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Securely Implementable Social Choice Rules with Partially Honest Agents

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Abstract

We define secure implementation with *partially honest* agents in a social choice model and we show that strategy-proofness is a necessary and sufficient condition for it. This result offers a behavioral foundation for *rectangularity*; and it remains valid even with only one partially honest agent. We apply the concept to a single-crossing voting environment, and we prove that it characterizes the family of augmented representative voter schemes. JEL Codes: C72, D03, D71, D82.

Key-words: Secure implementation; partial honesty; strategy-proofness; single-crossing preferences; representative (median) voter.

1 Introduction

It is well known in the literature on social choice that many strategy-proof mechanisms possess multiple Nash equilibria, some of which produce undesirable outcomes. This obviously creates problems when the mechanism is used in practice, since the designer cannot rule out completely the possibility of observing equilibrium behavior that is inconsistent with truth-telling and with the designer's objectives.

To deal with this problem and to provide a better foundation for implementation theory, Saijo, Sjöström, and Yamato (2007) have recently proposed the concept of *secure implementation*. Roughly speaking, a social choice rule is securely implementable if there exists a game form (mechanism) that simultaneously implements it in dominant strategy equilibria and in Nash equilibria.

Appealing as it might sound, secure implementation has been shown to be hard to achieve, especially in the context of voting, where non-pivotal voters are allowed to behave

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in arbitrary ways by the Nash equilibrium concept. This obviously renders the implementation of social goals a difficult task. A compelling example is provided by Saijo, Sjöström, and Yamato (2007) themselves, who show that on the classical single-peaked preference domain of public economics, if a Pareto efficient social choice rule is secure implementable, then it must be dictatorial.¹

This paper reexamines the concept of secure implementation in a social choice model where agents are *partially honest*. This behavioral hypothesis, which has been recently considered by Matsushima (2008), Dutta and Sen (2012) and Kartik, Tercieux and Holden (2014), among others, postulates essentially that economic agents in a mechanism design environment strictly prefer to report their true preferences whenever misrepresenting them does not produce a better social outcome according with their true preferences.

Apart from assuming partial honesty, the current work focuses on a model where each agent receives a preference ordering from an admissible preference domain over a finite set of alternatives. This constitutes each agent's private information. Society wishes to achieve a certain goal represented by a social choice rule, that is, by a mapping from the admissible preference profiles to the set of outcomes. To do that, every individual is required to submit simultaneously and independently a preference relation, which does not need to be the true one. Abstention is not permitted.

In this framework, the paper shows that a well known incentive compatibility property, namely, strategy-proofness, is necessary and sufficient for secure implementation. A previous characterization without partially honest agents, due to Saijo et al. (2007), has demanded in addition to strategy-proofness, the so-called *rectangularity* property. Thus, the result found here can be interpreted as providing a behavioral foundation for *rectangularity*, in the sense that under partial honesty, the condition is automatically satisfied. Moreover, the result remains valid even with only one partially honest agent. Thus, it also indicates that rectangularity might not be after all too strong.

After deriving the result pointed out above, the paper applies the concept of secure implementation with partially honest agents to a voting environment with single-crossing preferences. This preference domain plays an important role in political economics by guaranteeing the existence of Condorcet winners when convexity is violated. Moreover, as is known from Saporiti (2009), it also allows for the existence of appealing strategy-proof mechanisms, such as the median choice rule. The current paper proves that a social choice rule is securely implementable on a maximal single-crossing domain if and only if it is a member of the family of augmented representative voter schemes. This generalizes Saporiti's (2009) characterization by including within the family non-anonymous social

¹As is well known from Moulin (1980), this restricted domain allows for the existence of 'nice' strategy-proof social choice rules, such as the median voter scheme, which is also anonymous and efficient.

choice rules. It also stands in sharp contrast with Saijo et al.'s (2007) results for voting environments with single-peaked preferences and without partial honesty, where secure implementation comes at the price of democracy.

The rest of the paper is organized as follows. Section 2 presents the model, the notation and the main definitions. It also describes an example that is used throughout the paper to explain the intuitions behind the results. Section 3 contains the main contributions of this work, as well as an informal discussion of the results. All proofs are collected in Appendix A at the end of the paper.

2 Preliminaries

Consider a social choice environment with a finite set of agents $N = \{1, \dots, n\}$, $n \geq 2$. Let $X = \{x, y, z, \dots\}$ be a finite set of mutually exclusive alternatives, with $|X| > 2$.² Denote by \mathcal{R} the set of all complete and transitive binary relations on X , with generic element R , where P (resp., I) represents the strict (resp., indifference) preference relation induced by R . For any $R \in \mathcal{R}$ and any $Y \subseteq X$, define a *top* or *peak* alternative of R on Y as $\tau|_Y(R) \in \arg \max_Y (R)$. For simplicity, $\tau|_X(R) = \tau(R)$.

Let $\mathcal{D} \subset \mathcal{R}$ be the admissible domain of individual preferences, which is assumed to be the same for everybody and commonly known. Suppose each agent $i \in N$ is endowed with a preference relation $R_i \in \mathcal{D}$, which constitutes i 's private information. Let $\mathcal{D}^n = \prod_{i=1}^n \mathcal{D}$ be the set of all preference profiles $\rho = (R_1, \dots, R_n)$. As usual, $\rho_{-i} = (R_1, \dots, R_{i-1}, R_{i+1}, \dots, R_n)$; for each $\hat{R}_i \in \mathcal{D}$, $(\hat{R}_i, \rho_{-i}) = (R_1, \dots, R_{i-1}, \hat{R}_i, R_{i+1}, \dots, R_n)$; and, for every set $S \subseteq N$, $\rho_S = (R_i)_{i \in S} \in \prod_{i \in S} \mathcal{D} = \mathcal{D}^{|S|}$.

The problem for this society is to make a social choice from the set of alternatives X . Each agent is entitled to report a preference relation from the admissible set \mathcal{D} . These reports are intended to provide information about the true preferences of society, although agents' sincerity cannot be guaranteed. A social choice rule $f : \mathcal{D}^n \rightarrow X$ associates to each profile of reported preferences $\rho \in \mathcal{D}^n$ a unique social alternative $f(\rho) \in X$. Denote the range of f by $r_f = \{x \in X : \exists \rho \in \mathcal{D}^n \text{ such that } f(\rho) = x\}$. In the sequel, assume that f has a full range, so that $r_f = X$. In particular, given that $|X| > 2$, this rules out constant social choice functions.

