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Equilibrium Analysis and Experimental  
Evidence**

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# Political motivations and electoral competition: Equilibrium analysis and experimental evidence\*

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## Abstract

We study both theoretically and experimentally the complete set of Nash equilibria of a classical one-dimensional, majority rule election game with two candidates, who might be interested in power as well as in ideology, but not necessarily in the same way. Apart from obtaining the well known median voter result and the two-sided policy differentiation outcome, the paper uncovers the existence of two new equilibrium configurations, called ‘one-sided’ and ‘probabilistic’ policy differentiation, respectively. Our analysis shows how these equilibrium configurations depend on the relative interests in power (resp., ideology) and the uncertainty about voters’ preferences. The theoretical predictions are supported by the data collected from a series of laboratory experiments, as we observe convergence to the Nash equilibrium values at the aggregate as well as the individual levels in all treatments, and the comparative statics effects across treatments are as predicted by the theory.

*JEL Classification:* C72, C90, D72.

*Keywords:* Electoral competition, Power, Ideology, Uncertainty, Nash equilibrium, Experimental evidence.

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# 1 Introduction

The *spatial* theory of electoral competition begins with the seminal contributions of Hotelling (1929) and Downs (1957). The basic model considers a majority rule election where two political candidates compete for office by simultaneously and independently proposing a platform (e.g., an income tax rate) from a unidimensional policy space. The predictions of this model with respect to the policy platforms that could emerge in equilibrium depend crucially on candidates' motivations for running for office.

In this respect, the existing literature has almost always focused on a simple scenario where candidates possess *identical* electoral motivations. Moreover, it has frequently assumed as well that candidates are *single-minded*, in the sense that they are solely concerned about either winning the election and being in power or, alternatively, about the ideological position of the winning policy. We argue in this paper that, as a result of these simplifications, the theory has overlooked other interesting equilibrium predictions of the model, which we also show can be empirically supported with experimental data.

To be clear, the traditional assumptions about candidates' motivations do indeed provide significant insights on the possible outcomes of the electoral processes. Starting with the famous median voter result, the theory shows that if both candidates are *purely opportunistic*, in the sense that they choose their platforms with the sole purpose of maximizing the probability of winning the election, then so long as they share a common prior about the location of the median voter's preferred policy, the equilibrium platforms always converge to the center of the policy space; specifically, they coincide with the *estimated* median ideal point (Hotelling 1929; Downs 1957; Calvert 1985).<sup>1</sup>

By contrast, in the opposite case where both candidates are *purely ideological* and they select their platforms to minimize the utility loss generated by the distance between the location of the winning policy and their respective preferred positions or *ideologies*, the equilibrium platforms converge to the median ideal point only if the median voter's location is known with certainty (Calvert 1985; Roemer 1994). Otherwise, the election still has an equilibrium in pure strategies, but the corresponding platforms lie down on opposite sides of the estimated median position. (Roemer 1997).<sup>2</sup>

In this paper, we reconsider the equilibrium analysis of electoral competition by relaxing the previous hypotheses on candidates' motivations. To be more precise, we study a basic election game in which candidates are allowed to be concerned not only about

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<sup>1</sup>When candidates receive private signals about voters' preferences before choosing their platforms, as happens for example in the presence of private polling, the equilibrium is different, but policy platforms are still centrally located. See Bernhardt, Duggan and Squintani (2007, 2009a) for further details.

<sup>2</sup>Morton (1993) illustrates this type of two-sided policy differentiation equilibrium offering experimental evidence that suggests that uncertainty over voters' preferences is a major determinant of platform divergence when candidates are ideological.

winning the election, but also about the policy implemented afterwards, and not necessarily in the same way. This assumption, sometimes called *mixed* or *hybrid motivations*, was first suggested by Calvert (1985), and it has been recently used in a number of papers, such as Ball (1999), Groseclose (2001), Aragonés and Palfrey (2005), Duggan and Fey (2005), Saporiti (2008), Callander (2008), and Bernhardt, Duggan and Squintani (2009b). To the best of our knowledge, however, a full characterization of the set of Nash equilibria in the basic model is missing in the literature. The main goal here is precisely to fill that gap, and to test the theoretical predictions experimentally.

The main results obtained in this paper can be summarized as follows. When the value of being in office is the *same* for the two candidates, we find that both players announce either (i) a platform located on the estimated median ideal point (policy convergence) if the electoral uncertainty is *low* compared with the interest in office, or (ii) a platform located on their own ideological side (two-sided policy differentiation) if the uncertainty is *high*.<sup>3</sup> When, instead, candidates have *asymmetric* motivations, we still get that equilibrium platforms converge to the estimated median voter's ideal point for low levels of uncertainty. However, when the uncertainty increases (as the length of the interval over which the median is distributed increases), an equilibrium in pure strategies fails to exist. In this region, both candidates randomize optimally on one side of the median to avoid being copied and undercut by their rival (probabilistic differentiation). As the electoral uncertainty continues to increase, a pure strategy equilibrium is eventually reestablished and the two candidates assign all of the probability mass to a different platform. These are located initially on the same ideological side (one-sided policy differentiation), and then, as uncertainty further increases, on each candidate's own political ground (two-sided differentiation).

These theoretical results are supported by the experimental data we collect from a series of laboratory treatments. Firstly, we find in all treatments that the median behavior of the left- and the right-wing subjects converge to the Nash equilibrium values. This happens even in the probabilistic differentiation treatment, with a unique mixed strategy equilibrium (MSE). In that treatment, we observe that not only subjects' choices approximate the bounds and the median of the MSE support, but also that the empirical cumulative distributions are close to the theoretical ones, with the cumulative distribution of the left-wing players first-order stochastically dominating the distribution of the right-wing players.

Secondly, in the symmetric motivations treatments, we note that the 95% confidence intervals we construct around the medians shrink over time as well, indicating behavior

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<sup>3</sup>In this paper, candidates' preferred policies are assumed to be distributed on either side of the median ideal point, so that the ideology of one candidate lies on the left and the other on the right.

that is consistent with the Nash equilibrium not only at the aggregate level but also at the individual level. In the asymmetric treatments, with one-sided policy differentiation in either pure or mixed strategies, some noise in the individual choices persists even after sixty rounds (elections) of play. However, this is consistently skewed to the center of the policy space, and it diminishes over time.

Thirdly, we find that subjects' learning takes place mainly within the first ten periods (elections), and that most of that learning does not vanish as subjects interchange their roles between candidates of different ideologies. Finally, in line with the theory, the comparative statics analysis across treatments confirm the theoretical predictions that (i) an increase in the electoral uncertainty leads to an increase in policy divergence; (ii) policy convergence is reestablished as both candidates become more office-motivated; (iii) the extent of the empirical policy differentiation on either side of the median is independent of candidates' ideologies; and (iv) an asymmetric increase in candidates' interests in power leads to policy differentiation on one side of the median.

To conclude this section, we briefly discuss the main motivations for carrying out this research. First, from a conceptual point of view, the mixed motivations hypothesis is unquestionably more realistic than the alternative hypotheses mentioned before. In a modern democracy, it probably emerges naturally from the fact that candidates are usually representatives of complex political organizations. To elaborate, in real world politics to actually reach the stage of being in competition for public office, citizens must first be nominated candidates within the political parties; and for that to happen they need the support of regular party members, who are arguably much more concerned about the policies implemented after the election than about the actual winner of the contest. Thus, although politicians as other professionals might be more interested in their careers and, therefore, in winning the elections, it seems reasonable to expect that policy considerations will also enter into the candidate's payoff function with some weight.<sup>4</sup>

Obviously, these weights need not be the same for all candidates. They could depend, for instance, on the specific features of the political organization that the candidate represents, such as the number of regular members, the level of activism within the organization, the internal process to nominate candidates, etc. The value of winning the election might also vary depending on whether the party of the candidate is the incumbent in office or a challenger. In any case, the point to stress is that, as some casual evidence seems to point out, asymmetric electoral motivations might appear in reality quite frequently as well.<sup>5</sup>

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<sup>4</sup>Morton (1993), for instance, reports that experimental subjects place in the laboratory a weight of approximately 32 per cent on winning the election, and 68 per cent on the expected utility derived from the position of the implemented platform.

<sup>5</sup>An interesting case in that regard is the Radical and the Peronist Parties in Argentina, which have

Second, from a theoretical point of view, the mixed motivations hypothesis has been shown to have important implications for the predictive power of the theory of electoral competition. In effect, Ball (1999) finds out that, due to the discontinuities of the payoff functions created by the mixed motivations, the electoral contest with hybrid motives does not always possess a Nash equilibrium in pure strategies. This problem is further examined in Saporiti (2008), which shows that the blame for the instability can be entirely attributed to candidates' asymmetric motives.<sup>6</sup> Obviously, all this stands in sharp contrast with what happens in the extreme cases, where a pure strategy equilibrium always exists. Thus, it shows that the mixed motivations assumption is not vacuous. Apart from offering a more realistic description of electoral competition, the hybrid case provides a deeper understanding of it, uncovering features of the electoral process that cannot be appreciated in the extreme scenarios.

Finally, from an empirical point of view, this paper adds to the experimental literature on electoral competition, which has traditionally focused on testing platform convergence in the classical Downsian model with office-motivated candidates, ignoring the testable implications of other configurations of equilibrium platforms that emerge under the alternative hypotheses of candidates' motivations.<sup>7</sup> To the best of our knowledge, the only attempt in the literature to assess experimentally the implications of the ideological motivations was carried out by Morton (1993). But even in that article the analysis is confined to the symmetric and single-minded scenario. Our paper in this sense is the first one to fully examine in the lab the whole set of Nash equilibria, studying in a rich set of treatments not only convergence of subjects' behavior to the theoretical predictions, but also learning and a number of comparative statics effects resulting from changing the relative interests in power (resp., ideology) and the information about voters' preferences.

The rest of the paper is organized as follows. After briefly reviewing in Section 2 a number of papers closely related to our work, Section 3 presents the theoretical model. In Section 4 we derive the theoretical results, which are proved in Appendix A. Section 5 describes the experimental design. Section 6 displays the experimental evidence and discusses the empirical findings, which are classified in equilibrium convergence (Section 6.1), learning (Section 6.2), and comparison between treatments (Section 6.3). Additional data is provided in Appendix B. The paper ends in Section 7 with some final remarks.

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been a major inspiration for this paper. These two parties are the main political actors of the country. The Radical Party has been ever since its creation a strongly ideological party, whereas the Peronism has been a "movement," as Perón used to call it, basically motivated by being in power. Another example seems to be the Labour and the Conservative Parties in the UK during the years of Tony Blair.

<sup>6</sup>To be precise, the paper shows that in the mixed but *symmetric* motivation case, an equilibrium in pure strategies not only exists, but it is also unique.

<sup>7</sup>See McKelvey and Ordeshook (1990) for an overview of the early findings, and Schram (2002) and Palfrey (2005) for a more recent update.

## 2 Related literature

The literature on electoral competition is vast. We focus here only on those papers that are most relevant for our work. For a more comprehensive review, the suggested references are Osborne (1995), Roemer (2001) and Austen-Smith and Banks (2005).

On the theoretical front, this paper relates to two segments of the existing literature that deal with, respectively, elections with mixed motivations, and elections with advantaged candidates. In the first segment, we said already that the first reference is Calvert (1985), though he does not go beyond offering a continuity result according to which *small* departures from office motivation and certainty lead to only *small* departures from policy convergence. Ball (1999), Saporiti (2008), and Bernhardt et al. (2009b) further examine the implication of the assumption. The first two papers focus on equilibrium existence rather than the nature of the equilibrium policies, whereas the latter analyzes mainly the implication of symmetric mixed motivations on voters' welfare.

In addition to these articles, there is a large number of papers that adopt the mixed motivations assumption and simultaneously add other features to the basic framework. To mention a few, Aragonés and Palfrey (2005) study a general incomplete information model of candidate quality allowing for heterogeneity in valence, ideology, and motivations. Calender (2008) considers a model with either policy or office motivated candidates, private information about candidates' types, and partial commitment at the electoral stage. In a more significant departure, Roemer (1999) analyzes a model where parties represent different constituencies, or economic classes, with well defined policy preferences. Parties are also integrated by opportunistic individuals who desire only to win office. Roemer assumes that each party must reach inner-party unanimity to formulate a proposal, and he proves the existence of a so called *party unanimity Nash equilibrium*. Finally, Snyder and Ting (2002) model political parties as informative brands to voters, in a setup where candidates are driven by achieving office and, if elected, policy, and they need parties to credibly signal their true policy preferences.

Insofar as a relatively more office-motivated candidate has in equilibrium a higher probability of winning the election, this paper is also connected with the literature on elections with advantaged candidates. As Peress (2010) reckons, the theoretical literature on two candidate electoral competition is dominated by symmetric contests. However, starting with Ansolabehere and Snyder (2000), Groseclose (2001), and Aragonés and Palfrey (2002), there is now a sizeable literature that analyzes candidates' behavior in the presence of valence advantage. This includes the previous four articles plus several recent papers, such as Kartik and McAfee (2007), Ashworth and Bueno de Mesquita (2009), Iaryczower and Mattozzi (2009), and Hummel (2010), among others. An inter-

esting feature in some of these works is that, as happens in our case, equilibria in mixed strategies emerge because the advantaged candidate is willing to copy the position of the disadvantaged one, forcing the latter to randomize in order to not be predictable.

On the empirical front, our paper relates to the experimental literature in political economy that analyzes elections and candidate competition. First, there is a number of early laboratory tests, surveyed by McKelvey and Ordeshook (1990), that analyze the hypothesis of policy convergence to the median ideal point in the Downsian framework with purely office-motivated candidates. The stylized fact emerging from these studies is that convergence to the median voter occurs irrespective of the level of information (complete or incomplete) on ideal points and payoffs.

This early research has been later complemented by Morton (1993), who conducts a laboratory experiment to assess the hypothesis that platforms diverge when candidates are purely ideological and there is uncertainty about voters' preferences. She finds significant divergence in candidate positions, but less than the theory predicts, suggesting that subjects might enjoy non-monetary benefits from winning the election. More recently, Aragonés and Palfrey (2004) report experimental results about the effects of valence asymmetries on the location of the equilibrium policies. In line with the theory, the experimental evidence demonstrate that (i) candidates diverge from the center, with the weaker candidate diverging more than the stronger candidate; and (ii) as the distribution of voters becomes more spread out, both candidates go back to the center.

Finally, to the extent that some of the equilibria in the asymmetric motivation case are in mixed strategies, this paper also adds to the experimental economics literature that looks at how individuals behave in games with mixed strategy equilibria. Camerer (2003, ch. 3) provides an overview of the most relevant papers, with the main message being that although aggregate behavior is usually close to the equilibrium predictions, there are still significant deviations from them.

### 3 The Model

Two political candidates, indexed by  $i = L, R$ , compete in a winner-take-all election by simultaneously and independently announcing a platform  $x_i \in X = [0, 1]$ . The electorate is made up of a continuum of voters. Each voter is endowed with a preferred policy or *ideal point*  $\theta \in X$ , and with a preference relation over  $X$  represented by the utility (loss) function  $u_\theta(x) = -|x - \theta|$ , where  $|\cdot|$  denotes the absolute value on  $\mathbb{R}$ .

Due to the nature of voters' preferences (single-peaked and symmetric around  $\theta$ ), for every pair of proposals  $(x_L, x_R) \in X^2$  each voter votes sincerely for the platform closer to its ideal point, voting for the alternatives with equal probabilities when indifferent.



Given that there are only two candidates in this model and that each candidate enacts its proposed policy once elected, the assumption of sincere voting doesn't entail a significant loss of generality, because voting for the candidate whose position turns out to be the most preferred one is a weakly dominant strategy for each voter.

As is usual in the literature, candidate  $i$  wins the election if its platform  $x_i$  gets more than half of the votes, with ties broken by a fair coin toss. Apart from this uncertainty due to the possibility of a tie, candidates also have uncertainty about voters' preferences. We assume that the median voter's ideal point, denoted by  $\theta_m$ , is uniformly distributed over  $[1/2 - \beta, 1/2 + \beta]$ , with  $\beta > 0$ . This may be due to the fact that either (i) voters' preferences are fixed, but candidates perceive the fraction of types supporting their respective platforms with some noise, as happens for example in Roemer (2001, p. 45); or because (ii) voters' preferences actually change after candidates have announced their platforms, as is the case in Bernhardt, Duggan, and Squintani (2009b).

Regardless of the interpretation given to the model of electoral uncertainty, it transpires from our assumptions that the probability that candidate  $L$  attaches to winning the election is given by  $p(x_L, x_R) = \text{Prob}(\theta_m \in [0, \frac{x_L+x_R}{2}])$  if  $x_L \leq x_R$ , and by  $p(x_L, x_R) = \text{Prob}(\theta_m \in [\frac{x_L+x_R}{2}, 1])$  if  $x_L > x_R$ . Candidate  $R$ 's probability of winning is obviously  $1 - p(x_L, x_R)$ . Fig. 1 displays the domain of  $p(\cdot)$  and the different expressions of this function depending on the region of the strategy space  $X^2$  under consideration.

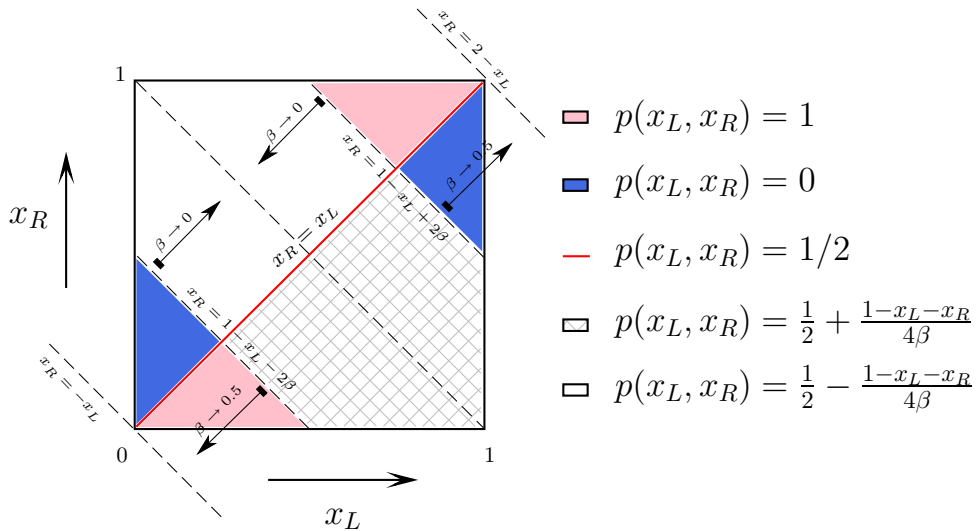


Figure 1: Probability of winning function for candidate  $L$ .

**Lemma 1** For any two platforms  $x_L < x_R$  (resp.,  $x_L > x_R$ ),  $p(x_L, x_R)$  is non-decreasing (resp., non-increasing) in  $x_i$ , for all  $i = L, R$ .

Lemma 1, whose proof follows immediately from the definition of  $p(\cdot)$  and is therefore omitted, reflects the spatial nature of electoral competition. Roughly speaking, it ensures that if one candidate moves its platform toward that of its opponent, then it does not decrease (and may increase) the probability with which it wins the election. Likewise, if it moves its platform away from its opponent's, then it does not increase (and may decrease) its probability of winning. Similarly, if a candidate's opponent moves toward (away from) the candidate's own platform, then the probability of winning does not increase (decrease). We invoke this result several times in the proofs.

As we said in the Introduction, candidates possess mixed or hybrid motives for running for office. That means that they are *office-motivated*, in the sense that they intrinsically value winning the election, and at the same time they are *policy-motivated* too, because they care about what policy is enacted after the election. Formally, the payoffs for candidate  $L$  and candidate  $R$  associated to any pair  $(x_L, x_R) \in X^2$  are given by, respectively,

$$\Pi_L(x_L, x_R) = p(x_L, x_R) \cdot (u_{\theta_L}(x_L) + \chi_L) + [1 - p(x_L, x_R)] \cdot u_{\theta_L}(x_R), \quad (1)$$

and

$$\Pi_R(x_L, x_R) = [1 - p(x_L, x_R)] \cdot (u_{\theta_R}(x_R) + \chi_R) + p(x_L, x_R) \cdot u_{\theta_R}(x_L), \quad (2)$$

where  $\theta_i$  stands for candidate  $i$ 's ideological (preferred) position on  $X$ , and  $\chi_i > 0$  denotes candidate  $i$ 's payoff for being in power (office rents).<sup>8</sup> Note that Hotelling (1929)-Downs' (1957) office motivation hypothesis, according to which candidates maximize the probability of winning the election, is obtained from the previous specification of the payoffs by letting  $\chi_i$  be arbitrarily large for all  $i$ . Likewise, Wittman's (1983) entirely ideological candidates follow from the same particular by setting out the rents  $\chi_i$  of both candidates equal to zero.

In this paper, we assume that candidates' ideological positions are distributed on either side of the (expected) median voter's ideal policy, i.e.,  $\theta_L < 1/2 < \theta_R$ ; and we identify the half-open interval  $[0, 1/2)$  (resp.,  $(1/2, 1]$ ) with the left-wing (resp., right-wing) candidate's ideological side. In addition, to rule out uninteresting equilibria with large electoral uncertainty and no trade-off between power and ideology, the essence of this investigation, we assume that  $\beta < \bar{\beta} \equiv \min\{1/2 - \theta_L + \chi_L/2, \theta_R - 1/2 + \chi_R/2\}$ . If that were not the case, then in an equilibrium with differentiated policies at least one candidate would maximize its payoff at its preferred location  $\theta_i$ , independently of the position chosen by the other.

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<sup>8</sup>Using the specification of the mixed motivations found in Calvert (1985, p. 83) and Aragonés and Palfrey (2005, p. 98), Saporiti (2008, p. 834) shows that the relative value of winning the election can be defined as  $\chi_i = \frac{\lambda_i}{1-\lambda_i}$ , where  $\lambda_i \in (0, 1)$  is candidate  $i$ 's weight on the probability of winning the election, and  $1 - \lambda_i$  represents its weight on the expected utility for the pair of policies  $(x_L, x_R)$ .

Let  $\Delta$  be the space of probability measures on the Borel subsets of  $X$ . A mixed strategy for  $i$  is a probability measure  $\mu_i \in \Delta$ , with support  $\text{supp}(\mu_i) \equiv \{x \in X : \forall \epsilon > 0, \mu_i((x - \epsilon, x + \epsilon) \cap X) > 0\}$ . We extend each  $\Pi_i$  to  $\Delta^2$  by  $U_i(\mu_L, \mu_R) = \int_{X^2} \Pi_i(x_L, x_R) d(\mu_L(x_L) \times \mu_R(x_R))$ . Note that  $U_i$  is well defined because the set of discontinuities of  $\Pi_i$ , namely  $\{(x_L, x_R) \in X^2 : x_L = x_R \neq 1/2\}$ , has measure zero.

Let  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  denote a *mixed motivation election game*, and let  $\bar{\mathcal{G}} = (\Delta, U_i)_{i=L,R}$  be the mixed extension of  $\mathcal{G}$ . A Nash equilibrium of  $\bar{\mathcal{G}}$  is a pair of probability measures  $(\mu_L^*, \mu_R^*) \in \Delta^2$  such that for all  $(x_L, x_R) \in X^2$ ,  $U_L(\mu_L^*, \mu_R^*) \geq U_L(x_L, \mu_R^*)$  and  $U_R(\mu_L^*, \mu_R^*) \geq U_R(\mu_L^*, x_R)$ . We say that a Nash equilibrium  $(\mu_L^*, \mu_R^*) \in \Delta^2$  is a mixed strategy equilibrium (MSE) of  $\mathcal{G}$  if at least one candidate randomizes over two or more policies. Otherwise, if for all  $i = L, R$ ,  $\text{supp}(\mu_i^*) = \{x_i^*\}$  for some  $x_i^* \in X$ , then the profile  $(x_L^*, x_R^*)$  represents a pure strategy equilibrium (PSE) of  $\mathcal{G}$ .<sup>9</sup>

## 4 Equilibrium Analysis

We begin the equilibrium analysis noting that  $\mathcal{G}$  possesses neither a PSE where the left-wing candidate chooses a platform further to the right than the right-wing candidate's proposal, nor a PSE where one of the candidates wins the election for sure.

**Lemma 2** *If the strategy profile  $(x_L^*, x_R^*) \in X^2$  is a pure strategy equilibrium for the election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$ , then  $\theta_L < x_L^* \leq x_R^* < \theta_R$  and  $p(x_L^*, x_R^*) \in (0, 1)$ .*

The previous lemma, whose proof (as well as all other proofs of this section) is given in Appendix A, allows to focus the equilibrium analysis on the white and the red regions of the domain of  $p(\cdot)$  displayed in Fig. 1. In particular, it is used below to characterize each candidate's platform in a PSE with policy differentiation, and to provide a necessary condition for such an equilibrium to exist.

**Lemma 3** *The election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  has a pure strategy equilibrium with  $x_L^* < x_R^*$  only if  $\chi_L + \chi_R < 4\beta$ ,  $x_L^* = 1/2 - \beta + \chi_L/2$ , and  $x_R^* = 1/2 + \beta - \chi_R/2$ .*

The platforms characterized in Lemma 3 are a function of the electoral uncertainty  $\beta$  and the office rents  $\chi_i$ , and with the expected sign. All the rest equal, as the candidates become less certain about how moderate the median voter is (higher  $\beta$ ), they also become more polarized in their platform choice. On the contrary, a reduction of the uncertainty (resp., an increase in the office rents) moves both platforms towards the center of the political space.

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<sup>9</sup>When  $\mu \in \Delta$  assigns probability 1 to a single policy  $x \in X$ , we simply write  $x$  instead of  $\mu$ .

These platforms are obtained from the first-order conditions. That is, they are the stationary points of the conditional payoff functions. Unfortunately, Lemma 3 does not guarantee that these functions are quasi-concave. A case in point takes place when  $\chi_R = 0.2$ ,  $\chi_L = 0.6$ ,  $\beta = 0.25$ ,  $\theta_L = 0.2$ , and  $\theta_R = 0.9$ . For these values of the parameters, the conditional payoffs associated to the policy profile of Lemma 3, namely,  $(x_L^*, x_R^*) = (0.55, 0.65)$ , are illustrated in Figs. 2 and 3. Clearly, this profile is not a PSE, since  $x_R^* = 0.65$  does not maximize  $\Pi_R(0.55, x_R)$  over  $x_R \in [0, 1]$ . A bit of extra work shows that any other profile of pure strategies fails to be an equilibrium too.

