Measuring Vulnerability Using the Counting Approach

Indranil Dutta
Ajit Mishra

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The University of Manchester
Manchester M13 9PL
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Indranil Dutta            Ajit Mishra
University of Manchester, UK  University of Bath, UK

ABSTRACT

Vulnerability has become an integral part of any deprivation assessment. In this paper we take a fresh look at measuring vulnerability, where we separate out the identification part of whether an individual is vulnerable from the aggregation part as has been done in the multi-dimensional poverty context. In doing so, we have also been able to deal with one of the crucial problems that we see in the multi-dimensional context, which is that of weights used on the different dimensions under aggregation. We axiomatically characterise this new measure of vulnerability and thus also provide a theoretical underpinning to many of the empirical applications in this field. Further, we provide a real world application of the measure using data from Bangladesh.

Key Words: Poverty, Vulnerability, Uncertainty.


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1 Introduction

Vulnerability is fast becoming an integral part of any deprivation assessment. The Human Development Report (UNDP, 2014, p.10) noted that, “Vulnerability threatens human development and unless it is systematically addressed, by changing policies and social norms, progress will be neither equitable nor sustainable.” It was followed by the UN Sustainable Development Goals in 2015, which suggested as an integral part of poverty reduction, the implementation of a “...nationally appropriate social protection systems and measures for all, ... , and by 2030 achieve substantial coverage of the poor and the vulnerable.”\(^1\) While these goals to reduce vulnerability recognises the uncertainty individuals face in their daily livelihoods, implicit in them is the assumption that we are able to target the vulnerable effectively. There are, however, considerable debate over how to identify the vulnerable, without which effective targeting is difficult. In this paper we take a fresh look at this issue by providing a new method of identifying the vulnerable.

In recent years there has been significant development in our understanding of how to measure and estimate vulnerability (see Calvo (2018); Gallardo (2017) and Fuji (2016) for comprehensive reviews). Most of these measures conceptualise vulnerability as an ex-ante forward looking measure of poverty as opposed to the more static notions of poverty. The focus of many of these studies is on calibrating vulnerability through ‘aggregation’ of the deprivation faced by individuals under various shocks, which typically is measured by the expected level of poverty in the future.\(^2\) Research on income poverty (Sen 1976) and multi-dimensional deprivation measures (Alkire and Foster, 2011), however, has shown that along with ‘aggregation’ of deprivation, the ‘identification’ of the deprived should be an important aspect of any evaluation of deprivation. This holds true also for the measurement of vulnerability.

Identification rules are crucial when it comes to measuring the level of vulnerability consistently across time and space. Varying identification rules makes it difficult to determine whether the changes in vulnerability across time and space arises out of different identification rules or other factors. We also need to have a clear identification strategy if we want the vulnerability measure to have any prescriptive value.\(^3\) Thus without really identifying who is vulnerable, the whole exercise of calculating individual vulnerability becomes a little vacuous. The primary purpose of this paper is thus to provide a framework for identifying the vulnerable individuals in a society.

When it comes to identifying the vulnerable, the literature can be broadly divided in to three approaches. The first method considers the vulnerable as those who are poor and those who are living on the edge of poverty. In that spirit, the Human Development Report (UNDP 2014) identifies

\(^1\)More details about the Sustainable Development Goals can be found at https://sustainabledevelopment.un.org/sdgs
\(^2\)There is also a strong literature where vulnerability is defined as low expected utility in the future (Lighon and Schecter, 2003)
\(^3\)This has been highlighted by Sen (1978) for the case of income poverty.
the vulnerable as those who are “poor along with other groups such as women, immigrants, indigenous groups and older people.” In essence, it broadens the vulnerable population from the poor by including those who may not have adequate protection from different shocks. Building on this idea, Dang and Lanjouw (2017), Chakaravarty (2016), and de la Fuente et al. (2015) construct a vulnerability line which is similar in principle to a poverty line: any one with income below the vulnerability line will be considered vulnerable. For Chakaravarty (2016) existing poverty lines can be adjusted, based on risk preferences of individuals, to yield a vulnerability line which reflects the minimum level of income needed to be considered not vulnerable. On the other hand, Dang and Lanjouw (2017) and de la Fuente et al. (2015) define vulnerability lines based on the minimum income that guarantee individuals a non-poor status in the future too. If future vulnerability is considered to be, say 10 percent, then the vulnerability line is that level of income of the non-poor households which ensures that ten percent of them will be poverty in the future. This approach needs information on the future period to decide on the vulnerability line in the current period, thus going against the ex-ante nature of vulnerability. More importantly, under this approach the vulnerable are just a broader set of the poor people since the vulnerability line is just a certain percentage above the poverty line, and thus may not capture vulnerability.

It does not take into consideration that some of the poor of today may be able escape poverty tomorrow and join the ranks of non-poor.

The second method of identification in the literature is found mainly in empirical studies on vulnerability which use a threshold to identify the vulnerable. Under this approach, vulnerability levels are calibrated for all individuals and only those above a certain threshold are deemed vulnerable. Studies such as, Imai et al. (2011), Jha and Dang (2010), Christiaensen and Subbarao (2005), Chaudhuri (2003), Chaudhuri et al. (2002), have used a vulnerability level of 0.5 to classify individuals as vulnerable. However this need not be the case always. Ward (2016), for instance, estimating vulnerability for China has taken the threshold to be 0.33. This arbitrary setting of the vulnerability threshold has been criticised by Hohberg et al. (2018) as adhoc. Instead they propose a method for arriving at the optimal threshold level of vulnerability by minimising the error of identifying individuals as vulnerable who do not fall in to poverty in the future. While this method reduces the arbitrariness of choosing a threshold, as has been alluded in the earlier case too, the information required to arrive at the threshold may be too demanding in the context of an ex-ante measure. Thus, standing today, we may not be able to know who will be poor tomorrow among all those whom we consider

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1 For Dutta et al. (2012) the vulnerability line depends on the poverty line and the current standard of living. However they do not necessarily capture poor along with those on the edge of poverty as vulnerable.

2 That vulnerability is fundamentally different from just accounting for the larger set of the poor has been argued in Dutta et al. (2011). Empirically also studies such as Ersado (2008) have found vulnerability to be different from poverty.

3 Most studies assume vulnerability to vary between [0, 1] where 1 reflects full vulnerability and 0 reflects no vulnerability. Zhang and Wan (2008) report that identifying individuals as vulnerable only if they are over 0.5 of the vulnerability measure improves predictive power of the measure.
vulnerable today. More generally, this second approach suffers from the intuitively unsatisfactory position where individual vulnerability levels are calculated before identifying whether the individual is vulnerable or not. One would have presumed that, similar to most deprivation analysis, it would be natural to first identify the vulnerable before we estimate how much vulnerability they suffer from.

The third approach, implicitly considers an individual as vulnerable to poverty if in future there is any possibility of the individual falling below the poverty line (Calvo and Dercon, 2013 and Glewwe and Hall, 1998). This is broadly in line with what is known as the ‘union’ approach in the literature. This approach suffers from the undesirable possibility of considering individuals as vulnerable who may be really rich in general but in certain rare situations, may actually fall in to poverty. On the other hand, as is done in the ‘intersection’ approach, one could also consider the possibility that only those individuals are vulnerable who are certain to be poor in the future (Angelilo, 2014). In such case, we would miss out on a substantial number of individuals who may have a high probability of falling in to poverty in the future but not with full certainty. Relying only on the ‘intersection’ approach, thus, may not be very useful particularly for measuring vulnerability.

We contribute to the literature by considering a more general approach of identifying the vulnerable without it being informationally demanding. In doing so, we follow the steps of some of the recent literature in the area of poverty and deprivation which address the issue of identification. Alkire and Foster (2011), in the context of multi-dimensional deprivation, identify an individual as multi-dimensionally deprived based on the number of dimensions the individual is deprived in. This is known as the counting approach. In Foster (2007), an individual is identified as chronically poor based on the number of time periods they were poor. In either context, what the exact number of dimensions that the individual needs to be deprived in or what the exact number of periods the individual must be poor in to be identified as multi-dimensionally poor or chronically poor respectively, is determined by a cut-off parameter which reflects a value judgement of the policy maker or the analyst. Atkinson (2017) in his report of the monitoring of global poverty for the World Bank had recommended the counting approach as an extremely useful tool in deprivation analysis.