A mechanism Γ with consequences in X is a strategic game form $(S_i, \phi)_{i \in N}$, where S_i is the set of actions (pure strategies) of each agent $i \in N$, and $\phi : S \rightarrow X$ is an outcome function that associates an alternative from X with every action profile $s = (s_i, s_{-i}) \in S = \prod_{i \in N} S_i$. The mechanism Γ is called the direct mechanism associated with f if

²For every set A , $|A|$ stands for the cardinality of the set, and \bar{A} for the complement of A .

$S_i = \mathcal{D}$ for all $i \in N$, and $\phi = f$. A mechanism Γ together with a preference profile $\rho = (R_i, \rho_{-i}) \in \mathcal{D}^n$ induces a game $G = (\Gamma, \rho)$.

A Nash equilibrium for G is a strategy profile $s^* \in S$ such that for all $i \in N$, there does not exist $s_i \in S_i$ such that $\phi(s_i, s_{-i}^*) P_i \phi(s_i^*, s_{-i}^*)$. Similarly, the strategy profile $s \in S$ is said to be a dominant strategy equilibrium for $G = (\Gamma, \rho)$ if $s_i \in S_i$ is a dominant strategy for each agent $i \in N$, (i.e., if there does not exist $\hat{s}_i \in S_i$ such that $\phi(\hat{s}_i, \hat{s}_{-i}) P_i \phi(s_i, \hat{s}_{-i})$ for some $\hat{s}_{-i} \in S_{-i}$). Let $N(G)$ (resp., $DS(G)$) denote the set of Nash (resp., dominant strategy) equilibria of G .

A mechanism Γ **securely implements** the social choice rule f if for all $\rho \in \mathcal{D}^n$, (i) there exists $s \in DS(\Gamma, \rho)$ such that $f(\rho) = \phi(s)$, and (ii) for all $s \in N(\Gamma, \rho)$, $f(\rho) = \phi(s)$. In words, secure implementation requires that for each preference profile, there exists a dominant strategy equilibrium whose outcome coincides with the designer's objective (alternatively, is “ f -optimal”), and all Nash equilibria produce the f -optimal outcome.

A social choice rule $f : \mathcal{D}^n \rightarrow X$ is said to be strategy-proof if for all $i \in N$ and all $(R_i, \rho_{-i}) \in \mathcal{D}^n$, there is no $R'_i \in \mathcal{D}$ such that $f(R'_i, \rho_{-i}) P_i f(R_i, \rho_{-i})$. If a social choice rule f is not strategy-proof, then f is said to be manipulable by $i \in N$ at $(R_i, \rho_{-i}) \in \mathcal{D}^n$ via $R'_i \in \mathcal{D}$. The next example illustrates that although strategy-proofness is necessary for secure implementation (Saijo et al. 2007), it is not sufficient.

Example 1 Let $n = 3$ and $X = \{x, y, z\}$, with $z > y > x$. Suppose the admissible preference domain is $\mathcal{D} = \{xyz, xzy, zxy, zyx\}$, where each abc , $a, b, c \in X$, represents a preference ordering $P \in \mathcal{D}$ with the property that $a P b P c$.³ Let $\bar{\rho} = (xyz, xzy, zxy)$ be the profile of true preferences. Fix the median choice rule f , defined as $f(\rho) = \text{median}(\tau(P_1), \tau(P_2), \tau(P_3))$ for all $\rho \in \mathcal{D}^3$. As is shown later in Corollary 1, f is strategy-proof. Moreover, it is easy to show that the direct mechanism (\mathcal{D}, f) has a dominant strategy equilibrium at $\bar{\rho}$ given by the strategy profile $s^* = (xyz, xzy, zxy)$. Indeed, consider first the incentives of agent 1 to deviate from s^* . For each $s_{-1} \in \mathcal{D}^2$ such that $f(\cdot, s_{-1}) = \text{median}(\cdot, x, x)$, reporting any ordering $s_1 \in \mathcal{D}$ produces the same outcome $f(s_1, s_{-1}) = x$; in particular, that holds for s_1^* . On the other hand, for any $s_{-1} \in \mathcal{D}^2$ such that either $f(\cdot, s_{-1}) = \text{median}(\cdot, x, z)$, or $f(\cdot, s_{-1}) = \text{median}(\cdot, z, x)$, reporting 1's true preferences xyz generates a social choice that coincides with his preferred alternative. Thus, true-telling is a dominant strategy for agent 1; and, by the same token, so is for any other individual. However, as is illustrated in Table 1, the mechanism (\mathcal{D}, f) possess Nash equilibria where the outcomes differ from the true-telling equilibrium.⁴ A case in point is

³As a passing remark, notice that \mathcal{D} is not a single-peaked domain.

⁴For expositional convenience, in the payoff matrices it has been assumed that the preferences of all agents are represented by the same utility function, which gives a numerical value of 1 to the top alternative on X , 0 to the middle ranked alternative, and -1 to the bottom option. Apart from the payoffs, the cells also display the social choice, i.e., the median top corresponding to each strategy profile.

the profile $s = (zxy, zyx, zxy)$, which is a Nash equilibrium for $((\mathcal{D}, f), \bar{\rho})$ because any individual deviation from s is incapable of modifying the social choice $f(s) = z$. The same is true for any other profile corresponding to the pink cells of Table 1.⁵ Thus, the median choice rule is not securely implementable on this preference domain.⁶

		Agent 2			
		xyz	xzy	zxy	zyx
Agent 1	xyz	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>
	xzy	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>
	zxy	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	zyx	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
		xyz			
		Agent 3			
		xyz	xzy	zxy	zyx
Agent 1	xyz	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	xzy	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	zxy	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	zyx	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
		zxy			
		Agent 3			

		Agent 2			
		xyz	xzy	zxy	zyx
Agent 1	xyz	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>
	xzy	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>	1 1 0 <i>x</i>
	zxy	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	zyx	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
		xzy			
		Agent 3			
		xyz	xzy	zxy	zyx
Agent 1	xyz	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	xzy	1 1 0 <i>x</i>	1 1 0 <i>x</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	zxy	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
	zyx	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>	-1 0 1 <i>z</i>
		zyx			
		Agent 3			

Table 1: Example 1

As happens in other voting environments, the previous example illustrates that in the case under study, the direct mechanism admits undesirable Nash equilibrium outcomes when one or more agents are unable to influence the social choice given the strategies of the others. In those cases, Nash equilibrium allows individuals to report any preference relation as part of their best responses, and that creates unappealing results. Precisely, to rule out this, it is common in political economics to refine the equilibrium concept and to demand for example that voters do not play in equilibrium weakly dominated strategies.