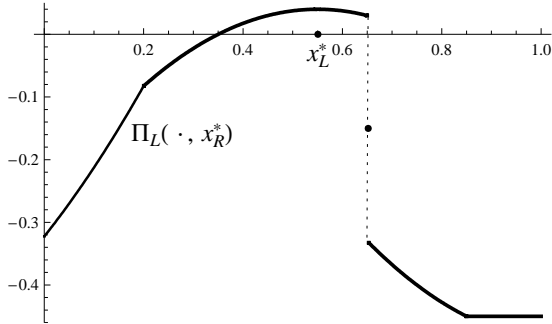


Figure 2: Left-wing candidate's conditional payoff function given  $x_R^* = 0.65$ .

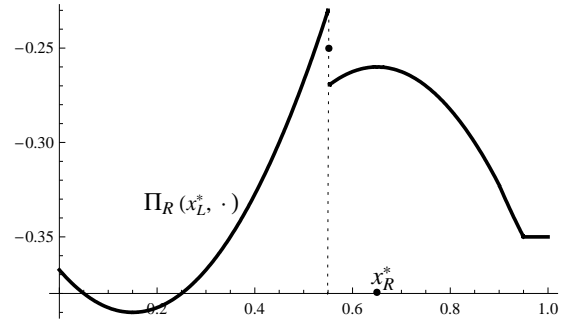


Figure 3: Right-wing candidate's conditional payoff function given  $x_L^* = 0.55$ .

Therefore, a sensible question to ask is what conditions prevent this from happening. The next three propositions are meant to shed some light into this inquiry. The first one offers necessary and sufficient conditions for policy convergence (i.e., equilibrium with identical platforms), which is the classical result of electoral competition.

**Proposition 1 (convergence)** *The election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  has a pure strategy equilibrium with  $x_L^* = x_R^* \equiv x^*$  if and only if  $x^* = 1/2$  and  $\chi_i \geq 2\beta$  for all  $i = L, R$ .*

The statement of Proposition 1 bears some similarity with Calvert's (1985) assertion that *small* departures from "office motivation and certainty" lead to only *small* departures from convergence. In line with that prediction, Proposition 1 asserts that both candidates will choose in equilibrium a platform located on the expected median ideal point if and only if the relative value of holding office  $\chi_i/2\beta$  is high enough for all  $i$ .

One way of interpreting this condition is as follows. In this paper the winner enjoys an extra payoff for being elected equal to  $\chi_i$ . From the candidates' viewpoint, however, hitting the median ideal point with a particular policy platform and actually winning the election has a chance of  $(2\beta)^{-1}$  (the inverse of the length of the support of  $\theta_m$ ). Therefore, the term  $\chi_i/2\beta$  can be viewed as the expected benefit for moving the platform one additional unit to the center. The cost of doing that is of course the additional unit of disutility created by the displacement towards the center and away from the candidate's

ideology. Thus, when  $\chi_i$  is large enough for all  $i$  (resp.,  $\beta$  is small enough), in the sense that  $\chi_i/2\beta \geq 1$ , the benefits of any such deviation to the center outweigh the costs and, consequently, candidates converge to the median voter's preferred policy.

An immediate implication of Proposition 1 and Lemma 3 is the following corollary.

**Corollary 1 (uniqueness)** *If the election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  possesses a pure strategy equilibrium, then the equilibrium is unique.*

The uniqueness result expressed in Corollary 1 is to some extent more general than the related results found in Saporiti (2008) and Bernhardt et al. (2009b), because the latter only refer to the homogeneous motivation case ( $\chi_L = \chi_R$ ), whereas the former also applies to cases where  $\chi_L$  is not necessarily equal to  $\chi_R$ . It is worth reminding, however, that the three models are different and, therefore, that the results are not directly comparable.

Our next proposition provides a necessary and sufficient condition for another well known configuration of equilibrium platforms (suggested first by Wittman (1983), and proved later by Roemer (1997)), where each candidate chooses a policy on its own ideological side.

**Proposition 2 (two-sided differentiation)** *The election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  has a pure strategy equilibrium with  $x_L^* < 1/2 < x_R^*$  if and only if  $\chi_i < 2\beta$  for all  $i = L, R$ .*

Thus, combining Propositions 1 and 2, the first conclusion that can be drawn here is that, when candidates possess identical motivations, these two results offer a full description of the equilibrium outcomes of the mixed motivation election game. To illustrate this, Fig. 4 displays the equilibrium platforms as a function of the electoral uncertainty  $\beta$ , and for a particular level of office rents  $\chi \equiv \chi_L = \chi_R$ .

As Proposition 1 points out, both policies are located at the estimated median voter's ideal point for any level of uncertainty lower than or equal to  $\chi/2$ . Above that threshold, Lemma 3 and Proposition 2 indicate that the equilibrium platforms lie down on each candidate's ideological ground, in accordance with the expressions  $x_L^* = 1/2 - \beta + \chi_L/2$  and  $x_R^* = 1/2 + \beta - \chi_R/2$ . That gives rise to a region of two-sided policy differentiation as is shown in the graph. The symmetric location of the policies about the median also implies that, in the identical motivation case, the probability of winning is the same for the two candidates.

Interestingly, when candidates hold asymmetric interests, Propositions 1 and 2 do not cover the whole spectrum of possibilities of the two-candidate electoral competition model. The main contribution of this paper is precisely to analyze what could happen in that case. To help the reader gain more insight about the equilibrium configurations that could arise in the asymmetric scenario, assume that  $\chi_L = 0.6$  and  $\chi_R = 0.05$ , and

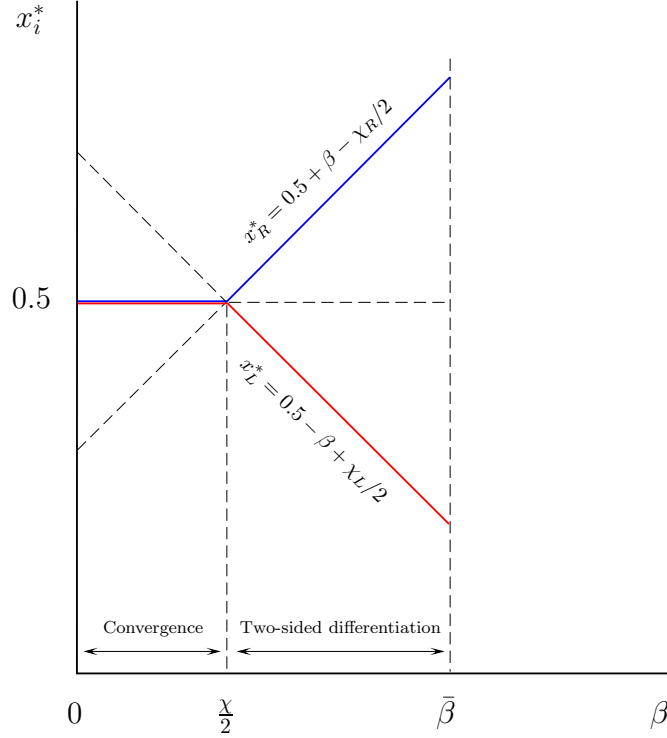


Figure 4: Symmetric case:  $\chi_L = \chi_R \equiv \chi$ .

suppose that  $\beta = 0.25$ ,  $\theta_L = 0.2$ , and  $\theta_R = 0.9$ . For these values of the parameters, Lemma 3 says that  $x_L^* = 0.55$  and  $x_R^* = 0.725$ . Figs. 5 and 6 confirm that these policies form in fact a pure strategy equilibrium.

Fig. 5 displays the left-wing candidate's conditional payoffs given  $x_R^* = 0.725$ . Likewise, Fig. 6 exhibits the right-wing candidate's payoffs given that the other candidate's chosen policy is  $x_L^* = 0.55$ . A simple inspection of the graphs shows that these platforms are best responses to each other, proving that  $(x_L^*, x_R^*) = (0.55, 0.725)$  is a PSE. This equilibrium is such that candidates locate on a different platform, but these platforms are on the same side of the median voter's ideal point (i.e.,  $1/2 < x_L^* < x_R^*$ ). In particular, being candidate  $L$  the most opportunistic of the two candidates,  $L$ 's proposal lies down on the other candidate's ideological ground. We refer to this kind of equilibria as pure strategy equilibria with *one-sided policy differentiation*.

The next proposition provides necessary and sufficient conditions for that equilibrium to occur. In words, when the right-wing candidate turns out to be the relatively more policy-concerned candidate, the conditions we state below require basically that level of uncertainty be (i) sufficiently low to ensure that  $L$ 's stationary point is still above  $1/2$ ; and (ii) high enough to discourage both players undercutting their equilibrium strategies, ensuring that  $\limsup_{x_R \rightarrow x_L^*} \Pi_R(x_L^*, x_R) \leq \Pi_R(x_L^*, x_R^*)$ , and ruling out cases like the example exhibited in Fig. 3. The interpretation of the conditions when the left-wing candidate

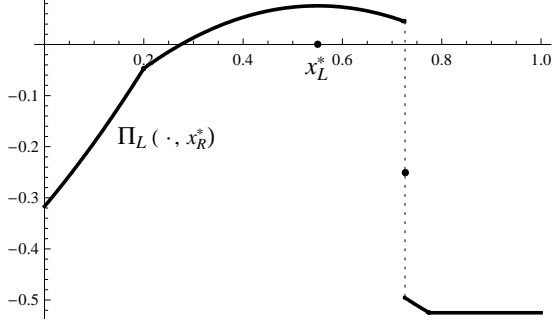


Figure 5: Left-wing candidate's conditional payoff function given  $x_R^* = 0.725$ .

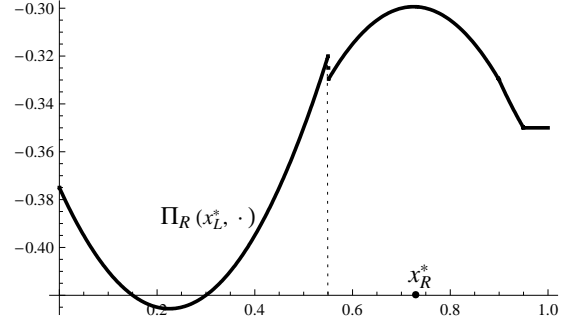


Figure 6: Right-wing candidate's conditional payoff function given  $x_L^* = 0.55$ .

is relatively more ideological is similar.

**Proposition 3 (one-sided differentiation)** *The election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  has a pure strategy equilibrium with  $1/2 < x_L^* < x_R^*$  (resp.,  $x_L^* < x_R^* < 1/2$ ) if and only if  $(\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2} \leq 2\beta < \chi_L$  (resp.,  $(\chi_R - \chi_L)/2 + (\chi_R \cdot \chi_L)^{1/2} \leq 2\beta < \chi_R$ ).*

Apart from the equilibrium with one-sided policy differentiation stated in the previous proposition, the asymmetric motivation model of electoral competition also admits mixed strategy equilibria. To analyze the properties of these equilibria, the following piece of notation is going to be helpful. First, denote the critical values of  $\beta$  stated in Proposition 3 by  $\beta_1^C \equiv \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_L \chi_R}}{2}$  and  $\beta_2^C \equiv \frac{\chi_R - \chi_L}{4} + \frac{\sqrt{\chi_L \chi_R}}{2}$ .

Second, consider the region of the strategy space where  $p(x_L, x_R) \in (0, 1)$ , as is shown in Fig.1. Within that region, for any  $x'_L < 1/2 + \beta - \chi_R/2 = x_R^*$  we have that<sup>10</sup>

$$\Pi_R(x'_L, x_R^*) = \frac{1}{4\beta} \left( \frac{1}{2} + \beta - x'_L + \frac{\chi_R}{2} \right)^2 + (x'_L - \theta_R),$$

and

$$\limsup_{x_R \rightarrow^- x'_L} \Pi_R(x'_L, x_R) = \left( \frac{1}{2} - \frac{1 - 2x'_L}{4\beta} \right) \chi_R + (x'_L - \theta_R).$$

Denote by  $\tilde{x}_L(\beta, \chi_R)$  the solution to  $\Pi_R(x'_L, x_R^*) - \limsup_{x_R \rightarrow^- x'_L} \Pi_R(x'_L, x_R) = 0$ .<sup>11</sup> The support of the mixed strategy equilibrium when the right-wing candidate is the relatively more ideological politician is characterized in the next proposition.

**Proposition 4 (probabilistic differentiation)** *If  $\chi_R/2 < \beta < \beta_1^C$ , the election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  has a mixed strategy equilibrium  $(\mu_L^*, \mu_R^*) \in \Delta^2$  with the property that,*

<sup>10</sup>As usual, “ $x \rightarrow^- y$ ” (resp., “ $x \rightarrow^+ y$ ”) indicates that  $x$  approaches  $y$  from the left (resp., right).

<sup>11</sup>Using the software Mathematica to solve this equation, it turns out that  $\tilde{x}_L(\beta, \chi_R) = 1/2 + \beta + 3/2\chi_R \pm \sqrt{2}\sqrt{2\beta\chi_R + \chi_R^2}$ .

- (a) If  $\beta \leq \frac{\chi_L + \chi_R}{4}$ , then  $\text{supp}(\mu_i^*) = [\underline{x}, \bar{x}]$  for all  $i = L, R$ , with  $\underline{x} = \tilde{x}_L(\beta, \chi_R)$  and  $\bar{x} = \frac{1}{2} + \beta - \frac{\chi_R}{2} = x_R^*$ ; and
- (b) If  $\beta > \frac{\chi_L + \chi_R}{4}$ , then  $\text{supp}(\mu_L^*) = [\underline{x}, \bar{x}]$  and  $\text{supp}(\mu_R^*) = [\underline{x}, \bar{x}] \cup \{x_R^*\}$ , with  $\underline{x} = \tilde{x}_L(\beta, \chi_R)$  and  $\bar{x} = \frac{1}{2} - \beta + \frac{\chi_L}{2} = x_L^*$ .

Obviously, an analogous characterization can be given for the case where the left-wing candidate is the relatively more ideological candidate. To do that, define  $\tilde{x}_R(\beta, \chi_L)$  as the solution to  $\Pi_L(x_L^*, x'_R) - \limsup_{x_L \rightarrow +x'_R} \Pi_L(x_L, x'_R) = 0$ . Then:

**Proposition 5 (probabilistic differentiation)** *If  $\chi_L/2 < \beta < \beta_2^C$ , the election game  $\mathcal{G} = (X, \Pi_i)_{i=L,R}$  has a mixed strategy equilibrium  $(\mu_L^*, \mu_R^*) \in \Delta^2$  with the property that,*

- (a) If  $\beta \leq \frac{\chi_L + \chi_R}{4}$ , then  $\text{supp}(\mu_i^*) = [\underline{x}, \bar{x}]$  for all  $i = L, R$ , with  $\underline{x} = \frac{1}{2} - \beta + \frac{\chi_L}{2} = x_L^*$  and  $\bar{x} = \tilde{x}_R(\beta, \chi_L)$ ; and
- (b) If  $\beta > \frac{\chi_L + \chi_R}{4}$ , then  $\text{supp}(\mu_R^*) = [\underline{x}, \bar{x}]$  and  $\text{supp}(\mu_L^*) = [\underline{x}, \bar{x}] \cup \{x_L^*\}$ , with  $\underline{x} = \frac{1}{2} + \beta - \frac{\chi_R}{2} = x_R^*$  and  $\bar{x} = \tilde{x}_R(\beta, \chi_L)$ .

As a matter of illustration, Tables 1 and 2 show the equilibrium distributions and the supports of the two types of MSE of a discrete version of the mixed motivation election game where both candidates choose their platforms from a grid of 101 locations, numbered from 0.00 to 1.00, and the parameters of the model adopt the following numerical values:  $\theta_L = 0.1$ ,  $\theta_R = 0.9$ ,  $\chi_L = 0.9$ ,  $\chi_R = 0.1$  and  $\beta = 0.15$  (resp.,  $\beta = 0.30$ ).<sup>12</sup>

Support	Left-wing candidate		Right-wing candidate	
	density	c.d.f.	density	c.d.f.
0.52	0.5529	0.5529	0.0919	0.0919
0.53	0.1048	0.6577	0.0117	0.1036
0.54	0.2295	0.8872	0.0409	0.1445
0.55	0.0000	0.8872	0.0000	0.1445
0.56	0.0887	0.9759	0.0225	0.1670
0.57	0.0000	0.9759	0.0000	0.1670
0.58	0.0229	0.9988	0.0117	0.1788
0.59	0.0012	1.0000	0.0000	0.1788
0.60	0.0000	1.0000	0.8212	1.0000

Table 1:  $\chi_L = 0.9$ ,  $\chi_R = 0.1$  and  $\beta = 0.15$ .

Support	Left-wing candidate		Right-wing candidate	
	density	c.d.f.	density	c.d.f.
0.59	0.2683	0.2683	0.0038	0.0038
0.60	0.1258	0.3941	0.0024	0.0062
0.61	0.1882	0.5823	0.0019	0.0081
0.62	0.0549	0.6372	0.0010	0.0091
0.63	0.1431	0.7803	0.0008	0.0098
0.64	0.2197	1.0000	0.0000	0.0098
0.65	0.0000	1.0000	0.0000	0.0098
0.75	0.0000	1.0000	0.9902	1.0000

Table 2:  $\chi_L = 0.9$ ,  $\chi_R = 0.1$  and  $\beta = 0.30$ .

To conclude this section, we plot in Fig. 7 the equilibrium platforms as a function of the electoral uncertainty for the case where candidates exhibit *asymmetric* motivations. As the graphs show, apart from a range of low and high levels of uncertainty, when

<sup>12</sup>The computations are carried out with the software GAMBIT (McKelvey, McLennan and Turocy 2010). Obviously, there are some differences between the (discrete) numerical results of Tables 1 and 2 and the (continuous) theoretical predictions of Proposition 4. However, these differences tend to vanish as the grid becomes finer.



candidates possess heterogeneous interests it is also possible to distinguish a range of moderate or intermediate levels that provides distinct equilibrium predictions. The three levels of electoral uncertainty are determined by the following ranges of values of  $\beta$ :

1. *low uncertainty*:  $0 \leq \beta \leq \min \left\{ \frac{\chi_L}{2}, \frac{\chi_R}{2} \right\}$ ;
2. *moderate uncertainty*:  $\min \left\{ \frac{\chi_L}{2}, \frac{\chi_R}{2} \right\} < \beta < \max \left\{ \frac{\chi_L}{2}, \frac{\chi_R}{2} \right\}$ ; and
3. *high uncertainty*:  $\max \left\{ \frac{\chi_L}{2}, \frac{\chi_R}{2} \right\} \leq \beta \leq \bar{\beta}$ .

As in the symmetric case, for low levels of uncertainty our model predicts that candidates converge to the estimated median voter's ideal point. However, as the length of the interval over which the median is distributed increases, there exists a range of intermediate levels of electoral uncertainty (namely, the values in Fig. 7a between  $\chi_R/2$  and  $\beta_1^C$ , and the values in Fig. 7b between  $\chi_L/2$  and  $\beta_2^C$ ) for which the mixed motivation election game fails to possess an equilibrium in pure strategies. Within that region, labeled in the graphs *probabilistic differentiation*, the game admits, as is shown in Saporiti (2008), an equilibrium in mixed strategies. Moreover, Prop. 4 states that the MSE support of both candidates is located on the same side of the median ideal point, as is illustrated by the grey areas in Figs. 7a and 7b.

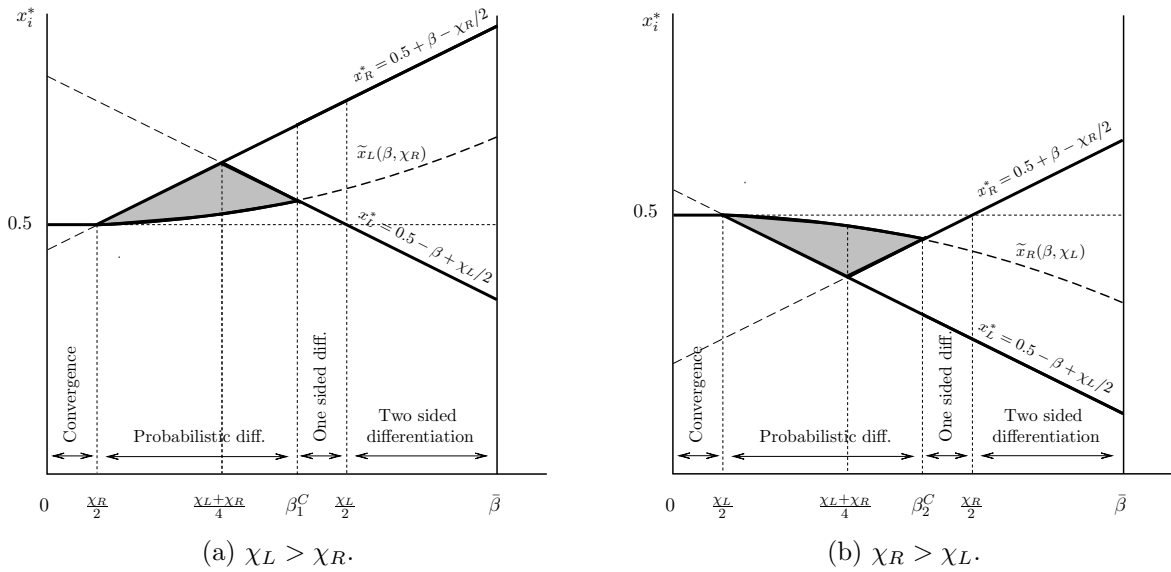


Figure 7: Asymmetric case.

As the electoral uncertainty continues increasing, it eventually surpasses either the critical threshold  $\beta_1^C$  if  $\chi_L > \chi_R$ , or the threshold  $\beta_2^C$  if  $\chi_R > \chi_L$ , and the existence of a pure strategy equilibrium is reestablished. For values of the uncertainty parameter above these thresholds and within the range of intermediate levels, Prop. 3 shows that a PSE not

only exists, but also that the corresponding equilibrium policies are placed on the same ideological ground, given rise to a region of *one-sided policy differentiation*. Afterwards, for high electoral uncertainty, the conditions of Prop. 2 hold, and each candidate chooses a policy on its own ideological side, mimicking the symmetric case.<sup>13</sup>

Finally, so long as PSE policies differ, the asymmetric motivation case predicts that the ideological (relatively more policy-concerned) candidate possess a lower probability of winning the election; or, to put it differently, that the opportunist (relatively more office-motivated) candidate enjoy an electoral advantage.

## 5 Experimental Design

In this section, we present a laboratory experiment designed to assess the theoretical predictions of the mixed motivation election game studied in Section 4. The experiment consisted of seven treatments, which were determined by varying the uncertainty parameter  $\beta$ , the ideologies  $\theta_i$  and the office rents  $\chi_i$ . For the convenience of the experimental subjects we considered only integer locations, numbered from 0 to 100, which required multiplying the relevant parameter values for  $\beta$ ,  $\theta$ , and  $\chi$  by 100. The values employed in each treatment, together with the corresponding equilibrium, are displayed in Table 3. The reader is referred to Table 1 for details of the MSE corresponding to Treatment 6.

Treatment	Uncertainty $\beta$	Ideology		Rents		NE Policy	
		$\theta_L$	$\theta_R$	$\chi_L$	$\chi_R$	$x_L^*$	$x_R^*$
1	2.5	10	90	10	10	50	50
2	15	10	90	10	10	40	60
3	15	10	90	40	40	50	50
4	15	34	66	10	10	40	60
5	35	10	90	10	10	20	80
6	15	10	90	90	10	MSE	
7	35	10	90	90	10	60	80

Table 3: Experimental treatments.

Subjects were told in the instructions a brief story of a town holding a two-candidate, majority rule election to select the location of a new post office on the high street. The subjects' task was to propose simultaneously and independently an integer number between 0 and 100 to locate the post office. They knew that voters were distributed uniformly across the 101 locations, and they were told that although each voter would vote for the proposal closer to its own location, for each profile of proposed locations

<sup>13</sup>As a matter of comparison, note that when  $\chi_L = \chi_R$ , all of the critical values of  $\beta$  indicated in Figs. 7a and 7b coincide, i.e.  $\beta_1^C = \beta_2^C = \chi_R/2 = \chi_L/2$ . That explains why Fig. 4 exhibits neither a region with a mixed strategy equilibrium, nor one with one-sided policy differentiation.

the percentage of votes received by each candidate was not known with certainty due to the existence of some uncertainty about voters' preferences. More precisely, denoting by  $P$  the expected percentage of votes for a candidate, subjects were told that the actual percentage for that candidate will be somewhere between  $P - \beta$  and  $P + \beta$ , with each value within that range being equally likely.

They were informed about the preferred location on the high street for each of the two candidates. In order to get convenient payoff values in the game we applied a linear transformation of the payoffs by adding, first, a positive constant of 90 to the loss function, and by multiplying then all payoffs by 10. Thus, subjects were told that they would receive a location payoff corresponding to 900 minus 10 times the distance between their ideal location ( $\theta$ ) for the post office and the location actually realized. In addition, the subjects were told that winning the election would provide to the winning candidate an extra payoff of  $\chi \cdot 10$ .

The locations were chosen by typing in a number on the decision screen. A screenshot of the interface is provided in Fig. 8. Before making their actual proposals, subjects were provided with the opportunity to use an *expected payoff calculator* (top half of the screen) in which they could enter several hypothetical locations for themselves and for their opponent and calculate the associated own payoff. This calculator offered subjects a convenient device for looking at the  $101 \times 101$  payoff matrix, but it makes no recommendation as how to play the game. There was no time limit for the subjects' decisions.

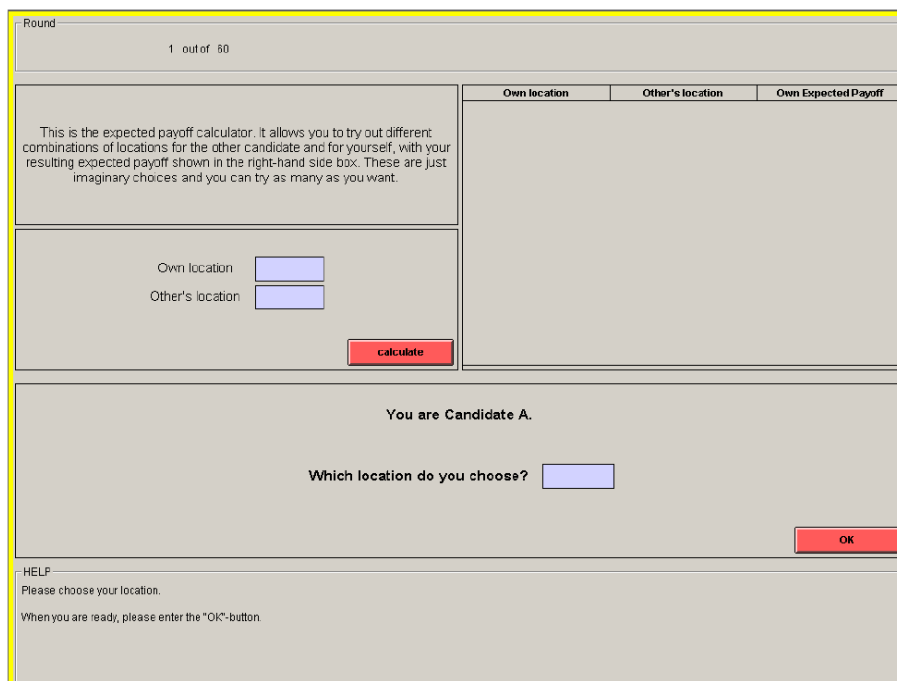


Figure 8: Decision interface.

