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Michal Król

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Product differentiation decisions under ambiguous consumer demand and pessimistic expectations*

Michał Król†

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ABSTRACT. This paper examines the effects of violating the common prior assumption embedded in the "Product differentiation and location decisions under demand uncertainty" model by Meagher and Zauner (Journal of Economic Theory [2004]). In particular, a situation is discussed in which the firms do not know the exact distribution of the location and price elasticity of consumer demand, but resolve the resulting ambiguity using the Arrow and Hurwicz α-maxmin criterion [3].
When the firms are sufficiently pessimistic (α is high enough), the results are in contrast with the existing literature. In particular, an increase of demand location uncertainty decreases the equilibrium product differentiation, as well as the resulting second-stage equilibrium prices and profits for any realisation of consumer demand, although the effect is dampened by a possibility of higher price elasticity of demand. Furthermore, pessimism could serve as a form of strategic deterrence, because any firm that can commit itself to a more pessimistic approach increases its equilibrium share of the market and becomes better off at the competitor's expense. However, this generates a Prisoner's Dilemma situation, since both firms lose when they both become more pessimistic, suggesting that the presence of ambiguity can make the market more competitive.

Keywords: Hotelling, uncertainty, non-unique prior

JEL Classification: C72, D43, D81, L13, R32

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†michal.krol@manchester.ac.uk, Economics, The University of Manchester, M13 9PL, UK

1 Introduction

Over the last few decades, the Hotelling model of spatial competition has been used to explain a wide variety of social phenomena, from the location of retail outlets to competition among political parties. A relatively new strand of the relevant literature investigates the effects of random demand fluctuations on the firms' location decisions. This typically entails introducing some form of demand uncertainty into a modified Hotelling setting. For instance, Balvers and Szerb [4] study the effect of random shocks to the products' desirability under fixed prices. Harter [9] examines the uncertainty in the form of a uniformly distributed random shift of the (uniform) customer distribution, where the firms locate sequentially. Other papers, such as [1], concentrate on the strategic effect of acquiring information about the demand through price-experimentation.

On the other hand, relatively few studies consider the effect of demand uncertainty in an otherwise unchanged Hotelling framework. Of those, Casado-Izaga [6] adopts the same form of uncertainty as Harter, but the duopolists locate simultaneously, before observing the actual customer distribution and then choosing prices. Meagher and Zauner [13] consider a similar setting, but succeed in parametrizing the support of the (uniform) random variable that shifts the customer distribution and report that demand uncertainty increases the equilibrium level of product differentiation. Finally, another study by Meagher and Zauner [12] (henceforth, MZ) considers a random shock arbitrarily (rather than uniformly) distributed on a fixed interval. Tractability of the model is maintained by assuming that the variance of the shock is small enough relative to the ex-post differentiation of tastes, so that no firm would ever choose to capture the entire market in any state of nature. Once again, it turns out that more uncertainty results in higher equilibrium level of product differentiation.

The intuition for those results is simple: if the demand is more likely to be located away from the centre of the market, then it is natural for the firms to venture into more distant areas and away from one another, relaxing the second-stage price competition. Nevertheless, one can conjure a similarly intuitive reasoning to the opposite effect if the firms are pessimistic, in the sense of always trying to prepare for the worst-case scenario. In such case, an increase in the range of possible demand variations means a player has to consider a potentially larger strategic advantage on the competitor's behalf, especially when the players are highly specialized, leaving more room for shifts in consumer preferences to favour one over the other. To insure against this threat, a pessimistic player might then want to make her product more similar to that of the rival when uncertainty increases.

