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Nutshell

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Correlated Equilibrium in a Nutshell

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Abstract: We analyze the concept of correlated equilibrium in the framework of two-player two-strategy games. This simple framework makes it possible to clearly demonstrate the characteristic features of this concept. We develop an intuitive and easily memorizable test for equilibrium conditions and provide a complete classification of symmetric correlated equilibria in symmetric games.

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Key words: games with communication, correlated strategies, correlated equilibrium, 2x2 games.

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1 Introduction

Correlated equilibrium, introduced by Aumann (1974), is a classical concept in game theory. A distinctive literature has emerged to address various facets of this concept, including a multi-stage extension (Forges, 1986), epistemic foundations (Aumann, 1987), a direct approach to existence (Hart and Schmeidler, 1989), communication mechanisms (Forges, 2000), perfection-type refinements (Dhillon and Mertens, 1996), emergence as limit of a class of learning dynamics (Hart and Mas-Colell, 2000), and computational methods (Papadimitriou and Roughgarden, 2008).

Introductory material on correlated equilibrium can be found practically in every modern text on game theory or microeconomics. Typically, the presentation of this subject is structured as follows. First a definition of this concept is given in a relatively general setting, and then the conceptual analysis proceeds nearly exclusively via examples. To the best of our knowledge, we cannot point to a source where the exposition deviates from this scheme and where a complete theory is provided, at least in a sufficiently narrow framework. The purpose of this note is to fill this gap.

As a vehicle for our analysis, we use the framework of two-player two-strategy games. It is well known that such games possess a fairly rich structure. They represent a convenient and simple model that makes it possible to clearly demonstrate a number of fundamental ideas, results and paradoxes of game theory, as reflected in the fact that some of the most widely invoked games in the social sciences belong to this class.

We begin with the definition of correlated equilibrium in a two-player two-strategy game. This notion is explained and interpreted by using Harsanyi's [6] "mediator". A system of linear inequalities is derived whose solutions are correlated equilibrium strategies. Then this system is transformed and represented in a compact and tractable form. Based on this representation, we develop an intuitive and easily memorizable algorithm allowing one to check whether a given correlated strategy forms a correlated equilibrium.

An outline of the algorithm is as follows. First the original game is reduced to an equivalent (having the same correlated equilibria) simple game. A game is called *simple* if the non-diagonal payoffs for both players are zero. After that the question boils down to the characterization of correlated equilibria for the given simple game. To analyze this

question we construct an auxiliary "test game", in which players 1 and 2 have strategies a_1, a_2 and b_1, b_2 , respectively. It turns out that the given correlated strategy represents a correlated equilibrium if and only if each of the strategy profiles (a_1, b_1) and (a_2, b_2) forms a Nash equilibrium in the test game.

We apply the above algorithm to describe all *symmetric* correlated equilibria in a *symmetric* simple two-strategy game. In this narrow, but still interesting, framework a correlated strategy can be specified by two numbers and thus can be represented by a point on the plane. Several cases are considered in which the set of correlated equilibria corresponds to a quadrangle, triangle, or segment in the plane. This leads to a complete theory of symmetric correlated equilibria in the simplest model we are dealing with.

In the authors' mind, the main motivation behind this novel treatment of correlated equilibrium is two-fold. First, it may pave the way for a more extensive use of correlated equilibrium in applied settings in economics and other social sciences. Second, it may facilitate the teaching of this concept at a level beyond illustrative examples.

The remainder of the paper is organized as follows. In Section 2 we introduce the concept of correlated equilibrium and derive the system of linear inequalities characterizing all correlated equilibria. In Section 3 we develop an algorithm based on the "test game" making it possible to verify that a given correlated strategy forms a correlated equilibrium. In Section 4 we provide a complete characterization of all symmetric correlated equilibria for a symmetric simple two-strategy game. An example in Section 5 concludes the paper.