In this paper we demonstrate that a similar framework may be quite appropriately used to identify the vulnerable, where the number of future income states where a person is deprived is used as a basis for the identification. We use the notion of income states to capture the different shocks individuals may face such as natural shocks like flood or drought or idiosyncratic shocks like health shocks. McCarthy et al. (2016) and Heltburg and Lund (2009) have used such information on discrete
shocks to study vulnerability. We propose a cut-off based on weighted sum of the number states the individual is likely to be deprived in the future. There can be several variants of this approach depending on the nature of these weights. In our context, probabilities associated with these states are taken as weights. We could also consider another variant where all states are given equal weights or the probability distribution over future states is uniform. Either of these methods can be used to identify the vulnerable individuals first, and then aggregation rules can be applied to compute the level of individual and societal vulnerability. To the best of our knowledge, using the counting based framework has so far not been applied to the context of identifying the vulnerable.

This paper can also be seen as providing a theoretical underpinning to many of the applied work in the context of vulnerability. As discussed earlier, most of these studies use a threshold level of vulnerability to determine who is vulnerable based on aggregating the deprivation arising from future distributions of income. Thus, under this approach, the identification function is also the same as their aggregation function. Hence, the two issues are not very distinct. There is, however, no absolute case for the identification and aggregation function to be the same since the purpose of identification and aggregation can be quite different. Here we provide an axiomatic framework which shows a different identification function compared to the aggregation function.

Further, we undertake an axiomatic characterisation of a class of vulnerability measures which are similar to the expected Foster Greer and Thorbecke (1984) (FGT) class. Although this class of measure is used in quite a few empirical papers, to the best of our knowledge an axiomatic characterisation has not been provided so far. In the next section we illustrate our concept of a vulnerability measure. Once we identify the individuals who are vulnerable, we engage in two levels of aggregation. First we aggregate the vulnerable individuals deprivation across the different states to find the individuals overall level of vulnerability. Next, given the individuals vulnerability, we use another aggregation rule to find the society’s vulnerability. In Section 3 we discuss some axioms and characterise the individual aggregation rule and in Section 4 we characterise the societal aggregation rule. An empirical application, using real world data from Bangladesh is provided in Section 5 of the paper. The final section draws the arguments of the paper together with some brief remarks about the further research.

9Apart from the studies referred in the previous paragraph, some of the other studies that employ the FGT class of measures are Zhang and Wan (2006), Azam and Imai (2009), Imai et al. (2010). Pritchett et al. (2000) uses the expected headcount in the multiperiod context.
2 The Concept of Vulnerability Measure

2.1 Notation

Consider a society of $N$ individuals. Suppose there be $m$ states of the world. Each individual $i$, faces a lottery, which shows the probability, $p_s$ and deprivation $d_s$ in each state $s$ of the world. Similar to Luce and Raiffa (1957) we represent the lottery as $L_i = (p_{i1}^1, d_{i1}^1; p_{i2}^2, d_{i2}^2; \ldots; p_{im}^m, d_{im}^m)$, where $m \geq 2$ and $\forall s, d_s \in [0, 1], p_s \geq 0$, and $\sum_{s=1}^{m} p_s = 1$. The probability vector associated with lottery $L$ is represented as $p_L = (p_{i1}^1, p_{i2}^2, \ldots; p_{im}^m)$, where $p_L \in P$ is the set of all probability distributions over the $m$ states. While for each individual we consider the deprivation profile $d_i = (d_{i1}^1, d_{i2}^2, \ldots; d_{im}^m)$ as given, the deprivation profile could differ between individuals. Let $L$ be the set of all such lotteries an individual may face different lotteries such as $L^i = (p_{i1}^1, d_{i1}^1; p_{i2}^2, d_{i2}^2; \ldots; p_{im}^m, d_{im}^m)$, and $L^0 = (p_{01}^1, d_{01}^1; p_{02}^2, d_{02}^2; \ldots; p_{0m}^m, d_{0m}^m)$. The individual has a complete and transitive preference ordering $\succ^i$ over $L$.

In this paper, to focus on the identification part we consider a cut-off based on the weighted average of the number of states in which the individual is deprived. Therefore, associated with lottery $L$ for individual $i$ we create a $(1 \times m)$ vulnerability identification vector $r = (r^1, r^2, \ldots; r^m)$ based on the following rule

$$
\forall s, r^{is} = \begin{cases} 
1 & \text{if } d^{is} > 0 \\
0 & \text{otherwise}
\end{cases}.
$$

It is clear that $r^i$ basically partitions the states into deprived and non-deprived states. To reduce notational burden, when there is no scope of confusion, we shall drop the superscript $i$, from the lotteries and the associated probabilities and deprivations when dealing with an individual.

The ‘counting’ function is defined as $\rho : L \rightarrow [0, 1]$, where $\rho(L) = 1$ would mean that an individual facing lottery $L$, is certainly deprived in the future whereas $\rho(L) = 0$, that the individual is certainly not deprived. We use a specific function, $\rho^E$, which is defined as

**Definition 1** $\rho^E$ is a counting function such that for any individual (i) $\rho^E(L) > \rho^E(L')$ iff $p_L \cdot r > p_L' \cdot r$ and (ii) $\rho^E(L) = \rho^E(L')$ iff $p_L \cdot r = p_L' \cdot r$.

We identify any individual facing lottery $L$ as vulnerable if the scalar product $p_L \cdot r \geq \theta$, where $\theta \in [0, 1]$ is the cutoff point. This means when $\theta = 0$, we consider the individual vulnerable if they are poor in any one future state with positive probability. On the other hand when $\theta = 1$ the individual is deemed vulnerable if he is poor in all future states. Once we identify the individual as vulnerable, we then use the lottery the individual faces to come up with the level of vulnerability an individual faces. Thus for each individual $i$, vulnerability is measured by $V^i : [0, 1] \times L \rightarrow \mathbb{R}_+$. In addition if
\[ \rho(L) < \theta, \text{ then } V^i(L) = 0. \] Throughout the paper when we discuss the vulnerability of one individual, we shall often denote the vulnerability measure is, \( V^i(L) \) as \( V(L) \).

For the society with \( N \) individuals and \( m \) future states of the world, we have a \( N \times m \) vulnerability matrix which we denote as \( M^{N,m} \). Each cell of the matrix shows the probability and deprivation associated with an individual in that state. Thus each row of the matrix lists the probability and deprivation of one individual over the different states which is effectively the lottery that the individual faces. Let \( \Phi \) denote the set of all such matrices. The societal vulnerability is a function \( V^S: \Phi \rightarrow \mathbb{R}_+ \).

### 2.2 The Measure

In this section we present our vulnerability measure in three steps and then illustrate it with an example. Suppose an individual faces a lottery \( L \). The first step focuses on identifying whether an individual is vulnerable or not and hence included in our overall vulnerability index. If the individual is identified as vulnerable from the first step i.e. \( p^L \cdot r \geq \theta \), then in the second step, we aggregate his deprivation across all the states and his vulnerability is given as

\[
V(L) = \sum_{s=1}^{m} p_s (d^s)^\alpha \quad \text{if } p^L \cdot r \geq \theta
\]

where \( \alpha \geq 0 \). Note that when \( \alpha = 0 \), there will be no difference between the aggregation rule proposed in (1) and the identification rule \( \rho^E(L) \).

Once each individual’s vulnerability has been computed, in the third step, we aggregate over all individuals to find out the societal level of vulnerability. Let \( M^{N,m} \) represent the societal deprivation matrix of \( N \) individuals over \( m \) future states. Then the societal vulnerability measure can be represented as

\[
V^S(M^{N,m}) = \frac{1}{N} \sum_{i=1}^{N} V^i(L)
\]

Before we proceed to the axiomatic characterisation of our measure, we can highlight an important feature of the identification strategy. When we talk about vulnerability, we often refer to the probability that the individual will be deprived in future. The common intuition uses the crudest (coarsest) partition of states; deprived or not deprived. Our identification method formalizes this intuition. Even when there are several states, while identifying whether an individual is vulnerable or not, we suppress the extent of deprivations in different states and consider the total probability that an individual is likely to be in a deprived state.
2.3 Illustrated Examples

Example 1 Consider the following vulnerability matrix with $N = 3$ and $m = 3$:

$$M^{3,3} = \begin{pmatrix}
\text{State 1} & \text{State 2} & \text{State 3} \\
0.3,0.1 & 0.6,0 & 0.1,0.7 \\
0.2,0 & 0.1,0 & 0.7,0.8 \\
0.5,0.6 & 0.25,0.4 & 0.25,0.2 \\
\end{pmatrix}$$

where in each cell the first number is the probability and the second is the deprivation faced by an individual. Thus the lotteries faced by each of the individuals are, $L^1 = (0.3,0.1;0.6,0;0.1,0.7)$, $L^2 = (0.2,0;0.1,0;0.7,0.8)$, and $L^3 = (0.5,0.6;0.25,0.4;0.25,0.2)$. Thus $r^1 = \{1,0,1\}$, $r^2 = \{0,0,1\}$, and $r^3 = \{1,1,1\}$.

