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Michal Król

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Economics
School of Social Sciences
The University of Manchester
Manchester M13 9PL

‘Everything must go!’ - Cournot as a Stable Convention within Strategic Supply Function Competition

Michał Król*

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Abstract

This paper considers competition in supply functions in a homogeneous goods market. It demonstrates that when firms are few, production costs rise steeply and are largely sunk, a restriction of the players’ strategy sets equivalent to Cournot competition constitutes an instance of the von Neumann-Morgenstern Stable Set. Specifically, any player who believes others will compete à la Cournot (but is ignorant of their exact quantity choices) finds a strategy admissible if and only if it is within her restricted strategy set. In fact, under capacity constraints Cournot may constitute the unique set-valued solution satisfying these conditions. It also follows that Cournot then fullfills the ‘preparation’ requirement of Voorneveld [2004] and has the three defining characteristics of the Self-Admissible Set by Brandenburger, Friedenberg, and Keisler [2008].

KEYWORDS: Cournot competition, von Neumann-Morgenstern Stable Set, supply function competition

1 Introduction

A common argument against the Cournot model of oligopolistic competition is that it is ‘right for the wrong reasons’, in the sense that it leads to plausible comparative statics results, but does so, unrealistically, without allowing the firms to make any pricing decisions.[1] This was often contrasted with the alternative price competition framework proposed by Bertrand. In the words of Shapiro [1989], ‘A common view is that pricing competition more accurately reflects actual behavior, but the predictions of Cournot’s theory are closer to matching the evidence’. Indeed, the existence (Novshek [1985]) and uniqueness (Friedman

*Department of Economics, The University of Manchester, michal.krol@manchester.ac.uk

[1]The phrase ‘right for the wrong reasons’ was first used by W. Fellner in *Competition Among The Few* (1949) with regards to the Cournot tatonement process, and the firms’ myopic conjectures turning out to be correct in equilibrium. However, the same phrase captures the essence of the critique, initiated by Bertrand, of excluding the price as a strategic variable.

[1977]) of Cournot equilibria hold under fairly general conditions, and there is considerable evidence for the resulting testable implications of the model, such as relatively high price-cost margins that, in addition, are decreasing in the number of firms and demand elasticity.[2] This is in contrast with survey-based studies, which indicate that decision makers generally consider the price, rather than the quantity of output, to be the key strategic variable (see Aiginger [1999]).

Existing literature attempts to resolve the above difficulty in a way accurately summed up by Vives [1989]:

> The fact that the Cournot model does not explain the way prices are set, the implicit assumption that output is auctioned efficiently, is certainly a drawback in descriptive terms but [...] price setting models may boil down to Cournot outcomes. The quantity setting model can then be viewed as a reduced form of a more complex and realistic multistage game.

In other words, although firms do not directly compete in quantities alone, their seemingly more complex strategic interaction may implicitly comply with the simplified Cournot framework. This is often demonstrated by imposing a dynamic structure on the original static problem, e.g. by introducing an initial stage of the game at which the firms commit to a certain mode of competition (price or quantity), as in Singh and Vives [1984]. A similar approach was taken by Kreps and Scheinkman [1983], who considered capacity precommittment at stage one, followed by competition in prices. They found that the Cournot equilibrium outcome is also the unique subgame-perfect equilibrium (SPNE) outcome of the two-stage game, in the sense that firms choose the Cournot equilibrium quantities as their capacities at stage one, and then name prices equal to the demand price of the aggregate capacity / output. However, the result relies on mixed strategy play in some subgames, as well as a particular ('efficient') rationing rule, applied when the lower-priced firm is unable to satisfy the entire demand due to the self-imposed capacity constraint (see Davidson and Deneckere [1986]). Although both difficulties are resolved in Moreno and Ubeda [2006], where firms set reservation, rather than exact prices, other problems remain. First, for a continuum of stage one capacity choices, the subsequent price equilibria still exhibit total sales below the aggregate capacity. Thus, the two-stage game is incompatible with Cournot off the equilibrium path, or, using the terminology of Tirole [1988], does not have a 'Cournot reduced form'. In addition, the scope of these results in support of Cournot is limited to those real-world situations that match the dynamic structure superimposed on the original, static problem, specifically the availability of capacity pre-committments prior to competition in prices.

[2] I will not attempt to review the vast literature of relevance in this respect, but see e.g. studies in empirical Industrial Organization by Aiginger [1996], Brander and Zhang [1990], Domowitz et al. [1987], Genesove and Mullin [1998], Haskel and Martin [1994], and those in experimental Game Theory by Feinberg and Husted [1993], Raab and Schipper [2009], Morrison and Kamarei [1990].

An interesting alternative to the above dynamic treatment is to consider static models with rich strategy sets, encompassing both Cournot and Bertrand strategies. In particular, Klemperer and Meyer [1986] considers a simple union of quantity and price strategies, while Klemperer and Meyer [1989] allows for more general 'supply-functions', where each firm simultaneously commits to a price-contingent supply schedule, specifying the quantity of output it would be willing to provide at every possible selling price. Typically, this results in a large range of potential equilibrium outcomes - see e.g. Grossman [1981]. However, Klemperer and Meyer show that the number of equilibria is dramatically reduced in the presence of demand uncertainty. Furthermore, it turns out that when firms are few and production costs steeply rising, the surviving equilibria exhibit supply functions that are steep, i.e. close to the 'vertical' Cournot-like committment to a given quantity regardless of the price.

Another way of reducing the number of supply function equilibria, alternative to introducing demand uncertainty, has been to impose the equilibrium refinement condition of coalition-proofness (Bernheim et al. [1987]). It turns out that when the number of firms is sufficiently large (Delgado and Moreno [2004]), and their capacities are not too asymmetric (Delgado [2006]), the Cournot outcome is the only coalition-proof equilibrium.

Most importantly, though, all of the studies discussed above, both dynamic and static, have one common feature. They provide support for the Cournot *outcome*, by showing that it may follow as an equilibrium of more general games which involve pricing decisions. However, this only strengthens the proposition that Cournot is 'right', while not refuting the claim that this is so 'for the wrong reasons'. In contrast, the present paper aims to provide motivation for the entire Cournot *game* (not just its equilibrium outcome), as a 'convention' within supply function competition. More specifically, the purpose is to show that, under certain conditions, a restriction of the players' strategy sets equivalent to competition à la Cournot could constitute a (set-valued) solution to the supply function competition game.

To this end, I use a recent re-interpretation of one of game theory's earliest solution concepts, namely the von Neumann - Morgenstern (vN-M) Stable Set, generalized by Luo [2001, 2009] to allow for dominance relations that are conditional on the set of available alternatives. The specific conditional dominance relation defined here on the set of all strategy profiles (for the purpose of the non-cooperative oligopoly problem) is based on the notion of admissibility (see e.g. Samuelson [1992]). In particular, a restriction of the players' strategy sets is said to be 'stable' when a strategy is admissible with respect to this restriction if and only if it does not violate it. The purpose of this specification is to capture the idea that, for Cournot to constitute a stable convention, firms should be willing to adhere to it so long as they believe that it is universally accepted, whatever the exact conventional actions of the other firms (i.e. their chosen quantities of output) might be. Hence the use of weak dominance / admissibility - to allow for the fact that players take all of the others' conventional strategies into account (see Kohlberg and Mertens [1986], and further discussion in Section 2.4 of the paper).

It turns out that for Cournot to be stable in the above manner, it is necessary and sufficient that three conditions hold. The first two of those, that the number of firms is not too big and production costs steeply rising, are in line with the results of Klemperer and Meyer, possibly due to ignorance about the counterparts' quantity choices playing a role similar to demand uncertainty in their model. The last condition is that production costs are sufficiently 'sunk'. This means that building the capacity to support one's supply schedule comes at a cost that is, to an extent, impossible to recover even when the capacity is not subsequently utilized in full due to the market-clearing price falling short of the one required (as per the chosen supply schedule).
If, in addition to those requirements, the capacity that firms can build is subject to constraints that fall within a certain range, then every stable convention within supply function competition is equivalent to a stable convention within quantity competition à la Cournot. In fact, when the firms' Cournot best-response functions are sufficiently steep, the only such convention amounts to the entire Cournot game.

Intuitively, competition à la Cournot means the firms' chosen outputs are always entirely sold. This avoids the inefficiency of wasting resources on the production of unsold output (or building the required capacity) - a fact of particular appeal when the sunk costs are large. Similarly, the Cournot model, in contrast with Bertrand, has been traditionally motivated by the presence of steeply rising costs or capacity constraints (Shapiro [1989]). What this paper hopes to accomplish is to provide such commonly held views and intuitions with a strictly formal justification.

While the structure of the problem at hand does not permit the use of alternative solution concepts, the present solution exhibits a number of desirable properties embedded in other set-valued frameworks. In particular, it satisfies the 'preparation' requirement of Voorneveld [2004] - that each player's restricted strategy set contains at least one best reply to whatever belief she may hold consistent with the restrictions imposed on the strategy sets of others. In addition, under the derived stability conditions, Cournot has the three defining properties of a (finite) Self-Admissible Set by Brandenburger et al. [2008], a recently developed concept which has already attracted considerable interest of game-theorists, but so far has been applied mostly to relatively simple canonical games (see Brandenburger and Friedenberg [2010]).

The underlying supply function framework is enjoying renewed attention of researchers as an apt description of competition in markets ranging from wholesale electricity to airline tickets, while the 'dual' model of demand schedule competition has several financial applications (see Vives [2011] for an overview). In fact, the current study relaxes the common requirement that supply functions are continuous. Together with allowing for sunk costs, this makes the model applicable to an even wider range of problems, including online auctions (see Section 2.3). In all those situations, the entire Cournot game (rather than just its equilibrium outcome) can constitute a stable solution of a model which does include pricing decisions, thereby making Cournot right 'for the right reasons'.

2 The Model

2.1 Industry

There are $n \geq 2$ identical firms operating in a market for a homogeneous good with an inverse demand schedule $P(\cdot)$ that is twice continuously differentiable, strictly decreasing and known to all firms with certainty. In addition, the demand satisfies the Hahn [1962] condition, i.e.:

$$P'(Q) + QP''(Q) < 0, \text{ for all } Q \geq 0 \tag{1}$$

In the context of the Cournot competition game, (1) is equivalent to the firms' individual marginal revenues being decreasing in the rivals' aggregate output. For the production technology specified below, this ensures that the firms' reaction functions are continuous and downward sloping, so that a Cournot equilibrium exists (see Novshek [1985]). A sufficient condition for (1) to hold is $P''(Q) \leq 0$, i.e. that the demand is concave.

The firms' production technology is represented by a twice continuously differentiable non-decreasing convex cost function C.
Let $q^c > 0$ be the firms' production capacity, and assume that C satisfies:

$$C'(0) \in [P((n-1)q^m), P(0)), \ q^m = \arg\max_{q \in [0, q^c]} \{qP(q) - C(q)\} \tag{2}$$

This makes the monopoly-optimal output q^m positive, since $P(0) > C'(0)$. The other part of (2) stipulates that $C'(0) \geq P((n-1)q^m)$, which is, in turn, necessary and sufficient for all $q \in [0, q^m]$ to be rationalizable in the Cournot game. This is because, due to decreasing Cournot best-replies, any quantity in excess of q^m (the best response to zero output by rivals), but no greater than q^c, is strictly dominated. However, zero is in turn each player's best-response to all $(n-1)$ counterparts acting as monopolists, since the Cournot marginal revenue at zero output is then $P((n-1)q^m) \leq C'(0)$. This ensures that any $q \in (0, q^m)$ is in turn a best-response to some level of the competitors' aggregate output between 0 and $(n-1)q^m$.
All in all, this means that Cournot players would restrict attention to quantities $q \in [0, q^m]$, and we will take this to be the strategy set in the Cournot game corresponding to the industry just described.

2.2 Strategies and Payoffs

Each firm $i \in N = \{1, 2, ..., n\}$ sets a non-decreasing, right-continuous supply schedule $s_i : [0, P(0)] \to [0, q^c]$ simultaneously with the counterparts, specifying the quantity of output it offers to provide at every given price. Note that, in contrast with the majority of existing literature, the supply schedule need not be continuously differentiable[3]. Before

[3] Attempts at relaxing this restriction typically come at a cost of other simplifying assumptions, see e.g. the 'piecewise-linear' specification in Baldick et al. [2004].

discussing the additional applications of the model that this permits, it should be noted how the process of market clearing is affected by the change. To this end, for any supply function profile $\mathbf{s} = \{s_i(\cdot)\}_{i \in N}$, a market price p and a subset of players $A \subseteq N$, define the following 'aggregate supply' functions:

$$S_A(\mathbf{s}, p) = \sum\nolimits_{i \in A} s_i(p), \ \ S_A^-(\mathbf{s}, p) = \begin{cases} \lim_{p' \to p^-} S_A(\mathbf{s}, p') & \text{for } p \in (0, P(0)] \\ 0 & \text{for } p = 0 \end{cases}$$

Note that for $p > 0$ the existence of the left-sided limit involved in the above is guaranteed by the monotonicity of each $s_i(\cdot)$, and hence the aggregate supply schedule of any subset of players as well. The market-clearing price $p^*(\mathbf{s})$ then satisfies:

$$D(p^*) \in \left[S_N^-(\mathbf{s}, p^*), S_N(\mathbf{s}, p^*)\right], \ \ D(\cdot) = P^{-1}(\cdot) \tag{3}$$

When the aggregate supply $S_N(\mathbf{s}, \cdot)$ is continuous at p^*, (3) is equivalent to the usual requirement for the market-clearing price to equalize demand and supply. However, with any of the individual supply schedules discontinuous, it can be that $S_N(\mathbf{s}, \cdot)$ exhibits a jump at p^* that takes it above the demand curve without ever crossing it, in which case p^* is the market-clearing price.

Due to the monotonicity of $S_N(\mathbf{s}, \cdot)$ and $D(\cdot)$, a price $p^* \in [0, P(0)]$ satisfying (3) always exists, and is also unique. To see why, consider an alternative price $p > p^*$, so that $S_N^-(\mathbf{s}, p) \geq S_N(\mathbf{s}, p^*)$ and $D(p) < D(p^*)$. Consequently, it cannot be the case that $D(p^*) \leq S_N(\mathbf{s}, p^*)$ and $D(p) \geq S_N^-(\mathbf{s}, p)$, so p and p^* cannot both satisfy (3).

As there is a possibility of excess supply at the market-clearing price p^*, some firms may not be able to sell as much as $s_i(p^*)$. Let $x_i(\mathbf{s}) \leq s_i(p^*)$ denote the quantity of output that firm i is actually able to sell at market clearing.