After all participants made their actual choices, in each round subjects found a feedback screen with their chosen location, the location chosen by the other candidate, and the resulting own payoff, denominated in points. Subjects were recommended to transcribe the results of each round from the feedback window on a provided logsheet.

In each treatment there were 2 or 3 sessions, each comprising 60 rounds (elections). At the beginning of each session, subjects were randomly and anonymously matched into pairs. Within each pair, one subject was assigned the role of candidate A, whereas the other played the role of candidate B. Subjects were informed that they would not know who of the other people in the room they were paired with, and that matched pairs would remain fixed for the entire session. They were also aware that their initial roles would be swapped after round 30. This swapping allowed us to study some aspects of the learning by the subjects, particularly the transfer of insights from one role to the other. It also removed possible concerns about payoff asymmetries present in some of the treatments.

The experiment was carried out in the Spring of 2010 in the Centre for Experimental Economics of the University of York. Subjects were recruited from a university-wide pool of undergraduate and postgraduate students using Greiner's (2004) Online Recruitment System for Economic Experiments (ORSEE). The experiment was programmed and conducted with the software Z-Tree (Fischbacher 2007).

Upon arrival, subjects were assigned to a computer terminal and they were given a set of written instructions.<sup>14</sup> After reading the instructions, they were allowed to ask questions by raising their hands and speaking with the experimenter in private. To ensure that subjects understood the decision situation and the mechanics of payoff calculations, all participants answered several computerized test questions. The experiment did not proceed until every subject had answered these questions correctly. Subjects were not allowed to communicate directly with one another, and they only interacted indirectly via the decisions they entered in the computer terminals.

Subjects were informed that the points accumulated throughout the 60 rounds would determine, together with a given exchange rate, their monetary payoffs. A typical session lasted approximately 2 hours. The average payment of each treatment, the exchange rate, and the number of sessions, participants, and pairs are all summarized in Table 4.

## 6 Experimental Evidence

In this section we focus on the positions of the left-wing (henceforth Left) and right-wing (henceforth Right) players as well as the average absolute distance between these positions and the equilibrium predictions. Appendix B at the end of the paper presents

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<sup>14</sup>A copy of the instructions is available from the authors upon request.

Treatment	Sessions	Subjects	Pairs	Exchange rate (GBP per 1000 points)	Average payment (GBP)
1	3	26	13	0.60	19.80
2	2	20	10	0.60	19.80
3	2	20	10	0.50	21.00
4	2	20	10	0.45	21.30
5	2	20	10	0.60	19.81
6	2	20	10	0.40	24.56
7	3	30	15	0.40	24.85

Table 4: Overview of the experiment.

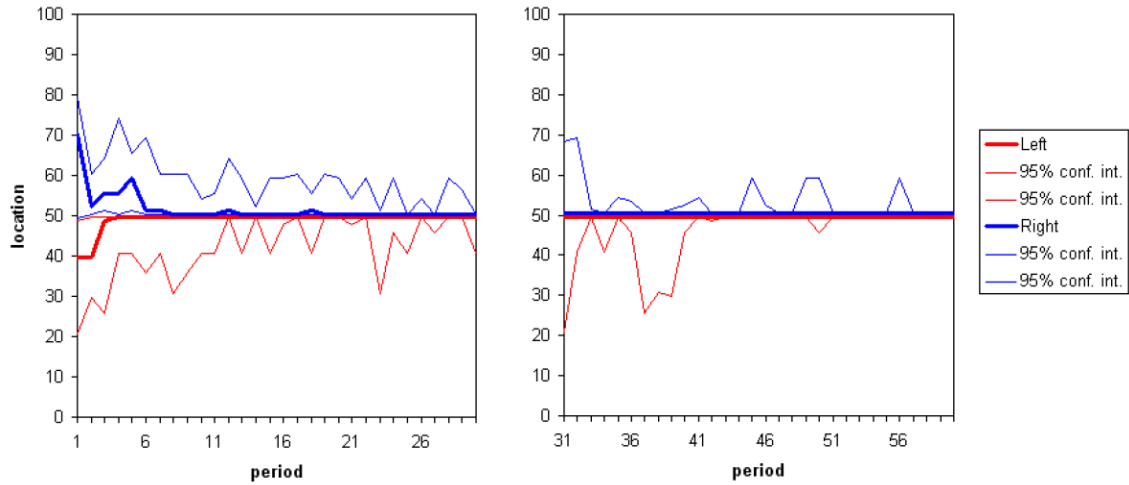
two sets of detailed tables. The first set (Tables 10a-10g) gives information for these variables for all single periods as well as subintervals of the 60 period experiment. The second set (Tables 11a-11g) shows details for each matching pair for selected intervals.

## 6.1 Equilibrium convergence

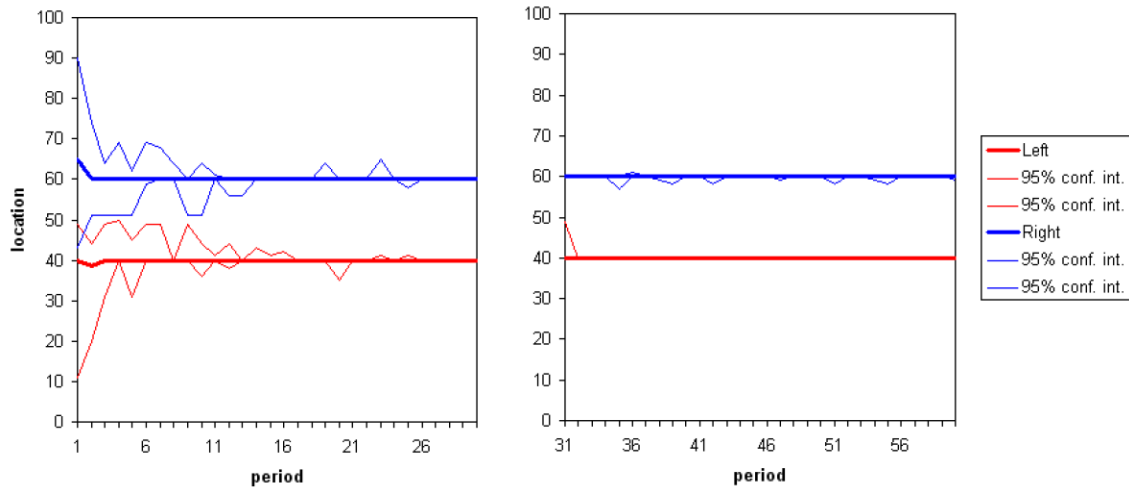
First, we look at the location choices of the Left and the Right players in the various treatments described in Table 3, comparing them with the values predicted by the Nash equilibrium.

Figure 9 shows for each treatment for which a PSE exists the per period median location of the Left and the Right players, as well as the 95% confidence intervals. These confidence intervals are determined as follows. Depending on the treatment, for each period there are between ten and fifteen independent observations (pairs). Using these observations as the unit of analysis, for every possible value  $m$  between 0 and 100, we test the null hypothesis (two-sided binomial test) that  $m$  is the median, i.e., that the probability to observe a location choice below  $m$  equals the probability to observe one above  $m$ . The alternative hypothesis is that the median has either a lower or a higher value than  $m$ , i.e., that these probabilities are not equal. For any given value  $m$ , the null hypothesis is rejected if there are too few or too many observations on one side of  $m$ .

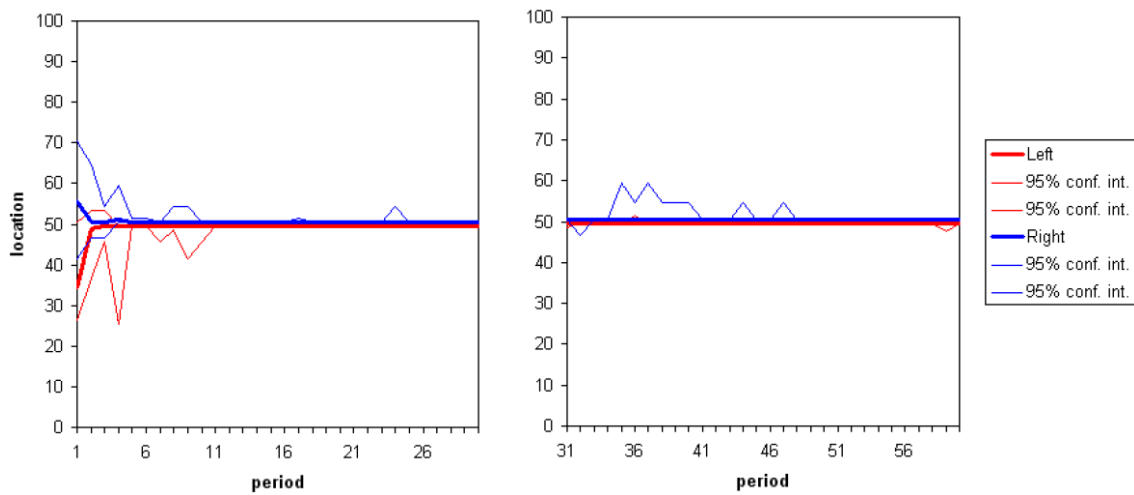
Two main conclusions emerge from the graphs. On one hand, in Treatments 1 to 5 (Figs. 9a-9e) not only the median locations converge to the equilibrium values predicted by the theory, but also the 95% confidence intervals shrink over time. On the other hand, in Treatment 7 (Fig. 9f) with one-sided policy differentiation, although the median locations of the Left and the Right players converge to the equilibrium, the 95% confidence intervals of both players tend to be skewed towards the center of the policy space. This suggests that although most of the players behaved in the lab as the theory predicts, some Left as well as some Right players deviated and they tended to stay towards the left of the theoretical predictions and closer to the center even after 60 periods of play.



(a) Treatment 1:  $x_L^* = x_R^* = 50$ .

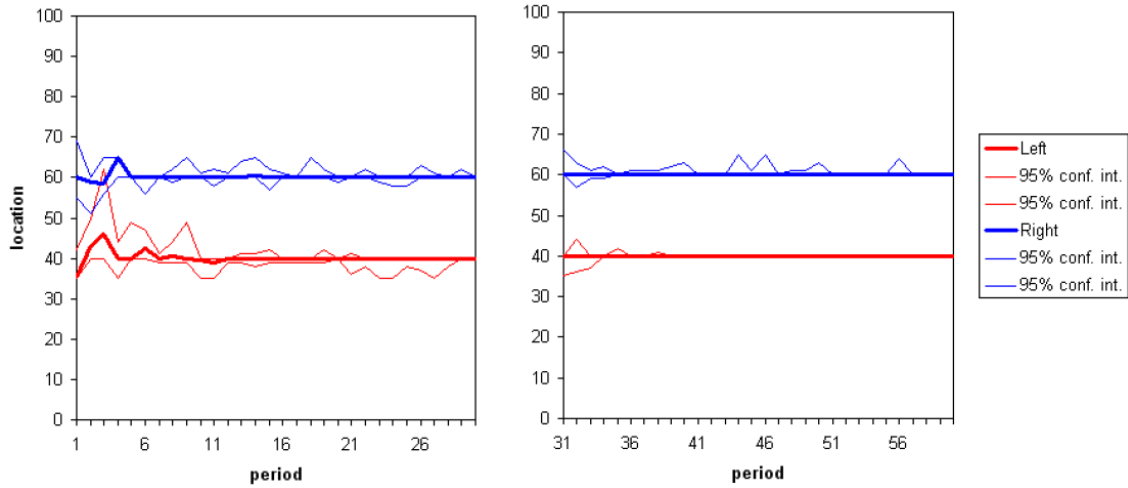


(b) Treatment 2:  $x_L^* = 40$  &  $x_R^* = 60$ .

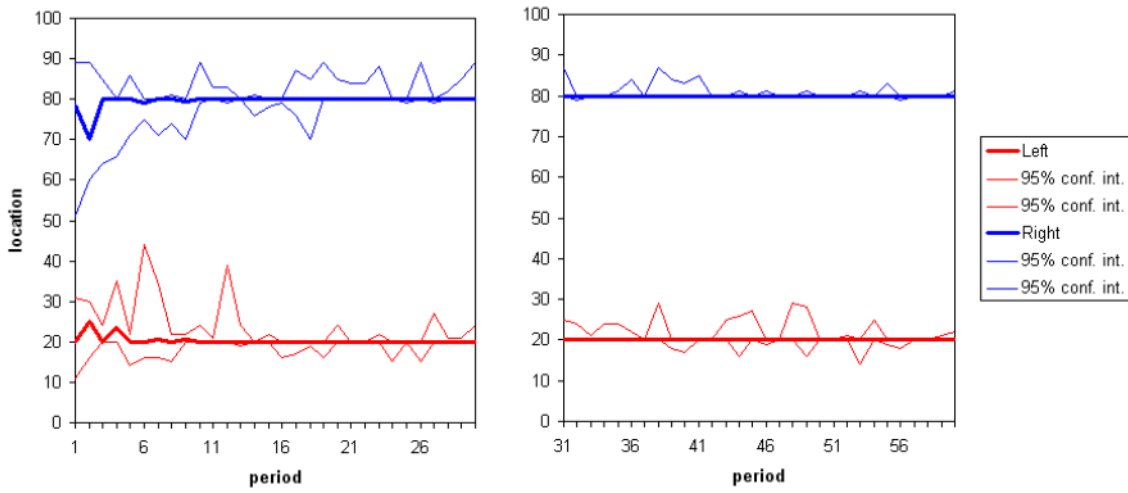


(c) Treatment 3:  $x_L^* = x_R^* = 50$ .

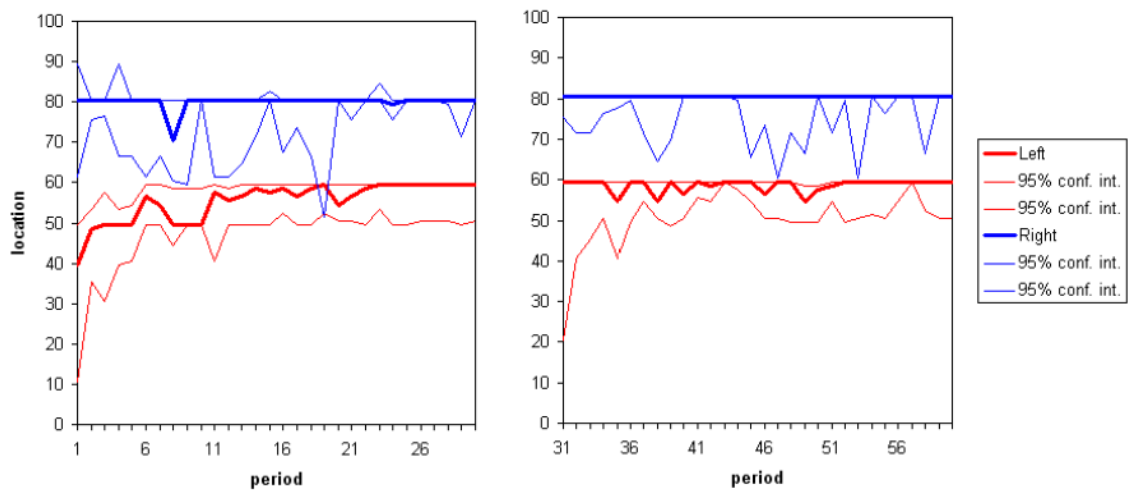
Figure 9: Median locations and 95% confidence intervals.



(d) Treatment 4:  $x_L^* = 40$  &  $x_R^* = 60$ .



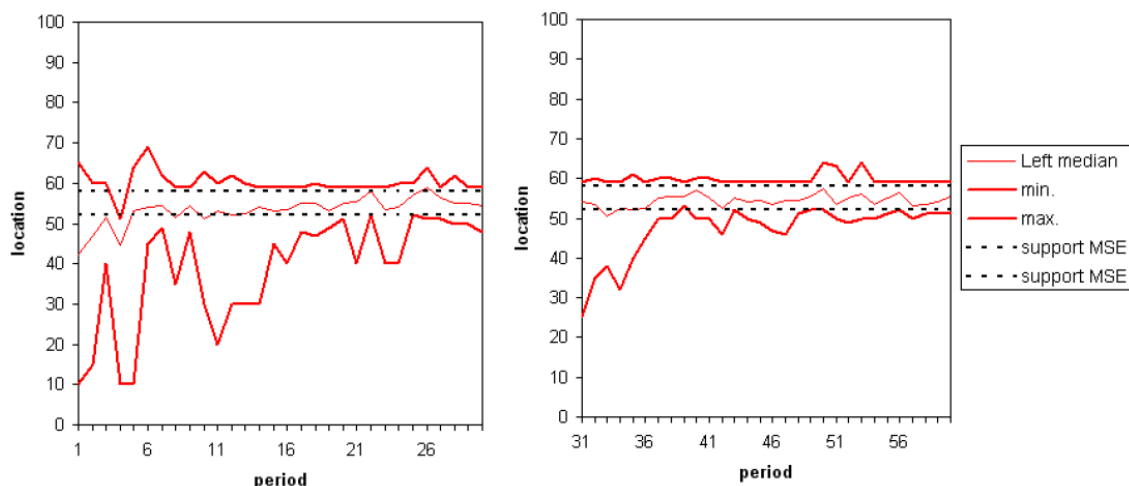
(e) Treatment 5:  $x_L^* = 20$  &  $x_R^* = 80$ .



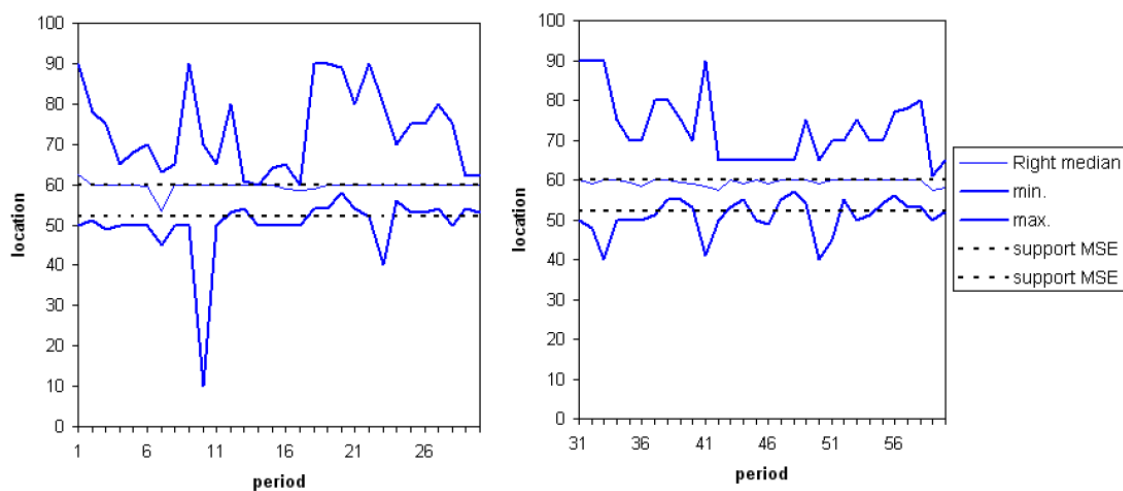
(f) Treatment 7:  $x_L^* = 60$  &  $x_R^* = 80$ .

Figure 9: Median locations and 95% confidence intervals (continued).

As to Treatment 6, notice that this case is different because the unique Nash equilibrium of the game is in mixed strategies. Therefore, besides the median locations of the Left and the Right players in each period, in Figs. 10a and 10b we also display the minimum and the maximum values of their locations, and we compare these values with the theoretical lower and upper bounds of the MSE support.



(a) Median, minimum and maximum locations of the Left players.



(b) Median, minimum and maximum locations of the Right players.

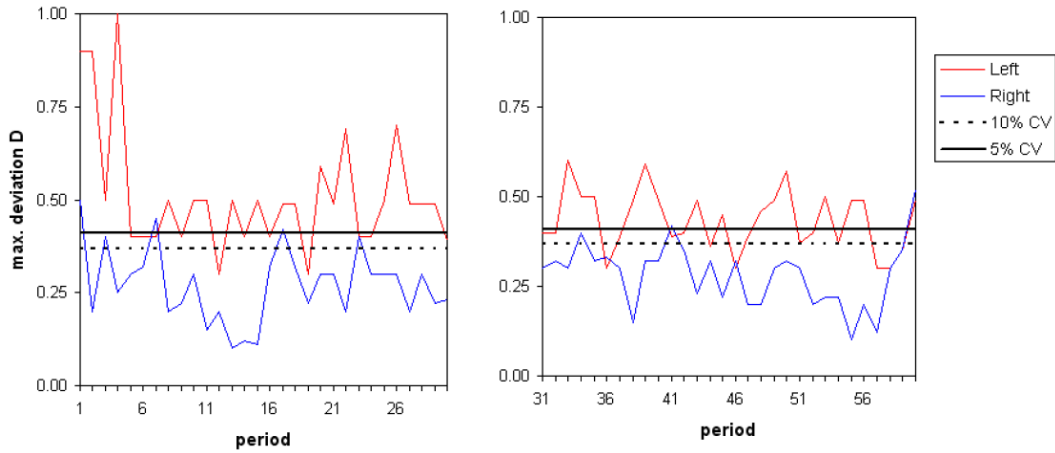
Figure 10: Treatment 6.

We find that the median of the Left (resp. Right) players converges to 55 (resp. 60), which is close to (resp. coincides with) the median location of the MSE (52 and 60, for Left and Right players respectively). Moreover, the pictures show that the minimum and the maximum locations chosen in the lab approximate the bounds of the MSE support, which ranges from 52 to 59 for the Left player, and from 52 to 60 for the Right player.

Since the median and support measure just some aspects of the distributions, to further assess the differences between the empirical and the theoretical distributions,



we apply the Kolmogorov-Smirnov test, considering for each period ten independent observations for the Left players and ten observations for the Right players. The test statistic, denoted by  $D$ , represents the maximum deviation between the empirical and the theoretical cumulative distributions. The null hypothesis is that these distributions are identical. The alternative hypothesis is that they are not the same. The critical values to reject the null hypothesis at 5% and 10% significance levels are, respectively, 0.410 and 0.368 (see Siegel 1988), with values of  $D$  above the critical values leading to the rejection of the null hypothesis.



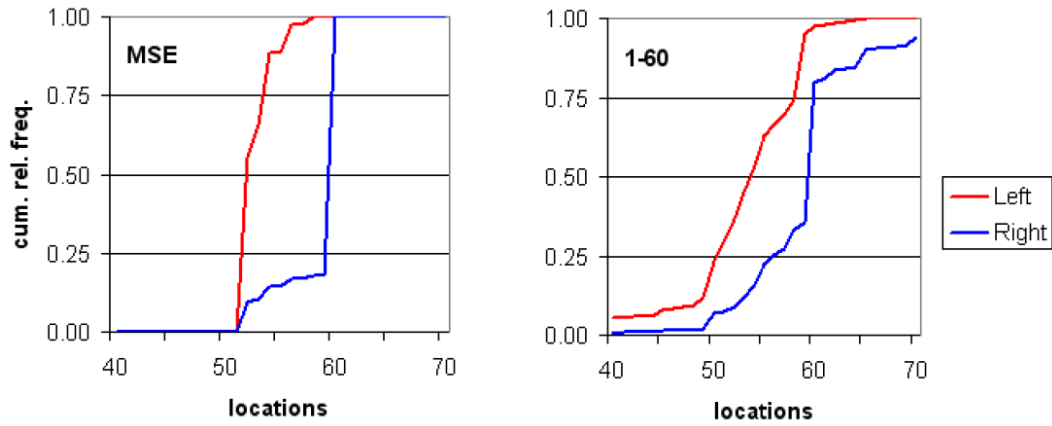
(c) Maximum deviation from the cumulative equilibrium distributions in each period.

Figure 10: Treatment 6 (continued).

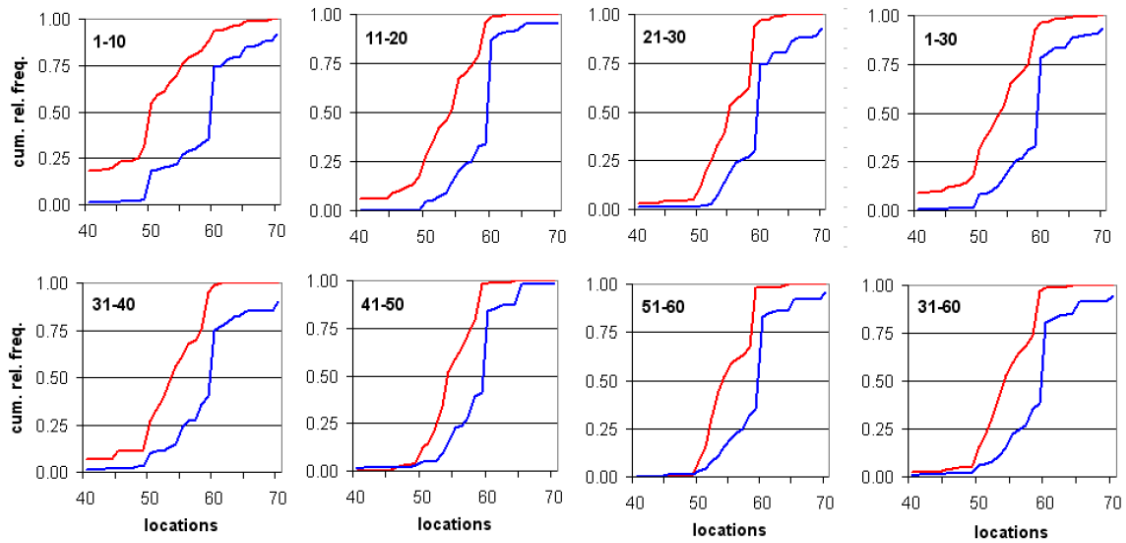
Focusing on each of the 60 periods separately, Fig. 10c shows that we cannot reject the null hypothesis in most of the periods for the Right players. Specifically, using the 5% critical value, the MSE distribution cannot be rejected in 27 of the first 30 periods, and 28 of the last 30 periods. For the Left players, however, the picture is somewhat different. Still at 5% significance, the MSE distribution cannot be rejected in 11 periods in the first half of the experiment and 15 periods in the second half.