First let us begin with Stage 1. Let the cut-off point $\theta = 0.5$. $p^{L^1} = \{0.3,0.6,0.1\}$ and the scalar product $p^{L^1} \cdot r^1 = 0.1 < 0.5$. Hence individual 1 is not considered vulnerable. On the other hand $p^{L^2} = \{0.2,0.1,0.7\}$ and $p^{L^2} \cdot r^2 = 0.7 > 0.5$ which implies that individual 2 is vulnerable. Similarly for the third individual, $p^{L^3} = \{0.5,0.25,0.25\}$ and $p^{L^3} \cdot r^3 = 1 > 0.5$ which indicates that individual 3 is also vulnerable.

In stage 2 using (1) and $\alpha = 1$, $V^1(L^1) = 0$, $V^2(L^2) = 0.56$ and $V^3(L^3) = 0.45$.

In stage 3, the overall societal vulnerability will be $V^S(M^{3,3}) = 1/3(V^1(L^1)+V^2(L^2)+V^3(L^3)) = 0.34$.

In the next example we show why the proposed measure will be different from the notion of vulnerability as expected poverty. For any lottery $L = (p_1,d^1; p_2,d^2; \ldots; p_m,d^m)$, the vulnerability of an individual based on the expected poverty is given by

$$E_p(L) = \sum_{s=1}^{m} p_s (d^s)^\alpha$$

where $\alpha \geq 0$. Note that this is similar to the vulnerability measure under the counting approach (1) when $\theta = 0$ and $\alpha = 1$. Thus an individual is considered vulnerable under the expected poverty approach if they are deprived in any state.

Example 2 Suppose the lotteries faced by an individual are: $L = (5/6,0.3;1/6,0;0,0.95)$, and $L' = (0,0.3;2/3,0;1/3,0.95)$. Note that the deprivation profile is the same in both the lotteries. Let $\theta = 0.5$ as in the previous example.

The vulnerability for these lotteries under the expected poverty approach (3) are $E_p(L) = 0.25 < 0.316 = E_p(L')$.

For calibrating vulnerability under the counting approach, note $r = \{1,0,1\}$, $p^L = \{5/6,1/6,0\}$, and
\( \mathbf{p}^{k'} = \{0, 2/3, 1/3\} \). Then \( p^{k'} \cdot \mathbf{r} = 5/6 > 0.5 \) and \( p^{k'} \cdot \mathbf{r} = 0.33 < 0.5 \). Given \( \theta = 0.5 \), thus \( V(L) > 0 \), where as \( V(L') = 0 \).

Thus under the expected poverty approach lottery \( L^1 \) has lower vulnerability than \( L^2 \), where as under the counting approach to vulnerability suggested in this paper we see \( L^1 \) with a higher vulnerability. This reversal occurs because in the latter approach, the identification method rules out individual 2 as vulnerable. Therefore, the ranking of lotteries by expected poverty approach to vulnerability would be very different from the ranking provided by the measure proposed in this paper.

3 Individual’s Measure of Vulnerability

Properties of the vulnerability measures will differ based on whether they are aimed at the individual level or the societal level. We first postulate the axioms for the vulnerability at the individual level and then the axioms related to the overall society measure is considered. The individual vulnerability measure will have an identification rule, and also the aggregation rule across states. Here we characterise these functions separately.

3.1 Characterisation of the Counting Function

The first axiom just captures the notion that if two lotteries that have only one state of positive deprivation with positive probability, then the lottery which has a higher probability of the deprived state should be ranked higher in terms of vulnerability compared to the other lottery. Thus if the lottery with the lower probability in the deprived state is considered to be vulnerable, so should the other.

**Axiom 1** Axiom of Single State Deprivation (A1): Consider a lottery \( L \) such that \( p_k \cdot r^k > 0 \), and \( \forall s \neq k, p_s \cdot r^s = 0 \). Let \( L' \) be such that \( p'_{k'} \cdot r^{k'} > 0 \), and \( \forall s \neq k', p'_s \cdot r^s = 0 \). If \( p'_{k'} > p_k > 0 \) \( [p'_{k'} = p_k > 0] \) then \( \rho(L') > \rho(L) \) \( [\rho(L') = \rho(L)] \).

The intuition for the next axiom is quite straight forward. Consider a lottery with deprivation in \( k \) states. If we have another lottery with deprivation in an additional state besides the \( k \) states, where probability of the additional state of deprivation has been transferred from one of the previous \( k \) deprived states, then potential vulnerability of this lottery should be the same as the previous lottery. Before we state the axiom, let us provide a further definition. Let any given lottery \( L \), with \( k \) deprived states with positive probability as \( L_k \).
**Definition 2** We say $L_{k+1}'$ is derived through a probability transfer from $L_k$ where $L_k = (p_1, d_1; p_2, d_2; ...; p_k, d_k, p_{k+1}; p_{k+1}, d_{k+1}; ..., p_m, d_m)$, and $L_{k+1}' = (p_1, d_1; p_2, d_2; ...; p_k - \delta, d_k; p_{k+1} + \delta, d_{k+1}; ..., p_m, d_m)$, $p_k > \delta > 0$ and $d_k > 0$, $d_{k+1} > 0$.

Given this definition we can now state the axiom formally.

**Axiom 2** Axiom of Invariance to Probability Transfers (A2): Suppose $L_{k+1}'$ is derived through a probability transfer from $L_k$, where $k < m$. Then $\rho(L') = \rho(L)$.

Given these two axioms we can show the following result.

**Theorem 1** A counting function $\rho$ satisfies Axioms of Single State Deprivation (A1) and Axiom of Invariance to Probability Transfers (A2) iff $\rho = \rho^E$.

Proof: Only if.

Suppose $\rho$ satisfies A1 and A2. Then given any two lotteries $L$ and $L'$ we show the following:

Case I:

$$\mathbf{p}^L \cdot \mathbf{r} > \mathbf{p}^{L'} \cdot \mathbf{r} \implies \rho(L) > \rho(L')$$

We apply the method of induction. Let $n$ stand for the number of deprived states with positive probability.

Suppose $n = 1$. Consider two lotteries $L_1$ and $L_1'$. Suppose for $L_1$ the deprived state with positive probability is $s$ and for $L_1'$ it is $s'$. In this case

$$\mathbf{p}^{L'} \cdot \mathbf{r} > \mathbf{p}^{L'} \cdot \mathbf{r} \implies p_s^L > p_{s'}^L$$

where $p_s^L$ is the probability of state $s$ in lottery $L$. Then from axiom A1 we can show that

$$p_s^L > p_{s'}^L' \implies \rho(L) > \rho(L')$$

Suppose it is true for $n = k < m$ states, which implies that for any lottery $L$ and $L'$

$$\sum_{n=1}^{k} p_n^L r^n > \sum_{n=1}^{k} p_n^{L'} r^n \implies \rho(L) > \rho(L'). \tag{4}$$

Now consider $n = k + 1 \leq m$, we have to show that

$$\sum_{n=1}^{k+1} p_n^L r^n > \sum_{n=1}^{k+1} p_n^{L'} r^n \implies \rho(L) > \rho(L').$$

Consider any lottery, $L(k+1) = (p_1, d_1; p_2, d_2; ...; p_k, d_k; p_{k+1}, d_{k+1}; ..., p_m, d_m)$ such that (i) $\forall s$, $s < k + 1$, $p_s > 0$ and $d_s > 0$ and (ii) $\forall s$, $s > k + 1$, $p_s = 0$ or $d_s = 0$. Now construct a lottery $\hat{L}(k)$
from $L(k+1)$ such that $\tilde{L}(k) : (p^L_1, d^1; p^L_2, d^2; \ldots; p^L_k, d^k; 0, d^{k+1}; \ldots; p^L_m, d^m)$, where $\tilde{p}_k = (p^L_k + p^L_{k+1})$.