A particularly useful illustration of the described mechanism is offered by the online sports betting industry. In general, the betting market is associated with a large degree of product differentiation, as different bookmakers specialize in different sports, types of bets and outcomes of a particular sporting event (for instance, some are known to offer better prices on the favorites, others on the underdogs of a competition). This gives them flexibility in balancing the bets, while avoiding head-to-head competition in terms of the overall house edge[1], which a 'recreational' bettor will find difficult to compare between bookmakers with different relative prices, particularly for events with several possible outcomes, such as horse races. Those events are characterized by a significant increase of the uncertainty about bettors' preferences shortly before the start of the competition, as evidenced by intensified trade and increased price volatility in betting exchanges (see Smith et al. [14] for more information). Interestingly, this coincides with the odds baskets offered by different bookmakers ceasing to be differentiated (with a smaller house edge), i.e. everyone quotes exactly the same price on every horse.

The explanation might lie in the fact that bookmakers are unlikely to know the exact distribution of the punters' betting preferences, although they might be able to place them within a certain range (for instance, an outsider will not suddenly become the clear favorite of a race). Furthermore, a unique feature of the gambling industry is its extremely pessimistic approach to uncertainty, manifested by the traditional objective of 'balancing the books', i.e. effectively focusing on the worst-case scenario in which the outcome that attracted the largest volume of bets is realized. Assuming the bookmakers extend this attitude to uncertainty about consumer preferences, an increase in the range of possible demand variations will make it more undesirable for any particular firm to differentiate its odds from those of the competitors, because there is more room for the betting patterns to shift 'against it', in the sense of the books becoming more unbalanced due to bettors switching to and from other bookmakers in search of better odds. Consequently, losses associated with the success of the most excessively backed contestant are becoming more severe.

The existing literature outlined earlier is unable to accommodate the above mechanism, because of its reliance on the common prior assumption in modelling demand uncertainty. On the one hand, it seems reasonable to assume that the firms are not completely certain of the exact consumer preferences at the time of designing the product or choosing the location of their outlet. On the other hand, for all firms to unanimously form precise probabilistic estimates of all potential demand realizations may be too much to ask of an industry. Thus, the present paper considers

[1] i.e. the percentage of bettors' money the bookmaker is aiming to keep as revenue regardless of the outcome of the event

an altogether different scenario, where the firms are ignorant of the distribution of demand fluctuations and possibly differ in their resolution of the resulting ambiguity. In particular, the setting is the same as in MZ, except the demand is allowed to vary not only in location, but also in its price elasticity, as captured by the transportation cost parameter[2]. Furthermore, the firms know only the support of the distribution of those changes, which of course requires a different payoff specification for the reduced location game. Instead of calculating the expected value of the second stage Nash Equilibrium profits for a particular location-pair, the firms consider a weighted average of the highest and lowest of those profits, i.e. use the Arrow-Hurwicz α-maxmin criterion [3] to resolve the ambiguity[3]. The definition of uncertainty must also change. In MZ it was specified as the variance of the distribution of the shock shifting consumer preferences, which is not applicable in the absence of a common prior. Instead, an increase of demand uncertainty will be modelled *via* spreading the support of the random demand fluctuations, which is closer to the approach taken in [13]. The terms 'ambiguity' and 'uncertainty' will be used interchangeably in the context of the present model, while 'uncertainty' will be exclusive to MZ.

It turns out that an increase of uncertainty about the demand's location decreases the equilibrium product differentiation (and with it, the resulting second-stage prices and profits) when the firms are sufficiently pessimistic, in the sense of assigning a high enough weight to the worst-profit scenario. This is because a pessimistic player effectively assumes that the consumer preferences will move in a way offering a strategic advantage to the counterpart ahead of the second-stage price competition. By locating closer to the rival, he partly insures against this possibility, because even if the customers find the product of the other firm more suitable, his own one, being similar, is not so badly handicapped.

Surprisingly, the effect of pessimistic expectations is moderated by uncertainty about the price elasticity of demand (represented by a possibility of lower transport costs) despite the fact that the transportation cost parameter has no effect on location decisions in the Hotelling framework. This is because a pessimist will expect to see unfavorable consumer preferences combined with competitive pricing due to high elasticity of demand. A possibility of lower costs will make the price competition in such worst-case scenario even more intense, so that the pessimistic outcome of the

[2] The total consumer demand is, by assumption, completely inelastic in the Hotelling framework. However, when the transport cost parameter decreases, the individual demand of each firm for given locations and the counterpart's price becomes more elastic in the firm's own price, which is what is henceforth meant by price elasticity.

