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Interest On Reserves As A Main Monetary Policy Tool

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Abstract

This paper examines the potential role of the interest on reserves as a main monetary policy tool, in a model of financial intermediation with financial and nominal frictions calibrated to US data (1985-2018). The interest on reserves is shown to affect financial spreads and real economic activity, through its effect on the banking system's reserves (*balance sheet channel*) and the price of safe liquid assets (*intertemporal Euler equation channel*). It is shown to provide, (i) determinacy when the interest rate is pegged, independently of the size of reserves, bank capital restrictions, or fiscal policy; (ii) similar welfare improvements to an optimal Taylor rule; (iii) an alternative main monetary policy tool for driving the economy out of recessions when the conventional interest rate is trapped at the zero-lower-bound.

JEL Classification Numbers: E31, E32, E44, E52, E50, G28

Keywords: Interest on reserves; monetary policy; excess reserves; credit risk; balance sheet; Euler equation; welfare; zero-lower bound.

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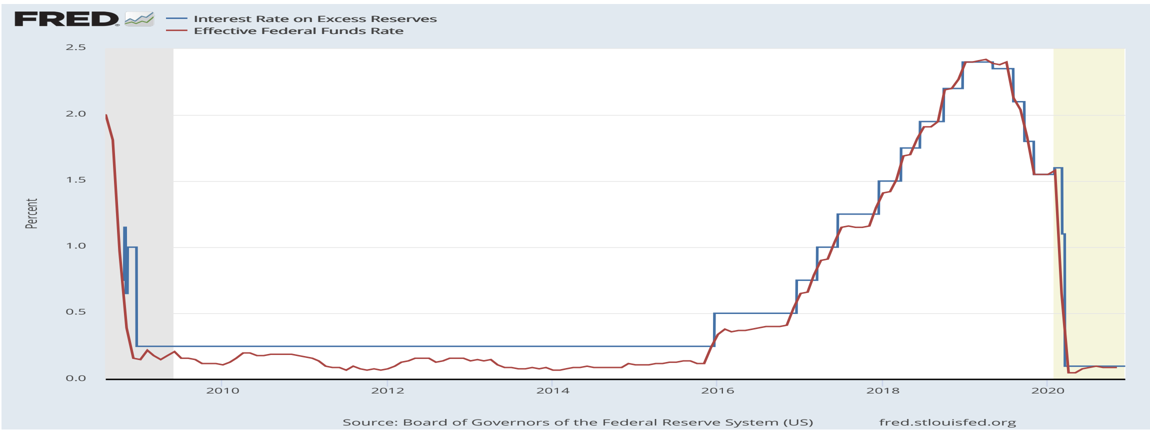
1 Introduction

Since the Great Recession the US Fed placed a new policy focus on the role of the interest on reserves. From its introduction however, in 2008 and up to recently following the 2019 pandemic of Covid-19, the interest on reserves has not been allowed to deviate away from the average federal funds rate, nor has it been truly tested as an independent policy tool. This is shown in Figure 1, where the interest on reserves is shown to follow very closely the effective federal funds rate, although the latter rate occasionally trends at a slightly lower value.[1] Similarly, most economic models assume that monetary policy is typically conducted by an interest rate rule and even the most recent literature, that examines the role of the interest on reserves, focuses mainly on its contribution as a complementary, (though independent), monetary policy tool that can affect the opportunity cost of holding liquidity in the form of reserves, or other safe liquid assets, (Ireland (2014); Cochrane (2014); Güntner (2015); Dressler and Kersting (2015); Hall and Reis (2016); Ennis (2018); Lenel et al. (2019); Piazzesi et al. (2019))[2].

Figure 1: Interest on Reserves and Effective FFR



This paper examines the potential contribution of the interest on reserves as a main monetary policy tool. The literature closer to it are the recent papers by Piazzesi et al. (2019); Diba and Loisel (2020, 2021), and Benigno and Benigno (2021), but this paper differs in both focus and modelling framework. This literature examines the role of the interest on reserves and the supply of reserves, when these are backed by fiscal policy, or when the reserves supplied by the central bank are supported by bond creation. The latter is a key assumption in this literature, because bonds either act as a collateral asset, (Piazzesi et al. (2019); Benigno and Benigno (2021)), or they reduce directly the cost

[1]This is because not all lending banks are entitled to the interest on reserves offered by the Fed and therefore they may not all have a strong incentive not to fall below the floor set by the interest on reserves, (Bowman et al., 2010; Goodfriend, 2015).

[2]Earlier contributions include, Hall (2002); Goodfriend (2011); Curdia and Woodford (2011); Bech and Klee (2011); Dutkowsky and VanHoose (2011); Kashyap and Stein (2012), among many other interesting papers.

of banking (Diba and Loisel (2020, 2021)). This implies a strong interdependent demand for reserves and bonds that leads to a less than perfect pass-through of the policy rate on aggregate demand, as the latter becomes a function of the spread between the policy rate and the interest on reserves.

In this paper, although banks have access to bonds and this facilitates the existence of a risk-free benchmark bond rate, at equilibrium the net supply of reserves and bonds is zero, which means that banks can hold reserves only by converting deposits into reserves. Holding reserves in this model, does not act explicitly as a collateral, or reduce directly the cost of banking; instead the desire of banks to hold reserves arises because of economic uncertainty that raises the risk of loan repayment. When this risk is high, banks become reluctant to make loans, and given the fixed supply of reserves, they increase their reserves-to-deposits ratio. To determine, in a meaningful way, the desire of banks to convert deposits into reserves, rather than loans, there is a cost associated with holding excess reserves above the level reflecting the risk of borrowers. Adding this cost ensures that banks only hold excess reserves based on the risk they perceive. When excess reserves match the probability of loan default, this cost is eliminated, but when the risk perceived by banks is higher than their existing reserves, banks increase their excess reserves-to-deposits ratio. Thus, the benefit of holding reserves in this model, is that it reduces the potential losses of banks coming from the default of risky loans. Since deposits and a fixed supply of reserves and bonds are the safe liquid assets in this model, we can examine the net effect of the interest on reserves as a main monetary policy tool, in the absence of an active fiscal or quantitative easing policy. The focus of this paper is also motivated by the prevailing view following the financial crisis, that banks became reluctant to make loans, not only because of their desire to increase their collateral (in terms of reserves or bonds), but mainly because of the high perceived riskiness in the credit markets during that time.[3]

As with this literature, in this paper there is also a disconnect between the policy rate and the relevant rate affecting aggregate demand, the deposit rate. However, here this disconnect arises from risk and the opportunity cost of holding reserves. Since, for simplicity, only banks can hold bonds and the household's safe asset is deposits, aggregate demand is shown to be affected by the deposit rate. However this rate is shown to be driven endogenously by the spread between the interbank rate and the interest on reserves, and risk. When the risk of default is eliminated from the model, this disconnect is also eliminated, but only if the interest on reserves is also set equal to the interbank rate. Otherwise this disconnect is still driven by the opportunity cost of holding reserves, (the spread between the interbank rate (bond rate) and the interest on reserves).

[3]The assumption of a zero net supply of reserves by the central bank, also avoids issues relating to whether the FED's supply of reserves, was close to satiation following the massive scale quantitative easing in the aftermath of the financial crisis, (see for example, Diba and Loisel (2020); Benigno and Benigno (2021)).

Another key difference between this literature and this paper, is the approach to the interest on reserves as a monetary policy tool. This recent literature uses the interest on reserves, either as 'a corridor system', where the central bank maintains a fixed spread between the interest on reserves and the interbank rate and allows the reserves to adjust endogenously (Piazzesi et al. (2019); Diba and Loisel (2020)); or as 'a floor system', where the growth of reserves is exogenously set and the interest on reserves follows a rule, where the latter usually responds to inflation, or mimics a Taylor rule, (Piazzesi et al. (2019); Diba and Loisel (2020); Benigno and Benigno (2021)).

As in most of this literature, by the 'interest rate', this paper also refers to the interbank rate, which is also the discount rate, and thus here the 'corridor system' is also formed by the spread of this rate and the interest on reserves.[4] However, since the supply of reserves is fixed in this paper, our 'corridor system' is one where when this spread is fixed, it is the reserves-to-deposits ratio that adjusts endogenously. Thus an important difference here is that the endogenous adjustment of reserves comes through the demand for reserves by banks, rather than the supply of central bank reserves. More importantly, reserves are not used as a policy instrument in this paper. Within this framework, we examine the following monetary policy scenaria: (a) the interest rate follows an optimal Taylor rule with no use of the interest on reserves, which we use as our benchmark case for welfare analysis; (b) a 'zero-spread corridor system' where the interest on reserves is equal to the optimal interest rate (thus the spread is zero) and the demand for reserves by banks adjusts endogenously; (c) a floor system where the interest rate is pegged to its steady state and monetary policy is driven entirely by an interest on reserves rule, (three rules are examined); (d) a zero-lower-bound scenario, where the 'zero-spread corridor system' (described in b), is trapped in the zero-lower bound region, and the central bank switches to a 'floor system', where the interest on reserves drives monetary policy. Another difference with the existing literature, is that in this paper the interest on reserves rules that we consider in the floor system, respond to key credit market variables, (credit, default risk or the output gap), rather and than mimic a Taylor rule as in most of these papers. This means that the interest on reserves can also be used along with a Taylor rule, but as an independent monetary policy tool targeting different variables.

To examine the role of the interest on reserves in this framework, the paper introduces a new Keynesian DSGE model of financial intermediation with financial and liquidity frictions, where banks form micro-founded decisions about their supply of loans, the demand for deposits and their optimal level of excess reserves, all from one maximization problem and subject to, (i) their balance sheet, (ii) the endogenous risk of default of their borrowers, (iii) a fixed required reserve ratio and (iv) a fixed

[4] For a paper where the interbank rate differs to the discount rate see Bigio and Sannikov (2021)

bank capital requirement ratio. The model introduced here makes it possible to examine explicitly the effects that the interest on reserves has on financial spreads and real economic activity, through its two key transmission channels: (i) the level of excess reserves and hence credit supply: a *balance sheet channel* and (ii) the deposit rate, which in this model sets the price of the main safe liquid asset held by households (deposits/savings): an *Euler equation channel*.

Using this framework, the paper first shows that under a floor system where the interest rate is pegged, an interest on reserves rule can, by itself, provide determinacy through its effect on the deposit rate. This is shown to be true even when the *reserves-to-deposits* ratio and the *bank capital-to-loan* ratio are fixed to their steady state value or zero and even when risk is eliminated from the model. Therefore, the innovation here is that local determinacy can hold independently of bank capital constraints, or changes in the cost, or the size (scarce or abundant) of central bank, or commercial bank, balance sheets, (Ennis (2018); Williamson (2019); Lenel et al. (2019); Piazzesi et al. (2019); Diba and Loisel (2020)). Also determinacy here does not require the central bank to set both the interest on reserves and the supply of reserves, Diba and Loisel (2020) and it also holds independently of risk, or properties of the fiscal theory of the price level, (Cochrane (2014); Hall and Reis (2016)).[5] Instead, determinacy here depends simply on whether the interest on reserves offered to banks can affect the intertemporal choices of households and the reserve-to-deposit ratio, by affecting the return on liquid safe assets available to them, the deposit rate in this model. The mechanism can be explained as follows. Reserves in this paper depend on the demand for other safe assets (deposits), and the level of risk of loan default. The deposit rate is shown to be affected mainly by the opportunity cost of holding reserves (the spread between the interest rate and the interest on reserves), and risk (the gap between the excess-reserves-to-deposits ratio and the perceived probability of default, as explained above). It is shown that even when the risk is eliminated, the interest on reserves still affects the deposit rate, through the former effect. The deposit rate in turn affects the intertemporal choices of households, (consumption and aggregate demand), but also the level of deposits, and thus the reserves-to-deposits ratio, given a fixed supply of reserves. Thus by affecting the deposit rate, the interest on reserves determines the equilibrium in the goods and the loan and money markets and thus it provides determinacy through the *Euler equation channel* and indirectly through the *balance sheet channel*.

The model is calibrated to US data, for the period 1985 Q1 to 2018 Q4, so as to evaluate the

[5] Cochrane (2014) indicates that conventional theories cannot explain how at times of large balance sheets the interest on reserves can determine inflation, but shows that this is possible with the backing of fiscal policy and the fiscal theory of the price level. In a similar spirit, Hall and Reis (2016) use a simple model of the fiscal theory of the price level to show that an interest payment on reserves can pin down a unique equilibrium of the price level based on arbitrage between only two periods.

potential welfare contribution from using optimal interest on reserves rules as the main (and single) monetary policy tool, in relation to that achieved by an optimal Taylor rule. It is shown that although, overall, Taylor rules perform best at normal times, simple optimal rules where the interest on reserves responds to key macroeconomic variables, such as credit, risk of default, or the output gap, can replace the conventional interest rate and provide similar welfare improvements to an optimal Taylor rule even during normal times (i.e. based on the calibrated baseline model).

