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ABSTRACT: We prove existence of the maximin (efficient incentive compatible) value allocation in an economy with countably infinite states.

KEYWORDS: value allocation; asymmetric information; ambiguity; countable states; existence.

JEL Classification: D5 · D81 · D82 · D86

1. Introduction

De Castro and Yannelis (2009) introduce a private information maximin value allocation notion in an exchange economy with partition type differential information and ambiguity. The economy was assumed to be finite. That is, containing a finite number of states and commodities, while being comprised of a finite number of non-Bayesian agents.

Angelopoulos and Koutsougeras (2014) enrich and generalize this maximin general equilibrium concept in various aspects. First, they assume arbitrary information exchange protocols within agents' coalitions, so that agents' prior beliefs and maximin utilities become coalitional dependent, accommodating any kind of information for individuals as group members, not necessarily and only their private information. In this way, the private (information) maximin value allocation becomes just a special (yet, notable) case. Second, with these conceptual tools in hand, the authors of this paper introduce both ex ante and interim maximin value allocation notions. Finally, they allow for an infinite dimensional commodity space in the underlying economy.

In this paper we restrict our attention to the (ex ante) private maximin value allocation only, which is worth pursuing for the following sequence of reasons of increasing significance:

To begin with, it is a Shapley (1969) value allocation notion, hence it is a fair cooperative equilibrium concept. Indeed, each agent is assigned with a utility level, which is the expected marginal contribution of this agent to all the coalitions he participates.

Thereafter, it is a direct extension of the private (Bayesian) value allocation of Krasa and Yannelis (1994, 1996). Thus, within it, fairness is strengthened; better informed agents are assigned with higher utility.

Unlike the private value allocation, however, the maximin value allocation exists without being necessarily tied with the private information measurability assumption on agents' net trades (see in Angelopoulos and Koutsougeras, 2014). Thereby, all the maximin (Pareto efficient) value allocations are taken into account and there is no efficiency loss in equilibrium.

More importantly, for the accomplishment of incentive compatibility of this notion[1], less informational measurability restrictions are imposed. That is, assuming only informational measurable initial endowments, every maximin value allocation is maximin efficient incentive compatible (see again in Angelopoulos and Koutsougeras, 2014). In that sense, no conflict between efficiency and incentive compatibility occurs.

Many real life economies are better modeled and explained via a non-finite number of states, countable or uncountable depending on the case. In the maximin preferences framework, nevertheless, this poses elemental analytical obstacles: agents' maximin utilities (minimized over the states) are not well defined to begin with. Existence issues of the maximin value allocation with a non - finite set of states were raised in Angelopoulos and Koutsougeras (2014). This paper attempts to deal with this matter when, specifically, the set of states is countable.

In sections 2 and 3 the appropriate analytical framework is established; the ambiguous economy and the maximin value allocation are, respectively, defined. The existence result of the paper is provided in section 4. Existence of the maximin value allocation is proved by truncating the countability of the set of states; a technique also adopted in He and Yannelis (2013), but for the maximin Walrasian expectations equilibrium. In section 5 we conclude.

2. The Ambiguous Economy

The ambiguous economy is a two (ex ante - ex post) period exchange economy, within which the non-Bayesian asymmetrically informed agents are, in particular, maximin ambiguity averse. There is a finite number of individuals (maximin agents) participating into the ambiguous economy, who are allowed to cooperate and form alliances. Their trade (contract writing) occurs in Euclidean spaces. They write their consumption contacts facing, specifically, countable infinitely many states of nature of the world.

[1]Incentive compatibility is a contract theoretic property of an allocation. It is originated by the contracts' ex post fulfillment issue. See, for example, in the introduction of de Castro et al (2011) or of Angelopoulos and Koutsougeras (2014).

We construct such an economy following the footsteps of de Castro and Yannelis (2009), de Castro et al. (2011, 2012), Angelopoulos and Koutsougeras (2014) and He and Yannelis (2013).

$I = \{1, 2, ..., s\}$ is the finite set of agents of the economy and an $S \in \mathcal{P}(I)$ is a coalition of agents. There is a finite number, l, of commodities traded in the market and $\mathbb{R}^l$ is the economy's commodity space. The underlying state contingent uncertainty, state dependent randomness and informational structure in the economy are summarized by a countable set of states $\Omega = \mathbb{N} = \{\omega_n\}_{n\in\mathbb{N}}$. $\mathcal{F} = \mathcal{P}(\Omega)$ is the natural σ - algebra of Ω, containing all the events of the economy.

$\mathbb{R}^l$, Ω and $\mathcal{F}$ are common (factors) to all the economy's agents. Agents, particularly, are assigned with the following differential characteristics:

1. Informational sets. Π_i is the agent's i partition of Ω and $\mathcal{F}_i \subseteq \mathcal{F}$ is the same agent's σ - algebra, generated by Π_i. Both of them interchangeably represent the private (or asymmetric) information of the i agent. It is specifically assumed that the agent's i partition of $\mathbb{N}$ contains countable infinitely many finite (only) sets. That is, the states between which the agent i cannot distinguish are always of finite number. In other words, the privately informed agent i cannot be "too uninformed". Finally, it is maintained that the agent i retains his private information within his coalitions. There are no underlying information exchange protocols inside agents' groups. Consequently, the agents' priors and preferences are not coalitional dependent[2].
2. Prior beliefs. The (σ - additive) probability measure $q_i : \mathcal{F}_i \to [0, 1]$ is the informationally restricted private prior of the i agent. By definition, q_i satisfies the following incompleteness property: $q_i(B_i)$ may be unknown for a $\emptyset \neq B_i \subset A_i \in \mathcal{F}_i$, even though $q_i(A_i)$ is provided (known) by q_i. That is, the economy's agents may be unable to completely form a prior belief. In other words, agents face ambiguity.
3. Preferences, consumption sets and endowments. With $u_i(\omega) := u_i^\omega : \mathbb{R}^l_+ \to \mathbb{R}_+$, $\omega \in \Omega$, we denote the agent's i random state dependent (r.s.d.) utility function(s), representing the same agent's preferences (over r.s.d. consumption). For each agent i, u_i^ω, $\omega \in \Omega$, is taken to be continuous and concave. Additionally, for each agent i,