[3] For the model discussed here, the α-maxmin profits coincide with the α-Maxmin Expected Utility (as in Ghirardato et al. [8]) of a risk-neutral agent.

uncertainty becomes even more threatening. However, it also becomes more costly to insure against this threat, because a larger reduction of product differentiation is required in order to achieve the same second-stage profit improvement in those unfavorable circumstances, resulting in a more significant reduction of one's strategic advantage in case of the demand being favorably located. Consequently, a firm must be more pessimistic in order to continue to decrease product differentiation in response to an increase of uncertainty about the demand's location.

The results are, to a large extent, robust to a change in timing, such that the pricing decisions are made before the resolution of the uncertainty. Despite the existence of multiple price equilibria for 'not too asymmetric' locations, both the highest and lowest possible equilibrium prices are decreasing in demand location uncertainty when the firms are pessimistic, despite the effect being dampened by uncertainty about transport costs. Thus, uncertainty about the placement of consumer demand still makes pessimistic producers more competitive, although less so when faced with a possibility of a highly price elastic demand.

Finally, whenever a particular firm adapts a more pessimistic approach, its equilibrium location is further towards the competitor's end of the market, with the rival withdrawn into his own hinterland. As a result, the pessimistic firm becomes better off in equilibrium at the counterpart's expense, regardless of the eventual demand realisation, i.e. of whether or not the firm's pessimistic expectations prove justified. This suggests that if the duopolists could commit themselves to an attitude of 'preparing for the worst', then they might use pessimism as a form of strategic deterrence, preventing the competitor from targeting their own market niche.

However, this also creates a prisoner's Dilemma situation, because when both firms become more pessimistic, product differentiation decreases and both fall victim of the intensified price competition in the second stage of the game. In a sense, their self-imposed pessimism becomes a self-fulfilling prophecy. Alternatively, rather than through deliberate commitment, a pessimistic approach towards uncertainty could become prevalent through the elimination of firms who fail to adapt it and do worse relative to the competitors. Either way, on this evidence the presence of ambiguously distributed demand fluctuations makes the product market more competitive.

2 The Model

As indicated above, the setting is, in general, the same as in MZ. In the first stage of the game, two firms simultaneously choose locations x_1, x_2 (without loss of generality set $x_1 \leq x_2$) and then proceed to simultaneous setting of their respective prices p_1, p_2 in the second stage. As usual, a consumer located at x chooses to buy a unit of the good from firm $i \in \{1, 2\}$, so as to minimize the total purchase cost of $p_i + t(x_i - x)^2$, where $t > 0$ is the transportation cost parameter. The good costs nothing to produce and the consumers are uniformly distributed on the interval $\left[M - \frac{1}{2}, M + \frac{1}{2}\right]$, where the duopolists get to know the value of M, as well as t, once they choose the locations, but before setting prices. Initially, all they know is that the joint probability distribution of (M, t) has support $[-L, L] \times [t_0, 1]$, where $L \in \left[0, \frac{1}{2}\right]$ and $t_0 \in (0, 1]$[4]. The difference from MZ is introducing uncertainty about transportation costs (MZ assumes $t = 1$), as well as the fact that the exact probability distribution of (M, t) is unknown and so is the expected value of the second stage Nash Equilibrium profits. Instead, the players' payoffs in the reduced location game are given by a weighted average of the lowest and highest possible profits, i.e. are computed using the Arrow/Hurwicz α-maxmin criterion instead of the expected value. More specifically, let $\pi_i^*(x_1, x_2, M, t)$ be the second-stage unique Nash Equilibrium profit associated with a particular location-pair and demand realization. Then the first-stage payoffs are given by:

$$\Pi_i(x_1, x_2) = \alpha \left[\min_{(M,t) \in [-L,L] \times [t_0,1]} \pi_i^*(x_1, x_2, M, t) \right] + \\ + (1 - \alpha) \left[\max_{(M,t) \in [-L,L] \times [t_0,1]} \pi_i^*(x_1, x_2, M, t) \right]$$

where $\alpha \in [0, 1]$ is a parameter representing the degree of the duopolists' pessimism.