On the one hand, $x_i(\mathbf{s})$ must at least equal the left-sided limit of $s_i(\cdot)$ at p^*, because it could not be the case that some output is sold at the market-clearing price, while some remains unsold despite being available for sale below that price. In addition, $x_i(\mathbf{s})$ must not be smaller then the minimum residual demand $D(p^*) - S_{N \setminus \{i\}}(\mathbf{s}, p^*)$, i.e. what is left after all the other players sell their entire outputs.

On the other hand, it may not be the case that $x_i(\mathbf{s})$ exceeds the maximum residual demand $D(p^*) - S_{N \setminus \{i\}}^-(\mathbf{s}, p^*)$, i.e. what remains once each of the counterparts only sells the left-sided limit of her supply at p^*. Thus, $x_i(\mathbf{s})$ lies within the following interval:

$$\left[\max\left\{S_{\{i\}}^-(\mathbf{s}, p^*), D(p^*) - S_{N \setminus \{i\}}(\mathbf{s}, p^*)\right\}, \min\left\{s_i(p^*), D(p^*) - S_{N \setminus \{i\}}^-(\mathbf{s}, p^*)\right\}\right] \tag{4}$$

Observe that when all supply schedules are continuous at p^*, we have:

$$S_{\{i\}}^-(\mathbf{s}, p^*) = s_i(p^*) = D(p^*) - S_{N \setminus \{i\}}(\mathbf{s}, p^*) = D(p^*) - S_{N \setminus \{i\}}^-(\mathbf{s}, p^*)$$

so that $x_i(\mathbf{s}) = s_i(p^*)$.

When the supply schedule of at most one firm (i) is discontinuous at p^*, it follows from (3) that:

$$D(p^*) - S_{N\backslash\{i\}}(\mathbf{s}, p^*) = D(p^*) - S^-_{N\backslash\{i\}}(\mathbf{s}, p^*) \in \left[S^-_{\{i\}}(\mathbf{s}, p^*), s_i(p^*)\right]$$

which implies that $x_i(\mathbf{s}) = D(p^*) - S_{N\backslash\{i\}}(\mathbf{s}, p^*)$, i.e. firm i only gets what remains after all counterparts have sold their enire outputs.

Finally, suppose the supply schedule of more than one firm is discontinuous at p^*, leading to excess supply at p^*. Let N_0 denote the set of those firms. Then (4) is a proper interval for all $i \in N_0$ (while $x_i(\mathbf{s}) = s_i(p^*)$ for all $i \in N\backslash N_0$). The exact value of $x_i(\mathbf{s})$ for $i \in N_0$ is then determined by a sharing rule, specifying how the $D(p^*) - S_{N\backslash N_0}(\mathbf{s}, p^*)$ part of the demand is to be distributed among the firms in N_0. Since the results of this paper turn out to hold for any such sharing rule, it is left unspecified throughout the remainder of the text.

With the firms' sales $x_i(\mathbf{s})$ determined, the resulting profits are:

$$\pi_i(\mathbf{s}) = p^*(\mathbf{s})\, x_i(\mathbf{s}) - \left[(1-\gamma)\, C(x_i(\mathbf{s})) + \gamma C(s_i(P(0)))\right], \ \gamma \in [0,1] \tag{5}$$

The way the costs are incorporated in (5) is meant to reflect the idea that firms need to build capacity to support their commitment to the chosen supply schedule, and that the cost of doing so is at least partly sunk, i.e. independent of their eventual sales. For instance, suppose there are two firms, the first of which offers to supply up to ten units of output (given a sufficiently high price), while the second one only offers to provide up to six units. Since the first firm needs to build more capacity in order to make its commitment viable, its costs could be higher even when the eventual market price is such that both firms only sell six units of output (i.e. only the second firm utilizes its full capacity). This would be the case whenever part of the cost of having built the excess capacity of four units is sunk and impossible to recover once the extra capacity is known to be redundant.

The degree to which costs are sunk in this manner is represented by the parameter $\gamma \in [0,1]$ in the profit formula (5). When $\gamma = 0$, costs are given by $C(x_i(\mathbf{s}))$, i.e. are entirely 'sales-dependent'. In contrast, when $\gamma = 1$, costs equal $C(s_i(P(0)))$, i.e. are completely 'sunk' at the point of submitting a supply schedule. More specifically, in the latter scenario costs depend only on the maximum quantity $s_i(P(0))$ that firm i is willing to provide (recall s_i is defined for $p \in [0, P(0)]$ and is non-decreasing). It is as if in order to commit to providing a certain quantity of output (at some price) the firm first had to produce it, where the associated cost cannot be recovered if some of the output remains unsold. As γ decreases, so does the fraction of the cost that becomes lost in this manner for any unsold output.

2.3 Example: Online Auctions

Consider a homogeneous good market where every buyer seeks to purchase one unit of the good, so long as the required price does not exceed her individual valuation. Suppose these valuations give rise to an aggregate demand function as specified in Section 2.1[4], and that the good is sold via an online auction platform, e.g. eBay. Each seller operating in this market is at liberty to set up an arbitrary number of auctions with possibly different reservation prices, and will generally be inclined to do so simultaneously with the competitors. This is because auctions typically have a fixed duration (usually one week), and it is considered optimal for an auction to end during the peak-time of buyer activity (typically Sunday evening). In fact, this pattern is promoted by auction sites, which often reduce seller fees for auctions starting / finishing at those times (e.g. the weekend 'no-insertion fees' promotion by eBay). Hence, competing auctions are usually launched at roughly the same time.

The majority of existing literature, empirical and theoretical alike, treats online auctions as independent of one another, in the sense that there is no strategic interaction between sellers, each acting as a monopolist in their own market (see a comprehensive review by Bajari and Hortacsu [2004]). Of the few studies that challenge this assumption in what is known as 'competing auctions theory', Peters and Severinov [2006] describe a (weak) perfect Bayesian equilibrium where buyers always bid with the minimum increment on the auction with the lowest 'standing' bid (which may entail switching between auctions), as opposed to bidding one's true valuation straight away in any single auction, which does not constitute an equilibrium in this setting. Indeed, this tendency in buyers' behaviour is supported by the empirical analysis of Anwar et al. [2006], as is the fact that final selling prices in competing auctions (those simultaneously offering close substitutes) tend to be uniform. In fact, assuming bidders do behave in this way also makes the sellers' strategic interaction consistent with the model described in Section 2.2.

To see why, observe first that by setting up an arbitrary number of auctions with possibly different reserves, each seller effectively submits a supply schedule, contingent on the final (uniform) selling price in the sellers' common market. For instance, suppose a seller sets up two auctions (each for a single unit of the good), with reservation prices of 1 and 2 respectively. The corresponding supply schedule $s_i(p)$ assigns zero to all $p \in [0, 1)$, 1 to $p \in [1, 2)$ and 2 to all values of p not smaller than 2. It is easy to see that any choice of auctions / reserves results in a supply schedule that is non-decreasing and right-continuous.[5]

Suppose further there is a second seller who sets up two auctions, both with a reservation

[4] In accordance with common practice, we take a non-discrete $P(\cdot)$ to be a good approximation of the demand of a large (finite) number of potential buyers, though a small-number toy example is used in what follows for illustrative purposes.

[5] Technically, in this case each seller's strategy set comprises only those non-decreasing, right-continuous supply schedules which are also step functions. All results derived in the paper would continue to hold under this restriction.

price of 3, and that there are exactly three buyers with valuations not smaller than this value (i.e. $D(3) = 3$). Thus, buyers will first start outbidding one another in the auction of the first seller with reserve equal to 1, until the standing bid reaches 2. At this point, rather than go above this value, one of the outbidded buyers will place a bid equal to the reserve of 2 in the first seller's other auction. However, since there will remain at least one outbidded buyer with valution not smaller than 3, this buyer will want to bid higher in one of the two 'active' auctions. This will continue until the standing bids in both of these two auctions are equal to 3, at which point there will be exactly one outbidded buyer with valution not smaller than 3. This buyer will then meet the reserve of 3 in one of the auctions of the second seller, and the bidding will end. Thus, the first seller succeeds in selling both units of the good she put up for sale, while the counterpart only sells one out of two. All trade occurs at a uniform price of 3.

Let us see how the same outcome follows directly as an application of the formulae in Section 2.2. First, observe that $p^* = 3$ satisfies condition (3), as we then have $D(p^*) = 3$, $S_N^-(\mathbf{s}, p^*) = 2$ and $S_N(\mathbf{s}, p^*) = 4$. Note that any $p < 3$ implies $S_N(\mathbf{s}, p) \leq 2 < D(p) \geq 3$, while any $p > 3$ leads to $S_N^-(\mathbf{s}, p) = 4 > D(p) \leq 3$, i.e. $p^* = 3$ is the unique price satisfying (3). We then have:

$$\begin{aligned} S_{\{1\}}^-(\mathbf{s}, p^*) &= S_{N\backslash\{2\}}^-(\mathbf{s}, p^*) = 2,\ S_{\{2\}}^-(\mathbf{s}, p^*) = S_{N\backslash\{1\}}^-(\mathbf{s}, p^*) = 0 \\ s_1(p^*) &= S_{N\backslash\{2\}}(\mathbf{s}, p^*) = s_2(p^*) = S_{N\backslash\{1\}}(\mathbf{s}, p^*) = 2 \end{aligned}$$

so that, based on (4), the quantities sold by the two sellers are determined as follows:

$$\begin{aligned} x_1(\mathbf{s}) &\in [\max\{2, 3-2\}, \min\{2, 3-0\}] \equiv \{2\} \\ x_2(\mathbf{s}) &\in [\max\{0, 3-2\}, \min\{2, 3-2\}] \equiv \{1\} \end{aligned}$$

Note, however, that should the first seller increase the reserve in either of their two auctions to 3, then buyers would outbid one another in the other (lowest-priced) auction until the standing bid reaches 3, at which point the two outbidded buyers would each bid the reserve of 3 in one of the remaining three auctions. As these auctions belong to different sellers, each of them now ends up selling at least one of the total three units of output being traded. Formally, we have $x_i(\mathbf{s}) \in [1, 2]$ for $i = 1, 2$, subject to $x_1(\mathbf{s}) + x_2(\mathbf{s}) = D(p^*) = 3$, where the exact sales are then determined by the (unspecified) sharing rule.

Finally, suppose there is a fixed fee of 0.5 that sellers need to pay in order to set up each auction, while the (constant) unit cost of producing the good is 1.5. The profits of firm i are then given by:

$$\pi_i(\mathbf{s}) = p^*(\mathbf{s})\, x_i(\mathbf{s}) - [1.5 \times x_i(\mathbf{s}) + 0.5 \times (\text{no. of auctions})]$$

which coincides with (5) for $\gamma = 1/4$. In other words, the total unit cost of output is $1.5 + 0.5 = 2$, a quarter of which is 'sunk', as the auction fees cannot be recovered if the reserve is not met and the item remains unsold.

In fact, the proportion of costs that are sunk could be even greater if the goods need to be produced ‘upfront’, i.e. prior to setting up the auction. This could be e.g. because the production process takes plenty of time, and so must be scheduled in advance to allow prompt delivery of the item in the event of it getting sold. Once an item that has already been produced fails to sell, some of its production cost may be impossible to recover. This could be, for example, because it is a perishable good that becomes worth less with time and hence must then be sold at a discounted price or even discarded. Even when that is not the case, delaying the sale until a subsequent auction causes an irreversible decrease of the present discounted value of the potential revenue, which constitutes a form of sunk costs and can be captured by the parameter γ.

2.4 Solution Concept

Let $S = \times_{i\in N} S_i$ denote the set of strategy profiles, comprising strategies defined in Section 2.2 (in this case $S_1 = S_2 = ... = S_n$). We will refer to any product set $S' = \times_{i\in N} S'_i \subseteq S$ as a *convention*, where $S'_{-i} = \times_{j\neq i} S'_j$, and any strategy $s_i \in S'_i$ will be called *conventional* (as opposed to *unconventional* strategies $s_i \in S_i \backslash S'_i$).

Definition 2.1 *A strategy $s_i \in S_i$ is* ***admissible*** *with respect to a convention S' when there exists no strategy $s'_i \in S'_i$ that weakly dominates s_i with respect to S'_{-i}, i.e. one that satisfies:*

$$\forall \mathbf{s}'_{-i} \in S'_{-i} : \pi_i \left(s'_i, \mathbf{s}'_{-i}\right) \geq \pi_i \left(s_i, \mathbf{s}'_{-i}\right)$$

where the inequality is strict for some $\mathbf{s}'_{-i} \in S'_{-i}$. Otherwise, s_i is ***inadmissible*** *w.r.t. S'.*

I will now define the solution concept used in the present paper, then illustrate it with an example, discuss its properties and formally relate it to the generalized vN-M Stable Set by Luo [2001, 2009].

Definition 2.2 *A convention S' is* ***stable*** *when a strategy is conventional if and only if it is admissible with respect to S'.*

To begin with, observe that Definition 2.2 comprises two requirements:

1. That every conventional strategy is admissible with respect to S', i.e. that it is not weakly dominated by another conventional strategy w.r.t. the set S'_{-i} of other players’ conventional strategy profiles. This will be referred to as the *internal stability* requirement.

2. That every strategy that is not conventional is not admissible w.r.t. S', i.e. that it is weakly dominated by some conventional strategy. This will be referred to as the *external stability* requirement.

These two conditions taken together also mean that no $s'_i \in S'_i$ is weakly dominated w.r.t. S'_{-i} by a $s_i \in S_i \backslash S'_i$. If such a s_i did exist, then the conventional strategy which weakly dominates s_i by virtue of the external stability requirement would weakly dominate s'_i w.r.t. S'_{-i}, in violation of internal stability.

Example 2.1 *Consider the following two-player normal-form game:*

	D	**E**	**F**
A	$0,6$	$\mathbf{2,5}$	$\mathbf{1,6}$
B	$4,2$	$0,4$	$3,0$
C	$1,1$	$\mathbf{1,5}$	$\mathbf{4,3}$

Initially, no strategies can be eliminated as (weakly) dominated, but once we eliminate B, D becomes weakly dominated by F, and if we eliminate D, then B is strictly dominated by C. Hence, the convention $\{A, C\} \times \{E, F\}$ satisfies external stability. Since neither of the conventional strategies is then weakly dominated by another, internal stability is also satisfied, so that the convention $\{A, C\} \times \{E, F\}$ is stable.

