations structure for the banking sector, whereby each generation of banks operates for two periods, similar to Clerc et al. (2015). Banks that start operating in period t receive a starting net worth from the household which owns the bank, and pay dividends, attract deposits, and purchase corporate securities. Next period, at the beginning of period $t+1$, returns are realized, the resulting bank net worth is paid out to the household to which the bank belongs, and the bank stops operating.[13]

Specifically, bank $j \in [0,1]$ enters period t with net worth $n_{j,t}^b$, which is provided by the owners of bank j (households), and is the same for all banks. In order to allow bank j to choose the degree of risk it takes with its balance sheet, bank j pays dividends $\eta_{j,t}$ to its shareholders in the first period of its existence.[14] These dividend payments, however, are subject to quadratic adjustment costs as in Jermann and Quadrini (2012).[15]

$$f(\eta_{j,t}) = \frac{1}{2} \kappa_\eta (\eta_{j,t} - \hat{\eta})^2, \tag{20}$$

where $\hat{\eta}$ is a target level for dividends, as quadratic adjustment costs are zero in that case. Hence,

[13]The ex-post real returns on deposits are realized following productivity shocks. The return on corporate securities is subject to the same shocks but additionally a multiplicative idiosyncratic risk shock. Note that by idiosyncratic risk shocks we mean bank-specific shocks to their return on the corporate securities, but the realization of the shock that each bank gets is drawn from the same distribution (Bernanke et al., 1999; Clerc et al., 2015). We interpret these shocks as reduced form for heterogeneity in banks' lending portfolio. Depending on the realization of the idiosyncratic shock, it is possible that a bank-specific share of the aggregate gross return on corporate securities is lost, as in Clerc et al. (2015). These authors refer to this shock as a portfolio returns shock.

[14]Banks choose how to fund a given amount of corporate securities through debt and equity but in our model there is no risk-taking choice on the asset side since there is only one type of asset, as we will see below.

[15]Dividend adjustment costs capture preferences for dividend smoothing (Lintner, 1956). We explain the implications of adopting dividend adjustment costs for the probability of bank default in the steady state at the end of this subsection.

after paying dividends, equity $e_{j,t}$ is given by:

$$e_{j,t} = n^b_{j,t} - \eta_{j,t} - f(\eta_{j,t}), \tag{21}$$

The bank is subject to the following minimum equity-deposit ratio, which implies that the amount of deposits $d_{j,t}$ that can be raised by bank j is limited by the amount of equity $e_{j,t}$:[16]

$$e_{j,t} \geq \kappa_t d_{j,t}, \tag{22}$$

We model market power for banks in the market for deposits assuming a Dixit–Stiglitz framework, similar to Güntner (2011), Gerali et al. (2010) and Damjanovic et al. (2020).[17] In this framework, each bank j is the unique provider of deposits of type j, which implies that bank j has the market power to set the nominal interest rate $R^{n,d}_{j,t}$ on deposits of type j, and provides any amount of deposits demanded at that interest rate. Therefore, the aggregate household's demand for deposits at bank j is given by:

$$d_{j,t} = \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t}\right)^{-\epsilon^d} d_t, \tag{23}$$

where d_t are the aggregate deposits in the economy and $\epsilon^d < -1$ (Gerali et al., 2010; Damjanovic et al., 2020). $R^{n,d}_t$ is given by:

$$R^{n,d}_t = \left[\int_0^1 \left(R^{n,d}_{j,t}\right)^{1-\epsilon^d} dj\right]^{1/(1-\epsilon^d)}. \tag{24}$$

After setting the nominal interest rate $R^{n,d}_{j,t}$, bank j obtains an amount of deposits $d_{j,t}$. Deposits $d_{j,t}$ and equity $e_{j,t}$ are then used to finance the acquisition of corporate securities $s^k_{j,t}$ at price q^k_t. Therefore, bank j's period t balance sheet constraint is given by:

$$q^k_t s^k_{j,t} + \eta_{j,t} + f(\eta_{j,t}) = n^b_{j,t} + d_{j,t} = n^b_{j,t} + \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t}\right)^{-\epsilon^d} d_t, \tag{25}$$

where we substituted equation (21) for $e_{j,t}$ and equation (23) for $d_{j,t}$. Similarly, we can write the equity-deposit constraint (22) as:

$$n^b_{j,t} - \eta_{j,t} - f(\eta_{j,t}) \geq \kappa_t \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t}\right)^{-\epsilon^d} d_t. \tag{26}$$

[16] In the absence of this constraint, the bank could raise deposits, and pay out all funds as dividends without investing any funds in corporate securities.

[17] In a previous version of our paper we had a perfectly competitive market for deposits, and found that the steady state spread between the return on corporate securities and the interest rate on deposits is always negative. As this is not in line with the data, we introduce market power for banks in the market for deposits to obtain a positive steady state credit spread.

Net worth $n^b_{j,t+1}$ of bank $j \in [0,1]$ in period $t+1$ equals the difference between the gross return R^k_{t+1} on corporate securities $q^k_t s^k_{j,t}$ and the gross return $R^{n,d}_{j,t}/\pi_{t+1}$ on deposits $d_{j,t}$. Bank j, however, receives a multiplicative idiosyncratic shock $\omega^b_{j,t+1}$ at the beginning of period $t+1$ on the aggregate return R^k_{t+1} on its corporate securities. Each bank's shock is drawn from the same log-normal distribution with cumulative density function $F^b\left(\omega^b_{j,t+1}\right)$ with mean μ^b_{t+1} and standard deviation σ^b_{t+1}. Hence bank j's gross return on corporate securities is $\omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t}$, as in Bernanke et al. (1999). Therefore, bank j's net worth in period $t+1$ is given by:

$$n^b_{j,t+1} = \omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{R^{n,d}_{j,t}}{\pi_{t+1}} d_{j,t} = \omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t, \qquad (27)$$

where we substituted equation (23). Bank j defaults if the idiosyncratic shock $\omega^b_{j,t+1}$ is such that bank j's return on assets is below the return on deposits. Hence we can define a cutoff value $\bar{\omega}^b_{j,t+1}$ below which bank j defaults:

$$\bar{\omega}^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} = \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t \implies \bar{\omega}^b_{j,t+1} = \frac{\frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t}{R^k_{t+1} q^k_t s^k_{j,t}}. \qquad (28)$$

We can now distinguish two cases: $\omega^b_{j,t} \geq \bar{\omega}^b_{j,t}$, in which case the bank pays out the remaining funds to its shareholders, and $\omega^b_{j,t} < \bar{\omega}^b_{j,t}$, in which case bank j defaults and does not pay out dividends in period $t+1$. In that case, the bank is taken over by the deposit insurance agency, which tries to recoup the bank's assets, but faces verification costs to be described below.

Bank profits $\Pi^b_{j,t+1}\left(\omega^b_{j,t+1}\right)$ in period $t+1$ for a given value of the idiosyncratic shock $\omega^b_{j,t+1}$ can be written as:

$$\Pi^b_{j,t+1}\left(\omega^b_{j,t+1}\right) = \max\left[\omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{R^n_{j,t}}{\pi_{t+1}} d_{j,t}, 0\right] = \max\left[\omega^b_{j,t+1} - \bar{\omega}^b_{j,t+1}, 0\right] R^k_{t+1} q^k_t s^k_{j,t},$$

where we substituted equation (28). Because we know the distribution $F^b\left(\omega^b_{j,t+1}\right)$, we can calculate the expected profit conditional on the realization of the aggregate return on securities R^k_{t+1} and inflation π_{t+1}, see Appendix A2.1:

$$\Pi^b_{j,t+1} = \int_{\bar{\omega}^b_{j,t+1}}^{\infty} \left(\omega^b_{j,t+1} - \bar{\omega}^b_{j,t+1}\right) f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} = \left[\Omega^b_{t+1} - \Gamma^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1} q^k_t s^k_{j,t}, \qquad (29)$$

where $\Gamma^b\left(\bar{\omega}^b_{j,t+1}\right)$ is defined as in Bernanke et al. (1999):

$$\Gamma^b\left(\bar{\omega}^b_{j,t+1}\right) = G^b\left(\bar{\omega}^b_{j,t+1}\right) + \bar{\omega}^b_{j,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right], \qquad (30)$$

where $G^b\left(\bar{\omega}^b_{j,t+1}\right) \equiv \int_0^{\bar{\omega}^b_{j,t+1}} \omega^b_{j,t+1} f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1}$ and $\int_{\bar{\omega}^b_{j,t+1}}^{\infty} f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1} = 1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)$. Finally, $E_{t+1}\left(\omega^b_{j,t+1}\right) = \int_0^{\infty} \omega^b_{j,t+1} f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1} = \Omega^b_{t+1}$ denotes the unconditional expectation of $\omega^b_{j,t+1}$, which is the same for all banks. We will assume that $\mu_b = -(1/2)\left(\sigma^b\right)^2$, which ensures that the unconditional expected value of $\omega^b_{j,t+1}$, $\bar{\Omega}^b$ is equal to one in the steady state (Bernanke et al., 1999).

The banks' objective function is given by the sum of today's dividends $\eta_{j,t}$ and expected (discounted) profits (29):

$$\eta_{j,t} + E_t \left\{ \beta\Lambda_{t,t+1} \left[\Omega^b_{t+1} - \Gamma^b \left(\frac{\frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t}{R^k_{t+1} q^k_t s^k_{j,t}} \right) \right] R^k_{t+1} q^k_t s^k_{j,t} \right\}. \tag{31}$$

where we used equation (28) to substitute out $\bar{\omega}^b_{j,t+1}$. The banks' optimization problem is now given by the maximization of (31), subject to the balance sheet constraint (25) and the equity-deposit constraint (26). To find bank j's optimal choices, we set up the Lagrangian:

$$\begin{aligned} \mathcal{L} \;\; = \;\; & \eta_{j,t} + E_t \left\{ \beta\Lambda_{t,t+1} \left[\Omega^b_{t+1} - \Gamma^b \left(\frac{\frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t}{R^k_{t+1} q^k_t s^k_{j,t}} \right) \right] R^k_{t+1} q^k_t s^k_{j,t} \right\} \\ & + \psi^b_t \left\{ n^b_{j,t} - \eta_{j,t} - f(\eta_{j,t}) + \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t} \right)^{-\epsilon^d} d_t - q^k_t s^k_{j,t} \right\} \\ & + \psi^d_t \left\{ n^b_{j,t} - \eta_{j,t} - f(\eta_{j,t}) - \kappa_t \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t} \right)^{-\epsilon^d} d_t \right\}, \end{aligned}$$

where ψ^b_t is the Lagrangian multiplier on bank j's balance sheet constraint (25), and ψ^d_t the Lagrangian multiplier on bank j's equity-deposit constraint (26). This generates the following first order conditions:

$$s^k_{j,t} \; : \; \psi^b_t = E_t \left\{ \beta\Lambda_{t,t+1} \left[\Omega^b_{t+1} - \Gamma^b \left(\bar{\omega}^b_{j,t+1} \right) \right] R^k_{t+1} \right\} + E_t \left\{ \beta\Lambda_{t,t+1} \left[1 - F^b \left(\bar{\omega}^b_{j,t+1} \right) \right] \bar{\omega}^b_{j,t+1} R^k_{t+1} \right\}, \tag{32}$$

$$\eta_{j,t} \; : \; 1 = \left(\psi^b_t + \psi^d_t \right) \left[1 + f'(\eta_{j,t}) \right], \tag{33}$$

$$R^{n,d}_{j,t} \; : \; -\epsilon^d \left(\psi^b_t - \kappa_t \psi^d_t \right) = \left(1 - \epsilon^d \right) E_t \left\{ \beta\Lambda_{t,t+1} \left[1 - F^b \left(\bar{\omega}^b_{j,t+1} \right) \right] \frac{R^{n,d}_{j,t}}{\pi_{t+1}} \right\} \tag{34}$$

where we used $\frac{d\Gamma^b(\omega)}{d\omega} = 1 - F^b(\omega)$; see Appendix A2.2. In addition, observe that the balance sheet constraint (25) is always binding, as the right hand side of equation (32) is always larger than zero. Meanwhile, the equity-deposit constraint is occasionally binding, which is captured by the following equation:

$$\psi^d_t \left(n^b_{j,t} - \eta_{j,t} - f(\eta_{j,t}) - \kappa_t \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t} \right)^{-\epsilon^d} d_t \right) = 0. \tag{35}$$

To understand the intuition behind the first order condition for corporate securities (32), we substitute equation (30) to obtain:

$$\psi_t^b = E_t \left\{ \beta \Lambda_{t,t+1} \left[\Omega_{t+1}^b - G^b \left(\bar{\omega}_{j,t+1}^b \right) \right] R_{t+1}^k \right\}, \tag{36}$$

The left hand side of equation (36) represents the marginal cost from attracting an additional unit of corporate securities, as doing so tightens intermediaries' balance sheet constraint which is captured by its shadow value ψ_t^b. The marginal benefit from an additional unit of corporate securities is the expected value of the idiosyncratic shock conditional on survival, which is equal to $\Omega_{t+1}^b - G^b \left(\bar{\omega}_{j,t+1}^b \right)$, multiplied by the aggregate return on corporate securities R_{t+1}^k.[18] To convert in terms of today's utility, we multiply by households' stochastic discount factor $\beta \Lambda_{t,t+1}$ and take the expected value.

The intuition behind the first order condition for dividends (33) is straightforward. The left hand side represents the marginal benefit from an additional unit of dividends that is paid out to households. The marginal cost is on the right hand side, and comes from the fact that an additional unit of dividends tightens banks' balance sheet constraint, the shadow value of which is ψ_t^b, as well as the equity-deposit constraint, the shadow value of which is ψ_t^d. The amount by which it is tightened, however, is not $\psi_t^b + \psi_t^d$, but $\left(\psi_t^b + \psi_t^d \right) \left[1 + f' \left(\eta_{j,t} \right) \right]$ since paying out an additional unit in dividends also decreases banks' net worth as a result of dividend adjustment costs by an amount $f' \left(\eta_{j,t} \right)$.

The intuition behind the first order condition for the nominal interest rate on deposits (34) can be explained in the following way. The left hand side represents the net marginal benefit from an increase in the nominal interest rate, which consists of two components. First, an increase in the nominal interest rate increases the volume of deposits, which relaxes the banks' balance sheet constraint (25), everything else equal. The amount by which the constraint is relaxed is equal to the shadow value ψ_t^b from an additional unit of deposits multiplied by the amount by which deposits increase (as a result of an increase in the interest rate), which is the elasticity of substitution $-\epsilon^d > 0$. However, attracting additional deposits also implies a tightening of the equity-deposit constraint (26). The amount by which the constraint is tightened is equal to the additional equity that is required, which is a fraction κ_t of the additional deposits $-\epsilon^d$, and is multiplied by the shadow value ψ_t^d of an additional unit of equity. Therefore, the net marginal benefit from an increase in the nominal interest rate is $-\epsilon^d \left(\psi_t^b - \kappa_t \psi_t^d \right)$.

The marginal cost from an increase in the nominal interest rate on deposits can be found on the right hand side of equation (34), and comes from the fact that an increase in the deposit interest rate not only increases interest payments directly, but also indirectly through an increase in the volume of deposits, see equation (23). The total effect on interest payments is given by the term $1 - \epsilon^d$ multiplied by the current (expected) real interest rate $\frac{R_{j,t}^{n,d}}{\pi_{t+1}}$ and corrected for the probability $1 - F^b \left(\bar{\omega}_{j,t+1}^b \right)$ that bank j survives the idiosyncratic shock.

[18] Remember that $G \left(\bar{\omega}_t^b \right) \equiv \int_0^{\bar{\omega}_t^b} \omega_t^b f^b \left(\omega_t^b \right) d\omega_t^b$ denotes the expected value of the idiosyncratic shock conditional on the bank not surviving, in which case bank j's assets are seized by the deposit insurance agency of the government and no longer accrue to banks' owners.

We prove in Appendix A2.3 that all banks choose the same quantities and interest rate in equilibrium, i.e. $s_{j,t}^k = s_t^k$, $\eta_{j,t} = \eta_t$, and $R_{j,t}^{n,d} = R_t^{n,d}$. Therefore, we drop the j subscript going forward. We then calculate the default rate, the fraction of banks that default, which occurs if $\omega_t^b < \bar{\omega}_t^b$. We denote this rate by $\Delta_t^b = \int_0^{\bar{\omega}_t^b} f^b\left(\omega_t^b\right) d\omega_t^b = F^b\left(\bar{\omega}_t^b\right)$. Since households own a diversified portfolio of banks, every household receives the same level of profits from the banks that survive the idiosyncratic shock. After receiving these payouts, households provide the new generation of bankers an amount that is equal to a fraction θ^b of the profits of the banks that survived the idiosyncratic shock (Gertler and Karadi, 2011). In addition, they provide a fraction χ^b of previous period aggregate net worth to the new generation of bankers. Therefore, the net worth for a bank from the new generation (as well as aggregate net worth for the new generation of bankers) is given by:

$$n_t^b = \theta^b\left[\Omega_t^b - \Gamma^b\left(\bar{\omega}_t^b\right)\right] R_t^k q_{t-1}^k s_{t-1}^k + \chi^b n_{t-1}^b, \tag{37}$$

More detailed mathematical derivations are provided in Appendix A2.4.

Finally, we show that in the absence of dividend adjustment costs there is no effect from changes in the standard deviation of banks' idiosyncratic shocks on the steady state probability of bank default when the equity-deposit constraint (26) is not binding. To see this, first observe from equation (33) that $\psi_t^b = 1$ when the equity-deposit constraint (26) is not binding, i.e. $\psi_t^d = 0$, and when dividend-adjustment costs are zero, i.e. $\kappa_\eta = 0$. Solving for the nominal interest rate on deposits from households' first order condition for deposits (4) and substituting the resulting expression in banks' first order condition for the nominal interest rate on deposits gives the following expression:

$$1 = \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \frac{E_t\left\{\beta \frac{\Lambda_{t,t+1}}{\pi_{t+1}}\left[1 - F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}}{E_t\left\{\beta \frac{\Lambda_{t,t+1}}{\pi_{t+1}}\left[1 - \gamma F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}}.$$

This expression boils down to the following expression in the non-stochastic steady state:

$$1 = \left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\left[\frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)}\right],$$

from which we immediately see that $F^b\left(\bar{\omega}^b\right)$ will not change with the standard deviation of banks' idiosyncratic shocks σ^b for $\gamma < 1$, a feature which we think is unrealistic.

2.4 Government

2.4.1 Fiscal authority

The government purchases a constant fraction of output: $g_t = (\bar{g}/\bar{y})\, y_t$, where $\bar{g}$ and $\bar{y}$ denote steady state government spending and output, respectively. The government is engaged in (partial) deposit insurance. When a bank defaults, the deposit insurance agency closes the bank down and takes over its assets. This agency incurs costs T_t^{dia}, both from (partially) reimbursing depositors as well as from the fact that a fraction μ^{dia} of recouped assets of bank j are lost (because of costs that arise upon recouping and selling assets of the banks that have defaulted, see Bernanke et al. (1999); Clerc et al. (2015)):

$$\begin{aligned} T_t^{dia} &= (1-\gamma)\int_0^1\int_0^{\bar{\omega}_t^b} R_t^d d_{j,t-1} f^b\left(\omega_t^b\right) d\omega_t^b dj - \int_0^1\int_0^{\bar{\omega}_t^b} \omega_t^b R_t^k q_{t-1}^k s_{j,t-1}^k f^b\left(\omega_t^b\right) d\omega_t^b dj \\ &+ \mu^{dia}\int_0^1\int_0^{\bar{\omega}_t^b} \omega_t^b R_t^k q_{t-1}^k s_{j,t-1}^k f^b\left(\omega_t^b\right) d\omega_t^b dj \\ &= (1-\gamma) F^b\left(\bar{\omega}_t^b\right) R_t^d d_{t-1} - \left(1-\mu^{dia}\right) G^b\left(\bar{\omega}_t^b\right) R_t^k q_{t-1}^k s_{t-1}^k, \end{aligned} \tag{38}$$

where $R_t^d \equiv \frac{R_{t-1}^{n,d}}{\pi_t}$ denotes the real return on deposits in case of no default, and where $G\left(\bar{\omega}_t^b\right) = \int_0^{\bar{\omega}_t^b} \omega_t^b f^b\left(\omega_t^b\right) d\omega_t^b$. Intermediate steps in this calculation can be found in the Appendix.

Total government expenditures in period t are equal to $g_t + T_t^{dia}$. These expenditures are paid by raising lump sum taxes T_t from households.[19] Hence the government budget constraint is given by:

$$T_t = g_t + T_t^{dia}. \tag{39}$$

2.4.2 Monetary authority

The monetary authority is in charge of setting the nominal interest rate R_t^n on the risk-free asset a_t that is in zero net supply. It does so by employing a standard Taylor rule with interest rate smoothing parameter ρ_r:

$$R_t^n = (1-\rho_r)\left[\bar{R}_n + \kappa_\pi\left(\pi_t - \bar{\pi}\right) + \kappa_y \log\left(y_t/y_{t-1}\right)\right] + \rho_r R_{t-1}^n + \varepsilon_{r,t}, \tag{40}$$

where $\varepsilon_{r,t} \sim N(0,\sigma_r^2)$ and κ_π and κ_y are the weights given to inflation deviations and the growth rate of output.

2.5 Market clearing

Market clearing in the capital market occurs when total securities ($s_t^k \equiv \int_0^1 s_{j,t}^k dj$) equal the aggregate capital stock k_t:

$$s_t^k = k_t. \tag{41}$$

In equilibrium, output is used for consumption by households, investment by capital producers, government spending, price-adjustment costs, and deadweight costs from the verification of

[19] Observe that households therefore do not internalize the fact that they have to pay for their own (partial) deposit insurance.

bankers $\mu^{dia} G^b \left(\bar{\omega}_t^b\right) R_t^k q_{t-1}^k s_{t-1}^k$ that report that they cannot repay their loans:

$$y_t = c_t + i_t + g_t + \frac{\kappa_p}{2}\left(\pi_t - \pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^2 y_t + \mu^{dia} G^b \left(\bar{\omega}_t^b\right) R_t^k q_{t-1}^k s_{t-1}^k. \tag{42}$$

2.6 The role of limited liability in banks' lending decisions

To further highlight the role that limited liability plays in our model, we will now compare the first order conditions from Section 2.3 with those obtained from a model where banks are not subject to limited liability, a case we will refer to as "unlimited liability". In this model, banks have to repay their creditors even in case the return on their assets is not sufficient. In this last case, the owners of the bank (households) can be forced to reimburse the creditors.[20,21] We show the results for the case where the equity-deposit constraint (26) is binding, i.e. $\psi_t^d > 0$, as this will turn out to be the case in (most of) the simulations in Section 4. We show in Appendix A6 that the results carry over in case the constraint is not binding, i.e. $\psi_t^d = 0$. For analytical convenience, we assume in the current and next section that dividend adjustment costs are zero, i.e. $f\left(\eta_{j,t}\right) = 0$. We will lift this restriction when we discuss the numerical results in Section 4. Finally, we will employ the superscripts ULL and LL, respectively, to indicate the case of unlimited liability and the case of limited liability, respectively.

First, we show in Appendix A5 that the first order condition for dividends under unlimited liability is the same as the first order condition under limited liability, equation (33).[22] Next, we compare the first order condition for the nominal interest rate on deposits under unlimited liability and limited liability, respectively:

$$\psi_t^{b,ULL} = \frac{\kappa_t}{1+\kappa_t} + \frac{1}{1+\kappa_t}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right), \tag{43}$$

$$\psi_t^{b,LL} = \frac{\kappa_t}{1+\kappa_t} + \frac{1}{1+\kappa_t}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\frac{E_t\left\{\beta\frac{\Lambda_{t,t+1}}{\pi_{t+1}}\left[1 - F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}}{E_t\left\{\beta\frac{\Lambda_{t,t+1}}{\pi_{t+1}}\left[1 - \gamma F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}}, \tag{44}$$

where the derivation of equation (43) can be found in Appendix A5. The first order condition (44) is obtained from the first order condition for the nominal interest rate on deposits under limited liability (34) in the following way. First, we substitute $\Delta_t^b = F^b\left(\bar{\omega}_t^b\right)$ into households' first order condition for deposits (4), after which we solve for the nominal interest rate on deposits $R_t^{n,d}$ and substitute the resulting expression into equation (34). Then, we solve for ψ_t^d from the first order condition for dividends (33) and substitute to obtain first order condition (44).

[20]The equivalent maximization objective of (31) under unlimited liability is $\eta_{j,t} + E_t\left[\beta\Lambda_{t,t+1}\left(\Omega_{t+1}^b R_{t+1}^k q_t^k s_{j,t}^k - R_{j,t}^{n,d} d_{j,t}/\pi_{t+1}\right)\right]$, as we now integrate the idiosyncratic shock $\omega_{j,t+1}^b$ from 0 to ∞, rather than from $\bar{\omega}_{j,t+1}^b$ to ∞.

[21]However, observe that under limited liability, households also effectively reimburse creditors in the presence of deposit insurance, as payments from the deposit insurance agency to creditors are financed by levying lump sum taxes on households.

[22]For comparability, we assume that the equity-deposit constraint is also binding in the unlimited liability case.

Before we compare equations (43) and (44), let us first discuss the economic intuition behind first order condition (44). The left hand side of equation (44) denotes the marginal benefit from a relaxation of the balance sheet constraint (25) by an additional unit of deposits – through an increase in the nominal interest rate on deposits, see equation (23) – as an additional unit of deposits allows the bank to expand corporate securities by an amount $1+\kappa_t$.[23] The right hand side denotes the marginal cost from an additional unit of deposits, which is a weighted average of the need to retain an additional amount of equity κ_t (normalized by $1+\kappa_t$), and the need to pay additional interest, which is equal to the real interest rate multiplied by the probability $1-F^b\left(\bar{\omega}^b_{t+1}\right)$ that the bank survives the idiosyncratic shock (and discounted using the households' stochastic discount factor $\beta\Lambda_{t,t+1}$), which is normalized by $1+\kappa_t$, the amount by which corporate securities expand as a result of an additional unit of deposits.

When $\gamma = 1$, households fully price in the probability of default. In that case, the first order condition under limited liability (44) exactly coincides with that under unlimited liability (43). However, when $0 \leq \gamma < 1$, we see that $\psi_t^{b,LL} < \psi_t^{b,ULL}$ since $E_t\left\{\beta\frac{\Lambda_{t,t+1}}{\pi_{t+1}}\left[1-\gamma F^b\left(\bar{\omega}^b_{t+1}\right)\right]\right\} > E_t\left\{\beta\frac{\Lambda_{t,t+1}}{\pi_{t+1}}\left[1-F^b\left(\bar{\omega}^b_{t+1}\right)\right]\right\}$. The reason is that limited liability generates moral hazard under (partial) deposit insurance (Kareken and Wallace, 1978): since households are (partially) reimbursed in case of default, the probability of bank default is not fully incorporated in the interest rate at which creditors are willing to lend to the bank. As a result, the cost from acquiring an additional unit of deposits decreases relative to unlimited liability, which induces banks to acquire more deposits (through raising the interest rate), which increases the cut-off value $\bar{\omega}^b_{t+1}$ in equation (28), everything else equal, and therefore the probability of default $F^b\left(\bar{\omega}^b_{t+1}\right)$.

However, despite the cost of an additional unit of deposits being lower under limited liability, there is a second effect from limited liability that induces banks to *reduce* credit provision to the real economy, everything else equal. To explain this, let us start by writing down the first order condition for corporate securities under unlimited and limited liability, respectively:

$$\psi_t^{b,ULL} = E_t\left[\beta\Lambda_{t,t+1}\Omega^b_{t+1}R^k_{t+1}\right], \tag{45}$$

$$\psi_t^{b,LL} = E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1}-G^b\left(\bar{\omega}^b_{j,t+1}\right)\right]R^k_{t+1}\right\}, \tag{46}$$

The intuition behind equation (45) is that the marginal cost ψ_t^{ULL} from a tightening of bank j's balance sheet constraint (25) must be equal in equilibrium to the marginal benefit from an additional unit of corporate securities, which is equal to the unconditional expected value $\Omega^b_{t+1} = \int_0^\infty \omega^b_{t+1} f^b\left(\omega^b_{t+1}\right) d\omega^b_{t+1}$ of the idiosyncratic shock ω^b_{t+1} multiplied by the aggregate return on capital R^k_{t+1} (and discounted using the households' stochastic discount factor $\beta\Lambda_{t,t+1}$).

The marginal benefit is different under limited liability, see equation (46). In contrast with the unlimited liability case, we see that the returns from an additional unit of corporate securities do not always accrue to bank j under limited liability, as they accrue to the government's

[23] Substitution of banks' binding equity-deposit constraint (22) into the banks' balance sheet constraint $q^k_t s^k_{j,t} = e_{j,t} + d_{j,t}$ immediately allows us to write $q^k_t s^k_{j,t} = (1+\kappa_t)\, d_{j,t}$.

deposit insurance agency in case of a default by bank j. This is captured by the fact that we subtract $G^b\left(\bar{\omega}^b_{t+1}\right)$ from Ω^b_{t+1} in equation (46), where $G\left(\bar{\omega}^b_{j,t}\right) = \int_0^{\bar{\omega}^b_{j,t}} \omega^b_{j,t} f^b\left(\omega^b_{j,t}\right) d\omega^b_{j,t}$ denotes the expected value of the idiosyncratic shock conditional on default by bank j, which represents the states of the world in which bank j's assets are seized by the deposit insurance agency of the government.

Therefore, we can immediately see that the case of $\gamma = 1$ represents the worst case from the perspective of bank j, everything else equal (relative to the case $0 \leq \gamma < 1$ and unlimited liability). The marginal cost of an additional unit of deposits is the same as under unlimited liability, but the benefits from an additional unit of corporate securities do not always accrue to bank j (unlike the case of unlimited liability). Since the marginal benefit from an additional unit of corporate securities is smaller than under unlimited liability, bank j will acquire fewer corporate securities, everything else equal. Since all banks choose an identical allocation in equilibrium, less credit provision will lead to a lower capital stock.

For $0 \leq \gamma < 1$, it remains the case that the marginal benefit from an additional unit of corporate securities under limited liability will always be smaller than that under unlimited liability, everything else equal. However, in this case the effect on credit provision to the real economy is ambiguous because of moral hazard, which arises because the marginal cost from an additional unit of deposits (to finance an additional unit of corporate securities) is lower than under unlimited liability.

3 Analytical results

In this section we derive analytical results for the non-stochastic steady state of the model. The non-stochastic steady state suffices to properly capture the role that idiosyncratic shocks have on long-run outcomes. In contrast to aggregate shocks, which are absent in the non-stochastic steady state, idiosyncratic shocks still arrive for all bankers that are in the second period of their existence. In addition, the idiosyncratic realizations of these shocks differ between different bankers. In particular, every period bankers that receive an idiosyncratic shock $\omega^b < \bar{\omega}^b$ will default, despite the economy being in the non-stochastic steady state. However, as the idiosyncratic shocks have a stationary distribution, aggregate variables remain at their steady state values. We will derive the analytical results for the case where the equity-deposit constraint (26) is binding, which will turn out to be the case in (most of) our numerical simulations in Section 4. We show in Appendix A9 the analytical results for the case where the equity-deposit constraint (26) is not binding.