[2]Of course, one can proceed as in Angelopoulos and Koutsougeras (2014): allow individuals to exchange information within their groups by obeying to arbitrary information sharing rules and, thereby, examine generalized informational aspects of the (ex ante) maximin value allocation.

the family of utility functions $\{u_i^\omega : \mathbb{R}_+^l \to \mathbb{R}_+, \omega \in \Omega\}$ is assumed to be uniformly bounded. An $x_i(\omega) \in \mathbb{R}_+^l$, $\omega \in \Omega$, is a r.s.d. consumption bundle of the i agent. Then, an $\{x_i(\omega)\}_{\omega\in\Omega} := x_i \in (\mathbb{R}_+^l)^\infty$ is a r.s.d. consumption plan of the i agent. Followingly, $X_i = \{x_i : x_i \in (\mathbb{R}_+^l)^\infty\} \subset (\mathbb{R}_+^l)^\infty$ is the (feasible) r.s.d. consumption set of this agent[3]. An $e_i(\omega) \in \mathbb{R}_+^l$, $\omega \in \Omega$, is a r.s.d. initial endowment of the agent i and $\{e_i(\omega)\}_{\omega\in\Omega} := e_i \in \ell_+^1 \cap X_i \neq \emptyset$ is the same agent's r.s.d. initial endowment plan[4]. Agents' preferences over r.s.d. consumption bundles give rise to their maximin preferences as well, over r.s.d. consumption plans. Agent's i maximin preferences are represented by his maximin (expected) utility function $\upsilon_i : X_i \to \mathbb{R}_+$, which is defined by

$$\upsilon_i(x_i) = \sum_{A_i\in\Pi_i} [\min_{\omega\in A_i} u_i^\omega(x_i(\omega))] q_i(A_i).$$

Agents are thought to be rational, that is, utility maximizers. Hence, agents are maximin utility maximizers as well. Lastly, agents are assumed to have monotone (increasing) maximin preferences[5].

Remark 1 The formulation above was established in de Castro and Yannelis (2010) with a finite Ω. With a countable Ω, He and Yannelis (2013) (well) define and use it as well, in a close (but different) to ours manner[6]. In our setting, this format is well defined because, for each agent i: (i) each $A_i \in \Pi_i$ is finite (so the minimum in the expression above is attained) and (ii) $\{u_i^\omega : \mathbb{R}_+^l \to \mathbb{R}_+, \omega \in \Omega\}$ is uniformly bounded (so the the sum above is finite). $\square$

[3]For each agent i, X_i is presumed to be an uncountably infinite set.

[4]Note that $(\mathbb{R}^l)^\infty = \mathbb{R}^{l\times\infty}$ is an infinite dimensional Euclidean space (i.e., of dimension $l.\infty = \infty$). It is the space of all $l \times \infty$ matrices with real entries. Equivalently viewed, it is the space of all sequences of real $l \times 1$ vectors. Henceforth, ℓ^1 is thought of as the subspace of $(\mathbb{R}^l)^\infty$ containing all the summable sequences of real $l \times 1$ vectors.

[5]First the column wise and then the coordinate wise ordering is assumed on $(\mathbb{R}^l)^\infty$.

[6]He and Yannelis (2013) define (on $\Omega \times \mathbb{R}_+^l$) one r.s.d. utility function for each agent, instead of defining a class of r.s.d. utility functions (one for each state) for each agent. They also use a different assumption to derive the fact that all the elements of an agent's partition are finite. The essential difference, however, is that He and Yannelis (2013) allow for the maximin utility of an agent to be infinity.

Remark 2 It is worth observing that if (i) agents are assumed to have private information measurable r.s.d. utility functions, i.e., *if for any* $i \in I$ *we have that:* $\omega, \bar{\omega} \in A_i \in \Pi_i$, *then* $u_i^{\omega}(x_i(\omega)) = u_i^{\bar{\omega}}(x_i(\bar{\omega}))$ and (ii) agents' priors are assumed to be non - informationally restricted and of full support, the previous formula reduces to the standard Bayesian (or subjective) expected utility. As it was to be expected: (i) the private information measurability condition is necessary (and unavoidable) in the Bayesian context, while (ii) Bayesian agents (i.e., agents with Bayesian preferences) are accommodated in our model as a special case. □

We have finally derived the following ambiguous economy:

$$\mathcal{E} = \{ \ (\mathbb{R}^l)^{\infty} \ ; \ (\Omega, \mathcal{F}) \ ; \ (\ [\ \mathcal{F}_i(\Pi_i), X_i, e_i, \upsilon_i(u_i^{\omega}, q_i) \ ] : i \in I \ ) \ \}.$$

A r.s.d. allocation (contract) of $\mathcal{E}$ is a list of all the economy's agents' r.s.d. consumption plans. It is notated as

$$x = (x_1, x_2, ..., x_i, ..., x_s) \in X = \prod_{i \in I} X_i \subset ((\mathbb{R}_+^l)^{\infty})^s$$

and is said to be feasible if $\sum\limits_{i \in I} x_i = \sum\limits_{i \in I} e_i \iff \sum\limits_{i \in I} x_i(\omega) = \sum\limits_{i \in I} e_i(\omega)$, for all $\omega \in \Omega$.

According to the feasibility condition, the market (i.e., the economy) clears without free disposal.

3. The maximin value allocation

Upon the ambiguous economy constructed in the previous section, we can now define the (ex ante private) maximin value allocation of Angelopoulos and Koutsougeras, 2014.

First, however, we have to define a Shapley (1953) - value - solvable maximin transferable utility (MTU) game.