Definition 2.2 attempts to capture a key feature of the classic characterization of social conventions (as laid down by David Hume and, later, David Lewis), namely the fact that for a convention to be stable / sustainable, it must be in everyone's best interest to adhere to it so long as they believe others to do the same (see Rescorla [2011] for an overview of the related literature). However, the present notion extends this to allow for the fact that conventions need not determine agents' activities exactly, but may allow for flexibility of behaviour within the limits of convention. Such an approach is related to the more recent concept of 'constitutive conventions' by Marmor [2009], which are said to 'constitute' a social practice by defining the rules of how to engage in it correctly. For instance, chess players are only allowed to make certain moves as a matter of convention, but are still free to shape the course of the game so long as they play by the rules. This kind of flexibility within the convention is crucial for the present study, as it strives to provide motivation for quantity competition in general, rather than some specific quantity choices, as a convention within supply function competition.

The question is then whether it is indeed in everyone's interest not to violate a convention that is stable in the sense of Definition 2.2. This should be the case so long as all players believe that their counterparts will adhere to it, i.e. that they will choose *some* conventional strategies, whatever these might be. For instance, consider a population of individuals, each of whom chooses how much of her time to devote to work, and how much of the resulting income to declare for tax purposes. Suppose it is a convention to declare one's income truthfully and hence pay the full tax amount due, and observe that this still leaves people free to choose how much they wish to work. However, in reality we would not expect an individual's decision to obey the convention to be based on estimating how much income every other person in the economy is going to earn / declare. Instead, people would choose to pay their taxes simply because they believe this is what (almost?) everybody else does, whatever their exact income might be.

The way this reasoning is embedded in Definition 2.2 may be illustrated by revisiting Example 2.1. Suppose the column player expects the counterpart to choose one of the conventional strategies $\{A, C\}$, but is ignorant of their respective likelihoods. It would not then be in the column player's interest to violate the convention by opting for strategy D, as to do so would be worse than playing the conventional strategy F, just as a risky prospect that yields 6 or 1 with unknown probabilities is worse than one that gives 3 in the event where the other gives 1. Thus, by requiring that all unconventional strategies are also inadmissible (as per the external stability condition), Definition 2.2 ensures that it is in everyone's interest to obey the convention, so long as others are expected to play conventional strategies, but nothing more is known about their exact actions. What the internal stability condition does is, in turn, to make such expectations consistent with the avoidance of inadmissible strategies, by requiring these to be absent from any stable convention. Combined with external stability, this ensures that a player who takes all of the others' conventional strategies (and none of their unconventional ones) into account, will be subject to similar expectations from the counterparts, who in their turn will take all of the player's conventional strategies (and none of her unconventional ones) into consideration. These mutual expectations turn out to be correct, in the spirit of Nash. Indeed, we have:

Remark 2.1 *A strict Nash Equilibrium in pure strategies is equivalent to a singleton stable convention.*

In addition to that, the internal / external stability terms, as explained above, suggest that a direct parallel can be drawn between the present concept and one of the earliest and most profound formalizations of 'standards of behaviour'. This is detailed below.

Relationship to vN-M Stable Set

Definition 2.2 can be linked to the vN-M stable set concept, by expressing it using the *general systems* framework proposed by Luo [2001]. To see this, consider a general system:

$$\left(S, \{\succ^A\}_{A \subseteq S}\right) \tag{6}$$

where $\succ^A$ is a *conditional dominance relation* on S defined as follows:

Take $\mathbf{s}^1, \mathbf{s}^2 \in S$, where $\mathbf{s}^j = \{s_i^j\}_{i \in N}$, and let $A_{-i} \equiv \{\mathbf{s}_{-i} \in S_{-i} \mid \exists s_i \in S_i : (s_i, \mathbf{s}_{-i}) \in A\}$.

We then have $\mathbf{s}^1 \succ^A \mathbf{s}^2$ if and only if for some $i \in N$ it is true that:

$$\forall \mathbf{a}_{-i} \in A_{-i} : \pi_i\left(s_i^1, \mathbf{a}_{-i}\right) \geq \pi_i\left(s_i^2, \mathbf{a}_{-i}\right)$$

where the inequality is strict for some $\mathbf{a}_{-i} \in A_{-i}$.

The *general stable set* $S' \subseteq S$ of the general system (6) is then defined (for an arbitrary conditional dominance relation) as the vN-M stable set of an abstract game $(S, \succ^{S'})$, i.e. one in which the *unconditional* dominance relation coincides with $\succ^{S'}$. That is to say, S' must satisfy:

1. [vN-M internal stability] $\nexists \mathbf{s}, \mathbf{s}' \in S' : \ \mathbf{s} \succ^{S'} \mathbf{s}'$
2. [vN-M external stability] $\forall \mathbf{s} \in S \backslash S' \ \exists \mathbf{s}' \in S' : \ \mathbf{s}' \succ^{S'} \mathbf{s}$

For the particular conditional dominance relation $\left\{\succ^A\right\}_{A \subseteq S}$ specified above, we then have:

Proposition 2.1 *Any stable convention S' is a vN-M stable set of an abstract game $(S, \succ^{S'})$, and any general vN-M stable set S' of a general system $(S, \left\{\succ^A\right\}_{A \subseteq S})$ is a stable convention.*

Proof. See the Appendix. ■

3 Results

We now proceed to derive the conditions for a restriction of the players' strategy sets equivalent to Cournot competition to constitute a stable convention. To this end, it is first necessary to explain what is meant by 'Cournot-equivalent' in the context of supply function competition.

In principle, what is needed is that every supply schedule within the convention corresponds to some quantity of output in Cournot, in the sense that the outcome of any profile of supply schedules will coincide with that of their corresponding outputs under Cournot competition, in terms of the quantities sold, selling prices and profits made by each player. In addition, as detailed in Section 2.1, the Cournot strategy set of each player is $[0, q^m]$, where q^m is the monopoly-optimal output. Thus, it is required that for every $q \in [0, q^m]$ there exists some conventional supply schedule such that q is its corresponding Cournot strategy. More formally:

Definition 3.1 *A convention S' is* ***Cournot-Equivalent*** *when there exists a collection of surjective functions $\{\varphi_i : S'_i \to [0, q^m]\}_{i \in N}$ such that for any supply function profile $\mathbf{s} = \{s_i(\cdot)\}_{i \in N} \in S'$ and every player $i \in N$ we have $x_i(\mathbf{s}) = \varphi_i(s_i(\cdot))$ and:*

$$\pi_i(\mathbf{s}) = \varphi_i(s_i(\cdot)) \, P(\textstyle\sum_{j \in N} \varphi_j(s_j(\cdot))) - C(\varphi_i(s_i(\cdot)))$$

We may now state the main result of the paper.

Proposition 3.1 *There is at most one Cournot-Equivalent convention S' that is also stable in the sense of Definition 2.2. This convention is characterized by:*

$$\forall i \in N : S_i' = \{s_i(\cdot) \in S_i : s_i(P(s_i(P(0)) + (n-1)q^m)) = s_i(P(0)) \leq q^m\} \tag{7}$$

A necessary and sufficient condition for S' to be stable is:

$$P(nq^m) + q^m P'(nq^m) \geq (1-\gamma) C'(q^m) \tag{8}$$

Proof. See the Appendix. ■

In words, choosing a supply schedule s_i within the stable Cournot-Equivalent convention S' is no different from selecting a quantity $q \in [0, q^m]$ in Cournot, where q is equal to the maximum output that could be supplied under s_i. That is to say, the surjective function $\varphi_i(\cdot)$ of Definition 3.1 satisfies $\varphi_i(s_i(\cdot)) = s_i(P(0))$ for all $s_i(\cdot) \in S_i'$. The maximum output $s_i(P(0))$ of every player $i \in N$ is sold entirely at the corresponding demand price, precisely as in Cournot. This is because, according to (7), every player is willing to provide up to $s_i(P(0))$ even when everybody else supplies the monopoly-optimal output, making the market-clearing price as low as $P(s_i(P(0)) + (n-1)q^m)$. In short, when the players' supply schedules are 'competitive', in the sense that they commit themselves to selling their maximum quantities even at relatively low prices, then these quantities become the effective strategic variables.

It now becomes easier to understand condition (8), which is necessary and sufficient for the above convention to be stable in the sense of Definition 2.2. The condition states that the marginal revenue of a Cournot player when everyone (including the player) produces the monopoly-optimal output is no smaller than a fraction $(1-\gamma)$ of the player's marginal cost. In fact, under assumption (1), the left-hand side of (8) is decreasing in both the output of the player in question and that of her competitors. Together with $C''(\cdot) \geq 0$, this implies that the inequality holds, more generally, for everyone producing *no more* than q^m, and becomes strict when one or more players produce strictly less than q^m (see proof of Proposition 3.1). This can be given the following interpretation.

Suppose player i decides to commit to supplying up to a certain (maximum) quantity of output $s_i(P(0)) \leq q^m$, and expects others to choose supply schedules that are sufficiently 'competitive' to fully sell their associated maximum quantities, whatever these might be. Thus, player i is effectively a monopolist faced with the (residual) demand that remains after the aggregate (maximum) quantity of other players $Q_{-i} = \sum_{j \in N \setminus \{i\}} s_j(P(0))$ has beed sold ($Q_{-i} \in [0, (n-1)q^m]$). By choosing her exact supply schedule, player i effectively determines the quantity q_i that she will sell at the corresponding demand price $P(q_i + Q_{-i})$. Hence, the player's decision problem is the same as in Cournot, but for two exceptions. First, q_i must not exceed $s_i(P(0))$, and second, a fraction γ of production cost is sunk, i.e. depends on $s_i(P(0))$, but not q_i. It is as if a Cournot player only had to pay a fraction $(1-\gamma)$ of the cost of her output, provided it does not exceed $s_i(P(0))$. Condition (8) then states that the corresponding marginal profit is positive for $q_i < s_i(P(0))$, i.e. the player would always want to produce up to $s_i(P(0))$, regardless of the exact value of Q_{-i}. This means it is best for the player to make her supply schedule as competitive as those of the competitors, so as to sell the maximum quantity $s_i(P(0))$ for any $Q_{-i} \in [0, (n-1)q^m]$. In other words, it is in the interest of every player to compete à la Cournot, so long as others are believed to do so.

It is clear that condition (8) is more easily satisfied when γ is larger, i.e. production costs are, to a greater extent, sunk. This is intuitive, since the main feature and appeal of Cournot competition is that the players' outputs always get sold in their entirety. The reason why this is particularly important when costs are largely sunk, is that it is then not worth witholding some of the output from sale in order to maintain high prices, because the cost of this unsold output will, in large part, be impossible to recover. Thus, even in case of low (residual) demand it is better to sell maximum output at whatever price it takes to do so, which is precisely what Cournot competition guarantees.

As for the impact of the number of firms, it follows from the above discussion that, since an increase in n increases the others' aggregate monopoly-optimal output, the left-hand side of inequality (8) is decreasing in n. Hence, when the number of players is large, it is more difficult for Cournot to meet the stability requirements of Definition 2.2, which may be interpreted as follows. When facing a large number of competitors, players need to consider the possibility of a large aggregate supply driving the price down to a level at which any revenue lost due to witholding some of the output from sale is smaller than the proportion of the associated cost that may be recovered when the produce remains unsold. Thus, it may then be beneficial to select an unconventional supply schedule that offers a way of insuring against low prices, by stipulating that less than $s_i(P(0))$ is to be offered for sale in the event of the others' aggregate output bringing the market-clearing price down to a certain level. Such unconventional strategies may hence become admissible, violating the external stability requirement.

A similar reasoning may be applied to consider the effect of changes in the production technology. When the marginal costs $C'(\cdot)$ rise more steeply and / or are larger to begin with, the monopoly-optimal output q^m is smaller, so that the lower-bound on Cournot prices increases, and there is less reason to insure against low prices by means of an unconventional supply schedule.
To link this more directly to condition (8), suppose that (contrary to assumption) $n = 1$, and that we have $\gamma = 0$ and $q^m < q^c$. In such a case, the fact that q^m is monopoly-optimal means (8) must be satisfied as an equality. Thus, the change in costs just decribed would result in an increase of $C'(q^m)$, accompanied by an equal increase of the left-hand side of (8), representing the marginal revenue at q^m. However, for $n \geq 2$ there would be an additional effect of the (maximum) aggregate supply of the competitors $(n-1)q^m$ going down. This, as already noted, would further increase the marginal revenue, so its overall change would be larger than that of the right-hand side of (8). Clearly, the positive effect of the change in costs on the stability of Cournot extends to $\gamma \in (0, 1]$, which only dampens

the increase of $C'(q^m)$ on the right-hand side of (8). In fact, we have seen that a larger γ makes stability more likely.

These results are similar to those of Klemperer and Meyer [1989], established under demand uncertainty. They found that when the number of firms is small and the (increasing) marginal cost curves are steep relative to demand, the equilibrium supply functions are steep, i.e. resemble our Cournot-Equivalent supply functions that are vertical above a relatively low price-threshold. Interestingly, Klemperer and Meyer rely on demand uncertainty for their results, which means that equilibrium strategies must perform well under different levels of consumer demand. In comparison, the current results are obtained under uncertainty *about the actions of the competitors*. Hence, each strategy in a stable Cournot-Equivalent convention must perform well against different levels of the competitors' aggregate output, causing variations in (residual) demand similar to demand uncertainty.

We illustrate this discussion with an example of linear demand and constant unit costs.

Example 3.1 *Suppose that $n > 2$, $q^c \to \infty$ (no capacity constraints), $P(Q) = \alpha - \beta Q$ and $C(q) = cq$, where $\alpha > c > 0$ and $\beta > 0$. The assumptions of Section 2.1 are satisfied and it follows that $q^m = (\alpha - c)/2\beta$. Consequently, condition (8) simplifies to:*

$$c/\alpha \geq (n-1)/(n-1+2\gamma)$$

In other words, Cournot competition is a stable convention when the unit cost of production is sufficiently large relative to consumer demand, where the required cost threshold is increasing in the number of players and decreasing in the degree to which costs are sunk.

It has been demonstrated that, under condition (8), (7) is the unique Cournot-Equivalent stable convention. However, the question that remains open is whether there are then any stable conventions that are not Cournot-Equivalent. Alternatively, can anything be said about the set of all existing stable conventions? It turns out that an affirmative answer to the last question can be given when a stronger variant of condition (8) holds.