Then from axiom A2, $\rho(L(k+1)) = \rho(\tilde{L}(k))$. Suppose by repeated transfer of probability we arrive at $\tilde{L}(1) : (\tilde{p}_1, d^1; 0, d^2; \ldots; 0, d^m)$, where $\tilde{p}_1 = \sum_{n=1}^{k+1} p_n$. From axiom A2 we can derive,

$$\rho(L(k+1)) = \rho(\tilde{L}(1)).$$

Similarly for a lottery $L'(k+1) : (p'_1, d^1; p'_2, d^2; \ldots; p'_{k+1}, d^{k+1}; \ldots; p'_m, d^m)$ through repeated transfer of probability we can arrive at $\tilde{L}'(1) : (\tilde{p}'_1, d^1; 0, d^2; \ldots; 0, d^m)$, where $\tilde{p}'_1 = \sum_{n=1}^{k+1} p'_n$. From axiom A2 we know

$$\rho(L'(k+1)) = \rho(\tilde{L}(1)).$$

Suppose, without loss of generality, $p^L_1 \geq p'^L_1$, then using the axiom A1, (5) and (6) we can claim

$$p^L_1 > p'^L_1 \implies \rho(\tilde{L}(1)) > \rho(\tilde{L}'(1)),
\sum_{n=1}^{k+1} p_n > \sum_{n=1}^{k+1} p'_n \implies \rho(L(k+1)) > \rho(L'(k+1)).$$

Case II:

$$p^L \cdot r = p'^L \cdot r \implies \rho(L) = \rho(L').$$

The proof is similar to Case I and is omitted.

It can be easily checked that the sufficient conditions are satisfied. \hfill \blacksquare

This, however, is not the only ‘counting’ function that can be used. For instance, we can have a counting function based on the number of deprived states the individual faces. In that case an individual is identified as vulnerable if the number of the deprived states with positive probability of occurrence exceeds a given level. Thus it is quite similar to the counting approach in the context of multi-dimensional deprivation. In Appendix B we provide a characterisation of such a counting function.

### 3.2 Axioms for the Individual’s Vulnerability Measure

We bring the identification part along with the notion of vulnerability in the following two axioms.

Suppose for an individual with lottery $L$, after applying an identification rule (using $\rho^E(L)$), the individual is identified as vulnerable, i.e. $V(L) > 0$. Then, any other individual with lottery $L'$, such that $\rho^E(L') > \rho^E(L)$, must also be identified as vulnerable, i.e. $V(L') > 0$. \footnote{This has a similar appeal to the Poverty Consistency Axiom proposed in Lasso de la Vega (2009).}
Axiom 3  **Axiom of Consistency (A3)** Consider two lotteries $L$ and $L'$, such that $\rho^F(L') > \rho^F(L)$. If $V(L) > 0$, then $V(L') > 0$.

The next axiom identifies whether individuals should considered as vulnerability or not for the extreme values of $\rho^F(L)$. When there is no possibility of being poor in the future, then the individual should not be identified as vulnerable. On the other hand, if the individual is definitely going to be poor in the future then we should identify him as vulnerable.

Axiom 4  **Axiom of Identification (A4)** Consider two lotteries $L$ and $L'$ such that $\rho^F(L) = 1$, and $\rho^F(L') = 0$, then $V(L) > 0$, and $V(L') = 0$ respectively.

Our next axiom follows the equivalent of the focus axiom that has been put forth in Alkire and Foster (2010) and Foster (2007), in the context of multi-dimensional deprivation and chronic poverty. What we are trying to capture through this axiom is the equivalent of the focus axiom in the context of vulnerability. In the Alkire and Foster (2011) approach there is a cut-off based on the number of dimensions one must be poor to be considered as deprived in that context. In the current context, we have a cut-off based on the probability that the person is poor in the future. So for instance, consider a lottery $L = (0.15, 1; 0.2, 1; 0.1, 1; 0.25, 0; 0.2, 0)$ faced by an individual. The individuals is deprived in the first three states only. Now if we consider the cut-off level of $\rho^F(L)$ to be deemed vulnerable as 0.4, then obviously the person is vulnerable. This is intuitive since the total probability that the individual will be deprived in the future is 0.45.

Axiom 5  **Focus Axiom (A5):** Consider any $L = (p_1, d_1; p_2, d_2; \ldots; p_m, d_m)$ such that $\rho^F(L) < \theta$. Then $V(L) = 0$.

The axiom states that vulnerability emanating from any lottery not making the cutoff is zero. We shall demonstrate later that the Focus axiom can be derived from more primitive properties.

The next two axioms that we present is similar to those of Dutta Foster and Mishra (2012). First, we define the convex combination of two lotteries as follows:

**Definition 3** Suppose $L = (p_1, d_1; p_2, d_2; \ldots; p_m, d_m)$ and $L' = (p'_1, d_1; p'_2, d_2; \ldots; p'_m, d_m)$. Then $\lambda L + (1 - \lambda)L' = (\lambda p_1 + (1 - \lambda)p'_1, d_1; \lambda p_2 + (1 - \lambda)p'_2, d_2; \ldots; \lambda p_m + (1 - \lambda)p'_m, d_m)$, where $0 < \lambda < 1$.

The following axiom states that the vulnerability of a convex combination of lotteries should be the same as the convex combination of the vulnerability of each of the lotteries.

Axiom 6  **Axiom of Decomposability (A6):** Consider any two deprivation lotteries $L$ and $L'$ such that $V(L) > 0$ and $V(L') > 0$. Then $V(\lambda L + (1 - \lambda)L') = \lambda V(L) + (1 - \lambda)V(L')$. 

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The implication of this axiom would be to make the vulnerability measure linear in probabilities. It will thus generate the von Neuman-Morgenstern expected utility structure for the vulnerability measure.

The intuition for our next axiom comes from the well known monotonicity axiom for poverty (Sen, 1976). In the context of vulnerability, it means that vulnerability should increase when the probability of a bad state occurring increases relative to a better state. It implies that to reduce vulnerability we have ensure that the individual becomes less likely to fall in to high deprivation states.

**Axiom 7** *Axiom of Monotonicity (A7):* Consider two lotteries $L = (p_1, d_1; \ldots; p_k, d_k; \ldots; p_m, d_m)$ and $L' = (p_1, d_1; \ldots; p_k + \delta, d_k; \ldots; p_m, d_m)$, $\delta > 0$ such that $p_k > 0$, $p_k' > 0$, $d_k \geq d_k' > 0$. Then $V(L) < V(L').$

Keeping with the previous theme of transfers of probability across states, the next axiom addresses the question on the change in vulnerability due to probability transfers when there is a reduction in deprivation across all states. This reduction in deprivation can happen due to economic development or better provision of insurance options to protect against different shocks. The formal statement of the axiom is as follows

**Axiom 8** *Axiom of Scale Variant Transfer (A8):* Consider lotteries $L = (p_1, d_1; \ldots; p_k, d_k; \ldots; p_m, d_m)$, $L' = (p_1, d_1; \ldots; p_k + \delta, d_k; \ldots; p_m, d_m)$, $\tilde{L} = (p_1, \tilde{d}_1; \ldots; p_k, \tilde{d}_k; \ldots; p_m, \tilde{d}_m)$, $\tilde{L}' = (p_1, \tilde{d}_1; \ldots; p_k + \delta, \tilde{d}_k; \ldots; p_m, \tilde{d}_m)$, such that $p_k > 0$, $p_k' > 0$, $d_k \geq d_k'$, $p_k > \delta > 0$ and $\forall s \tilde{d}_k = \lambda d_k$, $0 < \lambda \leq 1$. Then $V(\tilde{L}') - V(\tilde{L}) = g(\lambda)(V(L') - V(L))$, where $0 < g(\lambda) \leq 1$.

We compare the probability transfers under two situations reflected in the lotteries $L$ and $\hat{L}$ where in $\hat{L}$ the deprivation levels are proportionally lower across all the states relative to $L$. The axiom implies that as the deprivation decreases across the board, the change vulnerability due to transfers also reduces. Thus a society with better social safety nets, should see a lower increase in vulnerability when individuals become more prone to higher deprivation, compared to a society without such safety nets.