An alternative to equilibrium refinements comes from recent research on implementation theory, which departs from the classical theory by assuming that people face an intrinsic lying cost that holds them back from misreporting, at least to some extent

⁵Note in the Table that truth-telling is not the unique dominant strategy equilibrium; in effect, so is any profile associated with the blue cells. However, all of them lead to the same social choice x .

⁶A similar conclusion applies to other members of the family of social choice rules characterized in Proposition 1.

(Matsushima 2008, Dutta and Sen 2012, and Kartik et al. 2014).⁷ In the context of mechanism design, this implies that individuals have preferences not just on the set of outcomes, but also directly on the messages that they are required to send. Specifically, agents are assumed to be partially honest, in the sense that they strictly prefer to report the true state rather than a false state when misreporting does not provide any individual gains from the chosen outcome.

To be more formal, suppose $\rho = (R_i, \rho_{-i}) \in \mathcal{D}^n$ is the profile of true preferences over X . For each agent $i \in N$, define a complete and transitive preference relation \succsim_i^ρ on $\mathcal{D} \times X$ as follows. For all $\hat{\rho}_{-i} \in \mathcal{D}^{n-1}$, and all $\hat{R}_i \in \mathcal{D}$, with $\hat{R}_i \neq R_i$,

(i) If $f(R_i, \hat{\rho}_{-i}) \succ f(\hat{R}_i, \hat{\rho}_{-i})$, then $(R_i, f(R_i, \hat{\rho}_{-i})) \succ_i^\rho (\hat{R}_i, f(\hat{R}_i, \hat{\rho}_{-i}))$,

(ii) Otherwise, $(R_i, f(R_i, \hat{\rho}_{-i})) \succsim_i^\rho (\hat{R}_i, f(\hat{R}_i, \hat{\rho}_{-i}))$ if and only if $f(R_i, \hat{\rho}_{-i}) \succ f(\hat{R}_i, \hat{\rho}_{-i})$,

where \succ_i^ρ denotes the antisymmetric part of \succsim_i^ρ . Let $\succsim^\rho = (\succsim_i^\rho)_{i \in N}$ be the profile of preferences over the augmented set $\mathcal{D} \times X$.

Example 2 (*Continued Example 1*). Table 2 reproduces Example 1's payoffs when all individuals are partially honest. Following Kartik et al. (2014), this is done by adding an $\epsilon > 0$ to the individual payoffs when the agent makes an honest report. The table displays the best responses of players 1, 2 and 3, in yellow, green and blue, respectively, and it shows that there exists a unique dominant strategy equilibrium, which is also the unique Nash equilibrium of the game. That's actually the strategy profile $s^* = (xyz, xyz, zxy)$, where each agent announces his true preferences. Thus, when all agents are partially honest, the median choice rule is securely implementable in this example.

More generally, a mechanism Γ is said to **securely implements** the social choice rule f **when individuals are partially honest** if for all $\rho \in \mathcal{D}^n$, (i) there exists $s \in DS(\Gamma, \rho, \succsim^\rho)$ such that $f(\rho) = \phi(s)$, and (ii) for all $s \in N(\Gamma, \rho, \succsim^\rho)$, $f(\rho) = \phi(s)$, where $DS(\Gamma, \rho, \succsim^\rho)$ (resp., $N(\Gamma, \rho, \succsim^\rho)$) represents the set of dominant strategy equilibria (resp., Nash equilibria) with partially honest players. Specifically, a strategy profile $s \in DS(\Gamma, \rho, \succsim^\rho)$ (resp., $s \in N(\Gamma, \rho, \succsim^\rho)$) if for all $i \in N$ there does not exist $\hat{s}_i \in S_i$ such that for some $\hat{s}_{-i} \in S_{-i}$, $(\hat{s}_i, \phi(\hat{s}_i, \hat{s}_{-i})) \succ_i^\rho (s_i, \phi(s_i, \hat{s}_{-i}))$ (resp., there does not exist $\hat{s}_i \in S_i$ such that $(\hat{s}_i, \phi(\hat{s}_i, s_{-i})) \succ_i^\rho (s_i, \phi(s_i, s_{-i}))$), where s_i is what agent i considers "truthful" at ρ .

The next section explores the implementation concept just defined in both, an abstract setting with no specific restriction on the preference domain, and in a model that generalizes Examples 1 and 2.

⁷Evidence on lying costs comes from lab experiments. A robust result is that many subjects misreport their private information to their own advantage, but that a significant number refrains from reporting the payoff maximizing type, and that some subjects are fully honest (Abeler, Becker and Falk 2014).

		2						2						
1		xyz	xzy	zxy	zyx	1		xyz	xzy	zxy	zyx			
	xyz	1+ ϵ	1	0	x		xyz	1+ ϵ	1+ ϵ	0	1+ ϵ	1	0	x
	xzy	1	1	0	x		xzy	1	1+ ϵ	0	1	1	0	x
	zxy	1	1	0	x		zxy	1	1+ ϵ	0	-1	0	1	z
	zyx	1	1	0	x	zyx	1	1+ ϵ	0	-1	0	1	z	
		xyz						xzy						
		3						3						
		2						2						
1		xyz	xzy	zxy	zyx	1		xyz	xzy	zxy	zyx			
	xyz	1+ ϵ	1	ϵ	x		xyz	1+ ϵ	1+ ϵ	ϵ	-1+ ϵ	0	1+ ϵ	z
	xzy	1	1	ϵ	x		xzy	1	1+ ϵ	ϵ	-1	0	1+ ϵ	z
	zxy	-1	0	1+ ϵ	z		zxy	-1	ϵ	1+ ϵ	-1	0	1+ ϵ	z
	zyx	-1	0	1+ ϵ	z	zyx	-1	ϵ	1+ ϵ	-1	0	1+ ϵ	z	
		zxy						zyx						
		3						3						

Table 2: Continued Example 1

3 Results

The following result offers a necessary and sufficient condition for secure implementation with partial honesty that is domain independent, in the sense that it doesn't depend on the structure of the set of admissible preferences. Instead, it holds for every preference domain that allows for the existence of strategy-proof social choice rules.