Let the MTU game $\Gamma = (I, V_{\lambda,\upsilon}, Sh)$. Γ is a cooperative (coalitional) game, allowing for side payments among its finitely many players $1, 2, ..., s \in I$. Within Γ, the players' payoffs are identified with maximin utilities and:

1. υ is the set of all the players' maximin utility functions. The players' maximin utilities $\upsilon_i(\cdot)$, $i \in I$, become common scaled (hence, interpersonally comparable) and transferable by a personal factor $\lambda_i \geq 0$ assigned to each player i, such that not all λ_i are equal to zero. $\lambda \in \mathbb{R}^s_+ \setminus \{0\}$ is the vector of all the players' factors. At the same time, λ_i is the player's i weight to Γ, so that $\sum_{i \in I} \lambda_i = 1$. Finally, $V(\lambda, \upsilon) := V_{\lambda,\upsilon} : 2^I \to \mathbb{R}_+$ is a set function, called the maximin characteristic function of Γ. It measures the gain (i.e., the maximin utility level) of every coalition $S \subseteq I$[7]. The $V_{\lambda,\upsilon}$ of Γ must have a specific (any) functional form, satisfying monotonicity, superadditivity and normalized to become zero for the empty set (coalition).
2. $\mathcal{V}$ is the class of all the $V_{\lambda,\upsilon}$ of Γ. Then, $Sh : \mathcal{V} \to \mathbb{R}^s_+$ is the maximin Shapley value function of Γ, assigning: (i) to Γ the maximin Shapley (1953) value $Sh(V_{\lambda,\upsilon})$, which is a vector of $\mathbb{R}^s_+$ and (ii) to each player i of Γ the respective coordinate $Sh_i(V_{\lambda,\upsilon})$ of the previous vector. The latter is the maximin Shapley value of the i player, a proposed (positive) maximin utility level to be received by this player. For each player i of Γ, his $Sh_i(V_{\lambda,\upsilon})$ is given by the formula

$$Sh_i(V_{\lambda,\upsilon}) = \sum_{S \subseteq I, i \in S} \tfrac{(|S|-1)!(|I|-|S|)!}{|I|!}[V_{\lambda,\upsilon}(S) - V_{\lambda,\upsilon}(S \setminus \{i\})], \text{ where } |I| = s,$$

which conveys the following interpretation: each player i is assigned with a maximin utility $Sh_i(V_{\lambda,\upsilon}) \in \mathbb{R}_+$, which is the expected marginal contribution of this player to all the different(ly sized) coalitions S he becomes a member of. Therefore, for any $V_{\lambda,\upsilon} \in \mathcal{V}$, $Sh(V_{\lambda,\upsilon})$ is a fair solution to Γ. $Sh(V_{\lambda,\upsilon})$ is a normative solution as well, satisfying: (i) (group rationality) $\sum_{i \in I} Sh_i(V_{\lambda,\upsilon}) = V_{\lambda,\upsilon}(I)$ and (ii) (individual rationality) $Sh_i(V_{\lambda,\upsilon}) \geq V_{\lambda,\upsilon}(\{i\})$, for all $i \in I$.

Now, by associating $\mathcal{E}$ with Γ and by appropriately defining the $V_{\lambda,\upsilon}$ of Γ, i.e., by attaching a specific functional form to $V_{\lambda,\upsilon}$, we define the maximin value allocation of $\mathcal{E}$ as follows:

[7]Hence, $V_{\lambda,\upsilon}$ measures the worth or power of every coalition.

Definition Let the $V_{\lambda,v}$ of Γ be given for every coalition $S \subseteq I$ by

$$V_{\lambda,v}(S) = \max_{x_i \in X_i} \sum_{i \in S} \lambda_i \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} u_i^{\omega}(x_i(\omega))] q_i(A_i),$$

$$\text{subject to } \sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega), \text{ for all } \omega \in \Omega.$$

Then, an allocation $x \in X$ of $\mathcal{E}$ is said to be an (*ex ante private*) *maximin value allocation* if the following two conditions are satisfied:

1. $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$, for all $\omega \in \Omega$.
2. For all $i \in I$, we have that $\lambda_i \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} u_i^{\omega}(x_i(\omega))] q_i(A_i) = Sh_i(V_{\lambda,v})$,

where $\lambda_i \geq 0$ for all i and $\sum_{i \in I} \lambda_i = 1$. □

The following series of remarks is in order:

Remark 3 The maximin value allocation is a cardinal value allocation. It is easy to show that the group rationality (individual rationality, respectively) of $Sh(V_{\lambda,v})$ guarantees the maximin Pareto efficiency (maximin individual rationality, respectively) of the maximin value allocation. Let $\Pi_i(\omega)$ be the element of the agent's i partition Π_i containing ω, the state realized in the economy's second period. Then, by appropriately adjusting Theorem 2 of Angelopoulos and Koutsougeras (2014), it is straightforward to prove that (i) the maximin efficiency and (ii) the condition *if for any* $i \in I$ *we have that:* $\omega, \bar{\omega} \in A_i \in \Pi_i$, *then* $e_i(\omega) = e_i(\bar{\omega})$[8] secure the transfer

[8] This is the private information measurability assumption, imposed on the agents' initial endowments. Private information measurable agents' consumption is not, on the other hand, demanded.

maximin coalitional incentive compatibility of the maximin value allocation[9]. $\square$

Remark 4 It can be easily verified that the $V_{\lambda,v}$ of Γ is (indeed) monotone, superadditive and becomes zero for the empty coalition (of none agent). A coalition S obtains for its members a total gain of $V_{\lambda,v}(S)$. By the way $V_{\lambda,v}$ is defined, it is secured that every coalition S of $\mathcal{E}$ acts rationally, i.e., maximizes its maximin utility, subject to the feasibility of consumption within S. The coalition's S maximin utility is its agents' aggregate common scaled (and weighted) maximin utility. $\square$