First, we show that the absolute level of credit provision can be ranked for the case of full deposit insurance, unlimited liability, and no deposit insurance. Then, we develop comparative dynamics for the parameters that reflect the extent of bank regulation: the volatility of the idiosyncratic shocks to the return on bankers' assets (σ^b) and the extent to which deposits are insured (γ).

σ^b represents a measure of the asset risk that banks are allowed to incur by regulation. For example, interest rate ceilings in the US were effectively preventing banks from lending to riskier borrowers (since they would not be properly compensated) before the 1978 the Marquette vs. First of Omaha ruling which effectively set off a competitive wave of deregulation (Sherman, 2009). Therefore, we interpret σ^b as a measure of the degree to which banks are regulated, with a low standard deviation representing a regime with heavy regulation, and a large standard deviation representing a regime with light regulation.[24]

γ represents the fraction of deposits that are not covered by the government's insurance system (modern banks have forms of debt not covered by deposit insurance: such as interbank loans, bank bonds and shadow banking).

We start by writing down the non-stochastic steady state equations of the banks' first order condition for corporate securities (36), for dividends (33), the nominal interest rate on deposits (34), the households' first order condition for saving through deposits (4), and the return on corporate securities (13):

$$\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k = \left(\frac{\bar{\kappa}}{1+\bar{\kappa}}\right)\frac{1}{1+\kappa_\eta\left(\bar{\eta}-\hat{\eta}\right)}+\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\frac{1-F^b\left(\bar{\omega}^b\right)}{1-\gamma F^b\left(\bar{\omega}^b\right)}, \quad (47)$$

$$\bar{\psi}^b+\bar{\psi}^d = \frac{1}{1+\kappa_\eta\left(\bar{\eta}-\hat{\eta}\right)}, \quad (48)$$

$$\bar{\psi}^b = \left(\frac{\bar{\kappa}}{1+\bar{\kappa}}\right)\frac{1}{1+\kappa_\eta\left(\bar{\eta}-\hat{\eta}\right)}+\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\frac{1-F^b\left(\bar{\omega}^b\right)}{1-\gamma F^b\left(\bar{\omega}^b\right)}, \quad (49)$$

$$1 = \beta\left[1-\gamma F^b\left(\bar{\omega}^b\right)\right]\bar{R}^d, \quad (50)$$

$$\bar{R}^k = \alpha\bar{m}c\bar{z}\bar{k}^{\alpha-1}\bar{h}^{1-\alpha}+1-\delta, \quad (51)$$

where we define the real interest rate on deposits as $\bar{R}^d \equiv \frac{\bar{R}^{n,d}}{\bar{\pi}}$. Note that the ex-ante and ex-post interest rates coincide in a non-stochastic steady state. The right hand side of equations (47) and (49) are obtained in the following way: first, we solve for $\bar{\psi}^d$ from the first order condition for dividends (33), and substitute the resulting expression into the first order condition for banks' nominal interest rate on deposits (34). This allows us to obtain an expression for $\bar{\psi}^b$, after which we substitute the expression that we obtain for $\frac{\bar{R}^{n,d}}{\bar{\pi}}$ from the households' first order condition for saving through deposits (4). This results in equation (49), which we subsequently substitute into the first order condition for corporate securities to obtain equation (47). Also observe that we have substituted the marginal product of capital (14) into the expression for the ex post return on corporate securities to obtain equation (51).

We will make two simplifying assumptions in this section to facilitate analytical tractability, but which do not affect our results qualitatively. First, we temporarily assume that dividend adjustment costs are zero by setting $\kappa_\eta = 0$. Second, we temporarily assume that households' labor supply is fixed at $\bar{h} = 1$, and that the wage rate adjusts such that intermediate goods

[24]Note that it may not be always the case that more stringent regulation reduces risk, see for example Laeven and Levine (2009), who explain that higher capital requirements might induce banks to lend to riskier borrowers.

producers hire all labor in equilibrium. Doing so allows us to immediately infer from equation (51) that the return on corporate securities and the stock of physical capital are inversely related under changes in either σ^b or γ. Also observe that the change in capital leads to a change in investment and output of the same sign, since $\bar{i} = \delta\bar{k}$ and $\bar{y} = \bar{z}\bar{k}^{\alpha}$.

In addition, we assume that the unconditional credit spread $\bar{R}^k - \bar{R}^d$ is always positive, and check that $\bar{R}^k - \bar{R}^d > 0$ always holds in the numerical simulations in Section 4. In that case, we can immediately infer that the steady state cut-off value $\bar{\omega}^b = \frac{\bar{R}^d \bar{d}}{\bar{R}^k \bar{k}} < 1$.

Finally, we combine the first order conditions for the nominal interest rate on deposits (43) and corporate securities (45) under unlimited liability, and find that the steady state return on corporate securities under unlimited liability (when dividend adjustment costs are zero) is given by:

$$\bar{R}^k\Big|^{ULL} = \frac{1}{\beta}\left[\frac{\bar{\kappa}}{1+\bar{\kappa}} + \frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\right] > 1. \tag{52}$$

The fact that $\bar{R}^k > 1$ can be observed from remembering that $\epsilon^d < -1$. Therefore, $\left(\epsilon^d - 1\right)/\epsilon^d > 1$, which in turn ensures that the term inside the square brackets is larger than one. Observing that households' subjective discount factor $\beta < 1$ then ensures that $\bar{R}^k > 1$.

3.1 The level of credit provision to the real economy

We start this section by looking at the level of credit provision to the real economy under the case where there is unlimited liability, the case of limited liability with full deposit insurance ($\gamma = 0$), and the case of limited liability without deposit insurance ($\gamma = 1$). Specifically, we will prove that credit provision under limited liability and full deposit insurance is always larger than or equal to credit provision under unlimited liability, which in turn is always larger than or equal to credit provision under limited liability and no deposit insurance:

Proposition 1. *Credit provision under limited liability and full deposit insurance is always larger than or equal to credit provision under unlimited liability, which in turn is always larger than or equal to credit provision under limited liability and no deposit insurance:*

$$\bar{k}\Big|^{LL}_{\gamma=0} \geq \bar{k}\Big|^{ULL} \geq \bar{k}\Big|^{LL}_{\gamma=1},$$

where we recall from the market clearing condition for corporate securities (41) *that credit provision to the real economy $\bar{s}^k$ is in equilibrium equal to the stock of physical capital $\bar{k}$.*

Proof. We start by rewriting the first order condition for corporate securities (47) under limited liability (denoted by superscript LL) and with dividend adjustment cost $\kappa_\eta = 0$ in the following way:

$$\bar{R}^k\Big|^{LL} = \frac{1}{1 - G^b\left(\bar{\omega}^b\right)}\left\{\frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)} \cdot \bar{R}^k\Big|^{ULL} + \frac{1}{\beta}\left(\frac{\bar{\kappa}}{1+\bar{\kappa}}\right)\frac{(1-\gamma)\,F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)}\right\}, \tag{53}$$

where we employed equation (52).

We now compare the return on corporate securities under unlimited liability with the return on corporate securities under limited liability in the absence of deposit insurance ($\gamma = 1$).

Lemma 1. *In the absence of deposit insurance ($\gamma = 1$), the return on capital under limited liability $\bar{R}^k\Big|_{\gamma=1}^{LL}$ is always larger than or equal to the return on capital under unlimited liability $\bar{R}^k\Big|^{ULL}$:*

$$\bar{R}^k\Big|_{\gamma=1}^{LL} \geq \bar{R}^k\Big|^{ULL}.$$

Proof. We start by substituting $\gamma = 1$ into equation (53) to write the return on capital under limited liability and no deposit insurance $\bar{R}^k\Big|_{\gamma=1}^{LL}$ as:

$$\bar{R}^k\Big|_{\gamma=1}^{LL} = \frac{1}{1 - G^b\left(\bar{\omega}^b\right)} \cdot \bar{R}^k\Big|^{ULL} \geq \bar{R}^k\Big|^{ULL},$$

since $G^b\left(\bar{\omega}^b\right) \equiv \int_0^{\bar{\omega}^b} \omega f\left(\omega\right) d\omega \leq \int_0^{\infty} \omega f\left(\omega\right) d\omega = 1$. □

The intuition behind this result is the following. In the absence of deposit insurance, depositors price in the probability of default. Therefore, the marginal cost from an additional unit of deposits is the same as under unlimited liability. However, unlike the case with unlimited liability, there are states of the world where the return on corporate securities is equal to zero, namely when the bank cannot meet its liabilities and defaults. The existence of these states reduces the marginal benefit from an additional unit of corporate securities by a factor $1 - G^b\left(\bar{\omega}^b\right)$ with respect to unlimited liability. As a result, banks decrease the size of their balance sheets, which raises the return on corporate securities $\bar{R}^k$.

Next, we prove that the return on capital under limited liability and full deposit insurance is always less than or equal to the return on capital under unlimited liability.

Lemma 2. *Under full deposit insurance ($\gamma = 0$), the return on capital under limited liability $\bar{R}^k\Big|_{\gamma=0}^{LL}$ is always less than or equal to the return on capital under unlimited liability $\bar{R}^k\Big|^{ULL}$:*

$$\bar{R}^k\Big|_{\gamma=0}^{LL} \leq \bar{R}^k\Big|^{ULL}.$$

Proof. Substitution of $\gamma = 0$ into equation (53) does not lead to a straightforward expression from which we can immediately infer Lemma 2. Therefore, we follow an alternative strategy to prove Lemma 2. To do so, let us first observe that when $\sigma^b \downarrow 0$, the probability of default $F^b\left(\bar{\omega}^b\right) \downarrow 0$ and the expected value of the idiosyncratic shock conditional on default $G^b\left(\bar{\omega}^b\right) \downarrow 0$. As a result, we see that:

$$\lim_{\sigma^b \downarrow 0} \bar{R}^k\Big|_{\gamma=0}^{LL} = \bar{R}^k\Big|^{ULL}.$$

Next, we prove below in Proposition 2 that $\frac{d\bar{R}^k}{d\sigma^b} < 0$ by using the expression for the return on corporate securities under limited liability (47). In addition, observe that expression (47) is continuous in σ^b for $\sigma^b > 0$ and observe from equation (52) that the return on corporate securities under unlimited liability is constant.[25] Therefore, we can conclude that since the return on corporate securities under limited liability is equal to the return on corporate securities under unlimited liability when the probability of default is zero, and since the return on corporate securities under limited liability is decreasing with σ^b while it is constant under unlimited liability, it must be the case that $\bar{R}^k\Big|_{\gamma=0}^{LL} < \bar{R}^k\Big|^{ULL}$ for values of $\sigma^b > 0$. Therefore, we conclude that $\bar{R}^k\Big|_{\gamma=0}^{LL} \leq \bar{R}^k\Big|^{ULL}$ for $\sigma^b \geq 0$. This proves the proposition. □

The intuition behind this result is the following. The introduction of limited liability and full deposit insurance introduces moral hazard (Kareken and Wallace, 1978). As a result, the marginal cost from raising an additional unit of deposits decreases by a factor $1-F^b\left(\bar{\omega}^b\right)$ (relative to unlimited liability), since depositors do not price in the probability of default. This can be seen from the first term on the right hand side of equation (53). Simultaneously, the introduction of limited liability generates states of the world in which the cash flows from corporate securities do not accrue to the bank. This reduction in the marginal benefit of an additional unit of corporate securities is equal to the expected value of the idiosyncratic shock conditional on default $G^b\left(\bar{\omega}^b\right)$, and shows up in the denominator of equation (53). It turns out that the expected value $G^b\left(\bar{\omega}^b\right)$ is always smaller than or equal to the probability of default $F^b\left(\bar{\omega}^b\right)$ since $G^b\left(\bar{\omega}^b\right) \equiv \int_0^{\bar{\omega}^b} \omega^b f^b\left(\omega^b\right) d\omega^b \leq \bar{\omega}^b \int_0^{\bar{\omega}^b} f^b\left(\omega^b\right) d\omega^b = \bar{\omega}^b F^b\left(\bar{\omega}^b\right) \leq F^b\left(\bar{\omega}^b\right)$, since $\bar{\omega}^b \leq 1$. In other words, the marginal benefit from an additional unit of corporate securities decreases by less than the marginal cost from an additional unit of deposits, as the second term on the right hand side of equation (53) will turn out to be close to zero.[26] As a result of the fact that banks' marginal costs of deposit funding decreases by more than the rate at which banks' expected return on corporate securities decreases, banks expand the balance sheet, which in turn drives down the return on corporate securities $\bar{R}^k$.[27]

Now, we are ready to prove Proposition 1 with the help of Lemmas 1 and 2. To do so, we observe from equation (51) that the return on corporate securities and the stock of physical capital are inversely related. Therefore, we infer from Lemma 1 that $\bar{k}\Big|^{ULL} \geq \bar{k}\Big|_{\gamma=1}^{LL}$. Similarly, we infer from Lemma 2 that $\bar{k}\Big|_{\gamma=0}^{LL} \geq \bar{k}\Big|^{ULL}$. This concludes the proof. □

[25] Remember that $F^b\left(\bar{\omega}^b\right) \equiv \int_0^{\bar{\omega}^b} f(\omega)\, d\omega$ and $G^b\left(\bar{\omega}^b\right) \equiv \int_0^{\bar{\omega}^b} \omega f(\omega)\, d\omega$ with $f(\omega)$ the lognormal probability density function, which is continuous in σ^b for $\sigma^b > 0$.

[26] We will see in Section 4.2 that $\bar{\kappa}$ is a number close to zero.

[27] In related work by Afanasyeva and Guntner (2020), banks can price in an extension of the balance sheet due to credit market power. This incentivizes them to raise their borrowers and thus their own leverage, for example in response to a monetary expansion, even without limited liability.

3.2 Impact of risk on the steady state

In the previous section, we proved that credit provision under limited liability and full deposit insurance is always larger than or equal to credit provision under unlimited liability, which in turn is always larger than credit provision under limited liability in the absence of deposit insurance. We now focus on how credit provision, the return of corporate securities and welfare change with the standard deviation σ^b of the idiosyncratic bankers' shock.

In doing so, we will assume that the change in the probability of default $F^b\left(\bar{\omega}^b\right)$ and the expected value of the idiosyncratic shock conditional on default $G^b\left(\bar{\omega}^b\right)$ increase with σ^b, despite the fact that $\frac{d\bar{\omega}^b}{d\sigma^b}$ can be negative:

$$\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} = \Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\cdot\frac{\frac{\sigma^b}{\bar{\omega}^b}\cdot\frac{d\bar{\omega}^b}{d\sigma^b}+\frac{1}{2}\left(\sigma^b\right)^2-\log\left(\bar{\omega}^b\right)}{\left(\sigma^b\right)^2} > 0, \tag{54}$$

$$\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} = \Phi'\left(\frac{\log\left(\bar{\omega}^b\right)-\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\cdot\frac{\frac{\sigma^b}{\bar{\omega}^b}\cdot\frac{d\bar{\omega}^b}{d\sigma^b}-\frac{1}{2}\left(\sigma^b\right)^2-\log\left(\bar{\omega}^b\right)}{\left(\sigma^b\right)^2} > 0, \tag{55}$$

In other words, we assume that the direct effect from an increase in σ^b will trump the indirect effect that might arise through $\frac{d\bar{\omega}^b}{d\sigma^b}$. We show in the main text and Appendix A1 that these assumptions always hold in our numerical simulations.

We start by looking at how credit provision to the real economy changes with risk (σ^b). Before we do so, observe from the first order condition for corporate securities under unlimited liability (52) that $\left.\frac{d\bar{R}^k}{d\sigma^b}\right|^{ULL} = 0$. Therefore, we know that credit provision to the real economy is constant under unlimited liability: $\left.\frac{d\bar{k}}{d\sigma^b}\right|^{ULL} = 0$. Proposition 2, however, shows that credit provision to the real economy increases under full deposit insurance and limited liability:

Proposition 2. *Under full deposit insurance ($\gamma = 0$), the stock of physical $\bar{k}$ increases under limited liability when volatility σ^b increases:*

$$\frac{d\bar{k}}{d\sigma^b} > 0.$$

Proof. We show in Appendix A7 that implicit differentiation of equation (47) with respect to σ^b gives the following expression for $\frac{d\bar{R}^k}{d\sigma^b}$ when $\gamma = 0$ and $\kappa_\eta = 0$:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\sigma^b} = \frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}\left[\frac{1}{\epsilon^d}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\right] < 0,$$

where $\Phi\left(...\right)$ denotes the cumulative density function of the standard normal distribution. Therefore, $\Phi'\left(...\right) > 0$. In addition, we know that $\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} > 0$ by assumption (54) and that $\epsilon^d < -1$. Therefore, we immediately see that $\frac{d\bar{R}^k}{d\sigma^b} < 0$. Since the return on corporate securities and the stock of physical capital are inversely related, we conclude that $\frac{d\bar{k}}{d\sigma^b} > 0$. □

To better understand the intuition behind this result, we implicitly differentiate equation (47) with respect to σ^b, and set $\gamma = 0$ and $\kappa_\eta = 0$. The resulting expression is given by:

$$\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b} = \frac{1}{\beta \left[1 - G^b\left(\bar{\omega}^b\right)\right] \bar{R}^k} \left[\beta \bar{R}^k \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \frac{1}{1+\bar{\kappa}} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \beta \bar{R}^d \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}\right],$$

where we remember that under full deposit insurance $\beta \bar{R}^d = 1$. The intuition behind the above expression is as follows. The first term on the right hand side captures the fact that an increase in risk increases the states of the world in which the return on corporate securities is zero, since $\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} > 0$ by assumption (55). This causes banks to reduce credit provision, everything else equal, which increases the return on corporate securities. The second term on the right hand side captures the fact that an increase in risk decreases banks' marginal cost from an additional unit of deposits, as there are more states of the world in which they will default, i.e. $\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} > 0$. Lower marginal funding costs causes banks to increase credit provision, everything else equal, which decreases the return on corporate securities. Since we show in Appendix A7, that $\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} = \bar{\omega}^b \left[\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \Phi'\left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\right]$, and prove in Appendix A7 that $\frac{\bar{R}^d}{1+\bar{\kappa}} = \bar{R}^k \bar{\omega}^b$, we can see that the effect from more states in which the return on corporate securities will be zero is dominated by the decrease in banks' marginal expected funding costs. In other words: an increase in risk decreases the marginal cost from an additional unit of deposits by more than the decrease in the marginal benefit from an additional unit of corporate securities. As a result, banks further expand the balance sheet, which drives down the aggregate return on corporate securities.

Next, we look at Proposition 3, which allows us to prove that credit provision to the real economy under limited liability always decreases with σ^b in the absence of deposit insurance ($\gamma = 1$):

Proposition 3. *Under limited liability, in the absence of deposit insurance ($\gamma = 1$), the stock of physical $\bar{k}$ decreases when volatility σ^b increases:*

$$\frac{d\bar{k}}{d\sigma^b} < 0.$$

Proof. We show in Appendix A7 that implicit differentiation of equation (47) with respect to σ^b gives the following expression for $\frac{d\bar{R}^k}{d\sigma^b}$ when $\gamma = 1$:

$$\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b} = \frac{1}{1 - G^b\left(\bar{\omega}^b\right)} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} > 0,$$

since $\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} > 0$ by assumption (55). Therefore, we see that $\frac{d\bar{R}^k}{d\sigma^b} > 0$, which implies that $\frac{d\bar{k}}{d\sigma^b} < 0$. □

Since the probability of default is priced in by depositors, an increase in risk does not change the marginal cost from an additional unit of deposits. However, the marginal benefit from an additional unit of corporate securities decreases, as the expected value of the idiosyncratic shock conditional on default $G^b\left(\bar{\omega}^b\right)$ increases, which decreases the expected return on corporate securities $\left[1 - G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k$, everything else equal. In response, banks decrease credit provision to the real economy, which raises the aggregate return on corporate securities $\bar{R}^k$ in equilibrium. Therefore, higher idiosyncratic risk decreases credit provision to the real economy in the absence of deposit insurance, and through that channel investment and output. This sharply contrasts with the case of full deposit insurance ($\gamma = 0$), where credit provision to the real economy, and investment, increase.

Therefore, we do not only see that credit provision under limited liability and full deposit insurance is always larger or equal to credit provision under limited liability and no deposit insurance, we also see that the *difference* between the two increases with risk: credit provision under full insurance increases with risk, whereas it decreases in the absence of deposit insurance. Unsurprisingly, we find in Appendix A8 that the sign of the change in credit provision to the real economy is ambiguous for intermediate values of $0 < \gamma < 1$.

Finally, we look at the welfare implications of an increase in the volatility of σ^b. To do so, we first remember that welfare is only increasing in consumption, as we temporarily assume in this section that labor is supplied inelastically by households. Therefore, to study how welfare changes as a result of an increase in σ^b, it suffices to study how consumption changes. Before we do so, let us first remember that changes in investment and output have the same sign as the change in the capital stock, since $\bar{i} = \delta\bar{k}$ and $\bar{y} = \bar{z}\bar{k}^\alpha$. We are now ready to show how consumption and welfare change as a result of an increase in σ^b.

Corollary 1. *The change in consumption and welfare as a result of an increase in σ^b is ambiguous, and given by:*

$$\frac{d\bar{c}}{d\sigma^b} = C \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \mu^{dia}\bar{R}^k\bar{k} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b}.$$

with C given by:

$$C = \alpha\bar{c} - (1-\alpha)\,\bar{i} - (1-\alpha)\,\mu^{dia}G^b\left(\bar{\omega}^b\right)(1-\delta)\,\bar{k} \lesseqgtr 0. \tag{56}$$

Proof. See Appendix A7. □

The intuition behind the expression in Corollary 1 can be explained in the following way. First, observe that the direct effect from an increase in risk on welfare is negative, as higher risk increases the fraction of banks that default ex post, which increases the expected value of the idiosyncratic shock conditional on default, $\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} > 0$. As a result, deadweight costs increase, which implies that fewer final goods are available for consumption, see the aggregate resource constraint (42). Therefore, consumption decreases, which negatively affects welfare. This effect is captured by the second term in Corollary 1.

Second, we observe that changes in the capital stock affect welfare, which is captured by the first term in Corollary 1. However, the sign of the direct effect from a change in the capital stock is ambiguous since the sign of C is ambiguous. This ambiguity is better understood by rewriting the steady state aggregate resource constraint (42) in the following way:

$$\bar{c} = \left[1 - \left(\frac{\bar{g}}{\bar{y}}\right)\right] \bar{z}\bar{k}^{\alpha} - \delta\bar{k} - \mu^{dia} G^{b}\left(\bar{\omega}^{b}\right) \bar{R}^{k}\bar{k}. \tag{57}$$

From this expression, we see that an increase in capital $\bar{k}$ has two opposite direct effects on consumption. First, more capital directly increases output, and therefore consumption, everything else equal, which is captured by the first term in expressions (56) and (57). Second, an increase in capital increases steady state investment and the deadweight costs from from default, which everything else equal leaves fewer final goods available for consumption. This is captured by the second and third term in expressions (56) and (57). Therefore, the total effect from an increase in capital on consumption depends on whether the positive effect from higher output dominates the negative effect from higher investment and deadweight costs.

3.3 Impact of deposit insurance on the steady state

In this section we will perform a similar analysis as in the previous section, but we now investigate the general equilibrium effects of a change in the fraction of deposits not reimbursed by the deposit insurance agency (γ).

Proposition 4. *The direct effect of an increase in γ is to increase the steady state return on deposits $\bar{R}^d$. Furthermore, $\bar{R}^d$ is increasing in the probability of default:*

Proof. Implicit differentiation of equation (50) with respect to γ shows that the change in the real interest rate on deposits is given by:

$$\frac{1}{\bar{R}^{d}} \cdot \frac{d\bar{R}^{d}}{d\gamma} = \underbrace{\frac{F^{b}\left(\bar{\omega}^{b}\right)}{1 - \gamma F^{b}\left(\bar{\omega}^{b}\right)}}_{\text{direct effect}} + \frac{\gamma}{1 - \gamma F^{b}\left(\bar{\omega}^{b}\right)} \cdot \frac{dF^{b}\left(\bar{\omega}^{b}\right)}{d\gamma}. \tag{58}$$

We can see that the first term is larger than zero, which concludes the first part of the proof. In addition, since $\frac{\gamma}{1-\gamma F^{b}(\bar{\omega}^{b})} > 0$, we see that $\bar{R}^d$ increases when $\frac{dF^{b}(\bar{\omega}^{b})}{d\gamma} > 0$ and decreases when $\frac{dF^{b}(\bar{\omega}^{b})}{d\gamma} < 0$. Therefore, the steady state return on deposits is increasing in the probability of default, which concludes the second part of the proof. □

We also see from expression (58) that the overall (i.e. direct plus indirect) change in the return on deposits is increasing in γ for a given change in the probability of default $\frac{dF^{b}(\bar{\omega}^{b})}{d\gamma}$.

In what follows, we will assume that the direct effect from an increase in γ – the first term in expression (58) – ensures that the total effect from an increase in γ is an increase in the steady state return on deposits $\bar{R}^d$, which is also confirmed in the section with the numerical results:

$$\frac{d\bar{R}^d}{d\gamma} > 0. \tag{59}$$

However, not only does the change in the return on deposits $\frac{d\bar{R}^d}{d\gamma}$ depends on the change in the probability of default $\frac{dF^b(\bar{\omega}^b)}{d\gamma}$, it turns out that the change in the probability of default also depends on the change in the return on deposits. This leads to the emergence of a negative feedback loop, in which a higher interest rate on deposits increases the probability of default, which in turn increases interest rates even further:

Corollary 2. *There exists a feedback loop between the real interest rate on deposits $\bar{R}^d$ and the probability of default $F^b\left(\bar{\omega}^b\right)$.*

Proof. We already saw in Proposition 4 how the return on deposits is affected by a change in the probability of default. Remember, however, that the probability of default $F^b\left(\bar{\omega}^b\right)$ depends on the cut-off value $\bar{\omega}^b \equiv \frac{\bar{R}^d}{\bar{R}^k} \cdot \bar{x}^b$ where $\bar{x}^b \equiv \bar{d}/\bar{k}$ is the deposits-assets ratio. Therefore, we can write the change in the probability of default as:

$$\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} = \frac{1}{\sigma^b}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\left[\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma} - \frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}\right],$$

where we observe that $\frac{d\bar{x}^b}{d\gamma} = 0$, since $\bar{x}^b = 1/\left(1+\bar{\kappa}\right)$ when the equity-deposit constraint (26) is binding. We show in Appendix A7 that we can rewrite this equation in the following way:

$$\left[1+\frac{1}{\sigma^b\epsilon^d}\cdot\frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\right]\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} = \frac{1}{\sigma^b}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)B\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma} > 0, \tag{60}$$

where $B > 0$ and where we check in Appendix A1 that the coefficient in front of $\frac{dF^b(\bar{\omega}^b)}{d\gamma}$ is always larger than zero in our numerical simulations.

Therefore, we see that an increase in the interest rate on deposits increases the probability of default, everything else equal. The increase in the probability of default then increases the interest rate on deposits via expression (58), which in turn leads to a second round increase in the probability of default. This proves the existence of a feedback loop between the interest rate on deposits and the probability of default. □

Corollary 3. *When the probability of default is increasing in γ, the amplification cycle increases the return on corporate securities nonlinearly in γ.*

Proof. We show in Appendix A7 that the return on corporate securities is given by:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma} = \frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}\left\{\frac{1}{\epsilon^d}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} + \left(\frac{\epsilon^d-1}{\epsilon^d}\right)\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}\right\}. \tag{61}$$

Before we discuss the nonlinearity, we first make an observation about the return on corporate securities. We see that the change in the return on corporate securities can be decomposed into two effects. First, a higher probability of default decreases banks' expected funding costs, everything else equal, as a result of which the return on corporate securities decreases This is captured by the first term, where we remember that $\epsilon^d < -1$. However, an increase in the interest rate on deposits increases banks' expected funding costs, everything else equal, as a result of which the return on corporate securities increases. This effect is captured by the second term on the right hand side of expression (61).

Now, substitution of the expression for the change in the return on deposits (58) into expression (61) provides us with the following expression:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma} = \left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)}\left\{\underbrace{\left(\frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)}\right)F^b\left(\bar{\omega}^b\right)}_{\substack{\text{direct effect}\\ \text{from increase in } \gamma\\ \text{on deposit rate}}} + \left[\gamma\left(\frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)}\right) - \frac{1}{1 - \epsilon^d}\right]\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}\right\}. \tag{62}$$

From this expression, we clearly see that both the term related to the direct effect from an increase in γ on the deposit rate (first term), as well as the coefficient that is in front of the change in the probability of default (second term) are increasing in γ everything else equal. Therefore, the return on corporate securities will increase in γ for a given change in the probability of default $\frac{dF^b(\bar{\omega}^b)}{d\gamma}$. □

The intuition of the previous results can be explained in the following way. We see from Proposition 4 that the direct effect from an increase in γ is that the interest rate on deposits increases, as a smaller fraction of deposits is now insured against default. We then see from expression (60) that higher funding costs increase the probability of default, everything else equal. We then see from expression (58) that an increase in the probability of default further increases the return on deposits, which in turn leads to an even higher probability of default (60). As a result, a feedback loop between the probability of default and the (expected) return on deposits emerges. Therefore, we see that increasing γ might be counterproductive in reducing the probability of default (and the fraction of banks that default ex post, which are the same in the non-stochastic steady state), despite reducing moral hazard by raising banks' marginal cost from an additional euro of deposits, and lead to an *increase* in the probability of default in equilibrium.

Observe, however, that in the presence of dividend adjustment costs ($\kappa_\eta > 0$), there is a counter-effect to the feedback loop, as a higher marginal cost from an additional unit of deposits (ψ_t^b) induces banks to reduce dividend payments everything else equal, see equation (49). Therefore, the probability of default decreases, everything else equal. We show in the next section that the negative feedback amplification cycle between deposit rates and the probability of default is numerically relevant. Hence, our results provide a new argument in favor of deposit insurance, namely the elimination of the feedback loop between banks' funding costs and the probability of default that causes credit provision, investment and output to decrease. This argument is different from the well-known Diamond and Dybvig (1983) argument that deposit insurance eliminates bank runs (which are absent in our model). Note that our mechanism does not rely on a strategic decision by depositors concerning the ability to withdraw. In the Diamond-Dybvig model, deposit insurance prevents bank runs in equilibrium. As mentioned, bank runs are absent from our model. Nonetheless, our feedback loop generated by the interaction between funding costs and the probability of default means that when funding costs get high, banks reduce lending - and this leads to a similar outcome as quantitative rationing. However, in our case the feedback loop is generated by the interaction between funding costs and the probability of default, without any strategic considerations.

4 Quantitative results and discussion

In this section, we perform numerical simulations (with positive dividend adjustment costs and endogenous labor supply, in contrast to Section 3), and discuss our results in detail, including comparative statics on the degree of deposit insurance and deadweight costs, as well as their welfare and policy implications.

As a baseline, we consider the case of unlimited liability, whereby bank owners entirely reimburse depositors in case of bank default. We compare this to the full deposit insurance case ($\gamma = 0$), which corresponds to the situation of advanced economies between the 1930s and 1970s. During this time, deposit insurance had already been implemented, and banks were still mainly relying on deposits to finance themselves. We also consider the case of no deposit insurance ($\gamma = 1$) which corresponds to most historical economies prior to the 1930s, plus many non-Western economies until later. Finally, we consider an intermediate case, which applies to advanced economies today, in which financial institutions (both banks and the shadow banking system) only partially rely on deposit-insured liabilities for funding ($\gamma = 0.5$). Figure 1 illustrates the four cases that we consider.