Theorem 1 *A mechanism Γ securely implements the social choice rule f when individuals are partially honest if and only if f is strategy-proof.⁸*

One could misread Theorem 1 and conclude erroneously that it improves upon Saijo et al. (2007) by ensuring secure implementation without their rectangular condition. However, a more accurate interpretation of this result is that it provides a behavioral foundation for Saijo et al.'s (2007) rectangular property, in the sense that under individual behavior consistent with partial honesty, the condition becomes automatically satisfied.

So long as there could be different rationales for rectangularity, one of which is partial honesty, Saijo et al.'s (2007) characterization of securely implementable social choice functions remains the more general one. But Theorem 1 is nevertheless interesting because of the behavioral content that it provides to their otherwise abstract condition. It

⁸The discussion in the paragraph following Figure 1, as well as the proof of the Theorem 1 make clear that the result holds even if there exists only one partially honest individual.

is, of course, an empirical matter to verify whether partial honesty is actually observed in experiments and field data.

To gain more insight about how partial honesty relates with Saijo et al.'s (2007) condition, recall that a social choice function f satisfies the **rectangularity property** if for all $\rho, \hat{\rho} \in \mathcal{D}^n$, if $f(R_i, \hat{\rho}_{-i}) I_i f(\hat{R}_i, \hat{\rho}_{-i})$ for all $i \in N$, then $f(\rho) = f(\hat{\rho})$. Clearly, Example 1's social choice rule does not satisfy rectangularity. In effect, consider the left-hand table of Figure 1, which reproduces one of the payoffs matrices of Table 1. For simplicity, only the information relevant for the present analysis has been kept. Take the preference profiles $\bar{\rho} = (xyz, xzy, zxy)$ and $\rho = (zxy, zyx, zxy)$. Recall that the former is the truth-telling dominant strategy equilibrium, and the second one of the Nash equilibria. As the figure shows, $f(zxy, zyx, zxy) = f(zxy, xzy, zxy) = f(xyz, zyx, zxy) = z$, implying that individual deviations are useless to modify the social choice. Therefore, for all $i = 1, 2, 3$, $f(\bar{P}_i, \rho_{-i}) I_i f(P_i, \rho_{-i})$. However, $f(\rho) = z \neq x = f(\bar{\rho})$, contradicting rectangularity. That explains why the game form allows two different equilibrium outcomes, one that coincides with the designer's objective (blue cell), and the other not (pink cell).

What if agents are partially honest? As is illustrated in Figure 1b, it is still the case that $f(zxy, zyx, zxy) = f(zxy, xzy, zxy) = f(xyz, zyx, zxy) = z$. However, it does not follow from that that all agents are indifferent between these outcomes. On the contrary, their preference for honesty implies that they are strictly better off by reporting their true preference ordering. Therefore, rectangularity holds; and the social choice rule, which is strategy-proof in this example, is securely implementable.

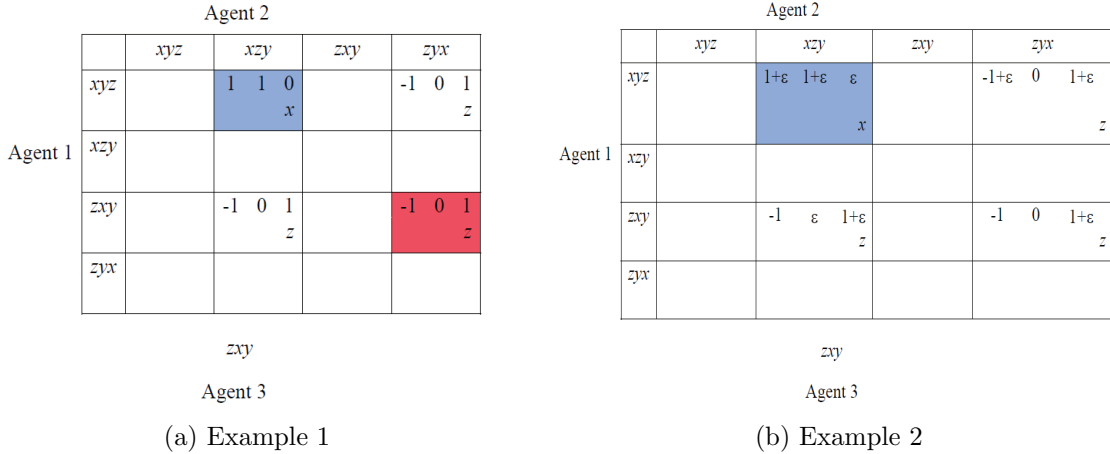


Figure 1: Rectangularity with and without partial honesty

What's more, notice that this is also the case even if only one agent is partially honest: just repeat the analysis above after deleting the extra payoff $\epsilon > 0$ in all but one agent's payoffs, say for example individual 1. This suggests that rectangularity might actually

be not as demanding as one could initially imagine before having a behavioral foundation of it. Highlighting this is probably the main contribution of this work.

We now apply the previous result to a single-crossing voting environment that generalizes the example provided in Section 2.⁹ To be precise, let $\mathcal{P} \subset \mathcal{R}$ be the universal set of strict preference relations on X . An admissible domain $\mathcal{D} \subset \mathcal{P}$ is said to have the **single-crossing property** if there exists a linear order $>$ on X and a linear order \succ on \mathcal{D} such that for all $x, y \in X$ and all $P, P' \in \mathcal{D}$, (1) if $y > x$, $P' \succ P$ and $y P x$, then $y P' x$, and (2) if $y > x$, $P' \succ P$ and $x P' y$, then $x P y$.¹⁰ Fix a maximal set $\mathcal{D} \subset \mathcal{P}$ of single-crossing preferences, with $X(\mathcal{D}) = \{\tau(P) \in X, P \in \mathcal{D}\}$.¹¹ A particular instance of this preference domain was given in Example 1.