Proposition 3.2 *Let $q_{br}(Q) = \arg\max_{q\in[0,q^m]} qP(q+Q) - C(q)$, and suppose we have:*

$$P(nq^c) + q^c P'(nq^c) \geq (1-\gamma) C'(q^c) \tag{9}$$

A convention S' is then stable only if for some $q^l, q^h \in [0, q^m]$, $q^l < q^h$, $Q^h = (n-1)q^h$ and $Q^l = (n-1)q^l$, the following three conditions hold:

$$\begin{aligned}
(1) \quad &: \quad \forall i \in N : S_i' = \{s_i(\cdot) \in S_i : s_i(P(s_i(P(0)) + Q^h)) = s_i(P(0)) \in [q^l, q^h]\} \\
(2) \quad &: \quad q^l = q_{br}(Q^h) \text{ and } q^h = q_{br}(Q^l) \\
(3) \quad &: \quad 2[P(q^l + Q^h) - C'(q^h)] \leq -q^h P'(q^l + Q^h)
\end{aligned}$$

Proof. See the Appendix. ■

Let us examine the three conditions of Proposition 3.2 in turn. The first one is similar to (7), i.e. the way the stable Cournot-Equivalent convention is specified in Proposition 3.1. That is to say, any stable convention comprises those supply schedules that will sell their maximum quantities regardless of the conventional strategies selected by others. As explained before, any such convention is therefore equivalent to the firms each choosing a Cournot quantity from within a specified range, where the difference from Proposition 3.1 is that this range $[q^l, q^h]$ may be a proper subset of $[0, q^m]$.

Condition (2) further restricts the values that q^l and q^h can take. Specifically, the minimum quantity q^l must be the Cournot best-response to the highest possible aggregate output Q^h of the competitors. Conversely, q^h must be the best-response to minimum total output Q^l of the counterparts. This means that any $q \in \left(q^l, q^h\right)$ in turn satisfies $q_{br}\left(Q\right) = q$ for some $Q \in \left(Q^l, Q^h\right)$, and that any $q < q^l$, as well as any $q > q^h$, are outperformed by q^l and q^h respectively when others produce $Q \in \left[Q^l, Q^h\right]$. Thus, every stable convention of the strategic supply function game must be equivalent to a stable convention in Cournot, in the sense that every $q \notin [q^l, q^h]$ is weakly dominated in Cournot by some $q \in [q^l, q^h]$, whereas no $q \in [q^l, q^h]$ weakly dominates another quantity within the same range.

Nevertheless, for the convention to be stable not only in Cournot, but also within supply function competition, it must satisfy an additional condition (3), which is, roughly speaking, that the convention is not 'too small', in the sense of q^l and q^h being close together. To see this, we can write condition (3) as:

$$P(q^l + Q^h) + q^h P'(q^l + Q^h) - C'(q^h) \leq q^h P'(q^l + Q^h)/2 \tag{10}$$

and observe that, when q^l becomes close to q^h, the left-hand side of (10) is close to the marginal profit of a Cournot player producing q^l when everyone else produces q^h, which equals zero by virtue of $q^l = q_{br}(Q^h)$, and is therefore larger than the negative term on the right-hand side. The reason why this scenario needs to be ruled out is that marginal profits are then just slightly below zero even when everyone produces q^h, which means it might be profitable to sell more than q^h so long as this is done not by decreasing the market-clearing price, but rather by means of capturing some of the competitors' demand at existing prices. Thus, the purpose of condition (3) is to make it unprofitable to offer in excess of q^h via an unconventional supply schedule that is sufficiently competitive to 'price-undercut' the competitors' conventional ones, preventing them from utilizing their full capacities.

Finally, it is worth noting the role of the production capacity constraint, determined by the parameter q^c, in narrowing down the set of all stable conventions in the above manner. Condition (9) is obtained by substituting q^c for q^m in condition (8) of Proposition 3.1. Thus, it follows from the discussion of the latter that (9) becomes more difficult to satisfy when q^c increases, which is interesting in the light of the common view (e.g. Maggi [1996], Shapiro [1989]) that Cournot is an apt description of oligopolistic competition when firms are faced with capacity constraints. In fact, when $q^m = q^c$, i.e. q^c is sufficiently low to make the constraint binding under monopoly, conditions (8) and (9) coincide. In such a case, either there is no stable Cournot-Equivalent convention, or not only one does exist, but also every stable convention within supply function competition is equivalent to some stable restriction of the Cournot game. However, it should be noted that q^c may not be too low either, as this would contradict the assumption that $C'(0) \geq P((n-1)q^m)$ for $q^m = q^c$.

The next proposition shows that, subject to firms' reaction functions satisfying an additional requirement, conditions (1) – (3) of Proposition 3.2 are sufficient to further restrict the set of stable conventions to a single one - the Cournot-Equivalent convention of Proposition 3.1.

Proposition 3.3 *Suppose that condition* (9) *of Proposition 3.2 holds, and that we have:*

$$\left|q'_{br}(Q)\right| > 1/(n-1) \;\; \textit{for all } Q \in \left((n-1)q^{\min}, (n-1)q^{\max}\right) \tag{11}$$

where $q^{\max} = D(C'(0))/(n-1)$ *and* $q^{\min}$ *is the largest* q *that solves* $q_{br}((n-1)q) = q^{\max}$ *The Cournot-Equivalent convention of Proposition 3.1 is then the unique stable convention.*

Proof. See the Appendix. ■

As $q^{\max}$ satisfies $P((n-1)q^{\max}) = C'(0)$, it represents the smallest quantity that, when produced by each of a player's $(n-1)$ counterparts, induces the player to produce nothing. Similarly, $q^{\min}$ is the largest quantity that, when produced by others, makes it optimal for the player to produce $q^{\max}$. Hence, $Q \in \left((n-1)q^{\min}, (n-1)q^{\max}\right)$ implies $q = q_{br}(Q)$ must be an interior solution, i.e. we must have $P(q+Q) + qP'(q+Q) = C'(q)$. Using implicit function theorem then yields:

$$\left|q'_{br}(Q)\right| = \frac{P'(q+Q) + qP''(q+Q)}{2P'(q+Q) + qP''(q+Q) - C''(q)} \tag{12}$$

The numerator of the above fraction represents the change in marginal revenue brought about by a marginal increase of the competitors' aggregate output, which we know to be negative under the present assumptions. The denominator, in turn, comprises the (negative) change in marginal revenue resulting from a marginal increase of a firm's *individual* output, net of the associated (non-negative) change of its marginal cost. Consequently, condition (11) is easier satisfied when the marginal cost function $C'(\cdot)$ is less steep. In fact, it is clear that when marginal costs are constant and demand is concave, (12) is smallest when $qP''(q+Q) = 0$, in which case $|q'_{br}(Q)| = 1/2$.

Corollary 3.1 *For constant marginal costs and concave demand, condition* (11) *of Proposition 3.3 reduces to* $n > 3$.

We may illustrate these considerations by revisiting the previous example of constant marginal costs and linear demand.

Example 3.2 *Consider the model in Example 3.1, except that* q^c *is now finite. Condition (9) of Proposition 3.2 then becomes* $q^c \leq q_h^c = [\alpha - (1-\gamma)c] / [\beta(n+1)]$, *and we have* $q^m \leq q^c \Leftrightarrow q^c \geq q_m^c = (\alpha - c)/2\beta$. *But when* $q^c < q_m^c$, *i.e. the production capacity constraint is binding under monopoly, we also need* $q^c \geq q_l^c = (\alpha - c)/[\beta(n-1)]$ *in order to satisfy the assumption* $C'(0) \geq P(q^m)$.

It follows that for $\gamma > 2(\alpha - c)/[c(n-1)]$ *we have* $q_h^c > q_l^c$. *In other words, when* γ, n *and* c *are large relative to* α, *then for a range of values of* q^c *every stable convention must satisfy the requirements of Proposition 3.2. If, in addition,* $n > 3$, *then the only stable convention is the Cournot-Equivalent one described in Proposition 3.1.*

Other Solution Concepts

It would be useful to formally assess the robustness of these conclusions to using some of the existing set-valued solution concepts instead of the notion of stability defined for the purpose of the present paper. Unfortunately, these concepts are typically defined under specific assumptions regarding the nature of the player's strategy sets, that are not satisfied by infinite sets of supply functions. Nevertheless, an informal discussion situating the present findings within two recent frameworks may be of interest.

The first of those is the Preparation Set by Voorneveld [2004], defined as a product set of non-empty, compact subsets of the players' compact Hausdorff topological strategy spaces that satisfies the 'preparation' requirement. This, in the words of the author, is 'a standard rationality condition, stating that [...] each player's set of recommended strategies must contain at least one best reply to whatever belief he may have that is consistent with the recommendations to the other players'. It is not difficult to see that any stable convention would satisfy this requirement, whatever (probabilistic) belief a player may hold regarding the conventional strategies to be chosen by others. If the payoff of any given unconventional strategy is at least matched by that of some conventional one, for any conventional strategy profile of other players, then the same should apply to the respective (expected) payoffs of these two strategies for any (probabilistic) belief regarding the conventional strategies to be chosen by others. Consequently, the unconventional strategy in question cannot be the unique best reply to the player's belief, as at least one must lie within the convention. However, the converse is not true, i.e. not every convention satisfying the preparation requirement is stable. For instance, suppose the payoff of the row-player associated with strategy-profile $\{C, E\}$ in Example 2.1 is equal to -1 instead of 1. Then strategy B can no longer lie outside the convention, as it is no longer (weakly) dominated by C w.r.t. $\{E, F\}$. Hence, $\{A, C\} \times \{E, F\}$ is no longer stable, though it still satisfies preparation, because A is better than B when E is believed to be more likely than F, whereas C in no worse than B in the opposite case.

The other solution concept is the Self-Admissible Set (SAS) by Brandenburger et al. [2008], which would have been an obvious alternative to the present stability notion due to its use of admissibility, if not for the fact that it is only defined for finite strategy sets

(although the authors mention the extension to infinite games as a possible direction of future work). Importantly, the framework allows for mixed strategies, so that the notion of admissibility specified in Definition 2.1 would have to be modified accordingly. In particular, a strategy s_i would be said to be mixed-strategy admissible w.r.t. S' when there is no mixed-strategy with support in S_i' that weakly dominates s_i w.r.t. S_{-i}'. Clearly, any strategy that is not (pure-strategy) admissible must not be mixed-strategy admissible, so that any externally stable convention would maintain this property if the alternative definition of admissibility was used. In addition, as previously mentioned, any given Cournot quantity $q \in [0, q^m]$ is a strict best-response to some level $Q \in [0, (n-1)q^m]$ of the competitors' aggregate output, making it mixed-strategy admissible in Cournot. By the same token, for the Cournot-Equivalent convention S' of Proposition 3.1, every $s_i'(\cdot) \in S_i'$ such that $s_i'(P(0)) = q$ is mixed-strategy admissible w.r.t. S'. Hence, S' continues to be stable under the alternative definition of admissibility.
In addition, this means that for any $s_i'(\cdot) \in S_i'$ there exists no mixed-strategy that plays some $s_i(\cdot) \in S_i \backslash S_i'$ with positive probability and results in an expected payoff no smaller than that of $s_i'(\cdot)$ for any $\mathbf{s}_{-i}' \in S_{-i}'$. This is because instead of every such $s_i(\cdot)$, the mixed-strategy in question could play the conventional strategy which weakly dominates $s_i(\cdot)$ w.r.t. S_{-i}'. The resulting improved mixed-strategy could still not match the payoff of $s_i'(\cdot)$ w.r.t. S_{-i}' due to $s_i'(P(0))$ outperforming any other quantity $q \in [0, q^m]$ in Cournot for some $Q \in [0, (n-1)q^m]$. At a stroke, this ensures that all three defining characteristics of a (finite) Self-Admissible Set are met by S'. That is to say, no $s_i'(\cdot) \in S_i'$ is weakly dominated by a mixed strategy with support in S_i, whether w.r.t. S_{-i}' or S_{-i}; and no pure strategy in support of a mixed strategy that matches the payoff of $s_i'(\cdot)$ w.r.t. S_{-i} is excluded from S_i'. However, it should be noted that one can give examples of conventions in finite games that satisfy the current stability requirements, but not those of a SAS, or vice-versa, i.e. the two do not generically coincide.

Overall, while for technical reasons it is difficult to evaluate the paper's predictions under different set-valued solution concepts, the present solution possesses a number of features that are considered desirable in some of the most prominent game-theoretic frameworks.

4 Concluding Remarks

The paper addressed the long-standing controversy associated with the Cournot model of oligopolistic competition, namely the fact that it is 'right for the wrong reasons', in the sense that its plausible predictions follow from an unrealistic model set-up, ignoring the fact that players make pricing, as well as quantity decisions. However, as opposed to existing literature, the paper did not strive to offer further support to the Cournot *outcome* - as an equilibrium of a more complex price-quantity game. Instead, it was demonstrated that the entire Cournot *game* can constitute a set-valued solution of the strategic supply function model. The circumstances in which this was shown to occur, often associated with Cournot on an intuitive level, were now given a strictly formal justification.

The specific set-valued solution used to obtain the results is one of the oldest formalizations of 'standards of behaviour': the von Neumann - Morgenstern Stable Set, in its recently generalized form, and for a dominance relation set to incorporate the admissibility requirement well-established in non-cooperative game theory. In particular, a convention restricting the players' strategy sets is said to be stable when all conventional strategies (but no unconventional ones) are admissible with respect to the conventional strategies of others. In fact, this means the present solution exhibits a number of desirable properties embedded in other solution concepts, such as 'preparation' in the sense of Voorneveld [2004] and all characteristics of a (finite) Self Admissible Set by Brandenburger et al. [2008].

It follows that a convention equivalent to Cournot can be stable when costs are sufficiently 'sunk', in the sense that building the capacity to support a chosen supply schedule requires expenditure that cannot be wholly recovered even when the capacity is not subsequently utilized in full due to an insufficient market price. Furthermore, stability of Cournot is facilitated when marginal costs are initially high and rise steeply, and when the number of firms is low, which is in line with the classic results of Klemperer and Meyer. If, in addition to those requirements, the capacity that firms can build is adequately constrained, then every stable convention within supply function competition is equivalent to some stable convention within Cournot. In fact, when the firms' Cournot best-response functions are sufficiently steep, the only such convention amounts to the entire Cournot game.

What drives the results is the fact that when costs are large and mostly sunk, it becomes vital to utilize full capacity at whatever price it takes to do so, which in any event cannot be too low with the number of rival firms, and hence the potential aggregate output, relatively small. Hence, players then adopt an *'Everything must go!'* approach, setting supply schedules sufficiently price-competitive to sell their maximum quantities regardless of the exact conventional strategies selected by the counterparts. These quantities thus become the effective strategic variables, and the complex strategic interaction at hand simplifies to Cournot competition as a matter of convention.