The final axiom is the normalization axiom, which reflects the intuition that if a state which happens with certainty has the highest deprivation level, then vulnerability should be maximum. Similarly on the other hand if deprivation of the certain state is at the lowest then vulnerability should be also at minimum.

**Axiom 9** *Axiom of Normalization (A9):* Let $L = (p_1, d_1; \ldots; p_m, d_m)$, with, $d_k = 1$ and $d_k' = 0$. Then if $p_k = 1$, $V(L) = 1$. If $p_k' = 1$ then $V(L) = 0$. 

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3.3 Characterisation of the Individual Measure

In this section we characterise the two broad class of vulnerability measures presented in Section 2. Before we characterise the measures, we demonstrate some of the key implications of the axioms.

First, we show that Axiom of Decomposability (A6) implies continuity of the vulnerability function $V(L)$.

**Lemma 1** Axiom of Decomposability (A6) implies that the vulnerability function $V(L)$ is continuous.

Proof: Please see Appendix A

Next, we demonstrate that the focus axiom, which is one of the key to the counting based approaches, can be derived from other more primitive axioms.

**Lemma 2** Axiom of Consistency (A3) and Axiom of Identification (A4) $\implies$ Focus (A5).

Proof: Please see Appendix A

We first characterise a general measure based on (1) in the following Theorem.

**Theorem 2** A vulnerability index $V$ of a given lottery $L$ satisfies Axiom of Consistency (A3), Axiom of Identification (A4), Axiom of Decomposability (A6), Axiom of Monotonicity (A7) iff

$$V(L) = \begin{cases} \sum_{s=1}^{m} p_s f(d^s) & \text{if } p^L \cdot r \geq \theta \\ 0 & \text{otherwise} \end{cases}$$

where $f(d^s)$ is monotonic and continuous.

Proof: We start by proving the necessary condition. First consider any lottery $L = (p_1, d_1; \ldots; p_m, d^m)$ faced by an individual where $p^L \cdot r \geq \theta$. From Lemma 1 of Dutta et al. (2012) we know that A6 implies that

$$V(L) = \sum_{s=1}^{m} p_s f(d^s). \quad (7)$$

Consider two lotteries $L = (p_1, d_1; \ldots; p_{k'}, d^{k'}; p_k, d^k; \ldots; p_m, d^m)$ and $L' = (p_1, d_1; \ldots; p_{k'} + \delta, d^{k'}; p_k - \delta, d^k; \ldots; p_m, d^m)$, such that $p_{k'} > 0; p_k > 0, d^{k'} \geq d^k > 0$. Then using A7 and (7) and cancelling terms, we can show

$$p_{k'} f(d^{k'}) + p_k f(d^k) < (p_{k'} + \delta) f(d^{k'}) + (p_k - \delta) f(d^{k'})$$

$$\implies f(d^k) < f(d^{k'}). \quad (8)$$
Given that \( d^{k'} \geq d^k \), and (8) holds for any arbitrary \( k' \) and \( k \), one can infer that \( f(d^s) \) is monotonic.

From Lemma 1 we know that \( V(L) \) is continuous. Given (7) and \( p \in [0,1] \), Lemma 1 implies that that \( f(d^s) \) must be continuous.

We know from Lemma 2, Axioms A3 and A4 implies Axiom A5. For any lottery \( L' \) such that \( p^{L'} \cdot r < \theta \), thus, \( V(L') = 0 \). It can be easily checked that the sufficient conditions are satisfied. \( \blacksquare \)

Next we characterise a more specific functional form of the deprivation function \( f(d^s) \) which is represented in (1).

**Theorem 3** A measure of vulnerability, \( V \), of an individual satisfies Axiom of Consistency (A3), Axiom of Identification (A4), Axiom of Decomposability (A6), Axiom of Monotonicity (A7), Axiom of Scale Variant Transfer (A8) and Axiom of Normalization (A9) iff:

\[
V(L) = \begin{cases} 
\sum_{s=1}^{m} p_s (d^s)^\alpha & \text{if } p^{L'} \cdot r \geq \theta \\
0 & \text{otherwise}
\end{cases}
\]

Proof: First lets start with the necessary conditions. Using Axioms A3, A4, A7, and Theorem 1, we can show for any lottery, \( L = (p_1, d^1; \ldots; p_m, d^m) \) such that \( p^{L} \cdot r \geq \theta \),

\[
V(L) = \sum_{s=1}^{m} p_s f(d^s). \tag{9}
\]

where \( f(d^s) \) is monotonic. Using Axiom A9 we can demonstrate \( f(0) = 0 \) and \( f(1) = 1 \).

Now consider a lottery \( L' = (p_1, d^1; \ldots; p_k + \delta, d^k; p_l - \delta, d^l; \ldots; p_m, d^m) \) obtained from lottery \( L = (p_1, d^1; \ldots; p_m, d^m) \) through a transfer of probability between two states, and through a similar transfer we obtain \( \hat{L}' = (p_1, \hat{d}^1; \ldots; p_k + \delta, \hat{d}^k; p_l - \delta, \hat{d}^l; \ldots; p_m, \hat{d}^m) \) from \( \hat{L} = (p_1, \hat{d}^1; \ldots; p_k, \hat{d}^k; p_l, \hat{d}^l; \ldots; p_m, \hat{d}^m) \), where \( p_k > 0, p_l > 0, d^k \geq d^l, p_l > \delta > 0 \) and \( \forall s \hat{d}^s = \lambda d^s, 0 < \lambda \leq 1 \). Using Axiom A8 and (9), we can show

\[
\delta (f(\hat{d}^k) - f(\hat{d}^l)) = \delta g(\lambda)(f(d^k) - f(d^l)).
\]

where \( 0 < g(\lambda) \leq 1 \). Given \( \forall s \hat{d}^s = \lambda d^s \), from the above equation we can obtain

\[
f(\lambda d^k) - g(\lambda)f(d^k) = f(\lambda d^k) - g(\lambda)f(d^l) \tag{10}
\]

Since \( f(d) \) is monotonic and (10) should hold for all \( d^k \) and \( d^l \), therefore it must be the case that for all \( d \)

\[
f(\lambda d) = g(\lambda)f(d) \tag{11}
\]
Suppose $d = 1$, given $f(1) = 1$, it implies that $f(\lambda) = g(\lambda)$. Replacing this in (11) we get

$$f(\lambda d) = f(\lambda)f(d).$$

(12) is a Cauchy equation, whose general solution, for $d > 0$, is given by (Aczel 1966)

$$f(d) = d^\alpha$$

where $\alpha \geq 0$. Thus from (9) and (13) we can show $V(L) = \sum_{s=1}^{m} p_s(d^\alpha)$, $\alpha \geq 0$. As earlier for any lottery $L'$ such that $p^{L'} \bullet r < \theta$, we know from Axioms A3 and A4 and Lemma 1, $V(L') = 0$. Further it can be easily checked that the sufficient conditions are satisfied.

4 Societal Measure of Vulnerability

Once the individual vulnerability measure is computed, we can then measure the level of vulnerability for the society. Although the domain of the societal measure of vulnerability is the set of vulnerability matrices denoted by $\Phi$, in calculating the overall vulnerability we take the approach recommended in Dutta, Pattanaik and Xu (2003) for deprivation analysis. We first find the the individual vulnerability and then aggregate over the individual vulnerability measures for the societal vulnerability. For any individual $i$, $0 < V^i(L) < 1$, then the societal vulnerability measure $V^S : [0, 1]^N \rightarrow \mathbb{R}_+$.

Next, we consider the axioms on the societal vulnerability measure and characterise the societal measure.

4.1 Axioms on the Societal Measure

Since each row of the vulnerability matrix represents a lottery faced by an individual, interchanging of the rows should not affect the overall vulnerability. We say that matrix $M'^{N,m}$ is obtained from $M^{N,m}$ by the permutation of rows if only the rows are interchanged with everything else remaining same.

**Axiom 10 Axiom of Symmetry (A10):** Consider two matrices $M^{N,m}$ and $M'^{N,m}$ where $M'^{N,m}$ is obtained from $M^{N,m}$ through a permutation of rows. Then $V^S(M^{N,m}) = V^S(M'^{N,m})$.

This axiom will imply that the individual vulnerability functions are the same for all the people. In other words if two individuals face the same lottery then their vulnerability should be same.