[4] The assumption that transportation costs are always no greater than 1 can be imposed without loss of generality. The assumption $L \leq 1/2$ was imposed in MZ for the purpose of mathematical tractability and is equally useful here.

3 Results

The second-stage unique Nash Equilibrium profits are exactly the same as the ones derived in MZ, i.e.:

$$\pi_i^*(x_1, x_2, M, t) = \begin{cases} t(x_2 - x_1)\left[1 + 2(-1)^i (M - \overline{x})\right] & (-1)^i (M - \overline{x}) \geq 3/2 \\ t(x_2 - x_1)\left[3(-1)^i + 2(M - \overline{x})\right]^2/18 & (M - \overline{x}) \in (-3/2, 3/2) \\ 0 & \text{otherwise} \end{cases}$$

where $\overline{x} = (x_1 + x_2)/2$. The first (topmost) segment of the above piecewise function corresponds to firm i capturing the entire market (later referred to as "monopolistic equilibrium"), while the middle segment is where firm i shares the market with the rival ("competitive equilibrium"). It is immediately clear that $\pi_i^*(x_1, x_2, M, t)$ is increasing in t and straightforward to verify that it is also decreasing in M for $i = 1$ and increasing in M for $i = 2$. In other words, the second stage equilibrium profit of the firm located on the left declines as the customers are located further and further to the right. Similarly for the firm located on the right when the consumer preferences shift leftward. Consequently, we have:

$$\Pi_i(x_1, x_2) = \alpha\pi_i^*\left(x_1, x_2, -L[-1]^i, t_0\right) + (1 - \alpha)\pi_i^*\left(x_1, x_2, L[-1]^i, 1\right)$$

Note that a player can always ensure a positive α-maxmin profit, by locating at $x_i = -x_{-i}$, so that $\overline{x} = 0$ and $(M - \overline{x}) \in (-3/2, 3/2)$ for both $M = -L$ and $M = L$ (recall $L < 1/2$ by assumption), i.e. there is a competitive equilibrium in both the highest-profit and lowest-profit scenarios.

We will now show that any Nash Equilibrium of the reduced location game must satisfy $(M - \overline{x}) \in (-3/2, 3/2)$ for $M \in \{-L, L\}$, i.e. that it must result in the best and worst second-stage equilibria being competitive. To this end, consider the following cases:

1. Player 1 captures the entire market for $M = -L$, while player 2 does the same for $M = L$. This is impossible, since:

   $$(L + \overline{x} \geq 3/2 \ \wedge \ L - \overline{x} \geq 3/2) \Leftrightarrow L - 3/2 \geq |\overline{x}|$$

   which is false by the assumption of $L < 1/2$.

2. The same player i monopolizes the market for both $M = -L$ and $M = L$. This cannot constitute a Nash Equilibrium of the reduced location game, because the other player is able to improve on her zero payoff.

3. Player 1 monopolizes the market in the highest-profit scenario of $M = -L$ and a competitive equilibrium follows for $M = L$. The α-maxmin profit of player 2 then equals:

$$\Pi_2^{0c}(x_1, x_2) = \alpha \times 0 + (1-\alpha)(x_2 - x_1)\left[3 + 2(L - \overline{x})\right]^2 / 18$$

For $\alpha = 1$ the above is equal to 0, so that player 2 benefits from re-locating to $x_2 = -x_1$ and ensuring competitive equilibria for $M \in \{-L, L\}$. As for the case of $\alpha < 1$, differentiating the above expression with respect to x_2 gives:

$$(1-\alpha)(3 + 2L + x_1 - 3x_2)\left[3 + 2(L - \overline{x})\right]$$

we have $3 + 2(L - \overline{x}) > 0$, since $(L - \overline{x}) \in (-3/2, 3/2)$. The only stationary point left is therefore $x_2^s = 1 + (2L + x_1)/3$, so that $\overline{x} = (3 + 2L + 4x_1)/6$ and the monopolistic equilibrium at $M = -L$ implies:

$$L + (3 + 2L + 4x_1)/6 \geq 3/2 \Leftrightarrow x_1 \geq \underline{x}_1 = (3 - 4L)/2$$