2 Correlated strategies and correlated equilibrium

Consider a two-player two-strategy game with strategy sets $A = \{a_1, a_2\}$, $B = \{b_1, b_2\}$ and payoff functions $u(a, b)$, $v(a, b)$. A correlated strategy

$$\gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}, \quad \gamma_{ij} \geq 0, \quad \sum_{ij=1,2} \gamma_{ij} = 1, \quad (1)$$

is a probability distribution on the set of strategy profiles (a_i, b_j) of the two players. The number γ_{ij} is the probability that the randomly chosen strategy profile (a, b) of the players

is (a_i, b_j) :

$$\gamma_{ij} = \Pr\{(a, b) = (a_i, b_j)\}.$$

A correlated strategy may be regarded as a *randomized plan of coordinated actions* of both players, which is performed by using communication (involving, for example, a "mediator").

For the correlated strategy γ , the expectations of the payoffs $u(a, b)$ and $v(a, b)$ are given by

$$u(\gamma) = Eu(a, b) = \sum_{i,j=1,2} \gamma_{ij}u(a_i, b_j), \quad v(\gamma) = Ev(a, b) = \sum_{i,j=1,2} \gamma_{ij}v(a_i, b_j).$$

Consider the framework for analyzing correlated strategies involving a *mediator* due to J.C. Harsanyi [6]. Suppose the players use some correlated strategy γ defined by probabilities γ_{11} , γ_{12} , γ_{21} , and γ_{22} . Let us imagine that there is a mediator selecting the strategy profile (a, b) at random according to the probabilities γ_{ij} , so that $\gamma_{ij} = \Pr\{(a, b) = (a_i, b_j)\}$. If the random strategy profile (a, b) happens to be (a_i, b_j) , the mediator recommends Player 1 to play a_i and Player 2 to play b_j . The mediator communicates a_i (only) to Player 1 and b_j (only) to Player 2 and each player does not know what has been recommended to the other.

For our further analysis, we will need to consider the conditional expectations:

$$E[u(a', b)|a = a_i] \text{ and } E[v(a, b')|b = b_j] \tag{2}$$

for any $a' \in A$ and $b' \in B$. In the former expectation, a' and $a = a_i$ are fixed, and b is random. The expectation is taken under the *conditional distribution of b given $a = a_i$* . In the latter expectation, b' and $b = b_j$ are fixed, and a is random. The expectation is taken under the *conditional distribution of a given $b = b_j$* .

Let us explain the meaning of the conditional expectations (2), say, the former. Suppose the mediator has announced a_i to Player 1, i.e., the event $\{a = a_i\}$ has occurred. Assume that for one reason or another, Player 1 decides to disobey the mediator's recommendation and to play another strategy a' , not a_i . Then expected payoff Player 1 gets (provided that Player 2 obeys the recommendations of the mediator) is the conditional expectation of $E[u(a', b)|a = a_i]$. The conditional expectation $E[v(a, b')|b = b_j]$ has the analogous meaning.

A correlated strategy γ is said to form a *correlated equilibrium* if

$$E[u(a_i, b)|a = a_i] \geq E[u(a', b)|a = a_i] \text{ for all } i \text{ and } a'; \quad (3)$$

$$E[v(a, b_j)|b = b_j] \geq E[v(a, b')|b = b_j] \text{ for all } j \text{ and } b'. \quad (4)$$

Then conditions (3) and (4) — termed *strategic incentive constraints* — say that, in the correlated equilibrium, it is optimal for each player to obey the mediator's recommendations. Condition (3) states that, given $a = a_i$, the maximum conditional expected payoff of Player 1 is attained when he plays a_i . Analogously, given $b = b_j$, the maximum expected payoff of Player 2 is attained when she plays b_j . In this sense, the equilibrium correlated strategy γ is *self-enforcing*.