Finally, a non-linear version of this model is used to examine how the conventional interest rate and the interest on reserves perform at the zero-lower bound under occasionally binding constraints.[6] Adapting the algorithm of Guerrieri and Iacoviello (2015) for solving dynamic models with occasionally binding constraints, we examine a scenario of a 'corridor system' where the interest on reserves is equal to the interest rate and the latter follows a Taylor rule. We then allow the economy to be affected by a combination of a financial shock and a negative preference shock, that trap endogenously the interest rate in the zero-lower bound region for four quarters. It is shown that if monetary policy switches from this zero-spread corridor system to a floor system, where an interest on reserves rule, responding to credit conditions, targets the deposit rate, then the economy is driven out of the zero-lower-bound region within one quarter. This is achieved if the interest on reserves is reduced substantially (offering a negative rate) below the interest rate. Through the *Euler* and *balance sheet channels* explained above, this reduces the deposit rate and encourages consumption, while it reduces the reserves-to-deposits ratio, as it penalises excess reserves. In this example, as deposits earn almost a zero return and reserves are penalised heavily, loans and economic activity are shown to gradually increase. A key question here is what happens to the interest rate and particularly how this scenario would change if there was also an increase in the bond supply, where these released reserves could be diverted into, instead of loans. The paper demonstrates that for this policy to be most effective, the equilibrium deposit rate that results from the interest on reserves policy, should also set the ceiling for the federal funds rate. In other words, trapped in a zero-lower bound scenario where banks are reluctant to make loans, the monetary authority should aim to use the interest on reserves to target the price of all short safe assets, including deposits, bills and notes. This latter policy would also discourage an excessive investment in short safe bonds, as an alternative higher yielding safe asset, or for collateral purposes in such times, (Lenel et al. (2019); Piazzesi et al. (2019)).

The rest of the paper is organized as follows. Section 2, introduces the main framework and derives the decisions of households, firms and banks and defines the aggregate equilibrium. Section 3, examines

[6] In a different framework, where households derive utility directly from real reserves and the latter also reduces banking costs and the borrowing costs of firms, Diba and Loisel (2020, 2021) focus on determinacy at the zero-lower bound and the 2008-2015 episode. They show that local-equilibrium determinacy is achieved when both the interest rate on reserves and the supply of reserves are set exogenously.

the conditions for the local determinacy of the model. Section 4, calibrates the baseline model to US data (1985Q1-2018Q4). Section 5, uses the calibrated model of Section 4, to perform a numerical welfare analysis for optimal interest on reserves policy rules, under the assumption that the interest on reserves is (i) a 'complementary' policy tool to the Taylor rule and (ii) a 'main' or 'single' monetary policy tool. Section 6, uses a non-linear version of the model to examine the potential role of the interest on reserves in the zero-lower bound region. This scenario is examined under both a flexible and a fixed reserves-to-deposits ratio and it is compared to the case of conventional monetary policy. Finally, section 7 concludes.

2 The Model

This paper introduces a closed economy, dynamic stochastic general equilibrium (DSGE) model of financial intermediation with, financial and liquidity frictions, nominal frictions, a cost channel, a balance sheet channel and fixed bank capital requirements. The economy consists of dynasties of households that live forever, a continuum of differentiated intermediate goods sectors, each populated by a firm and a bank, a competitive final good sector and a central bank. Each intermediate goods firm borrows from its local bank to fund wage payments to households.[7] Its production is subject to an idiosyncratic shock, which makes loan repayment risky requiring a fraction of its output as collateral. Banks fund loans from household deposits and bank capital (equity). They can also borrow from the central bank, or hold short government bonds, at the interbank rate. For simplicity, only banks have access to bond and they use this for potential borrowing purposes. Money (notes and coins) is not examined explicitly in this model, but it is approximated by reserves.[8] Banks select their optimal level of excess reserves, the deposit rate, the loan rate, and short government debt, all from one maximization problem and subject to, their balance sheet, the endogenous default risk of their borrowers, and the fixed required reserve and bank capital requirement ratios imposed by the monetary authority. Loans are determined by both demand and supply conditions. The supply of reserves is fixed and in equilibrium borrowing from the central bank (or short government debt) is also zero and thus the two main safe liquid assets for banks are, reserves and deposits/savings, whereas for households only the latter.[9]. Bank equity holders absorb the cost of the default of borrowers. The

[7]For simplicity and since the focus of this paper is on the financial intermediaries, firms use only labour in their production.

[8]Cash plays no key role in this model. In fact, unlike the Great Depression in the 1930's, where cash withdrawals increased dramatically, pushing up the currency ratio to very high levels, the latter ratio even fell during the Great Recession, pointing to no substantial evidence of large bank withdrawals during the recent crisis.

[9]With no loss in generality, the assumption that deposits are assumed to be the main risk-free assets in the model implies that deposits are fully covered by a government scheme. The paper could be extended to account explicitly for a

supply of bank equity is determined by the fixed bank capital requirement ratio. Intermediate goods firms select prices, employment and loans, taking the loan rate offered by the bank as given.

2.1 Households

Households maximize their expected lifetime utility,

$$U_t = \mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \xi_t \left[\frac{c_t^{1-\sigma}}{1-\sigma} - v \frac{h_t^{1+\eta}}{1+\eta} \right], \tag{1}$$

where $\mathbb{E}_t$ the expectations operator, $\beta \in (0,1)$, is the discount factor, v, σ, $\eta > 0$ and ξ_t is a preference shock. At the beginning of each period households choose between two assets: risk-free bank deposits, d_t, and risky bank equity, e_t, (both defined in real terms) and spend their remaining income on a consumption basket, c_t. Their real wage income is, $w_t h_t$, where w_t is their real wage and h is their employment hours. They also receive gross interest payments on deposits from the previous period, R_{t-1}^D and, with a probability $(1 - \Phi_t[\varepsilon_t^*])$, they also receive a gross bank equity return R_{t-1}^E, which is adjusted for the rate of growth (volatility) of returns, $a^E(R_{t-1}^E - R_{t-2}^E)$, where $a^E > 0$.[10] In acquiring bank equity investors incur a fixed cost, ϱ^E, which is proportional to the opportunity cost of their bank capital investment, determined by the deposit rate R_t^D and the volume of bank equity they buy, e_t.[11] $\Phi_t[\varepsilon_t^*]$ is the probability of credit default of borrowers (derived below). Households also receive aggregate real profits from firms and financial intermediaries, $V_t = \sum \Pi_t^s$, $s \in \{f, b\}$.

Households maximize (1) subject to,

$$\begin{aligned} c_t + d_t + e_t = w_t h_t + \frac{R_{t-1}^D d_{t-1}}{\pi_t} + (1 - \Phi_t[\varepsilon_t^*]) \left(R_{t-1}^E + a^E (R_{t-1}^E - R_{t-2}^E) \right) \frac{e_{t-1}}{\pi_t} \\ - \varrho^E R_{t-1}^D \frac{e_{t-1}}{\pi_t} + V_t, \end{aligned} \tag{2}$$

where p_t is the price of the final good and $\pi_t \equiv \frac{p_t}{p_{t-1}}$ is the gross inflation rate. The first order conditions are,

$$c_t^{-\sigma} \xi_t = \beta \mathbb{E}_t \left(\frac{R_t^D c_{t+1}^{-\sigma} \xi_{t+1}}{\pi_{t+1}} \right), \tag{3}$$

$$w_t = v h_t^\eta c_t^\sigma, \tag{4}$$

$$R_t^E = \frac{a^E}{1+a^E} R_{t-1}^E + \frac{(1+\varrho^E)}{(1+a^E)(1-\Phi_t[\varepsilon_t^*])} R_t^D. \tag{5}$$

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deposit insurance covering part of potential deposit losses, however this would not change the qualitative results in this paper and thus it is omitted for simplicity.

[10] This adjustment serves mainly to produce some persistence in the equity returns that can improve the fit of the data. Setting $a^E = 0$ does not affect the qualitative results in the paper.

[11] This assumption is used to improve the calibration of the spreads.

Equations (3) and (4) describe the household's Euler equation and labour supply respectively. Equation (5) is the return on bank equity required by households and provided by banks, shown to be a markup over the risk-free deposit rate. Thus, the risk-free deposit rate determines the intertemporal choices of consumers and sets the benchmark rate for the equity rate.

2.2 Final Goods Firm

The competitive final good firm assembles all intermediate goods, $y_{j,t}$, $j \in (0,1)$, to produce a final output, y_t, which then sells at the price p_t. This is produced using a CES technology with Dixit-Stiglitz (1977) preferences, $y_t = \left(\int_0^1 y_{j,t}^{\frac{\lambda_p - 1}{\lambda_p}} dj \right)^{\frac{\lambda_p}{\lambda_p - 1}}$, where $\lambda_p > 1$, is the elasticity of substitution between differentiated intermediate goods. The corresponding demand for each intermediate good j is, $y_{j,t} = y_t \left(\frac{p_{j,t}}{p_t} \right)^{-\lambda_p}$, where $p_{j,t}$, is the price set by intermediate firm j and $p_t = \left(\int_0^1 p_{j,t}^{1-\lambda_p} dj \right)^{\frac{1}{1-\lambda_p}}$ is the average price index.

2.3 Intermediate Goods Firms

The production of each intermediate good j relies on average labour hours, h_t, and it is subject to both an aggregate supply shock, A_t, and an idiosyncratic risk shock, $\varepsilon_{j,t}$.

$$y_{j,t} = \varepsilon_{j,t} A_t h_t, \tag{6}$$

where, A_t, follows an $AR(1)$ process, $\log A_t = \rho_A \log A_{t-1} + \epsilon_t^A$, and ϵ_t^A is an i.i.d. shock, with standard deviation σ_A and mean $A = 1$. The idiosyncratic shock, $\varepsilon_{j,t}$, is uniformly distributed over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$, with a constant variance and a mean of unity, so that at the symmetric aggregate equilibrium, productivity is A_t.[12] Each firm j uses loans $l_{j,t}$ and a fraction ζ of past output, $y_{j,t-1}$, to cover its working capital, in real terms[13],

$$l_{j,t} = w_t h_t - \zeta y_{j,t-1}. \tag{7}$$

In a good state the firm repays the bank the full borrowing cost, $R_t^L l_{j,t}$, where R_t^L is the gross loan rate, as set by banks (derived below). In the event of default the bank seizes a fraction χ of the firm's final output, $y_{j,t}$, as collateral.[14] Default occurs when the real value of seizable collateral is less than the amount that needs to be repaid, $\chi y_{j,t} \leqslant R_t^L l_{j,t}$. Using equations (6) and (7), the cut-off value

[12]The assumption that the idiosyncratic shock $\varepsilon_{j,t}$ follows a uniform distribution, is only to facilitate a tractable probability of default with no loss in generality.

[13]Using a fraction of past output to provide some internal funds can be seen as a simplified proxy to net worth and it mainly serves to improve the calibration of the model. Eliminating this type of net worth, by setting $\zeta = 0$, does not affect the qualitative results of the paper.

[14]See also Agénor et al. (2014)

below which the firm defaults is,

$$\varepsilon_{j,t}^{*} = \frac{R_t^L l_{j,t}}{\chi y_t}, \tag{8}$$

where, $y_t = A_t h_t$. Price setting is based on Calvo-type contracts, where ω_p firms keep their prices fixed, while the rest $(1 - \omega_p)$ of firms adjust prices optimally. Each firm j maximizes its discounted expected stream of profits,

$$\max_{P_{j,t+s}} \mathbb{E}_t \sum_{s=0}^{\infty} \omega_p^s \lambda_{t,t+s} \{\Pi_{j,t+s}^f\}$$

subject to, $\Pi_{j,t}^f = \frac{P_{j,t}}{P_t} y_{j,t} - mc_t y_{j,t}$ and equations (6-8), and by taking the loan rate offered by the bank R_t^L, as given. The discount factor, $\lambda_{t,t+s} = \beta^s \frac{c_{t+1}^{-\sigma} \xi_{t+1}}{c_t^{-\sigma} \xi_t}$ is equal to that of the firm owners (households). The firm's cost minimization problem is based on average operational costs and thus based on the mean value of the idiosyncratic shock, (which is unity), hence real marginal cost is,[15]

$$mc_t = \frac{R_t^L w_t}{A_t}. \tag{9}$$

From the firm's maximization problem the new Keynesian Phillips curve equation is,[16]

$$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p \widehat{mc}_t, \tag{10}$$

where $k_p = (1 - \omega_p)(1 - \omega_p \beta)/\omega_p$. Equation (10) is a standard new Keynesian Phillips curve with a cost channel, $mc_t[R_t^L]$, however, as shown below, the loan rate here is driven by the default risk and other key financial variables.

2.4 The Banks

The financial sector consists of many local branch banks $j \in (0, 1)$, each making loans to their respective sector firm j.[17] Each bank j raises funds from, household deposits $d_{j,t}$ at the gross return R_t^D, and bank equity $e_{j,t}$ at the gross return R_t^E, determined by the demand for equity by households, in eq. 5. It can also borrow from the central bank an amount $B_{j,t} < 0$, at the interbank rate R_t, or alternatively, it can

[15] Any changes in risk are captured in the loan rate; see derivation below.

[16] Hats, $\widehat{X}$, denote log-linearizations from steady state. The firm's maximization problem is standard, though here we also use the distribution properties of the idiosyncratic shock, $\mathbb{E}_t \varepsilon_{j,t} = 1$, and $\int_0^1 \varepsilon_{j,t} = 1$. Also, in this section of the paper, where $\pi = 1$, the log-linearized aggregate equilibrium implies that any price dispersion terms are eliminated.