The α-maxmin profit of player 1 equals:

$$\Pi_1^{cm}(x_1, x_2) = \alpha t_0 (x_2 - x_1)\left[2(L - \overline{x}) - 3\right]^2/18 + \\ + (1-\alpha)(x_2 - x_1)\left[1 + 2(L + \overline{x})\right]$$

differentiating with respect to x_1 and substituting x_2^s for x_2 we obtain:

$$\phi(x_1) = -\frac{16\alpha t_0}{81} x_1^2 + \left(\left[2 + \frac{4(8L-15)t_0}{81}\right]\alpha - 2\right) x_1 + \\ + \frac{2\left[81L - 2(L-3)(4L-3)t_0\right]\alpha}{81} - 1 - 2L + \alpha$$

which is quadratic concave with a maximum at:

$$x_1 = \left(\left[81 + 2(8L - 15)t_0\right]\alpha - 81\right)/16 t_0 \alpha < \underline{x}_1$$

Furthermore:

$$\phi(\underline{x}_1) = 2L - 4(1-\alpha) - (2L + \left[2 + L(16L/9 - 4)\right] t_0)\alpha$$

which is negative under the assumptions on α, L and t_0. Hence, the derivative $\phi(\cdot)$ is negative, i.e. whenever player 2 is satisfied with her current location, player 1 wants to move leftward. Consequently, there can be no location equilibrium with player 1 monopolizing the market for $M = -L$ and a competitive equilibrium at $M = L$. Similarly for the opposite case of player 2 monopolizing the market for $M = L$ and a competitive equilibrium at $M = -L$.

The only remaining possibility is that the price equilibrium is competitive for $M \in \{-L, L\}$, in which case payoffs equal:

$$\Pi_i^{cc}(x_1, x_2) = \alpha t_0 (x_2 - x_1) \left[(-1)^i (3 - 2L) - 2\overline{x}\right]^2 /18+ \\ + (1-\alpha)(x_2 - x_1)\left[(-1)^i (3 + 2L) - 2\overline{x}\right]^2 /18$$

The corresponding first order conditions are:

$$\partial \Pi_1^{cc}/\partial x_1 = [1 + (t_0 - 1)\alpha]\,[4L^2 + (3 + 3x_1 - x_2)(3 + x_1 + x_2)] - \\ - 4L(3 + 2x_1)\,[(t_0 + 1)\alpha - 1] = 0$$

$$\partial \Pi_2^{cc}/\partial x_2 = [1 + (t_0 - 1)\alpha]\,[4L^2 - (3 + x_1 - 3x_2)(x_1 + x_2 - 3)] + \\ + 4L(2x_2 - 3)\,[(t_0 + 1)\alpha - 1] = 0$$

This has three possible solutions, two of which fail to satisfy the competitive equilibrium condition $(M - \overline{x}) \in (-3/2, 3/2)$ for $M \in \{-L, L\}$ (see online appendix). The remaining solution is:

$$x_1^* = -x_2^* = \frac{(3 + 2L)^2 (1 - \alpha) + (3 - 2L)^2 t_0 \alpha}{4\,[2L([t_0 + 1]\,\alpha - 1) - 3 - 3(t_0 - 1)\alpha]} \tag{1}$$

which gives both players an α-maxmin profit equal to:

$$\Pi^* = \frac{\left[(3 + 2L)^2 (1 - \alpha) + (3 - 2L)^2 t_0 \alpha\right]^2}{36\,[3 + 3(t_0 - 1)\alpha - 2L([t_0 + 1]\,\alpha - 1)]}$$