Remark 5 When agents have monotone maximin preferences, it can be easily understood that the following property is valid for every (feasible and maximin individually rational Pareto optimal) maximin value allocation of $\mathcal{E}$: *Every coalition maximizes its maximin utility subject to its consumption constraints* if and only if *every agent in a coalition independently maximizes his maximin utility subject to the feasibility of consumption within this coalition.* This allows us to deduce that if $x \in X$ is a maximin value allocation of $\mathcal{E}$, then the $V_{\lambda,v}$ of Γ ends up being defined by

$$V_{\lambda,v}(S) = \max_{x_i \in X_i} \sum_{i \in S} \lambda_i \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} u_i^{\omega}(x_i(\omega))] q_i(A_i) =$$

$$= \sum_{i \in S} \lambda_i \max_{x_i \in X_i} \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} u_i^{\omega}(x_i(\omega))] q_i(A_i) =$$

$$= \sum_{i \in S} \lambda_i \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} \left(\max_{x_i(\omega)} u_i^{\omega}(x_i(\omega)) \right)] q_i(A_i),$$

$$\text{subject to} \sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega), \text{ for all } \omega \in \Omega,$$

$$\text{that is, finally, by } \; V_{\lambda,v}(S) = \sum_{i \in S} \lambda_i \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} u_i^{\omega}(x_i^{\star}(\omega))] q_i(A_i). \; \square$$

[9]The reader is referred to Angelopoulos and Koutsougeras (2014), for the definition of the notions of maximin: Parato optimality, individual rationality and (transfer coalitional) incentive compatibility of an allocation. De Castro and Yannelis (2010) first introduced the maximin version of these properties an allocation should (desirably) satisfy. Angelopoulos and Koutsougeras (2014) enrich them and provide a stronger notion of incentive compatibility.

Remark 6 Within any maximin value allocation, a coalition S cannot redistribute among its members its allocated maximin utility. That is to say, side payments are not allowed within a maximin value allocation[10]. Finally, although an agent's i λ_i may be zero, the maximin value allocation may still generate a strictly positive maximin utility for this agent. However, an agent's zero weight leads, irrespective of the agent's maximin utility level, to the same agent's zero Shapley value. □

Remark 7 Assuming that: (i) the agents' maximin expected utilities are specified to Bayesian ones as in Remark 2 and (ii) the agents' consumption and initial endowments (i.e., net trades) are, specifically, private information measurable, the definition above reduces to the one of the private (Bayesian) value allocation of Krasa and Yannelis (1994). □

4. Existence

The analysis has revealed that although we begun with a finite dimensional Euclidean space, $\mathbb{R}^l$, as the economy's commodity space, $\mathcal{E}$ ended up being defined with the infinite dimensional Euclidean commodity space $(\mathbb{R}^l)^\infty$. $(\mathbb{R}^l)^\infty$ is a separable, partially ordered vector space. The product topology, that is, the topology of point wise (or coordinate wise) convergence, can be supplied to $(\mathbb{R}^l)^\infty$. $(\mathbb{R}^l)^\infty$, however, does not carry any norm. Thus, the standard separable Banach space methods are not applicable for existence purposes, now that the economy is underpinned by countably many states of nature.

Other techniques, therefore, have to be adopted. Towards this objective, instead of truncating the infinite dimension of the commodity space à la Bewley (1972), we truncate (as in He and Yannelis, 2013) the countability of the set of states. By doing so, the infinite - dimensionality of the commodity space is automatically reduced (to sequential finite dimensions) as well.

To be more precise: (i) We sequentially reduce the economy into its Ω - finite traces, (ii) we prove existence of the maximin value allocation in each one of these

[10]Despite it being associated with a λ - transferable utility game.

truncated economies and (iii) we use limiting arguments to prove existence of the maximin value allocation under the desirable infinity of Ω.

Therefore, given the economy with countable states, we first have to appropriately define a sequence of (truncated) economies with finitely many states. We begin by considering the (countable) set of all the finite subsets of $\Omega = \mathbb{N}$. We then assume any sequence E_n, $n \in \mathbb{N}$, of (not all the) finite subsets of Ω, satisfying the following condition:

$(\mathcal{C})$ For each $n \in \mathbb{N}$, for each $i \in I$ and for each $A_i \in \Pi_i$, we have that either $A_i \cap E_n = \emptyset$ or $A_i \subseteq E_n$.

Finally, for each $n \in \mathbb{N}$, we construct a truncated ambiguous economy $\mathcal{E}^n$ with a finite number of states, such that the finite set of states of $\mathcal{E}^n$ coincides with the term E_n of the aforementioned sequence. Given $\mathcal{E}$, therefore, we construct a sequence $\{\mathcal{E}^n\}_{n\in\mathbb{N}}$ of ambiguous economies containing a finite set of states as follows:

For each $n \in \mathbb{N}$, we define the economy $\mathcal{E}^n$ as

$$\mathcal{E}^n = \{\ (\mathbb{R}^l)^{|\Omega^n|}\ ;\ (\Omega^n, \mathcal{F}^n)\ ;\ (\ [\ \mathcal{F}_i^n(\Pi_i^n), X_i^n, e_i^n, v_i^n((u_i^\omega)^n, q_i^n)\ ] : i \in I\ )\ \},$$

where $\Omega^n = E_n$, $\mathcal{F}^n = \{A \subseteq \Omega^n : A \in \mathcal{F}\} \subset \mathcal{F}$ [11] and for each agent i:

1. The private informational sets Π_i^n and $\mathcal{F}_i^n$ are now finite and defined as

$$\Pi_i^n = \{A_i \subseteq \Omega^n : A_i \in \Pi_i\} \subset \Pi_i \text{ and } \mathcal{F}_i^n = \mathcal{F}_i^n(\Pi_i^n) = \{A_i \subseteq \Omega^n : A_i \in \mathcal{F}_i\} \subset \mathcal{F}_i,$$

i.e., they are the restrictions of Π_i and $\mathcal{F}_i$ to Ω^n respectively[12 13].

2. The private prior q_i^n is now a finitely additive probability measure, defined as $q_i^n = q_i|_{\Omega^n}$, satisfying $q_i^n(A_i \in \Pi_i^n) = q_i(A_i \in \Pi_i)$.

3. $X_i^n = X_i|_{\Omega^n} = \{\{x_i(\omega)\}_{\omega\in\Omega^n} := x_i^n \mid x_i^n \in (\mathbb{R}_+^l)^{|\Omega^n|}\} \subset (\mathbb{R}_+^l)^{|\Omega^n|}$, so that $X_i^n \subset X_i$; it is further assumed that X_i^n is convex and (naturally) that $\{e_i(\omega)\}_{\omega\in\Omega^n} := e_i^n \in X_i^n$.