In Figures 10d and 10e we continue the analysis of Treatment 6, presenting the empirical cumulative distributions for the 60 period interval as a whole and for a number of different subintervals. In conformity with the theory, the graphs show that the cumulative distribution of the Left players first-order stochastically dominates the distribution of the Right players. But when the Kolmogorov-Smirnov test is applied to these subintervals of the 60 periods (see Fig. 10f), we see that the null hypothesis of the empirical distributions being indistinguishable from the MSE distributions must be rejected in every single case. This means that the empirical distributions of the Left and the Right players are indeed statistically different from the theoretical ones.

The question, then, is how substantial these differences are. To answer that question,



(d) Cumulative relative frequencies.



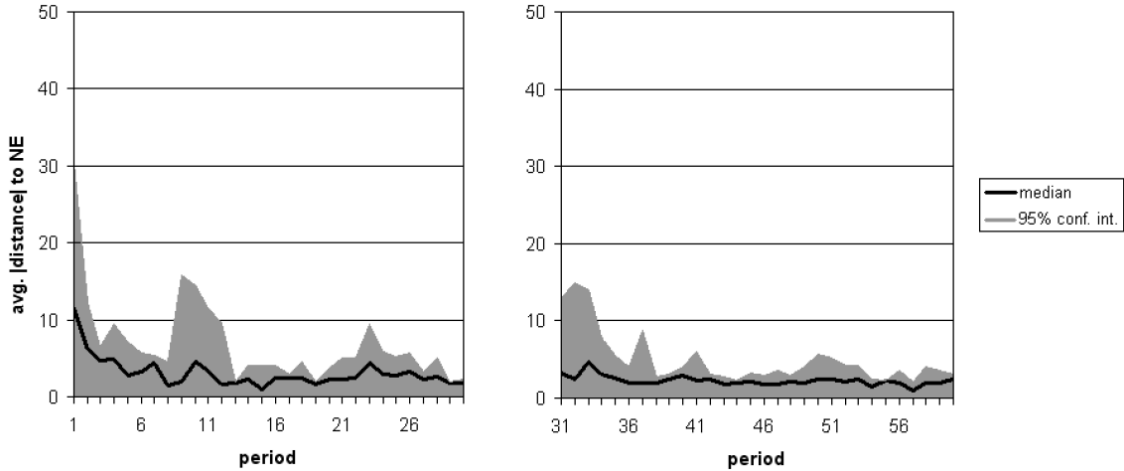
(e) Cumulative relative frequencies in different subintervals.

Figure 10: Treatment 6 (continued).

in every period we take the empirical distribution of the ten Left (Right) players, and we compute for each of these players how many locations they would need to move to reach the MSE distribution (allowing for fractions of players). To do this, we only stretch, squash and shift the empirical distribution, thus preserving the order of the location choices. That is, if player  $i$  had chosen a location smaller (greater) than player  $j$ , then after all moves have been made to reach the MSE distribution, player  $i$  still has a location smaller (greater) than or equal to player  $j$ . Once the number of locations each player would need to move to reach the equilibrium distribution has been found, in any given period we take the average number of moves of the Left and the Right player in each matching pair as the distance between the empirical and the theoretical distributions. This provides for each period ten independent observations for this distance. Figure 10g shows that the median distance as well as the 95% confidence interval diminish over time,

	max. dev. D		critical values	
	Left	Right	5%	10%
<b>1-10</b>	0.59	0.26	0.14	0.12
<b>11-20</b>	0.38	0.16	0.14	0.12
<b>21-30</b>	0.50	0.26	0.14	0.12
<b>1-30</b>	0.37	0.22	0.08	0.07
<b>31-40</b>	0.33	0.25	0.14	0.12
<b>41-50</b>	0.37	0.23	0.14	0.12
<b>51-60</b>	0.37	0.17	0.14	0.12
<b>31-60</b>	0.35	0.21	0.08	0.07
<b>1-60</b>	0.35	0.21	0.06	0.05

(f) Kolmogorov-Smirnov test.



(g) Median average absolute distance from the MSE distribution (with 95% confidence interval).

Figure 10: Treatment 6 (continued).

and that in the last subinterval, i.e., in periods 51-60, on average the median distance to be moved is only 2.0 locations. This means that although the empirical distributions of the Left and the Right players are statistically different from the theoretical ones, these differences are relatively small.

To conclude Section 6.1, we compare the positions of the Left players with the locations of the Right players. For each treatment and each matching pair, we compute the average position of the Left and of the Right players in different intervals. Thus, depending on the treatment, for each interval we have ten to fifteen independent observations, each of them being a matched pair. We use the Wilcoxon signed-ranks test to assess whether we can reject the null hypothesis that the position of the Left players is equal to that of the Right players. The results (one- or two-tailed tests as indicated by  $H_1$ ) are shown in Table 5.

As we see, in each treatment where the Left players would be expected to be on the left of the Right players (i.e. in Treatments 2, 4, 5, 6, and 7) this was indeed what

	treat1	treat2	treat3	treat4	treat5	treat6	treat7
$H_0$	L=R	L=R	L=R	L=R	L=R	L=R	L=R
$H_1$	L<>R	L<R	L<>R	L<R	L<R	L<R	L<R
<b>1-10</b>	1% (<)	0.1%	1% (<)	0.1%	0.1%	0.1%	0.00%
<b>11-20</b>	1% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
<b>21-30</b>	10% (<)	0.1%	1% (<)	0.1%	0.1%	0.1%	0.00%
<b>1-30</b>	1% (<)	0.1%	1% (<)	0.1%	0.1%	0.1%	0.00%
<b>31-40</b>	10% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
<b>41-50</b>	no diff.	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
<b>51-60</b>	10% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
<b>31-60</b>	10% (<)	0.1%	no diff.	0.1%	0.1%	0.1%	0.00%
<b>1-60</b>	1% (<)	0.1%	5% (<)	0.1%	0.1%	0.1%	0.00%

Table 5: Players' median locations (significance levels for rejection of  $H_0$ ).

happened. Note that in Treatment 6 according to the MSE predictions it can happen that a Left player chooses a location to the right of the Right player as the supports of the equilibrium distributions overlap. Nevertheless, for each of the intervals considered the expected mean location for the Left player is to the left of that of the Right player.

In Treatments 1 and 3 the Left and the Right players were supposed to converge to the same location. Nevertheless, the table shows that the position of the Left players was often significantly to the left of that of the Right players in these two treatments. Note that although statistically significant, these deviations were not widespread, as was shown above in Figure 9 by the convergence of the medians to the Nash equilibrium. In as far as there were deviations from the PSE in these treatments, they tended to be towards the left for Left players and towards the right for Right players. This may be explained by a bias induced by the subjects' ideology, or by the out-of-equilibrium incentives.<sup>15</sup>

## 6.2 Learning

The main message of Section 6.1 is clear: convergence to the Nash equilibrium is almost perfect in most of the treatments.<sup>16</sup> We now take a closer look at this result. More precisely, we want to know when this convergence takes place. For each treatment, we distinguish the 30 periods before the swapping of the roles and the 30 periods after the swap. We also split these intervals into smaller subintervals of ten periods. For every matching pair, we compute for each subinterval the average absolute distance from

<sup>15</sup>If the opponent chooses the PSE location, then deviating from the PSE towards a subject's own ideology leads to a less steep decline in payoffs than a deviation in the opposite direction.

<sup>16</sup>We also considered Quantal Response Equilibria (QRE). For each treatment we estimated the QRE choice intensity parameter by minimizing the error of the QRE strategy profiles with the empirical distribution observed in periods 41-60. Using this free parameter we obtain errors for the QRE that are only marginally below those for the Nash equilibrium predictions, and this slightly better fit is achieved by using widely different choice intensity parameter values across treatments. Further details are available from the authors upon request.

the Nash equilibrium, and we test whether these distances are different in two specified intervals.

To do this, we use the one-tailed Wilcoxon signed-ranks test, distinguishing 1% and 5% significance levels. This is a non-parametric statistical test to assess whether there is a difference in the median of two related samples. The only assumption made about the underlying distribution is that these differences are independent observations from a symmetric distribution. The null hypothesis is that the median difference between the pairs of observations is zero. The alternative hypothesis in Table 6 is that the median of the interval that comes later is lower than that of the earlier interval, reflecting the learning and adaptive behavior of the experimental subjects.

The results are reported in Table 6. In each box, we compare the average absolute distance in the intervals indicated on the left-hand side to those indicated at the top of the box. Thus, if we consider for instance Treatment 1 (first box), we see that the average absolute distance from the PSE is smaller in periods 11-20 (first column at the top) than in periods 1-10 (first row on left-hand side) at the 1% significance level. For Treatment 6 we present two boxes: the first box (treat6a) shows the distance from the MSE support, whereas the second (treat6b) shows the distance from the entire distribution.

First, we ask whether there has been a significant amount of learning over the entire experiment. As the tables show, learning did happen since in every treatment the average absolute distance from the Nash equilibrium is statistically significantly smaller in the last ten periods, i.e., in periods 51-60, than in the first ten periods.

Second, we ask in which periods the average absolute distance actually decreases. Looking at the main diagonal of the tables, it turns out that except in Treatment 7, where it seems that learning happened between periods 11 and 20, in the rest of the treatments learning took place mainly in the first ten periods (elections), which was also the most active interval in terms of subjects' use of the expected payoff calculator.<sup>17</sup>

Third, we ask whether players after swapping their roles between periods 30 and 31 succeed in transferring some of their findings from before the swapping to after the swapping. The answer is largely affirmative as the distance from the Nash equilibrium is smaller in periods 31-40 than in periods 1-10 for all treatments except Treatment 1.

Finally, fourth, we test whether the swapping as such led to an increase in the distance from the NE right after the swapping. As we see in Table 7, in some treatments there

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<sup>17</sup>Both findings are confirmed by the OLS regressions displayed in Table 9 of Appendix B, which regress the position of the Left players, the Right players, and the average absolute distance from the equilibrium as dependent variables, against the inverse of time  $1/t$  as the only independent variable. The table show all the coefficients and the t-statistics obtained in these regressions. As we see, almost all coefficient are significant, and in particular the slope coefficients for the distance from the equilibrium are significant with the expected sign for all treatments in periods 1-30 as well as in periods 31-60.

		to periods:						
		treat1	11-20	21-30	31-40	41-50	51-60	31-60
from	1-10		yes (1%)	yes (1%)	no	yes (1%)	yes (1%)	
	11-20			no	no	no	yes (5%)	
	21-30				no	no	yes (1%)	
	31-40					yes (1%)	yes (1%)	
	41-50						no	
	1-30							yes (1%)
from	1-10		yes (1%)	yes (1%)	yes (1%)	yes (1%)	yes (1%)	
	11-20			no	no	no	no	
	21-30				no	no	no	
	31-40					no	no	
	41-50						no	
	1-30							yes (1%)
from	1-10		yes (1%)	yes (1%)	yes (5%)	yes (1%)	yes (1%)	
	11-20			no	no	no	no	
	21-30				no	no	no	
	31-40					yes (1%)	yes (5%)	
	41-50						no	
	1-30							no
from	1-10		yes (1%)	yes (1%)	yes (1%)	yes (1%)	yes (1%)	
	11-20			no	no	no	yes (1%)	
	21-30				no	no	no	
	31-40					no	yes (5%)	
	41-50						no	
	1-30							yes (1%)
from	1-10		yes (1%)	yes (1%)	yes (1%)	yes (5%)	yes (5%)	
	11-20			no	no	no	no	
	21-30				no	no	no	
	31-40					no	no	
	41-50						no	
	1-30							yes (5%)
from	1-10		yes (1%)	yes (1%)	yes (5%)	yes (1%)	yes (1%)	
	11-20			no	no	no	no	
	21-30				no	no	no	
	31-40					yes (5%)	yes (5%)	
	41-50						no	
	1-30							yes (5%)
from	1-10		yes (1%)	yes (1%)	yes (5%)	yes (1%)	yes (5%)	
	11-20			no	no	no	yes (1%)	
	21-30				no	yes (10%)	yes (5%)	
	31-40					yes (5%)	yes (1%)	
	41-50						no	
	1-30							yes (5%)
from	1-10		no	yes (1%)	yes (5%)	yes (1%)	yes (1%)	
	11-20			yes (1%)	no	yes (5%)	yes (1%)	
	21-30				no	no	no	
	31-40					no	yes (5%)	
	41-50						no	
	1-30							yes (1%)

Table 6: Decrease in the average absolute distance from the Nash equilibrium.

is an increase in the distance from the equilibrium if the intervals considered are 1 or 5 periods before the swap, but not considering such a ten period interval.

		to:			
from:	treat1	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			yes (5%)	
from:	treat2	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			no	
from:	treat3	31-40	31-35	31	
	21-30	no			
	26-30		yes (5%)		
	30			yes (5%)	
from:	treat4	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			no	
		to:			
from:	treat5	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			no	
from:	treat6a	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			yes (5%)	
from:	treat6b	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			yes (5%)	
from:	treat7	31-40	31-35	31	
	21-30	no			
	26-30		no		
	30			no	

Table 7: Increase in the average absolute distance from the Nash equilibrium.

### 6.3 Comparisons between treatments

Apart from testing whether the observed choices converge to the Nash equilibrium, the data set obtained from the lab is also used to perform several ‘comparative statics’ tests across treatments. For expositional convenience, all the pair-wise comparisons are illustrated in Fig. 11, where a double arrow relating any two treatments is used to indicate a direct statistical comparison between them.

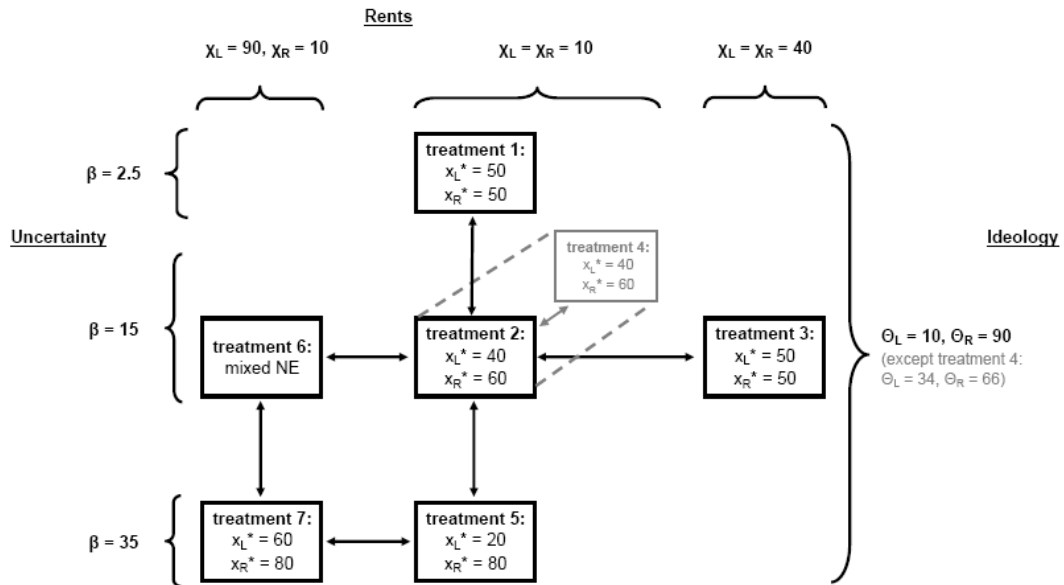


Figure 11: Overview of the comparisons between Treatments.

The comparative statics tests carried out in this subsection focus on three variables, namely, the position of the Left players, the position of the Right players, and the average absolute distance from the Nash equilibrium. In each treatment, we compute the average value of these variables for different subintervals and for the whole session. We have,

depending on the treatment, between ten and fifteen independent observations, and we use the robust rank-order test to compare the samples between two treatments, distinguishing 1%, 5% and 10% significance levels.<sup>18</sup> The results found are reported in Table 8. Table 8a concerns the positions of the Left players, Table 8b the location of the Right players, and Table 8c shows the average absolute distance from the Nash equilibrium.

Left	treatments							
	1 v. 2	2 v. 5	1 v. 5	2 v. 4	2 v. 3	2 v. 6	5 v. 7	6 v. 7
$H_0$	$L(1) = L(2)$	$L(2) = L(5)$	$L(1) = L(5)$	$L(2) = L(4)$	$L(2) = L(3)$	$L(2) = L(6)$	$L(5) = L(7)$	$L(6) = L(7)$
$H_1$	$L(1) > L(2)$	$L(2) > L(5)$	$L(1) > L(5)$	$L(2) <> L(4)$	$L(2) < L(3)$	$L(2) < L(6)$	$L(5) < L(7)$	$L(6) < L(7)$
1-10	5%	1%	1%	no diff.	1%	1%	1%	no diff.
11-20	1%	1%	1%	no diff.	1%	1%	1%	no diff.
21-30	1%	1%	1%	no diff.	1%	1%	1%	no diff.
1-30	1%	1%	1%	no diff.	1%	1%	1%	no diff.
31-40	5%	1%	1%	no diff.	1%	1%	1%	no diff.
41-50	1%	1%	1%	no diff.	1%	1%	1%	10%
51-60	1%	1%	1%	no diff.	1%	1%	1%	no diff.
31-60	1%	1%	1%	no diff.	1%	1%	1%	no diff.
1-60	1%	1%	1%	no diff.	1%	1%	1%	no diff.

(a) Left players' positions across treatments.

Right	treatments							
	1 v. 2	2 v. 5	1 v. 5	2 v. 4	2 v. 3	2 v. 6	5 v. 7	6 v. 7
$H_0$	$R(1) = R(2)$	$R(2) = R(5)$	$R(1) = R(5)$	$R(2) = R(4)$	$R(2) = R(3)$	$R(2) = RL(6)$	$R(5) = R(7)$	$R(6) = R(7)$
$H_1$	$R(1) < R(2)$	$R(2) < R(5)$	$R(1) < R(5)$	$R(2) <> R(4)$	$R(2) > R(3)$	$R(2) <> R(6)$	$R(5) <> R(7)$	$R(6) < R(7)$
1-10	5%	1%	1%	no diff.	1%	no diff.	no diff.	1%
11-20	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
21-30	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
1-30	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
31-40	1%	1%	1%	no diff.	1%	no diff.	1% (>)	1%
41-50	1%	1%	1%	no diff.	1%	no diff.	1% (>)	1%
51-60	1%	1%	1%	no diff.	1%	no diff.	no diff.	1%
31-60	1%	1%	1%	no diff.	1%	no diff.	1% (>)	1%
1-60	1%	1%	1%	no diff.	1%	no diff.	5% (>)	1%

(b) Right players' positions across treatments.

distance	treatments							
	1 v. 2	2 v. 5	1 v. 5	2 v. 4	2 v. 3	2 v. 6	5 v. 7	6 v. 7
$H_0$	$d(1) = d(2)$	$d(2) = d(5)$	$d(1) = d(5)$	$d(2) = d(4)$	$d(2) = d(3)$	$d(2) = d(6)$	$d(5) = d(7)$	$d(6) = d(7)$
$H_1$	$d(1) <> d(2)$	$d(2) <> d(5)$	$d(1) <> d(5)$	$d(2) <> d(4)$	$d(2) <> d(3)$	$d(2) <> d(6)$	$d(5) <> d(7)$	$d(6) <> d(7)$
1-10	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	1% (<)
11-20	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	5% (<)	10% (<)
21-30	10% (>)	no diff.	no diff.	no diff.	no diff.	no diff.	5% (<)	no diff.
1-30	no diff.	no diff.	no diff.	no diff.	no diff.	no diff.	5% (<)	5% (<)
31-40	10% (>)	no diff.	no diff.	no diff.	no diff.	2% (<)	5% (<)	no diff.
41-50	no diff.	no diff.	no diff.	no diff.	no diff.	2% (<)	10% (<)	no diff.
51-60	no diff.	no diff.	no diff.	no diff.	no diff.	1% (<)	no diff.	no diff.
31-60	10% (>)	no diff.	no diff.	no diff.	no diff.	2% (<)	10% (<)	no diff.
1-60	no diff.	10% (<)	no diff.	no diff.	no diff.	5% (<)	5% (<)	10% (<)

(c) Average absolute distance from the Nash equilibrium across treatments.

Table 8: Differences between treatments.

First, to assess the impact on policy divergence of an increase in the electoral uncertainty, Treatment 1 is compared with Treatments 2 and 5, respectively, and Treatment 2 is compared with Treatment 5. In each of these treatments, the ideologies and the office

<sup>18</sup>This test statistic has the advantage that it compares the median of two unrelated samples without making any assumptions about the higher moments of the distribution of the two samples. The critical values are taken from Feltovich (2003).



rents remain constant, whereas the electoral uncertainty gradually increases, leading to increasing policy divergence in theory. The results are shown in the second, third and fourth columns of Tables 8a-8c. In conformity with the theory, in all cases and in every interval the null hypothesis that there is no difference between the positions of the Left (resp. Right) players across the treatments is rejected at 1% or 5% significance levels, with the alternative hypothesis being in the direction predicted by the theory.

As to the average absolute distance from the Nash equilibrium, the tests indicate no significant differences in most of the intervals. However, looking at the whole session, Treatment 5 appears to show less convergence to the Nash equilibrium than Treatment 2, albeit only at 10% significance level, although both treatments deal with the same type of equilibrium, namely, two-sided differentiation. We conjecture that the reason could be that the equilibrium associated with the parameter values of Treatment 5 (i.e.,  $x_L^* = 20$  and  $x_R^* = 80$ ) is somewhat more extreme than the one corresponding to Treatment 2 (i.e.,  $x_L^* = 40$  and  $x_R^* = 60$ ), and that some of the subjects may have been concerned about choosing such extreme policies.<sup>19</sup>

Second, by varying the ideologies, the comparison of Treatments 2 and 4 offers the chance to see whether the two-sided differentiation effect present in Treatment 2 is independent (in the linear, risk-neutral case) of the degree of ideological polarization  $\theta_R - \theta_L$ . In conformity with the theory, in every interval the null hypothesis that there is no difference between the positions of the Left (resp. Right) players and between the average absolute distances cannot be rejected at 1% and 5% significance levels.

Third, the issue of whether policy convergence is re-established as candidates become more office-motivated is investigated by comparing Treatments 2 and 3. The results show that in every interval the positions of the Left (resp. Right) players in Treatment 2 are statistically different at 1% significance level from the positions of the Left (resp. Right) players in Treatment 3, which is again consistent with the theory. Moreover, there are no statistically significant differences in these two treatments with respect to the average absolute distances from the Nash equilibrium.

Fourth, to assess the change in policy differentiation that results from raising the office rents of one of the candidates while keeping the other constant, Treatment 5 is contrasted with Treatment 7. The theory predicts no changes in the location of the Right candidate, and a move of the Left candidate from the left-hand side to the right-hand side of the median voter. The experimental results are mixed. On the one hand, in every interval the positions of the Left players in Treatment 7 are statistically different at 1% significance level from the positions of the Left players in Treatment 5. On the other hand, however,

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<sup>19</sup>Recall that the analysis assumes risk neutrality, which might not have been the case in the lab for at least some of the subjects.

contrary to the theoretical prediction, we find significant differences in the Right players' positions in several intervals, including the last 30 periods (at 1%) and the whole session (at 5%). The data shows the locations of these players in Treatment 5 tend to be more extreme. Consistent with our previous results, convergence to the NE is also worse in Treatment 7 than in Treatment 5. In the whole session as well as in several subintervals, there are significant differences (at 5 and 10%) in the average absolute distances from the NE, with the distance in Treatment 5 tending to be smaller.

Fifth, we compare Treatment 6, in which there is no PSE, with Treatments 2 and 7, to detect any significant variations in subjects' behavior in the absence of a PSE. For a start, one should expect less convergence in Treatment 6.<sup>20</sup> Nevertheless, we see that in the first twenty periods the distance from equilibrium is smaller in Treatment 6 than in Treatment 7. More interesting are the location choices of the Left and Right players. Comparing Treatment 6 with Treatment 2, we observe that the Left players in the latter, in which office rents are lower, choose locations to the left of those in Treatment 6. For Right players we do not see a difference between these two treatments, which seems related to the fact that the expected median in Treatment 6 is 59 whereas it is 60 in Treatment 2. Comparing Treatment 6 with Treatment 7, in which uncertainty has increased, we see that there are no significant differences in the Left players' behavior between these treatments (recall the expected median in the former is 53 and in the latter 60), whereas the Right players, as predicted, choose locations more to the right in Treatment 7.

## 7 Final remarks

This paper constitutes the first attempt to study both theoretically and experimentally the complete set of Nash equilibria of a classical one-dimensional, majority rule election game with two candidates, who might be interested in power as well as in ideology, but not necessarily in the same way. We provided a full characterization of the set of Nash equilibria, showing how the different equilibrium configurations depend on the relative interests in power (resp., ideology) and the uncertainty about voters' preferences; and we examined the empirical relevance of these theoretical predictions through a series of laboratory experiments. The experimental data show convergence to the Nash equilibrium values at the aggregate as well as the individual levels in all treatments, and the comparative statics effects across treatments are as predicted by the theory.

Despite these positive results, and despite the fact that the model considered here seems rich enough to pick up several interesting features of electoral competition that

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<sup>20</sup>With a small number (ten) of per period observations one cannot expect to hit the equilibrium distribution exactly.

had been overlooked in the literature, there are a number of issues that may require more attention in future work. First, the assumption of risk neutrality (with respect to the distance  $|x - \theta_i|$ ), embedded into the assumption of Euclidean preferences of Section 3, entails a loss of generality in the analysis. This is because in spite of being ideologically different, risk averse candidates tend to move closer to each other and toward to the center. Indeed, given the position of one candidate, the rival chooses a less differentiated platform when it is risk averse than when it is risk neutral because it must compensate a higher utility loss due to the risk aversion with a rise in the probability of winning the contest.

Second, we noted in the experimental evidence that convergence to the Nash equilibrium is not equally precise across treatments. In particular in the asymmetric treatments, where the candidates have different motives, we saw more noise at the individual level than in the symmetric ones. It may be interesting to investigate the causes of this difference in the degree of convergence across treatments, and to find out, for example, whether this observation that there is less convergence in the asymmetric treatments is due to the fact that the theoretical predictions implied one-sided policy differentiation or just to the fact that these equilibria are not symmetric around the center. Further experiments may shine some light on this matter.