Denote by μ_{ij} the conditional probability that $a = a_i$ given $b = b_j$ and by ν_{ji} the conditional probability that $b = b_j$ given $a = a_i$, then

$$\begin{aligned} \mu_{ij} = \Pr\{a = a_i|b = b_j\} &= \frac{\Pr\{a = a_i, b = b_j\}}{\Pr\{b = b_j\}} = \frac{\gamma_{ij}}{\gamma_{1j} + \gamma_{2j}} \text{ and} \\ \nu_{ji} = \Pr\{b = b_j|a = a_i\} &= \frac{\Pr\{b = b_j, a = a_i\}}{\Pr\{a = a_i\}} = \frac{\gamma_{ij}}{\gamma_{i1} + \gamma_{i2}}. \end{aligned} \quad (5)$$

It is convenient to write conditions (3) and (4) in the following form:

$$E[u(a_i, b) - u(a', b)|a = a_i] \geq 0, \quad E[v(a, b_j) - v(a, b')|b = b_j] \geq 0 \text{ for all } i, j \text{ and all } a' \neq a_i, b' \neq b_j.$$

Express the conditional expectations through conditional probabilities (5) and multiply the first inequality by $\gamma_{i1} + \gamma_{i2}$ and the second by $\gamma_{1j} + \gamma_{2j}$. This yields

$$[u(a_i, b_1) - u(a', b_1)]\gamma_{1i} + [u(a_i, b_2) - u(a', b_2)]\gamma_{2i} \geq 0, \quad (6)$$

$$[v(a_1, b_j) - v(a_1, b')]\gamma_{1j} + [v(a_2, b_j) - v(a_2, b')]\gamma_{2j} \geq 0. \quad (7)$$

3 Correlated equilibrium test

Let us introduce the notation:

$$u_{ij} = u(a_i, b_j), \quad v_{ij} = v(a_i, b_j), \quad a' = a_k, \quad b' = b_m.$$

Then the inequalities (6) and (6) can be written for all $i, k \neq i$ and $j, m \neq j$ as

$$(u_{i1} - u_{k1})\gamma_{i1} + (u_{i2} - u_{k2})\gamma_{i2} \geq 0, \quad (v_{1j} - v_{1m})\gamma_{1j} + (v_{2j} - v_{2m})\gamma_{2j} \geq 0 \quad (8)$$

and a correlated strategy $\gamma = (\gamma_{ij})$ forms a correlated equilibrium in a game with payoffs $u_{ij} = u(a_i, b_j)$, $v_{ij} = v(a_i, b_j)$ if and only if the four inequalities in (8) hold.

From (8), we recover directly the well known result that the set of correlated equilibria is a compact and convex set (Myerson, 1991).

Consider a new game, called the *reduced version* of the original one, in which the payoff matrix is

$u_1 = u_{11} - u_{21}, v_1 = v_{11} - v_{12}$	$0, 0$
$0, 0$	$u_2 = u_{22} - u_{12}, v_2 = v_{22} - v_{21}$

It has zero non-diagonal payoffs. Such games are called *simple*. Observe that the conditions (8) for the original game and for its reduced version coincide. Hence both games have the same correlated equilibria!

Let us examine correlated equilibria in a simple game

u_1, v_1	$0, 0$
$0, 0$	u_2, v_2

(9)

For such games, conditions (8) characterizing correlated equilibria take on the following form

(I) $u_1\gamma_{11} \geq u_2\gamma_{12}$,	(II) $u_2\gamma_{22} \geq u_1\gamma_{21}$,
(III) $v_1\gamma_{11} \geq v_2\gamma_{21}$	(IV) $v_2\gamma_{22} \geq v_1\gamma_{12}$

(10)

Consider the following auxiliary game, which will be called the *test game*:

	b_1	b_2
a_1	$u_1\gamma_{11}, v_1\gamma_{11}$	$u_1\gamma_{21}, v_2\gamma_{21}$
a_2	$u_2\gamma_{12}, v_1\gamma_{12}$	$u_2\gamma_{22}, v_2\gamma_{22}$

(11)

Proposition. *The correlated strategy $\gamma = (\gamma_{ij})$ forms a correlated equilibrium in the simple game (9) if and only if the strategy profiles (a_1, b_1) and (a_2, b_2) form Nash equilibria in the test game.*

Proof. To prove the proposition, observe that

- (I) holds if and only if a_1 is the best response to b_1 ;
- (II) holds if and only if a_2 is the best response to b_2 ;
- (III) holds if and only if b_1 is the best response to a_1 ;
- (IV) holds if and only if b_2 is the best response to a_2 .