The next axiom captures the notion that we divide the individuals in to different groups then the overall vulnerability should be the sum of the vulnerability of the different groups.
Axiom 11 Axiom of Societal Decomposability (A11): Consider three matrices $M^N,m$, $M^{n_1,m}$ and $M^{n_2,m}$, where $N = n_1 + n_2$. Then $V^S(M^N,m) = \frac{n_1}{N}V^S(M^{n_1,m}) + \frac{n_2}{N}V^S(M^{n_2,m})$.

4.2 Characterisation of the Societal Measure

We now characterise the overall societal measure which is similar to (2).

**Theorem 4** A societal measure of vulnerability, $V^S$, satisfies Axiom of Symmetry (A10) and Axiom of Societal Decomposability (A11) iff:

$$V^S = \frac{1}{N} \sum_{i=1}^{N} V(L^i),$$

where $L^q$ is the lottery faced by individual $q$.

Proof: Repeated application of axiom A11, will yield

$$V^S = \frac{1}{N} \sum_{i=1}^{N} V^i(L^i). \quad (14)$$

Due to axiom A10, we can show that $V^i(L^i) = V(L^i)$. Applying this in (14) would yield the result.

In light of Theorem 3, we can have intuitive interpretations of some of the measures discussed earlier. From Theorem 2 we know that for any individual $i$, faced with lottery $L^i$, the vulnerability is given by $V(L^i) = \sum_{s=1}^{m} p_s^L(d^s)^\alpha$, if the individual is identified as vulnerable. Now consider the case where $\alpha = 0$. From Theorem 3 the societal vulnerability can be written as

$$V^S = \frac{1}{N} \sum_{i=1}^{N} \sum_{s=1}^{m} p_s^L^i. \quad (15)$$

Let $q$ be the number of individuals who have been identified as vulnerable then we can write (15) as

$$V^S = \frac{q}{N} \sum_{i=1}^{q} \sum_{s=1}^{m} p_s^L^i = H.I_0 \quad (16)$$

where $H$ is the head count ratio of the vulnerable, and $I_0$ is the average probability with which people will fall in to poverty in the future and thus reflects the intensity of the vulnerability. In this sense, the measure is very similar to the adjusted head count ratio proposed by Alkire and Foster (2011) in the context of multi-dimensional poverty.
We can easily expand this analysis for \( \alpha > 0 \). In that context we can decompose the societal vulnerability in a similar manner to (16). Thus

\[
V^S = \frac{\sum_{i=1}^{q} \sum_{s=1}^{m} P_s(d^i)^\alpha}{q} = H.I_\alpha
\]  

(17)

where \( I_\alpha \) is the intensity of vulnerability and \( H \) is the head count ratio. When \( \alpha = 1 \), \( I_\alpha \) captures the average deprivation people will suffer in the future.

5 Empirical Application

In this section we apply the vulnerability measure developed in this paper on a data from a hamlet, Hrishipara, near Kapasia sub-district of Dhaka division, in Bangladesh. Our data comes from the Hrishipada Daily Diary project where households were asked to keep a record of the daily income and expenditures. It covers 50 households, from May 2015 to October 2016. Not all households came under the survey at the same time and there is considerable variation in terms of the number of observations that we have for each household. In a series of blogs Rutherford (2017a, 2017b, 2016a, 2016b) provided a detailed structure of the data and how one can use it to track financial transactions of the poor.\(^{11}\) We also have the information on household size, which we report in the Appendix (Table B1). The household size shown is an average of the number of people in the household over the sample period. For household 1, for instance, the household size is 1.2, which reflects the fact that around 20 percent of the time, there is a second person in the house. The total number of individuals in our sample is around 201 which amounts to around 4 members for each household.

Our interest here is to assess the vulnerability of individuals to income poverty. We define income as any earnings during the sample period, which includes wages from jobs and profits from businesses, loans taken and savings withdrawals, and gifts and transfer received. For the self-employed, we do subtract from their income, the legitimate expenses that one has to incur to keep their business running. For instance, we deduct the costs of the products sold for shop keepers, similarly we deduct the cost of repairing vehicles for households who ply their vehicles to earn money. We include the income of all the individuals in the household income. The income of the household divided by the household size gives us the individual income. To smoothen out the noise in daily income, we aggregate the daily information in to weekly income. The minimum weekly observation is 2 and the maximum is 77. The average weekly income for individuals in our sample is around 1900 taka (Bangladesh’s currency). This is slightly above $3.20 per day, which is the median poverty line for

\(^{11}\)More information on the financial diary project and data can be found at https://sites.google.com/site/hrishiparadailydiaries/home
low middle income countries (Ferreira 2017).

The income shocks that individuals face are divided into four states, with one state capturing all the positive shocks and the rest three states capturing negative shocks of varying intensity: (i) extreme shocks, (ii) moderate shocks, (iii) mild shocks. We classify the individual under extreme shock if the weekly income falls under $13.3, which is equivalent to $1.90 per day. Similarly, the moderate shock is for weekly incomes between $13.3 and $22.4, which is equivalent to $3.20 per day. The mild shock is when weekly income is between $3.20 per day and $5.50 per day, which is equivalent to between $22.4 and $38.5. For the positive shock state we consider any income above $38.5 per week. These are standard poverty lines considered by the World Bank (2017) in their assessment of poverty. Using current exchange rates, in terms of Bangladesh currency, individuals with weekly income not more than 1100 taka is considered to be under extreme shock, individuals with income between 1100 and 1860 taka is considered to be under moderate shock, individuals with weekly income between 1860 and 3200 taka is under mild shock and any individuals with income above 3200 taka as with no shock.

To calibrate the deprivation in each of the different states, we take 3200 taka per week as the cut-off point. One justification for this is that we take this to be the minimum income above which all individuals are guaranteed to be non-poor in the future. It is similar in spirit to Dang and Lanjouw (2017) where they consider the vulnerability line to be that level of income which ensures that anyone earning at least that amount will not fall into poverty in the future. We take the average income in each state and then calculate the normalised gap from the cut-off point. Therefore, for all the states with negative shocks, the deprivation will range between [0, 1] and for the state with positive shocks, the deprivation would be considered as zero. What is important to note is that the states are nested in terms of the deprivation. This is because we have assumed that income below, say for instance, 1100 taka, will be in the extreme shock state, incomes between 1100 and 1860 taka will be in the moderate shock state and so on. Thus if the household faces deprivation over all the different states, the average income in the extreme shock state would be lower than the average income in the moderate shock state which in turn would be lower than the mild state. It however could be that the household just faces moderate shocks among the negative shocks. Thus average income in the other two states would be zero. Due to informational constraints we have framed the different states in this nested manner, however, our theoretical model is more general framework where there is no need for such restrictions. For instance, if we can classify the income based on different shocks such as health shocks, or unemployment shocks or natural shocks (flood or drought), then there is no reason why these different states should be nested in the way that we see in our application.

We use a simple frequency based rule to calculate probabilities associated with the different states.
For any individual the probability of a state is the total number of weeks the income was in that range, divided by the total number of weeks the individual earned income. Thus, each individual has a different probability distribution over the states depending on the weeks it has earned the incomes in the different ranges. Note that individuals in the same household, will have the same probabilities of the states. Unlike average income, or deprivation, the probabilities of the states are not nested. Hence for instance moderate income shocks may have a lower probability than either the extreme and the mild shock states.

To calibrate the societal vulnerability we proceed in three steps. First for each individual, any positive deprivation in any state gets assigned a value of one. Intuitively it means that the individual is considered to be fully deprived in that state. We then take the expected value of the deprivation for each household, it is the $\rho^E(L)$ referred previously. Thus the deprivation in each state gets weighted with the probability of each state, which is the level of vulnerability that individuals would face if they were fully deprived in each state. We can now decide the level of first cut-off ($\theta$) of this full vulnerability. Thus individuals below $\theta$, would be considered not vulnerable enough even under full deprivation in each of the states it is deprived, to be considered as vulnerable in our analysis. Second, once the cut-off is decided, then the individuals with less than that cut-off are assigned a value of zero for all their states. Now we calibrate the weighted deprivation for each individual based on this updated information. This is the vulnerability to poverty that each individual faces. Thus individuals who were not deprived at all, and individuals below the cut-off will have a vulnerability level of zero. Third, since each member of a household have the same level of vulnerability, we multiply the individual vulnerability with the household size to get the total vulnerability of all the individuals in that household. The average of the vulnerability of all the individuals in the society gives the societal vulnerability.