[17] This is only a simplification assumption, with no loss in generality, since at equilibrium, all firms and banks behave similarly.

hold short government debt, $B_{j,t} > 0$, and receive R_t. Bank j's, real profit maximization problem is,

$$\Pi^b_{j,t} = \widetilde{r}_{j,t} d_{j,t} R^{IOR}_t + \int_{\varepsilon^*_{j,t}}^{\bar{\varepsilon}} R^L_t l_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}} \chi y_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t} - R^D_t d_{j,t} - R^E_t e_{j,t} \tag{11}$$

$$+ R_t B_{j,t} - \varrho^L_t l_{j,t} - \varrho^D d_{j,t} - \frac{\theta}{2} d_{j,t} (\widetilde{s}_{j,t} - \widetilde{s}_{t-1})^2 - \frac{\psi}{2} d_{j,t} (\widetilde{s}_{j,t} - \varphi \int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}} f(\varepsilon_{j,t}) d\varepsilon_{j,t})^2,$$

where $\widetilde{r}_{j,t}$ is the *reserves-to-deposits* ratio, that is the fraction of deposits that the bank holds as total reserves, $r_{j,t} = \widetilde{r}_{j,t} d_{j,t} = (\widetilde{s}_{j,t} + \widetilde{\varsigma}) d_{j,t}$. Total reserves, $r_{j,t}$, consist of required reserves $\varsigma_{j,t} = \widetilde{\varsigma} d_{j,t}$, where $\widetilde{\varsigma}$ is the fixed *required reserve ratio* set by the monetary authority, and excess reserves, $s_{j,t} = \widetilde{s}_{j,t} d_{j,t} = (\widetilde{r}_{j,t} - \widetilde{\varsigma}) d_{j,t}$, where $\widetilde{s}_{j,t}$ is the *excess reserves to deposits* ratio. Both required and excess reserves receive the interest on reserves, R^{IOR}_t. The remaining deposits, $(1 - \widetilde{r}_{j,t}) d_{j,t}$ together with bank equity $e_{j,t}$ and loans from the central bank $B_{j,t} < 0$, can be used for loans to the real sector, $l_{j,t}$, at the gross borrowing cost, R^L_t, (see eq 12). From the loan market, the bank receives $\int_{\varepsilon^*_{j,t}}^{\bar{\varepsilon}} R^L_t l_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t}$ in a good state, or the collateral in times of a bad state, $\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}} \chi y_{j,t} f(\varepsilon_{j,t}) d\varepsilon_{j,t}$, where $f(\varepsilon_{j,t})$ is the probability density function of the uniform distribution. $\varrho^L_t l_{j,t}$ captures other loan-related costs, (including transaction, monitoring costs and other loan-related innovations) and ϱ^D is a similar fixed cost for managing deposits.[18] To ensure banks cannot increase excess reserves indefinitely and that excess reserves are well-behaved, two quadratic adjustment costs are also employed, the last two terms in (11). The first is an excess reserve adjustment cost, with $\theta > 0$, associated with the bank adjusting the fraction of the deposits it holds as excess reserves, $\widetilde{s}_{j,t}$, away from the previous period's equilibrium level of that fraction, $\widetilde{s}_{t-1}$. The second cost, (last term), is more important in this paper and captures the fact that for any given level of deposits, it is costly for banks to hold a fraction of excess reserves above the level reflecting the probability of default of borrowers. The elasticity $\psi > 0$ determines the size of this cost, while $\varphi > 0$ determines how sensitive this cost is to the level of risk. Note that the term, $\widetilde{s}_{j,t} - \varphi \int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}} f(\varepsilon_{j,t}) d\varepsilon_{j,t}$, penalizes holdings of excess reserves below as well as above the levels reflecting loan defaults. This, as shown in equation (16) below, implies that excess reserves are determined both by the opportunity cost of holding reserves, $R_t - R^{IOR}_t$, but also by the level of perceived risk in the economy, Φ_t, which can rise or fall, as it is endogenous and time-varying in this model. In determining the optimal level of excess reserves, the bank also takes into account the fraction of deposits that potentially can be lost in loans in a bad state.[19] Bank j

[18] ϱD is also used for calibrating the steady state value of the deposit rate.

[19] For details see in the Appendix A.2

maximizes real profits, (11), subject to,

$$l_{j,t} \leq (1-\widetilde{r}_{j,t})d_{j,t} + e_{j,t} - B_{j,t}, \tag{12}$$

$$e_{j,t} = \gamma l_{j,t}, \tag{13}$$

Equation (12), is the bank's balance sheet constraint, whereas (13) sets the bank capital-to-loan requirement ratio to a fixed level γ. Since raising funds through equity is more costly for the bank ($R_t^E > R_t$), bank equity is issued merely to satisfy regulatory bank capital requirements and thus it is always binding in this model.[20] From the above problem the bank calculates the equilibrium loan rate, taking the policy rate and the bank equity rate as given[21]

$$R_t^L = R_t + \gamma(R_t^E - R_t) + \frac{\chi A_t}{w_t}\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\Phi_t^2 + \varrho_t^L \tag{14}$$

where Φ_t, is the probability of credit default,

$$\Phi_t = \int_{\underline{\varepsilon}}^{\varepsilon_t^*} f(\varepsilon_t) d\varepsilon_t = \frac{\varepsilon_t^* - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}},$$

and $\varepsilon_t^* = \frac{R_t^L l_t}{\chi y_t}$ is the cut-off point. The deposit rate is,

$$R_t^D = R_t - (R_t - R_t^{IOR})\widetilde{r}_t - \frac{\theta}{2}(\widetilde{s}_t - \widetilde{s}_{t-1})^2 - \frac{\psi}{2}(\widetilde{s}_t - \varphi\Phi_t)^2 - \varrho^D, \tag{15}$$

and the fraction of deposits the bank desires to holds as excess reserves is, the excess reserves-to-deposits ratio, is

$$\widetilde{s}_t = \rho_{\widetilde{s}}\widetilde{s}_{t-1} - \frac{R_t - R_t^{IOR}}{\theta + \psi} + \frac{\psi\varphi}{\theta+\psi}\Phi_t + \frac{\chi A_t}{w_t}\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2(\theta+\psi)(1-\gamma)}\Phi_t^2. \tag{16}$$

where, $\widetilde{s}_t \equiv \widetilde{r}_t - \widetilde{\varsigma}$ and $\rho_{\widetilde{s}} = \frac{\theta}{\theta+\psi}$. From (14), the loan rate spread is shown to be positively related to the bank equity spread, $R_t^E - R_t$, the bank capital requirement, γ, the risk-related premium (third term in 14) and ϱ_t^L. The latter acts also as a credit spread shock, $\varrho_t^L = \varrho^L + \ln[\varrho_t^L]$, where $\ln[\varrho_t^L] = \rho_L \ln[\varrho_{t-1}^L] + \epsilon_t^L$, and ϵ_t^L is a normal random variable with zero mean and standard deviation σ_L. From (12), (15) and (16), an increase in the interest on reserves is shown to reduce the opportunity cost of holding reserves, $(R_t - R_t^{IOR})$, and this acts as '*a subsidy*' to banks for holding excess reserves.[22] This encourages the accumulation of reserves which reduces the total amount of reserves available for

[20]The role of bank capital regulation is not the focus of this paper. For a model where the bank-capital constraint can be subject to the level of excess reserves see Ennis (2018).

[21]The maximization problem is described in more detail in Appendix A.2

[22]For this 'subsidy' effect, see also Dutkowsky and VanHoose (2011), Ireland (2014), Güntner (2015)

credit to firms, which in this model is shown to induce a bank balance sheet effect. It is also shown to encourage a higher return on liquid assets, the deposit rate here, which is key to the household's intertemporal decisions, as it reduces their consumption and their potential demand for credit. Both of these effects are eliminated when the interest rate is set equal to the reserve rate, $R_t^{IOR} = R_t$. Risk in this model is also shown to affect the deposit rate and excess reserves. A higher level of risk, Φ_t, makes safe liquid assets more desirable, thus increasing their return and the excess reserves-to-deposits ratio. However, risk also affects excess reserves through a second channel in this model, which comes from the assumption that in selecting their desired level of excess reserves, banks internalise the effect that their decision about reserves has on the potential loan losses in a bad state, (last term in 16).[23]

2.5 Conventional Monetary Policy and Aggregate Equilibrium

In the baseline model the interest rate, R_t, follows a conventional Taylor rule,

$$R_t = R^{(1-\phi)} R_{t-1}^{\phi} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{x_t}{x} \right)^{\phi_x} \right]^{(1-\phi)}, \tag{17}$$

where, $y_t/y_t^* \equiv x_t$ is the output gap, $\phi \in (0,1)$ and $\phi_\pi, \phi_x > 0$. The interest rate affects the spreads and the cost of borrowing, through the loan rate and the deposit rate.[24] Alternative monetary policy rules are examined in later sections.

At the aggregate equilibrium, aggregate demand is determined by consumption, $y_t = c_t$. On the production side equilibrium also requires that $\int_0^1 \left(\frac{p_{j,t}}{p_t} \right)^{-\lambda_p} y_t = \int_0^1 \varepsilon_{j,t} A_t h_t$. Using the distribution properties of the idiosyncratic shocks (that imply $\int_0^1 \varepsilon_{j,t} = 1$), aggregate output is, $y_t = A_t h_t / V_t$, where $V_t \equiv \int_0^1 \left(\frac{p_{j,t}}{p_t} \right)^{-\lambda_p}$ is the price dispersion index. At the aggregate equilibrium the loan and deposit markets are also in equilibrium and there is no borrowing from the central bank, $B_t = 0$. This implies that labour demand is, $l_t = w_t h_t - \zeta y_{t-1} = (1 - \widetilde{r}_t) d_t + e_t$, where the latter is the loan supply from the balance sheet, whereas deposits are determined endogenously. The aggregate equilibrium and the full log-linearised system of the baseline model are described in Appendix A.3.

3 Local determinacy

We first show that the interest on reserves can provide determinacy, even when we assume a fixed policy rate and a fixed level of reserves. Although reserves play a role for determinacy, the innovation

[23] For the derivation of this effect see in Appendix A.2

[24] Dreschler, Savov and Schnal (2017), show that the policy rate can also affect the deposits channel through the monopolistic power of banks. This channel is relaxed here as it is not the focus of this paper.

shown here is independent of the level of reserves-to-deposits, or the degree of bank capital restrictions, but instead it depends on whether the interest on reserves can affect the intertemporal choices of households by affecting the return on liquid safe assets, the deposit rate. To demonstrate this, consider a simple rule, where the interest on reserves responds to the output gap,[25]

$$R_t^{IOR} = R^{IOR} \left(\frac{x_t}{x}\right)^{\mu^x} \tag{18}$$

where, μ^x, is a policy response parameter to be determined optimally in section 5.

Proposition 1. *The interest on reserves can provide determinacy by affecting the return on the safe liquid assets, the deposit rate. This result is independent of bank capital restrictions, credit risk, fiscal policy, or the supply of reserves. For $R^{IOR} \neq R$, determinacy is also independent of the reserves-to-deposits ratio. A necessary condition for determinacy in this model is that $\mu^x \neq 0$ and thus, $\partial \widehat{R}_t^D / \partial \widehat{R}_t^{IOR} \neq 0$, that is, that the interest on reserves can affect the deposit rate when it is set to respond to changes in the output gap, or other key macroeconomic variables.*

Proof. A formal proof of Proposition 4 is provided in Appendix A.4, and supported by Table 3 and Figure 2 □

As shown in Appendix A.4, this result is independent of the fiscal theory of the price level, or financial frictions and hence it works even when risk is fully removed, ($\Phi = \widehat{\Phi}_t = 0$ and $\widehat{R}_t^L = \widehat{R}_t = 0$). It is also holds under a fixed *reserves-to-deposits* ratio, $\widetilde{r}_{j,t}$, or when bank capital constraints are relaxed, ($\gamma = 0$). It also holds independently of any adjustment costs in the deposit rate, excess reserves, or internal funding, and thus it still holds when, $\psi = \varphi = \zeta = 0$. When all these assumptions are employed, a necessary condition for local determinacy is that,

$$1 - \left(\widetilde{r} - \frac{(R - R^{IOR})}{\theta}\right) \frac{R^{IOR}}{R^D} \mu^x \left(1 - \frac{1}{\beta}\right) > 1.$$

From this it follows that when the policy rate and the reserves-to-deposits ratio, are fixed to their steady state values, or zero, implying $R \geqslant 0$, ($\widehat{R}_t = 0$), and $\widetilde{r} \geqslant 0$, ($\widehat{\widetilde{r}}_t = 0$), respectively, then for $R^{IOR} \neq R$, a necessary condition for a determinate equilibrium is that $\mu^x \neq 0$. In particular,

(i) if, $\widetilde{r} > \frac{(R - R^{IOR})}{\theta}$, or if the interest rate is fixed to zero, $R = 0$, then a necessary condition for a unique equilibrium is, $\mu^x < 0$.

[25]Here I purposely choose a rule that does not respond to a financial variable, so as to show that this effect can also hold independently of financial frictions.

(ii) if, $0 \leq \widetilde{r} < \frac{(R - R^{IOR})}{\theta}$, then a necessary condition for a unique equilibrium is simply that $\mu^x > 0$. In all cases, $\mu^x \neq 0$ implies, (from 15), that $\partial \widehat{R}_t^D / \partial \widehat{R}_t^{IOR} \neq 0$, and thus that the interest on reserves can provide determinacy through its effect on the deposit rate. The latter affects the equilibrium in the goods market through the *Euler equation channel*, and the loan and money markets by affecting the demand for deposits and thus the reserves-to-deposits ratio through the *balance sheet channel*.