Let us describe an easily memorizable algorithm for constructing the test game. Suppose we are given a simple game and a correlated strategy

$$\begin{array}{|c|c|} \hline u_1, v_1 & 0, 0 \\ \hline 0, 0 & u_2, v_2 \\ \hline \end{array}, \quad \gamma = \begin{pmatrix} \gamma_{11} & \gamma_{12} \\ \gamma_{21} & \gamma_{22} \end{pmatrix}.$$

1st step. Based on the former matrix, construct the following one

$$\begin{array}{|c|c|} \hline u_1, v_1 & u_1, v_2 \\ \hline u_2, v_1 & u_2, v_2 \\ \hline \end{array}.$$

2nd step. *Transpose* the matrix γ , i.e. swap γ_{12} and γ_{21} :

$$\begin{pmatrix} \gamma_{11} & \gamma_{21} \\ \gamma_{12} & \gamma_{22} \end{pmatrix}.$$

3rd step. Multiply *element-wise* the last two matrices:

$$\begin{array}{|c|c|} \hline u_1\gamma_{11}, v_1\gamma_{11} & u_1\gamma_{21}, v_2\gamma_{21} \\ \hline u_2\gamma_{12}, v_1\gamma_{12} & u_2\gamma_{22}, v_2\gamma_{22} \\ \hline \end{array}$$

The result is precisely the payoff matrix for the test game!

The above analysis leads to the following procedure for the verification that some given correlated strategy γ forms a correlated equilibrium. 1) Reduce the given game to a simple one. 2) Construct the test game. 3) Check whether each of the strategy profiles (a_1, b_1) and (a_2, b_2) forms a Nash equilibrium. If the answer to the last question is positive, then γ is indeed a correlated equilibrium. If either (a_1, b_1) or (a_2, b_2) is not a Nash equilibrium, then γ is not a correlated equilibrium.

Consider an example. Let us verify that the correlated strategy

$$\gamma = \begin{pmatrix} \gamma_{11} = \frac{1}{3} & \gamma_{12} = 0 \\ \gamma_{21} = \frac{1}{3} & \gamma_{22} = \frac{1}{3} \end{pmatrix}. \quad (12)$$

forms a correlated equilibrium in the game

$$\begin{array}{|c|c|c|} \hline & b_1 & b_2 \\ \hline a_1 & u_{11} = 5, v_{11} = 1 & u_{12} = 0, v_{12} = 0 \\ \hline a_2 & u_{21} = 4, v_{21} = 4 & u_{22} = 1, v_{22} = 5 \\ \hline \end{array}.$$

The reduced simple game is as follows:

$u_1 = u_{11} - u_{21} = 1, v_1 = v_{11} - v_{12} = 1$	$0, 0$
$0, 0$	$u_2 = u_{22} - u_{12} = 1, v_2 = v_{22} - v_{21} = 1$

The test game (constructed by using the above algorithm) has the following payoff matrix:

	b_1	b_2
a_1	$u_1\gamma_{11} = \mathbf{1/3}, v_1\gamma_{11} = \mathbf{1/3}$	$u_1\gamma_{21} = \mathbf{1/3}, v_2\gamma_{21} = \mathbf{1/3}$
a_2	$u_2\gamma_{12} = 0, v_1\gamma_{12} = 0$	$u_2\gamma_{22} = \mathbf{1/3}, v_2\gamma_{22} = \mathbf{1/3}$

Both (a_1, b_1) and (a_2, b_2) are Nash equilibria, consequently, the correlated strategy (12) indeed forms a correlated equilibrium in the given game.