Indeed, it turns out that no unilateral deviation from x_i^* can result in a payoff higher than Π^*. Firstly, player 1 cannot deviate to a $x_1 < x_2^*$ and monopolize the market for $M = L$, since this requires:

$$-L + (x_1 + x_2^*)/2 \geq 3/2 \Leftrightarrow x_1 \geq 3 + 2L - x_2^*$$

while we have:

$$3 + 2L - x_2^* < x_2^* \Leftrightarrow \frac{(9\,[1 - t_0] + 4L\,[3 + L + 3(L - 1)t_0])\alpha - (3 + 2L)^2}{2L\,[(t_0 + 1)\alpha - 1] - 3[1 + (t_0 - 1)\alpha]} < 0$$

where both the numerator and the denominator of the above fraction are negative under the imposed parameter assumptions.

Player 1 can still deviate to a $x_1 < x_2^*$ such that he will monopolize the market for $M = -L$ only. However, it turns out that the first derivative of Π_1^{cm} with respect to x_1 is negative for $x_2 = x_2^*$ and $L + (x_1 + x_2^*)/2 > 3/2$. Similarly, when it is player 2 who monopolizes the market for $M = L$ only, i.e. when $L - (x_1 + x_2^*)/2 > 3/2$ and $-L - (x_1 + x_2^*)/2 \in (-3/2, 3/2)$, then the derivative of:

$$\Pi_1^{0c}(x_1, x_2^*) = \alpha \times 0 + (1-\alpha)(x_2^* - x_1)\left[-3 + 2(-L - (x_1 + x_2^*)/2)\right]^2/18$$

with respect to x_1 is positive. In other words, an optimal location x_1 to the left of x_2^* must be such that the resulting price equilibria are competitive for $M \in \{-L, L\}$. Out of all such locations, x_1^* is best, as it can be shown that $\partial\Pi_1^{cc}(x_1, x_2^*)/\partial x_1$ is negative for $x_1 < x_1^*$ and positive for $x_1 > x_1^*$, as long as $M - (x_1 + x_2^*)/2 \in (-3/2, 3/2)$ for $M \in \{-L, L\}$ (see the online appendix for details of all the algebraic derivations).

This means it is also impossible to benefit from deviating to a $x_1 > x_2^*$. To see this, observe that the game is symmetric, in the sense that $\Pi_2(x_1, x_2) = \Pi_1(-x_2, -x_1)$. As a consequence, for each $x_1 > x_2^*$ there exists a location $x_1' = x_2^* - (x_1 - x_2^*) < x_2^*$ giving a higher payoff, since:

$$\Pi_1(x_1', x_2^*) - \Pi_2(x_2^*, x_1) = \Pi_1(2x_2^* - x_1, x_2^*) - \Pi_1(-x_1, -x_2^*)$$

which means switching from x_1 to x_1' is equivalent to shifting the locations of both players rightward by the same distance of $2x_2^* > 0$. This in turn has the same effect as shifting the customer distribution leftward for locations fixed, thereby increasing the second-stage profit of player 1 for $M \in \{-L, L\}$. All in all, it is impossible for player 1 to gain by deviating from x_1^*, while the converse is true for player 2 (again, by symmetry of the game).

We now turn to the comparative statics of the above solution. Firstly, for symmetric locations the equilibrium product differentiation is $\Delta^* = 2x_2^*$, which is decreasing in α, t_0 and:

$$\frac{\partial\Delta^*}{\partial L} = \frac{[(t_0-1)\alpha + 1]\left[(3+2L)^2(1-\alpha) - (3-2L)^2 t_0\alpha\right]}{(3 + 3(t_0-1)\alpha - 2L[(t_0+1)\alpha - 1])^2} < 0 \Leftrightarrow$$

$$\Leftrightarrow \alpha > \widehat{\alpha} = \frac{(3+2L)^2}{(3+2L)^2 + (3-2L)^2 t_0}$$