The results could have important implications for the modelling of those industries where the apparent complexity of firms' decision problem makes Cournot seem inadequate, while allowing players the flexibility of a supply schedule renders equilibrium analysis intractable. For instance, we have seen how recent findings of competing auctions theory make the present model, with its sunk costs and possibly discontinuous supply schedules, applicable to the analysis of on-line auctions. Thus, Cournot may, in turn, constitute a framework that is viable here, where the role of the mythical 'Cournot auctioneer' is now played by independent auction bidders and sellers following an established convention. If the predictions of Cournot are found empirically correct for those types of goods where the derived stability conditions hold, one can at last reasonably maintain that this is so 'for the right reasons'.

Appendix: Proofs

Proof of Proposition 2.1. We will first prove that any stable convention is a vN-M stable set of the corresponding abstract game, and then establish that any general vN-M stable set of the specified general system is a stable convention.

(1) Suppose that $S' = \times_{i \in N} S'_i$ is a stable convention, and take any $\mathbf{s} \in S \backslash S'$. This must contain a strategy $s_i \in S_i \backslash S'_i$, which is, by Definition 2.2, weakly-dominated w.r.t. S'_{-i} by some $s'_i \in S'_i$. For any $\mathbf{s}'' = \{s''_j\}_{j \in N} \in S'$ such that $s''_i = s'_i$ we then have $\mathbf{s}'' \succ^{S'} \mathbf{s}$. Now take any $\mathbf{s}, \mathbf{s}' \in S'$ and observe that we cannot have $\mathbf{s}' = \{s'_j\}_{j \in N} \succ^{S'} \mathbf{s} = \{s_j\}_{j \in N}$, since this would imply that for some $i \in N$ the strategy $s_i \in S'_i$ is weakly dominated w.r.t. S'_{-i} by $s'_i \in S'_i$, which in turn would contradict the internal stability requirement of Definition 2.2. Thus, S' is a vN-M stable set of $(S, \succ^{S'})$.

(2) Suppose that S' is a general vN-M stable set of $(S, \{\succ^A\}_{A \subseteq S})$ and observe that it must then be in Cartesian product form, i.e.: $S' \equiv \times_{i \in N} S'_i$. Otherwise, we could choose a strategy profile $\mathbf{s} \in S \backslash S'$ consisting of strategies that are each part of some strategy profile in S'. By virtue of vN-M external stability, there would then exist a $\mathbf{s}' \in S'$ such that $\mathbf{s}' \succ^{S'} \mathbf{s}$, i.e. for some i the strategy $s_i \in S'_i$ would be weakly dominated by $s'_i \in S'_i$, which would contradict vN-M internal stability.
Suppose then that there exists a $s^1_i \in S'_i$ that weakly dominates some $s^2_i \in S'_i$ w.r.t. S'_{-i}. We could then take a pair of strategy profiles $\mathbf{s} = \{s_j\}_{j \in N}, \mathbf{s}' = \{s'_j\}_{j \in N}$, both in S', such that $s_i = s^1_i$ and $s'_i = s^2_i$, which would imply $\mathbf{s} \succ^{S'} \mathbf{s}'$, in violation of the vN-M internal stability of S'. Hence, S' must be internally stable in the sense of Definition 2.2.
Finally, suppose there exists some $s^0_i \in S_i \backslash S'_i$ that is admissible w.r.t. S' (i.e. S' is not externally stable in the sense of Definition 2.2), and take a $\mathbf{s} = \{s_j\}_{j \in N} \in S \backslash S'$ such that $s_i = s^0_i$ and $\mathbf{s}_{-i} \in S'_{-i}$. To satisfy vN-M external stability, we would need $\mathbf{s}'' \succ^{S'} \mathbf{s}$ for some $\mathbf{s}'' = \{s''_j\}_{j \in N} \in S'$. Since $s_i = s^0_i$ is admissible w.r.t. S', this would imply that $\mathbf{s}'' \succ^{S'} \mathbf{s}'$ for some $\mathbf{s}' \in S'$ which is identical to $\mathbf{s}$, except that $s'_i \neq s^0_i, s'_i \in S'_i$. However, this would contradict the vN-M internal stability of S'. Consequently, S' must be both internally and externally stable in the sense of Definition 2.2. ■

Proof of Proposition 3.1. Suppose S' is a Cournot-Equivalent Convention. As discussed in Section 2.1, it then follows from condition (1) that every strategy $s'_i \in S'_i$, being equivalent to some $q \in [0, q^m]$, is a best-response to some aggregate output of the counterparts, i.e. to some $s'_{-i} \in S'_{-i}$. Thus, s'_i cannot be weakly dominated by a $s_i \in S'_i$, i.e. it is admissible w.r.t. S'. Consequently, the internal stability requirement is satisfied.

We will now show that condition (8) is necessary for S' to be externally stable. To this end, consider two conventional strategy profiles $\mathbf{s}^m$ and $\mathbf{s}^0$, such that $\mathbf{s}^m = \{s^m_i(\cdot)\}_{i \in N} \in S'$ is equivalent to monopoly outputs by all players, i.e. $\varphi_i(s^m_i(\cdot)) = q^m$ for all $i \in N$, and $\mathbf{s}^0 = \{s^0_i(\cdot)\}_{i \in N} \in S'$ satisfies $\forall i \in N : \varphi_i(s^0_i(\cdot)) = 0$. Thus, it must be that for all $i \in N$

we have $s_i^0(P(0)) = 0$, so that the fact that $s_i^0(\cdot)$ is non-decreasing implies $s_i^0(p) = 0$ for all $p < P(0)$. In addition, $\forall i \in N : s_i^m(P(nq^m)) \geq q^m$, or else $\mathbf{s}^m$ could not result in everyone selling q^m at the corresponding demand price. Lastly, it cannot be that $s_i^m(P(nq^m)) > q^m$ for some $i \in N$, or else player i would sell more than q^m if others opted for $\mathbf{s}_{-i}^0$, contradicting the fact that a strategy profile comprising $s_i^m(\cdot)$ and $\mathbf{s}_{-i}^0$ is conventional, and hence Cournot-Equivalent. Thus, it must be that $s_i^m(P(nq^m)) = q^m$ for all i, and a similar reasoning establishes $s_i^m(P((n-1)q^m)) = q^m$, because $s_i^m(P((n-1)q^m)) > q^m$ would result in player i selling more than q^m when one of the competitors selects $s_j^0(\cdot)$ and others choose $s_j^m(\cdot)$. From the fact that $s_i^m(\cdot)$ is non-decreasing, we then have $s_i^m(p) = q^m$ for all $p \in (P(nq^m), P((n-1)q^m))$.

Now suppose a particular player $i \in N$ sets a supply schedule $\hat{s}_i(\cdot)$ that commits her to offer a quantity q^m if the price is at least equal to $\hat{p}$, where $\hat{p} \in (P(nq^m), P((n-1)q^m))$, and otherwise to offer nothing.

When other players select supply schedules $\mathbf{s}_{-i}^0$, player i succeeds in selling q^m at a price of $P(q^m)$. In the Cournot game, strategy q^m gives a strictly higher profit than any $q < q^m$ when others produce nothing. Thus, any conventional strategy $s_i(\cdot)$ that is no worse than $\hat{s}_i(\cdot)$ when others play $\mathbf{s}_{-i}^0$ must entail $\varphi_i(s_i(\cdot)) = q^m$.

In contrast, let $\hat{\mathbf{s}}^m$ denote a strategy profile which entails player i setting $\hat{s}_i(\cdot)$ and others choosing $\mathbf{s}_{-i}^m$. We have $S_N^-(\hat{\mathbf{s}}^m, \hat{p}) = (n-1)q^m$ and $S_N(\hat{\mathbf{s}}^m, \hat{p}) = nq^m$, and since $\hat{p} \in (P(nq^m), P((n-1)q^m))$ implies $D(\hat{p}) \in ((n-1)q^m, nq^m)$, $\hat{p}$ satisfies the market-clearing condition (3), i.e. $p^*(\hat{\mathbf{s}}^m) = \hat{p}$. Since the supply schedule of i (but not those of other players) is discontinuous at $\hat{p}$, we have:

$$x_i(\hat{\mathbf{s}}^m) = D(\hat{p}) - S_{N\setminus\{i\}}(\hat{\mathbf{s}}^m, \hat{p}) = D(\hat{p}) - (n-1)q^m \in (0, q^m)$$

Thus, $\hat{s}_i(\cdot)$ results in player i selling q^m when others play $\mathbf{s}_{-i}^0$, and less than q^m when others play $\mathbf{s}_{-i}^m$, which means $\hat{s}_i(\cdot)$ is not conventional, i.e. $\hat{s}_i(\cdot) \in S_i \setminus S_i'$. For S' to be externally stable, $\hat{s}_i(\cdot)$ would have to be weakly dominated by some $s_i(\cdot) \in S_i'$ w.r.t. S_{-i}'. However, any $s_i(\cdot)$ that could achieve this must at least match the profit resulting from $\hat{s}_i(\cdot)$ when others play $\mathbf{s}_{-i}^0$. As demonstrated above, this means it must be such that $\varphi_i(s_i(\cdot)) = q^m$, which in turn means that when others play $\mathbf{s}_{-i}^m$, $s_i(\cdot)$ gives a profit equal to $\pi_i(\mathbf{s}^m) = q^m P(nq^m) - C(q^m)$. This needs to be no smaller than:

$$\pi_i(\hat{\mathbf{s}}^m) = \hat{p}x_i(\hat{\mathbf{s}}^m) - [(1-\gamma)C(x_i(\hat{\mathbf{s}}^m)) + \gamma C(q^m)]$$

for all $\hat{p} \in (P(nq^m), P((n-1)q^m))$. Equivalently, fix a $\hat{q}_i \in (0, q^m)$, so that for $x_i(\hat{\mathbf{s}}^m)$ to equal $\hat{q}_i$, we must have:

$$D(\hat{p}) - (n-1)q^m = \hat{q}_i \Leftrightarrow \hat{p} = P(\hat{q}_i + (n-1)q^m)$$

Consequently, for S' to be externally stable, it is necessary that for all $\hat{q}_i \in (0, q^m)$, $\pi_i(\mathbf{s}^m)$ is no smaller than:

$$\pi_i(\hat{\mathbf{s}}^m(\hat{q}_i)) = P(\hat{q}_i + (n-1)q^m)\hat{q}_i - [(1-\gamma)C(\hat{q}_i) + \gamma C(q^m)]$$

Differentiating w.r.t. $\hat{q}_i$ yields

$$\partial\pi_i\left(\hat{\mathbf{s}}^m\left(\hat{q}_i\right)\right)/\partial\hat{q}_i = P\left(\hat{q}_i + (n-1)\,q^m\right) + P'\left(\hat{q}_i + (n-1)\,q^m\right)\hat{q}_i - (1-\gamma)\,C'\left(\hat{q}_i\right)$$

Thus, for $\hat{q}_i$ sufficiently close to q^m, $\pi_i\left(\hat{\mathbf{s}}^m\left(\hat{q}_i\right)\right)$ can become arbitrarily close to $\pi_i\left(\mathbf{s}^m\right)$, and $\partial\pi_i\left(\hat{\mathbf{s}}^m\left(\hat{q}_i\right)\right)/\partial\hat{q}_i$ can become arbitrarily close to:

$$P\left(nq^m\right) + P'\left(nq^m\right)q^m - (1-\gamma)\,C'\left(q^m\right) \tag{13}$$

Condition (8) states that (13) is non-negative, so if (8) was violated, $\partial\pi_i\left(\hat{\mathbf{s}}^m\left(\hat{q}_i\right)\right)/\partial\hat{q}_i$ would be negative, and $\pi_i\left(\hat{\mathbf{s}}^m\left(\hat{q}_i\right)\right)$ larger than $\pi_i\left(\mathbf{s}^m\right)$, for $\hat{q}_i$ sufficiently close to q^m. This means condition (8) is indeed necessary for the external stability of S'.
Furthermore, observe that (8) cannot be satisfied for $\gamma = 0$, as it then states that $P\left(nq^m\right) + P'\left(nq^m\right)q^m \geq C'\left(q^m\right)$. This in turn contradicts the assumption that $C'(0) \geq P\left((n-1)\,q^m\right)$, since $C'(0) \leq C'\left(q^m\right)$, $P\left(nq^m\right) < P\left((n-1)\,q^m\right)$ and $P'\left(nq^m\right) < 0$. Furthermore, Definition 3.1 states that for any $\mathbf{s} = \{s_i(\cdot)\}_{i\in N} \in S'$ and for all $i \in N$, $x_i(\mathbf{s}) = \varphi_i\left(s_i(\cdot)\right)$, which implies $p^*(\mathbf{s}) = P(\sum_{j\in N}\varphi_j(s_j(\cdot)))$. Hence, to ensure that the profit specification (5) coincides with Cournot for $\gamma \in (0,1]$, it is necessary that $\varphi_i\left(s_i(\cdot)\right) = s_i\left(P(0)\right)$ for all $\mathbf{s} \in S'$ and $i \in N$. Consequently, the only Cournot-Equivalent convention that satisfies the necessary conditions for stability is characterized by:

$$\forall i \in N : S_i' = \left\{s_i(\cdot) \in S_i : s_i\left(P\left(s_i\left(P(0)\right) + (n-1)\,q^m\right)\right) = s_i\left(P(0)\right) \leq q^m\right\} \tag{14}$$

In other words, a supply schedule is conventional if and only if its maximum quantity is sold even when everybody else sells the monopoly-optimal output. Note that in such case $s_i\left(P\left(s_i\left(P(0)\right) + (n-1)\,q^m\right)\right) < s_i\left(P(0)\right)$ would not allow player i to sell as much as $s_i\left(P(0)\right)$, contradicting $\varphi_i\left(s_i(\cdot)\right) = s_i\left(P(0)\right)$, whereas $s_i\left(P\left(s_i\left(P(0)\right) + (n-1)\,q^m\right)\right)$ cannot exceed $s_i\left(P(0)\right)$ due to $s_i(\cdot)$ being non-decreasing.