If at the steady state, the disconnect between the interest on reserves and the deposit rate is eliminated and the former equals the policy rate, $R = R^{IOR} = R^D$, then the determinacy condition is

$$1 - \widetilde{r}\mu^x \left(1 - \frac{1}{\beta}\right) > 1,$$

which means that local determinacy is attainable by an active interest on reserves rule, with a negative response coefficient, $\mu^x < 0$, and a positive (non-zero) reserves-to-deposits ratio. Thus, in this case, the level of the steady state reserves-to-deposits ratio becomes pivotal. This effect is not inconsistent with the findings in the existing literature that the level of reserves is crucial for determinacy, although here with a fixed supply of reserves, it is the demand for reserves in relation to deposits that matters for determinacy and only in the case of a zero spread between the interest on reserves and the interest rate, $R = R^{IOR}$.[26]

4 Baseline Calibration

Table 2 shows the key steady state parameters and moments of the model calibrated to US data, for the period, 1985(Q1) to 2018(Q4), using the baseline parameter values shown in Table 1.[27]. For this period, the average required reserves-to-deposits ratio is, $\widetilde{\varsigma} = 1.57\%$, and the average excess reserves-to-deposits ratio is, $\widetilde{s} \approx 5.42\%$, (which up to 2008(Q4) amounted to only 0.12%). Thus, the total reserves-to-deposits ratio is, $\widetilde{r} \approx 6.99\%$. The effective federal funds rate is $R = 3.64\%$, which implies a net quarterly rate of 0.0091. The average deposit rate spread, over the sample period is calibrated to, $R - R^D = 1.60\%$, (or 0.0040 quarterly), which implies, $R^D = 2.03\%$ (0.0051 quarterly) and $\beta = 0.9949$. The loan spread is calibrated to $R^L - R = 3.91\%$, (0.0097 quarterly), and the equity spread to, $R^E - R = 7.65\%$ (0.0191 quarterly). To match the Fed's policy we also set the reserve rate equal to the interest rate at the steady state baseline model, $R^{IOR} = R$. The annual steady state default probability is calibrated to $\Phi = 3.00$, (0.0077 quarterly), assuming a steady state fraction of

[26]See also Bratsiotis (2018).

[27]For more details on the data used see in Appendix A.1

Table 1: Baseline Parameters and Moments

Parameter	Value	Description
β	0.9949	Discount factor
η	1.00	Frisch elasticity of labor supply
σ	1.00	Elasticity of consumption in household's utility
λ_p	6.00	Elasticity of demand
ω_p	0.693	Price stickiness
A	1.00	Productivity mean
γ	0.10	Bank capital required ratio
ϕ_π	1.50	Interest rate responsiveness to inflation
ϕ_x	0.50	Interest rate responsiveness to output gap
ζ	0.430	Fraction of firm's internal funding
θ	1.24	Excess reserves dynamic adjustment cost
φ	7.000	Responsiveness to risk in cost of holding excess reserves
a^E	4.500	Adjustment elasticity of bank equity returns growth
υ	1.8776	Labour preference parameter
ψ	0.1567	Cost of excess reserves ratio exceeding default rate
ϱ^D	0.004	Deposit rate related cost
ϱ^L	0.0078	Loan rate related cost
ϱ^E	0.0151	Equity rate related cost
$\bar{\varepsilon}$	1.6018	Idiosyncratic risk: Upper bound
$\underline{\varepsilon}$	0.3982	Idiosyncratic risk: Lower bound
ρ_R	0.970	Autocorrelation of interest rate
ρ_A	0.720	Autocorrelation of Productivity Shock
ρ_L	0.700	Autocorrelation of loan rate spread shock
ρ_S	0.670	Autocorrelation of preference shock
σ_A	0.070	S.D. of productivity shock
σ_L	0.014	S.D. of loan rate spread shock
σ_S	0.055	S.D. of of preference shock

collateral, $\chi = 97$.[28] Deposits and loans are determined so that at equilibrium the loans demanded equal the loans supplied, $l^d \equiv wh = l^s \equiv \frac{1-\tilde{r}}{1-\gamma}d$, and calibrated so that at the aggregate equilibrium the loan-to-output ratio matches the US data for that period, $l/y = 0.38$. In Table 1, the moments of the three shocks, $(\rho_t^A, \rho_t^L, \rho_t^\xi, \epsilon_t^A, \epsilon_t^L, \epsilon_t^\xi)$, along with the parameters υ, ψ, ϱ^D, ϱ^E, ϱ^L, $\underline{\varepsilon}$ and $\bar{\varepsilon}$, (where the condition $Mean(\varepsilon) = (\bar{\varepsilon} + \underline{\varepsilon})/2 = 1$, must also be satisfied), are calibrated to match the data targets in Table 2. The rest of the parameter values in Table 1, follow largely the existing literature. The parameter υ is chosen so that at the steady state, $h = 0.66$ (Ravenna and Walsh (2006)). We also assume, $\lambda_p = 6.0$, that implies a quarterly price mark-up of 20 per cent, $(\vartheta_p = 1.20)$ and price stickiness of, ω_p=0.693, (Christiano et al. (2014)).

[28]See also Agénor et al. (2014).

Table 2: Calibrated Parameters

Parameter	Notation	US Data (1985-2018)	Baseline model
Required Reserves to Deposits Ratio	$\widetilde{\varsigma}$	1.57	1.57
Excess Reserves to Deposits Ratio	$\widetilde{s}$	5.42	5.42
Loan to GDP Ratio	(l/y)	38.7	38.7
Deposit Rate Spread	$R - R^D$	1.60	1.60
Bank Equity Rate Spread	$R^E - R$	7.64	7.64
Loan Rate Spread	$R^L - R$	3.92	3.92
std(inflation)/std(GDP)	σ_π/σ_y	0.22	0.27
std(loans)/std(GDP)	σ_l/σ_y	5.86	3.01
std(reserves)/std(GDP)	σ_s/σ_y	30.04	29.67
std(Loan Rate Spread)/std(GDP)	$\sigma_{(R^L-R)}/\sigma_y$	0.36	0.25
cor(inflation,GDP)	$cor(\pi, y)$	0.09	0.09
cor(loans,GDP)	$cor(l, y)$	0.33	0.27
cor(reserves,GDP)	$cor(s, y)$	-0.59	-0.50
cor(Loan Rate Spread,GDP)	$cor((R^L - R), y)$	-0.28	-0.10

All spreads and ratios are reported in annualized percentage points whereas all standard deviations are reported in quarterly percentage points.

5 Optimal Interest on Reserves Rules and Welfare

This section performs a numerical welfare analysis based on the calibrated baseline model of section 4, to examine how optimal interest on reserves policy rules, can perform (i) as a 'complementary' monetary policy tool', when the conventional interest rate Taylor rule is active and (ii) as the 'main' monetary policy tool, under a pegged interest rate. The central bank's objective function is derived by a second order approximation around the efficient steady state of the household's expected utility function (1), where the consumer's welfare losses are expressed as a fraction of steady state consumption[29],

$$\mathbb{W} = E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U}{U_c c} \right) = -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\frac{\lambda_p}{k_p} \right) \ \widehat{\pi}_t^2 \ + (\eta + \sigma) \left(\widehat{x}_t \right)^2 \right] \tag{19}$$

where, $\widehat{x}_t = \widehat{y}_t - \widehat{y}_t^*$ is the output gap and $\widehat{y}_t^*$ is the log deviation of the efficient output from its steady-state. The average welfare loss per period is given by, $\mathbb{L} = \frac{1}{2} \left[(\lambda_p/k_p) \ var(\widehat{\pi}_t) \ + (\eta + \sigma) \, var(\widehat{x}_t) \right]$. The net welfare gains are estimated based on the difference in consumer welfare losses between the baseline

[29]The derivation of the welfare loss function follows closely Ravenna and Walsh (2006), who also incorporates a monetary policy cost channel. As with the latter study, in the efficient steady state, price mark ups and financial distortions are eliminated through appropriate subsidies.

Table 3: Welfare Results: Taylor Rules and Interest on Reserves Rules

Policy	Welfare	Standard Deviations					
		$\widehat{x}_t$	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{R}_t^D$	$\widehat{R}_t^{IOR}$	$\widehat{s}_t$
Baseline Taylor Rule	ϕ_π=1.50	0.0540	0.0212	0.0041	0.0041	0.0041	2.300
($\widehat{R}_t^{IOR}$=$\widehat{R}_t$)	ϕ_x=0.50						
	CE(%) = –						
Optimal Taylor Rule	ϕ_π=50.00	0.0166	0.0070	0.0281	0.0282	0.0281	0.473
($\widehat{R}_t^{IOR}$=$\widehat{R}_t$)	ϕ_x=13.40						
	CE(%)=0.0058						
Optimal Taylor Rule*	ϕ_π=50.00	0.0173	0.0074	0.0299	0.0279	0.0000	1.923
($\widehat{R}_t^{IOR}=0$)	ϕ_x=13.20						
	CE(%)=0.0057						
Optimal IOR to $\widehat{l}_t$	$\phi_\pi = \phi_x = 0$	0.0239	0.0133	0.0000	0.0284	0.4049	26.24
($\widehat{R}_t$=0)	μ^l= -6.26						
eq.(20)	CE(%)=0.0042						
Optimal IOR to $\widehat{x}_t$	$\phi_\pi = \phi_x = 0$	0.0066	0.0095	0.0000	0.0289	0.5310	31.92
($\widehat{R}_t$=0)	μ^x= -0.80						
eq.(18)	CE(%)=0.0055						
Optimal IOR to $\widehat{\Phi}_t$	$\phi_\pi = \phi_x = 0$	0.0163	0.0080	0.0000	0.0289	0.4128	24.86
($\widehat{R}_t$=0)	μ^Φ= -3.0						
eq.(21)	CE(%)=0.0057						

CE(%): Consumption equivalent values reported as net welfare percentage gains from Baseline Taylor Rule. All cases assume $R^{IOR} = R$. The grid search for the optimal policy parameters targets standard deviations of similar size for the deposit rate.

policy rule $\mathbb{W}^B$ and the optimal policy rule $\mathbb{W}_t^O$,

$$CE = \left\{1 - \exp\left[(1-\beta)\left(\mathbb{W}_t^O - \mathbb{W}_t^B\right)\right]\right\} \times 100,$$

where the higher is CE, a consumption equivalent measure, the larger is the net welfare gain of the optimal policy rule from its baseline case.

Proposition 2. *When the interest rate is pegged, simple optimal reserve rate rules that respond to key macroeconomic variables, such as, credit, risk of loan default, or the output gap, can act as the main monetary policy tool. Such rules are welfare improving in relation to a conventional Taylor rule and can provide similar welfare improvements to an optimal Taylor rule.*

Proof. The proof of Proposition 2 is based on Table 3, in conjuction with Appendix A.4 and Figure 2. □

Table 3, examines Proposition 2, for the baseline calibrated model (with all three shocks active), using policy rules (17), (18), and the following two simple interest on reserves rules,

$$R_t^{IOR} = R^{IOR}\left(\frac{l_t}{l}\right)^{\mu^l} \tag{20}$$

$$R_t^{IOR} = R^{IOR}\left(\frac{\Phi_t}{\Phi}\right)^{\mu^\Phi} \tag{21}$$

For a transparent comparison of these policy rules to the optimal Taylor rule, the grid search for all optimal policy parameters targets standard deviations of similar size for the deposit rate, since this is a key variable here. As shown in Table 3, under normal economic conditions the optimal Taylor rule appears to performs best, particularly when the interest on reserves is also supporting it, (case 'Optimal Taylor Rule, $\widehat{R}_t^{IOR}$=$\widehat{R}_t$'). However, for the same economic conditions, but with a pegged interest rate, interest on reserves rules are also shown to be, (i) welfare improving with respect to the benchmark Taylor rule and (ii) capable of producing similar consumption equivalent (CE) welfare units as the optimal Taylor rule, as for example, in case 'Optimal IOR to $\widehat{\Phi_t}$', responding to risk). It must be stressed that in this case the interest rate is completely fixed to its steady state value $R_t = R$, which implies, $\widehat{R}_t$=0 and monetary policy is driven purely the interest on reserves rule, eq. (21). Figure 2, illustrates the impulse responses of a productivity shock for the baseline Taylor rule (Baseline TR), the optimal Taylor rule (Optimal TR) and the optimal interest rate on reserves rule responding to default risk (Optimal IOR to $\widehat{\Phi}_t$), using the policy parameter values as shown in Table 3.[30] In the 'Baseline TR' case, the shock reduces inflation and the output gap which reduces the policy rate and the deposit rate by similar amounts. The fall in these rates encourages consumption but reduces deposits and loans at equilibrium. The default probability of borrowers also falls as the loan-to-output ratio falls and this also reduces the equity and loan rate spreads. Comparing the 'Baseline TR' case with the optimal responses of the Taylor rule and the interest on reserves, Figure 2 shows that both 'Optimal TR' and 'Optimal IOR to $\widehat{\Phi}_t$' affect consumption, the output gap and inflation in a very similar fashion, which also explains their similar welfare effects, (see Table 3). However, this result is reached through different channels under the two policies.

The 'Optimal TR' case implies a much stronger response to stabilizing inflation and the output gap through the policy rate, which implies a substantial fall in the policy rate and the deposit rate by similar amounts. These effects encourage consumption while reduce the main cost of funds for loans, the deposit rate (R_t^D). At equilibrium this results in a smaller output gap reduction and higher levels of consumption, deposits and loans. A similar response, and welfare effect, is achieved through the use

[30] These optimal values are based on all shocks rather than just the productivity shock examined here.