Given any profile $\rho = (P_1, \dots, P_n) \in \mathcal{D}^n$, let $\ell_1, \ell_2, \dots, \ell_n$ be a relabeling of the set of agents N such that $\tau(P_{\ell_1}) \leq \tau(P_{\ell_2}) \leq \dots \leq \tau(P_{\ell_n})$. For any odd positive integer k , define the k -median function on X^k , $m^k : X^k \rightarrow X$, in such a way that for each $\mathbf{x} = (x_1, \dots, x_k) \in X^k$, $|\{x_i : m^k(\mathbf{x}) \geq x_i\}| \geq \frac{(k+1)}{2}$ and $|\{x_j : x_j \geq m^k(\mathbf{x})\}| \geq \frac{(k+1)}{2}$. A social choice rule $f : \mathcal{D}^n \rightarrow X$ is said to be an **augmented representative voter scheme** if for all $\rho \in \mathcal{D}^n$ and all $L, M \subset N$, there exist fixed ballots $\alpha_L, \alpha_M \in X(\mathcal{D})$ such that $\alpha_L \geq \alpha_M$ if $L \subseteq M$, and $f(\rho) = m^{2n-1}(\tau(P_1), \dots, \tau(P_n), \alpha_{\{\ell_1\}}, \alpha_{\{\ell_1, \ell_2\}}, \dots, \alpha_{\{\ell_1, \dots, \ell_{n-1}\}})$.

Proposition 1 *A social choice rule f is strategy-proof on a maximal single-crossing domain if and only if f is an augmented representative voter scheme.*

In spite of having this strong incentive compatibility property, Example 1 illustrates that none of these rules is securely implementable on single-crossing preferences. Fortunately, the conclusion changes quite dramatically with a little bit of honesty in society. Indeed, Theorem 1 and Proposition 1 together offer the following characterization result.

Theorem 2 *On a maximal single-crossing domain, a social choice rule f is securely implementable when individuals are partially honest if and only if f is an augmented representative voter scheme.*

Recall that a social choice rule $f : \mathcal{D}^n \rightarrow X$ is anonymous if $\forall \rho, \hat{\rho} \in \mathcal{D}^n$, $f(\rho) = f(\hat{\rho})$ if ρ is a permutation of $\hat{\rho}$. That is, a social choice rule is anonymous if the names of the individuals holding particular preferences are immaterial in deriving social choices. Notice that, since \mathcal{D}^n is a Cartesian product domain, if a profile ρ belongs to \mathcal{D}^n , then

⁹For specific applications, particularly on income taxation, see for instance Gans and Smart (1996), Austen Smith and Banks (1999), Persson and Tabellini (2000), and the references therein.

¹⁰For any $x, y \in X$, we write $x = y$ if and only if $\neg[x > y]$ and $\neg[y > x]$; and $x \geq y$ if and only if either $x = y$ or $x > y$. Similarly, for any two distinct preferences $P, P' \in \mathcal{D}$, $P \prec P'$ if and only if $\neg[P \succ P']$.

¹¹Recall that a set of single-crossing preferences \mathcal{D} is maximal if there does not exist $\mathcal{D}' \subset \mathcal{P}$ such that $\mathcal{D} \subset \mathcal{D}'$ and \mathcal{D}' is single-crossing. The largest size of \mathcal{D} is $|X| \cdot \frac{|X|-1}{2} + 1$ (Saporiti, 2009).

any of its permutations is also in \mathcal{D}^n . Thus, anonymity is non-vacuous in our framework. It is easy to show that an augmented representative voter scheme is anonymous only if for all $L, M \subset N$, $|L| = |M|$ implies that $\alpha_L = \alpha_M$.

With this in mind, the following is an immediate corollary of Theorems 1 and 2.

Corollary 1 *On a maximal single-crossing domain, an anonymous social choice rule f is securely implementable when individuals are partially honest if and only if there exist $n - 1$ fixed ballots $\alpha_1, \dots, \alpha_{n-1} \in X(\mathcal{D})$ such that for every preference profile $\rho \in \mathcal{D}^n$, $f(\rho) = m^{2n-1}(\tau(P_1), \dots, \tau(P_n), \alpha_1, \dots, \alpha_{n-1})$.*

The previous corollary generalizes the intuition about the median choice rule coming from Example 2.¹² Its main message as well as the message given by Theorem 2 contrasts sharply with Saijo et al.'s (2007) predictions for voting environments with single-peaked preferences. The reason lies obviously in the behavioral departure adopted by this work, according to which agents' preferences possess an extra bit of structure and satisfy partial honesty. It is left for future research to test whether this hypothesis finds any support in laboratory experiments. As is nicely explained by a recent paper on lying costs by Abeler, Becker and Falk (2014), there are good reasons to be optimistic about that. In any case, the only purpose of this paper has been to report the positive theoretical results associated with partial honesty and secure implementation.

A Appendix: Missing Proofs

Proof of Theorem 1. (Sufficiency). Let $\rho = (R_i, \rho_{-i}) \in \mathcal{D}^n$ be the true preference profile. Consider the direct mechanism (\mathcal{D}, f) . Since f is by hypothesis strategy-proof,

$$\forall i \in N, \forall \hat{\rho}_{-i} \in \mathcal{D}^{n-1}, \forall \hat{R}_i \in \mathcal{D}, f(R_i, \hat{\rho}_{-i}) R_i f(\hat{R}_i, \hat{\rho}_{-i}). \quad (1)$$

Using the definition of \succsim_i^ρ , (1) implies that

$$\forall i \in N, \forall \hat{\rho}_{-i} \in \mathcal{D}^{n-1}, \forall \hat{R}_i \in \mathcal{D}, \hat{R}_i \neq R_i, (R_i, f(R_i, \hat{\rho}_{-i})) \succsim_i^\rho (\hat{R}_i, f(\hat{R}_i, \hat{\rho}_{-i})).$$

Thus, $\rho \in DS((\mathcal{D}, f), \rho, \succsim^\rho)$, and consequently $N((\mathcal{D}, f), \rho, \succsim^\rho) \neq \emptyset$.

Suppose there exists a Nash equilibrium $\tilde{\rho} = (\tilde{R}_i, \tilde{\rho}_{-i}) \in N((\mathcal{D}, f), \rho, \succsim^\rho)$ such that $f(\tilde{\rho}) \neq f(\rho)$. (Otherwise, this part of the proof is complete.) By (1),

$$\forall i \in N, f(R_i, \tilde{\rho}_{-i}) R_i f(\tilde{R}_i, \tilde{\rho}_{-i}). \quad (2)$$

¹²Notice that the median choice rule is a particular member of the family of augmented representative voter schemes, where the fixed ballots are located on the lower and the upper bounds of X .