4.1 Solution procedure

Our solution procedure is complicated by the fact that we have an occasionally binding inequality constraint, namely the equity-deposit constraint (22). In the simulations below, we will solve the model as a function of σ^b, the standard deviation of the idiosyncratic shock to banks. For some values of σ^b the equity-deposit constraint (22) will be binding, while it will not for other values of σ^b. To properly take this inequality constraint into account, we adopt the following solution procedure.

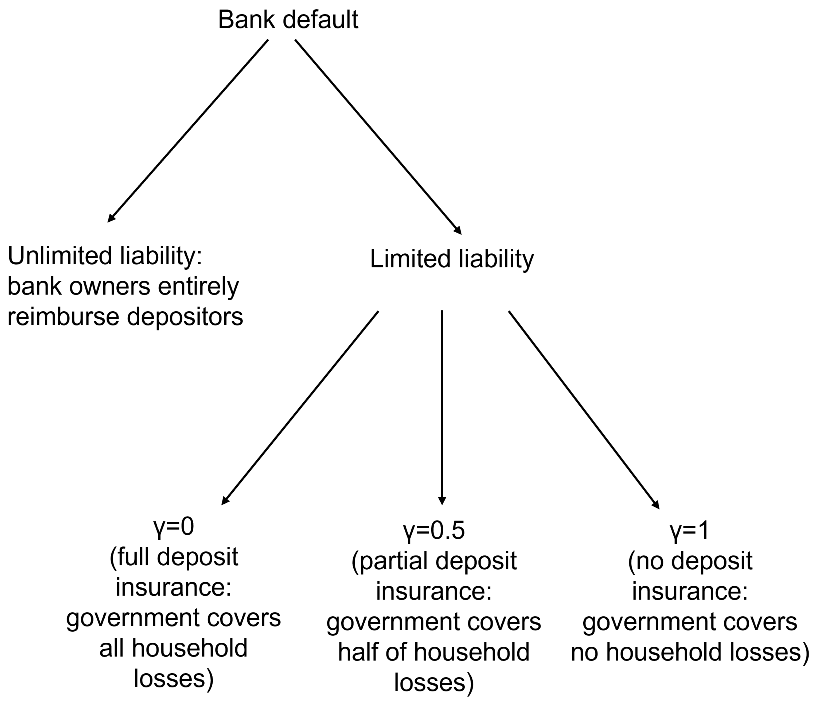

Figure 1: The four cases considered

First, for each value of σ^b we solve for the steady state without imposing the constraint (22). This gives us the solution for (among others) aggregate corporate securities, deposits, dividends, and equity. Second, we construct for each value of σ^b a two-dimensional grid with dividends $\bar{\eta}_j$ of bank j on one axis, and corporate securities $\bar{s}_j^k$ of bank j on the other.[28] Next, we calculate bank j's objective function (31) at each grid point $\left(\bar{\eta}_j, \bar{s}_j^k\right)$, which produces a surfplot (see Appendix A4 for an example).[29] While the surfplot features an interior solution for the unconstrained equilibrium, it might be the case that the global maximum of bank j's objective function is at the boundary where the equity-deposit constraint (22) is binding. To check this, we also calculate bank j's objective function at this boundary and determine the maximum value along this boundary. Afterwards, we compare this maximum value with the value of bank j's objective function at the interior solution. If the global maximum is at the interior solution, we check that the second order conditions are satisfied, as well as that the resulting equity-deposit ratio satisfies the inequality constraint (22).[30]

However, if the maximum value of the bank's objective function is at the boundary where

[28] We can use banks' balance sheet constraint (25) to solve for $R_{j,t}^{n,d}$, after which bank j effectively has two decision variables left, namely dividends $\eta_{j,t}$ and corporate securities $s_{j,t}^k$. Therefore, it suffices to create a two-dimensional grid with dividends $\bar{\eta}_j$ and corporate securities $\bar{s}_j^k$.

[29] Observe that bank j's objective function (31) features both aggregate variables as well as bank j variables.

[30] The second order conditions of the model can be found in Appendix A3.

the equity-deposit constraint (22) is binding, or if the equity-deposit constraint is violated at the interior solution, we solve the model with a binding equity-deposit constraint (22), a solution that we refer to as the constrained equilibrium. Afterwards, we check the second order conditions to ensure we have found a maximum.

We will see below that the equity-deposit constraint (22) will be binding for most values of σ^b. More details can be found in Appendix A4.

4.2 Calibration

A list of parameter values can be found in Table 1. Many are taken from Gertler and Karadi (2011); these include the household subjective discount factor β, the constant relative risk aversion (CRRA) coefficient σ_c, the inverse Frisch elasticity φ, the capital share of production α, the depreciation rate δ, the Taylor-rule coefficients κ_π and κ_y, the interest rate smoothing parameter ρ_r, the investment adjustment costs parameter κ_k, and the steady state proportion of government spending over output $\bar{g}/\bar{y}$. For the elasticity of substitution among intermediate goods producers ϵ, we take a value of 4. The habit formation parameter is equal to 0.8. We set the Rotemberg adjustment costs parameter κ_p equal to 34.952, which corresponds to a Calvo-probability of not being able to change prices of $\psi_p = 0.75$.[31]

Other parameters are obtained by targeting first order moments in the model version where the equity-deposit constraint (22) is not binding. We adjust the coefficient χ in front of the disutility from labor to set steady state labor supply equal to 1/3 and obtain a value of 3.0797. The financial sector parameters are calibrated in the following way. We numerically solve for the standard deviation of bankers' idiosyncratic shocks σ^b and the elasticity of substitution of deposits ϵ^d from equation (47), while matching the following three targets. First, we set an annual steady state default rate $\bar{\Delta}^b = F^b\left(\bar{\omega}^b\right)$ of 2.5% (Boissay et al., 2016). Second, we set the deposit-assets ratio $\bar{x}_b$ equal to 0.9, which implies a steady state ratio of equity over total assets $\bar{e}/\bar{q}^k\bar{s}^k$ equal to 0.1. To arrive at this number, we use the most recent leverage ratio data of the OECD (OECD, 2022). The OECD reports that the average deposit-assets ratio was 0.87 in the US in 2015, while the average value for all 35 OECD countries was 0.94. Therefore, we choose the mid-value of these two numbers. Third,we set the steady state credit spread $E\left[\bar{R}^k - \bar{R}^D\right]$ equal to 1.6%, which amounts to an annual credit spread of 6.4%. This is higher than the interest rate margin for US commercial banks of 4.6% (Corbae and D'Erasmo, 2021). However, we set the credit spread higher because the corporate securities in our model are more equity-like, since their cash flow consists of the after-wage profits of intermediate goods producers which directly change with productivity shocks and the price of capital, see also Gertler and Kiyotaki (2010) for this point. Fourth, we set the steady state equity-deposits $\bar{\kappa} = 0.087$, which implies that the minimum ratio of steady state equity over assets is 0.08, which is in line with Basel III

[31] See Ascari and Rossi (2012) who show that when gross steady state inflation $\bar{\pi}$ is equal to 1 (as we have), the linearized Phillips-curve from Calvo-pricing can be mapped to the Phillips-curve from Rotemberg pricing, and they provide an expression for how to obtain the Rotemberg adjustment costs parameter κ_p given the Calvo-parameter ψ_p and the elasticity of substitution ϵ.

requirements.[32] After matching these targets, we obtain $\bar{\omega}^b = 0.8861$, and $\sigma^b = 0.0480$. The last number is higher than in Mendicino et al. (2018), who find a standard deviation of 0.012 for mortgage banks and 0.027 for banks that lend to entrepreneurs. As our loans are more equity-like, we think a higher standard deviation σ^b is more appropriate. We set the mean μ_t^b of the log-normal distribution for the bankers' idiosyncratic shocks equal to $-(1/2)\,\sigma_t^b$, so that the unconditional expected value of ω_t^b is equal to 1 ($\Omega_t^b = 1$ in our notation, see Bernanke et al. (1999)). Finally, we set the deadweight costs from default μ^{dia} equal to 0.12, following Bernanke et al. (1999). This is also very close to the 0.125 that is employed by Damjanovic et al. (2020).

Casey and Dickens (2000) look at dividend payout ratios of banks in the US and find average values between 35% to 42% depending on the period (they look at several periods between 1982 and 1996). Abreu and Gulamhussen (2013) find a value of 35% before the financial crisis and 30% after the financial crisis. Therefore, we set steady state dividends equal to 35% of aggregate profits of surviving bankers $\Pi^b \equiv \left[\bar{\Omega}^b - \Gamma^b\left(\bar{\omega}^b\right)\right]\bar{R}^k\bar{q}^k\bar{s}^k$. We employ bank-specific dividend adjustment costs (20), and set $\kappa_\eta = 0.1$, which is close to the value used by Jermann and Quadrini (2012). Afterwards, we adjust $\hat{\eta}$ to match our calibration targets. We check in Section 4.5 that our results are not driven by this particular choice. We also set $\gamma = 1$ during the calibration of the model. Once we have determined all deep parameters in the calibration, we will vary γ in our numerical simulations in Section 4.3 and subsequent sections. The fraction χ^b of previous period aggregate net worth that goes to banks that start operating is equal to 0.40, after which we adjust the fraction θ^b of current profits that goes to starting banks so that the remaining steady state targets can be met. It turns out that $\theta^b = 0.7219$, which could suggest that banks are loss-making all the time. Observe, however, that financial intermediaries start their life by paying out dividends before they attract deposits and acquire corporate securities. So the net worth that remains in the bank is not equal to $\bar{n}^b$ but $\bar{e}$, and it turns out that the aggregate profits of the banking sector are larger than the initial amount of equity $\bar{e}$ with which they start operating.

4.3 Numerical solutions

We start by reporting numerical solutions for the non-stochastic steady state, which can be found in Figure 2, where we display the standard deviation σ^b of the bankers' idiosyncratic shock on the horizontal axis. The case with unlimited liability is represented by the black horizontal dotted line. There is no effect of risk on any variable for unlimited liability because banks must consider the entire distribution of idiosyncratic shocks. Note that the unlimited liability case differs from the limited liability case, because the owners of the banks (households) reimburse all depositors in case of shortfalls, while depositors either incur losses or are repaid by the government under

[32]Substitution of the equity-deposit constraint $\bar{d} = \bar{e}/\bar{\kappa}$, equation (22), into banks' balance sheet constraint $\bar{q}^k\bar{s}^k = \bar{e} + \bar{d}$, equation (25), shows that we can write $\bar{e} = \left(\frac{\bar{\kappa}}{1+\bar{\kappa}}\right)\bar{q}^k\bar{s}^k$. Setting $\frac{\bar{\kappa}}{1+\bar{\kappa}} = 0.08$ allows us to solve for $\bar{\kappa}$.

Parameter	Definition	Value
Households (HH)		
β	Subj. discount factor	0.99
σ_c	CRRA coefficient	1
υ	Habit formation	0.8
φ	Inverse Frisch elasticity	0.276
χ	Disutility of labor	3.0797
γ	Frac. of default costs for HH	1
Production Sector		
α	Capital income share	0.33
δ	Depreciation rate	0.025
ϵ	Elasticity of substitution between goods	4
κ_p	Price adj. costs	34.9515
κ_k	Investment adj. costs	1.728
Financial Intermediaries		
$\bar{\kappa}$	Minimum equity-deposit ratio	0.0870
σ^b	St. Dev. of banker shock	0.0480
$\bar{\Omega}^b$	Exp. value of banker shock	1
θ^b	Fraction of profits for new banks	0.7219
χ^b	Frac. of old net worth for new banker	0.40
μ^{dia}	Default verification costs	0.12
κ_η	Dividend adj. costs	0.1
$\hat{\eta}$	Dividend growth rate	0.2843
ϵ^d	Elasticity of substitution between deposits	-60.4033
Policy parameters		
ρ_r	Interest rate smoothing	0.8
κ_π	Interest rate rule weight on inflation	1.5
κ_y	Interest rate rule weight on output	0.125
$\bar{\pi}$	St. st. inflation	1
$\bar{g}/\bar{y}$	St. st. share of government spending	0.2

Table 1: List of calibrated parameters

limited liability. Figure 2 also shows the results for the several limited liability cases which we consider: full deposit insurance (grey solid line), partial insurance (black dashed line) and no insurance (black solid line).[33] The results are in line with those from the analytical section regarding the role of idiosyncratic risk. We observe that the default probability (as well as the fraction of banks that default ex post) always increases with the standard deviation of the bankers' idiosyncratic shock.[34] In addition, we see that the steady state probability of bank default under limited liability converges to the equilibrium allocation under unlimited liability when σ^b converges to zero.

In line with Section 3, it turns out that the equity-deposit constraint (22) is always binding, as we see that leverage is constant. Also in line with Section 3, it turns out that the return on corporate securities decreases under full deposit insurance (although this is not very visible in Figure 2 because of the large increase in the return on corporate securities for partial and no insurance). The decrease in the return on corporate securities under full deposit insurance occurs because credit expands under full insurance leading to higher investment (also output and consumption) which reduces the return on capital. The return on corporate securities increases under partial and no insurance, as credit contracts and investment falls with higher levels of risk. Dividends increase with full deposit insurance when σ^b increases, but decrease under partial and no deposit insurance. The return on deposits increases with σ^b under partial and no insurance because depositors demand compensation for the larger probability of default.

We also see from Figure 2 that when σ^b increases bank securities increase under full insurance, while falling under both partial and no insurance, all of which is in line with Propositions 2 and 3. The reason for the large difference between the full insurance case and the other two cases is the negative feedback loop between the interest rate on deposits and the probability of default, see Corollary 2, which causes the return on deposits and corporate securities under partial deposit insurance to increase by more than 1,000 basis points with respect to the case of full deposit insurance for $\sigma^b = 0.1$. In addition, observe that the reaction of output, consumption, investment and labor supply follow the same pattern as bank securities, for both partial and no insurance. Lending to the real economy is reduced, which results in lower investment and output. As a result, there are fewer goods for consumption, which is further crowded out by higher verification costs that result from a larger fraction of banks defaulting relative to $\gamma = 0$. Lower investment also leads to a lower capital stock, which results in lower wages. In response, households reduce labor supply.

Also observe that the quantitative difference between partial and no insurance for real economy variables such as bank securities, investment, output, and consumption is relatively small compared with the difference between these two cases on the one hand and the case of full deposit insurance on the other. The reason for the relatively small difference between partial and

[33] Therefore, the comparative statics results in Figure 2 correspond to a mapping of the analytical results of Section 3.

[34] In the non-stochastic steady state, the probability of default ex ante is equal to the fraction of banks that default ex post the realization of the idiosyncratic shock.

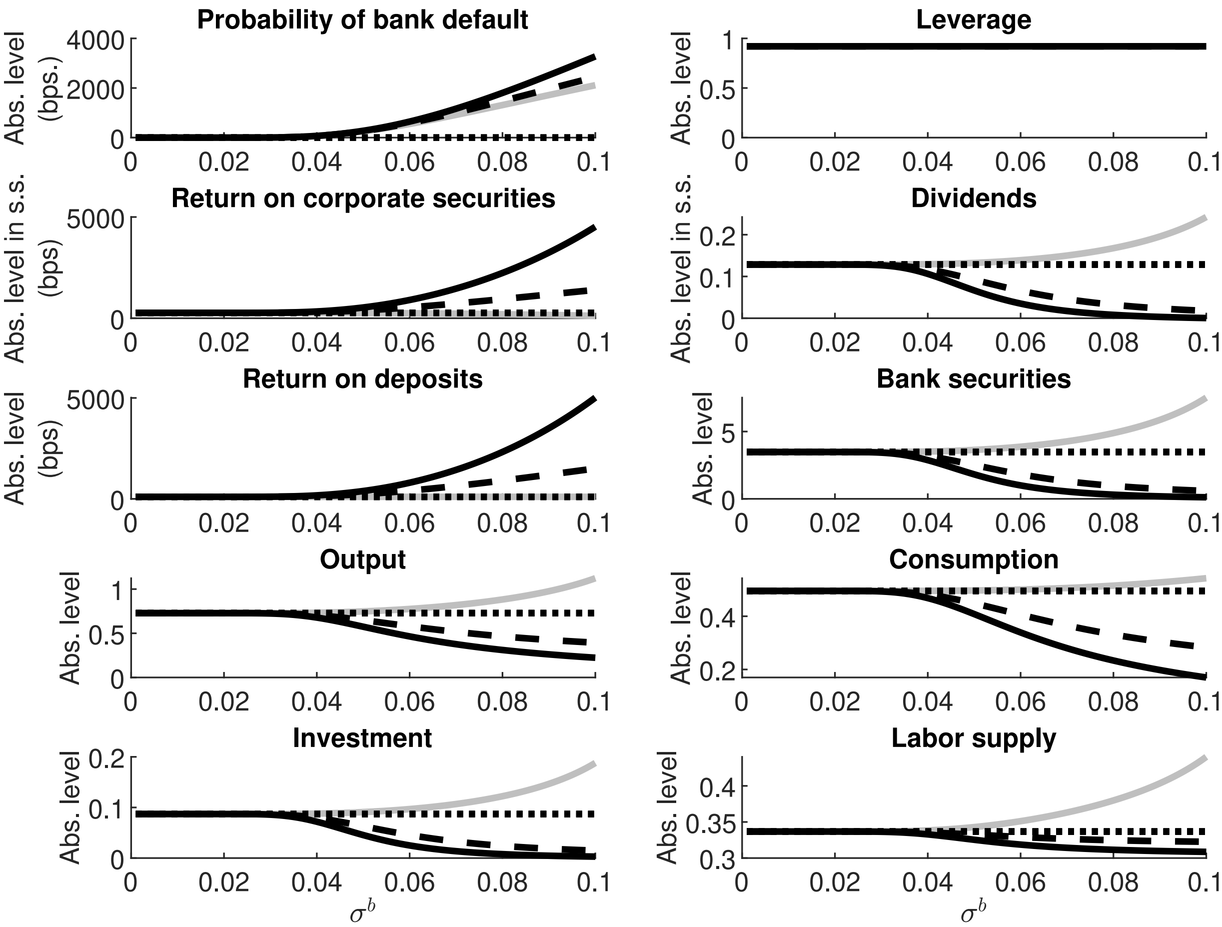

Figure 2: Steady state results for the model version with limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line) and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits refers to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refers to the volume of corporate securities $\bar{s}^k$ held by the banking system.

no insurance is that the return on corporate securities and the stock of capital are nonlinearly related, see equation (51). Therefore, the large increase in the return on corporate securities when moving from full deposit insurance to partial deposit insurance has a much larger effect on credit provision to the real economy than the effect on credit provision from moving from partial deposit insurance to no deposit insurance, despite the fact that the increase in the return on corporate securities is much larger when shifting from partial to no deposit insurance.

Finally, observe that the impact when moving from full deposit insurance to partial or no insurance has a large impact on macroeconomic variables: credit provision and investment fall by more than 90% with respect to unlimited liability for large values of σ^b, while consumption and output fall by approximately 50% with respect to unlimited liability for large values of σ^b. As explained above, this large decrease is ultimately the result of the feedback loop between the interest rate on deposits and the probability of default.

4.4 Welfare and policy implications

In this section we look at the consequences of an increase in the standard deviation of the idiosyncratic shock on welfare, which is defined as the sum of expected discounted utility as defined in expression (1).[35] The results can be found in Figure 3, where welfare is expressed relative to the unlimited liability case on the vertical axis (expressed in terms of the consumption equivalent ν, which we express in percentage points).[36] The figure displays the limited liability model version with full insurance $\gamma = 0$ (grey, solid line), partial insurance ($\gamma = 0.5$) (black, dashed line), and no insurance ($\gamma = 1$) (black, solid line). We do not display the case of unlimited liability, which would correspond to a horizontal line at zero. Observe that compared with Section 3, we now also have endogenous labor supply which directly affects households' welfare, see expression 1.

The results show that welfare always decreases with higher risk. However, the decrease in welfare is relatively limited for the case of full deposit insurance, whereas the decrease is substantially larger for partial and no deposit insurance. Although welfare always decreases with risk, as mentioned, the mechanisms are quite different between the full deposit insurance case on the one hand, and the partial and no deposit insurance case on the other. We start by focusing on the case $\gamma = 0$ in Figure 3. We see from Figure 2 that consumption increases with respect to consumption under unlimited liability, which everything else equal has a positive effect on welfare. However, the increase in consumption is relatively small, and is more than offset by a substantial increase in labor supply. A third factor that negatively affects welfare is the presence of deadweight costs from default, which increase with risk, and leave fewer final

[35]Steady state welfare is given by $\mathcal{W} \equiv \frac{U(\bar{c})-V(\bar{h})}{1-\beta}$, where $U(c) = \left((1-\upsilon)\,\bar{c}^{1-\sigma_c} - 1\right)/(1-\sigma_c)$ and $V(h) = \chi\bar{h}^{1+\varphi}/(1+\varphi)$.

[36]We follow Bianchi (2016) and define the consumption equivalent ν implicitly from the formula $\mathcal{W}^{LL} \equiv \frac{U(\bar{c}^{LL})-V(\bar{h}^{LL})}{1-\beta} = \frac{U(\bar{c}^{ULL}+\nu)-V(\bar{h}^{ULL})}{1-\beta}$, where $\mathcal{W}^{LL}$ is welfare under limited liability. LL refers to limited liability, and ULL refers to the unlimited liability case.

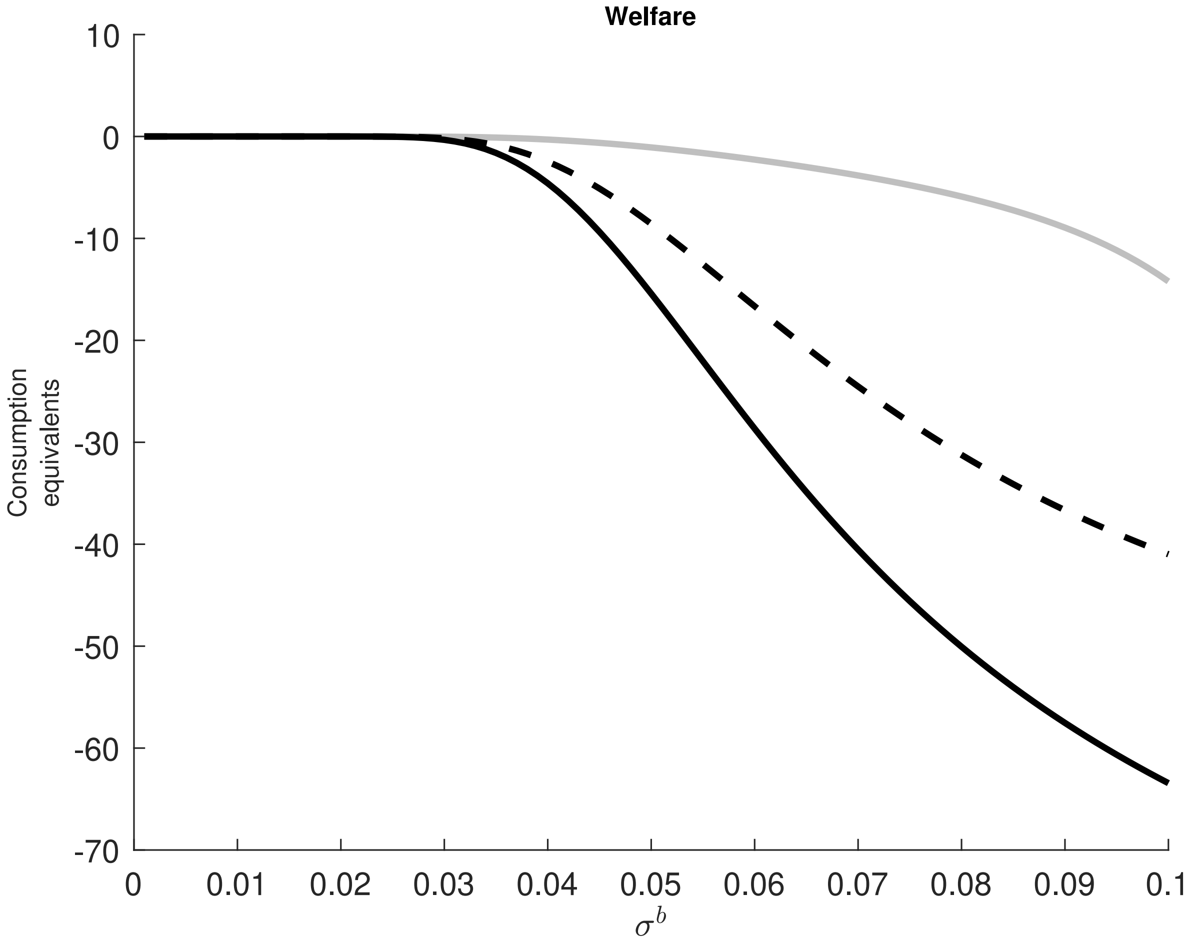

Figure 3: Steady state results for the model version with limited liability and full insurance ($\gamma = 0$) (grey, solid), limited liability and partial insurance ($\gamma = 0.5$) (black, dashed), and limited liability with no insurance ($\gamma = 1$) (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

goods for consumption, everything else equal, see the aggregate resource constraint (42) and Corollary 1.

For $\gamma = 0.5$ and $\gamma = 1$, welfare decreases much more with risk, and the more so for $\gamma = 1$. The mechanism behind this decrease, however, is different than under full deposit insurance: unlike the full deposit insurance case, we see from Figure 2 that consumption now decreases with σ^b, by approximately 50% for large values of σ^b, and is always below consumption under unlimited liability. This negative effect is mitigated by the fact that labor supply decreases with σ^b, and is below that under unlimited liability. However, the total effect on welfare is a decrease by approximately 40 and 60 percent consumption equivalents for large values of the standard deviation for partial and no deposit insurance, respectively. Such a drop is much larger than the typical change in welfare under short-run welfare analysis.[37] This is caused by the feedback loop between the probability of bank default and the interest rate on deposits, see Section 3.3, which ultimately causes the change in consumption between the full deposit insurance case on the one hand, and the partial and no deposit insurance case on the other to be large.

Next, we investigate in Figure 4 the role of deadweight costs (by varying μ^{dia}). As a reminder, μ^{dia} corresponds to the fraction of assets that cannot be recouped in the case of bank default (see section 2.4.1). In Figure 4, we display the welfare loss relative to the unlimited liability case on the vertical axis (expressed in terms of the consumption equivalent ν), with the deadweight costs parameter μ^{dia} and the standard deviation σ^b on the two horizontal axes. This 3D figure gives a mixed picture of the influence that risk and deadweight costs have on welfare. First, for small values of risk, the probability of default is approximately zero, and welfare is approximately the same as in the absence of limited liability. For small values of μ^{dia}, the deadweight costs from bank default are small, and welfare increases with risk to 10 percent consumption equivalents for $\mu^{dia} = 0$ and $\sigma^b = 0.1$. When μ^{dia} increases, deadweight costs increase, especially for larger values of risk. In that case, the negative impact of higher deadweight costs more than offsets the positive effects on consumption from higher risk, and welfare decreases. In case of large values of risk and high deadweight costs, we see that welfare sharply decreases with respect to unlimited liability, even hitting a drop in welfare of almost 80 percent consumption equivalents for $\mu^{dia} = 0.3$ and $\sigma^b = 0.1$. The reason why welfare losses become so large is the fact that deadweight costs not only increase with μ^{dia}, but also with credit provision to the real economy. And we already see in Figure 2 that credit provision increases nonlinearly for large values of risk under full insurance, thereby further increasing deadweight costs on top of the increase in μ^{dia}. This explains the sharp decrease in welfare in the full deposit insurance case when both μ^{dia} and risk are high.

Next, we look at 3D figures for $\gamma = 0.5$ (Figure 5) and $\gamma = 1$ (Figure 6). Just as in Figure 4, we still find that welfare is approximately equal to welfare under unlimited liability for small values

[37] Our welfare results refer to permanent consumption equivalents in the absence of limited liability. These results are quantitatively large but refer to the long run, so they are expected to be larger than short-run effects, which are typically less than one percent consumption equivalent, e.g. Bianchi (2016). Note that the large decrease in welfare is driven by a permanent increase in risk (the standard deviation is two times that under our baseline calibration).

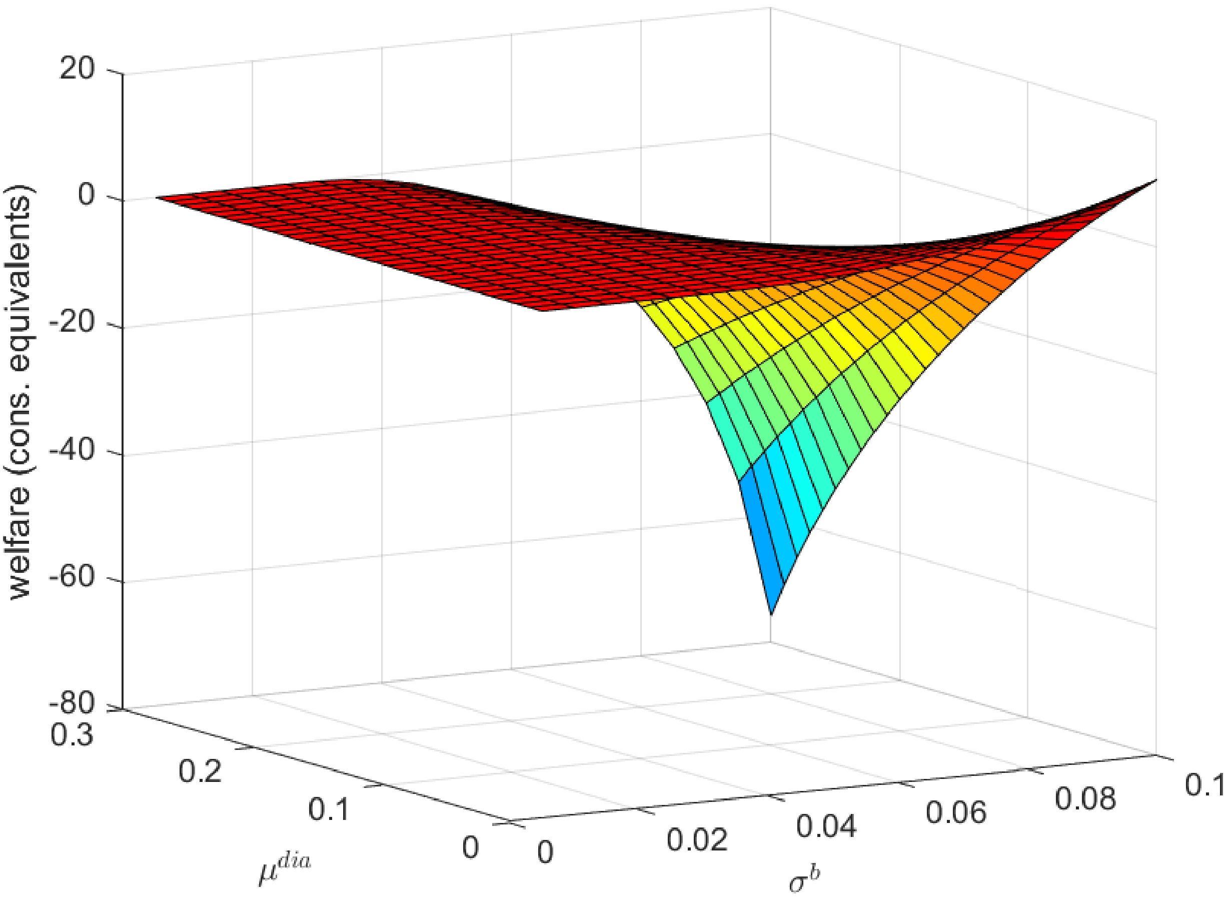

Figure 4: Steady state results for the model version with limited liability and full insurance $\gamma = 0$. One horizontal axis displays deadweight costs μ^{dia}, while the other horizontal axis features the standard deviation of the idiosyncratic bankers' shock, σ^b. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

of risk. Once we abolish the full deposit insurance scheme, and (partially) place default losses on depositors, welfare decreases for any value of μ^{dia}. Compared with Figure 4, the positive effect on the capital stock (for small values of μ^{dia}), and therefore on output, from higher idiosyncratic risk is eliminated, as deposit funding costs now increase and induce banks to reduce the size of their balance sheets. In the absence of a positive effect of idiosyncratic risk on output, only the negative effect on consumption that results from higher deadweight costs remains. Therefore, welfare will always decrease with higher idiosyncratic risk. Welfare also decreases with μ^{dia}, although the decrease along the μ^{dia} axis is substantially smaller than the decrease along the σ^b axis. Also observe that the cases with $\gamma = 0.5$ and $\gamma = 1$ differ quantitatively: welfare losses stay below 50 percent consumption equivalents for $\gamma = 0.5$, whereas they drop below 60 percent consumption equivalents for $\gamma = 1$.