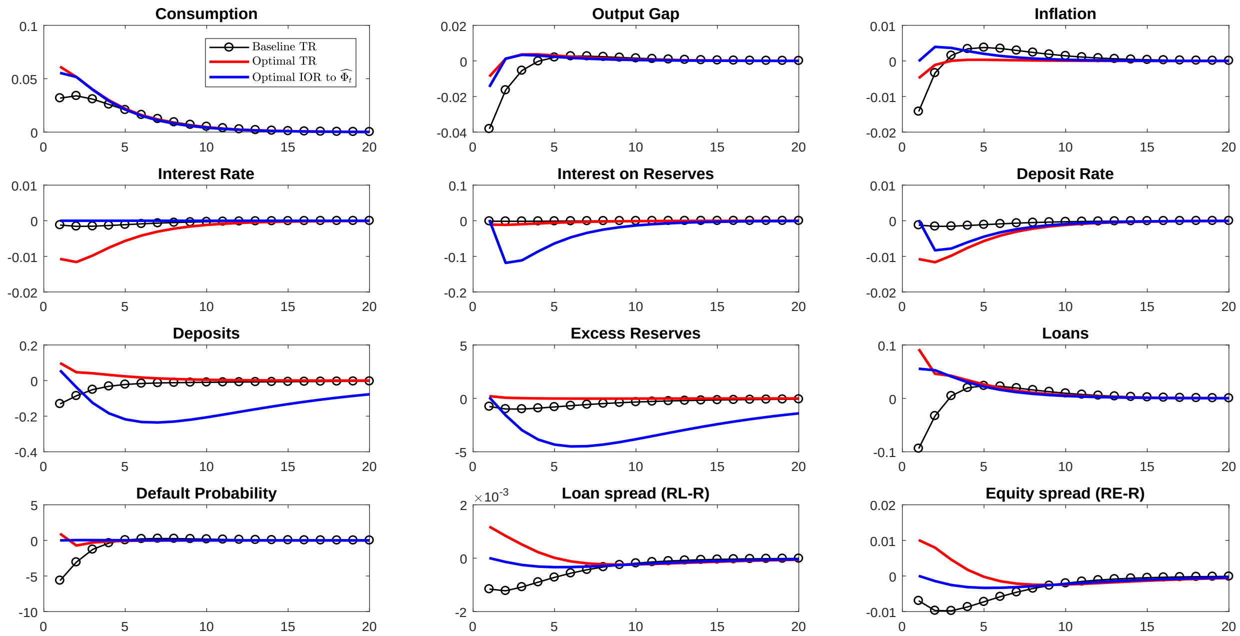

Figure 2: Taylor Rule and Interest on Reserves: A Productivity shock

of the optimal interest on reserve rule. However, since in the case 'Optimal IOR to $\widehat{\Phi}_t$', the interest rate is fixed ($\widehat{R}_t = 0$), the reduction in the deposit rate, and thus in the cost of loan funds, comes from a reduction in the interest on reserves. Moreover, the deposit rate here does not need to fall as much as with the optimal Taylor rule, because this policy also affects the level of reserves and thus it works also through the balance sheet channel, (see section 2.4). In this case, the fall in the interest on reserves, (below the fixed interest rate), acts as a tax on liquidity that creates a large gap between the fixed interest rate and the floor in the banking system, encouraging banks to release reserves towards loans, thus promoting both consumption and economic activity.

6 Occasional Binding Constraints and Monetary Policy at the ZLB

In this section we turn to assess the role of the interest on reserves when the economy is trapped in a zero-lower bound (ZLB) region. We consider a combination of a financial shock and a negative preference shock that are substantial to force the economy into the ZLB region for about four quarters. We then examine whether the interest on reserves can take over as the main monetary policy tool when the conventional interest rate is trapped in the ZLB region, unable to stimulate the economy.

To implement this we use the non-linear version of the model described in section 2, and the method of Guerrieri and Iacoviello (2015) for solving dynamic models with occasionally binding constraints. The advantage of using this method is that it produces high non-linearity while the duration of the economy being trapped in the ZLB regime depends on the state vector of all the endogenous variables in the model. Accordingly, we define

$$Z_t = \; R_{t-1}^{\phi} R^{(1-\phi)} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{x_t}{x} \right)^{\phi_x} \right]^{(1-\phi)}, \tag{22}$$

$$R_t = \max(Z_t, 1), \tag{23}$$

where Z_t is the notional policy instrument, as described by the Taylor Rule in section 2 and R_t is the actual interest rate, which is restricted by the occasionally binding constraint not to fall below 1, as shown in (23).[31] To make transparent the comparison between the conventional policy rate and the interest on reserves, but also to focus on the effects driven purely by the interest on reserves, independently of the way that risk affects the the level of reserves or the deposits rate, we update two equations of the baseline model in section 2, as follows. We assume that $\psi = 0$ and that banks do not internalise the effect of potential loan losses when setting their optimal reserves. Also for transparency, we assume that there are no fixed required reserves, (so that all reserves are excess reserves, $\widetilde{s}_t = \widetilde{r}_t$), and that in the cost of adjusting reserves, $\frac{\theta}{2} d_{j,t}(\widetilde{s}_{j,t} - \widetilde{s}_{t-1})^2$, the past target of the average level of excess reserves is fixed, (i.e. $\widetilde{s}_{t-1} = \widetilde{r}^*$).[32] Using these assumptions, together with $\varrho^d = 0$, the model in section 2, is updated with these two equations,

$$R_t^D = R_t - (R_t - R_t^{IOR})\widetilde{r}_t - \frac{\theta}{2}(\widetilde{r}_t - \widetilde{r}^*)^2, \tag{24}$$

$$\widetilde{r}_t = \widetilde{r}^* - \frac{R_t - R_t^{IOR}}{\theta}, \tag{25}$$

The two updated equations, (24) and (25), imply that at the steady state, where the interest rate equals the interest on reserves, the deposit rate is also equal to the former two, ($R = R^{IOR} = R^D$), and reserves are at their steady state. These assumptions imply that all three rates start from the same steady state value and thus make the net contribution of the interest on reserves policy with respect to the conventional policy rate, more transparent. The rest of the model remains the same as in section 2, with the exception that here we also allow the steady inflation to be $\pi > 0$, instead of unity. The full version of the non-linear model is described in Appendix A.5.

6.1 Main Monetary Policy Tool at the ZLB: The Conventional Interest Rate

We first consider the conventional case, where there is a 'zero-spread corridor system' where the interest on reserves is equal to interest rate, $R_t^{IOR} = R_t$, and the latter follows a standard Taylor rule.

[31]For transparency we also set, $\phi = 0$, as in Guerrieri and Iacoviello (2015).

[32]This is taken to be equal to the calibrated steady state of reserves, $\widetilde{r}^* = \; \widetilde{r} = 6.99$

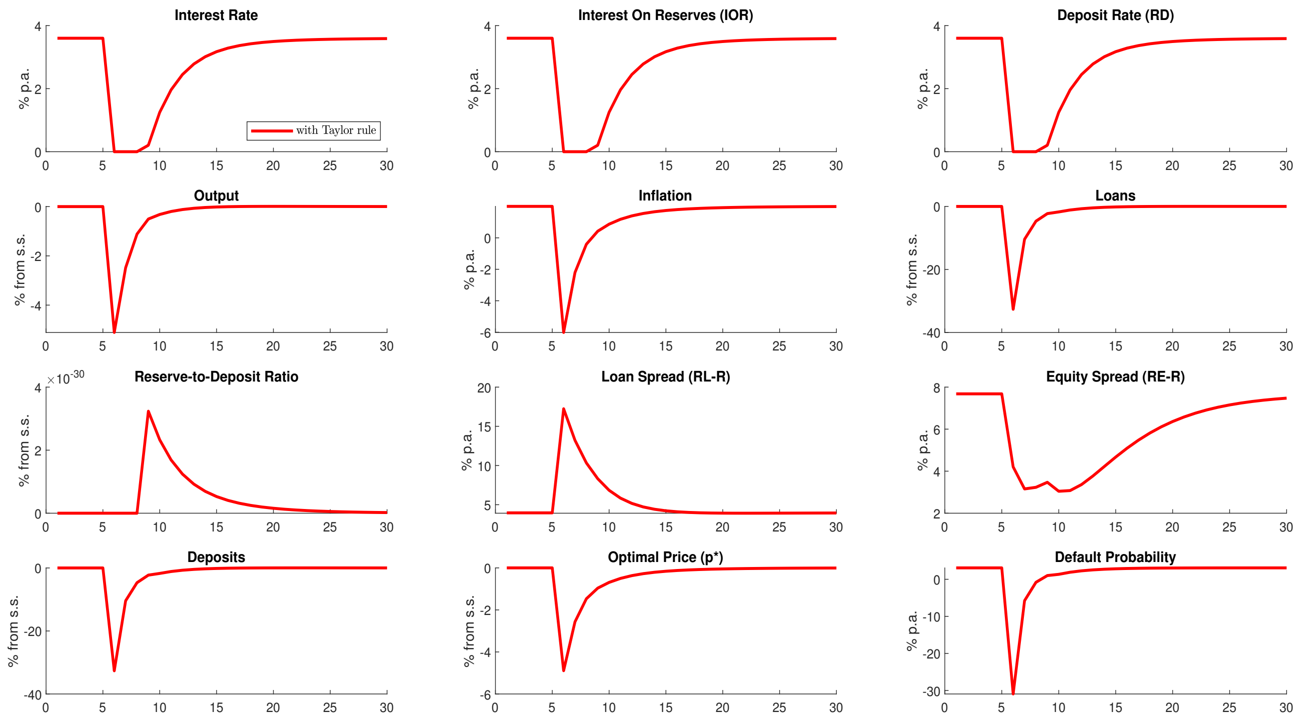

Figure 3: Occasionally Binding Constraints (OBC): Interest on Reserves follows Interest Rate

This implies that equations (24) and (25) reduce to, $R_t^{IOR} = R_t = R_t^D$, and $\widetilde{r}_t = \widetilde{r}^*$, and their respective steady states are, $R^{IOR} = R = R^D = \pi/\beta$. In this case, the intertemporal decisions of the household and thus the Euler equation, (3)-(5), are driven by the conventional interest rate, since $R_t^D = R_t$.

Figure 3 shows the effects of two shocks, a loan rate spread shock and a negative preference shock, hitting the economy simultaneously. The autocorrelation coefficients of the shocks are kept at their calibrated baseline values, $\rho_L = 0.70$ and $\rho_S = 0.67$, respectively, but their standard deviations are increased to, $\sigma_L = 0.035$ and $\sigma_S = 0.075$, respectively, to replicate the conditions required for a ZLB state. The shocks are shown to raise the loan spread and reduce loans by 32.6%.[33] With such a substantial reduction in loans, the exposure to risk also falls (default probability drops by approximately 7.5% quarterly), but this also reduces the return on bank equity. On impact, (within the first quarter) the shocks are shown to reduce GDP by 5.1% and deflate the economy by 6% per annum. These effects force the interest rate, endogenously (through the state vector of the model), to fall from its steady state and be trapped at the ZLB region for approximately 4 quarters, before the economy starts recovering again. As in this case the interest on reserves and the deposit rate follow the interest rate, and we have purposely removed the effect that risk may have on reserves and the interest on reserves, all three rates behave similarly at the ZLB region.

[33] Ivashina and Scharfstein (2010) show that new loans to large borrowers fell by 47% during the peak period of the financial crisis in the U.S. (fourth quarter of 2008), relative to the prior quarter.

6.2 Main Monetary Policy Tool at the ZLB: The Interest on Reserves

In this section, we re-examine the above scenario for the case where the central bank uses the interest on reserves as its main monetary policy tool and switches from a zero-spread corridor system to a floor system, where the interest on reserves targets the deposit rate, by using a rule that responds to credit conditions. The rationale here is that the monetary authority selects a policy rule for the interest on reserves with the view to affect the price of safe liquid assets (reserves and deposits). Thus, in a similar fashion that a central bank can use open market operations to affect the interbank market target rate, here it uses explicitly the interest on reserves to eventually affect the rate offered by banks on safe liquid assets, the deposit rate. Hence, although the main monetary policy tool is the interest on reserves, the policy instrument becomes the notional deposit rate.