We will now show that condition (8) is sufficient to ensure that the particular Cournot-Equivalent convention S', as characterized by (14), is externally stable.
To begin with, take any $s_i(\cdot) \in S_i \backslash S_i'$ such that $s_i\left(P(0)\right) \leq q^m$. We will demonstrate that $s_i(\cdot)$ is weakly dominated w.r.t. S_{-i}' by a $s_i'(\cdot) \in S_i'$ such that $s_i\left(P(0)\right) = s_i'\left(P(0)\right)$. To this end, let $\bar{q}_i = s_i\left(P(0)\right)$ and, for a particular $\mathbf{s}_{-i}' = \{s_j'(\cdot)\}_{j\in N\backslash\{i\}} \in S_{-i}'$, let $Q_{-i} = \sum_{j\in N\backslash\{i\}} s_j'\left(P(0)\right)$. In addition, let $\mathbf{s}' \in S'$ denote the strategy profile comprising $s_i'(\cdot)$ and $\mathbf{s}_{-i}'$, and let $\hat{\mathbf{s}} \in S\backslash S'$ denote the strategy profile comprising $s_i(\cdot)$ and $\mathbf{s}_{-i}'$. We then have:

$$\pi_i\left(\mathbf{s}'\right) = \bar{q}_i P\left(\bar{q}_i + Q_{-i}\right) - C\left(\bar{q}_i\right)$$

As for $\pi_i(\hat{\mathbf{s}})$, there are two possibilities.
First, that $\mathbf{s}_{-i}'$ is such that $s_i\left(P\left(\bar{q}_i + Q_{-i}\right)\right) = \bar{q}_i$. This means $p^*(\hat{\mathbf{s}}) = P\left(\bar{q}_i + Q_{-i}\right)$, at which price player i is commited to supply up to $\bar{q}_i$, and others are commited to supply up to Q_{-i}, since:

$$\forall j \in N\backslash\{i\} : P\left(\bar{q}_i + Q_{-i}\right) \geq P(s_j'\left(P(0)\right) + (n-1)\,q^m)$$

where the price at the RHS of the inequality is the one at which any conventional supply schedule $s'_j(\cdot)$ attains its maximum, in accordance with (14). Thus, for each $j \in N\backslash\{i\}$ we have $s'_j(p(\hat{\mathbf{s}})) = s'_j(P(0))$, which implies $S_N(\hat{\mathbf{s}}, p(\hat{\mathbf{s}})) = D(p(\hat{\mathbf{s}}))$ and $\pi_i(\hat{\mathbf{s}}) = \pi_i(\mathbf{s}')$. Another possibility is that $\mathbf{s}'_{-i}$ is such that $s_i(P(\bar{q}_i + Q_{-i})) < \bar{q}_i$. This must be the case for some $\mathbf{s}'_{-i}$, or a sufficiently large $Q_{-i} \in [0, (n-1)q^m]$, since otherwise $s_i(\cdot)$ would satisfy (14). We have $p(\hat{\mathbf{s}}) \geq P(\bar{q}_i + Q_{-i})$, so the argument used above for $p^*(\hat{\mathbf{s}}) = P(\bar{q}_i + Q_{-i})$ establishes that $S_{N\backslash\{i\}}(\hat{\mathbf{s}}, p) = Q_{-i}$ for any $p \geq P(\bar{q}_i + Q_{-i})$. This implies

$$\textstyle\sum_{j \in N\backslash\{i\}} x_j(\hat{\mathbf{s}}) = Q_{-i}$$

so that player i is effectively a monopolist with (residual) demand $D(p) - Q_{-i}$, i.e. choosing $s_i(\cdot)$ is equivalent to setting a quantity $q_i \leq \bar{q}_i$, where the resulting profit equals:

$$\hat{\pi}_i(q_i, Q_{-i}) = q_i P(q_i + Q_{-i}) - [(1-\gamma)C(q_i) + \gamma C(\bar{q}_i)] \tag{15}$$

Differentiating this w.r.t. q_i yields:

$$\partial\hat{\pi}_i(q_i, Q_{-i})/\partial q_i = P(q_i + Q_{-i}) + q_i P'(q_i + Q_{-i}) - (1-\gamma)C'(q_i) \tag{16}$$

Condition (8) states that (16) is non-negative for $q_i = q^m$ and $Q_{-i} = (n-1)q^m$. In addition, we have:

$$\partial^2\hat{\pi}_i(q_i, Q_{-i})/\partial q_i \partial Q_{-i} = P'(q_i + Q_{-i}) + q_i P''(q_i + Q_{-i}) < 0 \tag{17}$$

If $P''(q_i + Q_{-i}) \leq 0$, (17) is clearly satisfied due to $P(\cdot)$ being strictly decreasing. If $P''(q_i + Q_{-i}) > 0$, then (17) is implied by assumption (1), since:

$$(q_i + Q_{-i})P''(q_i + Q_{-i}) \geq q_i P''(q_i + Q_{-i})$$

Furthermore:

$$\partial^2\hat{\pi}_i(q_i, Q_{-i})/\partial q_i^2 = 2P'(q_i + Q_{-i}) + q_i P''(q_i + Q_{-i}) - (1-\gamma)C''(q_i) < 0 \tag{18}$$

due to the fact that $C''(q_i) \geq 0$ by assumption and $P'(q_i + Q_{-i}) < 0$. Together (17) and (18) imply that for $q_i < \bar{q}_i$, we have:

$$\partial\hat{\pi}_i(q_i, Q_{-i})/\partial q_i > P(nq^m) + q^m P'(nq^m) - (1-\gamma)C'(q^m) \geq 0$$

Consequently, for all $q_i < \bar{q}_i$:

$$\pi_i\left(\mathbf{s}'\right) = \bar{q}_i P(\bar{q}_i + Q_{-i}) - C(\bar{q}_i) > \hat{\pi}_i(q_i, Q_{-i})$$

i.e. $s'_i(\cdot)$ gives a higher payoff than $s_i(\cdot)$ w.r.t. all $\mathbf{s}'_{-i}$ such that $s_i(P(\bar{q}_i + Q_{-i})) < \bar{q}_i$. Overall, this means $s'_i(\cdot)$ weakly dominates $s_i(\cdot)$ w.r.t. S'_{-i}, i.e. $s_i(\cdot)$ is inadmissible w.r.t. S', in accordance with external stability of S'.

Finally, we will show that, when $q^m < q^c$ (the firms' capacity), any $s_i(\cdot) \in S_i \backslash S_i'$ such that $s_i(P(0)) > q^m$ is weakly dominated w.r.t. S'_{-i} by a $s_i'(\cdot) \in S_i'$ such that $s_i'(P(0)) = q^m$. The previous denotation of $\bar{q}_i$, $\mathbf{s}'_{-i}, \mathbf{s}', \hat{\mathbf{s}}$ and Q_{-i} is retained. We now have:

$$\pi_i\left(\mathbf{s}'\right) = q^m P\left(q^m + Q_{-i}\right) - C\left(q^m\right)$$

As for $\pi_i(\hat{\mathbf{s}})$, there are, once again, two possibilities.

First, that $\mathbf{s}'_{-i}$ is such that $\sum_{j\in N\backslash\{i\}} x_j(\hat{\mathbf{s}}) = Q_{-i}$, in which case player i is in effect a monopolist with (residual) demand $D(p) - Q_{-i}$, choosing a quantity q_i by means of its supply schedule (where q_i may now exceed q^m) and earning profits (15). When $q_i < q^m < \bar{q}_i$, the previous argument establishes that $\partial\hat{\pi}_i(q_i, Q_{-i})/\partial q_i > 0$, so that $\pi_i(\mathbf{s}') > \hat{\pi}_i(q_i, Q_{-i})$. When $q_i \in [q^m, \bar{q}_i]$, we have

$$\begin{aligned} \pi_i\left(\mathbf{s}'\right) &\geq q_i P\left(q_i + Q_{-i}\right) - C\left(q_i\right) \\ q_i P\left(q_i + Q_{-i}\right) - C\left(q_i\right) &\geq \hat{\pi}_i\left(q_i, Q_{-i}\right) \end{aligned}$$

where the first inequality is strict for $q_i \in (q^m, \bar{q}_i]$ (by virtue of q^m being monopoly-optimal), and the second one is strict for $q_i \in [q^m, \bar{q}_i)$. Thus, for all $q_i \in [q^m, \bar{q}_i]$, we have $\pi_i(\mathbf{s}') > \hat{\pi}_i(q_i, Q_{-i})$.

But suppose $\mathbf{s}'_{-i}$ is such that $\sum_{j\in N\backslash\{i\}} x_j(\hat{\mathbf{s}}) < Q_{-i}$. This means for some $j \in N\backslash\{i\}$ we must have $s_j'(p^*(\hat{\mathbf{s}})) < s_j'(P(0))$, so it follows from (14), that:

$$p^*(\hat{\mathbf{s}}) < \max_{j\in N\backslash\{i\}} P(s_j'(P(0)) + (n-1)\, q^m)$$

In addition, from the fact that $s_j'(P(0)) \leq q^m$ for all $j \in N\backslash\{i\}$, we have:

$$\max_{j\in N\backslash\{i\}} P(s_j'(P(0)) + (n-1)\, q^m) \leq P(q^m + \textstyle\sum_{j\in N\backslash\{i\}} s_j'(P(0))) = p^*\left(\mathbf{s}'\right)$$

Finally, since $C'(0) \geq P((n-1)\, q^m)$ and $C''(\cdot) \leq 0$ by assumption, it follows that, for all $q \in [q^m, \bar{q}_i]$, $C'(q) > p^*(\hat{\mathbf{s}})$. Combined with the fact that $p^*(\hat{\mathbf{s}}) < p^*(\mathbf{s}')$, as established above, this means $\pi_i(\mathbf{s}') > \pi_i(\hat{\mathbf{s}})$. The reason for this is that, compared with $s_i'(\cdot)$, $s_i(\cdot)$ results in selling a larger quantity of output at a lower price, where the new price is below the marginal cost level for all additional output being sold.

Thus, any $s_i(\cdot) \in S_i \backslash S_i'$ such that $s_i(P(0)) > q^m$ is inadmissible w.r.t. S', which completes the proof. ■

Proof of Proposition 3.2. Suppose S' is a stable convention and let:

$$q_i^l = \min_{s_i(\cdot)\in S_i'} s_i\left(P\left(0\right)\right),\ q_i^h = \max_{s_i(\cdot)\in S_i'} s_i\left(P\left(0\right)\right),\ Q_{-i}^h = \sum_{j\in N/\{i\}} q_j^h,\ Q_{-i}^l = \sum_{j\in N/\{i\}} q_j^l$$

We will first show that it is necessary that for some collection of pairs $\left\{\left(q_i^l, q_i^h\right)\right\}_{i\in N}$, S' satisfies the following three conditions for every $i \in N$:

$$\begin{aligned} S_i' &= \left\{s_i\left(\cdot\right) \in S_i : s_i\left(P(s_i\left(P\left(0\right)\right) + Q_{-i}^h)\right) = s_i\left(P\left(0\right)\right) \in [q_i^l, q_i^h] \subseteq [0, q^m]\right\} && \text{(C1)} \\ q_i^l &= q_{br}(Q_{-i}^h) \text{ and } q_i^h = q_{br}(Q_{-i}^l) && \text{(C2)} \\ \forall j &\in N/\{i\} : 2\left[P(q_i^l + Q_{-i}^h) - C'(q_j^h)\right] \leq -q_j^h P'(q_i^l + Q_{-i}^h) && \text{(C3)} \end{aligned}$$

Once this is achieved, we will show that it must also be that $\forall i \in N : q_i^l = q^l$ and $q_i^h = q^h$, where $q^l < q^h$, which amounts to Proposition 3.2.
To begin with, consider a supply schedule $s_i\left(\cdot\right) \in S_i$ such that for some $\mathbf{s}_{-i}' \in S_{-i}'$ we have $x_i\left(\hat{\mathbf{s}}\right) < s_i\left(P\left(0\right)\right)$, where $\hat{\mathbf{s}}$ denotes the strategy profile comprising $s_i\left(\cdot\right)$ and $\mathbf{s}_{-i}'$. Let $q_i = x_i\left(\hat{\mathbf{s}}\right)$, $Q_{-i} = \sum_{j\in N\setminus\{i\}} x_j\left(\hat{\mathbf{s}}\right)$ and $\bar{q}_i = s_i\left(P\left(0\right)\right)$, so that the profit of player i equals:

$$\hat{\pi}_i\left(q_i, Q_{-i}\right) = q_i P\left(q_i + Q_{-i}\right) - \left[\left(1-\gamma\right) C\left(q_i\right) + \gamma C\left(\bar{q}_i\right)\right]$$

In analogy with the proof of Proposition 3.1, for $q_i < \bar{q}_i \leq q^c$ we have:

$$\begin{aligned} \partial\hat{\pi}_i\left(q_i, Q_{-i}\right)/\partial q_i = P\left(q_i + Q_{-i}\right) + q_i P'\left(q_i + Q_{-i}\right) - \left(1-\gamma\right) C'\left(q_i\right) > \\ > P\left(nq^c\right) + q^m P'\left(nq^c\right) - \left(1-\gamma\right) C'\left(q^c\right) \geq 0 \end{aligned}$$

which is because assumptions (1) and $C''\left(q_i\right) \geq 0$ imply:

$$\begin{aligned} \partial^2\hat{\pi}_i\left(q_i, Q_{-i}\right)/\partial q_i \partial Q_{-i} = P'\left(q_i + Q_{-i}\right) + q_i P''\left(q_i + Q_{-i}\right) < 0 \\ \partial^2\hat{\pi}_i\left(q_i, Q_{-i}\right)/\partial q_i^2 = 2P'\left(q_i + Q_{-i}\right) + q_i P''\left(q_i + Q_{-i}\right) - \left(1-\gamma\right) C''\left(q_i\right) < 0 \end{aligned}$$

Observe that condition (9) and $P'\left(\cdot\right) < 0$ imply that $P\left(nq^c\right) \geq 0$ and that a strategy $s_i'\left(\cdot\right)$ such that $s_i'\left(0\right) = s_i\left(P(0)\right)$ would succeed in selling $\bar{q}_i > q_i$. Let $\mathbf{s}' \in S$ denote the strategy profile comprising $s_i'\left(\cdot\right)$ and $\mathbf{s}_{-i}'$. Clearly, for all $p \geq 0$ we have $S_N^-\left(\mathbf{s}', p\right) \geq S_N^-\left(\hat{\mathbf{s}}, p\right)$ and $S_N\left(\mathbf{s}', p\right) \geq S_N\left(\hat{\mathbf{s}}, p\right),$ so that $p^*\left(\mathbf{s}'\right) \leq P\left(q_i + Q_{-i}\right) = p^*\left(\hat{\mathbf{s}}\right).$ If $p^*\left(\mathbf{s}'\right) = P\left(q_i + Q_{-i}\right)$, other players sell $q_i + Q_{-i} - \bar{q}_i < Q_{-i}$. If $p^*\left(\mathbf{s}'\right) < P\left(q_i + Q_{-i}\right),$ then it follows from (4) that:

$$Q_{-i} \geq S_{N\setminus\{i\}}^-\left(\hat{\mathbf{s}}, p^*\left(\hat{\mathbf{s}}\right)\right) \geq S_{N\setminus\{i\}}\left(\hat{\mathbf{s}}, p^*\left(\mathbf{s}'\right)\right) \geq \sum_{j\in N\setminus\{i\}} x_j\left(\mathbf{s}'\right)$$

Thus, switching from strategy $s_i\left(\cdot\right)$ to $s_i'\left(\cdot\right)$ has the same effect as increasing q_i and making Q_{-i} no larger than before in profit specification $\hat{\pi}_i\left(q_i, Q_{-i}\right).$ Since $\partial\hat{\pi}_i\left(q_i, Q_{-i}\right)/\partial q_i > 0$ for $q_i < \bar{q}_i \leq q^c$, and $P'\left(\cdot\right) < 0$ implies $\partial\hat{\pi}_i\left(q_i, Q_{-i}\right)/\partial Q_{-i} = q_i P'\left(q_i + Q_{-i}\right) < 0$, it follows that $\pi_i\left(\hat{\mathbf{s}}\right) < \pi_i\left(\mathbf{s}'\right)$ for any $\mathbf{s}_{-i}'$ such that $x_i\left(\hat{\mathbf{s}}\right) < s_i\left(P\left(0\right)\right)$.