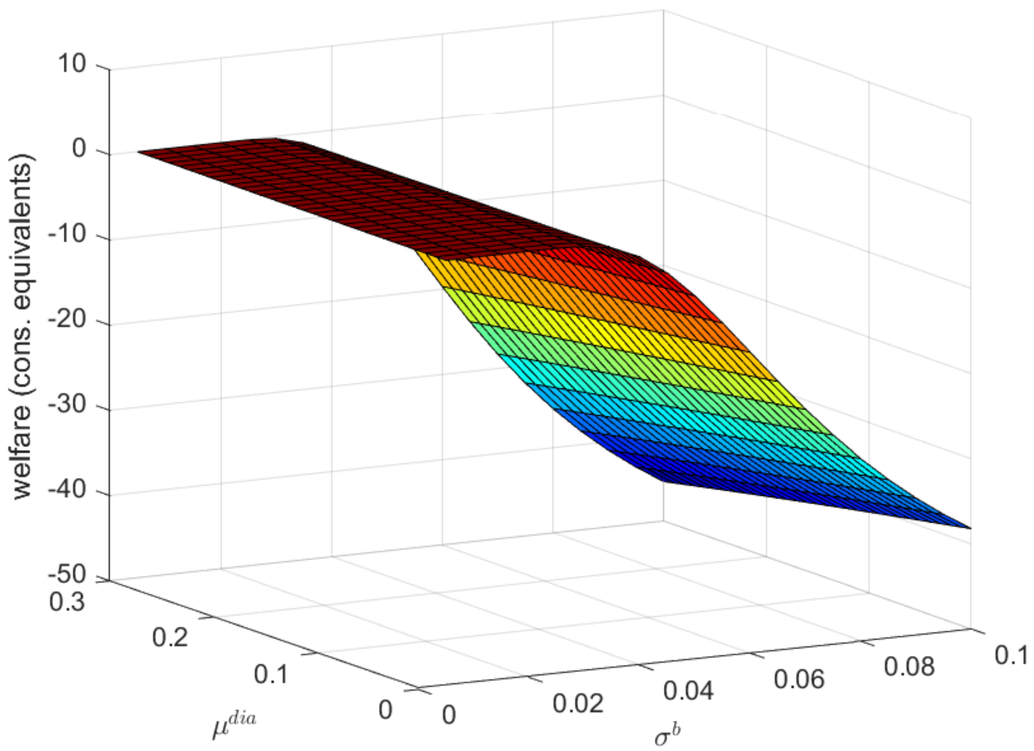

Figure 5: Steady state results for the model version with limited liability and partial deposit insurance ($\gamma = 0.5$). One horizontal axis displays deadweight costs μ^{dia}, while the other horizontal axis features the standard deviation of the idiosyncratic bankers' shock, σ^b. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

In conclusion, the influence of risk on welfare foremost depends on the degree to which banks are financed through liabilities covered by deposit insurance. Specifically, we find that an increase in γ sharply decreases credit provision, investment and output. Therefore, in economies for which banks also rely on other forms of debt funding such as wholesale and long-term debt, such as advanced economies after the 1980s, regulators should try to minimize risk as much as possible, as welfare decreases with the standard deviation. This conclusion does not depend on the presence of deadweight costs from default, as a higher standard deviation not only amplifies the negative effects on consumption through higher deadweight costs, but also eliminates the positive effect on consumption from higher output. Only in an economy where full deposit insurance is applied to the liabilities of financial intermediaries that lend to non-financial corporations is it possible for an increase in risk to have a positive effect on the economy and welfare. However, this is only the case when deadweight costs are small.

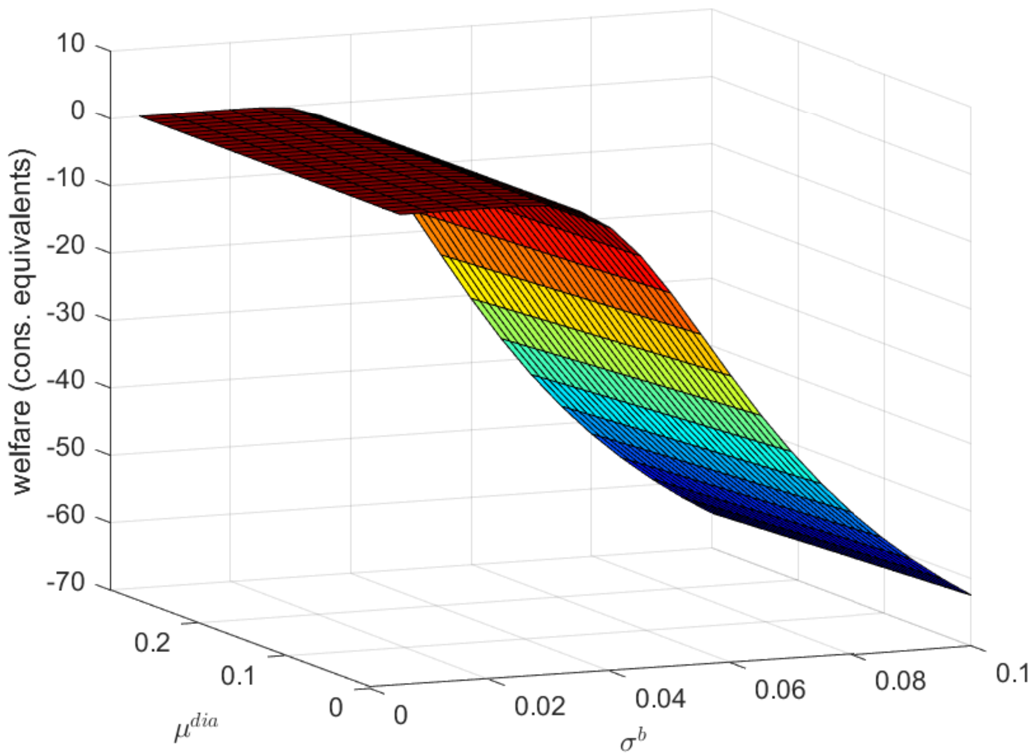

Figure 6: Steady state results for the model version with limited liability and partial deposit insurance ($\gamma = 1$). One horizontal axis displays deadweight costs μ^{dia}, while the other horizontal axis features the standard deviation of the idiosyncratic bankers' shock, σ^b. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

4.5 Robustness checks

We have performed robustness checks to confirm the numerical relevance of the negative feedback loop between deposit rates and probability of bank default. We do so in Appendix A1 by reproducing Figures 2 and 3 for different values of deadweight costs μ^{dia}, and for different target values for steady state dividends over aggregate bank profits in the calibration of our model. We continue to find that the negative feedback loop between banks' funding costs and the probability of bank default, which results from reducing moral hazard, remains present for these alternative calibrations.

Finally, remember from Section 3 that there is a counter-effect that diminishes the strength of the feedback loop, everything else equal: increasing γ raises the marginal cost $\bar{\psi}^b$ from attracting an additional unit of deposits, which tightens the banks' balance sheet constraints (25) and thereby raises the marginal cost from paying an additional unit of dividends. This suggests that the parameter that affects the marginal cost from changing dividends, κ_η, might have a first order effect on the strength of the feedback loop between deposit rates and the probability of default. To check the role that this parameter plays, we redo our simulations from Section 4.3 in Appendix A1 for $\kappa_\eta = 0.05$ and $\kappa_\eta = 0.15$. We find from these robustness checks that our results regarding the feedback loop continue to hold, i.e. the feedback loop between deposit rates and the probability of default continue to dominate the effects from lower dividend payments.

5 Conclusion

In this paper we have employed a DSGE model with banks benefiting from limited liability to investigate how risk, captured by the standard deviation of idiosyncratic shocks to banks' return on assets, affects long-run macroeconomic outcomes. The combination of limited liability and full deposit insurance gives rise to moral hazard, since higher risk allows banks to increase profits when good outcomes happen, while deposit insurance allows banks to continue to finance their balance sheets at the risk-free interest rate (despite a higher probability of bank default) (Kareken and Wallace, 1978). As a result, banks' expected profitability *conditional* on surviving the idiosyncratic shock increases, everything else equal, which leads to an expansion of credit provision, investment and output. We show not just quantitatively but also analytically that credit provision is always higher under full deposit insurance (relative to unlimited liability) and is unequivocally increasing in risk. Similarly, we show that credit provision is always lower in the absence of deposit insurance (relative to unlimited liability) and is unequivocally decreasing in risk.

Moral hazard can be reduced by increasing the fraction of deposits that are not reimbursed in case of bank default. As a result, creditors internalize the probability of bank default, which forces banks to reduce dividends. Everything else equal, this should reduce the probability of bank default. However, we find that the default probability actually *increases* in equilibrium, because of a feedback loop between banks' funding costs and the probability of default. Creditors pricing in the probability of default raises banks' funding costs. As a result, banks' (expected) profitability decreases, which in turn increases the probability of default with respect to the full deposit insurance case. Therefore, creditors further increase interest rates, which further raises the probability of default and then have amplification effects. As a result, credit provision to the real economy and investment decrease by approximately 90% (with respect to unlimited liability) for large values of idiosyncratic risk, as a result of which consumption and output decrease by approximately 50%.

The traditional argument for deposit insurance is to prevent bank runs (Diamond and Dybvig, 1983). Our results provide an additional reason: deposit insurance eliminates the feedback loop between banks' funding costs and bank default probability. Therefore, financial instability (as defined by the fraction of defaulting banks) decreases, despite leading to higher moral hazard.

Finally, we investigate welfare for different combinations of risk, deposit insurance, and deadweight costs from default, and find that welfare always decreases with risk, except under full deposit insurance when deadweight costs are small. In that case, output expands sufficiently to allow consumption to increase, despite a larger fraction of output being absorbed by deadweight costs.

References

Abreu, J.F., Gulamhussen, M.A., 2013. Dividend payouts: Evidence from U.S. bank holding companies in the context of the financial crisis. Journal of Corporate Finance 22, 54–65.

Afanasyeva, E., Guntner, J., 2020. Bank market power and the risk channel of monetary policy. Journal of Monetary Economics 111, 118–134.

Aghion, P., Angeletos, G.M., Banerjee, A., Manova, K., 2010. Volatility and growth: Credit constraints and the composition of investment. Journal of Monetary Economics 57, 246–265.

Aghion, P., Banerjee, A., et al., 2005. Volatility and growth. Oxford University Press.

Ascari, G., Rossi, L., 2012. Trend Inflation and Firms Price-Setting: Rotemberg Versus Calvo. Economic Journal 122, 1115–1141. doi:j.1468-0297.2012.02517.x.

Bernanke, B.S., Gertler, M., Gilchrist, S., 1999. The financial accelerator in a quantitative business cycle framework, in: Taylor, J.B., Woodford, M. (Eds.), Handbook of Macroeconomics. Elsevier. volume 1. chapter 21, pp. 1341–1393.

Bezemer, D., Samarina, A., Zhang, L., 2020. Does mortgage lending impact business credit? Evidence from a new disaggregated bank credit data set. Journal of Banking and Finance 113. URL: https://doi.org/10.1016/j.jbankfin.2020.105760.

Bianchi, J., 2016. Efficient Bailouts? American Economic Review 106, 3607–3659. URL: https://ideas.repec.org/a/aea/aecrev/v106y2016i12p3607-59.html.

Bloom, N., 2014. Fluctuations in uncertainty. Journal of Economic Perspectives 28, 153–76.

Boissay, F., Collard, F., Smets, F., 2016. Booms and Banking Crises. Journal of Political Economy 124, 489–538. URL: https://ideas.repec.org/a/ucp/jpolec/doi10.1086-685475.html, doi:10.1086/685475.

Calomiris, C.W., Haber, S.H., 2015. Fragile by design: The political origins of banking crises and scarce credit. volume 48. Princeton University Press.

Casey, K.M., Dickens, R., 2000. The effects of tax and regulatory changes on commercial bank dividend policy. The Quarterly Review of Economics and Finance 40, 279–293.

Choudhary, M.A., Limodio, N., 2022. Liquidity risk and long-term finance: Evidence from a natural experiment. The Review of Economic Studies 89, 1278–1313.

Christiano, L., Motto, R., Rostagno, M., 2014. Risk Shocks. American Economic Review 104, 27–65.

Clerc, L., Derviz, A., Mendicino, C., Moyen, S., Nikolov, K., Stracca, L., Suarez, J., Vardoulakis, A.P., 2015. Capital Regulation in a Macroeconomic Model with Three Layers of Default. International Journal of Central Banking 11, 9–63.

Corbae, D., D'Erasmo, P., 2021. Capital Buffers in a Quantitative Model of Banking Industry Dynamics. Econometrica 89, 2975–3023.

Damjanovic, T., Damjanovic, V., Nolan, C., 2020. Default, Bailouts and the Vertical Structure of Financial Intermediaries. Review of Economic Dynamics 38, 154–180. URL: https://ideas.repec.org/a/red/issued/18-105.html, doi:10.1016/j.red.2020.04.002.

Dávila, E., Goldstein, I., 2021. Optimal deposit insurance. Technical Report. National Bureau of Economic Research.

De Roux, N., Limodio, N., 2023. Deposit insurance and depositor behavior: Evidence from Colombia. Review of Financial Studies .

Diamond, D.W., Dybvig, P.H., 1983. Bank Runs, Deposit Insurance, and Liquidity. Journal of Political Economy 91, 401–419. URL: https://ideas.repec.org/a/ucp/jpolec/v91y1983i3p401-19.html, doi:10.1086/261155.

Diamond, D.W., Rajan, R.G., 2011. Fear of Fire Sales, Illiquidity Seeking, and Credit Freezes. The Quarterly Journal of Economics 126, 557–591. URL: https://ideas.repec.org/a/oup/qjecon/v126y2011i2p557-591.html.

Elenev, V., Landvoigt, T., Nieuwerburgh, S.V., 2021. A Macroeconomic Model With Financially Constrained Producers and Intermediaries. Econometrica 89, 1361–1418. URL: https://ideas.repec.org/a/wly/emetrp/v89y2021i3p1361-1418.html, doi:10.3982/ECTA16438.

Gerali, A., Neri, S., Sessa, L., Signoretti, F.M., 2010. Credit and Banking in a DSGE Model of the Euro Area. Journal of Money, Credit and Banking 42, 107–141.

Gertler, M., Karadi, P., 2011. A model of unconventional monetary policy. Journal of Monetary Economics 58, 17–34.

Gertler, M., Karadi, P., 2013. QE 1 vs. 2 vs. 3. . . : a framework for analyzing large-scale asset purchases as a monetary policy tool. International Journal of Central Banking 9, 5–53.

Gertler, M., Kiyotaki, N., 2010. Financial intermediation and credit policy in business cycle analysis, in: Friedman, B.M., Woodford, M. (Eds.), Handbook of Monetary Economics. Elsevier. volume 3. chapter 11, pp. 547–599.

Gertler, M., Kiyotaki, N., Queralto, A., 2012. Financial crises, bank risk exposure and government financial policy. Journal of Monetary Economics 59, 17–34.

de Groot, O., 2014. The Risk Channel of Monetary Policy. International Journal of Central Banking 10, 115–160.

Güntner, J.H., 2011. Competition among banks and the pass-through of monetary policy. Economic Modelling 28, 1891–1901.

Jermann, U., Quadrini, V., 2012. Macroeconomic effects of financial shocks. American Economic Review 102, 238–71.

Kareken, J.H., Wallace, N., 1978. Deposit Insurance and Bank Regulation: A Partial-Equilibrium Exposition. The Journal of Business 51, 413–438. URL: https://ideas.repec.org/a/ucp/jnlbus/v51y1978i3p413-38.html, doi:10.1086/296006.

Laeven, L., Levine, R., 2009. Bank governance, regulation and risk taking. Journal of Financial Economics 93, 259–275. URL: https://ideas.repec.org/a/eee/jfinec/v93y2009i2p259-275.html.

Lintner, J., 1956. Distribution of incomes of corporations among dividends, retained earnings, and taxes. The American economic review 46, 97–113.

Mendicino, C., Suarez, J., Supera, D., 2018. Optimal dynamic capital requirements. Journal of Money, Credit and Banking 50, 1271–1297.

Obstfeld, M., Rogoff, K., 1996. Foundations of international macroeconomics. MIT press.

OECD, 2022. Banking sector leverage (indicator). URL: https://data.oecd.org/corporate/banking-sector-leverage.htm.

Ranciere, R., Tornell, A., Westermann, F., 2008. Systemic crises and growth. The Quarterly Journal of Economics 123, 359–406.

Reinhart, C.M., Rogoff, K.S., 2009. This time is different: Eight centuries of financial folly. Princeton University press.

Rotemberg, J., 1982. Monopolistic Price Adjustment and Aggregate Output. Review of Economic Studies 49, 517–531.

Schularick, M., Taylor, A.M., 2012. Credit booms gone bust: Monetary policy, leverage cycles, and financial crises, 1870-2008. American Economic Review 102, 1029–61.

Sherman, M., 2009. A short history of financial deregulation in the United States. Center for economic and policy research 7.

Contents

Appendix

to *"The long-run effects of risk: An equilibrium approach"*

C. van der Kwaak[1], J. Madeira[2], N. Palma[3]

[1]University of Groningen
[2]Department of Economics and BRU-Business Research Unit, ISCTE-University Institute of Lisbon
[3]University of Manchester; Instituto de Ciências Sociais, Universidade de Lisboa; CEPR

A1 Appendix figures

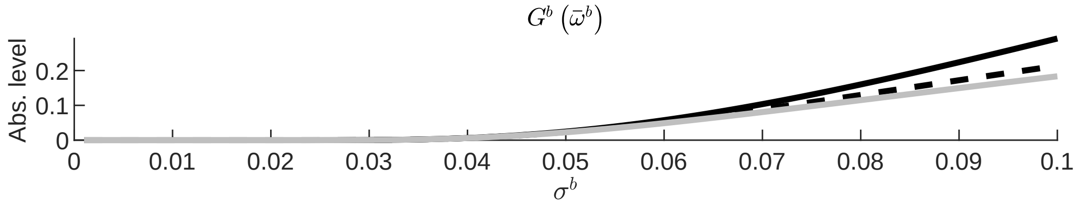

Figure A1: Steady state results for $G^b\left(\bar{\omega}^b\right)$ belonging to the simulations in Figure 2. On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The figure clearly shows that $G^b\left(\bar{\omega}^b\right)$ is always increasing in σ^b (horizontal axis).

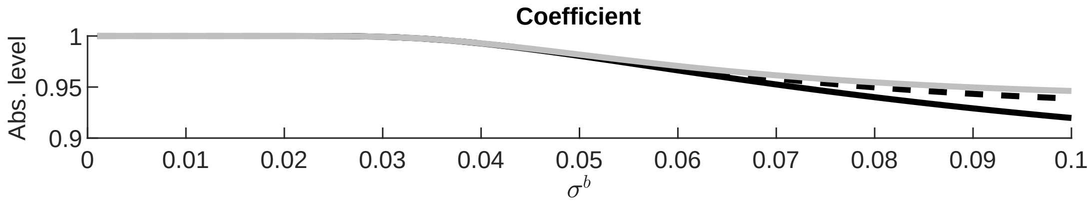

Figure A2: Steady state results for the coefficient $1+\frac{1}{\sigma^b\epsilon^d}\cdot\frac{\bar{\omega}^b}{1-G^b(\bar{\omega}^b)}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)$ in front of the change in the probability of default $\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}$ in equation (60) of Corollary 2 for $\gamma=0$ (gray, solid line), $\gamma=0.5$ (black, dashed line), and $\gamma=1$ (black, solid line). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The figure clearly shows that the coefficient is always larger than zero, as assumed in Corollary 2.

Robustness check: $\mu^{dia} = 0.06$

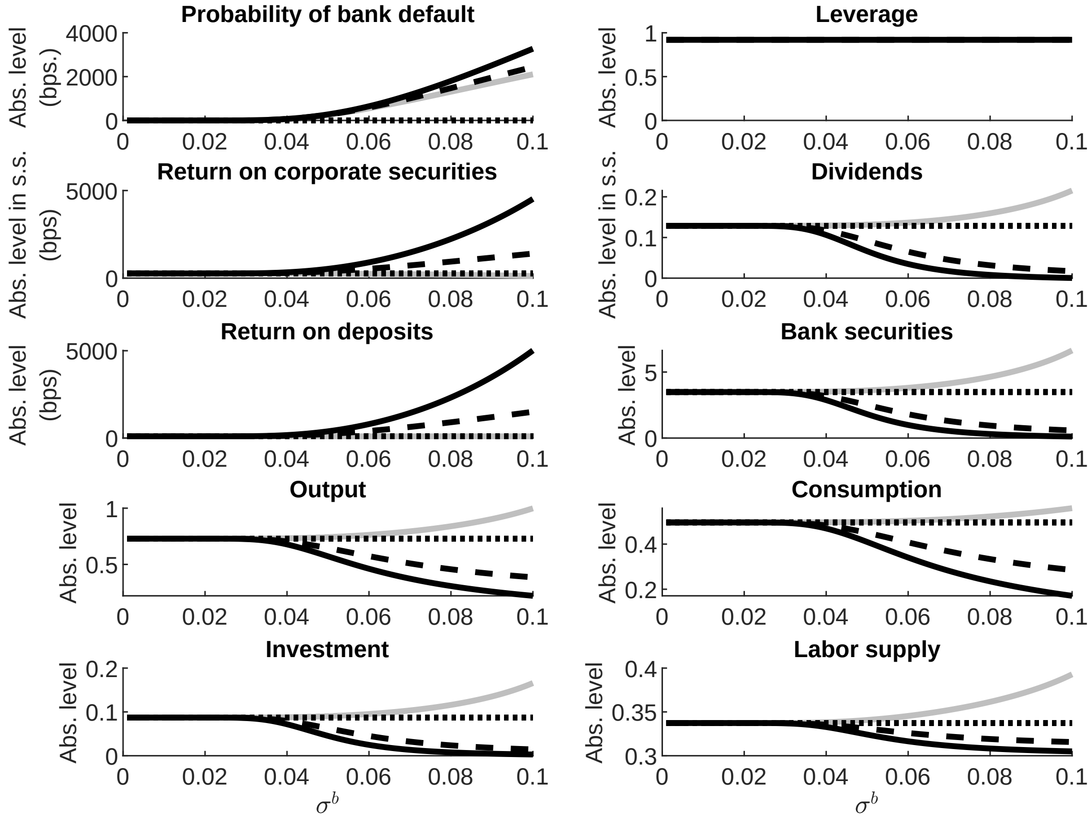

Figure A3: Steady state results for the model version with $\mu^{dia} = 0.06$, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line), and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits refers to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refer to the volume of corporate securities $\bar{s}^k$ held by the banking system.

Robustness check: $\mu^{dia} = 0.06$

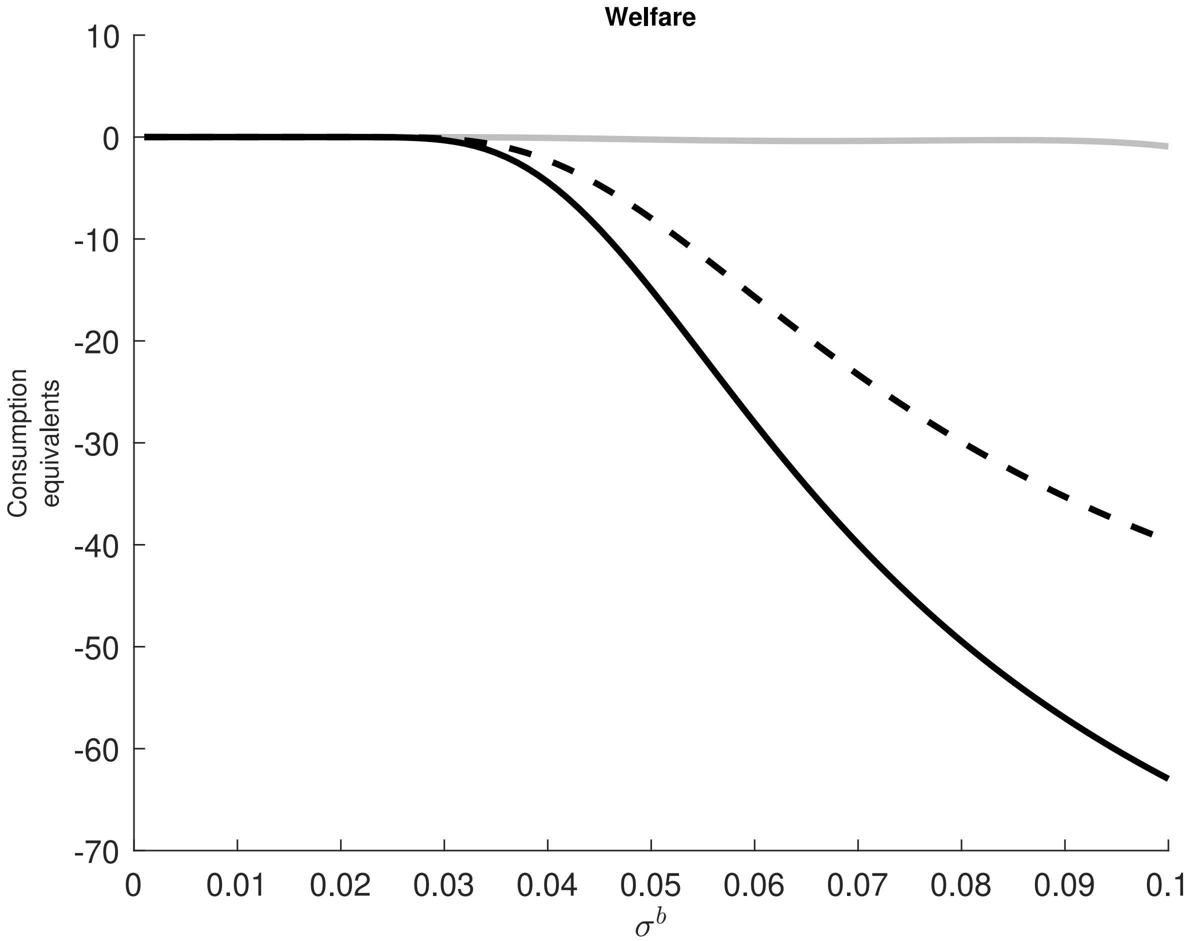

Figure A4: Steady state results for the model version with $\mu^{dia} = 0.06$, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), and limited liability and $\gamma = 1$ (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

Robustness check: $\mu^{dia} = 0.18$

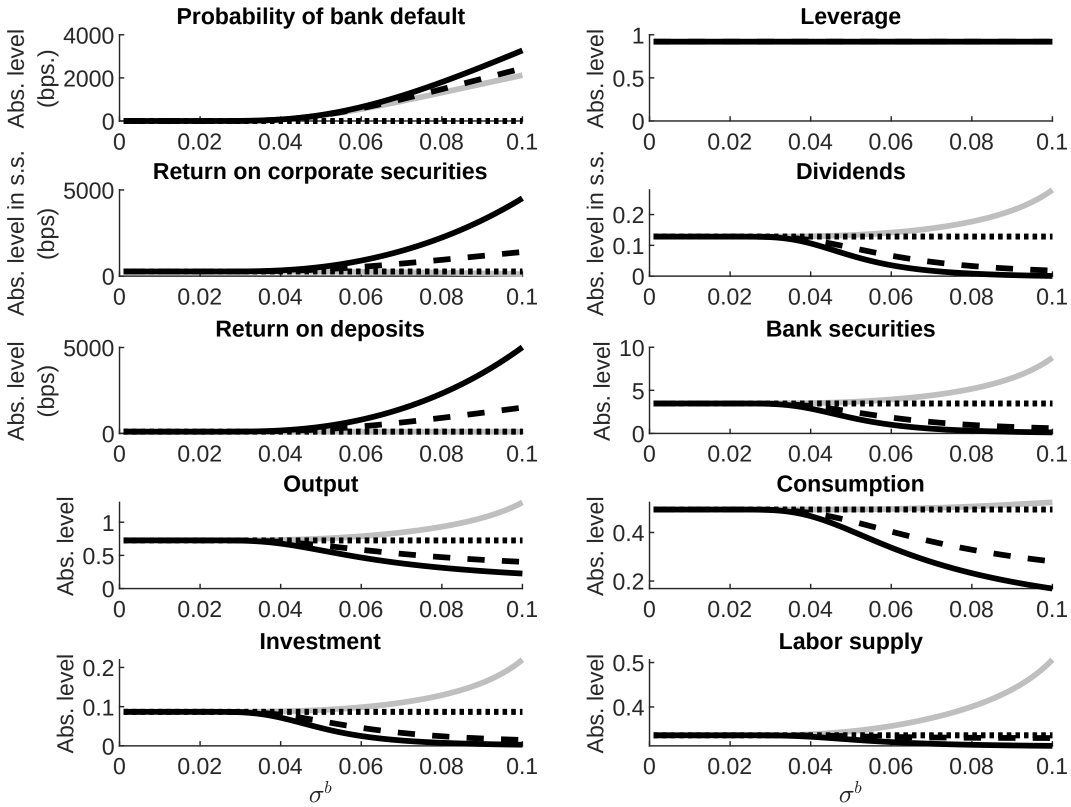

Figure A5: Steady state results for the model version with $\mu^{dia} = 0.18$, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line), and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits refers to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refer to the volume of corporate securities $\bar{s}^k$ held by the banking system.