Proposition 3. *During economic conditions that force the interest rate to be trapped in the zero-lower bound region, the interest on reserves can take over as the main monetary policy tool and stimulate the economy by aiming to reduce the price of safe liquid assets (the deposit rate here) and the level of reserves, so as to encourage a credit expansion. For such a policy to be most effective, the price of safe liquid assets targeted by the interest on reserves, should also set the ceiling target for the interbank rate, (i.e.* $R_t \leq R_t^D$*)*

Proof. This is provided by the example shown below and demonstrated in figure 4. □

Given the nature of this model, consider the interest on reserves rule described in (20), in section 5,

$$R_t^{IOR} = R^{IOR} \left(\frac{l_t}{l} \right)^{\mu^l}.$$

This rule, together with equations (24) and (25), imply that the interest on reserves deviates from the interest rate and the deposit rate. The main monetary tool is the interest on reserves, but the notional policy instrument becomes the deposit rate, thus the occasionally binding constraint is,

$$Z_t = R - (R_t - R_t^{IOR})\widetilde{r}_t - \frac{\theta}{2}(\widetilde{r}_t - \widetilde{r}^*)^2, \tag{26}$$

$$R_t = R_t^D = \max(Z_t, 1), \tag{27}$$

where Z_t is the notional deposit rate, and R_t^D is the actual deposit rate, which is restricted here by the occasionally binding constraint not to fall below 1. In addition, in accordance with Proposition 3, equation (27) also assumes that for this policy to be most effective, the deposit rate as determined by

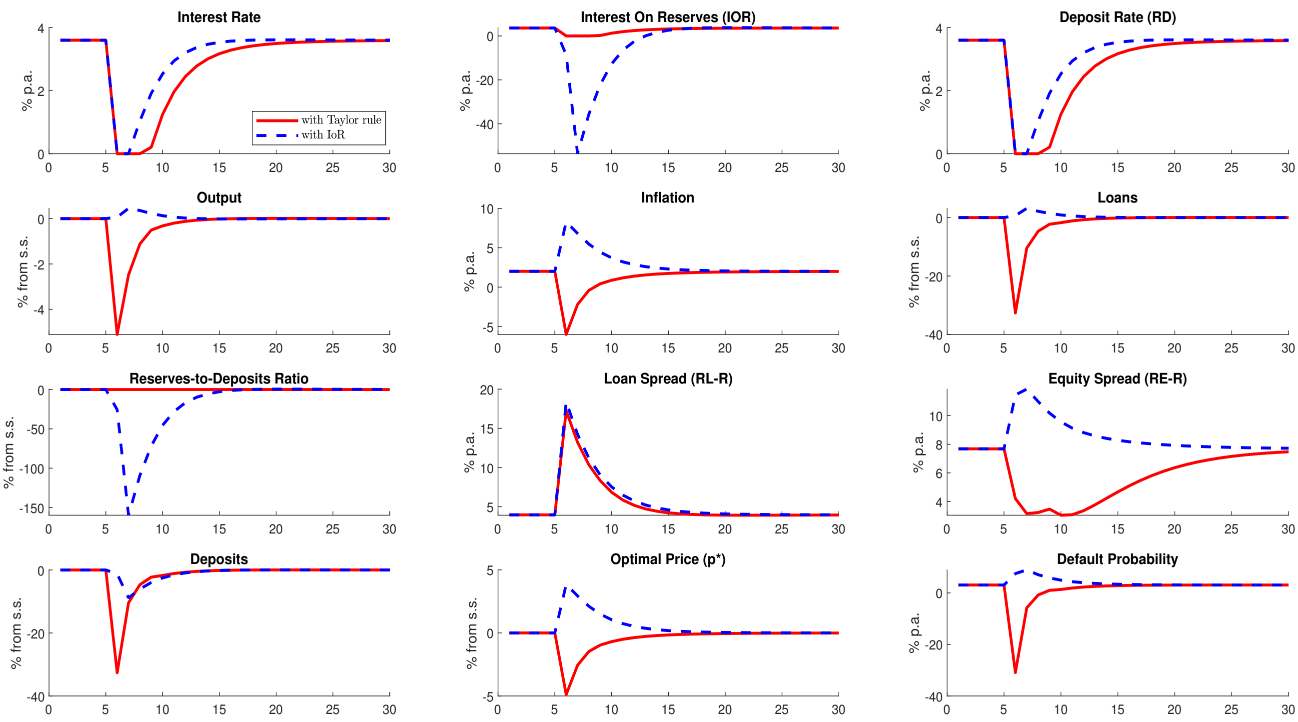

Figure 4: OBC: Interest on Reserves as Leading Monetary Tool

the interest on reserves, must set the ceiling for other safe assets, the interest rate on bonds here; thus for simplicity we set, $R_t = R_t^D$.[34] Intuitively, this is also to avoid the problem that if the interbank rate is higher than the deposit-savings rate, the private sector may turn to the accumulation of bonds as an investment, or collateral (Lenel et al. (2019)), in such times, undermining the release of credit to the real sector. However, such a large increase in the demand for bonds, could only be supported with the willingness of the Treasury to issue large amounts of government securities and thus support the economy through a fiscal expansion.

As figure 4 shows, the shocks initially force the deposit rate into the ZLB region, just as in the case where the conventional interest rate was the main monetary policy tool (red solid line). However here, the response of the interest on reserves rule to the fall in loans ($\mu^l = -4.5$), is to decrease the interest on reserves by a substantial amount, (-13.25% per quarter), so as to (i) increase substantially the cost of holding reserves while, (ii) reduce the real price of safe liquid assets (deposit rate). In this example, the reserves-to-deposits ratio falls by 159% from its steady state value, while the return on deposits and bonds are around zero. These two effects imply that as the heavily penalised reserves are forced to be reduced, banks have no alternative safe liquid asset to invest in for a higher return, (since the interest rate follows closely the deposit rate), and thus loans divert to the real sector, as the high loan spread becomes attractive. As a result, the deposit rate and hence the interest rate do not get trapped

[34]In this example we retain the assumption that at the steady state, $R^{IOR} = R^D = R = \pi/\beta$ and $\widetilde{r} = \widetilde{r}^*$. This together with the proposition that the interest rate must be set to follow the deposit rate, (rather the Taylor rule), implies that the deposit rate and the policy rate follow equation (26), starting from the fixed (steady state) value of $R^{IOR} = R^D = R$.

in the ZLB region, but instead they bounce back much faster, within a quarter here, and the economy avoids a significant recession. GDP is shown to rise by 0.45% from steady state as loans rise by 3.15%. Equity rate also rises to 11.85% (per annum) and as economic activity picks up and loans increase, the risk also rises by just over 2% per quarter. Deposits fall by 32%, therefore the rise in loans is met from the large drop in reserves and thus in the reserves-to-deposits ratio. Finally, since here the interest rate follows the deposit rate, which is driven by an interest on reserves rule targeting credit market conditions, rather than inflation (as with the Taylor rule), interest rates do not rise in line with rising prices, before the credit market and the economy have been fully stimulated. Such policy, therefore, could tend to encourage price growth and inflation. The model estimates that inflation can rise by 8.2% per annum.[35]

6.3 The Interest on Reserves with a Fixed Reserves-to-Deposits Ratio

In the previous example, the credit expansion following the interest on reserves policy was met by a substantial drop in the reserves-to-deposit ratio. This, as illustrated in figure 4, implied that the effectiveness of the interest on reserves relied heavily on the reserves channel. However, as shown earlier, the effectiveness of the interest on reserves in this paper can hold independently of the level of reserves. To test this proposition, this section replicates our interest on reserves policy at the ZLB scenario, illustrated in figure 4, under the assumption that the reserves-to-deposits ratio is fixed to its steady state value. As figure 5 shows, the interest on reserves policy is still effective in this case. This is because the interest on reserves does not work only by discouraging reserves, but simultaneously, by setting the price of liquid safe assets which determine the deposits and loans markets. As before, (in figure 4), the interest on reserves policy is shown to drive the economy out of the ZLB region, by making loans more attractive than holdings reserves. The only difference here is that since reserves cannot fall in relation to deposits, give the pegged reserves-to-deposits ratio, but loans are, in relative terms, an attractive option for both banks and the real sector, the increase in the loan supply must be met by an increase in deposits matched by reserves. This is particular true given that bank capital and borrowing from the central bank are also assumed to be fixed in our example. In this example, the 3.15% increase in loans is met entirely by an increase in deposits.

Finally, outside the occasional binding constraint framework that we examine here, where both the interest rate and the deposit rate are restricted not to fall below unity, negative interest rates and deposit rates could also be targeted by the interest on reserves policy pursued by central banks. In this

[35] In reading figure 4, it is worth noting that in this simple model of working capital with a strong cost channel effect, inflation can overreact to a fast recovery accompanied by a sudden and fast rise of interest and loan rates from ZLB levels.

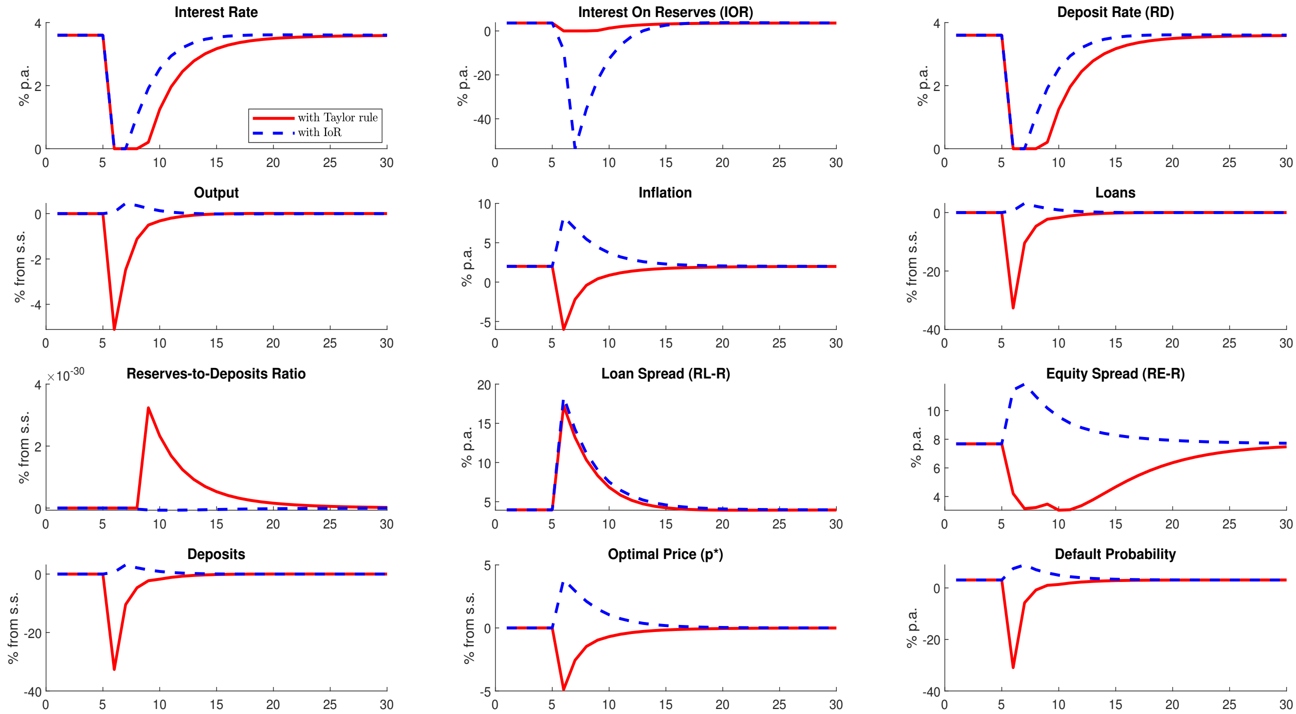

Figure 5: OBC: Interest on Reserves as Leading Monetary Tool: fixed reserves-to-deposits ratio

case, a negative real deposit-savings rate, that also sets the ceiling for the interest rate, as explained earlier, could help avert an over-accumulation of safe assets in the banking system, thus encouraging a faster credit expansion.

7 Concluding Remarks

This paper analyses the potential role of the interest on reserves as a leading monetary policy tool. It is shown that although the Taylor rule, responding to inflation and the output gap, performs best during normal times, the interest on reserves can also act as a leading monetary tool that can provide, independently, determinacy and deliver welfare improving results that could match those of an optimal Taylor rule. The paper also points to the potential role of the interest on reserves during periods where the interest rate is trapped at the zero-lower bound. When the interest rate cannot stimulate economic activity by affecting the price of interbank lending, the interest or reserves can do so by affecting simultaneously the cost of excess reserves and the price of safe assets and thus the reserves-to-deposits ratio. This result is shown to be effective when the conventional interest rate is trapped at the ZLB region, independently of the reserves-to-deposits ratio. Conversely, 'tightening monetary policy without draining reserves' is also possible with the use of the interest on reserves, irrespective of whether excess reserves are large or small, as supported by some international evidence, (Bowman et al., 2010).

The results in this paper have been demonstrated using interest on reserves rules that respond to three specific variables: the level of credit, credit risk, and the output gap. In practice, depending on the nature of shocks affecting the economy and the model considered, central banks may set the interest on reserves to respond to other macro or financial variables. Such policy rules may add new channels through which the interest on reserves can affect the economy. However, given the nature of this monetary policy tool, the two channels which are expected to remain central to the effectiveness of the interest on reserves, are (i) the *intertemporal Euler equation channel*, that is, through its effect on the price of safe liquid assets, which also affects the intertemporal decisions of households, and (ii) the *balance sheet channel*, that is, through its effect on the level of reserves and deposits which affects banks' decisions on loans to firms and households.

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A APPENDIX

A.1 Calibration Data

The calibration uses US data for the period: 1985 Q1 to 2018 Q4.

GDP:* Gross Domestic Product, Billions of Dollars, Quarterly, (Annual Rate, FRED)

FFR: Average effective federal funds rate (FRED)

Loan spread: Moody's Seasoned BAA Corporate Bond Minus Federal Funds Rate. (FRED)

Equity spread: Return on Average Equity for all U.S. Banks, Minus *FFR* (FRED)

Deposit Rate: on M2 Deposits, percent, (Divisia).

Deposit Spread: Deposit Rate on M2 Deposits Minus Federal Funds Rate.