4 Symmetric correlated equilibria in symmetric games

For a symmetric game, $v(a, b) = u(b, a)$ for all a, b , i.e. in our notation, $v_{ij} = u_{ji}$, $i, j = 1, 2$. If the original game

u_{11}, u_{11}	u_{12}, u_{21}
u_{21}, u_{12}	u_{22}, u_{22}

is symmetric, then so is the reduced one:

$u_1 = u_{11} - u_{21}, v_1 = u_1$	$0, 0$
$0, 0$	$u_2 = u_{22} - u_{12}, v_2 = u_2$

A correlated strategy γ is called *symmetric* if the matrix γ is symmetric: $\gamma_{12} = \gamma_{21}$. Consider a simple symmetric game

u_1	0	(13)
0	u_2	

(we write only the payoffs of the first player). If a correlated strategy $\gamma = (\gamma_{ij})$ is symmetric, then the four conditions (10) hold if and only if (I) and (II) hold (conditions (III) and (IV) coincide with (I) and (II) because $u_1 = v_1, u_2 = v_2$ and $\gamma_{12} = \gamma_{21}$).

Our next goal is to characterize symmetric correlated equilibria in simple symmetric games (13).

For symmetric correlated strategies γ , it will be convenient to use the following notation

$$\gamma = \begin{pmatrix} \rho_1 & \theta \\ \theta & \rho_2 \end{pmatrix}, \quad \rho_1 \geq 0, \quad \rho_2 \geq 0, \quad \theta \geq 0, \quad \rho_1 + \rho_2 + 2\theta = 1. \quad (14)$$

where $\rho_1 = \gamma_{11}, \rho_2 = \gamma_{22}$ and $\theta = \gamma_{12} = \gamma_{21}$.

For a simple game (13) and a symmetric correlated strategy (14) the test game is also symmetric and it takes on the following form:

	a_1	a_2	
a_1	$u_1\rho_1$	$u_1\theta$.
a_2	$u_2\theta$	$u_2\rho_2$	

(15)

Conditions (I) and (II) mean that a symmetric correlated strategy γ forms a correlated equilibrium if and only if both (a_1, a_1) and (a_2, a_2) are NE in the test game, i.e.

$$(I) \quad u_1\rho_1 \geq u_2\theta, \quad (II) \quad u_2\rho_2 \geq u_1\theta. \quad (16)$$

For a simple symmetric game (13) and symmetric correlated strategy (14) we can express θ as

$$\theta = (1 - \rho_1 - \rho_2)/2,$$

then conditions (16) can be written as

$$\rho_1(2u_1 + u_2) + \rho_2u_2 \geq u_2, \quad \rho_1u_1 + \rho_2(2u_2 + u_1) \geq u_1. \quad (17)$$

So a correlated strategy (14) forms a correlated equilibrium if and only if the pair of non-negative numbers (ρ_1, ρ_2) satisfies the system of linear inequalities (17) and

$$\rho_1 + \rho_2 \leq 1.$$

Let us describe the set of such pairs of numbers (ρ_1, ρ_2) , representing them as points on the plane. Without loss of generality, we shall assume that $u_1 \geq u_2$.