Third, an important element of our design is the expected payoff calculator. The calculator provided information about the available payoffs. This information is usually presented in the form of a payoff matrix in experimental settings. We had not made the entire  $101 \times 101$  payoff matrix available for practical reasons. Instead, the calculator allowed the subjects to observe snapshots of the underlying payoff matrix. However, it did not create any bias, in the sense that it did not induce the subjects to examine any particular areas of the strategy space. Subjects had to enter explicitly the location choices for themselves and for their opponents, and the calculator only provided factual information about the corresponding payoffs, without suggesting any kind of recommendation. Having said that, we acknowledge that it may be interesting to consider alternative experimental designs in this type of electoral games, in particular designs in which information about the strategic environment is conveyed in a less convenient way to the subjects.

Finally, our experimental design treats voters as artificial actors, as seems conventional in the literature. It would seem interesting, however, to organize an experiment in which the voters are experimental subjects as well. This has been done in some of the early papers about the median voter outcome, and it should be easier to implement nowadays thanks to the communication tools (such as iPhones, iPads, etc.) currently available. This may be interesting from a methodological viewpoint, as well as to assess related issues such as private polling and voter turnout (especially in cases of similar platforms).

## References

- Ansolabehere, S., and J. Snyder (2000) Valence politics and equilibrium in spatial election models, *Public Choice*, 103, 327-36.
- Aragones, E., and T. Palfrey (2002) Mixed equilibrium in a Downsian model with a favorite candidate, *Journal of Economic Theory*, 103, 131-161.
- Aragones, E., and T. Palfrey (2004) The effect of candidate quality on electoral equilibrium: An experimental study, *American Political Science Review*, 98, 77-90.
- Aragones, E., and T. Palfrey (2005) Electoral competition between two candidates of different quality: The effects of candidate ideology and private information, *Social Choice and Strategic Decisions*, by D. Austen-Smith and J. Duggan, Eds., New York: Springer-Verlag, 93-112.
- Ashworth, S., and E. Bueno de Mesquita (2009) Elections with platform and valence competition, *Games and Economic Behavior*, 67, 191-216.
- Austen-Smith, D., and J. Banks (2005) *Positive Political Theory II: Strategy and Structure*, Ann Arbor: The University of Michigan Press.
- Ball, R. (1999) Discontinuity and nonexistence of equilibrium in the probabilistic spatial voting model, *Social Choice and Welfare*, 16, 533-555.
- Bernhardt, D., Duggan, J., and F. Squintani (2007) Electoral competition with privately informed candidates, *Games and Economic Behavior*, 58, 1-29.
- Bernhardt, D., Duggan, J., and F. Squintani (2009a) Private polling in elections and voter welfare, *Journal of Economic Theory*, 144, 2021-2056.
- Bernhardt, D., Duggan, J., and F. Squintani (2009b) The case for responsible parties, *American Political Science Review*, 103, 570-587.
- Callender, S. (2008) Political motivations, *Review of Economic Studies*, 75, 671-697.
- Calvert, R. (1985) Robustness of the multidimensional voting model: Candidate motivations, uncertainty, and convergence, *American Journal of Political Science*, 29, 270-286.
- Camerer, C. (2003) *Behavioral Game Theory: Experiments on Strategic Interaction*, Princeton: Princeton University Press.
- Downs, A. (1957) *An Economic Theory of Democracy*, New York: Harper Collins.

- Duggan, J., and M. Fey (2005) Electoral competition with policy-motivated candidates, *Games and Economic Behavior*, 51, 490-522.
- Feltoich, N. (2003) Nonparametric tests of differences in medians: Comparison of the Wilcoxon-Mann-Whitney and robust rank-order tests, *Experimental Economics*, 6, 273-297.
- Fischbacher, U. (2007) Z-tree: Zurich toolbox for ready-made economic experiments, *Experimental Economics*, 10, 171-178.
- Greiner, B. (2004) An online recruitment system for economic experiments, in: Kurt Kremer, Volker Macho, Eds., *Forschung und wissenschaftliches Rechnen 2003. GWDG Bericht 63*, Göttingen: Ges. für Wiss. Datenverarbeitung, 79-93.
- Groseclose, T. (2001) A model of candidate location when one candidate has a valence advantage, *American Journal of Political Science*, 45, 862-886.
- Hotelling, H. (1929) Stability in competition, *Economic Journal*, 39, 41-57.
- Hummel, P. (2010) On the nature of equilibria in a Downsian model with candidate valence, *Games and Economic Behavior*, 70, 425-445.
- Iaryczower, M., and A. Mattozzi (2009) On the nature of competition in alternative electoral systems, California Institute of Technology, typescript.
- Kartik, N., and P. McAfee (2007) Signaling character in electoral competition, *American Economic Review*, 97, 852-870.
- McKelvey, R., and P. Ordeshook (1990) A decade of experimental research on spatial models of elections and committees, *Advances in the Spatial Theory of Voting*, by Enelow, J., and M. Hinich, Eds., Cambridge: Cambridge University Press, 99-144.
- McKelvey, R., McLennan, A., and T. Turocy (2010) Gambit: software tools for game theory, Version 0.2010.09.01. Available online at <http://www.gambit-project.org>.
- Morton, R. (1993) Incomplete information and ideological explanations of platform divergence, *American Political Science Review*, 87, 382-392.
- Palfrey, T. (2005) Laboratory experiments in political economy, CEPS Working Paper No. 111.
- Peress, M. (2010) The spatial model with non-policy factors: A theory of policy-motivated candidates, *Social Choice and Welfare*, 34, 265-294.

- Osborne, M. (1995) Spatial models of political competition under plurality rule: A survey of some explanations of the number of candidates and the positions they take, *Canadian Journal of Economics*, 27, 261-301.
- Roemer, J. (1994) A theory of policy differentiation in single issue electoral politics, *Social Choice and Welfare*, 11, 355-380.
- Roemer, J. (1997) Political-economic equilibrium when parties represent constituents, *Social Choice and Welfare*, 14, 479-502.
- Roemer, J. (1999) The democratic political economy of progressive income taxation, *Econometrica*, 67, 1-19.
- Roemer, J. (2001) *Political Competition*, Cambridge: Harvard University Press.
- Saporiti, A. (2008) Existence and uniqueness of Nash equilibrium in electoral competition games: The hybrid case, *Journal of Public Economic Theory*, 10, 827-857.
- Schram, A. (2002). Experimental public choice, Tinbergen Institute Discussion Paper No. 106/1.
- Siegel, S. (1988) *Nonparametric Statistics for the Behavioral Sciences*, New York: McGraw-Hill.
- Snyder, J., and M. Ting (2002) An informational rationale for political parties, *American Journal of Political Science*, 46, 90-110.
- Wittman, D. (1983) Candidate motivation: A synthesis of alternative theories, *American Political Science Review*, 77, 142-157.

## A Appendix: Proofs

To simplify the notation, and given that the term  $u_{\theta_i}(x_j)$ ,  $i \neq j$ , of candidate  $i$ 's payoff function  $\Pi_i$  defined in (1) and (2) does not affect  $i$ 's optimal choices, in the rest of this appendix we work with the linear transformations  $\pi_i(x_i, x_j) \equiv \Pi_i(x_i, x_j) - u_{\theta_i}(x_j)$ .

**Proof of Lemma 2.** Let  $(x_L^*, x_R^*)$  be a PSE for  $\mathcal{G}$ . To see that  $p(x_L^*, x_R^*) \in (0, 1)$ , assume without loss of generality that  $p(x_L^*, x_R^*) = 1$ . Then, candidate  $R$ 's equilibrium payoff is  $\pi_R(x_L^*, x_R^*) = 0$ ; and it would be possible for  $R$  to increase its payoff by deviating to  $x_L^*$  (which would result in a payoff equal to  $\chi_R/2 > 0$ ), a contradiction.

Next, suppose that  $x_L^* < \theta_L$ . If  $x_R^* \geq \theta_L$ , it would be possible for  $L$  to increase its payoff by choosing  $\theta_L$ , because  $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [x_L^* + x_R^* - 2\theta_L + \chi_L] < p(\theta_L, x_R^*) \cdot [x_R^* - \theta_L + \chi_L] = \pi_L(\theta_L, x_R^*)$  (recall that, by Lemma 1,  $p(\theta_L, x_R^*) \geq p(x_L^*, x_R^*)$ ). Alternatively, if  $x_R^* < \theta_L$ , then: (i)  $L$  would profitably deviate to  $x_R^*$  if  $x_L^* < x_R^*$ , because  $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [x_L^* - x_R^* + \chi_L] < \chi_L/2$ ; (ii)  $R$  would find it beneficial to move to  $x_L^*$  if  $x_R^* < x_L^*$ , because  $\pi_R(x_L^*, x_R^*) = [1 - p(x_L^*, x_R^*)] \cdot [x_R^* - x_L^* + \chi_R] < \chi_R/2$ ; and (iii)  $L$  would do better by playing  $\theta_L$  if  $x_R^* = x_L^*$ , because  $\chi_L/2 < p(\theta_L, x_R^*) \cdot [\theta_L - x_R^* + \chi_L] = \pi_L(\theta_L, x_R^*)$ . Therefore,  $x_L^* \geq \theta_L$ .

Assume, by way of contradiction, that  $x_L^* = \theta_L$ . Then: (i) if  $x_R^* = \theta_L$ , candidate  $R$  can benefit by moving its proposal to  $x_R = \theta_L + \delta$ , with  $\delta > 0$  small, because  $\pi_R(x_L^*, x_R) = [1 - p(x_L^*, x_R)] \cdot (\chi_R + \delta) > \chi_R/2 = \pi_R(x_L^*, x_R^*)$ ; (ii) if  $x_R^* > \theta_L$ , candidate  $L$  would be able to increase its payoff by selecting  $x_L = \theta_L + \epsilon$ , which would result, given the assumption on  $\beta$  and for  $\epsilon > 0$  small enough, in a positive payoff change  $[p(x_L, x_R^*) - p(\theta_L, x_R^*)] \cdot [x_R^* - \theta_L + \chi_L] - \epsilon \cdot p(x_L, x_R^*)$ ;<sup>21</sup> finally (iii) if  $x_R^* < \theta_L$ ,  $R$  would find it profitable to deviate to  $\theta_L$  because  $\pi_R(x_L^*, x_R^*) = [1 - p(x_L^*, x_R^*)] \cdot (x_R^* - x_L^* + \chi_R) < \chi_R/2$ . Hence, from (i)-(iii), we conclude that  $x_L^* > \theta_L$ . A similar argument establishes that  $x_R^* < \theta_R$ .

To complete the proof, it remains to be shown that  $x_L^* \leq x_R^*$ . Assume, by way of contradiction, that  $x_L^* > x_R^*$ . There are three cases to consider.

**Case 1.** If  $x_R^* \in [0, \theta_L)$ , candidate  $L$  can deviate to  $\theta_L$  (recall that  $x_L^* > \theta_L$ ), which results in a payoff change equal to  $\pi_L(\theta_L, x_R^*) - \pi_L(x_L^*, x_R^*) = [p(\theta_L, x_R^*) - p(x_L^*, x_R^*)] \cdot [\theta_L - x_R^* + \chi_L] + p(x_L^*, x_R^*) \cdot (x_L^* - \theta_L) > 0$ , contradicting that  $x_L^*$  is candidate  $L$ 's best response to  $x_R^*$  (again  $p(\theta_L, x_R^*) - p(x_L^*, x_R^*) > 0$  because of Lemma 1).

**Case 2.** If  $x_R^* \in [\theta_L, 1/2)$ , then  $L$  can deviate to  $x_L = x_R^* + \epsilon$ ,  $\epsilon > 0$ , which results in a payoff change equal to  $\pi_L(x_L, x_R^*) - \pi_L(x_L^*, x_R^*) = p(x_L, x_R^*) \cdot (\chi_L - \epsilon) - p(x_L^*, x_R^*) \cdot [\chi_L - (x_L^* - x_R^*)]$ . By Lemma 1,  $p(x_L, x_R^*) \geq p(x_L^*, x_R^*)$ . Thus, for  $\epsilon$  small enough,  $\pi_L(x_L, x_R^*) > \pi_L(x_L^*, x_R^*)$ , implying that  $L$ 's deviation is profitable and, consequently, that  $(x_L^*, x_R^*)$  is not a PSE; a contradiction.

**Case 3.** Finally, if  $x_R^* \in [1/2, \theta_R)$ , then  $p(x_L^*, x_R^*) < 1/2$ ; and  $L$  can achieve a payoff greater than  $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [\chi_L - (x_L^* - x_R^*)]$  by choosing  $x_R^*$  (which actually offers a payoff of  $\chi_L/2$ ), contradicting the initial hypothesis that  $(x_L^*, x_R^*)$  is a PSE.

Therefore, from Cases 1-3, we conclude that  $x_L^* \leq x_R^*$ , as required. ■

**Proof of Lemma 3.** Let the profile  $(x_L^*, x_R^*) \in X^2$ , with  $x_L^* < x_R^*$ , be a PSE for  $\mathcal{G}$ . By Lemma 2,  $\theta_L < x_L^* < x_R^* < \theta_R$  and  $p(x_L^*, x_R^*) \in (0, 1)$ . Since the probability function  $p(\cdot)$  is continuous at  $(x_L^*, x_R^*)$ , there must exist  $\epsilon > 0$  sufficiently

<sup>21</sup>Note that  $p(\theta_L, x_R^*) \in (0, 1)$  because by hypothesis  $x_L^* = \theta_L$ . Hence, by Lemma 1,  $p(x_L, x_R^*) - p(\theta_L, x_R^*) > 0$ .

small such that, for all  $(x_L, x_R) \in R_\epsilon(x_L^*) \times R_\epsilon(x_R^*)$ ,  $\theta_L < x_L < x_R < \theta_R$  and  $p(x_L, x_R) \in (0, 1)$ , where  $R_\epsilon(x_i^*) \equiv (x_i^* - \epsilon, x_i^* + \epsilon)$ , with  $i = L, R$ . Thus, for any profile  $(x_L, x_R) \in R_\epsilon(x_L^*) \times R_\epsilon(x_R^*)$ , the left-wing candidate's payoff function can be written as  $\pi_L(x_L, x_R) = p(x_L, x_R) \cdot (x_R - x_L + \chi_L)$ , where  $p(x_L, x_R) = 1/2 + (x_L + x_R - 1)/4\beta$ .

Fix  $x_R^* \in R_\epsilon(x_R^*)$  and consider candidate  $L$ 's best response to  $x_R^*$  over  $R_\epsilon(x_L^*)$ , which is obtained by solving the problem  $\max_{x_L \in R_\epsilon(x_L^*)} \pi_L(x_L, x_R^*)$ . The first-order condition for this problem provides a stationary point  $1/2 - \beta + \chi_L/2$ . Note that this point actually maximizes  $\pi_L(\cdot, x_R^*)$  over  $R_\epsilon(x_L^*)$  because by hypothesis, for all  $x_L \in R_\epsilon(x_L^*)$ ,  $\pi_L(x_L^*, x_R^*) \geq \pi_L(x_L, x_R^*)$ ; i.e.,  $\pi_L(\cdot, x_R^*)$  has an interior maximum on  $R_\epsilon(x_L^*)$ . Moreover, since  $\pi_L(\cdot, x_R^*)$  is *strictly* concave on  $R_\epsilon(x_L^*)$ , with  $\partial^2 \pi_L(x_L, x_R^*) / \partial x_L^2 = -1/2\beta < 0$ , we have that  $x_L^* = 1/2 - \beta + \chi_L/2$ , as required. A similar argument shows that  $x_R^* = 1/2 + \beta - \chi_R/2$ .

Finally, the condition  $x_L^* > \theta_L$  (resp.,  $x_R^* < \theta_R$ ) is obtained from the early assumption about  $\beta$ , (namely,  $0 < \beta < \min\{1/2 - \theta_L + \chi_L/2, \theta_R - 1/2 + \chi_R/2\}$ ), whereas the condition  $\chi_L + \chi_R < 4\beta$  follows from the initial hypothesis, according to which  $x_L^* < x_R^*$ . Routine calculations also show that  $\chi_L + \chi_R < 4\beta$  implies that  $(x_L^* + x_R^*)/2 \in (1/2 - \beta, 1/2 + \beta)$ , so that  $p(x_L^*, x_R^*) \in (0, 1)$  as needed. ■

**Proof of Proposition 1.** To show sufficiency, fix the strategy profile  $(x_L^*, x_R^*) = (1/2, 1/2)$ , where both candidates propose the median voter's ideal point and receive a payoff of  $\pi_i(x_L^*, x_R^*) = \chi_i/2$ . Consider first a deviation for the left-wing candidate to any platform  $x'_L \in (\theta_L, 1/2)$ . For convenience, let's write  $x'_L = 1/2 - \delta$ , with  $\delta > 0$ . Routine calculations show that  $\pi_L(x'_L, x_R^*) \equiv \frac{\chi_L}{2} - \frac{\delta^2}{4\beta} + \left(\frac{1}{2} - \frac{\chi_L}{4\beta}\right) \delta > \chi_L/2$  if and only if  $\delta < 2\beta - \chi_L$ . However, the last inequality requires  $\delta < 0$  because by hypothesis  $\chi_L \geq 2\beta$ . Hence,  $\pi_L(x'_L, x_R^*) \leq \pi_L(x_L^*, x_R^*)$ . A similar argument proves that for any  $x'_R \in (1/2, \theta_R)$ ,  $\pi_R(x_L^*, x'_R) \leq \pi_R(x_L^*, x_R^*)$ . The careful reader should also check at this point that any deviation above  $1/2$  or below  $\theta_L$  (resp., below  $1/2$  or above  $\theta_R$ ) cannot raise candidate  $L$ 's (resp.,  $R$ 's) conditional payoff any further, proving in that way that the profile  $(x_L^*, x_R^*) = (1/2, 1/2)$  is a PSE for  $\mathcal{G}$ .

To show necessity, fix a PSE for  $\mathcal{G}$  with the property that  $x_L^* = x_R^* \equiv x^*$  for some  $x^* \in X$ . If  $x^* > 1/2$ , then candidate  $L$  can profitably deviate to  $1/2$ , because  $p(1/2, x^*) \in (1/2, 1]$  and therefore  $\pi_L(1/2, x^*) = p(1/2, x^*) \cdot [x^* - 1/2 + \chi_L] > 1/2 \cdot \chi_L = \pi_L(x^*, x^*)$ . A similar reasoning shows that candidate  $R$  can profitably deviate to  $1/2$  if  $x^* < 1/2$ . Therefore,  $x^* = 1/2$ .

Next, suppose that  $\chi_L < 2\beta$ , which in turn implies that  $1/2 + \chi_L/2 - \beta < 1/2$ . Since  $p(\cdot)$  is continuous at  $(1/2, 1/2)$  and strictly positive, there must exist  $\delta > 0$  such that for all  $x_L \in (1/2 - \delta, 1/2]$ ,  $p(x_L, 1/2) > 0$  and  $\pi_L(x_L, 1/2) = \left(\frac{1}{2} + \frac{x_L - 1/2}{4\beta}\right) \cdot (1/2 - x_L + \chi_L)$ . Simple calculations show that  $\pi_L(\cdot, 1/2)$  achieves a unique maximum over  $(1/2 - \delta, 1/2]$



at  $\hat{x}_L = 1/2 + \chi_L/2 - \beta$ , implying in particular that  $\pi_L(\hat{x}_L, 1/2) > \pi_L(1/2, 1/2)$ , a contradiction. Hence,  $\chi_L \geq 2\beta$ . A similar argument proves that  $\chi_R \geq 2\beta$ . ■

**Proof of Proposition 2.** To prove necessity, suppose  $\mathcal{G}$  has a PSE with the property that  $x_L^* < 1/2 < x_R^*$ . By Lemma 3,  $x_L^* = \frac{1}{2} - \beta + \frac{\chi_L}{2}$  and  $x_R^* = \frac{1}{2} + \beta - \frac{\chi_R}{2}$ . Therefore, using the initial hypothesis, it follows that  $\chi_i < 2\beta$  for all  $i = L, R$ .

To show sufficiency, fix the equilibrium candidate  $(x_L^*, x_R^*) = (\frac{1}{2} - \beta + \frac{\chi_L}{2}, \frac{1}{2} + \beta - \frac{\chi_R}{2})$ . By the initial hypothesis, i.e.,  $\chi_i < 2\beta$  for all  $i = L, R$ , it follows that  $x_L^* < 1/2 < x_R^*$ ,  $\chi_L + \chi_R < 4\beta$ , and  $p(x_L^*, x_R^*) \in (0, 1)$ . By the assumption on  $\beta$ ,  $\theta_L < x_L^*$  and  $x_R^* < \theta_R$ . Applying the reasoning of the proof to Lemma 3, for some  $\epsilon > 0$  such that  $R_\epsilon(x_L^*) \equiv (x_L^* - \epsilon, x_L^* + \epsilon) \subset (\theta_L, x_R^*)$ , we have that  $x_L^* = \arg \max_{x_L \in R_\epsilon(x_L^*)} \pi_L(x_L, x_R^*)$ , with  $\pi_L(x_L^*, x_R^*) = \frac{\chi_L}{2} + (\beta - \frac{\chi_R}{2}) + \frac{(\chi_L - \chi_R)^2}{16\beta}$ . Thus,  $\pi_L(x_L^*, x_R^*) > \chi_L/2 = \pi_L(x_R^*, x_R^*)$ .

Consider a deviation for the left-wing candidate to any platform  $x'_L \in [0, 1]$  different from  $x_L^*$  and  $x_R^*$ . On one hand, if  $p(x'_L, x_R^*) = 0$ , then  $\pi_L(x'_L, x_R^*) = 0 < \pi_L(x_L^*, x_R^*)$ , implying that the alternative policy does not raise  $L$ 's payoff. On the other hand, if  $p(x'_L, x_R^*) \in (0, 1]$ , two cases are in order:

**Case 1.** Assume  $x'_L \in (x_R^*, 1]$ . Then: (i) if  $p(x'_L, x_R^*) = 1$ , it must be the case that  $1 - (x'_L + x_R^*)/2 \geq 1/2 + \beta$ , which leads to the contradiction  $(x'_L - 1/2) + (\beta - \chi_R/2) \leq -2\beta$ , since the left-hand side of the previous inequality is strictly positive and the right-hand side is smaller than zero; alternatively (ii) if  $p(x'_L, x_R^*) \in (0, 1)$ , then  $\pi_L(x'_L, x_R^*) = \left(\frac{1}{2} + \frac{1-x'_L-x_R^*}{4\beta}\right) \cdot (x_R^* - x'_L + \chi_L)$ . Recall that  $1 - x'_L - x_R^* < 0$  and  $x_R^* - x'_L < 0$ , because  $x'_L > x_R^* > 1/2$ . Therefore,  $\pi_L(x'_L, x_R^*) < 1/2 \cdot \chi_L < \pi_L(x_L^*, x_R^*)$ , implying once again that candidate  $L$ 's deviation to  $x'_L$  is not beneficial.

**Case 2.** Suppose  $x'_L \in [0, x_R^*)$ . Then: (i) if  $p(x'_L, x_R^*) = 1$ , it must be that  $(x'_L + x_R^*)/2 \geq 1/2 + \beta$  and, consequently, that  $x'_L \geq 1/2 + \beta + \chi_R/2 > x_R^*$ , which supplies the desired contradiction (because by hypothesis  $x'_L < x_R^*$ ); alternatively (ii) if  $p(x'_L, x_R^*) \in (0, 1)$ , then: (ii.a) if  $\theta_L \leq x'_L < x_R^*$ , candidate  $L$ 's deviation payoff is  $\pi_L(x'_L, x_R^*) = \left(\frac{1}{2} + \frac{x'_L+x_R^*-1}{4\beta}\right) \cdot (x_R^* - x'_L + \chi_L)$ ; and, given that the function  $f(x_L) = \left(\frac{1}{2} + \frac{x_L+x_R^*-1}{4\beta}\right) \cdot (x_R^* - x_L + \chi_L)$  is strictly concave on  $x_L \in [\theta_L, x_R^*)$  and has a maximum at  $1/2 - \beta + \chi_L/2$ , we conclude that  $\pi_L(x'_L, x_R^*) < \pi_L(x_L^*, x_R^*)$ ; finally (ii.b) if  $0 \leq x'_L < \theta_L$ , it is easy to show that  $\pi_L(x'_L, x_R^*) < \pi_L(\theta_L, x_R^*) < \pi_L(x_L^*, x_R^*)$ , where the last inequality follows from the argument in (ii.a).

Summing up, Case 1 and Case 2 above, together with the fact that  $\pi_L(x_L^*, x_R^*) > \pi_L(x_R^*, x_R^*)$ , prove that  $x_L^* = \arg \max_{x_L \in [0, 1]} \pi_L(x_L, x_R^*)$ . A similar reasoning also shows that  $x_R^* = \arg \max_{x_R \in [0, 1]} \pi_R(x_L^*, x_R)$ . Therefore, the profile  $(x_L^*, x_R^*)$  is a PSE for  $\mathcal{G}$ . ■

**Proof of Proposition 3.** We prove the proposition for  $1/2 < x_L^* < x_R^*$ . The argument for  $x_L^* < x_R^* < 1/2$  is similar. First, assume the election game  $\mathcal{G}$  has a PSE with the property that  $1/2 < x_L^* < x_R^*$ . By Lemma 3,  $x_L^* = 1/2 - \beta + \chi_L/2$  and  $\chi_L + \chi_R < 4\beta$ . That implies that  $\frac{\chi_L}{2} > \beta > \frac{\chi_L + \chi_R}{4}$  and, therefore, that  $\chi_R < \chi_L$ . Using simple algebraic manipulation, it also follows that

$$\frac{\chi_L + \chi_R}{4} < \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_R \cdot \chi_L}}{2} < \frac{\chi_L}{2}. \quad (3)$$

Suppose, by way of contradiction, that  $2\beta < (\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2}$ . By definition,  $\pi_R(x_L^*, x_R^*) = \beta - (\chi_L - \chi_R)/2 + (\chi_L - \chi_R)^2/16\beta$ . Fix any  $x_R \in [1/2, x_L^*]$ . Candidate  $R$ 's payoff at  $(x_L^*, x_R)$  is  $\pi_R(x_L^*, x_R) = \left(\frac{1}{2} + \frac{x_L^* + x_R - 1}{4\beta}\right)(x_R - x_L^* + \chi_R)$ . Therefore,  $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R) = \frac{\chi_L \chi_R}{4\beta}$ . Notice that the difference between  $\pi_R(x_L^*, x_R^*)$  and  $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R)$  gives rise to a second-order polynomial equation in  $\beta$ , namely,  $4\beta^2 - 2\beta(\chi_L - \chi_R) + (\chi_L - \chi_R)^2/4 - \chi_L \cdot \chi_R$ , which has the following two roots:  $\frac{\chi_L - \chi_R}{4} \pm \frac{\sqrt{\chi_R \cdot \chi_L}}{2}$ . Therefore, for any  $\beta \in \left(\frac{\chi_L + \chi_R}{4}, \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_R \cdot \chi_L}}{2}\right)$ , we have that  $\pi_R(x_L^*, x_R^*) < \lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R)$ , contradicting that the strategy profile  $(x_L^*, x_R^*)$  is by hypothesis a PSE of  $\mathcal{G}$ . Hence,  $2\beta \geq (\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2}$ .