[11] That is, $\mathcal{F}^n$ is the power set (algebra) of Ω^n, which is the restriction of the power set (σ - algebra) $\mathcal{F}$ of Ω to Ω^n.

[12] Notice that condition $\mathcal{C}$ (i.e., the criterion of choosing the sequence $E_n = \Omega^n$, $n \in \mathbb{N}$) guarantees that Π_i^n is well defined, that is to say, Ω^n is well partitioned by every agent i. Consequently, condition $\mathcal{C}$ ensures that every truncated ambiguous economy $\mathcal{E}^n$ is well defined.

[13] Note also that $\mathcal{F}_i^n \subseteq \mathcal{F}^n$.

4. $\{(u_i^\omega)^n : \mathbb{R}_+^l \to \mathbb{R}_+, \omega \in \Omega^n\} \subset \{u_i^\omega : \mathbb{R}_+^l \to \mathbb{R}_+, \omega \in \Omega\}$, so that, for each $\omega \in \Omega^n$, $(u_i^\omega)^n = u_i^\omega : \mathbb{R}_+^l \to \mathbb{R}_+$(which is continuous, concave and has a uniform bound).
5. $\upsilon_i^n : X_i^n \to \mathbb{R}_+$, defined by $\upsilon_i^n(x_i^n) = \sum\limits_{A_i \in \Pi_i^n} [\min\limits_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i)$.

A r.s.d. allocation of $\mathcal{E}^n$ is denoted as

$$x^n = (x_1^n, x_2^n, ..., x_i^n, ..., x_s^n) \in X^n = \prod_{i \in I} X_i^n \subset ((\mathbb{R}_+^l)^{|\Omega^n|})^s$$

and is said to be feasible, if $\sum\limits_{i \in I} x_i^n = \sum\limits_{i \in I} e_i^n \iff \sum\limits_{i \in I} x_i(\omega) = \sum\limits_{i \in I} e_i(\omega)$, for all $\omega \in \Omega^n$.

We now state and prove the following lemma:

Lemma If the function $f : X \subseteq \mathbb{R}^{m<\infty} \to \mathbb{R}$ is continuous and bounded on X, then X is closed in $\mathbb{R}^m$ [14].

Proof. Assume that X is not closed. Then, we have that $\partial X \not\subset X$, i.e., there exists $y \in \partial X$, such that $y \notin X$. Considering the Euclidean norm on $\mathbb{R}^m$, let the continuous and bounded function $f : X \to \mathbb{R}$ be defined by $f(x) = \frac{1}{||x-y||}$. Note that f is continuous on X and because f is bounded on X we have that for all $x \in X$, $\frac{1}{||x-y||} \leq \epsilon$, $\epsilon > 0$. Consider now the (open) ball $B(y, \frac{1}{\epsilon})$, for any $\epsilon > 0$. Since $y \in \partial X$, it must be true that $X \cap B(y, \frac{1}{\epsilon}) \neq \emptyset$. This is turn implies that there exists $x \in X$ such that $x \in B(y, \frac{1}{\epsilon})$, so that $||x-y|| < \frac{1}{\epsilon}$, or that $\frac{1}{||x-y||} > \epsilon$, which is a contradiction. $\square$

We finally state and prove the following existence result:

[14] With respect to the standard topology of $\mathbb{R}^m$.

Theorem A maximin value allocation exists in $\mathcal{E}$.

Proof. step 1

We prove that a maximin value allocation exists in $\mathcal{E}^n$, $n \in \mathbb{N}$. Wlog, we write $\mathcal{E}^n$ as $\mathcal{E}^n = \{ \ (\mathbb{R}^l)^{|\Omega^n|} \ ; (X_i^n, e_i^n, \upsilon_i^n) : i \in I \ \}$, $n \in \mathbb{N}$, which can be directly related with the deterministic economy $\mathcal{E}' = \{\mathbb{R}^{k<\infty} \ ; (\mathbb{X}_i, \epsilon_i, w_i) : i \in I\}$. According to Emmons and Scafuri (1985), a cardinal value allocation exists in $\mathcal{E}'$ if for each agent i: (i) $\epsilon_i \in \mathbb{X}_i \subset \mathbb{R}^k_+$, (ii) $\mathbb{X}_i$ is closed, below bounded and convex in $\mathbb{R}^k$ and (iii) $w_i : \mathbb{X}_i \to \mathbb{R}_+$ is continuous and concave. Hence, a (cardinal) maximin value allocation exists in $\mathcal{E}^n$, $n \in \mathbb{N}$, if the same conditions are accordingly satisfied. Consider the Euclidean norm $|| \cdot ||$, the standard topology and the point wise ordering on any finite dimensional Euclidean space. Fix a $n \in \mathbb{N}$ and an agent $i \in I$ of the (fixed) economy $\mathcal{E}^n$. The continuity of u_i^ω, $\omega \in \Omega^n$, secures continuity for υ_i^n as well. Indeed: u_i^ω, $\omega \in \Omega^n$, is continuous on $\mathbb{R}^l_+$ iff for all $x_i(\omega) \in \mathbb{R}^l_+$, $\omega \in \Omega^n$ and for all $\epsilon > 0$, there exists $\delta(x_i(\omega), \epsilon) > 0$, such that for all $y_i(\omega) \in \mathbb{R}^l_+$, $\omega \in \Omega^n$, with $0 < ||y_i(\omega) - x_i(\omega)|| < \delta$, we have that $0 < |u_i^\omega(y_i(\omega)) - u_i^\omega(x_i(\omega))| < \epsilon$. Now, assume that υ_i^n is continuous on X_i^n. This would mean that for all $x_i^n \in X_i^n$ and for all $\epsilon > 0$, there exists $\delta(x_i^n, \epsilon) > 0$, such that for all $y_i^n \in X_i^n$, with $0 < ||y_i^n - x_i^n|| < \delta$, we have that

$$0 < | \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(y_i(\omega))] q_i^n(A_i) - \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) \ | < \epsilon, \text{ or}$$

$$0 < | \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(y_i(\omega)) - \min_{\omega \in A_i} u_i^\omega(x_i(\omega))] \ q_i^n(A_i) \ | < \epsilon, \text{ or}$$

$$0 < \sum_{A_i \in \Pi_i^n} | \min_{\omega \in A_i} u_i^\omega(y_i(\omega)) - \min_{\omega \in A_i} u_i^\omega(x_i(\omega))| \ q_i^n(A_i) < \epsilon, \text{ or}$$

$$0 < \sum_{A_i \in \Pi_i^n} \min_{\omega \in A_i} | \ u_i^\omega(y_i(\omega)) - u_i^\omega(x_i(\omega)) \ | \ q_i^n(A_i) < \epsilon.$$