*Required Reserves**: Required Reserves of all depository institutions, Billions of Dollars, Quarterly, (FRED)

*Excess Reserves**: Excess Reserves of all depository institutions, Millions of Dollars, Quarterly, (FRED)

*Deposits**: All Commercial Banks, Billions of U.S. Dollars, Quarterly, (FRED)

Population Level: Thousands of Persons, Quarterly, (FRED)

*Variables estimated as average quarterly growth of real per capita GDP

A.2 Bank's Optimization Problem

Each bank j keeps a fraction $\widetilde{r}_{j,t}$ of deposits as reserves, receiving R_t^{IOR} and uses the rest of funds, $(1-\widetilde{r}_{j,t})d_{j,t}+e_{j,t}+B_{j,t}$, as loans receiving loan rate R_t^L. Bank j maximizes,

$$\Pi_{j,t}^B = \widetilde{r}_{j,t}d_{j,t}R_t^{IOR} + \int_{\varepsilon_{j,t}^*}^{\bar{\varepsilon}} R_t^L l_{j,t} f(\varepsilon_{j,t})d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} \chi y_{j,t} f(\varepsilon_{j,t})d\varepsilon_{j,t} - R_t^D d_{j,t} - R_t^e e_{j,t} \tag{28}$$
$$- R_t B_{j,t} - \varrho_t^L l_{j,t} - \varrho^D d_{j,t} - \frac{\theta}{2}d_{j,t}(\widetilde{s}_{j,t}-\widetilde{s}_{t-1})^2 - \frac{\psi}{2}d_{j,t}(\widetilde{s}_{j,t} - \varphi\int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} f(\varepsilon_{j,t})d\varepsilon_{j,t})^2,$$

subject to,

$$l_{j,t} \leq (1-\widetilde{r}_{j,t})d_{j,t}+e_{j,t}+B_{j,t}, \tag{29}$$

$$e_{j,t} = \gamma l_{j,t}, \tag{30}$$

We can simplify the problem by using the definition of default, $\Phi_{j,t} = \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} f(\varepsilon_{j,t})d\varepsilon_{j,t}$, together with the properties of the uniform distribution and the fact that output is a function of labour and the latter depends on loans. First write, $\int_{\varepsilon_{j,t}^*}^{\bar{\varepsilon}} R_t^L l_{j,t} f(\varepsilon_{j,t})d\varepsilon_{j,t} + \int_{\underline{\varepsilon}}^{\varepsilon_{j,t}^*} \chi y_{j,t} f(\varepsilon_{j,t})d\varepsilon_{j,t} = R_t^L l_{j,t} -$

$\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}}(R^L_t l_{j,t} - \chi y_{j,t})f(\varepsilon_{j,t})d\varepsilon_{j,t}$. Use (7) and (8) to write $\varepsilon^*_{j,t}\chi A_t h_t = R^L_t l_{j,t}$ and substitute this and (6), $y_{j,t} = A_t\varepsilon_{j,t}h_t$, into $\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}}(R^L_t l_{j,t} - \chi y_{j,t})f(\varepsilon_{j,t})d\varepsilon_{j,t}$, to write $\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}}(\varepsilon^*_{j,t} - \varepsilon_{j,t})\chi A_t h_t f(\varepsilon_{j,t})d\varepsilon_{j,t}$. The latter is the potential bank's losses made during a bad state. Next we can use the fact that labour is hired using loans, to express the last term in terms of loans. From (7), $(l_{j,t} + \zeta y_{t-1})/w_t = h_t$, hence, $\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}}(\varepsilon^*_{j,t} - \varepsilon_{j,t})\chi A_t(\frac{l_{j,t}+\zeta y_{t-1}}{w_t})f(\varepsilon_{j,t})d\varepsilon_{j,t}$. Finally, we use the properties of $\varepsilon_{j,t}$, that is uniformly distributed over the interval $(\underline{\varepsilon}, \bar{\varepsilon})$, with a constant variance and a mean of unity, to write $\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}}(\varepsilon^*_{j,t} - \varepsilon_{j,t})\frac{\chi A_t l_{j,t}}{w_t}f(\varepsilon_{j,t})d\varepsilon_{j,t} = \Phi^2_{j,t}\frac{\bar{\varepsilon}-\underline{\varepsilon}}{2}\chi A_t(\frac{l_{j,t}+\zeta y_{t-1}}{w_t})$. Using the above information and the definition, $\widetilde{r}_{j,t} \ = \widetilde{s}_{j,t} + \widetilde{\varsigma}$, and also the assumption that in determining the optimal level of excess reserves the bank takes into account the fraction of deposits that potentially can be lost in loans in a bad state, (third term in equation below) bank $j's$ maximization problem becomes,

$$\Pi^B_{j,t} = (\widetilde{s}_{j,t} + \widetilde{\varsigma})d_{j,t}R^{IOR}_t + R^L_t l_{j,t} - \Phi^2_{j,t}\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\frac{\chi A_t(l_{j,t}[\widetilde{s}_{j,t}] + \zeta y_{t-1})}{w_t} - R^D_t d_{j,t} - R^E_t e_{j,t} \tag{31}$$
$$- R_t B_{j,t} - \varrho^D d_{j,t} - \varrho^L_t l_{j,t} - \frac{\theta}{2}d_{j,t}(\widetilde{s}_{j,t} - \widetilde{s}_{t-1})^2 - \frac{\psi}{2}d_{j,t}(\widetilde{s}_{j,t} - \varphi\int_{\underline{\varepsilon}}^{\varepsilon^*_{j,t}} f(\varepsilon_{j,t})d\varepsilon_{j,t})^2,$$

subject to,

$$l_{j,t} \leq (1 - \widetilde{s}_{j,t} - \widetilde{\varsigma})d_{j,t} + e_{j,t} + B_{j,t}, \tag{32}$$
$$e_{j,t} = \gamma l_{j,t}, \tag{33}$$

The Lagrangian of this problem is,

$$\mathbb{L}(B_{j,t}, l_{j,t}, d_{j,t}, \widetilde{s}_{j,t},) = (\widetilde{s}_{j,t} + \widetilde{\varsigma})d_{j,t}R^{IOR}_t + R^L_t l_{j,t} - \Phi^2_{j,t}\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\frac{\chi A_t(l_{j,t}[\widetilde{s}_{j,t}] + \zeta y_{t-1})}{w_t} - R^D_t d_{j,t}$$
$$- \gamma R^E_t l_{j,t} - R_t B_{j,t} - \varrho^L_t l_{j,t} - \varrho^D d_{j,t} - \frac{\theta}{2}d_{j,t}(\widetilde{s}_{j,t} - \widetilde{s}_{t-1})^2$$
$$- \frac{\psi}{2}d_{j,t}(\widetilde{s}_{j,t} - \varphi\Phi_{j,t})^2 + \boldsymbol{\lambda}_t\left[\frac{1}{1-\gamma}\left((1 - \widetilde{s}_{j,t} - \varsigma_t)d_{j,t} + l^{CB}_{j,t}\right) - l_{j,t}\right]$$

Each bank j therefore, fixes the supply of bank capital and lets the demand for equity from households, determine the equity rate. Taking the equity rate, R^E, and the policy rate, R, as given, the first order conditions are:

$B_{j,t} : \boldsymbol{\lambda}_t = (1 - \gamma)R_t$

$l_{j,t} : R^L_t - \lambda_t - \gamma R^E_t - \Phi^2_{j,t}\frac{(\bar{\varepsilon}-\underline{\varepsilon})}{2}\chi_t\frac{A_t}{w_t} - \varrho^L_t = 0$

$d_{j,t} : -R^D_t + R^{IOR}_t\widetilde{r}_{j,t} + \boldsymbol{\lambda}_t\frac{(1-\widetilde{r}_{j,t})}{1-\gamma} - \frac{\theta}{2}(\widetilde{s}_{j,t} - \widetilde{s}_{t-1})^2 - \frac{\psi}{2}(\widetilde{s}_{j,t} - \varphi\Phi_{j,t})^2 - \varrho^D = 0$

$\widetilde{s}_{j,t} : R^{IOR}_t d_{j,t} - \boldsymbol{\lambda}_t\frac{d_{j,t}}{1-\gamma} - \theta d_{j,t}(\widetilde{s}_{j,t} - \widetilde{s}_{t-1}) - \psi d_{j,t}(\widetilde{s}_{j,t} - \varphi\Phi_{j,t}) - \Phi^2_{j,t}\frac{(\bar{\varepsilon}-\underline{\varepsilon})}{2}\frac{\chi A_t}{w_t}\frac{\partial l_{j,t}}{\partial\widetilde{s}_{j,t}} = 0,$

where $\frac{\partial l_{j,t}}{\partial\widetilde{s}_{j,t}} = -d_{j,t}/(1 - \gamma)$. Using these equations and the fact that firms are symmetric, hence at equilibrium, the probability of default and decisions by all banks are identical, (so we drop the subscript j) and that there is no borrowing from the central bank, we obtain equations (14)-(5), in the text.

A.3 The Baseline Model: The full log-linearized system

The full log-linearized system of the baseline model is as follows.[36]. The rest of the parameter values are given in the baseline tables 2 and 1, of section 4.

The steady state

$A = 1$

$\varrho^L = 1$

$\xi = 1$

$\beta = 0.9949$

$\pi = 1.0$

$R = R^{IOR} = 1.0091$, based on the calibrated values of the baseline model, that also satisfy all the steady state calibrated spreads, $R^L - R$, $R - R^D$, $R^E - R$, while the steady state deposit rate is consistent with the Euler equation, $R^D = 1/\beta$, (for more details see section 4).

$R^D = 1/\beta = R - (R - R^{IOR})\widetilde{r} - \frac{\psi}{2}(\widetilde{s} - \varphi\Phi)^2 - \varrho^d$

$mc = 1/\vartheta_p, \qquad \vartheta_p = \frac{\lambda_p}{\lambda_p - 1}$

$w = \frac{Amc}{R^L}$

Using, $y = Ah$, and $h = (\frac{w}{vy})^{1/\eta}$

$v = \frac{h^{-(1+\eta)}mc}{R^L}$

$y = A\left(\frac{mc}{vR^L}\right)^{\frac{1}{(1+\eta)}}$

$x = \frac{y}{A}\vartheta_p^{\frac{1}{1+\eta}}$

$l = wh - \zeta y$

$d = \frac{1-\gamma}{1-\widetilde{r}}l$

$\widetilde{r} = \widetilde{s} + \widetilde{\varsigma}$

$\widetilde{s} = \varphi\Phi - (\frac{R - R^{IOR}}{\psi}) + \frac{\chi A}{\psi w(1-\gamma)}\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\Phi^2$

from $\Phi = \frac{\varepsilon^* - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$ *and* $\varepsilon^* = \frac{R^L l}{\chi y}$

$\bar{\varepsilon} = \frac{2\chi y(1-\Phi) - R^L l}{\chi y(1-2\Phi)}$

$\underline{\varepsilon} = 2 - \bar{\varepsilon}$

$R^E = \frac{R^D(1+\varrho^E)}{1-\Phi}$

$R^L = R + \gamma(R^E - R) + \frac{\chi A}{w}\frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\Phi^2 + \varrho^L.$

[36] Throughout this Appendix we have used $\sigma = 1$

The log-linearized dynamic system

At the aggregate equilibrium, aggregate demand is determined by consumption, $y_t = c_t$. On the production side equilibrium also requires that $\int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\lambda_p} y_t = \int_0^1 \varepsilon_{j,t} A_t h_t$. Using the distribution properties of the idiosyncratic shocks (that imply $\int_0^1 \varepsilon_{j,t} = 1$), aggregate equilibrium is, $y_t = A_t h_t / V_t$, where $V_t \equiv \int_0^1 \left(\frac{p_{j,t}}{p_t}\right)^{-\lambda_p}$ is the price dispersion index. At the aggregate equilibrium the loan market must also be in equilibrium, and deposits are determined endogenously. We also assume that at the aggregate equilibrium, there is no borrowing from the central bank so that $B_t = 0$. This implies that labour demand is, $l_t = w_t h_t - \zeta y_{t-1} = (1 - \widetilde{r}_t) d_t + e_t$, where the latter is the loan supply from the balance sheet. The model is log-linearized around its non-stochastic, zero inflation, flexible price steady state. The flexible price level of output is, $y_t^f = A_t \left(\vartheta_p R_t^{L,f}\right)^{-\frac{1}{1+\eta}}$, where $\vartheta_p = \lambda_p/(\lambda_p - 1)$ is the price mark-up and $R_t^{L,f}$ is the loan rate under flexible prices. The efficient level of output, free of both financial frictions (no cost channel) and nominal rigidities, is $y_t^* = A_t \left(\vartheta_p\right)^{-\frac{1}{1+\eta}} > y_t^f$.[37] Using these assumptions, the log-linearized aggregate equilibrium is summarized by 17 variables, $\{\widehat{x}_t, \widehat{y}_t, \widehat{\pi}_t, \widehat{w}_t, \widehat{l}_t, \widehat{d}_t, \widehat{R}_t^E, \widehat{R}_t^L, \widehat{\Phi}_t, \widehat{R}_t^D, \widehat{\widetilde{r}}_t, \widehat{\widetilde{s}}_t, \widehat{R}_t, \widehat{R}_t^{IOR}, \widehat{A}_t, \widehat{\varrho}_t^L, \widehat{\xi}_t\}$ and the following equations:

[37] See Ravenna and Walsh (2006).