Since $\tilde{\rho} \in N((\mathcal{D}, f), \rho, \succsim^\rho)$,

$$\forall i \in N, (\tilde{R}_i, f(\tilde{R}_i, \tilde{\rho}_{-i})) \succsim_i^\rho (R_i, f(R_i, \tilde{\rho}_{-i})),$$

which implies using the definition of \succsim_i^ρ that

$$\forall i \in N, f(\tilde{R}_i, \tilde{\rho}_{-i}) R_i f(R_i, \tilde{\rho}_{-i}). \quad (3)$$

Thus, combining (2) and (3), it follows that for all $i \in N$, $f(\tilde{R}_i, \tilde{\rho}_{-i}) I_i f(R_i, \tilde{\rho}_{-i})$.

Finally, by the definition of partial honesty, there exists $j \in N$ such that $(R_j, f(R_j, \tilde{\rho}_{-j})) >_j^\rho (\tilde{R}_j, f(\tilde{R}_j, \tilde{\rho}_{-j}))$, which stands in contradiction with the fact that $\tilde{\rho}$ is a Nash equilibrium for the game $((\mathcal{D}, f), \rho, \succsim^\rho)$. Therefore, the direct mechanism (\mathcal{D}, f) securely implements f .

(Necessity). By hypothesis, there exists a mechanism $\Gamma = (S_i, \phi)$ that securely implements f when agents are partially honest. That means that for all $\rho \in \mathcal{D}^n$, there exists $s \in DS(\Gamma, \rho, \succsim^\rho)$ such that $\phi(s) = f(\rho)$. Moreover, for all $s \in N(\Gamma, \rho, \succsim^\rho)$, $\phi(s) = f(\rho)$. Thus, Γ dominant strategy implements f when agents are partially honest.

Suppose f is not strategy-proof. Then, there exists $j \in N$, and $\rho' = (R'_j, \rho'_{-j})$ and $\rho'' = (R''_j, \rho''_{-j})$, with $\rho', \rho'' \in \mathcal{D}^n$, such that $f(R''_j, \rho''_{-j}) P'_j f(R'_j, \rho'_{-j})$. By hypothesis, there exists $s' = (s'_j, s'_{-j}) \in DS(\Gamma, \rho', \succsim^{\rho'})$ such that $\phi(s') = f(\rho')$, where s'_i is what agent i considers truthful at ρ' . Consider a dominant strategy $s''_j \in S_j$ at R''_j . Note that $(s''_j, s'_{-j}) \in DS(\Gamma, (R''_j, \rho''_{-j}), \succsim^{(R''_j, \rho''_{-j})})$. By dominant implementability, $\phi(s''_j, s'_{-j}) = f(R''_j, \rho''_{-j})$. By hypothesis, $\phi(s''_j, s'_{-j}) P'_j \phi(s'_j, s'_{-j})$. Using the definition of $\succsim_j^{\rho'}$,

$$(s''_j, \phi(s''_j, s'_{-j})) >_j^{\rho'} (s'_j, \phi(s'_j, s'_{-j})),$$

contradicting that $s' \in DS(\Gamma, \rho', \succsim^{\rho'})$. Therefore, f is strategy-proof. \blacksquare

Proof of Proposition 1. (Sufficiency). Suppose, by contradiction, that f is manipulable at $\rho \in \mathcal{D}^n$. Relabel N (if necessary) so that $\tau(P_1) \leq \tau(P_2) \leq \dots \leq \tau(P_n)$. By hypothesis, there exist $i \in N$ and $P'_i \in \mathcal{D}$ such that $f(P'_i, \rho_{-i}) P_i f(P_i, \rho_{-i})$. Hence, $f(P'_i, \rho_{-i}) \neq f(P_i, \rho_{-i}) \neq \tau(P_i)$. Without loss of generality, assume that $f(P_i, \rho_{-i}) > \tau(P_i)$.

By definition of f , $f(P_i, \rho_{-i}) \in X(\mathcal{D})$. Let $f(\rho) = \tau(P_j)$ for some $j \in N$, $j \neq i$,¹³ with

$$\underbrace{\tau(P_1) \leq \dots \leq \tau(P_i)}_{j-1} \leq \dots < \dots \leq \tau(P_j) = f(\rho) \leq \underbrace{\tau(P_{j+1}) \leq \dots \leq \tau(P_n)}_{n-j}. \quad (4)$$

¹³The other case, i.e., $f(\rho) = \alpha_L \in X(\mathcal{D})$ for some $L \subset N$, with $\alpha_L \neq \tau(P_j)$ for all $j \in N$, is proved in a similar fashion. Results are available from the author upon request.

Notice that f leaves $n - 1$ peaks and fixed ballots to each side of $f(\rho)$. From (4), only the first $j - 1$ individual peaks are lower than or equal to $f(\rho)$. Thus, there must be $n - j$ fixed ballots which are smaller than or equal to $f(\rho)$. Specifically,

$$\underbrace{\alpha_{\{1, \dots, n-1\}} \leq \dots \leq \alpha_{\{1, \dots, j\}}}_{n-j} \leq \tau(P_j) = f(\rho) \leq \underbrace{\alpha_{\{1, \dots, j-1\}} \leq \dots \leq \alpha_{\{1\}}}_{j-1}. \quad (5)$$

Denote $\rho' = (P'_i, \rho_{-i})$, and relabel N such that $\tau(P'_{\ell_1}) \leq \tau(P'_{\ell_2}) \leq \dots \leq \tau(P'_{\ell_n})$, where $\tau(P'_k) = \tau(P_k)$ for all $k \neq i$. Clearly, for all $k < j$, $k \neq i$, $\tau(P'_k) \leq \tau(P_j)$; and, for all $k > j$, $\tau(P'_k) \geq \tau(P_j)$. First, suppose that $\tau(P'_i) \leq f(\rho)$. That means that $\tau(P'_i) \leq \tau(P'_j)$; and, consequently, that agent i is a member of every set $L \in \{\{\ell_1, \dots, \ell_k\}, k = j - 1, \dots, n - 1\}$. Hence, $\alpha_{\{\ell_1, \dots, \ell_k\}} = \alpha_{\{1, \dots, k\}}$ for all $k = j - 1, \dots, n - 1$, which results by (5) in that

$$\underbrace{\alpha_{\{\ell_1, \dots, \ell_{n-1}\}} \leq \dots \leq \alpha_{\{\ell_1, \dots, \ell_j\}}}_{n-j} \leq \tau(P'_j) \leq \alpha_{\{\ell_1, \dots, \ell_{j-1}\}}. \quad (6)$$