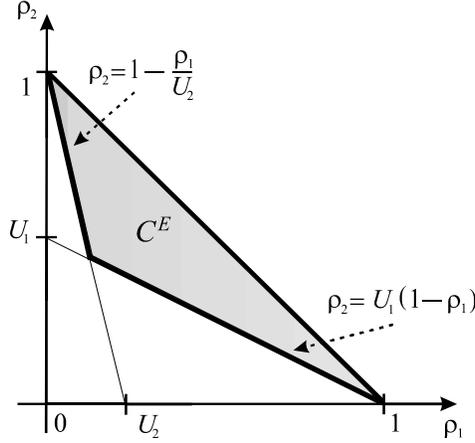
Case 1: $u_1 \geq u_2 > 0$. In this case, the inequalities (17) are equivalent to the following ones:

$$\frac{\rho_1}{U_2} + \rho_2 \geq 1, \quad \rho_1 + \frac{\rho_2}{U_1} \geq 1,$$

where

$$U_1 = \frac{u_1}{u_1 + 2u_2}, \quad U_2 = \frac{u_2}{2u_1 + u_2}.$$

The set of points (ρ_1, ρ_2) with $\rho_1 \geq 0$, $\rho_2 \geq 0$ satisfying these inequalities and $\rho_1 + \rho_2 \leq 1$ is the triangle C^E :



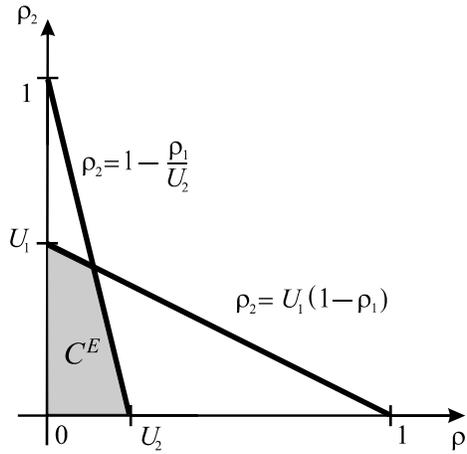
Case 2: $0 > u_1 \geq u_2$. Then the inequalities (17) can be transformed to the following ones:

$$\frac{\rho_1}{U_2} + \rho_2 \leq 1, \quad \rho_1 + \frac{\rho_2}{U_1} \leq 1, \quad (18)$$

where

$$U_1 = \frac{u_1}{u_1 + 2u_2}, \quad U_2 = \frac{u_2}{2u_1 + u_2}.$$

The set of points (ρ_1, ρ_2) with $\rho_1 \geq 0$, $\rho_2 \geq 0$ satisfying these inequalities is the quadrangle C^E :



Note that the inequality $\rho_1 + \rho_2 \leq 1$ follows automatically from each of the above two inequalities because $0 < U_2 < 1$ and $0 < U_1 < 1$.

Case 3: $u_1 > 0 > u_2$. Then the first of the equilibrium conditions (16) is always satisfied. From the second condition, it follows that $u_2\rho_2 = u_1\theta = 0$. Thus

$$\rho_2 = \theta = 0, \rho_1 = 1,$$

and so in this case the correlated equilibrium is given by

$$\gamma = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

which is nothing but a symmetric pure strategy Nash equilibrium (a_1, a_1) , in which both players play the strategy a_1 .

Case 4: $u_1 > u_2 = 0$. Condition (II) in (16) can be written as $0 = u_2\rho_2 \geq u_1\theta \geq 0$, from which we find that $\theta = 0$. The second condition is always satisfied, and so the set of correlated equilibria is given by

$$\gamma = \begin{pmatrix} \rho_1 & 0 \\ 0 & 1 - \rho_1 \end{pmatrix}, 0 \leq \rho_1 \leq 1.$$

This is the segment connecting the points $(1, 0)$ and $(0, 1)$ in the (ρ_1, ρ_2) -plane.