But consider a $\mathbf{s}'_{-i} \in S'_{-i}$ such that $x_i(\hat{\mathbf{s}}) = s_i(P(0))$. Clearly, if $p^*(\hat{\mathbf{s}}) = p^*(\mathbf{s}')$, then $\pi_i(\hat{\mathbf{s}}) = \pi_i(\mathbf{s}')$ (since $x_i(\mathbf{s}') = s_i(P(0))$). But suppose $p^*(\hat{\mathbf{s}}) > p^*(\mathbf{s}')$. This would require that:

$$S_N\left(\mathbf{s}', p^*\left(\mathbf{s}'\right)\right) = s_i(P(0)) + S_{N\backslash\{i\}}\left(\mathbf{s}', p^*\left(\mathbf{s}'\right)\right) \geq D\left(p^*\left(\mathbf{s}'\right)\right) > D\left(p^*(\hat{\mathbf{s}})\right)$$

Since $S_{N\backslash\{i\}}(\mathbf{s}', p^*(\mathbf{s}')) \leq S^-_{N\backslash\{i\}}(\mathbf{s}', p^*(\hat{\mathbf{s}})) = S^-_{N\backslash\{i\}}(\hat{\mathbf{s}}, p^*(\hat{\mathbf{s}}))$, we must then have:

$$\textstyle\sum_{j\in N\backslash\{i\}} x_j(\hat{\mathbf{s}}) \geq S_{N\backslash\{i\}}\left(\mathbf{s}', p^*\left(\mathbf{s}'\right)\right) > D(p^*(\hat{\mathbf{s}})) - s_i(P(0))$$

As this would contradict $x_i(\hat{\mathbf{s}}) = s_i(P(0))$, $p^*(\hat{\mathbf{s}}) > p^*(\mathbf{s}')$ is impossible.
All in all, this means that any supply schedule $s_i(\cdot) \in S_i$ such that for some $\mathbf{s}'_{-i} \in S'_{-i}$ we have $x_i(\hat{\mathbf{s}}) < s_i(P(0))$ is weakly dominated w.r.t. S'_{-i} by any $s'_i(\cdot) \in S_i$ such that $s'_i(0) = s_i(P(0))$. Any such $s_i(\cdot)$ is therefore inadmissible w.r.t. S'_{-i}, so from the fact that S' is stable it must be that $s_i(\cdot) \in S_i \backslash S'_i$. In other words, it is necessary for the stability of S' that for every $i \in N$ we have:

$$\forall s_i(\cdot) \in S'_i : s_i\left(P(s_i(P(0)) + \textstyle\sum_{j\in N/\{i\}} q^h_j)\right) = s_i(P(0)) \in \left[q^l_i, q^h_i\right] \tag{19}$$

where $q^l_j = \min_{s_j(\cdot)\in S'_j} s_j(P(0))$, $q^h_j = \max_{s_j(\cdot)\in S'_j} s_j(P(0))$. That is to say, any stable convention is Cournot-Equivalent (in the sense of Definition 3.1), except that players may need to choose their quantities of output out of sets different from $[0, q^m]$.
Next, let $Q^l_{-i} = \sum_{j\in N/\{i\}} q^l_j, Q^h_{-i} = \sum_{j\in N/\{i\}} q^h_j$, and observe that we cannot have $q_{br}(Q^h_{-i})$ strictly smaller than q^l_i for any $i \in N$. If this was the case, then we could take a strategy $s'_i(\cdot) \in S_i \backslash S'_i$ such that:

$$s'_i\left(P(s'_i(P(0)) + Q^h_{-i})\right) = s'_i(P(0)) = q_{br}(Q^h_{-i}) \tag{20}$$

For any $\mathbf{s}'_{-i} = \{s'_j(\cdot)\}_{j\in N\backslash\{i\}} \in S'_{-i}$ such that $Q_{-i} = \sum_{j\in N/\{i\}} s'_j(P(0)) = Q^h_{-i}$, $s'_i(\cdot)$ would then give a profit:

$$\hat{\pi}_i\left(q_{br}(Q^h_{-i}), Q^h_{-i}\right) = q_{br}(Q^h_{-i})P\left(q_{br}(Q^h_{-i}) + Q^h_{-i}\right) - C(q_{br}(Q^h_{-i})) \tag{21}$$

whereas any $s_i(\cdot) \in S'_i$, i.e. one satisfying (19), would yield:

$$\hat{\pi}_i\left(s_i(P(0)), Q^h_{-i}\right) = s_i(P(0))P\left(s_i(P(0)) + Q^h_{-i}\right) - C(s_i(P(0))) \tag{22}$$

Since $s_i(P(0)) \geq q^l_i > q_{br}(Q^h_{-i})$, and $q_{br}(Q^h_{-i}) = \arg\max_q \hat{\pi}_i\left(q, Q^h_{-i}\right)$, we have (21) > (22), i.e. $s'_i(\cdot) \in S_i \backslash S'_i$ could not be weakly dominated w.r.t. S'_{-i} by any $s_i(\cdot) \in S'_i$, violating the external stability of S'.

Similarly, we cannot have $q_{br}(Q^h_{-i}) > q^l_i$, as in such case we could take two strategies: $s_i(\cdot) \in S'_i$ such that $s_i(P(0)) = q^l_i$, and $s'_i(\cdot) \in S_i$ satisfying:

$$s'_i\left(P(s'_i(P(0)) + Q^h_{-i})\right) = s'_i(P(0)) = q_{br}(Q^h_{-i})$$

The payoff resulting from $s_i(\cdot)$ given $\mathbf{s}'_{-i} = \{s'_j(\cdot)\}_{j\in N\backslash\{i\}} \in S'_{-i}$ is then:

$$\hat{\pi}_i\left(q^l_i, Q_{-i}\right) = q^l_i P\left(q^l_i + Q_{-i}\right) - C(q^l_i) \tag{23}$$

where, as before, $Q_{-i} = \sum_{j\in N/\{i\}} s'_j(P(0))$. In contrast, when player i selects $s'_i(\cdot)$, she sells $q_{br}(Q^h_{-i})$ for any $\mathbf{s}'_{-i}$, whereas other players sell at most Q_{-i}. Thus, the profit of player i resulting from $s'_i(\cdot)$ is no smaller than:

$$\hat{\pi}_i\left(q_{br}(Q^h_{-i}), Q_{-i}\right) = q_{br}(Q^h_{-i})P\left(q_{br}(Q^h_{-i}) + Q_{-i}\right) - C(q_{br}(Q^h_{-i})) \tag{24}$$

For $Q_{-i} = Q^h_{-i}$ we have (24) > (23), since $q_{br}(Q^h_{-i}) = \arg\max_q \hat{\pi}_i\left(q, Q^h_{-i}\right)$. In addition, it has been established that $\partial^2\hat{\pi}_i(q_i, Q_{-i})/\partial q_i \partial Q_{-i} < 0$, so that (24) > (23) must follow for $Q_{-i} < Q^h_{-i}$ as well, i.e. for every $\mathbf{s}'_{-i} \in S'_{-i}$. Consequently, $q_{br}(Q^h_{-i}) > q^l_i$ would imply that $s'_i(\cdot)$ weakly dominates $s_i(\cdot) \in S'_i$ w.r.t. S'_{-i}, making the latter inadmissible and contradicting the stability of S'.
All in all, this means we must have $q_{br}(Q^h_{-i}) = q^l_i$ for every $i \in N$ in order for S' to be stable, and, by an analogous argument, we must also have $q_{br}(Q^l_{-i}) = q^h_i$. In other words, condition $(C2)$ is necessary for the stability of S'.
We now show that the converse of (19) is necessary as well, i.e. that $s_i(\cdot) \in S'_i$ is satisfied for any $s_i(\cdot) \in S_i$ such that:

$$s_i\left(P(s_i(P(0)) + \textstyle\sum_{j\in N/\{i\}} q^h_j)\right) = s_i(P(0)) \in \left[q^l_i, q^h_i\right] \tag{25}$$

Consider then a $s_i(\cdot) \in S_i \backslash S'_i$ that satisfies (25), so that for any $\mathbf{s}'_{-i} = \{s'_j(\cdot)\}_{j\in N\backslash\{i\}} \in S'_{-i}$ the resulting payoff is $\hat{\pi}_i(q_i, Q_{-i})$, where $q_i = s_i(P(0))$. Since $q_{br}(Q^h_{-i}) = q^l_i, q_{br}(Q^l_{-i}) = q^h_i$, we have $q_i \in \left[q_{br}(Q^h_{-i}), q_{br}(Q^l_{-i})\right]$ and $Q_{-i} \in \left[Q^l_{-i}, Q^h_{-i}\right]$. The fact that $q_{br}(\cdot)$ is continuous then implies that there must be a $Q^*_{-i} \in \left[Q^l_{-i}, Q^h_{-i}\right]$ such that $q_i = q_{br}(Q^*_{-i})$. Thus, given a $\mathbf{s}'_{-i} \in S'_{-i}$ such that $Q_{-i} = Q^*_{-i}$, $s_i(\cdot)$ gives a strictly higher profit than any $s'_i(\cdot) \in S'_i$ such that $s'_i(P(0)) \neq q_i$ and exactly the same profit as any $s'_i(\cdot) \in S'_i$ such that $s'_i(P(0)) = q_i$. Consequently, $s_i(\cdot) \in S_i \backslash S'_i$ cannot be weakly dominated w.r.t. S'_{-i} by a $s'_i(\cdot) \in S'_i$, violating the external stability of S'. This means condition $(C1)$ is necessary for the stability of S'.
Suppose now that conditions $(C1)$ and $(C2)$ hold, but condition $(C3)$ is violated, i.e. that for some $i, j \in N, i \neq j$ we have:

$$2\left[P(q^l_i + Q^h_{-i}) - C'(q^h_j)\right] > -q^h_j P'(q^l_i + Q^h_{-i}) \tag{26}$$

Given that, consider a strategy $s_j(\cdot) \in S_j \backslash S'_j$ of player j such that $s_j(0) = q_j^+ \geq q_j^h$, as well as two strategy profiles: $\mathbf{s}_{-j} = \{s_k(\cdot)\}_{k \in N \backslash \{j\}} \in S'_{-j}$ and $\mathbf{s}'_{-j} = \{s'_k(\cdot)\}_{k \in N \backslash \{j\}} \in S'_{-j}$ such that:

$$\sum_{k \in N \backslash \{j\}} s_k(P(0)) = Q^l_{-j} \text{ and } \sum_{k \in N \backslash \{j\}} s'_k(P(0)) = q^l_i - q^h_i + Q^h_{-j}$$

In addition, suppose that $s'_i(p) = 0$ for $p < P\left(q^l_i + Q^h_{-i}\right)$ and $s'_i(p) = q^l_i$ otherwise, which is consistent with $s'_i(\cdot) \in S'_i$. Since $q_{br}(Q^l_{-j}) = q^h_j$, we have:

$$\partial \hat{\pi}_j(q, Q^l_{-j})/\partial q \gtreqless 0 \Leftrightarrow q \lesseqgtr q^h_j \tag{27}$$

Let $q^-_j(q^+_j)$ denote a function that to every $q^+_j \geq q^h_j$ assigns the unique q^-_j that satisfies $q^-_j \leq q^h_j$ and:

$$\int_{q^-_j}^{q^+_j} \left[\partial \hat{\pi}_j(q, Q^l_{-j})/\partial q\right] dq = 0 \Leftrightarrow \hat{\pi}_j(q^-_j, Q^l_{-j}) = \hat{\pi}_j(q^+_j, Q^l_{-j})$$

When other players play $\mathbf{s}_{-j}$ and player j chooses $s_j(\cdot) \in S_j \backslash S'_j$, the latter will earn at least $\hat{\pi}_j(q^+_j, Q^l_{-j})$, so that any $\hat{s}_j(\cdot) \in S'_j$ that could weakly dominate $s_j(\cdot)$ w.r.t. S'_{-j} must satisfy $\hat{s}_j(P(0)) \in [q^-_j(q^+_j), q^h_j]$, or else we would have $\hat{\pi}_j(\hat{s}_j(P(0)), Q^l_{-j}) < \hat{\pi}_j(q^+_j, Q^l_{-j})$, i.e. $s_j(\cdot)$ would outperform $\hat{s}_j(\cdot)$ given $\mathbf{s}_{-j}$.
Suppose then others play $\mathbf{s}'_{-j}$, in which case player j must still be able to sell q^+_j, but others will not be able to sell as much as $q^l_i - q^h_i + Q^h_{-j}$, since this would require a price p_0 that satisfies:

$$p_0 = P(q^+_j + q^l_i - q^h_i + Q^h_{-j}) < P(q^h_j + q^l_i - q^h_i + Q^h_{-j}) = P(q^l_i + Q^h_{-i}) \Rightarrow s'_i(p_0) = 0$$