Output gap	$\widehat{x}_t = \widehat{y}_t - \widehat{A}_t$
Euler equation	$\widehat{x}_t = \mathbb{E}_t \widehat{x}_{t+1} - (\widehat{R}_t^D - \mathbb{E}_t \widehat{\pi}_{t+1} + \mathbb{E}_t \widehat{\xi}_{t+1} - \widehat{\xi}_t) + \mathbb{E}_t \widehat{A}_{t+1} - \widehat{A}_t$
Calvo NKPC	$\widehat{\pi}_t = \beta \mathbb{E}_t \widehat{\pi}_{t+1} + k_p (1+\eta) \widehat{x}_t + k_p \widehat{R}_t^L$
Wages	$\widehat{w}_t = (1+\eta)\widehat{y}_t - \eta \widehat{A}_t$
Loan demand	$\widehat{l}_t^d = \frac{wy}{Al}(\widehat{w}_t + \widehat{y}_t - \widehat{A}_t) - \frac{y}{l}\zeta \widehat{y}_{t-1}$
Loan supply	$\widehat{l}_t^s = \frac{d}{l(1-\gamma)}((1-\widetilde{r})\widehat{d}_t - \widetilde{r}\widehat{\widetilde{r}}_t)$
Default probability	$\widehat{\Phi}_t = \frac{R^L l/y}{\chi \Phi(\bar{\varepsilon} - \underline{\varepsilon})} \left(\widehat{R}_t^L + \widehat{l}_t - \widehat{y}_t \right)$
Equity rate	$\widehat{R}_t^E = \frac{a^E}{1+a^E} \widehat{R}_{t-1}^E + \frac{R^D(1+\varrho^E)}{R^E(1+a^E)(1-\Phi)} \left(\widehat{R}_t^D + \frac{\Phi}{(1-\Phi)} \right) \widehat{\Phi}_t$
Loan rate	$\widehat{R}_t^L = \frac{(1-\gamma)R}{R^L} \widehat{R}_t + \frac{\gamma R^E}{R^L} \widehat{R}_t^E + \frac{\chi A \Phi^2}{R^L w} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (2\widehat{\Phi}_t + \widehat{A}_t - \widehat{w}_t) + \frac{\widehat{\varrho}_t^L}{R^L}$
Deposit rate	$\widehat{R}_t^D = \frac{R}{R^D} \widehat{R}_t - \frac{\widetilde{r}(R\widehat{R}_t - R^{IOR}\widehat{R}_t^{IOR})}{R^D} - \frac{(R-R^{IOR})\widetilde{r}}{R^D} \widehat{\widetilde{r}}_t - \frac{\psi(\widetilde{er} - \varphi\Phi)}{R^d} (\widetilde{s}\widehat{\widetilde{s}}_t - \varphi \Phi \widehat{\Phi}_t)$
Interest rate	$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1-\rho_R)(\phi_\pi \widehat{\pi}_t + \phi_x \widehat{x}_t)$
Reserve rate	$\widehat{R}_t^{IOR} = \widehat{R}_t$ (baseline case)
Reserves	$\widehat{\widetilde{r}}_t = \frac{\widetilde{s}}{\widetilde{r}} \widehat{\widetilde{s}}_t$
Excess Reserves	$\widehat{\widetilde{s}}_t = \rho_{\widetilde{s}} \widehat{\widetilde{s}}_{t-1} + \frac{1}{\widetilde{s}(\theta+\psi)} \left(-R\widehat{R}_t + R^{IOR}\widehat{R}_t^{IOR} + \psi\varphi\Phi\widehat{\Phi}_t + \frac{\chi A \Phi^2}{w(1-\gamma)} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} (2\widehat{\Phi}_t + \widehat{A}_t - \widehat{w}_t) \right)$
Productivity shock	$\widehat{A}_t = \rho_A \widehat{A}_{t-1} + \epsilon_t^A$
Loan spread shock	$\widehat{\varrho}_t^L = \rho_{\varrho^L} \widehat{\varrho}_{t-1}^L + \epsilon_t^{\varrho^L}$
Preference shock	$\widehat{\xi}_t = \rho_\xi \widehat{\xi}_{t-1} + \epsilon_t^\xi$

A.4 Local determinacy

Proposition 1: *The interest on reserves can provide determinacy by affecting the return on the safe liquid assets, the deposit rate. This result is independent of bank capital restrictions, credit risk, fiscal policy, or the supply of reserves. For $R^{IOR} \neq R$, determinacy is also independent of the reserves-to-deposits ratio. A necessary condition for determinacy in this model is that $\mu^x \neq 0$ and thus, $\partial \widehat{R}_t^D / \partial \widehat{R}_t^{IOR} \neq 0$, that is, that the interest on reserves can affect the deposit rate when it is set to respond to changes in the output gap, or other key macroeconomic variables.*

Proof: Consider a very simplified version of the model where the interest rate is initially fixed to a non-negative level, thus, $R_t = R$, where $R \geqslant 0$, and $\widehat{R}_t = 0$. We assume no financial frictions or risk, no cost channel, and no bank capital requirements, implying $\widehat{\Phi}_t = 0$, $\widehat{R}_t^L = \widehat{R}_t$, $\widehat{R}_t^E = 0$, and $\gamma = 0$, $\zeta = 0$. Simplifying further, assume that there is a zero required reserve ratio, so that total reserves are equal to excess reserves, ($\widehat{\widetilde{r}}_t = \widehat{\widetilde{s}}_t$) and there are no costs involved with adjusting excess reserves implying, $\psi = \varphi = 0$. For transparency we also remove the persistence in excess reserves $\rho_{\widetilde{s}}\widetilde{s}_{t-1} = 0$ and since we want to focus on the stability properties of the model around its steady state, we also assume no shocks. Using these assumptions and setting, $\sigma = 1$, and $A = 1$, as employed in the baseline model, the log-linearized equation system in Appendix A.3 can be reduced to five simple equations,

$$\widehat{x}_t = \mathbb{E}_t\widehat{x}_{t+1} - \widehat{R}^D_t + \mathbb{E}_t\widehat{\pi}_{t+1},$$

$$\widehat{\pi}_t = \beta\mathbb{E}_t\widehat{\pi}_{t+1} + k_p\left(1+\eta\right)\widehat{x}_t,$$

$$\widehat{R}^D_t = \frac{\widetilde{r}R^{IOR}}{R^D}\widehat{R}^{IOR}_t - \frac{(R-R^{IOR})\widetilde{r}}{R^D}\widehat{\widetilde{r}}_t,$$

$$\widehat{\widetilde{r}}_t = \frac{R^{IOR}}{\widetilde{r}\theta}\widehat{R}^{IOR}_t,$$

$$\widehat{R}^{IOR}_t = \mu^x\widehat{x}_t.$$

Substituting $\widehat{R}^{IOR}_t$ and $\widehat{\widetilde{r}}_t$ into $\widehat{R}^D_t$ and then into $\widehat{x}_t$, we can write the above equations into a 2 x 2 vector system,

$$\begin{bmatrix} \mathbb{E}_t\widehat{x}_{t+1} \\ \mathbb{E}_t\widehat{\pi}_{t+1} \end{bmatrix} = A\begin{bmatrix} x_t \\ \widehat{\pi}_t \end{bmatrix}, \quad \text{where,} \quad A = \begin{bmatrix} 1+B & 0 \\ 0 & 1/\beta \end{bmatrix},$$

where,

$$B = \left(\widetilde{r} - \frac{(R-R^{IOR})}{\theta}\right)\frac{R^{IOR}}{R^D}\mu^x,$$

It follows that a necessary condition for local determinacy is that,

$$1 - \left(\widetilde{r} - \frac{(R-R^{IOR})}{\theta}\right)\frac{R^{IOR}}{R^D}\mu^x\left(1-\frac{1}{\beta}\right) > 1.$$

From this it follows that when $R \geqslant 0$, ($\widehat{R}_t = 0$), and $\widetilde{r} \geqslant 0$, ($\widehat{\widetilde{r}}_t = 0$), a necessary condition for a determinate equilibrium is, $\mu^x \neq 0$. In particular,

(i) if, $\widetilde{r} > \frac{(R-R^{IOR})}{\theta}$, or if the interest rate is fixed to zero, $R = 0$, then a necessary condition for a unique equibrium is, $\mu^x < 0$.

(ii) if, $0 \leq \widetilde{r} < \frac{(R-R^{IOR})}{\theta}$, then a necessary condition for a unique equilibrium is simply that $\mu^x > 0$.

In all cases, $\mu^x \neq 0$ implies, (from 15), that $\partial\widehat{R}^D_t/\partial\widehat{R}^{IOR}_t \neq 0$, and thus that the interest on reserves can provide determinacy through its effect on the deposit rate.

A.5 Occasional Binding Constraints and Monetary Policy at ZLB: The Full Non-Linear System

The full non-linear system of the model used for section 6 is as follows. The rest of the parameter values are given in the baseline tables 2 and 1, in section 4.

The steady state

$A = 1$

$\varrho^L = 1,$

$\xi = 1$

$\beta = 0.996036$

$\pi = 1.005$

$R = R^{IOR} = R^D = \pi/\beta$

$\vartheta_p = \frac{\lambda_p}{\lambda_p - 1}$

$p^* = \left(\frac{\pi - \omega\pi^{\lambda_p}}{(1-\omega)\pi}\right)^{\frac{1}{1-\lambda_p}}$

$V = \left(\frac{(1-\omega)p^{*-\lambda_p}}{1-\omega\pi^{\lambda_p}}\right)^{\frac{1}{1-\lambda_p}}$

$y = A\frac{h}{V}$

$x = \frac{y}{A}\vartheta_p^{\frac{1}{1+\eta}}$

$Q2 = \frac{\xi p^* \pi}{\pi - \omega\beta\pi^{\lambda_p}}$

$v = \frac{(1-\omega\beta\pi^{\lambda_p})Q2V^{-\eta}\left(\frac{y}{A}\right)^{-(1+\eta)}}{\vartheta_p \xi R^L}$

$l = v\left(\frac{y}{A}\right)^{(1+\eta)} V^\eta y - \zeta y$

$d = \frac{1-\gamma}{1-\widetilde{r}} l$

$\widetilde{r} = \widetilde{s} = \widetilde{r}^*$

$\bar{\varepsilon} = \frac{2\chi y(1-\Phi) - R^L l}{\chi y(1-2\Phi)}$

$\underline{\varepsilon} = 2 - \bar{\varepsilon}$

$R^E = \frac{R^D(1+\varrho^E)}{1-\Phi}$

$R^L = R + \gamma(R^E - R) + \frac{\chi A}{v\left(\frac{V}{A}\right)^\eta y^{(1+\eta)}} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2}\Phi^2 + \varrho^L.$

The non-linear dynamic system

Output gap	$x_t = \frac{y_t}{A_t} \vartheta_p^{\frac{1}{1+\eta}}$
Euler equation	$\frac{1}{x_t} = \frac{1}{x_{t+1}} \frac{\beta R_t^D A_t \xi_{t+1}}{\pi_{t+1} A_{t+1} \xi_t}$
Price dispersion	$V_t = \omega \pi_t^{\lambda_p} V_{t-1} + (1-\omega) p_t^{*-\lambda_p}$
Calvo aggregate dynamics	$1 = (1-\omega) p_t^{*(1-\lambda_p)} + \omega \pi_t^{(\lambda_p - 1)}$
Optimal price dynamics	$Q2_t = p_t^* \left(\xi_t + \frac{\omega \beta \pi_{t+1}^{\lambda_p - 1}}{p_{t+1}^*} Q2_{t+1} \right)$
	$Q2_t = v \xi_t \vartheta_p R_t^L V_t^\eta \left(\frac{y_t}{A_t} \right)^{(1+\eta)} + \omega \beta \pi_{t+1}^{\lambda_p} Q2_{t+1}$
Loan demand	$l_t = v \left(\frac{y_t}{A_t} \right)^{(1+\eta)} V_t^\eta y_t - \zeta y_{t-1}$
Loan supply	$l_t = d_t \frac{1-\widetilde{r}_t}{1-\gamma}$
Default probability	$\Phi_t = \frac{\frac{R_t^L l_t}{\chi y_t} - \underline{\varepsilon}}{\bar{\varepsilon} - \underline{\varepsilon}}$
Equity rate	$R_t^E = \frac{a^E}{1+a^E} R_{t-1}^E + \frac{(1+\varrho^E)}{(1+a^E)(1-\Phi_t)} R_t^D$
Loan rate	$R_t^L = R_t + \gamma_t (R_t^E - R_t) + \frac{\chi A_t}{2v \left(\frac{V_t}{A_t} \right)^\eta y_t^{(1+\eta)}} \frac{(\bar{\varepsilon} - \underline{\varepsilon})}{2} \Phi_t^2 + \varrho_t^L$
Deposit rate	$R_t^D = R_t - (R_t - R_t^{IOR}) \widetilde{r}_t - \frac{\theta}{2} (\widetilde{r}_t - \widetilde{r}^*)^2$
Interest rate	$R_t = R_{t-1}^\phi R^{(1-\phi)} \left[\left(\frac{\pi_t}{\pi} \right)^{\phi_\pi} \left(\frac{x_t}{x} \right)^{\phi_x} \right]^{(1-\phi)}$
Interest on reserves	$R_t^{IOR} = R^{IOR} \left(\frac{l_t}{l} \right)^{\mu^l}$
Reserves	$\widetilde{r}_t = \widetilde{r}^* - \frac{R_t - R_t^{IOR}}{\theta}$
Productivity shock	$log(A_t) = \rho_A log(A_{t-1}) + \epsilon_t^A$
Loan spread shock	$log(\varrho_t^L) = \rho_{\varrho^L} log(\varrho_{t-1}^L) + \epsilon_t^{\varrho^L}$
Preference shock	$log(\xi_t) = \rho_\xi log(\xi_{t-1}) + \epsilon_t^\xi.$