where $\widehat{\alpha}$ is decreasing in t_0 and increasing in L, so that for $\alpha > 4/(4+t_0)$ the equilibrium product differentiation is decreasing in L over the whole range of the parameter. The effect of the parameters on second stage equilibrium prices and profits is a consequence of their impact on Δ^*. As the price equilibria resulting from x_i^* are always competitive, we have:

$$\pi_i^*\left(x_1^*, x_2^*, M, t\right) = \Delta^* t \left[3\left(-1\right)^i + 2M\right]^2 /18, p_i^*\left(x_1^*, x_2^*, M, t\right) = \Delta^* t \left[3 + 2\left(-1\right)^i M\right]/3$$

and since $M \in [-1/2, 1/2]$, the effect of α, t_0 and L on π_i^* and p_i^* has the same sign as their effect on Δ^*.

The above results may be summarized as follows:

Proposition 1 *Consider a variant of the Hotelling duopoly game in which the joint distribution of transport costs and the median of the (uniform) consumer preferences is unknown with support on* $[t_0, 1] \times [-L, L]$*, where* $t_0 \in (0, 1]$ *and* $L \in \left[0, \frac{1}{2}\right]$*. Suppose the firms choose locations so as to maximize the* α*-maxmin value of the ex-post second-stage equilibrium profits. Then the unique equilibrium locations are given by* (1)*, while the corresponding product differentiation, as well as the second-stage equilibrium prices and profits, are all increasing in* L *for* α *sufficiently large.*

This shows that ambiguity attitudes determine the way in which the firms respond to changes in the spectrum of possible demand variations when their exact probability distribution is unknown. On the one hand, it follows from condition $\alpha > \widehat{\alpha}$ that firms taking an optimistic approach ($\alpha < 1/\left[1 + t_0\right]$) always respond to an increase of uncertainty by venturing further away from one another, which is consistent with MZ. On the other hand, when the duopolists are moderately pessimistic relative to the minimum transport costs ($\alpha \in \left[1/\left(1 + t_0\right), 4/\left(4 + t_0\right)\right]$), they initially decrease product differentiation when L increases, but reverse this tendency when uncertainty becomes sufficiently large. Finally, highly pessimistic firms ($\alpha > 4/\left[4 + t_0\right]$) always locate closer together when uncertainty increases, in contrast with MZ.

What drives the results is the fact that the players' α-maxmin approach prescribes them to resolve the uncertainty they face in different ways, in the sense that they proceed 'as if' they each had a different prior over M and were aiming to maximize the expected value of second-stage profits. Specifically, the player located on the left effectively assumes that the probability of a 'worst-case scenario', in which the demand is as far to the right as possible and transport costs are at their lowest, is equal to α. When taking a pessimistic approach, the player will then want to locate relatively far to the right and close to the competitor, thereby improving her strategic position (and the resulting Nash Equilibrium profits) in the lowest profit case of $M = L$. This will occur at the cost of losing some of the strategic advantage in case of a favorable demand realisation, but this has low-priority when α is high.

Crucially, the other player associates the same probability α with an opposite market scenario ($M = -L$), so that, when pessimistic, she will want to locate relatively far to the left and will mirror player 1's shift towards the counterpart, rather then respond by moving away in order to relax the resulting second-stage price competition[5]. This is also interesting in the context of the 'certainty' Hotelling game, which was analyzed for various, not necessarily symmetric, customer distributions (see, for example [2], [11]), but always based on the firms having exactly the same expectations regarding the distribution of customers across the space of tastes and possible states of nature. The present paper demonstrates that, starting from a common degree of ignorance, the firms may end up acting "as if" they maximized expected profits subject to non-identical priors.
In order to understand the effect of L on the equilibrium locations, it is helpful to observe that for $(M - \overline{x}) \in (-3/2, 3/2)$:

$$\partial \pi_1^* \left(x_1, x_2, M, t\right) / \partial x_1 = t(3 - 2M + x_1 + x_2)(2M - 3 - 3x_1 + x_2)/18$$