To carry out the second part of the proof, suppose  $(\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2} \leq 2\beta < \chi_L$ , and consider the equilibrium candidate  $(x_L^*, x_R^*) = (\frac{1}{2} - \beta + \frac{\chi_L}{2}, \frac{1}{2} + \beta - \frac{\chi_R}{2})$ . By the initial hypothesis and (3), we have that  $\chi_L + \chi_R < 4\beta$ . Therefore, since by assumption  $2\beta < \chi_L$ , it follows that  $\chi_R < 2\beta$  and, consequently, that  $1/2 < x_L^* < x_R^*$  and  $p(x_L^*, x_R^*) \in (0, 1)$ . Moreover, using the argument of the proof to Proposition 2,  $x_L^* = \arg \max_{x_L \in [0, 1]} \pi_L(x_L, x_R^*)$ . To show that  $x_R^* = \arg \max_{x_R \in [0, 1]} \pi_R(x_L^*, x_R)$  we proceed as follows. Firstly notice that, by applying the reasoning of the proof to Lemma 3, it can be shown that for some  $\epsilon > 0$  with the property that  $R_\epsilon(x_R^*) \equiv (x_R^* - \epsilon, x_R^* + \epsilon) \subset (x_L^*, \theta_R)$ ,  $\frac{1}{2} + \beta - \frac{\chi_R}{2} = \arg \max_{x_R \in R_\epsilon(x_R^*)} \pi_R(x_L^*, x_R)$ , with  $\pi_R(x_L^*, x_R^*) = \frac{\chi_R}{2} + (\beta - \frac{\chi_L}{2}) + \frac{(\chi_R - \chi_L)^2}{16\beta}$ . Secondly, to prove that  $\pi_R(x_L^*, x_R^*) > \frac{\chi_R}{2}$ , observe that  $\frac{\chi_R}{2} < \frac{\chi_L \chi_R}{4\beta}$  because  $\chi_L/2\beta > 1$ . Moreover, since  $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R) = \frac{\chi_L \chi_R}{4\beta}$ , it also follows that  $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R) > \frac{\chi_R}{2}$ . Thus, the desired result, i.e.,  $\pi_R(x_L^*, x_R^*) > \frac{\chi_R}{2}$  is obtained using the fact that, by hypothesis,  $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R) \leq \pi_R(x_L^*, x_R^*)$ . The rest of the proof follows the argument of the proof to Prop. 2 and is left to the readers.<sup>22</sup> ■

**Proof of Proposition 4.**<sup>23</sup> Under the hypothesis of Prop. 4, i.e.,  $\chi_R/2 < \beta < \beta_1^C$ , the existence of a MSE for the election game  $\mathcal{G} = (X, \Pi_i)$  follows from the following argument. First, by Prop. 1,  $\mathcal{G}$  does not possess a PSE with  $x_L = x_R$  because  $\chi_R < 2\beta$ . Second, notice that  $\beta < \beta_1^C$  implies  $\chi_L/2 > \beta$  (because  $\beta_1^C < \chi_L/2$ ). Thus, by Props.

<sup>22</sup>A complete version of it is available from the authors upon request.

<sup>23</sup>The proof of Proposition 5 is similar.

2 and 3, there exists no PSE with  $x_L < x_R$  either. But that means, by Lemma 2, that  $\mathcal{G}$  does not possess an equilibrium in pure strategies. Finally, remember that by Prop. 3 in Saporiti (2008), the mixed extension of  $\mathcal{G}$  is better reply secure; thereby  $\mathcal{G}$  must admit a Nash equilibrium where at least one candidate randomizes over two or more pure strategies.

Denote by  $(\mu_L^*, \mu_R^*) \in \Delta^2$  a MSE of  $\mathcal{G}$ , and let  $\underline{x}_i$  (resp.  $\bar{x}_i$ ) be the lower (resp. upper) bound of  $\text{supp}(\mu_i^*)$ . That is, let  $\underline{x}_i = \inf(\text{supp}(\mu_i^*))$  and  $\bar{x}_i = \sup(\text{supp}(\mu_i^*))$ , with  $i = L, R$ . The rest of the proof is organized in a series of claims.

**Claim 1**  $\text{supp}(\mu_R^*) \subseteq [1/2, \theta_R]$ .

Claim 1 is intuitive and follows from the fact that each location  $x_R$  smaller than  $1/2$  (resp. greater than  $\theta_R$ ) is strictly dominated for candidate  $R$  and, therefore, it's never played with positive probability in a MSE. For the sake of brevity, the details of the proof are left for the reader, and they are available from the author upon request.

**Claim 2**  $\mu_L^*(\underline{x}_L) < 1$ .

**Proof** Suppose not. Two cases are possible. First, if  $\underline{x}_L \leq \tilde{x}_L(\beta, \chi_R)$ , then  $R$ 's best response to  $\underline{x}_L$  is  $x_R^* = 1/2 + \beta - \chi_R/2$ . However, the profile  $(\underline{x}_L, x_R^*)$  can't be an equilibrium because under the hypothesis of Prop. 4,  $\mathcal{G}$  has no equilibrium in pure strategies. Second, if  $\tilde{x}_L(\beta, \chi_R) < \underline{x}_L \leq \theta_R$ ,<sup>24</sup> then  $R$ 's best response is to undercut  $L$ 's location by choosing a position just below  $\underline{x}_L$ , which is not well defined because the policy space is a continuum. ■

**Claim 3**  $\bar{x}_L \leq \bar{x}_R = x_R^*$ .

**Proof** To start, recall that a strategy profile  $(\mu_L^*, \mu_R^*)$  is a MSE of  $\mathcal{G}$  if and only if for each candidate  $i \neq j$ , (1)  $U_i(x, \mu_j^*) = U_i(y, \mu_j^*)$  for all  $x, y \in \text{supp}(\mu_i^*)$ , and (2)  $U_i(x, \mu_j^*) \geq U_i(y, \mu_j^*)$  for all  $x \in \text{supp}(\mu_i^*)$  and all  $y \notin \text{supp}(\mu_i^*)$ .

To prove the first part of Claim 3, note that if  $\bar{x}_L > \bar{x}_R$ , then candidate  $L$  can do better by undercutting  $\bar{x}_R$  from above, since for any  $\epsilon > 0$  such that  $\bar{x}_R < \bar{x}_L - \epsilon$

$$\begin{aligned} U_L(\bar{x}_L, \mu_R^*) &= \int_{x_R} \left( \frac{1}{2} + \frac{1 - x_R - \bar{x}_L}{4\beta} \right) \cdot (x_R - \bar{x}_L + \chi_L) \cdot d\mu_R^* < \\ &< \int_{x_R} \left( \frac{1}{2} + \frac{1 - x_R - (\bar{x}_L - \epsilon)}{4\beta} \right) \cdot (x_R - (\bar{x}_L - \epsilon) + \chi_L) \cdot d\mu_R^* = U_L(\bar{x}_L - \epsilon, \mu_R^*). \end{aligned}$$

To show the second part, i.e., that  $\bar{x}_R = x_R^*$ , consider two cases.

<sup>24</sup>Given that  $\text{supp}(\mu_R^*) \subseteq [1/2, \theta_R]$ , it's never optimal for  $L$  to play above  $\theta_R$ .

**Case 1.** Suppose  $\bar{x}_L < \bar{x}_R$ . On one hand, if  $\bar{x}_L \geq x_R^*$ , then  $\bar{x}_R > x_R^*$ . Consider any  $\epsilon > 0$  small enough such that  $\bar{x}_L < \bar{x}_R - \epsilon$ . Routine calculations show that

$$U_R(\mu_L^*, \bar{x}_R - \epsilon) - U_R(\mu_L^*, \bar{x}_R) = \frac{\epsilon}{4\beta} \cdot (2\bar{x}_R + \chi_R - 2\beta - (1 + \epsilon)),$$

which is strictly greater than zero because  $\bar{x}_R > 1/2 + \beta - \chi_R/2 = x_R^*$ , a contradiction.

On the other hand, if  $\bar{x}_L < x_R^*$ , then for any  $x_L \in \text{supp}(\mu_L^*)$ ,  $\pi_R(x_L, \bar{x}_R) \leq \pi_R(x_L, x_R^*)$ , with strict inequality if  $\bar{x}_R \neq x_R^*$  (recall  $\pi_R(x_L, \cdot)$  has a unique maximum at  $x_R^*$  above the diagonal). Integrating with respect to  $\mu_L^*$ , we have that  $U_R(\mu_L^*, \bar{x}_R) \leq U_R(\mu_L^*, x_R^*)$ , with strict inequality if  $\bar{x}_R \neq x_R^*$ . Hence, since  $\bar{x}_R \in \text{supp}(\mu_R^*)$ , it must be the case that  $\bar{x}_R = x_R^*$ .

**Case 2.** Suppose  $\bar{x}_L = \bar{x}_R \equiv \bar{x}$ . First, consider the case in which  $\bar{x} < x_R^*$ . For any  $x_L \in [\underline{x}_L, \bar{x}]$ ,  $\pi_R(x_L, \bar{x}) < \pi_R(x_L, x_R^*)$ . Integrating with respect to  $\mu_L^*$  and adding  $\mu_L^*(\bar{x}) \cdot \chi_R/2$  to both sides, we have

$$\underbrace{\int_{x_L \neq \bar{x}} \pi_R(x_L, \bar{x}) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}}_{=U_R(\mu_L^*, \bar{x})} < \int_{x_L \neq \bar{x}} \pi_R(x_L, x_R^*) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}. \quad (4)$$

Notice that  $\pi_R(\bar{x}, x_R^*) = \frac{1}{\beta} \left( \frac{x_R^* - \bar{x}}{2} + \frac{\chi_R}{2} \right)^2 > \frac{\chi_R}{2}$ . Therefore,

$$\underbrace{\int_{x_L \neq \bar{x}} \pi_R(x_L, x_R^*) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \pi_R(\bar{x}, x_R^*)}_{=U_R(\mu_L^*, x_R^*)} \geq \int_{x_L \neq \bar{x}} \pi_R(x_L, x_R^*) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}, \quad (5)$$

with strict inequality if  $\mu_L^*(\bar{x}) \neq 0$ . Thus, combining (4) and (5), we get that  $U_R(\mu_L^*, x_R^*) > U_R(\mu_L^*, \bar{x})$ , contradicting that  $\bar{x} \in \text{supp}(\mu_R^*)$ .

Second, consider the alternative case in which  $\bar{x} > x_R^*$ . Since  $\mu_L^*$  has at most countably many atoms and  $X$  is dense in the reals, assume without loss of generality that for some  $\epsilon > 0$  small enough,  $\mu_L^*(\bar{x} - \epsilon) = 0$ . Then,

$$\begin{aligned} U_R(\mu_L^*, \bar{x} - \epsilon) &= \int_{x_L}^{\bar{x} - \epsilon} \left( \frac{1}{2} + \frac{1 - x_L - (\bar{x} - \epsilon)}{4\beta} \right) \cdot (\bar{x} - \epsilon - x_L + \chi_R) \cdot d\mu_L^* + \\ &\quad + \int_{\bar{x} - \epsilon}^{\bar{x}} \left( \frac{1}{2} + \frac{x_L + (\bar{x} - \epsilon) - 1}{4\beta} \right) \cdot (\bar{x} - \epsilon - x_L + \chi_R) \cdot d\mu_L^* + \\ &\quad + \mu_L^*(\bar{x}) \cdot \left( \frac{1}{2} + \frac{2\bar{x} - \epsilon - 1}{4\beta} \right) \cdot (\chi_R - \epsilon), \end{aligned} \quad (6)$$

and

$$\begin{aligned}
U_R(\mu_L^*, \bar{x}) &= \int_{\underline{x}_L}^{\bar{x}-\epsilon} \left( \frac{1}{2} + \frac{1 - x_L - \bar{x}}{4\beta} \right) \cdot (\bar{x} - x_L + \chi_R) \cdot d\mu_L^* + \\
&+ \int_{\bar{x}-\epsilon}^{\bar{x}} \left( \frac{1}{2} + \frac{1 - x_L - \bar{x}}{4\beta} \right) \cdot (\bar{x} - x_L + \chi_R) \cdot d\mu_L^* + \mu_L^*(\bar{x}) \cdot \frac{\chi_R}{2}.
\end{aligned} \tag{7}$$

Note that the difference between the first term in the right hand side (henceforth, RHS) of the expression in (6) and the first term in the RHS of (7) is equal to

$$\frac{\epsilon}{4\beta} \cdot (2\bar{x} + \chi_R - 2\beta - (1 + \epsilon)) \cdot \int_{\underline{x}_L}^{\bar{x}-\epsilon} d\mu_L^*, \tag{8}$$

which is strictly positive for  $\epsilon < \bar{x} - x_R^*$  because by hypothesis  $\bar{x} > x_R^*$ .

Let's now consider the second term in the RHS of (6) and the second term in the RHS of (7). The difference between these two terms is equal to

$$\int_{\bar{x}-\epsilon}^{\bar{x}} \underbrace{\left( \frac{x_L + \bar{x} - 1}{2\beta} \right)}_{>0} \cdot \underbrace{(\bar{x} - x_L + \chi_R)}_{>\chi_R} \cdot d\mu_L^* + \frac{\epsilon}{4\beta} \cdot (1 + \epsilon - 2\bar{x} - \chi_R - 2\beta) \cdot \int_{\bar{x}-\epsilon}^{\bar{x}} d\mu_L^*. \tag{9}$$

Similarly, the difference between the last terms in the RHS of (6) and (7) is

$$\mu_L^*(\bar{x}) \cdot \left[ \left( \frac{1}{2} + \underbrace{\frac{2\bar{x} - \epsilon - 1}{4\beta}}_{>0} \right) \cdot (\chi_R - \epsilon) - \frac{\chi_R}{2} \right]. \tag{10}$$

Note that (9) and (10) are both continuous in  $\epsilon$ . Moreover, (9) is zero for  $\epsilon = 0$ , thereby it must be approximately zero for  $\epsilon > 0$  arbitrarily small. In addition, the expression in (10) is strictly positive for  $\epsilon = 0$  if  $\mu_L^*(\bar{x}) \neq 0$  (otherwise, if  $\mu_L^*(\bar{x}) = 0$ , then we can just ignore these terms); and by continuity it must be nonnegative for  $\epsilon$  sufficiently small. Hence, combining all this with (8), we conclude that for some  $\epsilon > 0$  small enough  $U_R(\mu_L^*, \bar{x} - \epsilon) > U_R(\mu_L^*, \bar{x})$ , contradicting that  $\bar{x} \in \text{supp}(\mu_R^*)$ . Therefore,  $\bar{x} = x_R^*$ .  $\blacksquare$

**Claim 4**  $\underline{x}_R = \underline{x}_L \equiv \underline{x} \geq 1/2$ .

**Proof** Assume, by way of contradiction,  $\underline{x}_R \neq \underline{x}_L$ . On one hand, if  $\underline{x}_R < \underline{x}_L$ , then by Claim 1,  $1/2 < \underline{x}_L \leq \theta_R$ , and therefore for any  $\epsilon > 0$  such that  $\underline{x}_R + \epsilon < \underline{x}_L$ ,  $U_R(\mu_L^*, \underline{x}_R) < U_R(\mu_L^*, \underline{x}_R + \epsilon)$ , because  $\underline{x}_R + \epsilon$  raises  $R$ 's probability of winning the election and, at the same time, it's closer to  $\theta_R$ . But that contradicts that by definition  $\underline{x}_R = \inf \text{supp}(\mu_R^*)$ .

On the other hand, if  $\underline{x}_R > \underline{x}_L$ , then we proceed as follows. Consider any  $\epsilon > 0$  such that  $\underline{x}_L + \epsilon < \underline{x}_R$ . Routine calculations show that

$$U_L(\underline{x}_L + \epsilon, \mu_R^*) - U_L(\underline{x}_L, \mu_R^*) = \frac{\epsilon}{4\beta} \cdot (1 - \epsilon - 2\underline{x}_L + \chi_L - 2\beta); \quad (11)$$

and, since by the definition of MSE we have that  $U_L(\underline{x}_L + \epsilon, \mu_R^*) \leq U_L(\underline{x}_L, \mu_R^*)$ , it follows that  $\underline{x}_L \geq (1 - \epsilon)/2 - \beta + \chi_L/2$  and, therefore, that  $\underline{x}_R > 1/2 - \beta + \chi_L/2$ , where the latter is obtained using the previous hypothesis that  $\underline{x}_R > \underline{x}_L$  and an  $\epsilon$  sufficiently small.

Fix any  $\hat{x}_R \in \text{supp}(\mu_R^*)$ . For each  $x_L < \hat{x}_R$ , the conditional payoff function  $\pi_L(x_L, \hat{x}_R) = [1/2 + (x_L + \hat{x}_R - 1)/4\beta](\hat{x}_R - x_L + \chi_L)$  has a unique maximum at  $x_L^* = 1/2 - \beta + \chi_L/2$ . Therefore,  $\pi_L(x_L^*, \hat{x}_R) \geq \pi_L(\underline{x}_L, \hat{x}_R)$ , with strict inequality if  $\underline{x}_L \neq x_L^*$ . Integrating with respect to  $\mu_R^*$ , we have  $U_L(x_L^*, \mu_R^*) \geq U_L(\underline{x}_L, \mu_R^*)$ , with strict inequality if  $\underline{x}_L \neq x_L^*$ . Hence, it must be that  $\underline{x}_L = x_L^*$ .

Recall that by hypothesis  $\underline{x}_R > \underline{x}_L$ ; and that by Claim 2 (resp. Claim 3)  $\bar{x}_L > \underline{x}_L$  (resp.  $x_R^* = \bar{x}_R \geq \bar{x}_L$ ). Moreover, it's easy to show that  $\underline{x}_R \leq \bar{x}_L$ .<sup>25</sup> Consider now an  $\epsilon > 0$  such that  $\underline{x}_L < \underline{x}_R - \epsilon$ . Then,

$$\begin{aligned} U_L(\bar{x}_L, \mu_R^*) &= \int_{\underline{x}_R}^{\bar{x}_L} \left( \frac{1}{2} + \frac{1 - \bar{x}_L - x_R}{4\beta} \right) (x_R - \bar{x}_L + \chi_L) d\mu_R^* + \mu_R^*(\bar{x}_L) \frac{\chi_L}{2} + \\ &+ \int_{\bar{x}_L}^{\bar{x}_R} \left( \frac{1}{2} + \frac{\bar{x}_L + x_R - 1}{4\beta} \right) (x_R - \bar{x}_L + \chi_L) d\mu_R^*, \end{aligned} \quad (12)$$

and

$$\begin{aligned} U_L(\underline{x}_R - \epsilon, \mu_R^*) &= \int_{\underline{x}_R}^{\bar{x}_L} \left( \frac{1}{2} + \frac{\underline{x}_R - \epsilon + x_R - 1}{4\beta} \right) (x_R - (\underline{x}_R - \epsilon) + \chi_L) d\mu_R^* + \\ &+ \mu_R^*(\bar{x}_L) \left[ \left( \frac{1}{2} + \frac{\underline{x}_R - \epsilon + \bar{x}_L - 1}{4\beta} \right) (\bar{x}_L - (\underline{x}_R - \epsilon) + \chi_L) \right] + \\ &+ \int_{\bar{x}_L}^{\bar{x}_R} \left( \frac{1}{2} + \frac{\underline{x}_R - \epsilon + x_R - 1}{4\beta} \right) (x_R - (\underline{x}_R - \epsilon) + \chi_L) d\mu_R^*. \end{aligned} \quad (13)$$

Notice that the difference between the first terms in the RHS of (12) and (13) is negative, since for all  $x_R \in [\underline{x}_R, \bar{x}_L)$  and all  $\epsilon > 0$  small enough, (i)  $\frac{1}{2} + \frac{1 - \bar{x}_L - x_R}{4\beta} < \frac{1}{2} + \frac{\underline{x}_R - \epsilon + x_R - 1}{4\beta}$ , and (ii)  $x_R - \bar{x}_L + \chi_L < x_R - (\underline{x}_R - \epsilon) + \chi_L$ .

<sup>25</sup>Otherwise, for any  $\hat{x}_R \in \text{supp}(\mu_R^*)$ ,  $\pi_L(\underline{x}_L, \hat{x}_R) > \pi_L(\bar{x}_L, \hat{x}_R)$ , and integrating with respect to  $\mu_R^*$  we would find the desired contradiction, i.e.  $U_L(\underline{x}_L, \mu_R^*) > U_L(\bar{x}_L, \mu_R^*)$ .

Similarly, the difference between the second terms is non-positive; that is,

$$\mu_R^*(\bar{x}_L) \left[ \frac{\chi_L}{2} - \left( \frac{1}{2} + \underbrace{\frac{x_R - \epsilon + \bar{x}_L - 1}{4\beta}}_{>0} \right) \underbrace{(\bar{x}_L - (x_R - \epsilon) + \chi_L)}_{>0} \right] \leq 0,$$

with strict inequality if  $\mu_R^*(\bar{x}_L) \neq 0$ . Finally, the difference between the last two terms in the RHS of (12) and (13) is also smaller than or equal to zero. Indeed, for all  $x_R \in (\bar{x}_L, \bar{x}_R]$ , the conditional payoffs are such that  $\pi_L(\bar{x}_L, x_R) \leq \pi_L(x_R - \epsilon, x_R)$ , since  $\pi_L(\cdot, x_R)$  has a unique maximum at  $x_L^* = x_L$  and decreases above  $x_L^*$  (recall  $x_L^* = x_L < \bar{x}_R = x_R^*$  implies that  $\beta > (\chi_L + \chi_R)/4$ ). Thus integrating with respect to  $\mu_R^*$  over  $(\bar{x}_L, \bar{x}_R]$  we get that  $\int_{\bar{x}_L}^{\bar{x}_R} \pi_L(\bar{x}_L, x_R) d\mu_R^* \leq \int_{\bar{x}_L}^{\bar{x}_R} \pi_L(x_R - \epsilon, x_R) d\mu_R^*$ , as required. And combining the three previous observations, it follows that  $U_L(\bar{x}_L, \mu_R^*) < U_L(x_R - \epsilon, \mu_R^*)$ , a contradiction. Hence,  $x_R = x_L \equiv x$ ; and by Claim 1,  $x \geq 1/2$ . ■

**Claim 5**  $x = \tilde{x}_L(\beta, \chi_R)$ .

**Proof** By Claims 1-4,  $\text{supp}(\mu_L^*) \subseteq [1/2, x_R^*]$  and  $\bar{x}_R > x$ ; hence,  $\mu_R^*(x) < 1$ . Assume, by contradiction,  $x > \tilde{x}_L(\beta, \chi_R)$ . (The other case is similar.) By the definition of MSE, for any  $\epsilon > 0$  small enough,  $U_R(\mu_L^*, x_R^*) \geq U_R(\mu_L^*, x - \epsilon)$ , where

$$\begin{aligned} U_R(\mu_L^*, x_R^*) &= \mu_L^*(x) \cdot \pi_R(x, x_R^*) + \\ &+ \int_{x_L \neq x} \left( \frac{1}{2} + \frac{1 - x_L - x_R^*}{4\beta} \right) \cdot (x_R^* - x_L + \chi_R) \cdot d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}, \end{aligned} \quad (14)$$

and

$$\begin{aligned} U_R(\mu_L^*, x - \epsilon) &= \mu_L^*(x) \cdot \pi_R(x, x - \epsilon) + \\ &+ \int_{x_L \neq x} \left( \frac{1}{2} + \frac{x - \epsilon + x_L - 1}{4\beta} \right) \cdot (x - \epsilon - x_L + \chi_R) \cdot d\mu_L^*. \end{aligned} \quad (15)$$

Note that since by hypothesis  $x > \tilde{x}_L(\beta, \chi_R)$ , we have that  $\limsup_{x_R \rightarrow -x} \pi_R(x, x_R) > \pi_R(x, x_R^*)$ . Therefore,

$$\mu_L^*(x) \cdot [\pi_R(x, x - \epsilon) - \pi_R(x, x_R^*)] > 0. \quad (16)$$

Applying once again the definition of a mixed strategy equilibrium, Claims 3 and 4

imply that  $U_R(\mu_L^*, x_R^*) = U_R(\mu_L^*, \underline{x})$ . Thus,

$$\begin{aligned} \int_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x}) d\mu_L^* &= \mu_L^*(\underline{x}) \cdot \left[ \pi_R(\underline{x}, x_R^*) - \frac{\chi_R}{2} \right] + \\ &+ \int_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*) d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \end{aligned} \quad (17)$$

If  $\underline{x} < \frac{1}{2} + \beta - \frac{\chi_R}{2} + (\chi_R - \sqrt{2\beta\chi_R})$ , then  $\pi_R(\underline{x}, x_R^*) > \frac{\chi_R}{2}$ . Hence, (17) implies that

$$\int_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x}) d\mu_L^* > \int_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*) d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \quad (18)$$

Notice that the left hand side of (18) is left continuous in  $x_R$  at  $\underline{x}$ , since  $\pi_R(x_L, \underline{x}) = \left( \frac{1}{2} + \frac{x_L + \underline{x} - 1}{4\beta} \right) \cdot (\underline{x} - x_L + \chi_R)$ , meaning that for  $\epsilon > 0$  sufficiently small,

$$\int_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x} - \epsilon) d\mu_L^* \geq \int_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*) d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \quad (19)$$

Thus, combining (16) and (19), it follows from (14) and (15) that  $U_R(\mu_L^*, x_R^*) < U_R(\mu_L^*, \underline{x} - \epsilon)$ , a contradiction.