Robustness check: $\mu^{dia} = 0.18$

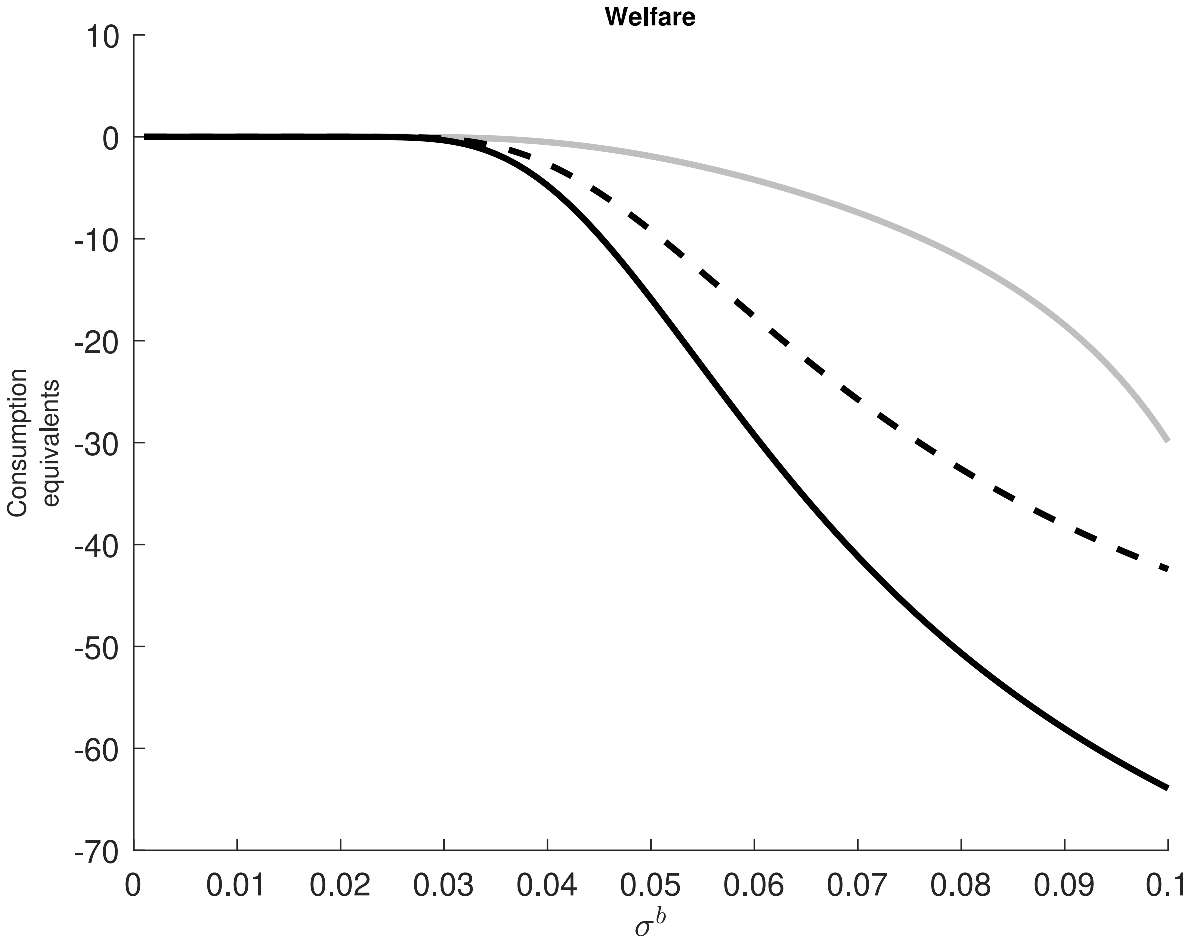

Figure A6: Steady state results for the model version with $\mu^{dia} = 0.18$, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), and limited liability and $\gamma = 1$ (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

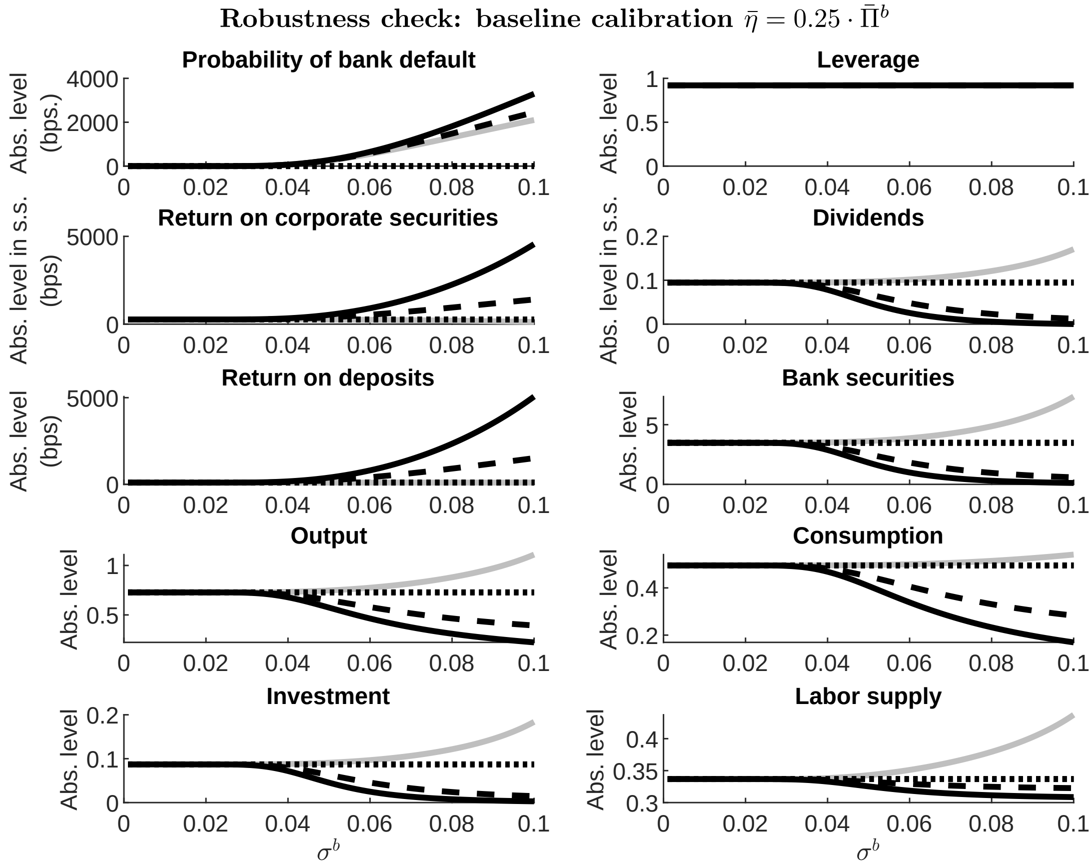

Figure A7: Steady state results for the model version with $\bar{\eta} = 0.25 \cdot \bar{\Pi}^b$ in the baseline calibration, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line), and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refer to the corporate securities $\bar{s}^k$ held by the banking system.

Robustness check: baseline calibration $\bar{\eta} = 0.25 \cdot \bar{\Pi}^b$

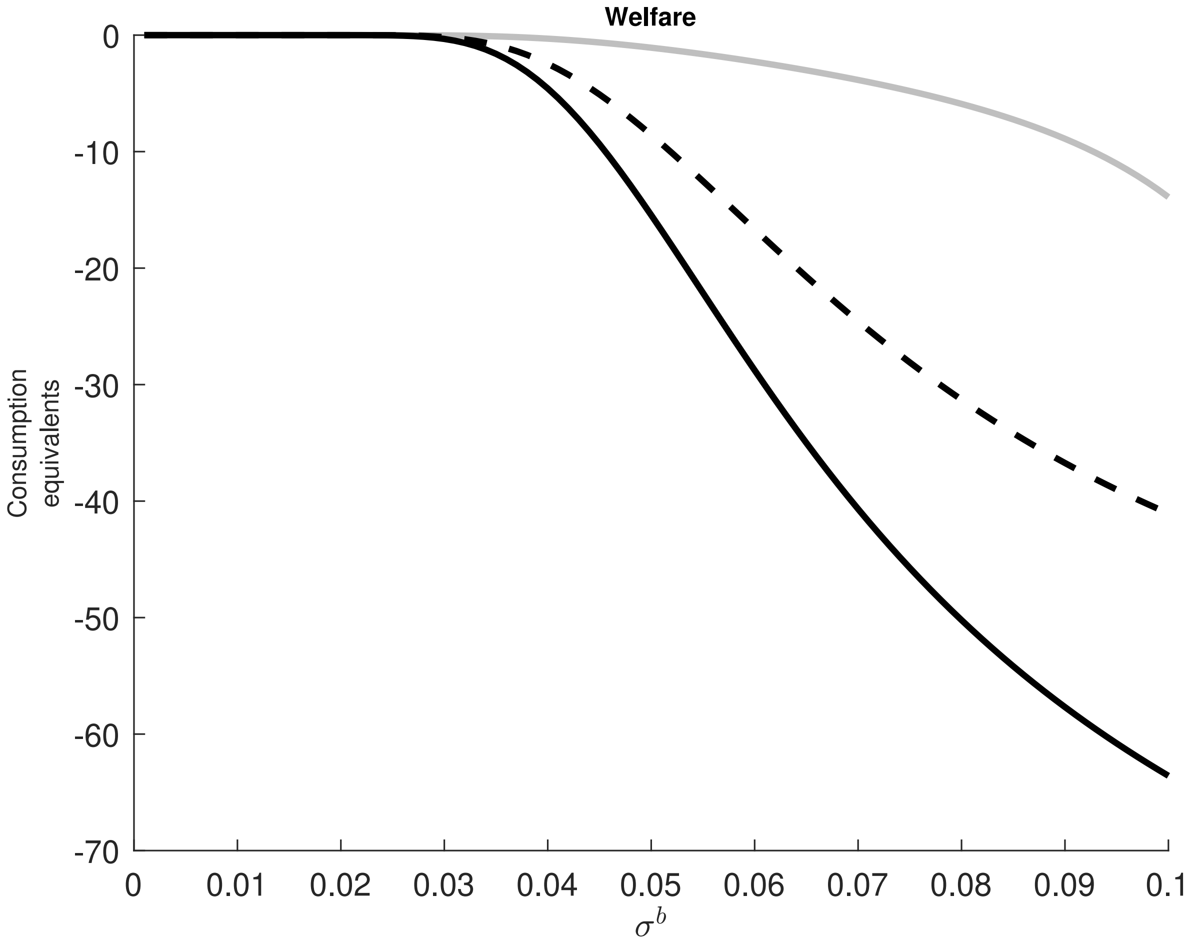

Figure A8: Steady state results for the model version with $\bar{\eta} = 0.25 \cdot \bar{\Pi}^b$ in the baseline calibration, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), and limited liability and $\gamma = 1$ (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

Robustness check: baseline calibration $\bar{\eta} = 0.45 \cdot \bar{\Pi}^b$

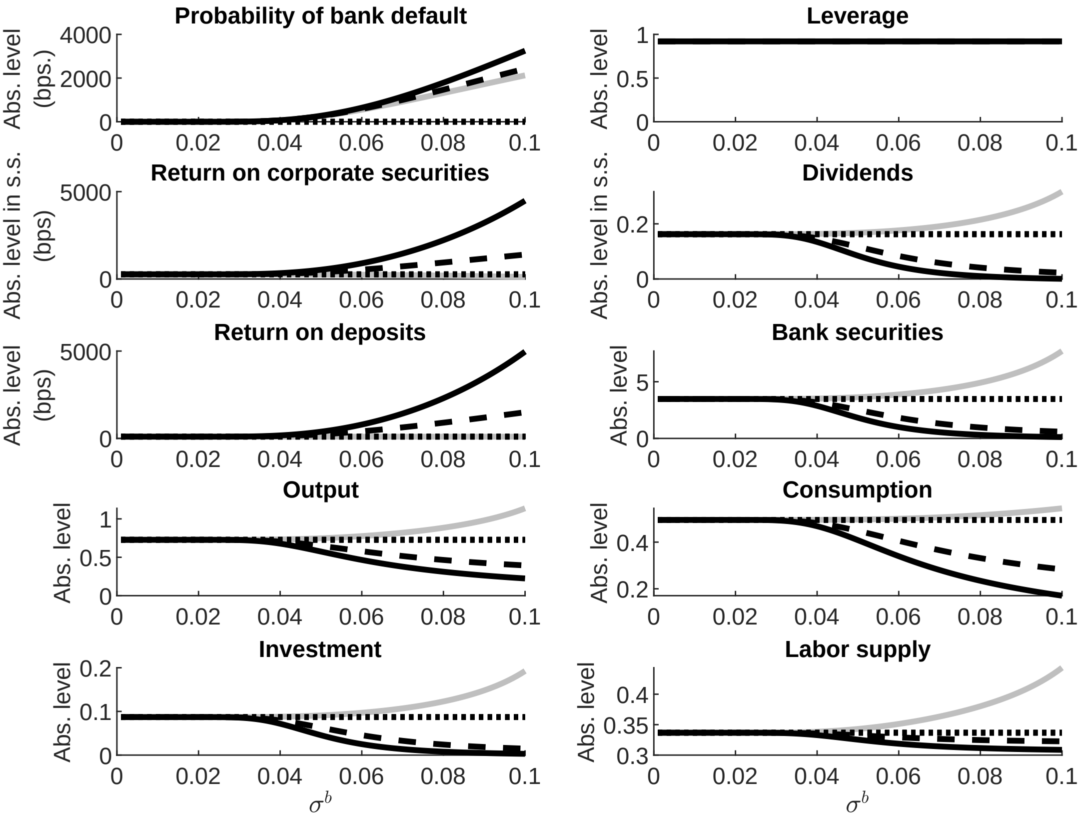

Figure A9: Steady state results for the model version with $\bar{\eta} = 0.45 \cdot \bar{\Pi}^b$ in the baseline calibration, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line), and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refer to the corporate securities $\bar{s}^k$ held by the banking system.

Robustness check: baseline calibration $\bar{\eta} = 0.45 \cdot \bar{\Pi}^b$

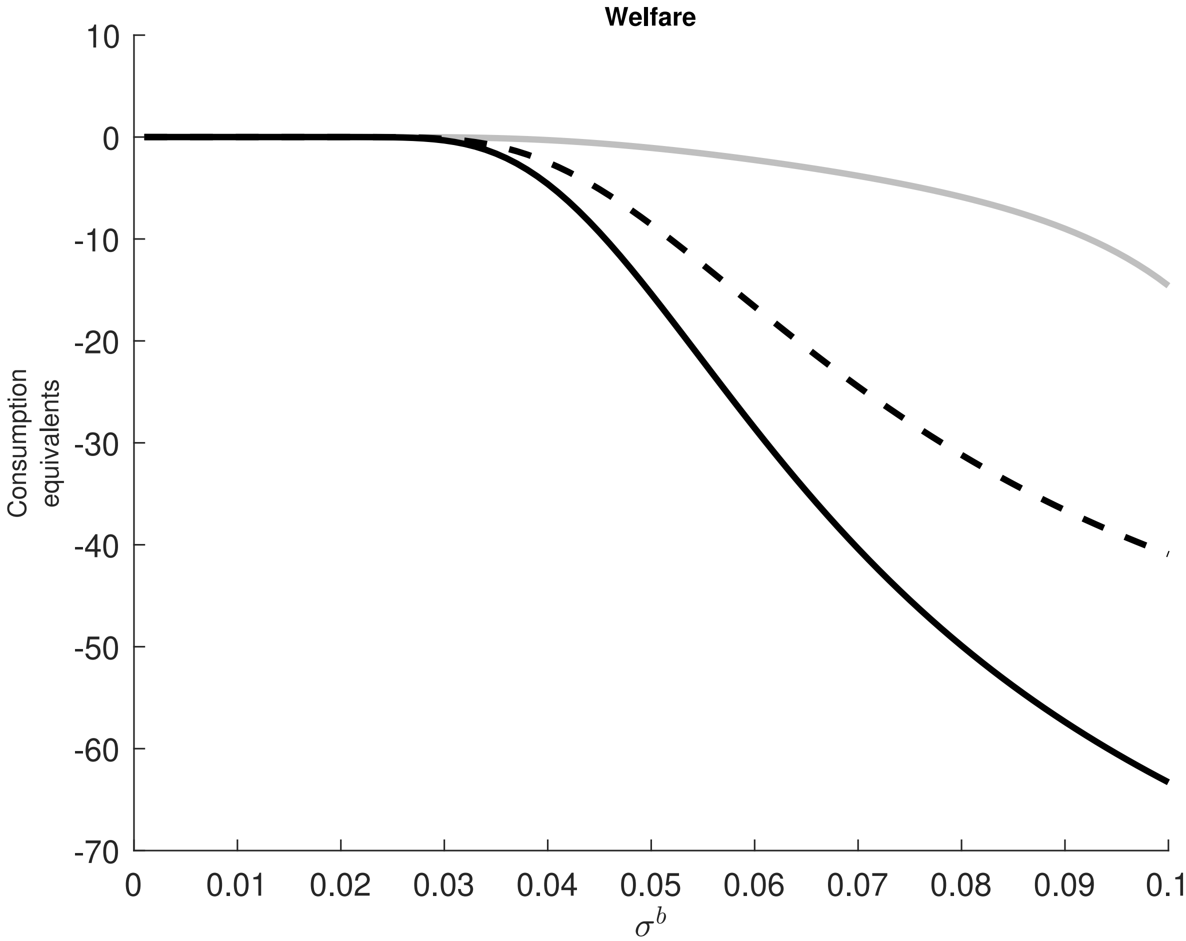

Figure A10: Steady state results for the model version with $\bar{\eta} = 0.45 \cdot \bar{\Pi}^b$ in the baseline calibration, limited liability and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), and limited liability and $\gamma = 1$ (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

Robustness check: $\kappa_\eta = 0.05$

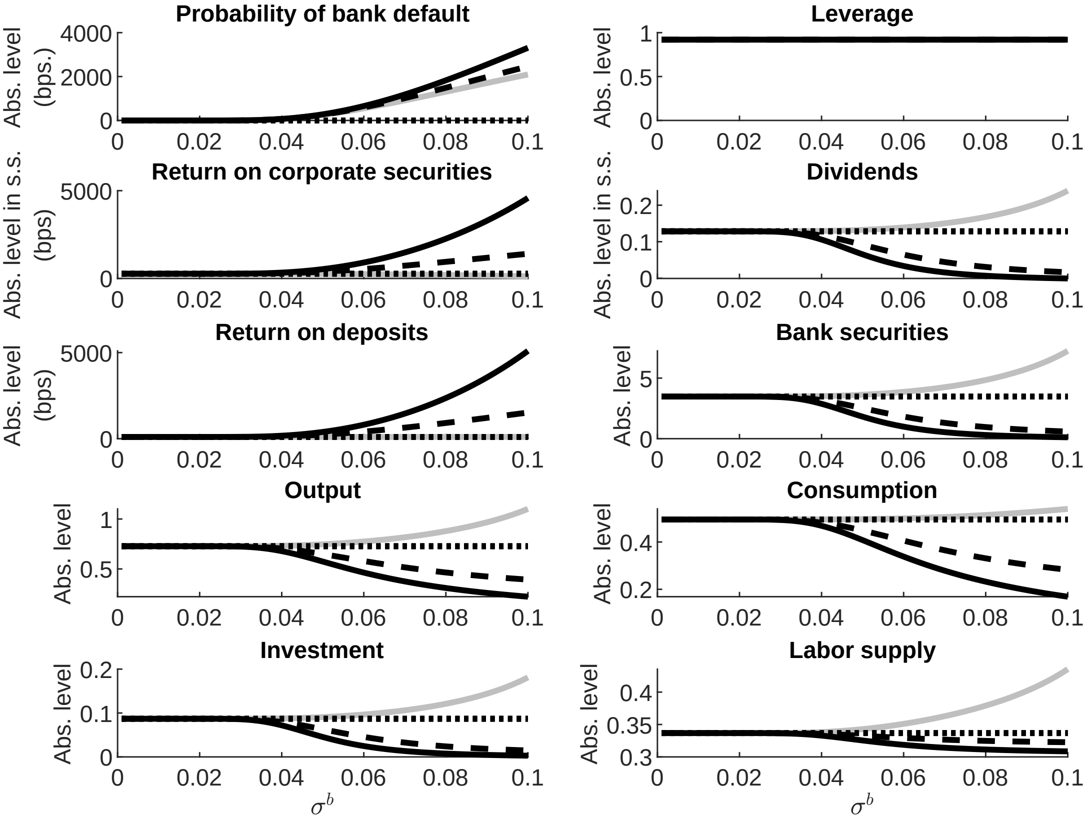

Figure A11: Steady state results for the model version with limited liability, $\kappa_\eta = 0.05$, and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line), and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits refers to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refer to the volume of corporate securities $\bar{s}^k$ held by the banking system.

Robustness check: $\kappa_\eta = 0.05$

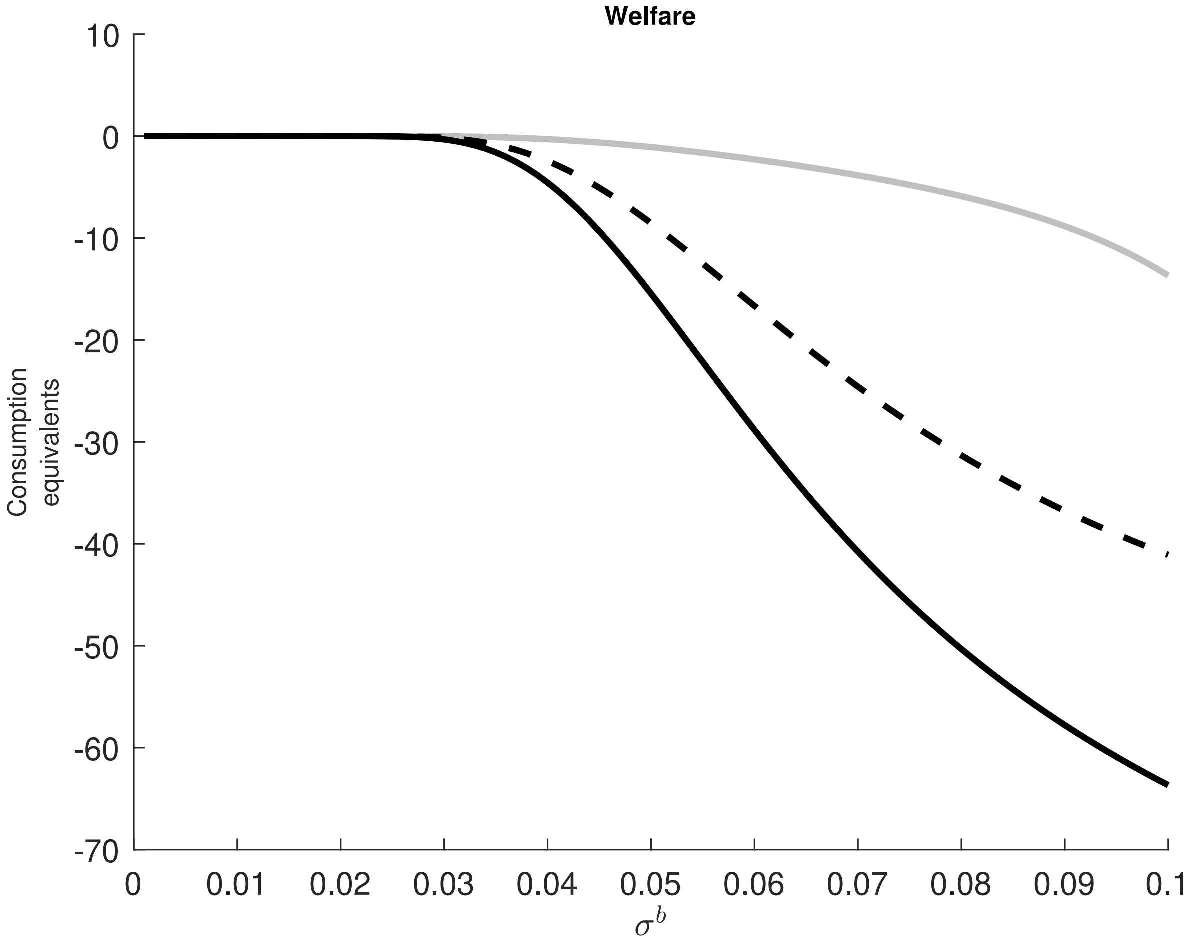

Figure A12: Steady state results for the model version with limited liability, $\kappa_\eta = 0.05$, and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), and limited liability and $\gamma = 1$ (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

Robustness check: $\kappa_\eta = 0.15$

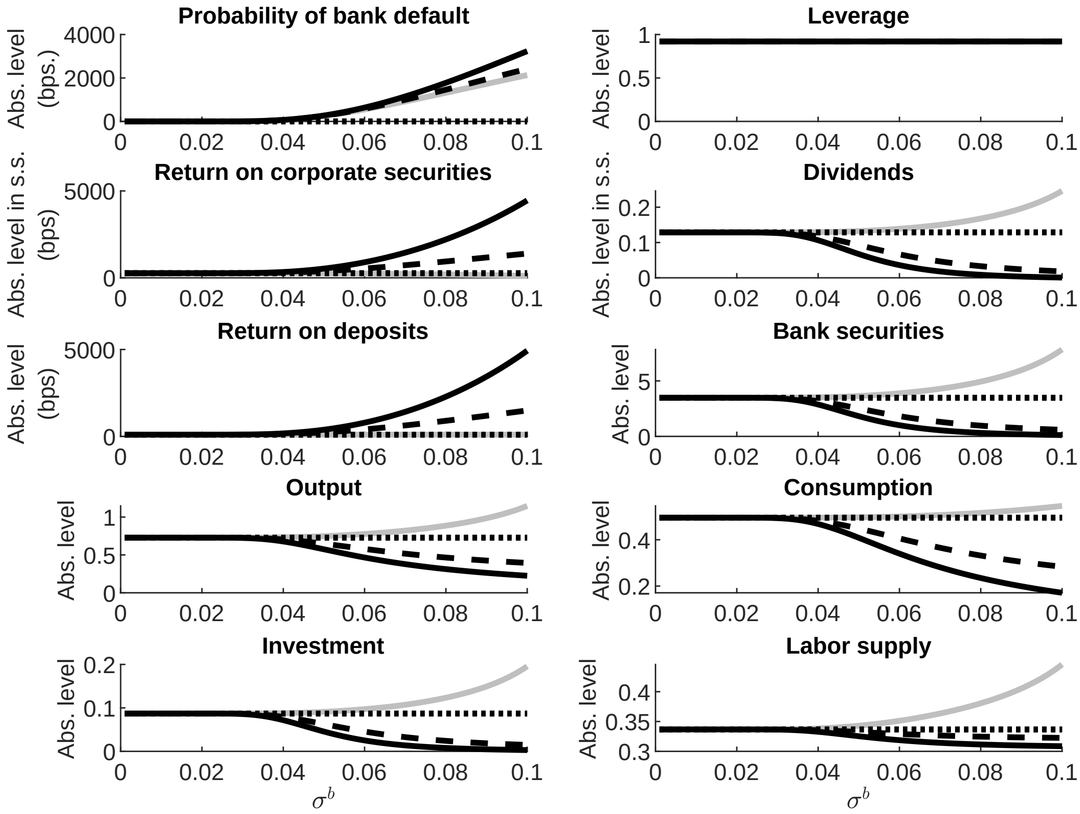

Figure A13: Steady state results for the model version with limited liability, $\kappa_\eta = 0.15$, and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), limited liability and $\gamma = 1$ (black, solid line), and the model version with unlimited liability (black, dotted). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The leverage ratio refers to the deposits-assets ratio $\bar{x}^b \equiv \frac{\bar{d}}{\bar{q}^k \bar{s}^k}$, the return on deposits refers to $\bar{R}^d = \frac{\bar{R}^{n,d}}{\bar{\pi}}$, and bank securities refer to the volume of corporate securities $\bar{s}^k$ held by the banking system.

Robustness check: $\kappa_\eta = 0.15$

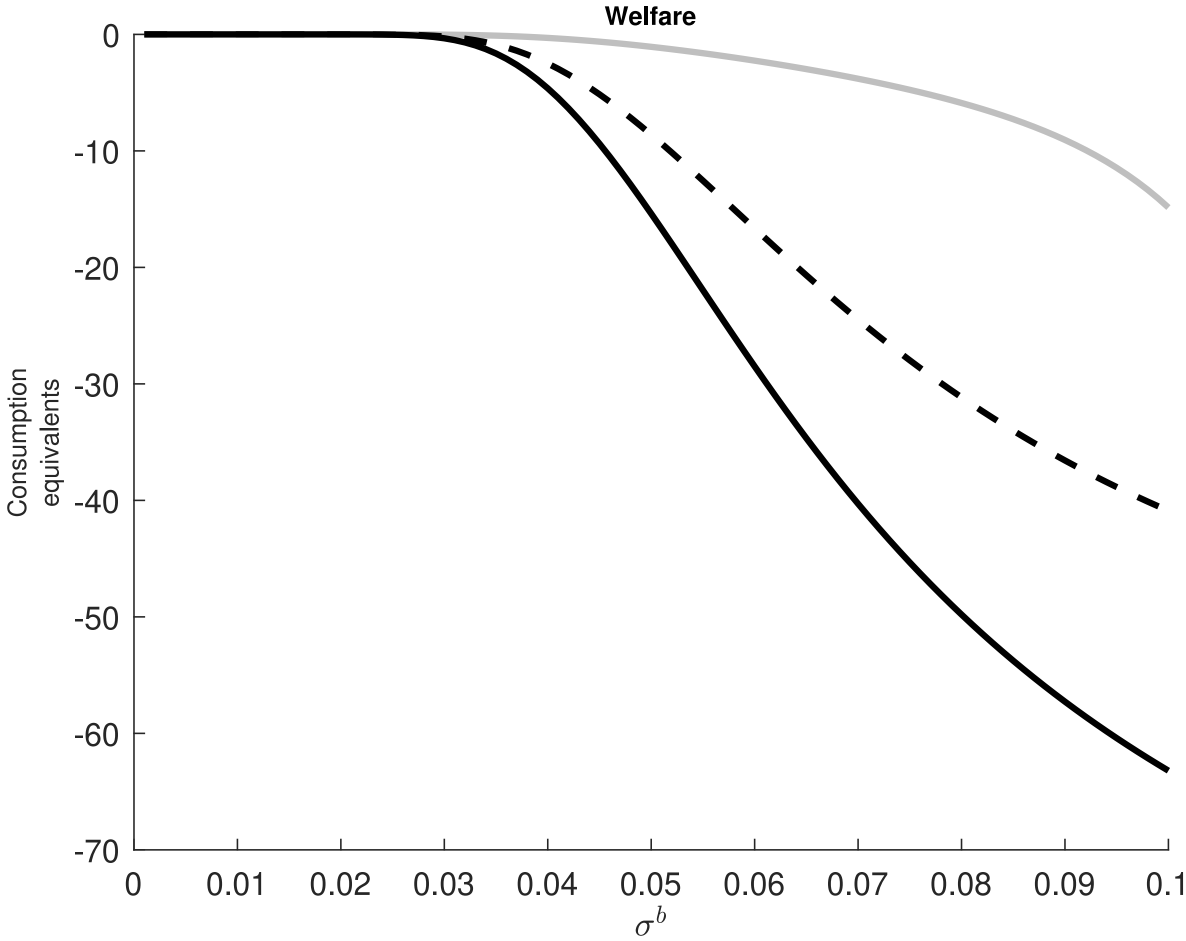

Figure A14: Steady state results for the model version with limited liability, $\kappa_\eta = 0.15$, and $\gamma = 0$ (grey, solid), limited liability and $\gamma = 0.5$ (black, dashed), and limited liability and $\gamma = 1$ (black, solid). On the horizontal axis the standard deviation of the idiosyncratic bankers' shock (σ^b) is displayed. The vertical axis features welfare in terms of consumption equivalents ν, which is expressed in percentage points.