Case 5: $u_1 = 0 > u_2$. Condition (I) in (16) is always satisfied. From (II) it follows that $0 \geq u_2\rho_2 \geq u_1\theta = 0$, from which we find that $\rho_2 = 0$. Thus in this case, $0 \leq \rho_1 \leq 1$, $\theta = (1 - \rho_1)/2$ and $\rho_2 = 0$. Consequently the set of correlated equilibria consists of correlated strategies of the form

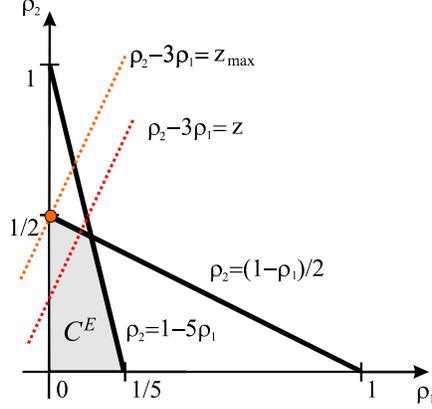
$$\gamma = \begin{pmatrix} \rho_1 & \frac{1-\rho_1}{2} \\ \frac{1-\rho_1}{2} & 0 \end{pmatrix}, 0 \leq \rho_1 \leq 1.$$

This set is represented by the line segment connecting the points $(0, 0)$ and $(1, 0)$ in the (ρ_1, ρ_2) -plane.

Case 6: $u_1 = u_2 = 0$. In this case any strategy (14) with (ρ_1, ρ_2) from the triangle $\{(\rho_1, \rho_2) : \rho_1 \geq 0, \rho_2 \geq 0, \rho_1 + \rho_2 \leq 1\}$ forms a correlated Nash equilibrium.

5 Example

Let us apply the above considerations to the analysis of a classical example: the "chicken game". This is a symmetric game where each player has two strategies a_1 "dare"



and a_2 "chicken out". The payoff matrix is as follows:

	a_1	a_2	
a_1	0, 0	7, 2	.
a_2	2, 7	6, 6	

We construct the reduced symmetric game (non-diagonal elements are subtracted from columns):

	a_1	a_2
a_1	$u_1 = -2$	0
a_2	0	$u_2 = -1$

Here, $u_1 < 0$, $u_2 < 0$, and so we are in the framework of case 2. We have

$$U_1 = \frac{u_1}{u_1 + 2u_2} = \frac{1}{2} \text{ and } U_2 = \frac{u_2}{2u_1 + u_2} = \frac{1}{5}.$$

By virtue of the results obtained, symmetric correlated equilibria (13) in this game correspond to those and only those pairs of non-negative numbers (ρ_1, ρ_2) that satisfy

$$5\rho_1 + \rho_2 \leq 1, \quad \rho_1 + 2\rho_2 \leq 1. \quad (19)$$

(see (18)). Geometrically, the set C^E of points $(\rho_1, \rho_2) \geq 0$ in the plane satisfying (19) is the quadrangle:

In the game at hand, the expected payoff for the correlated strategy (13) can be computed as follows:

$$u(\gamma) = 6\rho_2 + 9\theta = \frac{3}{2}(3 - 3\rho_1 + \rho_2).$$

It is of interest to examine which of the symmetric correlated equilibria maximizes the expected payoff of each player. Maximizing this function on the quadrangle C^E is equivalent to maximizing $z = \rho_2 - 3\rho_1$ on C^E . The analysis of the above diagram below (the dotted lines are level sets of the function z) shows that the maximum is attained at $(\rho_1, \rho_2) = (0, 1/2)$, for which

$$\gamma = \begin{pmatrix} 0 & 1/4 \\ 1/4 & 1/2 \end{pmatrix}.$$

and $u(\gamma) = 5\frac{1}{4}$.

This game does not have any symmetric pure strategy Nash equilibrium, but it has a unique symmetric mixed strategy Nash equilibrium, in which the players select a_1 and a_2 with probabilities $1/3$ and $2/3$ and the expected payoff of each player is $4\frac{1}{3}$. We can see that the correlated equilibrium described above yields a tangibly larger payoff of $5\frac{1}{4}$.

It is well known that Nash equilibria are quite often inefficient. An equilibrium might be a compromise, the achievement of which entails substantial losses to all the players. A correlated equilibrium makes it possible, by invoking a mediator, to reduce these losses. This illustrates in a game-theoretic setting the potential efficiency of intermediaries in resolving conflicts of interest.

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