To recap, the profile $\rho' = (P'_i, \rho_{-i})$ is such that $j - 1$ (resp., $n - j$) individual peaks are smaller (resp., greater) than or equal to $\tau(P'_j)$. In addition, (6) implies that $n - j$ (resp., $j - 1$) fixed ballots are located at or below (resp., above) $\tau(P'_j)$. (Recall that for all $L \subset \{\ell_1, \dots, \ell_{j-1}\}$, $\alpha_L \geq \alpha_{\{\ell_1, \dots, \ell_{j-1}\}}$.) Therefore, $f(\rho') = \tau(P'_j) = f(\rho)$, a contradiction.

Second, assume that $\tau(P'_i) > f(\rho)$. Notice that $\{\ell_1, \dots, \ell_{j-2}\} = \{1, \dots, i - 1, i + 1, \dots, j - 1\} \subset \{1, \dots, j - 1\}$. Therefore, (5) and the definition of f imply that

$$\underbrace{\alpha_{\{\ell_1\}} \geq \alpha_{\{\ell_1, \ell_2\}} \geq \dots \geq \alpha_{\{\ell_1, \dots, \ell_{j-2}\}}}_{j-2} \geq \tau(P'_j). \quad (7)$$

Combining (7) with the fact that $n - j + 1$ individual peaks of ρ' (namely, $\tau(P'_i)$ and $\{\tau(P'_k)\}_{k > j}$) are greater than or equal to $\tau(P'_j)$, it follows that $f(\rho') > \tau(P'_j) = f(\rho)$. Moreover, $P_j \succ P_i$. Otherwise, if $P_i \succ P_j$, then $\tau(P_i) < f(\rho)$ would imply by single-crossing that $\tau(P_i) P_j \tau(P_j)$, a contradiction. But then, since by hypothesis $f(\rho') P_i f(\rho)$, it follows from single-crossing that $f(\rho') P_j f(\rho)$, contradicting again that $\tau(P_j)$ is agent j 's most preferred alternative on X .

(Necessity). Fix a strategy-proof social choice function $f : \mathcal{D}^n \rightarrow X$. Following Saporiti (2009), f is tops-only and top-monotonic.¹⁴ Moreover, f is strategy-proof only if for all $i \in N$ and all $\rho \in \mathcal{D}^n$, $f(P_i, \rho_{-i}) = m^3(\tau(P_i), f(\underline{P}_i, \rho_{-i}), f(\overline{P}_i, \rho_{-i}))$, where \underline{P} (resp., \overline{P}) stands for the most leftist (resp., rightist) preference relation on X , so that

¹⁴A social choice rule f is tops-only if for all $\rho, \hat{\rho} \in \mathcal{D}^n$ such that $\tau|_{r_f}(P_i) = \tau|_{r_f}(\hat{P}_i)$ for all $i \in N$, $f(\rho) = f(\hat{\rho})$. On the other hand, f is top-monotonic if for all $i \in N$, all $(P_i, \rho_{-i}) \in \mathcal{D}^n$, and all $P'_i \in \mathcal{D}$ such that $\tau(P'_i) \geq \tau(P_i)$, $f(P'_i, \rho_{-i}) \geq f(P_i, \rho_{-i})$.

for any pair $x, y \in X$, $x \underline{P} y$ (resp., $y \overline{P} x$) if and only if $y > x$. Clearly, $\underline{P}, \overline{P} \in X(\mathcal{D})$. Moreover, $\tau(\underline{P}) = \min_{>} X = \underline{X}$ and $\tau(\overline{P}) = \max_{>} X = \overline{X}$.

For any $L \subset N$, define $\alpha_L = f(\underline{\rho}_L, \overline{\rho}_L)$. By top monotonicity, it is easy to prove that $\alpha_L \geq \alpha_M$ for all $L \subseteq M$, with $L, M \subset N$. Next, we show that $\alpha_L \in X(\mathcal{D})$. Consider first any $L = \{\ell\}$, with $\ell \in N$. Without loss of generality, assume that $f(\underline{P}_\ell, \overline{\rho}_{-\ell}) = z \neq \tau(P)$ for all $P \in \mathcal{D}$. Take a preference $P_\ell^\alpha \in \mathcal{P}$ with the property that $\tau(P_\ell^\alpha) = \tau(\underline{P}_\ell)$ and $\overline{X} P_\ell^\alpha z$. If $P_\ell^\alpha \in \mathcal{D}$, we are done. By tops-only, $f(P_\ell^\alpha, \overline{\rho}_{-\ell}) = z$. By unanimity over the range (which is implied by strategy-proofness), $f(\overline{\rho}) = \overline{X}$. Thus, agent ℓ can manipulate f at $(P_\ell^\alpha, \overline{\rho}_{-\ell})$ via \overline{P}_ℓ .

Instead, if $P_\ell^\alpha \notin \mathcal{D}$, then there must exist a $P^* \in \mathcal{D}$ such that $\tau(P^*) > \tau(\underline{P}_\ell)$ and $z P^* \overline{X}$. Let $P_\ell^\beta = \min_{>} \{P' \in \mathcal{D} : \tau(P') > \underline{X}\}$. Clearly, $z P_\ell^\beta \overline{X}$ because either P_ℓ^β coincides with P^* or $P_\ell^\beta \prec P^*$. Let $f(P_\ell^\beta, \overline{\rho}_{-\ell}) = z^\beta$. If $z > z^\beta$, agent ℓ would manipulate f at $(\underline{P}_\ell, \overline{\rho}_{-\ell})$ via P_ℓ^β . Similarly, if $z^\beta = \overline{X}$, then ℓ would manipulate f at $(P_\ell^\beta, \overline{\rho}_{-\ell})$ via \underline{P}_ℓ . Hence, $\overline{X} > z^\beta \geq z$.