From the last expression it is implied that for each $A_i \in \Pi_i^n$, we have that

$$0 < \min_{\omega \in A_i} | \ u_i^\omega(y_i(\omega)) - u_i^\omega(x_i(\omega)) \ | < \epsilon.$$

But this is true, because $0 < |u_i^\omega(y_i(\omega)) - u_i^\omega(x_i(\omega))| < \epsilon$, $\omega \in \Omega^n$. The concavity of u_i^ω, $\omega \in \Omega^n$, implies concavity for υ_i^n on X_i^n as well (besides, by assumption, X_i^n is convex in $(\mathbb{R}^l)^{|\Omega^n|}$). Indeed: Since u_i^ω, $\omega \in \Omega^n$, is concave on $\mathbb{R}_+^l$, it holds that for every $x_i(\omega), y_i(\omega) \in \mathbb{R}_+^l$, $\omega \in \Omega^n$ and for every $t \in [0,1]$ we have that

$$u_i^\omega(tx_i(\omega) + (1-t)y_i(\omega)) \geq tu_i^\omega(x_i(\omega)) + (1-t)u_i^\omega(y_i(\omega)).$$

It follows from the last expression and for an $A_i \in \Pi_i^n$ that

$$\min_{\omega \in A_i} u_i^\omega(tx_i(\omega) + (1-t)y_i(\omega)) \geq \min_{\omega \in A_i} [\ tu_i^\omega(x_i(\omega)) + (1-t)u_i^\omega(y_i(\omega))\ ] =$$

$$t \min_{\omega \in A_i} u_i^\omega(x_i(\omega)) + (1-t) \min_{\omega \in A_i} u_i^\omega(y_i(\omega))$$

and hence that

$$\min_{\omega \in A_i} u_i^\omega(tx_i(\omega) + (1-t)y_i(\omega))\ q_i^n(A_i) \geq$$

$$t \min_{\omega \in A_i} u_i^\omega(x_i(\omega))\ q_i^n(A_i) + (1-t) \min_{\omega \in A_i} u_i^\omega(y_i(\omega))\ q_i^n(A_i).$$

Finally, we conclude that

$$\sum_{A_i \in \Pi_i^n} [\ \min_{\omega \in A_i} u_i^\omega(tx_i(\omega) + (1-t)y_i(\omega))\ ]\ q_i^n(A_i) \geq$$

$$t \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) + (1-t) \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(y_i(\omega))] q_i^n(A_i), \text{ i.e., that}$$

$$\upsilon_i^n(tx_i^n + (1-t)y_i^n) \geq t\upsilon_i^n(x_i^n) + (1-t)\upsilon_i^n(y_i^n), \text{ for any } x_i^n, y_i^n \in X_i^n \text{ and } t \in [0,1].$$

By assumption, $e_i^n \in X_i^n$ (so that $X_i^n \neq \emptyset$). By construction, X_i^n is below (order) bounded by the zero vector. Since u_i^ω, $\omega \in \Omega^n$, is bounded on $\mathbb{R}_+^l$, υ_i^n is also bounded on X_i^n. Then, from the previous lemma, it follows that X_i^n is closed in $(\mathbb{R}^l)^{|\Omega^n|}$. Therefore, there exists an allocation $x^n \in X^n$ of the economy $\mathcal{E}^n$ such that:

1. $\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega)$, for all $\omega \in \Omega^n$,

2. for all $i \in I$, we have that $\lambda_i^n \sum\limits_{A_i \in \Pi_i^n} [\min\limits_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) = Sh_i(V^n_{\lambda^n, \upsilon^n})$, where:
   (i) $\lambda_i^n \geq 0$ for all i, with $\sum\limits_{i \in I} \lambda_i^n = 1$ and

   (ii) $Sh_i(V^n_{\lambda^n,\upsilon^n}) = \sum\limits_{S \subseteq I, i \in S} \frac{(|S|-1)!(|I|-|S|)!}{|I|!} [V^n_{\lambda^n,\upsilon^n}(S) - V^n_{\lambda^n,\upsilon^n}(S \setminus \{i\})]$

is the Shapley value of the i agent, derived from the truncated maximin TU game $\Gamma^n = (I, V^n_{\lambda^n,\upsilon^n}, Sh)$, whose characteristic function $V^n_{\lambda^n,\upsilon^n}$ is defined by

$$V^n_{\lambda^n,\upsilon^n}(S) = \max_{x_i^n \in X_i^n} \sum_{i \in S} \lambda_i^n \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i),$$

$$\text{subject to } \sum_{i \in S} x_i(\omega) = \sum_{i \in S} e_i(\omega), \text{ for all } \omega \in \Omega^n.$$

step 2

We approximate the existence of a feasible allocation in $\mathcal{E}$ by all the existing maximin value allocations in the sequence of economies $\{\mathcal{E}^n\}_{n \in \mathbb{N}}$. Consider a feasible allocation $x \in X$ of $\mathcal{E}$. From this allocation's feasibility condition

$$\sum_{i \in I} x_i(\omega) = \sum_{i \in I} e_i(\omega), \text{ for all } \omega \in \Omega,$$

the following condition is implied

$$\sum_{\omega \in \Omega} \sum_{i \in I} x_i(\omega) = \sum_{\omega \in \Omega} \sum_{i \in I} e_i(\omega) = e \ (< \infty, \text{ because } e_i \in \ell^1_+ \cap X_i, \text{ for all } i).$$