In our empirics we choose several different cut-off levels to assess the vulnerability. We start with the ‘union’ approach, where the household is considered vulnerable if it has a positive deprivation, thus $\theta > 0$. On the other hand for the ‘intersection’ approach, $\theta = 1$, where individuals are always facing negative shocks. Further we consider two other cases where $\theta \geq 0.5, \theta \geq 0.75$. One thing to note is that our analysis is based on the deprivation gap, which is when $\alpha = 1$. However it can be easily extended to other values of $\alpha$. The calibration of vulnerability for different levels of $\theta$ is presented in Table 1 below:

There are several interesting results that emerge from the calibrations. First, our calibrations, although based on a small specific sample, are not too off the mark for Bangladesh. The World Bank (2017) country profile for Bangladesh shows that 85 percent of the country would be considered poor if we take the cut-off to be 3200 taka per week (or $5.50 per day) as we have done here. Our results
Table 1: Levels of Vulnerability in Hrishipada, Bangladesh: 2015-16

<table>
<thead>
<tr>
<th></th>
<th>$\theta &gt; 0$</th>
<th>$\theta \geq 0.5$</th>
<th>$\theta \geq 0.75$</th>
<th>$\theta = 1$</th>
<th>Standard Approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>Societal Vulnerability</td>
<td>0.595</td>
<td>0.578</td>
<td>0.517</td>
<td>0.073</td>
<td>0.484</td>
</tr>
<tr>
<td>Intensity of Vulnerability</td>
<td>0.595</td>
<td>0.623</td>
<td>0.676</td>
<td>0.859</td>
<td>0.706</td>
</tr>
<tr>
<td>Head Count of Vulnerability</td>
<td>1.000</td>
<td>0.927</td>
<td>0.764</td>
<td>0.085</td>
<td>0.685</td>
</tr>
<tr>
<td>Number of Vulnerable (Households)</td>
<td>50</td>
<td>46</td>
<td>36</td>
<td>4</td>
<td>31</td>
</tr>
<tr>
<td>Number of Vulnerable (Individuals)</td>
<td>201.2</td>
<td>186.45</td>
<td>153.7</td>
<td>17.2</td>
<td>137.9</td>
</tr>
<tr>
<td>Median Vulnerability (Household)</td>
<td>2.284</td>
<td>2.284</td>
<td>2.284</td>
<td>0.000</td>
<td>2.055</td>
</tr>
<tr>
<td>Median Vulnerability (Individual)</td>
<td>0.576</td>
<td>0.576</td>
<td>0.572</td>
<td>0.000</td>
<td>0.576</td>
</tr>
<tr>
<td>Max Vulnerability (Household)</td>
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<td>5.525</td>
<td>5.525</td>
<td>5.207</td>
<td>5.525</td>
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<tr>
<td>Max Vulnerability (Individual)</td>
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<td>0.905</td>
<td>0.905</td>
<td>0.905</td>
<td>0.905</td>
</tr>
</tbody>
</table>

show that under the ‘union’ approach ($\theta > 0$) 100 percent of the sample is vulnerable. Given that this is a rural sample focussed on people participating in a microsaving scheme, the high vulnerability that we see in our sample is not surprising.

Second, there is a clear trend of societal vulnerability going down as $\theta$ increases. This is because progressively we are considering a smaller set of individuals in our vulnerability calculations. When we consider the union approach ($\theta > 0$), the vulnerability level is 0.60, which is considerably high, with all the households classified as vulnerable. However, as $\theta$ goes up, there is not much reduction in the level of vulnerability index initially. If we focus on $\theta \geq 0.5$, which is the cut-off considered by many empirical studies, the societal vulnerability is 0.58, which is not much different than when $\theta > 0$. As we move to $\theta \geq 0.75$ the over all index reduces by over 10 percent. When we consider $\theta = 1$, there are only 4 households, with a total of 17 individuals who are vulnerable, who will for certain be poor in the future.

Third, based on the equation (17) we decompose the societal vulnerability in to the head count ratio and the intensity of vulnerability. What is evident is that these two aspects work in opposite direction with the change in $\theta$. Thus as $\theta$ increases, the head count ratio decreases, whereas the intensity of vulnerability increases. Given that the vulnerability index does not change much with $\theta$, till we reach $\theta \geq 0.5$ it is not surprising that both head count and intensity also shows sharp changes around those values of $\theta$. But what is interesting is that the reduction in the head count dominates the increase in the intensity. As we move from $\theta \geq 0.5$ to the next cut-off of $\theta \geq 0.75$, while intensity increases by 8 percent, the headcount drops by around 18 percent. It is this large decrease in the headcount which leads to a reduction in vulnerability despite the increase in intensity of vulnerability.

Fourth, if we compare the standard approach in the literature to our method, we get very different results. Under the standard approach, vulnerability for each household is calibrated based on the

12Azam and Imai (2009) find that for rural areas in 2005 the proportion of the vulnerable 52 percent. In comparision, under our standard approach, we find around 68 percent of our sample is vulnerable with an overall society vulnerability to be around 0.48.
‘union’ criteria (i.e. \( \theta > 0 \)) and then an cut-off of 0.5 is applied. Thus, households with vulnerability less than 0.5, are not considered vulnerable. If we apply that method to our data, (Column 5 of Table 1), we find that the overall societal vulnerability is 0.48 and around 70 percent of the population is vulnerable. This is in sharp contrast to our results based on a \( \theta \geq 0.5 \) cut-off, where the head count is 92 percent and societal vulnerability is 0.60. Interestingly though we find the intensity of vulnerability to be way higher under the standard approach. This is because under that method all the households with low vulnerability are selectively pruned out.

Fifth, we also see the level of vulnerability of the individuals remain the same as we go through the various cutoffs until they drops out of our calibration when they fall below the threshold. What is important to keep in mind is that with the cutoffs, the pool of vulnerable individuals keep changing and not the states. Thus an extremely deprived individual will have the same vulnerability whether \( \theta > 0 \), or \( \theta \geq 0.5 \).

It is also interesting to note that household level vulnerability and individual level vulnerability will differ. While each member of a household may not have the highest vulnerability, if the household size is big, overall the household might be more vulnerable than another household with the highest individual vulnerability. In our sample, under \( \theta > 0 \), the most vulnerable household is household 15, with 8 members where each individual member has vulnerability of 0.69. On the other hand household 37 with 5 members, has the highest individual vulnerability, where each member has a vulnerability of 0.9. The median level of vulnerability at the household level is 5.56 holding steady for most values of \( \theta \) and for the individual it is 2.28.

6 Conclusion

Many empirical papers on vulnerability have used an identification strategy along with the FGT class of vulnerability measures to calibrate vulnerability. However, the identification strategy is not clear and the vulnerability measure used, although popular doesn’t have an axiomatic framework. Both the identification methods and the new measure of vulnerability discussed in this paper provide a theoretical underpinning to many of the empirical studies on measuring vulnerability.

The main innovation of the paper is in bringing a clear identification part to the measurement of vulnerability as has been done in the literature on poverty and multi-dimensional deprivation. Identification in measuring vulnerability is a highly active area of research and our paper contributes to that growing literature. As in the case of income poverty, we have argued that there can be multiple identification methods for vulnerability, each of which may have their own normatively appealing properties. We have provided characterisation of two possible identification methods - one in which all the states with positive deprivation is weighted according to their probability and another in
which the states are unweighted (see Appendix B). The latter is very similar to the counting method proposed in the multi-dimensional context.

We also conceptualize and characterise a new class of individual vulnerability measures which are based on the expected FGT class. The main difference is that the measure proposed in this paper is applied to only those who are identified as vulnerable. Using an example we have shown that the two measures of vulnerability, one based on the expected poverty from the FGT class of measures, and the other based on the counting approach to vulnerability, may not necessarily yield the same outcome, particularly for the case where the identification is not based on the union approach. As is the case for most deprivation measures, the focus axiom play a crucial role in ensuring that the vulnerability measure remains unaffected by those who are not identified as vulnerable.

Although we provided a real world application of our measure, it was mainly to demonstrate how the measure can be applied systematically. We cannot, using a survey based on just 50 households, make general statements for vulnerability in Bangladesh. However, we had clearly shown that the standard approach in the literature would give a significantly different results from our methods. In particular the current approach, based on adhoc rules, under-estimated vulnerability by a large margin. There is a lot of merit in the diary based methods in understanding the different shocks, but the measure proposed in this paper can be easily applied to any longitudinal data sets.
References.