Suppose $z^\beta = \tau(P_\ell^\beta)$. Then, $z^\beta > z$. Furthermore, there exists a $P_\ell^{\alpha'} \in \mathcal{D}$ such that $\tau(P_\ell^{\alpha'}) = \underline{X}$ and $z^\beta P_\ell^{\alpha'} z$. Indeed, to rule out $P_\ell^{\alpha'}$ from \mathcal{D} there should be a $P^{**} \in \mathcal{D}$ such that $\tau(P^{**}) > \underline{X}$ and $z P^{**} z^\beta$. By the definition of P_ℓ^β , $P^{**} \succ P_\ell^\beta$ (note that they cannot be equal because by hypothesis $z^\beta = \tau(P_\ell^\beta)$ and $z P^{**} z^\beta$); and, by single-crossing we would have that $z P_\ell^\beta z^\beta$, a contradiction. Thus, $P_\ell^{\alpha'} \in \mathcal{D}$. By tops-only, $f(P_\ell^{\alpha'}, \overline{\rho}_{-\ell}) = z$. Hence, agent ℓ can manipulate f at $(P_\ell^{\alpha'}, \overline{\rho}_{-\ell})$ via P_ℓ^β , a contradiction. Therefore, $z^\beta \neq \tau(P_\ell^\beta)$.

Consider a preference $P_\ell^{\alpha+1} \in \mathcal{P}$ such that $\tau(P_\ell^{\alpha+1}) = \tau(P_\ell^\beta)$ and $\overline{X} P_\ell^{\alpha+1} z^\beta$. If $P_\ell^{\alpha+1} \in \mathcal{D}$, by tops-only, $f(P_\ell^{\alpha+1}, \overline{\rho}_{-\ell}) = z^\beta$. Thus, agent ℓ can manipulate f at $(P_\ell^{\alpha+1}, \overline{\rho}_{-\ell})$ via \overline{P}_ℓ . On the contrary, if $P_\ell^{\alpha+1} \notin \mathcal{D}$, then we can repeat the previous argument and find a preference $P_\ell^{\beta+1} \in \mathcal{D}$ such that $\tau(P_\ell^{\beta+1}) > \tau(P_\ell^\beta)$ and $z^\beta P_\ell^{\beta+1} \overline{X}$. Since X is finite and in each step the top of the blocking ordering gets larger and larger, the sequence $\{\tau(P_\ell^{\beta+n})\}_{n=0}^\infty$ approaches $\tau(\overline{P}_\ell)$ as n increases. Hence, if we continue applying the same argument over and over again at some point we will either find the desired contradiction, or a preference $P_\ell^{\beta+n} \in \mathcal{D}$ such that (i) $\tau(P_\ell^{\beta+n}) = \tau(\overline{P}_\ell)$ and (ii) $z^{\beta+n-1} P_\ell^{\beta+n} \overline{X}$, which leads to a violation of strategy-proofness because $f(P_\ell^{\beta+n}, \overline{\rho}_{-\ell}) = \overline{X}$. Therefore, $f(\underline{P}_\ell, \overline{\rho}_{-\ell}) = \alpha_{\{\ell\}} \in X(\mathcal{D})$ for all $\ell \in N$.

Using the induction argument, assume that $\alpha_{\{\ell_1, \dots, \ell_k\}} = f(\underline{\rho}_{\{\ell_1, \dots, \ell_k\}}, \overline{\rho}_{\{\ell_{k+1}, \dots, \ell_n\}}) \in X(\mathcal{D})$ for some coalition of individuals $\{\ell_1, \dots, \ell_k\} \subset N$, with $k = 1, \dots, n-2$, and let's prove the claim for $\alpha_{\{\ell_1, \dots, \ell_{k+1}\}} = f(\underline{\rho}_{\{\ell_1, \dots, \ell_{k+1}\}}, \overline{\rho}_{\{\ell_{k+2}, \dots, \ell_n\}})$. Proceeding by way of contradiction, suppose that $\alpha_{\{\ell_1, \dots, \ell_{k+1}\}} \neq \tau(P)$ for all $P \in \mathcal{D}$. By top monotonicity, $\alpha_{\{\ell_1, \dots, \ell_k\}} > \alpha_{\{\ell_1, \dots, \ell_{k+1}\}}$, with strict inequality because $\alpha_{\{\ell_1, \dots, \ell_k\}} \in X(\mathcal{D})$ and $\alpha_{\{\ell_1, \dots, \ell_{k+1}\}} \notin X(\mathcal{D})$. As explained above, there must exist $P_{\ell_{k+1}}^\alpha \in \mathcal{D}$ and $P_{\ell_{k+1}}^\beta \in \mathcal{D}$

such that $\tau(P_{\ell_{k+1}}^\alpha) = \tau(P_{\ell_{k+1}}^\beta)$ and $\alpha_{\{\ell_1, \dots, \ell_k\}} P_{\ell_{k+1}}^\alpha f(\underline{\rho}_{\{\ell_1, \dots, \ell_k\}}, P_{\ell_{k+1}}^\beta, \bar{\rho}_{\{\ell_{k+2}, \dots, \ell_n\}})$. By topology, $f(\underline{\rho}_{\{\ell_1, \dots, \ell_k\}}, P_{\ell_{k+1}}^\alpha, \bar{\rho}_{\{\ell_{k+2}, \dots, \ell_n\}}) = f(\underline{\rho}_{\{\ell_1, \dots, \ell_k\}}, P_{\ell_{k+1}}^\beta, \bar{\rho}_{\{\ell_{k+2}, \dots, \ell_n\}})$. Hence, agent ℓ_{k+1} can manipulate f at $(\underline{\rho}_{\{\ell_1, \dots, \ell_k\}}, P_{\ell_{k+1}}^\alpha, \bar{\rho}_{\{\ell_{k+2}, \dots, \ell_n\}})$ via $\bar{P}_{\ell_{k+1}}$ (which results in $\alpha_{\{\ell_1, \dots, \ell_k\}}$ being chosen), a contradiction. Therefore, $\alpha_{\{\ell_1, \dots, \ell_{k+1}\}} \in X(\mathcal{D})$. The rest of the proof follows from Austen-Smith and Banks (2005, pp. 48-50). ■

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