This means that for each agent i, each $x_i(\omega)$, $\omega \in \Omega$, belongs in the compact rectangle $[0, e]$ of $\mathbb{R}^l$. Define now, for each agent i, the set

$$C_i = C = \{x_i : 0 \leq x_i(\omega) \leq e, \, \omega \in \Omega \ \} = [0, e]^\infty \subset (\mathbb{R}^l_+)^\infty.$$

Clearly, C is compact in $(\mathbb{R}^l)^\infty$ with respect to the product topology of $(\mathbb{R}^l)^\infty$. Certainly, the set $\prod\limits_{i \in I} C_i = C^{|I|=s}$ (which contains all the feasible allocations of $\mathcal{E}$) is also compact in $((\mathbb{R}^l)^\infty)^s$. Fix now again a $n \in \mathbb{N}$ and consider the corresponding (fixed) economy $\mathcal{E}^n$ with its (feasible) maximin value allocation $x^n \in X^n$. As previously,

$$\sum_{i\in I} x_i(\omega) = \sum_{i\in I} e_i(\omega), \text{ for all } \omega \in \Omega^n \Rightarrow \sum_{\omega\in\Omega^n}\sum_{i\in I} x_i(\omega) = \sum_{\omega\in\Omega^n}\sum_{i\in I} e_i(\omega) = e^n,$$

from which it is implied that for each agent i, each $x_i(\omega)$, $\omega \in \Omega^n$, belongs in the rectangle $[0, e^n]$ of $\mathbb{R}^l_+$. We define now the set

$$C_i^n = C^n = \{x_i^n : 0 \leq x_i(\omega) \leq e^n, \, \omega \in \Omega^n \} = [0, e^n]^{|\Omega^n|} \subset (\mathbb{R}^l_+)^{|\Omega^n|}.$$

Since by construction $[0, e^n] \subseteq [0, e]$ and, thus, $[0, e^n]^{|\Omega^n|} \subset [0, e]^\infty$, we conclude that for each agent i it holds that $x_i(\omega) \in [0, e]$, $\omega \in \Omega^n$ and $x_i^n \in C$. So that finally $x^n \in C^s$. Observe also that the agents' vector λ^n belongs in the unit $(s-1)$ - simplex of $\mathbb{R}^s$, which we denote as Δ. Then, by notating the maximin value allocation $x^n \in X^n$ of $\mathcal{E}^n$ with the augmented form $(x_1^n, ..., x_s^n \ , \lambda_1^n, ...\lambda_s^n) = (x^n, \lambda^n)$, we have that $(x^n, \lambda^n) \in C^s \times \Delta = K$. Evidently, K is compact in $((\mathbb{R}^l)^\infty)^s \times \mathbb{R}^s$, so that every sequence of K has a convergent subsequence in K. Consider the sequence $\{(x^m, \lambda^m) : m \in \mathbb{N}\}$ of K and its convergent subsequence $\{(x^n, \lambda^n) : n \in \mathbb{N}\}$ to the point (x, λ) of K. Since for each $n \in \mathbb{N}$ the maximin value allocation of the economy $\mathcal{E}^n$ belongs in K, we can wlog identify the previous (sub)sequence with the sequence of the existing maximin value allocations in the sequence of economies $\{\mathcal{E}^n\}_{n\in\mathbb{N}}$. Concluding, the sequence of the maximin value allocations (of the truncated economies of the original economy) converges to the point $(x, \lambda) \in K$, which is a feasible allocation of the default economy $\mathcal{E}$.

step 3

We verify that (x, λ) is a maximin value allocation for $\mathcal{E}$, i.e., that conditions 1, 2 of the definition in section 3 are satisfied in the limit of the sequence $\{(x^n, \lambda^n) : n \in \mathbb{N}\}$. Condition 1, i.e., the feasibility of the allocation (x, λ), was derived in step 2. For condition 2, we need to show that for the existing (by step 2) $\lambda_i \geq 0$ for all $i \in I$, with $\sum_{i\in I} \lambda_i = 1$, the following condition is satisfied

$$\lambda_i \sum_{A_i\in\Pi_i} [\min_{\omega\in A_i} u_i^\omega(x_i(\omega))] q_i(A_i) = Sh_i(V_{\lambda,v}), \text{ for all } i.$$

For all $n \in \mathbb{N}$ and for any i, we have proven that

$$\lambda_i^n \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) = Sh_i(V^n_{\lambda^n, \upsilon^n}).$$

Hence, for any i (and provided that the following limits exist), it holds that

$$\lim_{n\to\infty} \lambda_i^n \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) = \lim_{n\to\infty} Sh_i(V^n_{\lambda^n, \upsilon^n}), \text{ or that}$$

$$\lambda_i \lim_{n\to\infty} \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) =$$

$$\lim_{n\to\infty} \sum_{S \subseteq I, i \in S} \tfrac{(|S|-1)!(|I|-|S|)!}{|I|!} [V^n_{\lambda^n, \upsilon^n}(S) - V^n_{\lambda^n, \upsilon^n}(S \setminus \{i\})] =$$

$$\sum_{S \subseteq I, i \in S} \tfrac{(|S|-1)!(|I|-|S|)!}{|I|!} [\lim_{n\to\infty} V^n_{\lambda^n, \upsilon^n}(S) - \lim_{n\to\infty} V^n_{\lambda^n, \upsilon^n}(S \setminus \{i\})] = Sh_i(\lim_{n\to\infty} V^n_{\lambda^n, \upsilon^n}).$$

Since $\Omega = \mathbb{N}$, there exists an increasing (by containment) sequence $\{A_n\}_{n\in\mathbb{N}}$ of subsets of Ω, such that A_n is finite for all $n \in \mathbb{N}$ and $\bigcup_{n\in\mathbb{N}} A_n = \Omega$. Since $\{A_n\}_{n\in\mathbb{N}}$ satisfies condition $\mathcal{C}$, we do not loose in generality if we identify the sequence $\{A_n\}_{n\in\mathbb{N}}$ with the sequence $\{\Omega^n\}_{n\in\mathbb{N}}$. Then, the increasing set sequence $\{\Omega^n\}_{n\in\mathbb{N}}$ is above bounded by and convergent to Ω, i.e.,

$$\limsup_n \Omega^n = \liminf_n \Omega^n = \lim_n \Omega^n = \Omega.$$

It is then implied that $\lim_n \mathcal{E}^n = \mathcal{E}$ and in particular (for any i) that:

As $n \to \infty$, $u_i^\omega(x_i(\omega))$, $\omega \in \Omega^n \to u_i^\omega(x_i(\omega))$, $\omega \in \Omega$, (ii) $\lim_{n\to\infty} q_i^n(A_i) = q_i(A_i)$,

(iii) $\lim_{n\to\infty} \Pi_i^n = \Pi_i$ and (iv) $\lim_{n\to\infty} X_i^n = X_i$ (hence, $\lim_{n\to\infty} x_i^n = x_i$, for any $x_i^n \in X_i^n$).