Azam, Md. and K. Imai (2009), Vulnerability and Poverty in Bangladesh, Chronic Poverty Research Center, Manchester, Number 141.


A Appendix

Proof of Lemma 1: We demonstrate the result by contradiction using a diagrammatic proof. Given axiom A6, $V(L)$ is an affine function. Suppose, $V(L)$ is discontinuous at $L^3$ where $L^3 = (q_1, d^1; (1-q_1), 0)$. Now consider two lotteries, $L^1 = (p_1, d^1; (1-p_1), 0)$ and $L^2 = (p'_1, d^1; (1-p'_1), 0)$ where without loss of generality, $V(L^2) > V(L^1)$. Note that lotteries with two states can be represented in a line. From Figure 1, below we can see that $\alpha V(L^1) + (1-\alpha)V(L^2)$ will lie on the line AB, for different values of $\alpha$.

![Figure 1: Violation of Axiom of Decomposability.](image)

Let for $0 < \alpha = \alpha^* < 1$, $L^3 = \alpha^* L^1 + (1-\alpha^*)L^2$, i.e. $q_1 = \alpha^* p_1 + (1-\alpha^*)p'_1$. Then we can easily establish from Figure 1, given the discontinuity at $L^3$, that $\alpha^* V(L^1) + (1-\alpha^*)V(L^2) < V(\alpha^* L^1 + (1-\alpha^*)L^2)$, thus violating Axiom of Decomposability (A6).

Proof of Lemma 2: Let lottery $L$ is such that $\rho^E(L) = 0$, then from Axiom A4 we know that $V(L) = 0$. Further, for another lottery $T$ where $\rho^E(T) = 1$, from Axiom A4, $V(T) > 0$. Then given that $\rho^E(L) \in [0,1]$, there exists $0 \leq \theta = \theta^* \leq 1$, such that for $\rho^E(L) < \theta^*$, $V(L) = 0$ and for $\rho^E(L) \geq \theta^*$, $V(L) > 0$.

We now demonstrate uniqueness of $\theta^*$. Suppose there exists $\tilde{\theta}$, where $0 < \tilde{\theta} < \theta^*$, such that either: (Case i) if $0 < \rho^E(L) < \tilde{\theta}, V(L) > 0$ and $\tilde{\theta} \leq \rho^E(L) < \theta^*, V(L) = 0$) or (Case ii) (for $0 < \rho^E(L) < \tilde{\theta}, V(L) = 0$ and $\tilde{\theta} \leq \rho^E(L) < \theta^*, V(L) > 0$).
For (Case i), consider lotteries $L'$ and $L''$ where $\rho^E(L') = \tilde{\theta} - \epsilon < \tilde{\theta} = \rho^E(L'')$. Then by definition of $\tilde{\theta}$, $V(L') > 0$ and $V(L'') = 0$. This violates Axiom A3.

For (Case ii) if $0 < \rho^E(L) < \tilde{\theta}$, $V(L) = 0$ and $\tilde{\theta} \leq \rho^E(L) < \theta^*$, $V(L) > 0$, then we simply define $\theta = \tilde{\theta}$.

Therefore, there exists $\theta$, such that for all $\rho^E(L) < \theta$, $V(L) = 0$, which is Axiom A5. ■

B Appendix

In this appendix we provide a characterisation of an counting rule based on just the number of states in the future the person is deprived. Consider a lottery $L = (p_1^L, d_1^L; p_2^L, d_2^L; ...; p_k^L, d_k^L; \ldots; p_m^L, d_m^L)$. We define an associated vulnerability identification vector $\hat{r}^L = (\hat{r}^1_L, \hat{r}^2_L, ..., \hat{r}^m_L)$ where,

$$\forall s, \hat{r}^s_L = \begin{cases} 
1 & \text{if } p^s_L.d^s > 0 \\
0 & \text{otherwise}
\end{cases}.$$

This distinguishes the states which are deprived with positive probabilities. We define counting function as, $\rho^C$ as

**Definition 4** $\rho^C$ is a counting function such for any two lotteries $L$ and $L'$: (i) $\rho^C(L) > \rho^C(L')$ iff $\|\hat{r}^L\|^2 > \|\hat{r}'^L\|^2$ and (ii) $\rho^C(L) = \rho^C(L')$ iff $\|\hat{r}^L\|^2 = \|\hat{r}'^L\|^2$.

This is a pure counting function, where we differentiate the two lotteries based on the number of deprived states with positive probabilities. We characterise the counting function using the following axioms.

**Axiom 12** Monotonicity of deprived states. Let $L = (p_1^L, d_1^L; p_2^L, d_2^L; ...; p_{k+1}^L, d_{k+1}^L; ...; p_m^L, d_m^L)$, where $\forall s = 1, ..., k + 1, \hat{r}^s_L > 0$, and $L' = (p_1'^L, d_1'^L; p_2'^L, d_2'^L; ...; p_{k+1}'^L, d_{k+1}'^L; ...; p_m'^L, d_m'^L)$, where $\forall s = 1, ..., k, \hat{r}^s_L > 0$. Then $\rho(L) > \rho(L')$.

**Axiom 13** Strong Invariance to probability transfers. Let $L = (p_1^L, d_1; p_2^L, d_2; ..., p_k^L, d_k; ...; p_m^L, d_m)$, where $\hat{r}^k_L > 0$, and $L' = (p_1'^L, d_1; p_2'^L, d_2; ...; p_k'^L - \epsilon, d_k; ...; p_m'^L, d_m)$, where $p_k^L - \epsilon > 0$. Then $\rho(L) = \rho(L')$.

The intuition of the first axiom is that if a lottery has more deprived states with positive probability than another, it should have a higher counting to vulnerability than the other lottery.

The second axiom captures the idea that if you transfer some probability from a deprived state to any other state, then the counting function remains unchanged.
Theorem B.1 A counting function $\rho$ satisfies Axioms of Monotonicity of Deprived States (A1) and Axiom of Strong Invariance to Probability Transfers (A2) iff $\rho = \rho^C$.

Sketch of the Proof: Only if: Consider any two lotteries $L$ and $L'$. We will consider two cases: (A) where the number of deprived states with positive probability is different for the two lotteries and (B) where it is same.

Case A: Without loss of generalisation suppose, $\|\hat{\tau}^L\|^2 > \|\hat{\tau}^{L'}\|^2$, then clearly from Axiom A1, we can conclude that $\rho(L) > \rho(L')$.

Case B: Now suppose $\|\hat{\tau}^L\|^2 = \|\hat{\tau}^{L'}\|^2$.

(i) Let the number of deprived states with positive probability be $n < m$. Without loss of generality, let $\forall s = 1, ..., n, p_s^L \geq p_s^{L'}$, with strict inequality holding for some states, then it must be the case that $\forall s = n, ..., m, p_s^L \leq p_s^{L'}$ with strict inequality for some states. Then applying Axiom A2, repeatedly on $L$, where we transfer probability from deprived to non-deprived states we can retrieve $L'$. Thus $\rho(L) = \rho(L')$. On the other hand if $\exists s \in \{1, ..., n\}, p_s^L > p_s^{L'}$ and $\exists s' \neq s \in \{1, ..., n\}$, $p_{s'}^L < p_{s'}^{L'}$. Then again through repeated application of Axiom A2 on $L$, where we move probabilities to deprived and non deprived states, we can retrieve $L'$ and thus $\rho(L) = \rho(L')$.

(ii) Let $n = m$, then $\exists s \in \{1, ..., m\}, p_s^L > p_s^{L'}$ and $\exists s' \neq s \in \{1, ..., m\}, p_{s'}^L < p_{s'}^{L'}$. By repeated application of Axiom 2, where we move probabilities between deprived states, we can derive $L'$ from $L$ which implies $\rho(L) = \rho(L')$.

The If part of the proof is straight forward.

This characterises the counting function which is purely based on the number of deprived states with positive probability.

C Appendix

This Table contains more detailed information on the vulnerability for each household at the individual level.
### Table C: Vulnerability of household members in Hrishipara, Bangladesh: 2015-2016

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Mean 0.59 0.578 0.517 0.073 0.484 4.024