A2 Mathematical derivations

A2.1 Expression for banks' profits

We remember from Section 2.3 that expected profits in period $t+1$ are given by:

$$\Pi_{j,t+1}^{b} = \int_{\bar{\omega}_{j,t+1}^{b}}^{\infty} \left(\omega_{j,t+1}^{b} - \bar{\omega}_{j,t+1}^{b}\right) f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b} R_{t+1}^{k} q_{t}^{k} s_{j,t}^{k},$$

We can write this expression as:

$$\begin{aligned}
\Pi_{j,t+1}^{b} &= \int_{\bar{\omega}_{j,t+1}^{b}}^{\infty} \left(\omega_{j,t+1}^{b} - \bar{\omega}_{j,t+1}^{b}\right) f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b} R_{t+1}^{k} q_{t}^{k} s_{j,t}^{k} \\
&= \left[\int_{\bar{\omega}_{j,t+1}^{b}}^{\infty} \omega_{j,t+1}^{b} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b} - \bar{\omega}_{j,t+1}^{b} \int_{\bar{\omega}_{j,t+1}^{b}}^{\infty} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b}\right] R_{t+1}^{k} q_{t}^{k} s_{j,t}^{k} \\
&= \left[\int_{0}^{\infty} \omega_{j,t+1}^{b} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b} - \int_{0}^{\bar{\omega}_{j,t+1}^{b}} \omega_{j,t+1}^{b} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b}\right. \\
&- \left. \bar{\omega}_{j,t+1}^{b} \int_{\bar{\omega}_{j,t+1}^{b}}^{\infty} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b}\right] R_{t+1}^{k} q_{t}^{k} s_{j,t}^{k} \\
&= \left[E_{t+1}\left(\omega_{j,t+1}^{b}\right) - \int_{0}^{\bar{\omega}_{j,t+1}^{b}} \omega_{j,t+1}^{b} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b} - \bar{\omega}_{j,t+1}^{b} \int_{\bar{\omega}_{j,t+1}^{b}}^{\infty} f^{b}\left(\omega_{j,t+1}^{b}\right) d\omega_{j,t+1}^{b}\right] R_{t+1}^{k} q_{t}^{k} s_{j,t}^{k} \\
&= \left[\Omega_{t+1}^{b} - \Gamma^{b}\left(\bar{\omega}_{j,t+1}^{b}\right)\right] R_{t+1}^{k} q_{t}^{k} s_{j,t}^{k},
\end{aligned}$$

where the last line coincides with the final expression in equation (29).

A2.2 Proof that $\frac{d\Gamma^{b}(\omega)}{d\omega} = 1 - F^{b}(\omega)$

We start by proving that $\frac{d\Gamma^{b}(\omega)}{d\omega} = 1 - F^{b}(\omega)$. First, remember from the definition of $\Gamma^{b}(\omega)$ in expression (30) that its expression features $G^{b}(\omega)$ and $F^{b}(\omega)$, which are given by:

$$\begin{aligned}
G^{b}(\omega) &= \Phi\left(\frac{\log(\omega) - \frac{1}{2}\sigma^{2}}{\sigma}\right), \\
F^{b}(\omega) &= \Phi\left(\frac{\log(\omega) + \frac{1}{2}\sigma^{2}}{\sigma}\right),
\end{aligned}$$

where $\Phi(\cdot)$ denotes the standard normal cumulative density function. Therefore, we immediately see that $G^{b}(\omega) \leq F^{b}(\omega)$.

Next, we take the derivatives with respect to ω:

$$\begin{aligned}
\frac{dG^b(\omega)}{d\omega} &= \Phi'\left(\frac{\log(\omega)-\frac{1}{2}\sigma^2}{\sigma}\right)\cdot\frac{1}{\sigma\omega} = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\log(\omega)-\frac{1}{2}\sigma^2}{\sigma}\right)^2\right]\cdot\frac{1}{\sigma\omega}, \\
\frac{dF^b(\omega)}{d\omega} &= \Phi'\left(\frac{\log(\omega)+\frac{1}{2}\sigma^2}{\sigma}\right)\cdot\frac{1}{\sigma\omega} = \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\log(\omega)+\frac{1}{2}\sigma^2}{\sigma}\right)^2\right]\cdot\frac{1}{\sigma\omega},
\end{aligned}$$

Now we write out the expression for $\frac{dG^b(\omega)}{d\omega}$:

$$\begin{aligned}
\frac{dG^b(\omega)}{d\omega} &= \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}\left(\log^2(\omega)+\frac{1}{4}\sigma^4-\sigma^2\log(\omega)\right)\right]\cdot\frac{1}{\sigma\omega} \\
&= \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}\left(\log^2(\omega)+\frac{1}{4}\sigma^4+\sigma^2\log(\omega)-2\sigma^2\log(\omega)\right)\right]\cdot\frac{1}{\sigma\omega} \\
&= \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma^2}\left(\log(\omega)+\frac{1}{2}\sigma^2\right)^2+\log(\omega)\right]\cdot\frac{1}{\sigma\omega} \\
&= \frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2}\left(\frac{\log(\omega)+\frac{1}{2}\sigma^2}{\sigma}\right)^2\right]\exp\left[\log(\omega)\right]\cdot\frac{1}{\sigma\omega} \\
&= \omega\cdot\frac{dF^b(\omega)}{d\omega}. \qquad \text{(A1)}
\end{aligned}$$

Now we are ready to prove that $\frac{d\Gamma^b(\omega)}{d\omega} = 1 - F^b(\omega)$ by differentiating expression (30) with respect to ω:

$$\frac{d\Gamma^b(\omega)}{d\omega} = \frac{dG^b(\omega)}{d\omega} + 1 - F^b(\omega) - \omega\cdot\frac{dF^b(\omega)}{d\omega} = 1 - F^b(\omega),$$

since $\frac{dG^b(\omega)}{d\omega} = \omega\cdot\frac{dF^b(\omega)}{d\omega}$.

A2.3 Proof that all banks choose the same allocation and interest rate

We know from Gertler and Kiyotaki (2010); Gertler and Karadi (2011) and Damjanovic et al. (2020) that the shadow value of additional constraints that banks face are not bank-specific, which is why ψ_t^b and ψ_t^d are the same for all banks. Therefore, we can immediately see from the first order condition for dividends (33) that all banks will choose the same level of dividends in equilibrium, i.e. $\eta_{j,t} = \eta_t$. Next, we see from the first order condition for corporate securities (36) that all banks will choose the same ratio $\frac{\left(R_{j,t}^{n,d}\right)^{1-\epsilon^d}}{s_{j,t}^k}$ in equilibrium, which is the part of $\bar{\omega}_{j,t+1}^b$ that banks can choose in period t, see equation (28). Therefore, the (expected) value of $\bar{\omega}_{j,t+1}^b$ is the same across banks, and we can drop the subscript j.

Next, we see from the first order condition for the nominal interest rate on deposits (34) that all banks will choose the same nominal interest rate on deposits given that they choose the same (expected) value of $\bar{\omega}_{j,t+1}^b$, i.e. $R_{j,t}^{n,d} = R_t^{n,d}$. In that case, we can see from the equation that describes the demand for deposits of bank j, equation (23), that all banks will have the same level of deposits in equilibrium, i.e. $d_{j,t} = d_t$.

Finally, since all banks start with the same net worth $n^b_{j,t} = n^b_t$, pay the same dividends, and attract the same amount of deposits, we deduce from the balance sheet constraint (25) that all banks choose the same amount of corporate securities, $s^k_{j,t} = s^k_t$. Therefore, we can aggregate over all these variables and write down the banking sector first order conditions using aggregate corporate securities, dividends, and the nominal interest rate on deposits.

A2.4 Aggregation

Since we know from Appendix A2.3 that all banks receive the same amount of net worth, choose the same allocation for corporate securities, dividends in equilibirum, and attract the same ampount of deposits (because they set the same nominal interest rate on deposits), we can aggregate over all these variables and write down the aggregate profits of the banking sector Π^b_t:

$$\begin{aligned}
\Pi^b_t &= \int_0^1 \int_{\bar{\omega}^b_t}^{\infty} \left(\omega^b_t - \bar{\omega}^b_t\right) R^k_t q^k_{t-1} s^k_{j,t-1} f^b\left(\omega^b_t\right) d\omega^b_t dj \\
&= \int_0^1 \int_{\bar{\omega}^b_t}^{\infty} \left(\omega^b_t - \bar{\omega}^b_t\right) f^b\left(\omega^b_t\right) d\omega^b_t R^k_t q^k_{t-1} s^k_{j,t-1} dj \\
&= \int_0^1 \left[\Omega^b_t - \Gamma^b\left(\bar{\omega}^b_t\right)\right] R^k_t q^k_{t-1} s^k_{j,t-1} dj \\
&= \left[\Omega^b_t - \Gamma^b\left(\bar{\omega}^b_t\right)\right] R^k_t q^k_{t-1} \int_0^1 s^k_{j,t-1} dj \\
&= \left[\Omega^b_t - \Gamma^b\left(\bar{\omega}^b_t\right)\right] R^k_t q^k_{t-1} s^k_{t-1}. \qquad \text{(A2)}
\end{aligned}$$

We will assume that households whose bank did not default provide a fraction θ^b of the profits from the old bank as starting net worth to the new bank.

Households owning a bank that defaulted in period t, which amounts to a mass $F^b\left(\bar{\omega}^b_t\right)$, provide a fraction $\chi^b/F^b\left(\bar{\omega}^b_t\right)$ of previous period aggregate net worth n^b_{t-1} to the new bank. Therefore aggregate net worth $n^{b,n}_t$ of starting bankers is equal to:

$$n^{b,n}_t = \chi^b n^b_{t-1}. \qquad \text{(A3)}$$

So finally we can write down aggregate net worth n^b_t, which consists of a fraction θ^b of aggregate profits Π^b_t of non-defaulting banks (A2) and aggregate net worth $n^{b,n}_t$ of newly starting banks (A3):

$$n^b_t = \theta^b \Pi^b_t + n^{b,n}_t = \theta^b \left[\Omega^b_t - \Gamma^b\left(\bar{\omega}^b_t\right)\right] R^k_t q^k_{t-1} s^k_{t-1} + \chi^b n^b_{t-1}. \qquad \text{(A4)}$$

The period t costs for the deposit insurance agency are given by:

$$
\begin{aligned}
T_t^{dia} &= (1-\gamma)\int_0^1\int_0^{\bar{\omega}_t^b} R_t^D d_{j,t-1} f^b\left(\omega_t^b\right) d\omega_t^b dj - \int_0^1\int_0^{\bar{\omega}_t^b} \omega_t^b R_t^k q_{t-1}^k s_{j,t-1}^k f^b\left(\omega_t^b\right) d\omega_t^b dj \\
&+ \mu^{dia}\int_0^1\int_0^{\bar{\omega}_t^b} \omega_t^b R_t^k q_{t-1}^k s_{j,t-1}^k f^b\left(\omega_t^b\right) d\omega_t^b dj \\
&= (1-\gamma)\int_0^1\int_0^{\bar{\omega}_t^b} f^b\left(\omega_t^b\right) d\omega_t^b R_t^D d_{j,t-1} dj - \left(1-\mu^{dia}\right)\int_0^1\int_0^{\bar{\omega}_t^b} \omega_t^b f^b\left(\omega_t^b\right) d\omega_t^b R_t^k q_{t-1}^k s_{j,t-1}^k dj \\
&= (1-\gamma)\int_0^1 \left[1-F^b\left(\bar{\omega}_t^b\right)\right] R_t^D d_{j,t-1} dj - \left(1-\mu^{dia}\right)\int_0^1 G^b\left(\bar{\omega}_t^b\right) R_t^k q_{t-1}^k s_{j,t-1}^k dj \\
&= (1-\gamma)\left[1-F^b\left(\bar{\omega}_t^b\right)\right] R_t^D d_{t-1} - \left(1-\mu^{dia}\right) G^b\left(\bar{\omega}_t^b\right) R_t^k q_{t-1}^k s_{t-1}^k. \qquad \text{(A5)}
\end{aligned}
$$

A3 Second order conditions

In this section, we derive the second order conditions of the optimization problem of bank j. We will do so by separately considering the second order conditions for the case where the equity-deposit constraint (26) is binding and the case where it is not binding. Let me start by considering the case where the constraint is binding.

A3.1 The case $\psi_t^d > 0$

Before we derive the second order conditions for this case, it is useful to use the balance sheet constraint (25) and the equity-deposit constraint (26) to substitute away bank j's holdings of corporate securities $s_{j,t}^k$ and the nominal interest rate on deposits $R_{j,t}^{n,d}$. We start by using the binding equity-deposit constraint (26) to express the nominal interest rate on deposits as a function of dividends $\eta_{j,t}$:

$$
R_{j,t}^{n,d} = \left[\frac{n_{j,t}^b - \eta_{j,t} - f\left(\eta_{j,t}\right)}{\kappa_t \left(R_t^{n,d}\right)^{\epsilon^d} d_t}\right]^{-\frac{1}{\epsilon^d}}. \qquad \text{(A6)}
$$

From the above expression we clearly see that the nominal interest rate on deposits only depends on the volume of dividends $\eta_{j,t}$. Now that we have expressed the nominal interest rate on deposits in terms of dividends $\eta_{j,t}$, we can show that the number of corporate securities also only depends on the volume of dividends. To see, we rewrite the balance sheet constraint (25) in the following way:

$$
q_t^k s_{j,t}^k = n_{j,t}^b - \eta_{j,t} - f\left(\eta_{j,t}\right) + \left(\frac{R_{j,t}^{n,d}}{R_t^{n,d}}\right)^{-\epsilon^d} d_t. \qquad \text{(A7)}
$$

We see that the only bank-specific variables on the right hand side of the above expression are dividends $\eta_{j,t}$ and the nominal interest rate on bank deposits $R_{j,t}^{n,d}$. But since the nominal interest rate on bank deposits can be expressed in terms of dividends when the equity-deposit constraint (26) is binding, we conclude that the number of corporate securities can also be expressed solely in terms of dividends $\eta_{j,t}$.

Therefore, we can entirely express bank j's maximization objective in terms of dividends $\eta_{j,t}$, because we have employed the two constraints to which the bank is subject to eliminate the other two bank-specific variables, namely the number of corporate securities $s^k_{j,t}$ and the nominal interest rate on deposits $R^{n,d}_{j,t}$. Therefore, we can write bank j's maximization objective $H(\eta_{j,t})$ in the following way:

$$H(\eta_{j,t}) \equiv \eta_{j,t} + E_t \left\{ \beta \Lambda_{t,t+1} \left[\Omega^b_{t+1} - \Gamma^b \left(\bar{\omega}^b_{j,t+1} \right) \right] R^k_{t+1} q^k_t s^k_{j,t} \right\}, \tag{A8}$$

where $R^{n,d}_{j,t}$ and $s^k_{j,t}$ are given by equations (A6) and (A7), respectively, and $\bar{\omega}^b_{j,t+1}$ by equation (28).

Having eliminated $R^{n,d}_{j,t}$ and $s^k_{j,t}$ as independent variables from the maximization objective, we can simply find the first and second order conditions by differentiating with respect to dividends $\eta_{j,t}$. Before we do so, however, we calculate the derivative of $R^{n,d}_{j,t}$, $s^k_{j,t}$, and $\bar{\omega}^b_{j,t+1}$ with respect to $\eta_{j,t}$:

$$\frac{dR^{n,d}_{j,t}}{d\eta_{j,t}} = \frac{1}{\epsilon^d} \left(\frac{1 + f'(\eta_{j,t})}{\kappa_t \left(R^{n,d}_t \right)^{\epsilon^d} d_t} \right) \left(R^{n,d}_{j,t} \right)^{1+\epsilon^d}. \tag{A9}$$

Next, we differentiate $q^k_t s^k_{j,t}$ with respect to $\eta_{j,t}$ using bank j's balance sheet constraint (A7):

$$\begin{aligned} \frac{d\left(q^k_t s^k_{j,t}\right)}{d\eta_{j,t}} &= -\left[1 + f'(\eta_{j,t})\right] + \left(-\epsilon^d\right) \left(R^{n,d}_t \right)^{\epsilon^d} \left(R^{n,d}_{j,t} \right)^{-\epsilon^d - 1} d_t \cdot \frac{dR^{n,d}_{j,t}}{d\eta_{j,t}} \\ &= -\left[1 + f'(\eta_{j,t})\right] - \frac{1 + f'(\eta_{j,t})}{\kappa_t} \\ &= -\left(\frac{1 + \kappa_t}{\kappa_t} \right) \left[1 + f'(\eta_{j,t})\right]. \end{aligned} \tag{A10}$$

Finally, we calculate the derivative of $\bar{\omega}^b_{j,t+1}$ with respect to dividends $\eta^{j,t}$:

$$\begin{aligned} \frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}} &= \left(1 - \epsilon^d\right) \frac{\bar{\omega}^b_{j,t+1}}{R^{n,d}_{j,t}} \cdot \frac{dR^{n,d}_{j,t}}{d\eta_{j,t}} - \frac{\bar{\omega}^b_{j,t+1}}{q^k_t s^k_{j,t}} \cdot \frac{d\left(q^k_t s^k_{j,t}\right)}{d\eta_{j,t}} \\ &= \left(\frac{1 - \epsilon^d}{\epsilon^d} \right) \bar{\omega}^b_{j,t+1} \left(\frac{1 + f'(\eta_{j,t})}{\kappa_t \left(R^{n,d}_t \right)^{\epsilon^d} \left(R^{n,d}_{j,t} \right)^{-\epsilon^d} d_t} \right) + \frac{\bar{\omega}^b_{j,t+1}}{q^k_t s^k_{j,t}} \left(\frac{1 + \kappa_t}{\kappa_t} \right) \left[1 + f'(\eta_{j,t})\right] \\ &= \bar{\omega}^b_{j,t+1} \left(\frac{1 + f'(\eta_{j,t})}{\kappa_t} \right) \left[\left(\frac{1 - \epsilon^d}{\epsilon^d} \right) \frac{1}{\left(R^{n,d}_t \right)^{\epsilon^d} \left(R^{n,d}_{j,t} \right)^{-\epsilon^d} d_t} + \frac{1 + \kappa_t}{q^k_t s^k_{j,t}} \right]. \end{aligned} \tag{A11}$$

Now we can take the first order condition of bank j's maximization objective (A13) with respect to $\eta_{j,t}$:

$$
\begin{aligned}
H_{\eta_{j,t}} \equiv \frac{dH(\eta_{j,t})}{d\eta_{j,t}} &= 1 + E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - \Gamma^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1} \cdot \frac{d\left(q^k_t s^k_{j,t}\right)}{d\eta_{j,t}}\right\} \\
&- E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right] \cdot \frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}} \cdot R^k_{t+1} q^k_t s^k_{j,t}\right\} \\
&= 1 + E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - \Gamma^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1} \cdot \frac{d\left(q^k_t s^k_{j,t}\right)}{d\eta_{j,t}}\right\} \\
&+ E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right] \bar{\omega}^b_{j,t+1} R^k_{t+1} \cdot \frac{d\left(q^k_t s^k_{j,t}\right)}{d\eta_{j,t}}\right\} \\
&- \left(1-\epsilon^d\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right] \frac{\bar{\omega}^b_{j,t+1}}{R^{n,d}_{j,t}} \cdot \frac{dR^{n,d}_{j,t}}{d\eta_{j,t}} \cdot R^k_{t+1} q^k_t s^k_{j,t}\right\} \\
&= 1 + E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - G^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1}\right\} \cdot \frac{d\left(q^k_t s^k_{j,t}\right)}{d\eta_{j,t}} \\
&- \left(1-\epsilon^d\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right] \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{-\epsilon^d} d_t\right\} \cdot \frac{dR^{n,d}_{j,t}}{d\eta_{j,t}}
\end{aligned}
$$

Substitution of the expressions (A9) and (A10) gives the following expression for the derivative of $H(\eta_{j,t})$:

$$
\begin{aligned}
H_{\eta_{j,t}} &= 1 - \left(\frac{1+\kappa_t}{\kappa_t}\right)\left[1 + f'(\eta_{j,t})\right] E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - G^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1}\right\} \\
&+ \frac{1}{\kappa_t}\left[1 + f'(\eta_{j,t})\right]\left(\frac{\epsilon^d - 1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right] \frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\}. \qquad \text{(A12)}
\end{aligned}
$$

We can see that the accompanying first order condition, i.e. $H_{\eta_{j,t}} = 0$, coincides with the first order conditions in the main text when the equity-deposit constraint (26) is binding. To see this, we first solve for ψ^d_t from the first order condition for dividends (33). Next, we substitute this expression into the first order condition for the nominal interest rate on deposits (34), which gives the following expression:

$$
(1+\kappa_t)\,\psi^b_t = \frac{\kappa_t}{1 + f'(\eta_{j,t})} + \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right] \frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\}.
$$

Finally, we observe that we can write the first order condition for corporate securities (32) as:

$$
\psi^b_t = E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - G^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1}\right\},
$$

since $1 - \Gamma^b\left(\bar{\omega}^b_{j,t+1}\right) = G^b\left(\bar{\omega}^b_{j,t+1}\right) + \bar{\omega}^b_{j,t+1}\left[1 - F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]$. Substitution of this expression for ψ^b_t then gives the same first order condition as when $H_{\eta_{j,t}} = 0$ in expression (A12).

Now that we have found an expression for $H_{\eta_{j,t}}$, we can find the second order derivative $H_{\eta_{j,t}\eta_{j,t}}$ by differentiating (A12) with respect to $\eta_{j,t}$:

$$\begin{aligned} H_{\eta_{j,t}\eta_{j,t}} &= -\left(\frac{1+\kappa_t}{\kappa_t}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - G^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1}\right\} f''\left(\eta_{j,t}\right) \\ &+ \left(\frac{1+\kappa_t}{\kappa_t}\right)\left[1+f'\left(\eta_{j,t}\right)\right] E_t\left\{\beta\Lambda_{t,t+1}\frac{dG^b\left(\bar{\omega}^b_{j,t+1}\right)}{d\bar{\omega}^b_{j,t+1}}\cdot\frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}}\cdot R^k_{t+1}\right\} \\ &+ \frac{1}{\kappa_t}\left(\frac{\epsilon^d-1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\} f''\left(\eta_{j,t}\right) \\ &- \frac{1}{\kappa_t}\left[1+f'\left(\eta_{j,t}\right)\right]\left(\frac{\epsilon^d-1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\frac{dF^b\left(\bar{\omega}^b_{j,t+1}\right)}{d\bar{\omega}^b_{j,t+1}}\cdot\frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}}\cdot\frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\} \\ &+ \frac{1}{\kappa_t}\left[1+f'\left(\eta_{j,t}\right)\right]\left(\frac{\epsilon^d-1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\frac{1}{\pi_{t+1}}\cdot\frac{dR^{n,d}_{j,t}}{d\eta_{j,t}}\right\}. \end{aligned}$$

Substitution of expression (A1) allows us to write this as:

$$\begin{aligned} H_{\eta_{j,t}\eta_{j,t}} &= -\left(\frac{1+\kappa_t}{\kappa_t}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - G^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1}\right\} f''\left(\eta_{j,t}\right) \\ &+ \left(\frac{1+\kappa_t}{\kappa_t}\right)\left[1+f'\left(\eta_{j,t}\right)\right] E_t\left\{\beta\Lambda_{t,t+1}\bar{\omega}^b_{j,t+1}\cdot f^b\left(\bar{\omega}^b_{j,t+1}\right)\cdot\frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}}\cdot R^k_{t+1}\right\} \\ &+ \frac{1}{\kappa_t}\left(\frac{\epsilon^d-1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\} f''\left(\eta_{j,t}\right) \\ &- \frac{1}{\kappa_t}\left[1+f'\left(\eta_{j,t}\right)\right]\left(\frac{\epsilon^d-1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\cdot f^b\left(\bar{\omega}^b_{j,t+1}\right)\cdot\frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}}\cdot\frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\} \\ &+ \frac{1}{\kappa_t}\left[1+f'\left(\eta_{j,t}\right)\right]\left(\frac{\epsilon^d-1}{\epsilon^d}\right) E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\frac{1}{\pi_{t+1}}\cdot\frac{dR^{n,d}_{j,t}}{d\eta_{j,t}}\right\}, \end{aligned}$$

where $f^b\left(\bar{\omega}^b_{j,t+1}\right) \equiv \frac{dF^b\left(\bar{\omega}^b_{j,t+1}\right)}{d\bar{\omega}^b_{j,t+1}}$, and where $\frac{dR^{n,d}_{j,t}}{d\eta_{j,t}}$ and $\frac{d\bar{\omega}^b_{j,t+1}}{d\eta_{j,t}}$ are respectively given by expressions (A9) and (A11), respectively.

A3.2 The case $\psi^d_t = 0$

Again, we start from the maximization objective (A13):

$$H\left(\eta_{j,t}\right) \equiv \eta_{j,t} + E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} - \Gamma^b\left(\bar{\omega}^b_{j,t+1}\right)\right] R^k_{t+1} q^k_t s^k_{j,t}\right\}, \tag{A13}$$

However, now that the equity-deposit constraint (26) is not binding, we can no longer employ expression (A6) for the nominal interest rate on deposits, and can only employ the bank's balance sheet constraint (A7) to eliminate $q^k_t s^k_{j,t}$. We still replace $\bar{\omega}^b_{j,t+1}$ by equation (28). As a result, we have two independent variables, namely $R_{j,t}^{n,d}$ and $\eta_{j,t}$, which contrasts with the case where the equity-deposit constraint (26) is binding, in which case only $\eta_{j,t}$ is an independent variable.

We start by taking the first order conditions with respect to $\eta_{j,t}$ and $R_{j,t}^{n,d}$. To do so, we first calculate the partial derivative of the bank's balance sheet constraint (A7) with respect to $\eta_{j,t}$ and $R_{j,t}^{n,d}$:

$$\begin{aligned}\frac{\partial\left(q_t^k s_{j,t}^k\right)}{\partial \eta_{j,t}} &= -\left[1+f'\left(\eta_{j,t}\right)\right], &\text{(A14)}\\ \frac{\partial\left(q_t^k s_{j,t}^k\right)}{\partial R_{j,t}^{n,d}} &= -\epsilon^d\left(R_t^{n,d}\right)^{\epsilon^d}\left(R_{j,t}^{n,d}\right)^{-\epsilon^d-1} d_t, &\text{(A15)}\end{aligned}$$

Now we take the first order conditions with respect to dividends and the nominal deposit rate in the objective (A13) to obtain:

$$\begin{aligned}H_{\eta_{j,t}} &= 1-E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega_{t+1}^b-G^b\left(\bar{\omega}_{j,t+1}^b\right)\right]R_{t+1}^k\right\}\left[1+f'\left(\eta_{j,t}\right)\right], &\text{(A16)}\\ H_{R_{j,t}^{n,d}} &= -\epsilon^d E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega_{t+1}^b-G^b\left(\bar{\omega}_{j,t+1}^b\right)\right]R_{t+1}^k\right\}\left(R_t^{n,d}\right)^{\epsilon^d}\left(R_{j,t}^{n,d}\right)^{-\epsilon^d-1} d_t\\ &- \left(1-\epsilon^d\right)E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}_{j,t+1}^b\right)\right]\frac{1}{\pi_{t+1}}\right\}\left(R_t^{n,d}\right)^{\epsilon^d}\left(R_{j,t}^{n,d}\right)^{-\epsilon^d} d_t, &\text{(A17)}\end{aligned}$$

Setting the above two derivatives equal to zero results in the following first order conditions:

$$\begin{aligned}\eta_{j,t} &: E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega_{t+1}^b-G^b\left(\bar{\omega}_{j,t+1}^b\right)\right]R_{t+1}^k\right\}=\frac{1}{1+f'\left(\eta_{j,t}\right)}, &\text{(A18)}\\ R_{j,t}^{n,d} &: E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega_{t+1}^b-G^b\left(\bar{\omega}_{j,t+1}^b\right)\right]R_{t+1}^k\right\}=\left(\frac{\epsilon^d-1}{\epsilon^d}\right)E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}_{j,t+1}^b\right)\right]\frac{R_{j,t}^{n,d}}{\pi_{t+1}}\right\}. &\text{(A19)}\end{aligned}$$

We can retrieve the above conditions from the first order conditions (33) - (34) and (36). To do so, first observe that a non-binding equity-deposit constraint (26) implies that $\psi_t^d=0$. Then, from the first order condition for dividends (33), we immediately find:

$$\psi_t^b=\frac{1}{1+f'\left(\eta_{j,t}\right)}.$$

Substitution of this expression into the first order condition for corporate securities (36) immediately gives equation (A18). Equation (A19) is retrieved by substituting equation (36) and $\psi_t^d=0$ into the first order condition for the nominal interest rate on deposits (34).