The previous establish the fact that the expression

$$\lambda_i \lim_{n\to\infty} \sum_{A_i \in \Pi_i^n} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i^n(A_i) = Sh_i(\lim_{n\to\infty} V^n_{\lambda^n, \upsilon^n}),\ i \in I,$$

leads to the desirable expression $\lambda_i \sum_{A_i \in \Pi_i} [\min_{\omega \in A_i} u_i^\omega(x_i(\omega))] q_i(A_i) = Sh_i(V_{\lambda,\upsilon})$, $i \in I$.

The left hand side limit is the well defined maximin utility of each agent. The proof concludes by showing that the right hand side limit is well defined as well. For this, we have to prove that the $V_{\lambda,v}$ of Γ[15] exists (is well defined). For every $n \in \mathbb{N}$, consider the existing maximin value allocation (x^n, λ^n) of $\mathcal{E}^n$, in which (by definition) feasibility of consumption is satisfied within any coalition. Then, for any $S \subseteq I$,

$$\sum_{i\in S} x_i(\omega) = \sum_{i\in S} e_i(\omega), \text{ for all } \omega \in \Omega^n \Rightarrow \sum_{\omega\in\Omega^n}\sum_{i\in S} x_i(\omega) = \sum_{\omega\in\Omega^n}\sum_{i\in S} e_i(\omega) = \varepsilon^n,$$

which means that for every $i \in S$, each $x_i(\omega)$, $\omega \in \Omega^n$, belongs in the compact rectangle $[0, \varepsilon^n]$ of $\mathbb{R}^l$. For every $i \in S$, we know that u_i^ω, $\omega \in \Omega^n$ is continuous on $\mathbb{R}^l_+$, thus on $[0, \varepsilon^n] \subset \mathbb{R}^l_+$ as well. Then, for every $i \in S$ and for any $\omega \in \Omega^n$, it follows from the Weierstrass' Extreme Value Theorem that

$$\max_{x_i(\omega)\in[0,\varepsilon^n]} u_i^\omega(x_i(\omega)) = u_i^\omega(x_i^\star(\omega)) \text{ exists.}$$

For every $i \in S$, however, as $n \to \infty$, $u_i^\omega(x_i^\star(\omega))$, $\omega \in \Omega^n \to u_i^\omega(x_i^\star(\omega))$, $\omega \in \Omega$, thus $u_i^\omega(x_i^\star(\omega))$, $\omega \in \Omega$, exists as well[16]. Since for every $i \in S$ each $A_i \in \Pi_i$ is finite and the family $\{u_i^\omega : \mathbb{R}^l_+ \to \mathbb{R}_+, \omega \in \Omega\}$ is uniformly bounded, we conclude that

$$\text{both the } \min_{\omega\in A_i} u_i^\omega(x_i^\star(\omega)) \text{ exists and the } \sum_{A_i\in\Pi_i} [\min_{\omega\in A_i} u_i^\omega(x_i^\star(\omega))] q_i(A_i) \text{ (finitely) exists,}$$

for each $i \in S$. So that then the

$$\sum_{i\in S}\lambda_i \sum_{A_i\in\Pi_i} [\min_{\omega\in A_i} u_i^\omega(x_i^\star(\omega))] q_i(A_i) \text{ also exists.}$$

According to Remark 5, this finally means that

$$V_{\lambda,v}(S) = \max_{x_i\in X_i} \sum_{i\in S}\lambda_i \sum_{A_i\in\Pi_i} [\min_{\omega\in A_i} u_i^\omega(x_i(\omega))] q_i(A_i),$$

$$\text{subject to } \sum_{i\in S} x_i(\omega) = \sum_{i\in S} e_i(\omega), \text{ for all } \omega \in \Omega \text{ } (\Rightarrow \text{subject to } \sum_{\omega\in\Omega}\sum_{i\in S} x_i(\omega) = \sum_{\omega\in\Omega}\sum_{i\in S} e_i(\omega) = \varepsilon), \text{ exists.} \quad \square$$

[15]In the way it was specified in the definition of section 3.

[16]Note that $\lim_{n\to\infty} \varepsilon^n = \varepsilon = \sum_{\omega\in\Omega}\sum_{i\in S} e_i(\omega) = \sum_{\omega\in\Omega}\sum_{i\in S} x_i(\omega)$ ($< \infty$, since $e_i \in \ell^1_+ \cap X_i$, for all $i \in S$).

5. Conclusions

In this paper we allowed for countably infinite states in an ambiguous economy and established the viability of a maximin value allocation in it.

One may reasonably argue that the maximin value allocation is a pessimistic equilibrium notion. Nevertheless, as in de Castro and Yannelis (2009), de Castro et al. (2011, 2012), He and Yannelis (2013) and Angelopoulos and Koutsougeras (2014), pessimism turns out to be a normative attitude in general equilibrium terms. Indeed, maximin-pessimistic agents enjoy higher (first best) efficiency in equilibrium; the maximin value allocation exists without (necessarily) private information measurable consumption and initial endowments. On top of that, less maximin (efficient) value allocations are lost for the achievement of incentive compatibility of this concept.

Given that uncountable set of states arise naturally in many real life economies, the issue of examining the possibility of existence of a maximin (efficient incentive compatible) value allocation with a continuum of states bears considerable importance.

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