Alternatively, if  $\underline{x} \geq \frac{1}{2} + \beta - \frac{\chi_R}{2} + (\chi_R - \sqrt{2\beta\chi_R})$ , then

$$\pi_R(\underline{x}, x_R^*) \leq \frac{\chi_R}{2}; \quad (20)$$

and from (17) we have that

$$\int_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x}) d\mu_L^* \leq \int_{x_L \neq \underline{x}} \pi_R(x_L, x_R^*) d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2}. \quad (21)$$

Using again the continuity of  $\pi_R(x_L, x_R) = \left( \frac{1}{2} + \frac{x_L + x_R - 1}{4\beta} \right) \cdot (x_R - x_L + \chi_R)$  in  $x_R$  at  $\underline{x}$ , for  $\epsilon > 0$  small enough

$$\int_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x} - \epsilon) d\mu_L^* \approx \int_{x_L \neq \underline{x}} \pi_R(x_L, \underline{x}) d\mu_L^*. \quad (22)$$

By definition,  $\tilde{x}_L(\beta, \chi_R) \equiv 1/2 \cdot (1 + 2\beta + 3\chi_R - 2\sqrt{2}\sqrt{2\beta\chi_R + \chi_R^2})$ . Thus, since by the hypothesis of Prop. 4  $\chi_R < 2\beta$ , we have that  $\tilde{x}_L(\beta, \chi_R) > 1/2$ , which implies that  $\underline{x} > 1/2$  as well (recall we assumed before  $\underline{x} > \tilde{x}_L$ ). Hence, by the discontinuity of  $p(\cdot)$  at  $(\underline{x}, \underline{x})$ ,  $p(\underline{x}, \underline{x} - \epsilon)$  is well above  $1/2$ , meaning that for  $\epsilon > 0$  sufficiently close to zero

$$\pi_R(\underline{x}, \underline{x} - \epsilon) = \left( \frac{1}{2} + \frac{2\underline{x} - (1 + \epsilon)}{4\beta} \right) (\chi_R - \epsilon) > \frac{\chi_R}{2}. \quad (23)$$



Finally, from (17),

$$\mu_L^*(\underline{x}) \cdot \left[ \pi_R(\underline{x}, x_R^*) - \frac{\chi_R}{2} \right] + \int_{x_L \neq \underline{x}} [\pi_R(x_L, x_R^*) - \pi_R(x_L, \underline{x})] d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2} = 0; \quad (24)$$

and combining (20), (22) and (23) and comparing them with (24), the expression below

$$\begin{aligned} & \mu_L^*(\underline{x}) \cdot [\pi_R(\underline{x}, x_R^*) - \pi_R(\underline{x}, \underline{x} - \epsilon)] + \\ & + \int_{x_L \neq \underline{x}} [\pi_R(x_L, x_R^*) - \pi_R(x_L, \underline{x} - \epsilon)] d\mu_L^* + \mu_L^*(x_R^*) \frac{\chi_R}{2} \end{aligned} \quad (25)$$

turns out to be strictly smaller than zero. However, that means that  $U_R(\mu_L^*, x_R^*) < U_R(\mu_L^*, \underline{x} - \epsilon)$ , contradicting that  $x_R^* \in \text{supp}(\mu_R^*)$ . Therefore,  $\underline{x} = \tilde{x}_L(\beta, \chi_R)$ .  $\blacksquare$

**Claim 6** *If  $\beta \leq \frac{\chi_L + \chi_R}{4}$ , then  $\bar{x}_L = x_R^*$ .*

**Proof** Suppose, by way of contradiction, that  $\bar{x}_L < x_R^*$ . (Recall that by Claim 3,  $\bar{x}_L \leq x_R^*$ .) Then, for any  $x', x'' \in (\bar{x}_L, x_R^*)$ , with  $x' < x''$ , we have that  $\pi_R(x_L, x'') > \pi_R(x_L, x')$  for all  $x_L \in \text{supp}(\mu_L^*)$ , because  $\pi_R(x_L, \cdot)$  is strictly increasing on  $(\bar{x}_L, x_R^*)$ .<sup>26</sup> Integrating with respect to  $x_L$  over  $\text{supp}(\mu_L^*)$ , we get that  $U_R(\mu_L^*, x'') > U_R(\mu_L^*, x')$ ; and since this holds for any  $x' < x''$ , it follows that (i)  $R$  doesn't allocate probability mass on  $(\bar{x}_L, x_R^*)$ , and (ii) by Claim 3,  $\mu_R^*$  has an atom at  $x_R^*$ , i.e.,  $\mu_R^*(x_R^*) > 0$ . The rest of the proof shows that candidate  $L$  would profitably undercut  $x_R^*$  from below.

To do that, first we prove that  $\mu_R^*(\bar{x}_L) = 0$ . That follows by considering the difference between the left-wing candidate's conditional expected payoff at  $\bar{x}_L$  and at  $\bar{x}_L - \epsilon$ , with  $\epsilon > 0$  arbitrarily small, which is equal to

$$\begin{aligned} U_L(\bar{x}_L, \mu_R^*) - U_L(\bar{x}_L - \epsilon, \mu_R^*) &= \int_{\underline{x}}^{\bar{x}_L - \epsilon} [\pi_L(\bar{x}_L, x_R) - \pi_L(\bar{x}_L - \epsilon, x_R)] d\mu_R^* + \\ &+ \int_{\bar{x}_L - \epsilon}^{\bar{x}_L} [\pi_L(\bar{x}_L, x_R) - \pi_L(\bar{x}_L - \epsilon, x_R)] d\mu_R^* + \\ &+ \mu_R^*(x_R^*) [\pi_L(\bar{x}_L, x_R^*) - \pi_L(\bar{x}_L - \epsilon, x_R^*)] + \\ &+ \mu_R^*(\bar{x}_L) \left[ \frac{\chi_L}{2} - \pi_L(\bar{x}_L - \epsilon, \bar{x}_L) \right]. \end{aligned} \quad (26)$$

Using the continuity of the payoff function outside the main diagonal and the fact that  $\epsilon$  is by hypothesis arbitrarily small, the first three terms of the RHS of (26) are arbitrarily close to zero. Therefore, since  $\frac{\chi_L}{2} < \pi_L(\bar{x}_L - \epsilon, \bar{x}_L)$ , the fact that  $\bar{x}_L \in \text{supp}(\mu_L^*)$  implies that  $\mu_R^*(\bar{x}_L) = 0$ . (Otherwise, we would have that  $U_L(\bar{x}_L, \mu_R^*) < U_L(\bar{x}_L - \epsilon, \mu_R^*)$ , which would contradict that  $(\mu_L^*, \mu_R^*)$  is by hypothesis a MSE of  $\mathcal{G}$ .)

<sup>26</sup>In fact,  $\pi_R(x_L, \cdot)$  is strictly concave with a maximum at  $x_R^*$ .

Second, we work out candidate  $R$ 's probability mass on  $x_R^*$  by equalizing the left-wing candidate's conditional expected payoffs at  $\underline{x}$  and  $\bar{x}_L$ , which turns out to be

$$\mu_R^*(x_R^*) = \frac{\mu_R^*(\underline{x}) \left[ \frac{\chi_L}{2} - \pi_L(\bar{x}_L, \underline{x}) \right] + \int_{\underline{x}}^{\bar{x}_L} [\pi_L(\underline{x}, x_R) - \pi_L(\bar{x}_L, x_R)] d\mu_R^*}{\pi_L(\bar{x}_L, x_R^*) - \pi_L(\underline{x}, x_R^*)}. \quad (27)$$

Finally, notice that

$$\begin{aligned} U_L(x_R^* - \epsilon, \mu_R^*) - U_L(\bar{x}_L, \mu_R^*) &= \int_{\underline{x}}^{\bar{x}_L} \underbrace{[\pi_L(x_R^* - \epsilon, x_R) - \pi_L(\bar{x}_L, x_R)]}_{<0 \ \forall x_R \in (\underline{x}, \bar{x}_L)} d\mu_R^* + \\ &+ \mu_R^*(x_R^*) \underbrace{[\pi_L(x_R^* - \epsilon, x_R^*) - \pi_L(\bar{x}_L, x_R^*)]}_{>0 \text{ because } \pi_L'(\cdot, x_R^*) > 0}, \end{aligned} \quad (28)$$

and replacing (27) into (28), we get the desired contradiction, namely,  $U_L(x_R^* - \epsilon, \mu_R^*) > U_L(\bar{x}_L, \mu_R^*)$ . Therefore,  $\bar{x}_L = x_R^*$ . ■

**Claim 7** *If  $\beta > \frac{\chi_L + \chi_R}{4}$ , then  $\bar{x}_L = x_L^* < x_R^*$ .*

**Proof** The claim is proved following the same type of reasoning we have applied before in the proof of Claim 6. (The fact that  $x_L^* < x_R^*$  is shown in the proof of Lemma 3.) The only main difference is that the second term in the RHS of (28) is not anymore positive when  $\beta > \frac{\chi_L + \chi_R}{4}$ , because the conditional payoff function  $\pi_L(\cdot, x_R^*)$  is decreasing above  $x_L^*$ . That explains why undercutting the right-wing candidate's upper bound policy  $x_R^*$  is not anymore profitable for candidate  $L$ . ■

**Claim 8** *If  $\beta \leq \frac{\chi_L + \chi_R}{4}$ , then  $\text{supp}(\mu_i^*) = [\underline{x}, \bar{x}]$  for all  $i = L, R$ , with  $\underline{x} = \tilde{x}_L(\beta, \chi_R)$  and  $\bar{x} = \frac{1}{2} + \beta - \frac{\chi_R}{2} = x_R^*$ .*

**Proof** The fact that for all  $i$ ,  $\underline{x}_i = \tilde{x}_L(\beta, \chi_R)$  (respectively,  $\bar{x}_i = x_R^*$ ) follows from Claim 5 (respectively, from Claims 3 and 6). Thus, it remains to be shown that  $\text{supp}(\mu_i^*)$  is an interval. Without loss of generality, consider  $x \in (\underline{x}, \bar{x})$  and assume, by way of contradiction, that  $x \notin \text{supp}(\mu_R^*)$ . The other case, i.e.,  $x \notin \text{supp}(\mu_L^*)$ , is analogous.

By definition of  $\text{supp}(\mu_R^*)$ , there exists  $\epsilon > 0$  such that  $\mu_R^*([x - \epsilon, x + \epsilon] \cap X) = 0$ . Consider any two alternatives  $x', x'' \in [x - \epsilon, x + \epsilon]$ , with  $x' < x''$ . Since  $\pi_L(\cdot, x_R)$  is increasing for all  $x_R \in (x + \epsilon, x_R^*]$ , it is easy to show that  $U_L(x'', \mu_R^*) > U_L(x', \mu_R^*)$ . Therefore,  $x' \notin \text{supp}(\mu_L^*)$ ; and repeating the argument, it follows that  $\mu_L^*$  has an atom at  $x + \epsilon$ . But then  $R$  must find it profitable to undercut  $x + \epsilon$  from below (recall  $x + \epsilon > \tilde{x}_L$ ), contradicting that by hypothesis  $\mu_R^*([x - \epsilon, x + \epsilon] \cap X) = 0$ . ■

**Claim 9** *If  $\beta > \frac{\chi_L + \chi_R}{4}$ , then  $\text{supp}(\mu_L^*) = [\underline{x}, \bar{x}]$  and  $\text{supp}(\mu_R^*) = [\underline{x}, \bar{x}] \cup \{x_R^*\}$ , with  $\underline{x} = \tilde{x}_L(\beta, \chi_R)$  and  $\bar{x} = \frac{1}{2} - \beta + \frac{\chi_L}{2} = x_L^*$ .*

**Proof** The fact that  $\bar{x}_L = x_L^*$  follows from Claim 7. To show that  $\mu_R^*((x_L^*, x_R^*)) = 0$ , we use the argument of the proof of Claim 6. To be more precise, consider any  $x', x'' \in (\bar{x}_L, x_R^*)$ , with  $x' < x''$ . Since for all  $x_L \in [\underline{x}, \bar{x}]$ , the conditional payoff  $\pi_R(x_L, \cdot)$  is strictly increasing on  $(\bar{x}_L, x_R^*)$ , we have that  $\pi_R(x_L, x'') > \pi_R(x_L, x')$ . Integrating with respect to  $x_L$  over  $\text{supp}(\mu_L^*)$ , we get that  $U_R(\mu_L^*, x'') > U_R(\mu_L^*, x')$ . Hence, since the pair  $x' < x''$  was arbitrarily chosen, it follows that candidate  $R$  doesn't allocate probability mass on  $(\bar{x}_L, x_R^*)$ . The rest of the proof is similar to the proof of Claim 8. ■

## B Appendix: Data

Table 9 displays the ordinary least square (OLS) regressions  $y = a + b \cdot 1/t$  corresponding to the learning analysis of Section 6.2.

Table 10 shows for each treatment and for each period the mean position as well as the standard deviation for the Left player and for the Right player. The table also reports the average absolute distance from the Nash equilibrium as well as the standard deviation. For Treatment 6 we report the distance from the support as well as the distance from the entire equilibrium distribution. The table also provides averages of these statistics for selected intervals.

Finally, Table 11 shows for each treatment for selected intervals the average position of the Left and of the Right player for each matching pair.

			position Left		position Right		avg. distance NE	
			Coeff.	t stat	Coeff.	t stat	Coeff.	t stat
treat1	1-30	Intercept	48.475	75.539	53.415	95.008	4.041	10.240
		Slope	-14.483	5.232	8.418	3.471	14.631	8.594
	31-60	Intercept	49.510	137.199	51.677	117.054	2.297	5.430
		Slope	-8.202	5.269	6.222	3.267	7.573	4.151
treat2	1-30	Intercept	40.686	119.715	59.597	222.063	0.811	3.748
		Slope	-4.452	3.037	3.330	2.876	16.211	17.365
	31-60	Intercept	39.687	282.685	59.028	718.785	0.575	8.756
		Slope	2.599	4.291	0.617	1.742	2.076	7.324
treat3	1-30	Intercept	50.063	174.333	50.741	162.951	0.089	0.403
		Slope	-10.807	8.724	2.721	2.026	14.000	14.704
	31-60	Intercept	49.843	541.038	50.166	110.982	0.733	4.495
		Slope	-0.276	0.694	0.483	0.248	2.291	3.255
treat4	1-30	Intercept	39.864	73.908	60.472	259.823	1.459	6.495
		Slope	1.747	0.751	0.137	0.137	5.464	5.638
	31-60	Intercept	39.955	193.912	60.423	372.260	0.740	6.456
		Slope	0.587	0.661	1.127	1.609	2.081	4.211
treat5	1-30	Intercept	21.259	30.350	80.803	196.757	1.318	3.418
		Slope	3.711	1.228	-13.536	7.641	23.396	14.068
	31-60	Intercept	21.208	72.428	80.341	569.673	1.741	10.139
		Slope	5.246	4.153	0.642	1.055	3.046	4.112
treat6	1-30	Intercept	54.854	91.656	59.214	105.804	2.660	10.595
		Slope	-20.077	7.777	6.829	2.829	11.371	10.499
	31-60	Intercept	55.233	214.370	59.255	180.738	2.365	14.051
		Slope	-7.029	6.324	3.289	2.326	4.797	6.607
treat7	1-30	Intercept	55.817	103.139	74.854	94.680	6.548	14.957
		Slope	-26.060	11.163	0.031	0.009	14.378	7.613
	31-60	Intercept	55.881	128.415	75.989	159.904	4.797	18.131
		Slope	-10.818	5.763	-0.133	0.065	5.868	5.141

Table 9: OLS regressions  $y = a + b \cdot 1/t$ .

period(s)	position Left		position Right		avg. distance from NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	35.4	14.6	61.8	21.2	17.1	12.2
2	40.0	13.6	52.2	17.2	11.8	9.2
3	40.7	12.8	57.9	10.2	9.0	10.0
4	45.9	11.7	61.0	16.1	9.2	11.8
5	48.7	18.1	59.8	10.6	9.6	9.6
6	44.9	8.9	58.6	15.0	7.6	9.3
7	45.4	7.5	53.4	10.2	5.5	7.1
8	45.0	22.4	52.1	19.1	12.4	13.1
9	42.3	12.4	53.9	7.9	6.6	8.4
10	45.3	12.7	54.0	8.4	5.5	9.6
11	44.2	11.9	52.8	4.8	4.3	8.0
12	52.4	11.6	55.8	7.8	4.9	7.7
13	47.1	6.1	52.8	5.2	3.1	5.1
14	53.2	14.4	52.7	5.5	3.7	7.4
15	45.2	12.7	54.5	11.3	5.0	11.8
16	49.8	13.4	54.6	11.3	5.2	11.0
17	45.4	11.3	55.8	9.1	5.3	7.9
18	47.8	4.5	54.6	8.2	3.6	5.5
19	47.8	6.1	56.1	12.2	5.0	6.6
20	50.5	1.4	53.5	6.9	2.4	3.4
21	47.8	13.8	55.5	11.7	5.7	8.6
22	47.7	8.3	53.5	15.7	5.2	9.9
23	46.5	16.5	53.6	9.8	6.2	10.5
24	46.5	8.5	54.2	8.7	4.0	7.8
25	45.3	9.6	52.0	6.9	3.3	7.1
26	48.8	5.8	52.3	4.9	2.6	4.7
27	47.5	6.2	48.1	8.5	2.8	4.9
28	52.6	14.7	55.6	12.0	5.6	12.7
29	53.6	14.0	52.3	4.0	3.3	8.2
30	43.0	14.9	51.1	2.5	4.1	7.8
31	41.2	16.2	56.8	11.7	8.6	10.1
32	46.3	8.5	57.3	13.0	6.4	9.2
33	47.7	5.6	54.8	11.9	3.6	8.6
34	47.2	8.5	47.3	14.5	4.7	10.3
35	47.0	9.7	54.1	11.0	3.6	10.3
36	47.2	18.3	54.2	11.1	6.6	10.0
37	43.8	14.0	52.8	8.8	5.6	8.8
38	48.1	19.1	55.4	13.3	8.3	10.2
39	45.1	18.1	52.8	6.6	7.1	9.8
40	49.3	15.1	51.4	3.0	4.2	7.2
41	49.5	1.4	52.2	6.5	1.8	3.2
42	50.5	6.1	53.8	11.2	3.2	6.5
43	51.8	13.8	52.3	8.3	4.1	9.0
44	51.5	12.8	53.8	9.6	4.3	10.4
45	51.2	12.9	54.4	9.6	4.7	7.7
46	50.0	4.1	51.4	3.0	1.5	3.0
47	49.6	1.4	51.9	5.6	1.2	3.0
48	50.2	2.2	49.6	1.4	0.7	1.5
49	50.2	0.6	52.3	4.4	1.2	2.2
50	48.6	3.3	54.6	9.7	3.2	6.2
51	46.9	7.8	51.5	3.8	2.3	5.7
52	48.8	4.2	51.5	5.5	1.3	4.9
53	49.2	1.9	49.9	0.3	0.4	1.0
54	49.9	0.3	49.6	1.4	0.2	0.8
55	48.8	10.4	50.0	0.0	2.1	4.8
56	48.5	5.5	54.6	11.3	3.1	6.6
57	48.8	4.2	50.8	2.8	1.0	3.5
58	49.2	2.8	50.8	2.8	0.8	1.9
59	48.4	5.5	50.0	0.0	0.8	2.8
60	47.8	8.3	53.1	11.1	2.7	6.6
1-10	43.4	13.5	56.5	13.6	9.4	10.0
11-20	48.3	9.3	54.3	8.2	4.2	7.4
21-30	48.0	11.2	52.8	8.5	4.3	8.2
1-30	46.5	11.3	54.5	10.1	6.0	8.6
31-40	46.3	13.3	53.7	10.5	5.9	9.5
41-50	50.3	5.9	52.6	6.9	2.6	5.3
51-60	48.6	5.1	51.2	3.9	1.5	3.9
31-60	48.4	8.1	52.5	7.1	3.3	6.2
1-60	47.5	9.7	53.5	8.6	4.6	7.4

(a) Treatment 1.

Table 10: Players' positions and distance from the Nash equilibrium.

period(s)	position Left		position Right		avg. distance from NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	37.5	15.1	64.1	29.3	15.6	10.1
2	33.5	13.0	60.2	20.7	11.2	7.9
3	39.7	10.0	59.6	10.6	6.5	6.6
4	43.2	6.1	60.3	9.9	4.8	4.2
5	39.0	8.4	59.4	7.6	4.8	4.6
6	39.5	9.6	59.8	8.2	4.5	7.0
7	42.0	5.1	61.4	5.6	3.3	4.4
8	40.8	3.3	61.2	6.7	2.2	3.9
9	41.6	5.8	57.5	4.2	3.1	4.0
10	40.5	4.4	57.7	5.6	3.4	3.7
11	41.2	3.2	60.2	4.8	1.7	3.3
12	39.6	6.6	59.0	3.9	2.7	4.2
13	40.5	3.7	58.9	3.8	1.7	3.3
14	38.4	10.5	58.9	7.4	3.8	7.4
15	41.2	3.2	59.0	3.2	1.1	3.1
16	41.3	3.2	60.3	0.9	0.8	1.8
17	40.4	3.7	61.1	3.1	1.4	2.7
18	40.6	3.5	59.9	2.1	1.2	1.9
19	40.5	3.7	61.3	2.8	1.4	2.4
20	37.4	10.4	62.4	9.9	4.1	6.2
21	39.6	6.0	59.0	3.2	1.8	4.1
22	40.6	3.5	59.7	1.8	1.1	1.8
23	41.2	3.2	61.6	3.5	1.4	2.3
24	39.8	0.6	59.8	0.6	0.2	0.6
25	40.4	4.3	59.2	1.8	1.4	2.4
26	40.7	3.4	59.6	1.3	0.9	1.8
27	40.9	3.2	59.6	1.3	0.8	1.7
28	40.1	0.3	60.5	1.6	0.3	0.8
29	40.5	3.7	59.0	3.2	1.3	2.7
30	40.6	3.5	61.0	3.2	1.2	2.6
31	42.0	7.9	59.7	4.1	2.9	5.2
32	42.1	6.3	58.9	3.1	1.6	3.3
33	40.5	1.6	59.3	3.4	0.9	1.9
34	39.8	0.6	59.4	3.5	0.8	1.8
35	39.7	0.9	58.6	3.3	0.9	1.8
36	39.9	0.3	60.2	4.8	1.2	2.1
37	39.8	0.6	59.8	4.3	1.0	2.1
38	40.3	0.9	58.8	3.2	0.8	1.7
39	40.2	0.6	58.7	3.2	0.8	1.7
40	39.9	0.3	59.6	1.6	0.4	0.8
41	39.8	0.6	59.0	3.2	0.6	1.6
42	40.0	0.0	58.7	3.2	0.7	1.6
43	39.9	0.3	58.9	3.1	0.6	1.6
44	40.0	0.0	59.5	3.7	0.8	1.7
45	40.3	0.9	59.3	3.4	0.8	1.8
46	40.1	0.3	59.2	3.3	0.7	1.6
47	40.2	0.6	58.8	3.2	0.7	1.6
48	40.0	0.0	58.9	3.1	0.6	1.6
49	37.0	9.5	59.0	3.2	2.0	4.8
50	39.8	0.6	59.2	3.3	0.7	1.6
51	39.7	0.9	58.7	3.2	0.8	1.8
52	39.8	0.6	59.4	3.5	0.8	1.8
53	39.9	0.3	59.1	3.2	0.6	1.6
54	40.1	0.3	58.8	3.2	0.7	1.6
55	40.1	0.3	58.7	3.2	0.7	1.6
56	40.2	0.6	58.9	3.1	0.7	1.6
57	40.0	0.0	59.2	3.3	0.6	1.6
58	39.9	0.3	59.2	3.3	0.7	1.6
59	40.0	0.0	59.0	3.2	0.5	1.6
60	40.0	0.0	58.8	3.2	0.6	1.6
1-10	39.7	8.1	60.1	10.9	5.9	5.6
11-20	40.1	5.2	60.1	4.2	2.0	3.6
21-30	40.4	3.2	59.9	2.1	1.0	2.1
1-30	40.1	5.5	60.0	5.7	3.0	3.8
31-40	40.4	2.0	59.3	3.4	1.1	2.2
41-50	39.7	1.3	59.1	3.3	0.8	1.9
51-60	40.0	0.3	59.0	3.2	0.7	1.6
31-60	40.0	1.2	59.1	3.3	0.9	1.9
1-60	40.1	3.3	59.6	4.5	1.9	2.9

(b) Treatment 2.

Table 10: Players' positions and distance from the Nash equilibrium (continued).

period(s)	position Left		position Right		avg. distance from NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	38.2	16.0	53.1	18.8	14.6	8.2
2	47.8	7.5	53.3	15.4	6.8	6.1
3	46.3	13.1	50.8	4.5	4.4	6.0
4	44.0	13.1	53.1	4.6	5.1	7.0
5	50.4	1.7	50.2	2.2	1.0	1.5
6	49.8	0.8	51.8	4.7	1.1	2.3
7	47.9	4.5	50.4	1.6	1.4	3.0
8	45.8	12.6	49.9	6.1	3.6	7.7
9	48.0	4.0	51.3	2.5	1.7	2.6
10	48.5	2.8	50.6	1.9	1.1	2.2
11	50.4	1.4	51.0	2.8	0.9	2.0
12	49.4	1.6	50.1	4.7	1.4	2.1
13	50.0	0.0	50.9	2.8	0.5	1.4
14	50.0	0.0	50.9	2.5	0.5	1.3
15	49.8	0.6	51.0	2.8	0.6	1.4
16	49.7	0.7	50.1	0.3	0.2	0.3
17	49.4	2.0	50.5	1.8	0.9	1.4
18	49.2	2.5	49.5	1.6	0.8	2.0
19	49.9	0.3	53.4	13.1	2.5	6.3
20	49.0	3.2	49.9	0.3	0.6	1.6
21	49.0	3.2	53.4	13.0	2.8	7.8
22	49.0	3.2	49.3	1.9	0.9	1.7
23	49.0	3.2	52.0	6.3	1.5	3.4
24	49.5	1.3	55.5	15.7	3.0	7.8
25	48.9	3.1	50.0	0.0	0.6	1.6
26	49.9	0.6	50.0	0.0	0.2	0.2
27	49.9	0.6	50.0	0.0	0.2	0.2
28	50.1	0.3	50.9	2.8	0.5	1.4
29	49.9	0.3	50.1	0.3	0.1	0.3
30	50.0	0.0	50.1	0.3	0.1	0.2
31	49.3	3.7	52.4	7.9	2.2	4.1
32	49.5	1.6	46.6	13.4	3.1	6.2
33	50.2	1.9	47.0	9.5	1.9	4.7
34	50.0	0.0	48.0	6.3	1.0	3.2
35	50.0	0.0	52.5	4.2	1.3	2.1
36	49.6	3.7	52.0	4.8	1.9	3.5
37	50.5	1.6	53.6	8.1	2.1	4.8
38	50.6	1.6	53.5	9.4	2.1	4.7
39	50.5	1.6	52.5	6.3	1.5	3.2
40	49.5	1.6	51.5	3.4	1.0	2.4
41	49.5	1.3	49.5	3.7	1.0	2.2
42	49.5	1.6	50.4	1.6	0.6	1.2
43	49.5	1.6	50.6	1.6	0.6	1.6
44	50.0	0.0	51.5	3.4	0.8	1.7
45	50.0	0.0	50.5	1.6	0.3	0.8
46	50.0	0.0	49.5	3.7	0.8	1.7
47	50.3	1.3	51.5	3.4	1.0	2.1
48	50.1	0.3	50.4	1.6	0.4	0.8
49	49.9	0.3	50.0	0.0	0.1	0.2
50	50.0	0.0	49.4	1.6	0.3	0.8
51	50.0	0.0	50.0	0.0	0.0	0.0
52	49.5	1.6	50.4	1.6	0.6	1.0
53	49.5	1.6	49.8	0.4	0.4	0.8
54	49.5	1.6	52.0	6.0	1.3	3.8
55	49.7	0.9	49.9	0.3	0.2	0.6
56	50.0	0.0	49.8	0.6	0.1	0.3
57	49.5	1.6	50.0	0.0	0.3	0.8
58	49.8	0.6	46.0	12.6	2.1	6.3
59	48.7	3.2	46.0	12.6	2.7	6.3
60	49.5	1.6	50.1	0.3	0.3	0.9
1-10	46.7	7.6	51.5	6.2	4.0	4.7
11-20	49.7	1.2	50.7	3.3	0.9	2.0
21-30	49.5	1.6	51.1	4.0	1.0	2.5
1-30	48.6	3.5	51.1	4.5	2.0	3.0
31-40	50.0	1.7	51.0	7.4	1.8	3.9
41-50	49.9	0.6	50.3	2.2	0.6	1.3
51-60	49.6	1.3	49.4	3.5	0.8	2.1
31-60	49.8	1.2	50.2	4.3	1.0	2.4
1-60	49.2	2.3	50.7	4.4	1.5	2.7

(c) Treatment 3.