Thus, when $q^+_j - q^h_j \leq q^l_i$, we have:

$$S^-_N\left(\mathbf{s}, P(q^l_i + Q^h_{-i})\right) = q^+_j - q^h_i + Q^h_{-j},\ S_N\left(\mathbf{s}, P(q^l_i + Q^h_{-i})\right) = q^+_j + q^l_i - q^h_i + Q^h_{-j}$$

where $\mathbf{s} \in S \backslash S'$ is the strategy profile comprising $s_j(\cdot)$ and $\mathbf{s}'_{-j}$ (note that for $q^l_i = 0$ the reasoning below can be reproduced by using a $s'_i(\cdot)$ such that $s'_i(P(0))$ is marginally above 0). It follows that:

$$\begin{aligned} D\left(P(q^l_i + Q^h_{-i})\right) &= q^l_i + Q^h_{-i} \geq S^-_N\left(\mathbf{s}, P(q^l_i + Q^h_{-i})\right) = q^+_j - q^h_i + Q^h_{-j} \Leftrightarrow q^+_j - q^h_j \leq q^l_i \\ D\left(P(q^l_i + Q^h_{-i})\right) &= q^l_i + Q^h_{-i} \leq S_N\left(\mathbf{s}, P(q^l_i + Q^h_{-i})\right) = q^+_j + q^l_i - q^h_i + Q^h_{-j} \Leftrightarrow q^h_j \leq q^+_j \end{aligned}$$

so that $p^*(\mathbf{s}) = P(q^l_i + Q^h_{-i})$, and we have:

$$\pi_j(\mathbf{s}) = q^+_j P(q^l_i + Q^h_{-i}) - C(q^+_j)$$

But suppose player j chooses instead the strategy $\hat{s}_j(\cdot) \in S'_j$, where it must be that $\hat{s}_j(P(0)) \in [q_j^-(q_j^+), q_j^h]$ as explained above, and let $\hat{\mathbf{s}} \in S'$ denote the resulting strategy profile. We have:

$$\pi_j(\hat{\mathbf{s}}) = \hat{s}_j(P(0))\, P(\hat{s}_j(P(0)) + q_i^l - q_i^h + Q_{-j}^h) - C(\hat{s}_j(P(0)))$$

Since $q_j^h = q_{br}(Q_{-j}^l)$, for $Q_{-j}^l = q_i^l - q_i^h + Q_{-j}^h$ the maximum of $\pi_j(\hat{\mathbf{s}})$ is attained where $\hat{s}_j(P(0)) = q_j^h$, in which case $\pi_j(\hat{\mathbf{s}}) = \pi_j(\mathbf{s})$ for $q_j^+ = q_j^h$. We then have $\partial\pi_j(\mathbf{s})/\partial q_j^+ = P(q_i^l + Q_{-i}^h) - C'(q_j^+)$, which is positive for q_j^+ sufficiently close to q_j^h whenever (26) is true. For $Q_{-j}^l < q_i^l - q_i^h + Q_{-j}^h$, the fact that $\partial\hat{\pi}_j(q,Q)/\partial q\partial Q < 0$ implies that:

$$\partial\hat{\pi}_j(q, q_i^l - q_i^h + Q_{-j}^h)/\partial q < \partial\hat{\pi}_j(q, Q_{-j}^l)/\partial q$$

Thus, for q_j^+, and hence $q_j^-(q_j^+)$, sufficiently close to q_j^h it holds that $\partial\hat{\pi}_j(q_j^-, Q_{-j}^l)/\partial q_j^- < 0$, i.e. $\pi_j(\hat{\mathbf{s}})$ is maximized where $\hat{s}_j(P(0)) = q_j^-(q_j^+)$. We then have:

$$\pi_j(\mathbf{s}) - \pi_j(\hat{\mathbf{s}}) = q_j^+ P(q_i^l + Q_{-i}^h) - C(q_j^+) - \left[q_j^-(q_j^+) P(q_j^-(q_j^+) + q_i^l - q_i^h + Q_{-j}^h) - C(q_j^-(q_j^+))\right]$$

This is equal to 0 for $q_j^+ = q_j^h$, while differentiating w.r.t. q_j^+ yields:

$$\begin{aligned} &P(q_i^l + Q_{-i}^h) - C'(q_j^+) - q_j^{-\prime}(q_j^+) P(q_j^-(q_j^+) + q_i^l - q_i^h + Q_{-j}^h) - \\ &\qquad - q_j^-(q_j^+) P'(q_j^-(q_j^+) + q_i^l - q_i^h + Q_{-j}^h) q_j^{-\prime}(q_j^+) + C'(q_j^-(q_j^+)) q_j^{-\prime}(q_j^+) \end{aligned}$$

And evaluating this at $q_j^+ = q_j^h$ gives:

$$\begin{aligned} &P(q_i^l + Q_{-i}^h) - C'(q_j^h) + P(q_j^h + q_i^l - q_i^h + Q_{-j}^h) + q_j^h P'(q_j^h + q_i^l - q_i^h + Q_{-j}^h) - C'(q_j^h) = \\ &\quad = P(q_i^l + Q_{-i}^h) - C'(q_j^h) + P(q_i^l + Q_{-i}^h) + q_j^h P'(q_i^l + Q_{-i}^h) - C'(q_j^h) = \\ &\quad = 2\left[P(q_i^l + Q_{-i}^h) - C'(q_j^h)\right] + q_j^h P'(q_i^l + Q_{-i}^h) \end{aligned}$$

where we have used the facts that $q_j^-(q_j^h) = q_j^h$ and that $q_j^{-\prime}(q_j^h) = -1$ due to $\partial\hat{\pi}_j(q,Q)/\partial q$ being continuous and $\partial\hat{\pi}_j(q, Q_{-j}^l)/\partial q \gtreqless 0 \Leftrightarrow q \lesseqgtr q_j^h$. Observe that (26) states that the derivative $2\left[P(q_i^l + Q_{-i}^h) - C'(q_j^h)\right] + q_j^h P'(q_i^l + Q_{-i}^h)$ is positive.

Thus, under condition (26), a marginal increase of q_j^+ above q_j^h results in $\pi_j(\mathbf{s}) - \pi_j(\hat{\mathbf{s}}) > 0$, which makes it impossible for the strategy $s_j(\cdot) \in S_j \backslash S'_j$ to be weakly dominated by some $\hat{s}_j(\cdot) \in S'_j$ w.r.t. S'_{-j}, thereby contradicting the fact that S' is stable. Condition $(C3)$ is therefore necessary for stability to hold.

We now show that it is impossible to have $q_i^l = q_i^h$ for some $i \in N$. This would necessitate either $Q_{-i}^l = Q_{-i}^h$, or $Q_{-i}^l < Q_{-i}^h$ and $q_{br}(Q_{-i}^l) = q_i^l = q_i^h = 0$. In the former case, we would then need to have $q_j^l = q_j^h$ for all $j \in N$. In the latter case, we need $P(Q_{-i}^l) \leq C'(0)$, which would contradict the fact that for any $j \in N \backslash \{i\}$ such that $q_j^h > 0$ it must be that $P(q_j^h + Q_{-j}^l) + q_j^h P'(q_j^h + Q_{-j}^l) \geq C'(q_j^h)$, $q_j^h + Q_{-j}^l > Q_{-i}^l$. The only way to avoid contradiction would be that $q_j^l = q_j^h = 0$ for all $j \in N$, but then condition $(C2)$ would reduce to $q_{br}(0) = 0$, violating the assumption $C'(0) < P(0)$, or $q^m > 0$. Thus, to have $q_i^l = q_i^h$ for some $i \in N$, we must have $q_j^l = q_j^h > 0$ for all $j \in N$. This would imply:

$$\forall j \in N : P(Q) + q_j P'(Q) = C'(q_j), \text{ where } q_j = q_j^l = q_j^h, \; Q = \textstyle\sum_{j \in N} q_j$$

which would in turn require $q_1 = q_2 = ... = q_n$ to hold. But this means for any $i, j \in N$ we have:

$$P(q_i^l + Q_{-i}^h) - C'(q_j^h) = -q_j^h P'(q_i^l + Q_{-i}^h) > 0$$

contradicting condition $(C3)$.

Finally, we show that S' must be symmetric, i.e.:

$$\forall i \in N : \left\{q_i^l, q_i^h\right\} = \left\{q^l, q^h\right\} \text{ where } q^l, q^h \in [0, q^m]$$

Suppose the contrary, i.e. (without loss of generality) that for some $i, j \in N$ we have $q_i^l < q_j^l$. Observe this happens if and only if $q_i^h < q_j^h$, since otherwise we would have $Q_{-i}^h \leq Q_{-j}^h$, contradicting $q_i^l = q_{br}\left(Q_{-i}^h\right) < q_j^l = q_{br}(Q_{-j}^h)$ due to $q_{br}(\cdot)$ being a decreasing function. Consider first the case of $q_i^l = 0$, so that in order to have $q_i^l = 0 = q_{br}\left(Q_{-i}^h\right)$, we need $P\left(Q_{-i}^h\right) \leq C'(0)$, and for $q_j^l = q_{br}(Q_{-j}^h) > 0$ we require:

$$P(q_j^l + Q_{-j}^h) + q_j^l P'(q_j^l + Q_{-j}^h) = C'(q_j^l)$$

In order for both of these requirements to hold, it must be that:

$$q_j^l + Q_{-j}^h < Q_{-i}^h \Leftrightarrow q_j^l + q_i^h < q_j^h$$

However, to have $q_j^h = q_{br}(Q_{-j}^l) > 0$ and $q_i^h = q_{br}(Q_{-i}^l) > 0$, we need, respectively:

$$\begin{aligned} P(q_j^h + Q_{-j}^l) + q_j^h P'(q_j^h + Q_{-j}^l) &= C'(q_j^h) \\ P(q_i^h + Q_{-i}^l) + q_i^h P'(q_i^h + Q_{-i}^l) &= C'(q_i^h) \end{aligned}$$

Since $q_i^h < q_j^h$, this requires that $q_j^h + Q_{-j}^l < q_i^h + Q_{-i}^l \Leftrightarrow q_j^h + q_i^l = q_j^h < q_i^h + q_j^l$, contradicting $q_j^l + Q_{-j}^h < Q_{-i}^h$.

Similarly, in case of $q_i^l > 0$, in order to have $q_j^l = q_{br}(Q_{-j}^h) > 0$ and $q_i^l = q_{br}(Q_{-i}^h) > 0$, we need, respectively:

$$\begin{aligned} P(q_j^l + Q_{-j}^h) + q_j^l P'(q_j^l + Q_{-j}^h) &= C'(q_j^l) \\ P(q_i^l + Q_{-i}^h) + q_i^l P'(q_i^l + Q_{-i}^h) &= C'(q_i^l) \end{aligned}$$

Since $q_i^l < q_j^l$, this requires $q_j^l + Q_{-j}^h < q_i^l + Q_{-i}^h \Leftrightarrow q_j^l + q_i^h < q_i^l + q_j^h$, which contradicts the opposite inequality, necessary (as established above) to have $q_i^h < q_j^h$, $q_j^h = q_{br}(Q_{-j}^l) > 0$ and $q_i^h = q_{br}(Q_{-i}^l) > 0$. Thus, S' must be symmetric, which completes the proof. ■

Proof of Proposition 3.3. We have $q^{\max} = D\left(C'\left(0\right)\right) / \left(n-1\right)$, so that $q_{br}\left(Q\right) = 0$ if and only if $Q \geq \left(n-1\right) q^{\max}$. By assumption, $q^m \geq q^{\max}$, and we must also have $0 \leq q^{\min} < q^{\max}$ due to $q_{br}\left(\cdot\right)$ being decreasing.
Any stable convention S' must satisfy condition (2) of Proposition 3.2,which can only happen if for some q^l, q^h we have:

$$0 \leq q^l = q_{br}\left(\left(n-1\right) q^h\right) < q^h = q_{br}\left(\left(n-1\right) q^l\right) \leq q^m$$

Observe first that we have $q^l = 0$ if and only if $q^h = q^m$, since $q_{br}\left(\left(n-1\right) q^m\right) = 0$ and $q_{br}\left(\left(n-1\right) 0\right) = q^m$. Furthermore, any convention that satisfies condition (1) of Proposition 3.2 given $\left\{q^l, q^h\right\} = \{0, q^m\}$ is equivalent to the Cournot-Equivalent convention (7) of Proposition 3.1. We will demonstrate that under condition (11) no other convention satisfies the requirements of Proposition 3.2, specifically, that there exist no q^l, q^h such that we have:

$$0 < q^l = q_{br}\left(\left(n-1\right) q^h\right) < q^h = q_{br}\left(\left(n-1\right) q^l\right) < q^m \tag{28}$$

Suppose the contrary, i.e. that such q^l, q^h do exist, and observe that it must then be that $q^h < q^{\max}$ (or else we could not have $q^l = q_{br}((n-1)\, q^h) > 0$) and $q^l > q^{\min}$ (or else we could not have $q^h = q_{br}((n-1)\, q^l) < q^{\max}$). Consequently, $\left\{q^l, q^h\right\}$ must lay at the intersection of two functions: $q_1^h(q^l) = q_{br}\left(\left(n-1\right) q^l\right)$ and $q_2^h(q^l)$, which assigns to every $q^l \in \left(q^{\min}, q^{\max}\right)$ the value of q^h that solves $q^l = q_{br}\left(\left(n-1\right) q^h\right)$. Using implicit function theorem, we have:

$$\left(q_1^h(q^l) - q_2^h(q^l)\right)' = \left(n-1\right) q_{br}'((n-1)\, q^l) - 1/\left[\left(n-1\right) q_{br}'((n-1)\, q^h)\right]$$

Since $n \geq 2$ and $q_{br}'\left(\left(n-1\right) y\right) < 0$ for $y \in \left(q^{\min}, q^{\max}\right)$, it follows that:

$$\left(q_1^h(q^l) - q_2^h(q^l)\right)' < 0 \Leftrightarrow q_{br}'((n-1)\, q^l) q_{br}'((n-1)\, q^h) > 1/[(n-1)^2] \tag{29}$$

As $q^l, q^h \in \left(q^{\min}, q^{\max}\right)$, (29) is implied by condition (11) stated in Proposition 3.3. Hence, $q_1^h(q^l)$ and $q_2^h(q^l)$ intersect at most once for $q^l \in (q^{\min}, q^{\max})$.
But consider a solution $\{q^l, q^h\}$ to:

$$q_{br}\left(\left(n-1\right) q^l\right) = q^l = q^h$$

Since $q_{br}\left(\left(n-1\right) q^{\max}\right) = 0$, $q_{br}\left(\left(n-1\right) q^{\min}\right) = q^{\max}$, the solution $\{q^l, q^h\}$ must satisfy $q^l = q^h \in \left(q^{\min}, q^{\max}\right)$, as well as $q^l = q_{br}\left(\left(n-1\right) q^h\right)$, i.e. it must constitute the unique intersection of $q_1^h(q^l)$ and $q_2^h(q^l)$, as detailed above. However, $q^l = q^h$ means that requirement (28) does not hold, i.e. no $\left\{q^l, q^h\right\}$ to satisfy (28) may exist given (11). ■

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