The second derivatives are given by:

$$
\begin{aligned}
H_{\eta_{j,t}\eta_{j,t}} &= E_t\left[\beta\Lambda_{t,t+1}\left(\bar{\omega}^b_{j,t+1}\right)^2\cdot f^b\left(\bar{\omega}^b_{j,t+1}\right)R^k_{t+1}\right]\cdot\frac{\left[1+f'\left(\eta_{j,t}\right)\right]^2}{q^k_t s^k_{j,t}} \\
&- E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1}-G^b\left(\bar{\omega}^b_{j,t+1}\right)\right]R^k_{t+1}\right\}f''\left(\eta_{j,t}\right), \qquad \text{(A20)}\\
H_{\eta_{j,t}R^{n,d}_{j,t}} &= E_t\left[\beta\Lambda_{t,t+1}\bar{\omega}^b_{j,t+1}\cdot f^b\left(\bar{\omega}^b_{j,t+1}\right)\cdot\frac{\partial\bar{\omega}^b_{j,t+1}}{\partial R^{n,d}_{j,t}}\cdot R^k_{t+1}\right]\cdot\left[1+f'\left(\eta_{j,t}\right)\right], \qquad \text{(A21)}\\
H_{R^{n,d}_{j,t}R^{n,d}_{j,t}} &= \epsilon^d E_t\left[\beta\Lambda_{t,t+1}\bar{\omega}^b_{j,t+1}\cdot f^b\left(\bar{\omega}^b_{j,t+1}\right)\cdot\frac{\partial\bar{\omega}^b_{j,t+1}}{\partial R^{n,d}_{j,t}}\cdot R^k_{t+1}\right]\left(R^{n,d}_t\right)^{\epsilon^d}\left(R^{n,d}_{j,t}\right)^{-\epsilon^d-1}d_t\\
&+ \epsilon^d\left(\epsilon^d+1\right)E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1}-G^b\left(\bar{\omega}^b_{j,t+1}\right)\right]R^k_{t+1}\right\}\left(R^{n,d}_t\right)^{\epsilon^d}\left(R^{n,d}_{j,t}\right)^{-\epsilon^d-2}d_t\\
&+ \left(1-\epsilon^d\right)E_t\left[\beta\Lambda_{t,t+1}\cdot f^b\left(\bar{\omega}^b_{j,t+1}\right)\cdot\frac{\partial\bar{\omega}^b_{j,t+1}}{\partial R^{n,d}_{j,t}}\cdot\frac{1}{\pi_{t+1}}\right]\left(R^{n,d}_t\right)^{\epsilon^d}\left(R^{n,d}_{j,t}\right)^{-\epsilon^d}d_t\\
&+ \epsilon^d\left(1-\epsilon^d\right)E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\frac{1}{\pi_{t+1}}\right\}\left(R^{n,d}_t\right)^{\epsilon^d}\left(R^{n,d}_{j,t}\right)^{-\epsilon^d-1}d_t, \qquad \text{(A22)}
\end{aligned}
$$

where $\frac{\partial\bar{\omega}^b_{j,t}}{\partial R^{n,d}_{j,t}}$ is given by:

$$
\frac{\partial\bar{\omega}^b_{j,t}}{\partial R^{n,d}_{j,t}}=\frac{\bar{\omega}^b_{j,t+1}}{R^{n,d}_{j,t}}\left[1-\epsilon^d+\epsilon^d\frac{\left(R^{n,d}_t\right)^{\epsilon^d}\left(R^{n,d}_{j,t}\right)^{-\epsilon^d}d_t}{q^k_t s^k_{j,t}}\right].
$$

A4 Details of the solution procedure

To construct the grid for the evaluation of bank j's objective function (29), we first observe from the equity-deposit constraint (22) that bank j's equity cannot be negative, i.e. $\bar{e}_j \geq 0$. Therefore, we can find the maximum dividends $\bar{\eta}^{max}_j$ for bank j on our grid by setting $\bar{e}_j = 0$ in equation (21), after which we solve for $\bar{\eta}^{max}_j$. Note that when constructing the grid, we assume that net worth $\bar{n}^b$ is equal to aggregate net worth under the interior solution for which the equity-deposit constraint (22) is not binding. Afterwards, we find the maximum number of deposits possible on the grid through the equity-deposit constraint (22): $\bar{d}^{max}_j = \bar{e}^{max}_j/\bar{\kappa}$. Next, we calculate the maximum number of corporate securities by calculating $\bar{s}^{k,max}_j = \bar{e}^{max}_j + \bar{d}^{max}_j$. Afterwards, we construct a two-dimensional grid with dividends $\bar{\eta}_j$ of bank j on one axis, and corporate securities $\bar{s}^k_j$ of bank j on the other, with the aggregate banking variables at the interior solution.[4] The resulting surfplot of an individual bank's objective function (31) can be found in Figure A15.

[4] Observe that we can use banks' balance sheet constraint (25) to solve for $R^{n,d}_{j,t}$, after which bank j effectively has two decision variables left, namely dividends $\eta_{j,t}$ and corporate securities $s^k_{j,t}$. Therefore, it suffices to create a two-dimensional grid with dividends $\bar{\eta}_j$ and corporate securities $\bar{s}^k_j$.

Surfplot of bank j's objective function

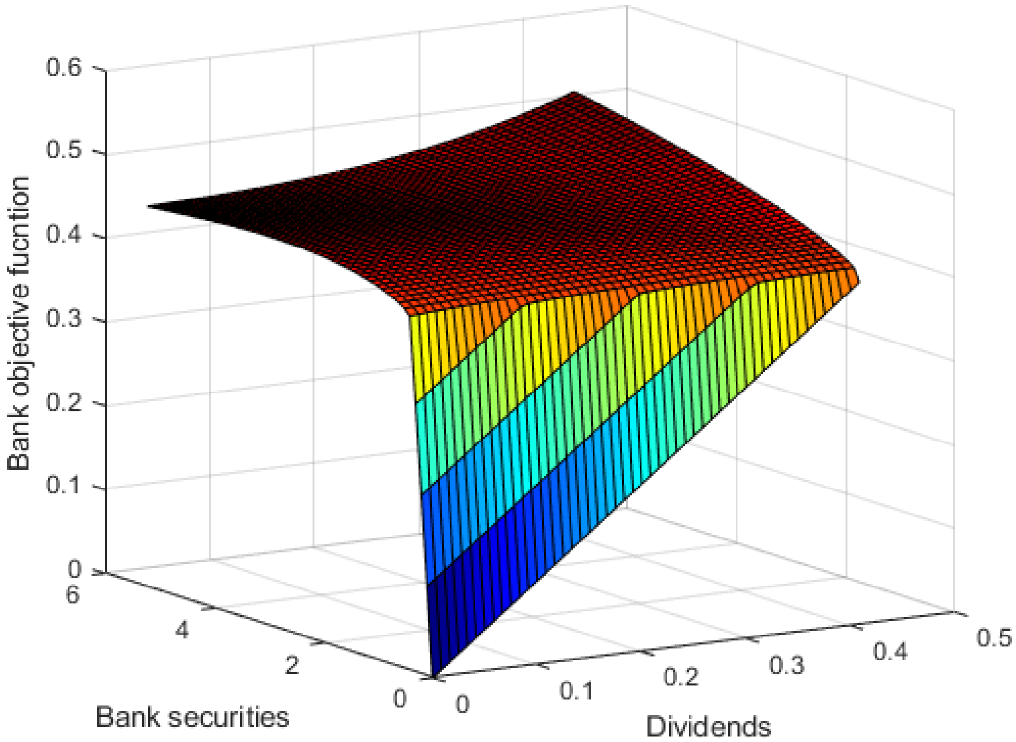

Figure A15: Surfplot of an individual bank's objective function (31) with bank j's dividends $\bar{\eta}_j$ and corporate securities $\bar{s}_j^k$ on the horizontal axes, and the objective function on the vertical axis (31) for $\sigma^b = 0.0480$, the standard deviation of the idiosyncratic shock in the baseline calibration.

A5 Unlimited liability

When there is unlimited liability, bankers will have to repay depositors from their own pockets in case the bank suffers losses. Hence bankers' profits in period $t+1$, conditional on the realization of $\omega^b_{j,t+1}$, are given by the same expression (A23) as under limited liability:

$$\Pi^b_{t+1}\left(\omega^b_{j,t+1}\right) = \omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t. \tag{A23}$$

Because we know the distribution $F^b\left(\omega^b_{j,t+1}\right)$, we can calculate the expected aggregate profit $\Pi^{b,ULL}_{j,t+1}$ of bank $j \in [0,1]$ conditional on the realization of the aggregate return on securities R^k_{t+1} and the inflation rate π_{t+1}:

$$\begin{aligned}
\Pi^{b,ULL}_{j,t+1} &= \int_0^\infty \left[\omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t\right] f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1} \\
&= \int_0^\infty \omega^b_{j,t+1} f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1} R^k_{t+1} q^k_t s^k_{j,t} - \int_0^\infty f^b\left(\omega^b_{j,t+1}\right) d\omega^b_{j,t+1} \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t \\
&= \Omega^b_{t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t, \qquad \text{(A24)}
\end{aligned}$$

Just as in the main text, bank j's optimization problem is given by the maximization of the sum of today's dividends and expected (discounted) profits (A24), subject to the balance sheet constraint (25), and the equity-deposit constraint (26). To find bank j's optimal choices, we set up the Lagrangian:

$$\begin{aligned}
\mathcal{L} = \; & \eta_{j,t} + E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1} R^k_{t+1} q^k_t s^k_{j,t} - \frac{1}{\pi_{t+1}} (R^{n,d}_t)^{\epsilon^d} (R^{n,d}_{j,t})^{1-\epsilon^d} d_t\right]\right\} \\
& + \psi^b_t \left\{ n^b_{j,t} + \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t}\right)^{-\epsilon^d} d_t - q^k_t s^k_{j,t} - \eta_{j,t} - f\left(\eta_{j,t}\right) \right\} \\
& + \psi^d_t \left\{ n^b_{j,t} - \eta_{j,t} - f\left(\eta_{j,t}\right) - \kappa_t \left(\frac{R^{n,d}_{j,t}}{R^{n,d}_t}\right)^{-\epsilon^d} d_t \right\},
\end{aligned}$$

where ψ^b_t is the Lagrangian multiplier on bank j's balance sheet constraint (25), and ψ^d_t the Lagrangian multiplier on bank j's equity-deposit constraint (26). This generates the following first order conditions:

$$s^k_{j,t} \; : \; \psi^b_t = E_t\left[\beta\Lambda_{t,t+1}\Omega^b_{t+1} R^k_{t+1}\right], \tag{A25}$$

$$\eta_{j,t} \; : \; 1 = \left(\psi^b_t + \psi^d_t\right)\left[1 + \kappa_\eta\left(\eta_{j,t} - \hat{\eta}\right)\right], \tag{A26}$$

$$R^{n,d}_{j,t} \; : \; \psi^b_t - \kappa_t \psi^d_t = \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) E_t\left[\beta\Lambda_{t,t+1}\frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right], \tag{A27}$$

together with the first order condition for the occasionally binding equity-deposit constraint:

$$\psi_t^d \left(n_{j,t}^b - \eta_{j,t} - f\left(\eta_{j,t}\right) - \kappa_t \left(\frac{R_{j,t}^{n,d}}{R_t^{n,d}} \right)^{-\epsilon^d} d_t \right) = 0. \tag{A28}$$

First, observe that the first order condition for dividends (A26) is the same as for the model with limited liability (33).

Second, since ψ_t^b and ψ_t^d are the same for all banks, we see from the first order condition for the nominal interest rate on deposits (A27) that all banks will choose the same nominal interest rate on deposits $R_{j,t}^{n,d}$ in equilibrium. In addition, we know that there is zero default risk for households in case of unlimited liability, as a result of which the nominal interest rate on deposits will be equal to the nominal interest rate on the risk-free asset, see equations (3) and (4). Therefore, we know that $E_t\left[\beta\Lambda_{t,t+1}\frac{R_{j,t}^{n,d}}{\pi_{t+1}}\right] = 1$, as a result of which the first order condition for the nominal interest rate on deposits (A27) collapses to:

$$\psi_t^b - \kappa_t \psi_t^d = \frac{\epsilon^d - 1}{\epsilon^d}. \tag{A29}$$

Next, we distinguish two cases. The first is one in which the equity-deposit constraint (26) does not bind, i.e. $\psi_t^d = 0$, and the second is the case where it binds, i.e. $\psi_t^d > 0$. Let us first consider the case where constraint (26) does not bind. In that case, we can write the first order condition for deposits (A29) in the following way by substituting $\psi_t^d = 0$:

$$\psi_t^b = \frac{\epsilon^d - 1}{\epsilon^d}. \tag{A30}$$

When the equity-deposit constraint (26) is binding, we solve for ψ_t^d from the first order condition for dividends (A26) to find:

$$\psi_t^d = \frac{1}{1 + \kappa_\eta\left(\eta_{j,t} - \hat{\eta}\right)} - \psi_t^b. \tag{A31}$$

Substitution of this expression for ψ_t^d into the first order condition for the nominal interest rate on deposits (A29) gives the following expression for the shadow value of intermediaries' balance sheet constraint (25):

$$\psi_t^b = \left(\frac{\kappa_t}{1+\kappa_t}\right) \frac{1}{1 + \kappa_\eta\left(\eta_{j,t} - \hat{\eta}\right)} + \frac{1}{1+\kappa_t}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right). \tag{A32}$$

A6 The role of limited liability in banks' lending decisions when $\psi_t^d = 0$

In this subsection, we show that the results from Section 2.6 regarding the role of limited liability carry over to the case where the equity-deposit constraint (26) is not binding, i.e. $\psi_t^d = 0$. In contrast to Section 2.6, we do not need to assume that dividend adjustment costs are zero to obtain analytical results.

Before we do so, let us first observe that the first order conditions for corporate securities under limited liability (32) and unlimited liability (A25) do not depend on ψ_t^d, and are therefore the same irrespective of whether the equity-deposit constraint (26) is binding or not. Therefore, the expressions (45) and (36) in Section 2.6 are the same when $\psi_t^d = 0$.

Therefore, we only need to derive the equivalent expressions for expressions (44) and (43). Let us first derive the equivalent expression for the first order condition for the nominal interest rate on deposits under limited liability, expression (44). To do so, we set $\psi_t^d = 0$ in equation (34), and solve for the shadow value of intermediaries' balance sheet constraint under limited liability $\psi_t^{b,LL}$ to get:

$$\psi_t^{b,LL} = \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \frac{E_t\left\{\beta \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \left[1 - F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}}{E_t\left\{\beta \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \left[1 - \gamma F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}}, \tag{A33}$$

where we substituted households' first order condition for deposits (4). We already derived the equivalent first order condition for the nominal interest rate on deposits under unlimited liability, see equation (A30).

Just as in the main text, we see that $\psi_t^{b,LL} < \psi_t^{b,ULL}$ when $0 \leq \gamma < 1$, since $E_t\left\{\beta \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \left[1 - \gamma F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\} > E_t\left\{\beta \frac{\Lambda_{t,t+1}}{\pi_{t+1}} \left[1 - F^b\left(\bar{\omega}_{t+1}^b\right)\right]\right\}$. Therefore, the intuition of the main text carries over to the case where the equity-deposit constraint (26) is not binding.

A7 Proofs of the Propositions in the main text

Proof of Proposition 2

Proof. We start by employing households' first order condition for deposits (50) to write equation (47) (with $\kappa_\eta = 0$), in the following way:

$$\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k = \frac{\bar{\kappa}}{1+\bar{\kappa}} + \frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\beta\left[1 - F^b\left(\bar{\omega}^b\right)\right]\bar{R}^d, \tag{A34}$$

Implicit differentiation with respect to σ^b gives the following expression:

$$\begin{aligned} -\beta\bar{R}^k \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} + \beta\left[1 - G^b\left(\bar{\omega}^b\right)\right] \cdot \frac{d\bar{R}^k}{d\sigma^b} \quad &= \quad -\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\beta\bar{R}^d \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} \\ &+ \quad \frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right)\beta\left[1 - F^b\left(\bar{\omega}^b\right)\right] \cdot \frac{d\bar{R}^d}{d\sigma^b}, \end{aligned}$$

Division of the left and right hand side by $\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k$ allows us to write this equation in the following way:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\sigma^b}=\frac{1}{\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k}\left[\beta\bar{R}^k\cdot\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b}-\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\bar{R}^d\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}\right.$$
$$\left.+\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{d\bar{R}^d}{d\sigma^b}\right]. \tag{A35}$$

Next, we use Lemma 3, which provides a relation between $\frac{dF^b(\bar{\omega}^b)}{d\sigma^b}$ and $\frac{dG^b(\bar{\omega}^b)}{d\sigma^b}$.

Lemma 3. *The following relation exists between* $\frac{dF^b(\bar{\omega}^b)}{d\sigma^b}$ *and* $\frac{dG^b(\bar{\omega}^b)}{d\sigma^b}$*:*

$$\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b}=\bar{\omega}^b\left[\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}-\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\right].$$

Proof. Let us start from expression (55):

$$\begin{aligned}\frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} &= \Phi'\left(\frac{\log\left(\bar{\omega}^b\right)-\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\cdot\frac{\frac{\sigma^b}{\bar{\omega}^b}\cdot\frac{d\bar{\omega}^b}{d\sigma^b}-\frac{1}{2}\left(\sigma^b\right)^2-\log\left(\bar{\omega}^b\right)}{\left(\sigma^b\right)^2}\\ &= \bar{\omega}^b\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\cdot\frac{\frac{\sigma^b}{\bar{\omega}^b}\cdot\frac{d\bar{\omega}^b}{d\sigma^b}+\frac{1}{2}\left(\sigma^b\right)^2-\log\left(\bar{\omega}^b\right)-\left(\sigma^b\right)^2}{\left(\sigma^b\right)^2}\\ &= \bar{\omega}^b\left[\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}-\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\right],\end{aligned} \tag{A36}$$

where we used expression (54) in going from the second to the third line. □

Substitution of Lemma 3 into expression (A35) allows us to write:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\sigma^b}=\frac{1}{\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k}\left\{\beta\left[\bar{R}^k\bar{\omega}^b-\frac{\bar{R}^d}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\right]\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}\right.$$
$$-\beta\bar{R}^k\bar{\omega}^b\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)$$
$$\left.+\frac{\beta\bar{R}^d}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\sigma^b}\right\}. \tag{A37}$$

Now we use Lemma 4 to rewrite $\frac{\bar{R}^d}{1+\bar{\kappa}}$:

Lemma 4. *We can write* $\frac{\bar{R}^d}{1+\bar{\kappa}}$ *in the following way:*

$$\frac{\bar{R}^d}{1+\bar{\kappa}}=\bar{R}^k\bar{\omega}^b.$$

Proof. We can infer from equation (28) that $\bar{R}^k \bar{\omega}^b = \frac{\bar{R}^d \bar{d}}{\bar{q}^k \bar{s}^k}$. Next, we substitute equation (22) into banks' balance sheet constraint (25) to write $\frac{\bar{d}}{\bar{q}^k \bar{s}^k} = \frac{1}{1+\bar{\kappa}}$. Substitution of the second expression into the first concludes the proof. □

Substitution of the expression for $\frac{\bar{R}^d}{1+\bar{\kappa}}$ from Lemma 4 allows us to rewrite equation (A37) in the following way:

$$\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b} = \frac{1}{\beta \left[1 - G^b\left(\bar{\omega}^b\right)\right] \bar{R}^k} \left\{ \beta \bar{R}^k \bar{\omega}^b \frac{1}{\epsilon^d} \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \beta \bar{R}^k \bar{\omega}^b \Phi' \left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b} \right) + \beta \bar{R}^k \bar{\omega}^b \left(\frac{\epsilon^d - 1}{\epsilon^d} \right) \left[1 - F^b\left(\bar{\omega}^b\right)\right] \cdot \frac{1}{\bar{R}^d} \cdot \frac{d\bar{R}^d}{d\sigma^b} \right\}.$$

Taking $\beta \bar{R}^k \bar{\omega}^b$ outside the brackets, we can write the above expression as:

$$\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b} = \frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)} \left\{ \frac{1}{\epsilon^d} \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \Phi' \left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b} \right) + \left(\frac{\epsilon^d - 1}{\epsilon^d} \right) \left[1 - F^b\left(\bar{\omega}^b\right)\right] \cdot \frac{1}{\bar{R}^d} \cdot \frac{d\bar{R}^d}{d\sigma^b} \right\}. \quad \text{(A38)}$$

Now, when there is full deposit insurance, we know that $\gamma = 0$, after which we can infer from households' first order condition (50) that $\bar{R}^d = 1/\beta$. Therefore, under full deposit insurance we have that $\frac{d\bar{R}^d}{d\sigma^b} = 0$. Substitution of $\frac{d\bar{R}^d}{d\sigma^b} = 0$ gives the expression in the proof of Proposition 2. □

Now that we have derived the expression from Proposition 2, we are ready to prove Proposition 3.

Proof of Proposition 3

Proof. First substitute $\gamma = 1$ into equation (47), after which we implicitly differentiate with respect to σ^b:

$$-\beta \bar{R}^k \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} + \beta \left[1 - G^b\left(\bar{\omega}^b\right)\right] \cdot \frac{d\bar{R}^k}{d\sigma^b} = 0.$$

Division by $\beta \left[1 - G^b\left(\bar{\omega}^b\right)\right] \bar{R}^k$ immediately leads to the desired expression. □

Proof of Corollary 1

Proof. We can write the steady state version of the aggregate resource constraint (42) in the following way:

$$\bar{c} = \bar{y} - \bar{i} - \left(\frac{\bar{g}}{\bar{y}}\right) \bar{y} - \mu^{dia} G^b\left(\bar{\omega}^b\right) \bar{R}^k \bar{k}, \quad \text{(A39)}$$

where we used the fact that $\bar{q}^k = 1$ and $\bar{s}^k = \bar{k}$. Next, we implicitly differentiate the above expression with respect to σ^b. Before doing so, however, remember that $\bar{y} = \bar{z}\bar{k}^{\alpha}\bar{h}^{1-\alpha}$ and $\bar{i} = \delta\bar{k}$. Remembering that $\bar{z} = 1$ and that labor $\bar{h}$ is inelastically supplied by households, we can immediately write that $\frac{d\bar{y}}{d\sigma^b} = \alpha\bar{y} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b}$ and $\frac{d\bar{i}}{d\sigma^b} = \delta\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b}$.

Finally, using equation (51), we can immediately write that $\frac{d\bar{R}^k}{d\sigma^b} = (\alpha - 1)\,\alpha\bar{m}c\bar{z}\bar{k}^{\alpha-1}\bar{h}^{1-\alpha} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} = (\alpha - 1)\left[\bar{R}^k - (1-\delta)\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b}$. Therefore, we can write that

$$\begin{aligned}\frac{d\left(\bar{R}^k\bar{k}\right)}{d\sigma^b} &= \bar{k}\,(\alpha-1)\left[\bar{R}^k - (1-\delta)\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} + \bar{R}^k\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} \\ &= \left[\alpha\bar{R}^k\bar{k} + (1-\alpha)(1-\delta)\,\bar{k}\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b}.\end{aligned}$$

Now we are ready to differentiate equation (A39) and substitute the above expressions for $\frac{d\bar{y}}{d\sigma^b}$, $\frac{d\bar{i}}{d\sigma^b}$, and $\frac{d\left(\bar{R}^k\bar{k}\right)}{d\sigma^b}$ to get:

$$\begin{aligned}\frac{d\bar{c}}{d\sigma^b} &= \left(1 - \frac{\bar{g}}{\bar{y}}\right)\alpha\bar{y} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \delta\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \mu^{dia}G^b\left(\bar{\omega}^b\right)\left[\alpha\bar{R}^k\bar{k} + (1-\alpha)(1-\delta)\,\bar{k}\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} \\ &- \mu^{dia}\bar{R}^k\bar{k} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} \\ &= \left(1 - \frac{\bar{g}}{\bar{y}}\right)\alpha\bar{y} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \alpha\delta\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - (1-\alpha)\,\delta\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} \\ &- \alpha\mu^{dia}G^b\left(\bar{\omega}^b\right)\bar{R}^k\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - (1-\alpha)\,\mu^{dia}G^b\left(\bar{\omega}^b\right)(1-\delta)\,\bar{k} \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \mu^{dia}\bar{R}^k\bar{k} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} \\ &= \alpha\left[\left(1 - \frac{\bar{g}}{\bar{y}}\right)\bar{y} - \delta\bar{k} - \mu^{dia}G^b\left(\bar{\omega}^b\right)\bar{R}^k\bar{k}\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} \\ &- \left[(1-\alpha)\,\delta\bar{k} + (1-\alpha)\,\mu^{dia}G^b\left(\bar{\omega}^b\right)(1-\delta)\,\bar{k}\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \mu^{dia}\bar{R}^k\bar{k} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} \\ &= \left[\alpha\bar{c} - (1-\alpha)\,\bar{i} - (1-\alpha)\,\mu^{dia}G^b\left(\bar{\omega}^b\right)(1-\delta)\,\bar{k}\right] \cdot \frac{1}{\bar{k}} \cdot \frac{d\bar{k}}{d\sigma^b} - \mu^{dia}\bar{R}^k\bar{k} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b},\end{aligned}$$

where we used that $\bar{i} = \delta\bar{k}$, and see that the above expression coincides with that in Proposition 1. □

Proof of Proposition 4

Proof. We start by rewriting equation (50) in the following way:

$$\bar{R}^d = \frac{1}{\beta\left[1 - \gamma F^b\left(\bar{\omega}^b\right)\right]}.$$

Implicit differentiation with respect to γ gives the following expression:

$$\begin{aligned}
\frac{d\bar{R}^d}{d\gamma} &= \frac{-1}{\beta\left[1-\gamma F^b\left(\bar{\omega}^b\right)\right]^2}\left[-F^b\left(\bar{\omega}^b\right)-\gamma\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}\right] \\
&= \frac{\bar{R}^d}{1-\gamma F^b\left(\bar{\omega}^b\right)}\left[F^b\left(\bar{\omega}^b\right)+\gamma\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}\right].
\end{aligned}$$

Division on the left and right hand side by $\bar{R}^d$ gives the expression of Proposition 4. □

Proof of Corollary 2

Proof. Let us start by reminding ourselves that the probability of default is given by the following expression:

$$F^b\left(\bar{\omega}^b\right)=\Phi\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right),$$

where $\Phi(...)$ denotes the cumulative density function of the standard normal distribution. Therefore, we can write the change in the probability of default in the following way:

$$\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}=\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\cdot\frac{1}{\sigma^b\bar{\omega}^b}\cdot\frac{d\bar{\omega}^b}{d\gamma}.$$

Now we remember that $\bar{\omega}^b \equiv \frac{\bar{R}^d}{\bar{R}^k}\bar{x}^b$. Implicit differentiation with respect to γ gives:

$$\begin{aligned}
\frac{d\bar{\omega}^b}{d\gamma} &= \frac{1}{\bar{R}^k}\bar{x}^b\cdot\frac{d\bar{R}^d}{d\gamma}+\frac{\bar{R}^d}{\bar{R}^k}\cdot\frac{d\bar{x}^b}{d\gamma}-\frac{\bar{R}^d}{\bar{R}^k}\bar{x}^b\cdot\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma} \\
&= \bar{\omega}^b\left[\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}-\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}\right],
\end{aligned}$$

where we remember that $\frac{d\bar{x}^b}{d\gamma}=0$ when moving from the first to the second line, since $\bar{x}^b = 1/\left(1+\bar{\kappa}\right)$ when the equity-deposit constraint (26) is binding. Substitution of this expression in the expression for $\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}$ gives the following expression:

$$\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}=\frac{1}{\sigma^b}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right)+\frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\left[\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}-\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}\right]. \tag{A40}$$

Next, we substitute expression (61) to calculate $\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}-\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}$, and obtain the following equation:

$$\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}-\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}=B\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}-\frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}\cdot\frac{1}{\epsilon^d}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}, \tag{A41}$$

where B is given by:

$$B = 1 - \frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \left[1 - F^b\left(\bar{\omega}^b\right)\right], \tag{A42}$$

Substitution of equation (A41) into expression (A40), and rearranging immediately gives the expression of Corollary 2:

$$\left[1 + \frac{1}{\sigma^b \epsilon^d} \cdot \frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)} \Phi'\left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\right] \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} = \frac{1}{\sigma^b} \Phi'\left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right) B \cdot \frac{1}{\bar{R}^d} \cdot \frac{d\bar{R}^d}{d\gamma}.$$

Finally, we need to prove that $B > 0$. To do so, we employ banks' first order condition for corporate securities (47) with $\kappa_\eta = 0$, which we can rewrite with the help of households' first order condition for deposits (50) into the following form:

$$\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right] \bar{R}^k = \frac{\bar{\kappa}}{1 + \bar{\kappa}} + \frac{1}{1 + \bar{\kappa}} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \beta\left[1 - F^b\left(\bar{\omega}^b\right)\right] \bar{R}^d. \tag{A43}$$

Division of left and right hand side by $\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]$ gives the following expression:

$$\bar{R}^k = \frac{1}{\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]} \cdot \frac{\bar{\kappa}}{1 + \bar{\kappa}} + \frac{1}{1 - G^b\left(\bar{\omega}^b\right)} \cdot \frac{1}{1 + \bar{\kappa}} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \left[1 - F^b\left(\bar{\omega}^b\right)\right] \bar{R}^d.$$

Next, we employ Lemma 4 to substitute $\frac{\bar{R}^d}{1+\bar{\kappa}} = \bar{R}^k \bar{\omega}^b$ in the second term on the right hand side, after which we shift this term to the left hand side to get:

$$\underbrace{\left\{1 - \frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \left[1 - F^b\left(\bar{\omega}^b\right)\right]\right\}}_{=B} \bar{R}^k = \frac{1}{\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]} \cdot \frac{\bar{\kappa}}{1 + \bar{\kappa}} > 0.$$

Since the right hand side is larger than zero, we immediately conclude that the term between square brackets, which is equal to B, must be larger than zero. □

Proof of Corollary 3

Proof. Implicit differentiation of banks' first order condition for corporate securities (A43) with respect to γ gives the following expression:

$$\begin{aligned} \frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\gamma} &= \frac{1}{\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right] \bar{R}^k} \left\{\beta \bar{R}^k \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\gamma} - \frac{1}{1 + \bar{\kappa}} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \beta \bar{R}^d \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} \right. \\ &+ \left. \frac{1}{1 + \bar{\kappa}} \left(\frac{\epsilon^d - 1}{\epsilon^d}\right) \beta\left[1 - F^b\left(\bar{\omega}^b\right)\right] \cdot \frac{d\bar{R}^d}{d\gamma} \right\}. \end{aligned}$$

Next, we use equation (A1) to write $\frac{dG^b\left(\bar{\omega}^b\right)}{d\gamma} = \bar{\omega}^b \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}$, and subsitute this expression to obtain:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma} = \frac{1}{\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k}\left\{\beta\left[\bar{R}^k\bar{\omega}^b-\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\bar{R}^d\right]\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} + \frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{d\bar{R}^d}{d\gamma}\right\}. \quad \text{(A44)}$$

Now we use Lemma 4 to rewrite the expression between square brackets:

$$\bar{R}^k\bar{\omega}^b-\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\bar{R}^d = \frac{\bar{R}^d}{1+\bar{\kappa}}-\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\bar{R}^d = \frac{1}{\epsilon^d}\cdot\frac{\bar{R}^d}{1+\bar{\kappa}}<0,$$

since $\epsilon^d < -1$. Substitution of this expression gives the following expression for the change in the return on corporate securities as a result of a change in the degree of deposit insurance γ:

$$\begin{aligned}\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma} &= \frac{1}{\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k}\left\{\frac{1}{\epsilon^d}\cdot\frac{\beta\bar{R}^d}{1+\bar{\kappa}}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}+\frac{1}{1+\bar{\kappa}}\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\left[1-F^b\left(\bar{\omega}^b\right)\right]\bar{R}^d\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}\right\}.\\ &= \left(\frac{\bar{R}^d}{\bar{R}^k\left(1+\bar{\kappa}\right)}\right)\frac{1}{1-G^b\left(\bar{\omega}^b\right)}\left\{\frac{1}{\epsilon^d}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}+\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}\right\}.\end{aligned} \quad \text{(A45)}$$

Finally, we employ Lemma 4 to replace $\frac{\bar{R}^d}{\bar{R}^k}\cdot\frac{1}{1+\bar{\kappa}}$ in equation (A45) by $\bar{\omega}^b$ to obtain:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma} = \frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}\left\{\frac{1}{\epsilon^d}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}+\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}\right\},$$

which coincides with expression (61) in Corollary 3. □

A8 Additional propositions

We start by proving that the interest rate on deposits increases with the probability of default when $\gamma > 0$:

Proposition 5. *For $\gamma > 0$, the interest rate on deposits $\bar{R}^d$ increases as a result of an increase the probability of default:*

$$\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\sigma^b} = \frac{\gamma}{1-\gamma F^b\left(\bar{\omega}^b\right)}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b}. \quad \text{(A46)}$$

Proof. Implicit differentiation of equation (50) immediately results in the above expression. □

Proposition 6. *For $\gamma > 0$, it is unclear whether the level of credit provision under limited* *liability is larger or smaller than credit provision under unlimited liability:*

$$\frac{d\bar{k}}{d\sigma^b} \lesseqgtr 0.$$

Proof. Looking at equation (A38), we see that the first two terms coincide with the expression for $\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b}$ from the proof of Proposition 2, and are therefore negative. However, we now have a third term, namely the change in the return on deposits, which will be increasing in σ^b as a result of a higher probability of default, see Proposition 5. Therefore, the sign of $\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b}$ is ambiguous, as a result of which the sign of $\frac{d\bar{k}}{d\sigma^b}$ will also be ambiguous. □

A9 Propositions and proofs for the case $\psi_t^d = 0$

In the main text, we discussed propositions for which we assumed the equity-deposit constraint (26) to be binding, i.e. $\psi_t^d > 0$. It turns out, however, that most propositions continue to hold for the case where the equity-deposit constraint is nog binding, i.e $\psi_t^d = 0$. In this section, we will provide the proofs of these propositions for the case where the equity-deposit constraint is not binding. We start by writing the counterparts to equations (47) - (49) for the case where the equity-deposit constraint is not binding.

$$\beta \left[1 - G^b\left(\bar{\omega}^b\right)\right] \bar{R}^k = \left(\frac{\epsilon_d - 1}{\epsilon_d}\right) \frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)}, \quad \text{(A47)}$$

$$\bar{\psi} = \frac{1}{1 + \kappa_\eta\left(\bar{\eta} - \hat{\eta}\right)}, \quad \text{(A48)}$$

$$\bar{\psi} = \left(\frac{\epsilon_d - 1}{\epsilon_d}\right) \frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - \gamma F^b\left(\bar{\omega}^b\right)}, \quad \text{(A49)}$$

(A50)

Proof of Proposition 1 for $\psi_t^d = 0$

Proof. We start by writing the expression for the return on corporate securities under unlimited liability with a equity-deposit constraint that is not binding, i.e. $\psi_t^d = 0$. To do so, we combine equations (A25) and (A30) to find that the steady state return on corporate securities under unlimited liability is given by:

$$\bar{R}^k\Big|^{ULL} = \frac{1}{\beta}\left(\frac{\epsilon^d - 1}{\epsilon^d}\right) > 1, \quad \text{(A51)}$$

since $\epsilon^d < -1$.