Table 10: Players' positions and distance from the Nash equilibrium (continued).

period(s)	position Left		position Right		avg. distance from NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	37.4	3.7	62.2	8.7	5.0	3.0
2	44.3	6.5	57.1	5.5	4.8	3.6
3	47.9	10.0	59.8	4.7	6.1	4.9
4	40.1	5.2	62.9	3.2	3.5	1.8
5	38.5	14.2	59.9	2.4	3.8	6.3
6	46.8	15.3	59.1	3.2	5.1	8.0
7	37.2	12.8	60.0	2.6	3.2	7.0
8	41.2	4.2	60.2	2.6	2.2	2.1
9	42.7	5.6	61.1	3.2	2.8	2.7
10	38.4	2.7	60.6	1.1	1.2	1.6
11	38.7	3.1	60.1	2.8	2.0	2.1
12	39.9	3.1	60.8	1.9	1.2	1.4
13	38.3	8.4	61.5	2.3	2.7	3.6
14	39.4	2.3	61.6	3.3	1.9	2.1
15	40.0	2.7	59.7	3.2	2.0	1.5
16	39.6	2.4	60.6	2.1	1.1	1.4
17	39.4	2.5	60.3	2.1	1.2	1.2
18	39.6	2.3	61.6	2.8	1.4	1.4
19	39.9	2.5	61.2	2.0	1.4	1.2
20	39.7	2.1	59.9	2.6	1.1	1.3
21	39.6	3.0	60.3	2.1	1.4	1.3
22	38.8	2.0	60.7	2.2	1.2	1.4
23	38.5	3.0	60.0	2.5	1.4	2.0
24	39.3	3.2	60.0	2.4	1.5	1.9
25	39.3	2.2	59.9	2.8	1.3	1.4
26	39.2	2.5	61.1	2.2	1.4	1.4
27	39.2	3.3	60.8	1.9	1.3	1.5
28	40.0	3.7	60.4	2.1	1.3	1.9
29	40.3	0.7	60.9	2.1	0.7	1.1
30	39.7	2.1	60.4	2.2	1.0	1.3
31	40.0	4.9	62.1	3.5	2.6	3.0
32	41.6	7.1	60.2	3.0	2.7	3.3
33	38.9	3.5	60.3	2.4	1.5	2.2
34	40.5	1.3	60.5	2.5	1.0	1.4
35	40.9	1.7	60.5	2.0	0.9	1.2
36	39.8	0.6	60.9	1.9	0.7	0.9
37	39.8	0.6	60.6	1.3	0.4	0.7
38	42.2	6.3	60.7	2.0	1.6	3.1
39	40.4	1.0	61.2	2.0	0.8	1.3
40	39.8	0.4	61.2	2.1	0.7	1.1
41	40.1	0.3	60.0	2.4	0.6	1.0
42	40.0	0.5	58.8	6.2	1.3	2.9
43	39.3	1.9	60.3	2.2	0.8	1.3
44	40.1	0.3	61.2	2.5	0.7	1.2
45	39.8	0.4	58.7	7.2	1.6	3.5
46	39.3	1.9	61.5	3.6	1.2	2.6
47	39.3	1.9	60.6	1.9	0.7	1.2
48	40.1	0.3	60.8	1.9	0.5	1.0
49	36.8	11.6	60.8	1.9	2.5	5.6
50	40.0	0.9	60.9	2.2	0.8	1.1
51	40.1	0.3	60.6	2.0	0.5	1.0
52	39.9	0.3	60.3	2.2	0.5	1.0
53	39.9	0.3	60.6	1.9	0.4	0.9
54	41.5	4.7	61.0	3.2	1.4	2.8
55	40.0	0.0	60.6	1.9	0.3	0.9
56	40.0	0.5	61.1	2.3	0.7	1.2
57	40.2	0.6	61.1	5.5	1.2	2.6
58	40.3	0.9	61.0	2.8	0.7	1.9
59	40.4	1.0	60.3	2.2	0.7	1.2
60	40.0	0.5	58.8	6.2	1.3	3.0
1-10	41.5	8.0	60.3	3.7	3.8	4.1
11-20	39.5	3.1	60.7	2.5	1.6	1.7
21-30	39.4	2.6	60.5	2.2	1.2	1.5
1-30	40.1	4.6	60.5	2.8	2.2	2.5
31-40	40.4	2.7	60.8	2.3	1.3	1.8
41-50	39.5	2.0	60.4	3.2	1.0	2.1
51-60	40.2	0.9	60.5	3.0	0.7	1.7
31-60	40.0	1.9	60.6	2.8	1.0	1.9
1-60	40.1	3.2	60.5	2.8	1.6	2.2

(d) Treatment 4.

Table 10: Players' positions and distance from the Nash equilibrium (continued).



period(s)	position Left		position Right		avg. distance from NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	22.2	9.1	68.8	23.8	27.2	7.9
2	24.0	7.7	71.6	13.3	9.7	4.8
3	21.6	3.7	76.8	10.5	4.8	3.9
4	31.7	21.4	76.0	8.1	8.4	10.2
5	18.2	4.7	78.6	6.3	4.1	3.3
6	28.1	23.6	78.0	5.8	7.5	11.9
7	22.8	8.0	78.1	7.9	5.0	4.8
8	25.2	23.1	77.8	7.7	7.0	12.5
9	21.2	6.1	77.7	6.6	3.9	3.7
10	21.7	4.3	80.0	9.9	3.8	5.0
11	19.8	3.8	81.0	3.6	1.9	2.1
12	24.5	9.9	80.7	3.9	3.9	5.1
13	21.1	3.7	81.0	3.2	1.6	2.0
14	19.7	3.1	78.2	7.6	2.8	3.4
15	26.6	22.5	77.7	6.7	5.8	10.8
16	19.2	3.5	80.0	2.4	1.5	1.7
17	20.0	3.3	81.0	4.8	2.4	2.3
18	19.7	2.0	78.3	8.2	2.9	4.0
19	19.0	2.1	80.9	6.1	2.3	3.0
20	22.8	10.0	81.4	3.7	3.0	5.5
21	20.5	1.6	80.9	3.3	1.1	1.9
22	19.9	4.5	81.4	3.4	2.0	3.4
23	20.7	5.3	82.1	4.8	2.4	4.6
24	18.7	3.4	79.9	2.1	1.3	2.1
25	19.4	2.0	73.9	22.7	4.6	11.9
26	19.0	4.2	82.3	5.3	2.5	3.9
27	26.0	19.3	80.2	2.6	4.4	9.2
28	19.9	2.8	80.6	2.0	1.2	1.7
29	19.4	3.4	82.3	5.0	2.1	3.3
30	20.0	4.1	82.8	5.0	2.6	4.0
31	27.1	22.0	81.3	4.4	5.5	10.6
32	26.1	19.1	79.9	2.2	4.1	9.2
33	20.3	2.7	80.3	1.8	1.1	1.5
34	21.3	2.5	80.6	3.5	1.5	2.5
35	21.4	2.1	80.2	2.4	1.3	1.8
36	20.5	2.1	80.7	3.2	1.4	1.9
37	19.7	1.9	79.8	0.6	0.6	0.9
38	21.8	4.4	81.8	3.8	2.0	3.4
39	20.2	3.9	80.5	2.8	1.7	2.2
40	21.8	10.3	80.9	3.8	3.1	5.1
41	21.4	6.7	81.9	3.1	2.3	3.4
42	21.4	6.8	80.5	2.7	1.9	3.2
43	22.1	7.6	79.1	3.3	2.7	3.9
44	21.3	7.7	80.1	4.5	3.1	5.1
45	22.7	7.0	80.5	2.4	2.1	3.5
46	21.1	6.9	80.6	1.7	1.9	4.0
47	22.9	9.5	80.3	0.9	1.7	4.7
48	23.1	6.9	79.5	1.6	2.1	3.4
49	21.4	8.1	80.4	1.2	2.5	3.6
50	21.6	7.0	80.6	2.4	1.8	3.3
51	22.0	7.1	80.3	0.9	1.5	3.5
52	22.6	7.2	80.0	0.5	1.4	3.6
53	20.9	8.7	81.1	3.6	2.7	4.5
54	22.9	8.0	80.3	0.7	1.8	3.9
55	21.9	8.6	81.2	2.4	2.3	4.8
56	18.6	3.5	79.5	2.0	1.3	2.3
57	22.2	10.1	80.4	1.3	2.2	4.9
58	22.8	9.6	80.2	1.4	2.0	4.7
59	22.8	9.7	79.6	1.8	2.2	4.6
60	21.3	12.0	80.7	2.0	3.1	5.8
1-10	23.7	11.2	76.3	10.0	8.1	6.8
11-20	21.2	6.4	80.0	5.0	2.8	4.0
21-30	20.4	5.1	80.6	5.6	2.4	4.6
1-30	21.8	7.5	79.0	6.9	4.4	5.1
31-40	22.0	7.1	80.6	2.9	2.2	3.9
41-50	21.9	7.4	80.4	2.4	2.2	3.8
51-60	21.8	8.5	80.3	1.7	2.0	4.3
31-60	21.9	7.7	80.4	2.3	2.1	4.0
1-60	21.8	7.6	79.7	4.6	3.3	4.6

(e) Treatment 5.

Table 10: Players' positions and distance from the Nash equilibrium (continued).

period(s)	position Left		position Right		average distance from support of NE		average distance from distr. NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	36.5	19.3	68.3	16.1	25.2	31.9	14.9	12.2
2	42.4	12.9	61.3	6.8	6.6	5.8	7.1	5.6
3	52.6	7.1	62.3	7.7	3.8	2.7	4.6	2.3
4	41.8	12.5	57.7	5.0	5.8	6.5	6.9	6.1
5	49.5	15.2	59.4	6.2	4.5	6.6	5.4	6.6
6	55.3	7.3	59.4	6.8	2.9	2.7	4.0	2.8
7	54.5	4.7	54.5	6.0	1.7	1.5	4.1	2.0
8	50.5	6.5	59.3	4.4	2.1	2.7	2.4	2.5
9	53.6	3.7	63.9	14.1	3.8	6.6	4.7	6.4
10	50.4	8.7	53.6	16.5	5.2	7.7	6.3	7.7
11	48.7	13.5	58.3	4.3	3.8	5.6	4.9	5.1
12	51.3	8.4	61.3	8.0	3.3	4.3	3.8	4.2
13	51.4	8.4	59.0	2.4	1.8	3.3	2.4	3.1
14	51.2	9.1	58.5	3.2	2.2	3.4	3.1	3.0
15	52.4	4.4	58.6	4.2	1.3	1.4	1.8	1.6
16	52.9	6.0	57.8	4.4	1.6	2.0	2.9	1.4
17	54.8	3.9	57.3	3.3	0.8	0.9	2.2	1.1
18	54.7	4.4	60.9	10.5	2.2	4.8	3.7	4.7
19	53.4	3.5	61.7	10.2	2.2	4.9	2.9	4.7
20	55.6	2.7	63.3	9.2	2.1	4.6	3.6	4.4
21	54.1	5.7	62.6	7.4	2.7	5.0	3.9	4.5
22	56.8	2.9	62.3	10.4	2.3	4.8	3.8	5.1
23	52.5	6.5	61.4	10.8	4.3	4.1	5.3	3.6
24	53.6	6.3	61.3	3.8	2.0	2.2	3.2	2.2
25	56.2	3.3	60.4	6.7	1.7	2.6	3.4	2.5
26	58.2	3.9	60.5	6.3	2.0	3.1	4.1	2.9
27	55.6	3.6	61.2	7.2	1.7	3.2	3.2	3.1
28	55.8	3.8	60.4	7.4	2.0	2.7	3.4	2.3
29	54.9	3.1	58.9	2.5	0.7	0.6	1.6	0.6
30	54.2	3.3	58.3	3.2	0.8	0.8	1.9	1.0
31	49.8	12.2	62.0	10.7	4.9	6.1	5.8	5.8
32	51.1	7.9	61.9	13.1	4.7	6.9	5.7	6.4
33	50.6	6.7	60.2	14.6	5.4	6.6	6.5	6.1
34	50.9	8.0	60.5	8.0	3.6	3.9	4.6	3.3
35	52.6	6.0	59.8	6.3	2.5	3.5	3.3	3.1
36	52.8	4.2	58.3	5.7	1.6	2.1	2.6	1.7
37	54.9	3.6	62.3	8.7	2.3	4.0	3.3	3.7
38	55.7	3.1	61.5	6.7	1.5	3.4	2.9	3.2
39	56.1	2.5	60.4	5.6	1.3	2.4	2.9	2.3
40	55.3	4.3	58.6	5.0	1.1	1.8	3.0	1.6
41	55.0	3.6	59.2	12.2	2.6	4.8	4.2	4.8
42	53.0	4.3	57.2	4.5	1.2	1.6	2.5	1.5
43	55.2	2.7	58.4	3.6	0.7	0.7	1.9	1.2
44	54.9	3.0	59.0	2.7	0.6	0.8	2.0	0.7
45	54.9	3.2	58.2	4.5	0.8	1.1	2.1	1.2
46	53.4	3.6	58.0	4.6	1.1	1.3	2.1	1.1
47	53.9	4.2	60.1	3.0	1.2	1.7	2.2	1.5
48	55.4	2.7	60.7	2.5	0.8	1.1	2.2	1.1
49	55.7	2.6	61.2	5.6	1.3	2.4	2.7	2.4
50	56.9	3.5	57.1	6.7	1.3	2.4	3.3	2.4
51	55.0	4.4	59.3	6.6	1.8	2.2	2.7	2.0
52	54.9	4.0	60.6	4.2	1.3	1.8	2.4	2.2
53	55.9	4.7	59.6	6.3	1.6	2.5	2.9	2.9
54	54.8	3.4	59.7	5.2	1.2	1.7	2.1	1.8
55	55.5	3.3	60.1	4.0	0.9	1.6	2.0	2.1
56	55.8	3.1	61.0	5.9	1.2	2.8	2.7	3.0
57	54.2	3.6	60.4	6.7	1.4	2.9	2.2	3.2
58	54.7	3.2	61.7	7.5	1.8	3.3	2.6	3.1
59	54.7	3.0	56.4	4.2	0.5	0.5	2.3	1.1
60	55.3	3.2	57.4	3.9	0.7	0.9	2.5	0.8
1-10	48.7	9.8	60.0	9.0	6.1	7.5	6.0	5.4
11-20	52.6	6.4	59.7	6.0	2.1	3.5	3.1	3.3
21-30	55.2	4.2	60.7	6.6	2.0	2.9	3.4	2.8
1-30	52.2	6.8	60.1	7.2	3.4	4.6	4.2	3.8
31-40	53.0	5.9	60.6	8.4	2.9	4.1	4.0	3.7
41-50	54.8	3.3	58.9	5.0	1.1	1.8	2.5	1.8
51-60	55.1	3.6	59.6	5.5	1.2	2.0	2.4	2.2
31-60	54.3	4.3	59.7	6.3	1.7	2.6	3.0	2.6
1-60	53.2	5.5	59.9	6.7	2.6	3.6	3.6	3.2

(f) Treatment 6.

Table 10: Players' positions and distance from the Nash equilibrium (continued).

period(s)	position Left		position Right		avg. distance from NE	
	mean	st. dev.	mean	st. dev.	mean	st. dev.
1	32.9	19.3	77.1	14.6	19.2	8.5
2	41.9	17.4	76.2	8.3	11.6	7.8
3	42.1	18.1	74.0	21.4	14.0	16.8
4	46.6	16.6	73.4	22.9	14.6	15.2
5	46.9	16.1	75.1	10.4	10.8	11.0
6	55.7	6.4	68.2	20.7	9.7	9.9
7	54.8	10.2	72.7	12.0	8.2	7.5
8	48.7	15.1	70.0	10.8	10.8	10.7
9	49.9	14.2	68.9	20.2	12.8	14.8
10	50.1	13.0	76.7	11.1	8.0	8.9
11	49.2	15.8	72.8	11.2	9.8	9.3
12	54.2	6.3	69.9	17.5	8.6	8.5
13	54.9	6.0	72.0	10.6	7.0	6.3
14	53.6	10.6	74.8	14.9	7.9	9.0
15	53.0	12.2	76.3	14.1	7.6	11.2
16	56.5	6.8	75.2	14.6	7.8	7.0
17	54.8	7.5	72.6	18.9	7.6	9.8
18	53.9	8.1	75.1	10.5	6.8	6.7
19	55.7	6.1	69.4	17.0	8.7	9.0
20	56.7	9.1	78.8	8.7	5.9	4.9
21	55.9	4.5	76.9	10.5	5.3	6.0
22	55.9	9.6	79.3	7.0	5.2	5.8
23	54.8	9.5	81.7	7.7	4.8	6.0
24	54.6	13.7	71.3	19.9	8.4	10.9
25	55.9	10.4	78.3	9.9	5.6	6.6
26	56.1	8.3	77.5	9.3	5.1	6.9
27	56.2	9.2	78.1	7.1	4.3	5.8
28	57.5	8.3	78.9	10.0	5.7	5.6
29	55.7	10.7	76.6	10.3	6.7	6.8
30	55.7	10.0	77.9	8.6	5.2	7.3
31	45.0	21.3	76.3	8.6	10.3	12.0
32	50.0	15.5	75.5	10.3	8.2	9.7
33	54.1	13.9	76.1	6.9	6.3	7.2
34	53.4	10.3	75.3	10.5	6.3	7.6
35	51.3	13.8	75.7	8.4	6.5	9.7
36	53.1	11.2	76.8	8.9	5.8	8.7
37	56.8	8.7	74.7	10.3	5.9	7.1
38	54.4	7.5	71.7	11.4	7.0	8.4
39	54.5	8.1	74.1	10.9	6.0	7.9
40	52.4	12.3	78.7	8.1	5.8	7.4
41	53.0	15.5	79.0	3.2	4.1	8.0
42	55.4	10.3	79.7	4.0	3.3	6.4
43	59.3	12.6	77.1	10.3	4.2	7.6
44	56.5	12.4	77.1	6.4	4.2	7.1
45	57.3	4.4	73.2	10.1	4.8	5.9
46	56.0	6.4	74.7	10.7	6.0	6.9
47	55.3	14.2	72.0	12.1	7.8	8.6
48	53.0	11.4	75.7	9.7	6.7	6.8
49	50.3	15.1	75.0	8.0	7.3	9.2
50	54.4	6.8	77.3	9.1	5.2	5.9
51	55.3	8.1	75.5	9.5	5.3	5.8
52	55.7	6.8	77.9	7.2	3.9	3.9
53	55.4	8.3	72.9	11.3	6.0	7.0
54	54.9	8.0	76.7	10.7	4.7	6.7
55	55.9	6.7	75.3	9.6	4.5	6.8
56	57.5	4.2	80.3	4.6	2.3	3.2
57	58.5	5.6	75.4	10.5	3.8	6.8
58	56.3	7.1	74.3	9.6	5.2	7.1
59	55.4	8.7	77.2	6.5	4.0	5.1
60	52.5	16.5	77.9	6.1	5.7	8.7
1-10	47.0	14.6	73.2	15.2	12.0	11.1
11-20	54.3	8.8	73.7	13.8	7.8	8.2
21-30	55.8	9.4	77.7	10.0	5.6	6.8
1-30	52.3	11.0	74.9	13.0	8.5	8.7
31-40	52.5	12.3	75.5	9.4	6.8	8.6
41-50	55.1	10.9	76.1	8.4	5.4	7.2
51-60	55.7	8.0	76.3	8.5	4.6	6.1
31-60	54.4	10.4	76.0	8.8	5.6	7.3
1-60	53.4	10.7	75.4	10.9	7.0	8.0

(g) Treatment 7.

Table 10: Players' positions and distance from the Nash equilibrium (continued).

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	41.2	56.6	47.9	57.0	44.6	56.8
2	49.3	52.0	50.0	50.0	49.7	51.0
3	49.3	53.0	49.2	50.7	49.3	51.9
4	47.3	55.7	43.3	50.0	45.3	52.8
5	46.7	53.0	58.8	54.8	52.8	53.9
6	48.8	52.0	50.0	50.0	49.4	51.0
7	49.9	50.0	50.0	50.0	50.0	50.0
8	47.2	56.3	51.5	52.1	49.3	54.2
9	45.5	55.0	49.7	51.0	47.6	53.0
10	48.5	51.2	50.0	50.0	49.3	50.6
11	50.0	50.2	50.0	50.0	50.0	50.1
12	50.3	55.6	41.5	52.4	45.9	54.0
13	31.0	68.5	37.5	64.5	34.3	66.5

(a) Treatment 1.

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	38.7	61.0	40.0	60.0	39.3	60.5
2	41.3	61.5	40.0	60.0	40.7	60.8
3	49.4	58.1	40.7	50.2	45.0	54.1
4	40.3	60.3	40.0	60.0	40.2	60.2
5	38.2	59.8	40.0	60.0	39.1	59.9
6	34.1	59.1	40.3	60.3	37.2	59.7
7	40.8	60.4	40.0	60.0	40.4	60.2
8	40.0	57.8	40.0	60.0	40.0	58.9
9	39.5	59.9	40.7	60.0	40.1	59.9
10	38.7	62.5	38.7	60.6	38.7	61.6

(b) Treatment 2.

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	48.8	52.9	50.0	50.0	49.4	51.5
2	48.9	50.9	50.0	50.0	49.4	50.5
3	44.9	52.7	50.0	50.0	47.5	51.4
4	50.0	50.3	49.4	46.5	49.7	48.4
5	49.0	51.0	49.7	49.8	49.4	50.4
6	48.6	51.1	49.4	52.0	49.0	51.6
7	50.0	50.2	49.7	50.0	49.9	50.1
8	49.5	50.5	50.2	50.0	49.8	50.2
9	48.0	50.7	49.6	53.1	48.8	51.9
10	48.5	50.8	50.1	50.8	49.3	50.8

(c) Treatment 3.

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	40.2	60.6	39.7	60.2	39.9	60.4
2	40.8	59.5	40.0	60.2	40.4	59.9
3	39.8	63.9	40.0	60.0	39.9	61.9
4	40.6	59.5	39.5	60.0	40.1	59.8
5	40.2	59.7	40.7	60.0	40.4	59.8
6	43.0	59.4	41.0	59.6	42.0	59.5
7	39.3	60.4	39.6	59.9	39.4	60.2
8	40.2	59.9	40.0	60.1	40.1	60.0
9	34.6	61.5	40.0	65.7	37.3	63.6
10	42.3	60.6	40.0	60.0	41.1	60.3

(d) Treatment 4.

Table 11: Players' average positions in the matching pairs.

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	19.8	76.6	37.9	79.8	28.8	78.2
2	21.0	79.6	20.0	80.0	20.5	79.8
3	22.0	77.1	15.5	84.1	18.8	80.6
4	20.5	77.3	20.0	80.0	20.3	78.7
5	20.5	79.4	22.0	80.0	21.3	79.7
6	23.5	81.3	20.9	80.5	22.2	80.9
7	30.3	85.0	22.3	80.0	26.3	82.5
8	19.2	80.0	20.0	79.9	19.6	79.9
9	20.0	76.8	20.4	80.0	20.2	78.2
10	20.6	77.3	20.0	80.0	20.3	78.7

(e) Treatment 5.

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	52.0	57.7	53.0	57.1	52.5	57.4
2	54.2	58.1	55.3	59.5	54.7	58.8
3	52.8	59.5	54.2	59.0	53.5	59.3
4	52.7	62.5	53.9	67.5	53.3	65.0
5	47.0	56.2	53.8	56.5	50.4	56.3
6	51.1	56.3	59.0	59.9	55.1	58.1
7	54.4	61.3	50.8	58.7	52.6	60.0
8	56.7	62.0	57.2	58.6	57.0	60.3
9	50.6	62.5	49.9	59.7	50.3	61.2
10	50.3	65.1	55.9	60.3	53.1	62.7

(f) Treatment 6.

matching pair	periods					
	1-30		31-60		1-60	
	Left	Right	Left	Right	Left	Right
1	55.8	80.2	60.0	80.0	57.9	80.1
2	52.5	80.8	60.5	77.0	56.5	78.9
3	51.7	71.3	53.3	79.0	52.5	75.2
4	49.9	76.9	49.4	68.4	49.7	72.7
5	58.0	76.8	56.7	70.5	57.4	73.7
6	61.5	78.3	56.6	72.5	59.0	75.4
7	51.0	80.0	60.7	80.0	55.8	80.0
8	56.1	74.4	44.3	80.5	50.2	77.5
9	51.3	64.5	52.2	76.0	51.7	70.1
10	51.6	73.3	54.5	73.9	53.1	73.6
11	56.8	80.3	60.0	80.0	58.4	80.2
12	43.5	66.8	57.7	80.0	50.6	73.4
13	49.3	70.7	38.5	77.1	43.9	73.9
14	58.2	80.7	60.0	80.0	59.1	80.3
15	38.0	68.2	52.2	64.6	45.1	66.4

(g) Treatment 7.

Table 11: Players' average positions in the matching pairs (continued).