Next, we set $\psi_t^d = 0$ in the first order condition for the nominal interest rate on deposits (34), solve for ψ_t^b, and substitute the resulting expression into the first order condition for corporate securities (36) to obtain:

$$E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1}-G^b\left(\bar{\omega}^b_{j,t+1}\right)\right]R^k_{t+1}\right\}=\left(\frac{\epsilon^d-1}{\epsilon^d}\right)E_t\left\{\beta\Lambda_{t,t+1}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\frac{R^{n,d}_{j,t}}{\pi_{t+1}}\right\} \quad \text{(A52)}$$

Next, we substitute $\Delta^b_t = F^b\left(\bar{\omega}^b_t\right)$ into households' first order condition for deposits (4), solve for the nominal interest rate on deposits, and substitute the resulting expression into equation (A52) to get the following expression:

$$E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega^b_{t+1}-G^b\left(\bar{\omega}^b_{j,t+1}\right)\right]R^k_{t+1}\right\}=\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\frac{E_t\left\{\frac{\beta\Lambda_{t,t+1}}{\pi_{t+1}}\left[1-F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\right\}}{E_t\left\{\frac{\beta\Lambda_{t,t+1}}{\pi_{t+1}}\left[1-\gamma F^b\left(\bar{\omega}^b_{j,t+1}\right)\right]\right\}}, \quad \text{(A53)}$$

which is in steady state given by:

$$\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k=\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\frac{1-F^b\left(\bar{\omega}^b\right)}{1-\gamma F^b\left(\bar{\omega}^b\right)}, \quad \text{(A54)}$$

Now we are in the position to write down the steady state return on corporate securities under limited liability:

$$\bar{R}^k\Big|^{LL}=\frac{1-F^b\left(\bar{\omega}^b\right)}{1-\gamma F^b\left(\bar{\omega}^b\right)}\cdot\frac{1}{1-G^b\left(\bar{\omega}^b\right)}\cdot\bar{R}^k\Big|^{ULL}, \quad \text{(A55)}$$

where we employed equation (A51).

Now we are ready to prove Lemma 1, which states that in the absence of deposit insurance ($\gamma = 1$), the return on capital under limited liability $\bar{R}^k\Big|^{LL}_{\gamma=1}$ is always larger than or equal to the return on capital under unlimited liability $\bar{R}^k\Big|^{ULL}$:

$$\bar{R}^k\Big|^{LL}_{\gamma=1}\geq\bar{R}^k\Big|^{ULL}.$$

Proof. Substitution of $\gamma = 1$ into equation (A55) allows us to write the return on capital under limited liability and full deposit insurance $\bar{R}^k\Big|^{LL}_{\gamma=1}$ as:

$$\bar{R}^k\Big|^{LL}_{\gamma=0}=\frac{1}{1-G^b\left(\bar{\omega}^b\right)}\cdot\bar{R}^k\Big|^{ULL}\geq\bar{R}^k\Big|^{ULL},$$

since $G^b\left(\bar{\omega}^b\right)\equiv\int_0^{\bar{\omega}^b}\omega f\left(\omega\right)d\omega\leq\int_0^{\infty}\omega f\left(\omega\right)d\omega=1.$ □

Next, we prove Lemma 2, which says that the return on capital under limited liability and full deposit insurance is always less than or equal to the return on capital under unlimited liability:

$$\bar{R}^k\Big|^{LL}_{\gamma=0}\leq\bar{R}^k\Big|^{ULL}.$$

Proof. Substitution of $\gamma = 0$ into equation (A55) allows us to write the return on capital under limited liability and full deposit insurance $\bar{R}^k\Big|_{\gamma=0}^{LL}$ as:

$$\bar{R}^k\Big|_{\gamma=0}^{LL} = \frac{1 - F^b\left(\bar{\omega}^b\right)}{1 - G^b\left(\bar{\omega}^b\right)} \cdot \bar{R}^k\Big|^{ULL} \leq \bar{R}^k\Big|^{ULL},$$

since $G^b\left(\bar{\omega}^b\right) \equiv \int_0^{\bar{\omega}^b} \omega f\left(\omega\right) d\omega \leq \int_0^{\bar{\omega}^b} \bar{\omega}^b f\left(\omega\right) d\omega \equiv \bar{\omega}^b F^b\left(\bar{\omega}^b\right) \leq F^b\left(\bar{\omega}^b\right)$ since $\bar{\omega}^b < 1$. ☐

Now, we can infer from Lemma 1 that $\bar{k}\Big|^{ULL} \geq \bar{k}\Big|_{\gamma=1}^{LL}$, and from Lemma 2 that $\bar{k}\Big|_{\gamma=0}^{LL} \geq \bar{k}\Big|^{ULL}$, since we know from equation (51) that the return on corporate securities and the stock of physical capital are inversely related. Together, these two (in)equalities prove Proposition 1. ☐

Proof of Proposition 2 for $\psi_t^d = 0$

Proof. First substitute $\gamma = 0$ into equation (A54), after which we implicitly differentiate with respect to σ^b:

$$-\beta\bar{R}^k \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} + \beta\left[1 - G^b\left(\bar{\omega}^b\right)\right] \cdot \frac{d\bar{R}^k}{d\sigma^b} = -\left(\frac{\epsilon_d - 1}{\epsilon_d}\right) \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b},$$

Division of the left and right hand side by $\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k$, and then employing equation (A54) on the right hand side allows us to write this equation in the following way:

$$\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b} = \frac{1}{1 - G^b\left(\bar{\omega}^b\right)} \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \frac{1}{1 - F^b\left(\bar{\omega}^b\right)} \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b},$$

Substitution of equation (A36) gives the following expression:

$$\frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\sigma^b} = \underbrace{\left[\frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)} - \frac{1}{1 - F^b\left(\bar{\omega}^b\right)}\right]}_{<0} \cdot \frac{dF^b\left(\bar{\omega}^b\right)}{d\sigma^b} - \frac{\bar{\omega}^b}{1 - G^b\left(\bar{\omega}^b\right)}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right) < 0,$$

where we remember that $\Phi\left(...\right) > 0$ denotes the cumulative density function of the standard normal distribution, and where the term inside the squared brackets is smaller than zero. The reason that the term inside the squared brackets is negative is the following. First, observe that $G^b\left(\bar{\omega}^b\right) \equiv \int_0^{\bar{\omega}^b} \omega^b f^b\left(\omega^b\right) d\omega^b \leq \bar{\omega}^b \int_0^{\bar{\omega}^b} f^b\left(\omega^b\right) d\omega^b = \bar{\omega}^b F^b\left(\bar{\omega}^b\right) \leq F^b\left(\bar{\omega}^b\right)$, where the last step follows from the fact $\bar{\omega}^b \leq 1$. Therefore, we have that $\frac{1}{1-G^b(\bar{\omega}^b)} \leq \frac{1}{1-F^b(\bar{\omega}^b)}$. Next, since $\bar{\omega}^b \leq 1$, it immediately follows that the term inside the squared brackets is negative, and we can conclude that Proposition 2 also holds when $\psi_t^d = 0$. ☐

Proof of Proposition 3 for $\psi_t^d = 0$

Proof. First substitute $\gamma = 1$ into equation (A54), after which we implicitly differentiate with respect to σ^b:

$$-\beta \bar{R}^k \cdot \frac{dG^b\left(\bar{\omega}^b\right)}{d\sigma^b} + \beta\left[1 - G^b\left(\bar{\omega}^b\right)\right] \cdot \frac{d\bar{R}^k}{d\sigma^b} = 0.$$

Division by $\beta\left[1 - G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k$ immediately leads to the desired expression in Proposition 3. □

Proof of Corollary 1 for $\psi_t^d = 0$

The proof for the expression in Corollary 1 does not depend on banks' first order conditions. Therefore, the proof for the case $\psi_t^d = 0$ is the same as for the case $\psi_t^d > 0$, and can be found in Appendix A7.

Proof of Proposition 4 for $\psi_t^d = 0$

The proof of Proposition 4 relies on households' first order condition for deposits (50), and does not depend on whether or not the equity-deposit constraint (26) is binding or not. Therefore, the proof in the main text carries over to the case with $\psi_t^d = 0$.

Proof of Corollary 2

Unlike the case where the equity-deposit constraint (26) is binding, we cannot prove the extistence of the feedback loop between the real interest rate on deposits and the probability of default. However, we can still prove the *possibility* of such a feedback loop to emerge, which we do in Corollary 4:

Corollary 4. *There exists the possibility of a feedback loop between the real interest rate on deposits $\bar{R}^d$ and the probability of default $F^b\left(\bar{\omega}^b\right)$.*

Proof. We already saw in Proposition 4 how the return on deposits is affected by a change in the probability of default. However, remember that the probability of default depends on the cut-off value $\bar{\omega}^b \equiv \frac{\bar{R}^d}{\bar{R}^k} \cdot \bar{x}^b$ where $\bar{x}^b \equiv \bar{d}/\bar{k}$ is the deposits-assets ratio. Therefore, we can write the change in the probability of default as:

$$\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma} = \frac{1}{\sigma^b}\Phi'\left(\frac{\log\left(\bar{\omega}^b\right) + \frac{1}{2}\left(\sigma^b\right)^2}{\sigma^b}\right)\left[\frac{1}{\bar{R}^d} \cdot \frac{d\bar{R}^d}{d\gamma} + \frac{1}{\bar{x}^b} \cdot \frac{d\bar{x}^b}{d\gamma} - \frac{1}{\bar{R}^k} \cdot \frac{d\bar{R}^k}{d\gamma}\right]. \tag{A56}$$

Therefore, we see that an increase in the interest rate on deposits increases the probability of default, everything else equal. The resulting increase in the probability of default then increases the interest rate on deposits via expression (58), which in turn leads to a second round increase in the probability of default. This proves the possibility of a feedback loop between the interest rate on deposits and the probability of default. □

Proof of Corollary 3 for $\psi_t^d = 0$

Proof. Substitution of households' first order condition for deposits (50) into equation (A54) gives the following equation:

$$\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k=\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\left[1-F^b\left(\bar{\omega}^b\right)\right]\bar{R}^d, \tag{A57}$$

Implicit differentiation with respect to γ gives the following expression:

$$-\beta\bar{R}^k\cdot\frac{dG^b\left(\bar{\omega}^b\right)}{d\gamma}+\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\cdot\frac{d\bar{R}^k}{d\gamma}=-\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\bar{R}^d\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}+\left(\frac{\epsilon^d-1}{\epsilon^d}\right)\beta\left[1-F^b\left(\bar{\omega}^b\right)\right]\cdot\frac{d\bar{R}^d}{d\gamma}.$$

We divide by $\beta\left[1-G^b\left(\bar{\omega}^b\right)\right]\bar{R}^k$, and employ equation (A57) to rewrite the above expression as:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}=\frac{1}{1-G^b\left(\bar{\omega}^b\right)}\cdot\frac{dG^b\left(\bar{\omega}^b\right)}{d\gamma}-\frac{1}{1-F^b\left(\bar{\omega}^b\right)}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}+\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}.$$

Using equation (A1), We know that $\frac{dG^b\left(\bar{\omega}^b\right)}{d\gamma}=\bar{\omega}^b\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}$, after which we obtain:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}=\underbrace{\left[\frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}-\frac{1}{1-F^b\left(\bar{\omega}^b\right)}\right]}_{<0}\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}+\frac{1}{\bar{R}^d}\cdot\frac{d\bar{R}^d}{d\gamma}, \tag{A58}$$

where we know that the expression between the squared brackets is negative because of the following. First, observe that $G^b\left(\bar{\omega}^b\right)\equiv\int_0^{\bar{\omega}^b}\omega^b f^b\left(\omega^b\right)d\omega^b\leq\bar{\omega}^b\int_0^{\bar{\omega}^b}f^b\left(\omega^b\right)d\omega^b=\bar{\omega}^bF^b\left(\bar{\omega}^b\right)\leq F^b\left(\bar{\omega}^b\right)$, where the last step follows from the fact $\bar{\omega}^b\leq 1$. Therefore, we have that $\frac{1}{1-G^b\left(\bar{\omega}^b\right)}\leq\frac{1}{1-F^b\left(\bar{\omega}^b\right)}$. Next, since $\bar{\omega}^b\leq 1$, it immediately follows that the term inside the squared brackets is negative.

Finally, we substitute the expression for the change in the return on deposits (58), which provides us with the following expression:

$$\frac{1}{\bar{R}^k}\cdot\frac{d\bar{R}^k}{d\gamma}=\underbrace{\frac{F^b\left(\bar{\omega}^b\right)}{1-\gamma F^b\left(\bar{\omega}^b\right)}}_{\substack{\text{direct effect}\\ \text{from increase in}\bar{R}^d}}+\left[\frac{\bar{\omega}^b}{1-G^b\left(\bar{\omega}^b\right)}-\frac{1}{1-F^b\left(\bar{\omega}^b\right)}+\frac{\gamma}{1-\gamma F^b\left(\bar{\omega}^b\right)}\right]\cdot\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}. \tag{A59}$$

We see that the term inside the squared brackets is negative for $\gamma=0$, while it is positive for $\gamma=1$. Therefore, it is likely that this term will switch from being negative to positive when γ increases. Therefore, for $\frac{dF^b\left(\bar{\omega}^b\right)}{d\gamma}>0$, larger values of γ will lead to a larger increase in the return on corporate securities, everything else equal. ∎

A10 Equilibrium & overview of first order conditions

Households' choice variables $\{c_t,h_t,d_t\}$, producers' choice variables $\{i_t,y_t,k_t\}$, bankers' choice variables $\left\{s_t^k,n_t^b,\bar{\omega}_t^b,x_t^b,\phi_t,\eta_t,e_t\right\}$, government's choice variables $\left\{T_t,T_t^{dia}\right\}$, prices

$\left\{q_t^k, r_t^k, R_t^k, w_t, R_t^D, mc_t, \pi_t, R_t^n, R_t^{n,d}\right\}$, shadow prices $\left\{\lambda_t, \psi_t^b, \psi_t^d\right\}$ exogenous processes $\{z_t, g_t, \kappa_t\}$.

A10.1 Households

$$\lambda_t = u'(c_t) = (c_t - \upsilon c_{t-1})^{-\sigma_c} - \beta\upsilon E_t\left[(c_{t+1} - \upsilon c_t)^{-\sigma_c}\right], \tag{A60}$$

$$\chi h_t^{\varphi} = \lambda_t w_t, \tag{A61}$$

$$1 = E_t\left[\beta\Lambda_{t,t+1}\frac{R_t^n}{\pi_{t+1}}\right], \tag{A62}$$

$$1 = E_t\left[\beta\Lambda_{t,t+1}\left[1 - \gamma F^b\left(\bar{\omega}_t^b\right)\right]\frac{R_t^{n,d}}{\pi_{t+1}}\right], \tag{A63}$$

where $\beta\Lambda_{t,t+s} = \beta\lambda_{t+s}/\lambda_t$ is the households' stochastic discount factor to discount cash flows from period $t+s$ to period t.

A10.2 Producers

$$y_t = z_t k_{t-1}^{\alpha} h_t^{1-\alpha}, \tag{A64}$$

$$r_t^k = \alpha mc_t y_t / k_{t-1}, \tag{A65}$$

$$w_t = (1-\alpha)\, mc_t y_t / h_t. \tag{A66}$$

$$R_t^k = \frac{r_t^k + (1-\delta)\, q_t^k}{q_{t-1}^k}, \tag{A67}$$

$$\kappa_p\left(\pi_t - \pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)\pi_t y_t = (1-\epsilon)\, y_t + \epsilon mc_t y_t + E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\kappa_p\left(\pi_{t+1} - \pi_t^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)\pi_{t+1} y_{t+1}\right]. \tag{A68}$$

$$k_t = (1-\delta)\, k_{t-1} + \left[1 - \frac{\kappa_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right] i_t, \tag{A69}$$

$$\frac{1}{q_t^k} = 1 - \frac{\kappa_k}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \kappa_k\frac{i_t}{i_{t-1}}\left(\frac{i_t}{i_{t-1}} - 1\right) + E_t\left[\beta\frac{\lambda_{t+1}}{\lambda_t}\frac{q_{t+1}^k}{q_t^k}\left(\frac{i_{t+1}}{i_t}\right)^2\kappa_k\left(\frac{i_{t+1}}{i_t} - 1\right)\right] \tag{A70}$$

A10.3 Bankers

$$q_t^k s_t^k + \eta_t + \frac{1}{2}\kappa_\eta\left(\eta_t - \hat{\eta}\right)^2 = n_t^b + d_t, \tag{A71}$$

$$\psi_t^b = E_t\left\{\beta\Lambda_{t,t+1}\left[\Omega_{t+1}^b - G^b\left(\bar{\omega}_{t+1}^b\right)\right]R_{t+1}^k\right\}, \tag{A72}$$

$$\psi_t^b + \psi_t^d = \frac{1}{1+\kappa_\eta\left(\eta_t - \hat{\eta}\right)}, \tag{A73}$$

$$\psi_t^b - \kappa_t\psi_t^d = E_t\left\{\beta\Lambda_{t,t+1}\left[1 - F^b\left(\bar{\omega}_{t+1}^b\right)\right]\frac{R_t^{n,d}}{\pi_{t+1}}\right\}, \tag{A74}$$

$$n_t^b = \theta^b\left[\Omega_t^b - \Gamma^b\left(\bar{\omega}_t^b\right)\right]R_t^k q_{t-1}^k s_{t-1}^k + \chi^b n_{t-1}^b, \tag{A75}$$

$$\bar{\omega}_t^b = \frac{\frac{1}{\pi_t}R_{t-1}^{n,d}x_{t-1}^b}{R_t^k}, \tag{A76}$$

$$x_t^b = \frac{d_t}{q_t^k s_t^k}, \tag{A77}$$

$$\phi_t = q_t^k s_t^k / e_t, \tag{A78}$$

$$e_t = n_t^b - \eta_t - \frac{1}{2}\kappa_\eta\left(\eta - \hat{\eta}\right)^2. \tag{A79}$$

$$0 = \psi_t^d\left(n_t^b - \eta_t - f\left(\eta_t\right) - \kappa_t d_t\right). \tag{A80}$$

A10.4 Government

$$T_t = T_t^{dia} + g_t, \tag{A81}$$

$$T_t^{dia} = (1-\gamma)F^b\left(\bar{\omega}_t^b\right)R_t^D d_{t-1} - \left(1-\mu^{dia}\right)G^b\left(\bar{\omega}_t^b\right)R_t^k q_{t-1}^k s_{t-1}^k, \tag{A82}$$

$$R_t^D = \frac{R_{t-1}^{n,d}}{\pi_t}. \tag{A83}$$

$$R_t^n = (1-\rho_r)\left[\bar{R}_n + \kappa_\pi\left(\pi_t - \bar{\pi}\right) + \kappa_y\log\left(y_t/y_{t-1}\right)\right] + \rho_r R_{t-1}^n + \varepsilon_{r,t}, \tag{A84}$$

A10.5 Exogenous processes

$$\log\left(z_t\right) = \rho_z\log\left(z_{t-1}\right) + \varepsilon_{z,t}, \tag{A85}$$

$$g_t = \left(\frac{\bar{g}}{\bar{y}}\right)y_t, \tag{A86}$$

$$\kappa_t = \bar{\kappa}. \tag{A87}$$

A10.6 Market clearing

$$s_t^k = k_t, \tag{A88}$$

$$y_t = c_t + i_t + g_t + \frac{\kappa_p}{2}\left(\pi_t - \pi_{t-1}^{\gamma_P}\bar{\pi}^{1-\gamma_P}\right)^2 y_t + \mu^{dia}G^b\left(\bar{\omega}_t^b\right)R_t^k q_{t-1}^k s_{t-1}^k. \tag{A89}$$

A11 Leibniz rule

Leibniz rule is given by:

$$\frac{d}{dx}\left(\int_{a(x)}^{b(x)} f(x,t)\,dt\right) = f(x,b(x))\cdot\frac{d}{dx}b(x) - f(x,a(x))\cdot\frac{d}{dx}a(x) + \int_{a(x)}^{b(x)}\frac{\partial f(x,t)}{\partial x}dt$$

Now we are going to calculate $\Gamma^j\left(\bar{\omega}_{t+1}^j\right)$ with $j \in (e,b)$, which is given by:

$$\Gamma^j\left(\bar{\omega}_{t+1}^j\right) = \int_0^{\bar{\omega}_{t+1}^j} \omega_{t+1}^j f^j\left(\omega_{t+1}^j\right)d\omega_{t+1}^j + \bar{\omega}_{t+1}^j\int_{\bar{\omega}_{t+1}^j}^{\infty} f^j\left(\omega_{t+1}^j\right)d\omega_{t+1}^j. \tag{A90}$$

We replace ω_{t+1}^j by x and $\bar{\omega}_{t+1}^j$ by $\bar{x}$ to obtain the following equation:

$$\Gamma^j(\bar{x}) = \int_0^{\bar{x}} x f^j(x)\,dx + \bar{x}\int_{\bar{x}}^{\infty} f^j(x)\,dx.$$

We assume that $F^j(x)$ is log-normal distributed with mean μ and variance σ^2. In that case we have that:

$$f^j(x) = \frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right).$$

Now we calculate the integrals:

$$\int_0^{\bar{x}} x f^j(x)\,dx = \int_0^{\bar{x}}\frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx$$

Now we perform a transform of variables by introducing $y = \log x \Rightarrow x = \exp(y)$. This results in the following differential: $dy = \frac{1}{x}dx \Rightarrow dx = xdy = \exp(y)dy$. We then also have to change the boundaries: $x = 0 \Rightarrow y = -\infty$ and $x = \bar{x} \Rightarrow y = \log\bar{x}$. Now we continue to calculate the integral:

$$\begin{aligned}
\int_0^{\bar{x}} x f^j(x)\,dx &= \int_0^{\bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx = \int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right)\exp(y)dy \\
&= \int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}+\frac{2\sigma^2 y}{2\sigma^2}\right)dy \\
&= \int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{y^2-2\left(\mu+\sigma^2\right)y+\mu^2}{2\sigma^2}\right)dy \\
&= \int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\left[y-\left(\mu+\sigma^2\right)\right]^2-\left(\mu+\sigma^2\right)^2+\mu^2}{2\sigma^2}\right)dy \\
&= \int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\left[y-\left(\mu+\sigma^2\right)\right]^2}{2\sigma^2}\right)\exp\left(\frac{\left(\mu+\sigma^2\right)^2-\mu^2}{2\sigma^2}\right)dy \\
&= \exp\left(\frac{2\mu\sigma^2+\sigma^4}{2\sigma^2}\right)\int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\left[y-\left(\mu+\sigma^2\right)\right]^2}{2\sigma^2}\right)dy
\end{aligned}$$

Now define a new variable $z = \left[y-\left(\mu+\sigma^2\right)\right]/\sigma \Rightarrow dz = dy/\sigma$. The boundaries then become $y=-\infty \Rightarrow z=-\infty$ and $y=\log\bar{x} \Rightarrow \bar{z} = \left[\log\bar{x}-\left(\mu+\sigma^2\right)\right]/\sigma$

$$\begin{aligned}
\int_0^{\bar{x}} x f^j(x)\,dx &= \exp\left(\frac{2\mu\sigma^2+\sigma^4}{2\sigma^2}\right)\int_{-\infty}^{\log \bar{x}} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\left[y-\left(\mu+\sigma^2\right)\right]^2}{2\sigma^2}\right)dy \\
&= \exp\left(\mu+\frac{\sigma^2}{2}\right)\int_{-\infty}^{\bar{z}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)dz \\
&= \exp\left(\mu+\frac{\sigma^2}{2}\right)\Phi\left[\frac{\log\bar{x}-\left(\mu+\sigma^2\right)}{\sigma}\right]
\end{aligned}$$

Now we calculate the second integral in equation (A90):

$$\begin{aligned}
\int_{\bar{x}}^{\infty} f^j(x)\,dx &= \int_{\bar{x}}^{\infty} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx \\
&= \int_0^{\infty} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx - \int_0^{\bar{x}} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx \\
&= 1-\int_0^{\bar{x}} \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx \qquad \text{(A91)}
\end{aligned}$$

Similarly to before, we perform a change in variables by defining $z = (\log x-\mu)/\sigma \Rightarrow dz = \frac{1}{\sigma x}dx$, while the boundaries change from $x=0 \Rightarrow z=-\infty$ and $x=\bar{x} \Rightarrow z = (\log\bar{x}-\mu)/\sigma$. We then get the following integral:

$$\begin{aligned}\int_{\bar{x}}^{\infty} f^j(x)\,dx &= 1-\int_0^{\bar{x}} \frac{1}{x\sigma\sqrt{2\pi}}\exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx \\ &= 1-\int_0^{(\log\bar{x}-\mu)/\sigma} \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{z^2}{2}\right)dz = 1-\Phi\left(\frac{\log\bar{x}-\mu}{\sigma}\right)\end{aligned}$$

Hence we can calculate $\Gamma^j(\bar{x})$, which is given by:

$$\Gamma^j(\bar{x}) = \exp\left(\mu+\frac{\sigma^2}{2}\right)\Phi\left[\frac{\log\bar{x}-\left(\mu+\sigma^2\right)}{\sigma}\right]+\bar{x}\left[1-\Phi\left(\frac{\log\bar{x}-\mu}{\sigma}\right)\right].$$

We now calculate $G^j\left(\bar{\omega}_t^j\right)$, where we replace $\bar{\omega}_t^j$ by $\bar{x}$:

$$G^j(\bar{x}) = \int_0^{\bar{x}} x f^j(x)\,dx = \exp\left(\mu+\frac{\sigma^2}{2}\right)\Phi\left[\frac{\log\bar{x}-\left(\mu+\sigma^2\right)}{\sigma}\right].$$

We now calculate $F^j\left(\bar{\omega}_t^j\right)$, where we replace $\bar{\omega}_t^j$ by $\bar{x}$:

$$F^j(\bar{x}) = \int_0^{\bar{x}} f^j(x)\,dx = \Phi\left(\frac{\log\bar{x}-\mu}{\sigma}\right).$$

Finally, we calculate the first derivative of $\Gamma^j(x)$ by applying the Leibniz rule:

$$\begin{aligned}\frac{d}{d\bar{x}}\Gamma^j(\bar{x}) &= \frac{d}{d\bar{x}}\left[\int_0^{\bar{x}} x f^j(x)\,dx+\bar{x}\int_{\bar{x}}^{\infty} f^j(x)\,dx\right] \\ &= \bar{x}f^j(\bar{x})+\int_{\bar{x}}^{\infty} f^j(x)\,dx-\bar{x}f^j(\bar{x}) \\ &= \int_{\bar{x}}^{\infty} f^j(x)\,dx = 1-F^j(\bar{x}). \qquad \text{(A92)}\end{aligned}$$

Finally we calculate the integral:

$$\int_{\bar{x}}^{\infty} x f^j(x)\,dx = \int_{\bar{x}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}}\exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right)dx$$

Again we perform a transform of variables by introducing $y=\log x \Rightarrow x=\exp(y)$. This results in the following differential: $dy=\frac{1}{x}dx \Rightarrow dx = xdy = \exp(y)dy$. We then also have to change the boundaries: $x=\bar{x} \Rightarrow y=\log\bar{x}$ and $x=\infty \Rightarrow y=\infty$. Now we continue to calculate the integral:

$$\begin{aligned}
\int_{\bar{x}}^{\infty} x f^{j}(x)\, dx &= \int_{\bar{x}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\log x-\mu)^2}{2\sigma^2}\right) dx = \int_{\log \bar{x}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(y-\mu)^2}{2\sigma^2}\right) \exp(y) dy \\
&= \int_{\log \bar{x}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\left[y-\left(\mu+\sigma^2\right)\right]^2}{2\sigma^2}\right) \exp\left(\frac{\left(\mu+\sigma^2\right)^2-\mu^2}{2\sigma^2}\right) dy \\
&= \exp\left(\mu+\frac{\sigma^2}{2}\right) \int_{\log \bar{x}}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{\left[y-\left(\mu+\sigma^2\right)\right]^2}{2\sigma^2}\right) dy
\end{aligned}$$

Again we define a new variable $z = \left[y-\left(\mu+\sigma^2\right)\right]/\sigma \Rightarrow dz = dy/\sigma$. The boundaries then become $y = \log \bar{x} \Rightarrow \bar{z} = \left[\log \bar{x} - \left(\mu+\sigma^2\right)\right]/\sigma$ and $y = \infty \Rightarrow z = \infty$

$$\begin{aligned}
\int_{\bar{x}}^{\infty} x f^{j}(x)\, dx &= \exp\left(\mu+\frac{\sigma^2}{2}\right) \int_{\bar{z}}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz \\
&= \exp\left(\mu+\frac{\sigma^2}{2}\right) \left[\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz - \int_{-\infty}^{\bar{z}} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right) dz\right] \\
&= \exp\left(\mu+\frac{\sigma^2}{2}\right) \left\{1-\Phi\left[\frac{\log \bar{x}-\left(\mu+\sigma^2\right)}{\sigma}\right]\right\}
\end{aligned}$$