which in turn is concave in M. In other words, as the demand shifts more and more to the right, the marginal gains from moving in the same direction increase by less and less. Consequently, when L increases, any additional benefits from re-locating rightward which this brings about in the lowest profit scenario are smaller than the corresponding additional losses in the $M = -L$ case, the more so the larger the value of L. Hence, any gains from reducing product differentiation due to an increase of uncertainty will be outweighed by losses, unless L is sufficiently small relative to α, i.e. the losses are small relative to the importance of gains for the player's decision variable. For this reason, the players' tendency to differentiate their products is weakened when uncertainty increases only as long as it is not too big, which could mean that it never happens ($\alpha < 1/\left[1 + t_0\right]$) or that it is the case for the entire range of $L \in [0, 1/2]$ ($\alpha > 4/\left[4 + t_0\right]$).

Returning to the sports betting example invoked in the introduction, it would appear that the bookmakers' degree of pessimism is large even relative to a considerably wide spectrum of possible demand variations. For this reason, an increase of demand uncertainty shortly before the start of a race makes them reduce the differentiation in the offered baskets of odds, despite the fact that this brings about a more competitive house edge. This is because any bookmaker who chose to offer different

[5] A notable caveat is that the players may not choose to re-locate to the other side of the competitor despite acting 'as if' the majority of consumers were bound to be located there. This is because doing so would cause the former lowest-profit demand realisation to become the highest-profit one and *vice versa*, i.e. the players would effectively switch their beliefs when switching sides.

odds from the rest would face a particularly bad worst-case scenario, in which the betting patterns shift significantly in a way favorable to the competitors, making his bets extremely unbalanced. In order to insure against this threat, it is better to offer the same product as everybody else, because without customers switching to and from other bookmakers in search of better odds any excessive volumes of bets resulting from a change in preferences will not be large enough to greatly unbalance the books. In other words, uncertainty then affects everyone in the same way and even an extreme pessimist sees no way of becoming disadvantaged[6].

It is interesting to observe that t_0, reflecting the degree of uncertainty about transportation costs, affects the equilibrium locations, despite the fact that the corresponding transport cost parameter t has no effect on location decisions under certainty. This was possibly why the potential role of uncertainty about transport costs (or, in general, about the price elasticity of consumer demand) has been ignored by the relevant literature (MZ assume $t = 1$). On the one hand, this seems reasonable, because if t does not affect location decisions under certainty, then it should not matter that the firms do not know its exact value, since the optimal choice will be the same regardless of what it is. On the other hand, it overlooks the potential interaction between two types of uncertainty: about the customers' locations and about the transport costs that they incur. In particular, if a certain realisation of consumer demand usually coincides with low transportation costs, then the resulting second-stage price competition is fierce and the equilibrium profits are low. Thus, a firm may choose not to locate in a way that would be advantageous in those circumstances if that means being further away from demand realizations associated with higher transportation costs and hence potentially more profitable. In a sense, locating under uncertainty is similar to designing a product to be sold in distinct markets, characterized by different consumer preferences and various degrees of price competition. It is therefore natural for the firms to target those of them where the consumers care more about the characteristics of the product than about its price, i.e. the ones which are less competitive.

In the current α-maxmin framework, the lowest-profit outcome entails transportation costs t_0, so that a reduction of this parameter would make the worst-case scenario even more of a threat. Despite that, the firms are less determined to insure against it by staying close together, because they would need to sacrifice more in the optimistic scenario in order to improve their situation in the pessimistic one. Thus,

[6]It may also be noted that no bets are accepted by traditional bookmakers after the start of the race, despite this form of betting being very popular in the betting exchanges. This may be due to the practical difficulties associated with coordinating their odds at a stage when betting patterns change within seconds.

uncertainty about the intensity of price competition dampens the negative effect of pessimism on strategic product differentiation, leading to the observed tradeoff between α and t_0. This is interesting, because the fact that re-scaling the transport costs fails to affect location choices in the classic Hotelling framework is somewhat paradoxical. In contrast, the current model shows that firms facing a possibility of lower transportation costs are more likely to venture out into more distant areas, relaxing the intensified price competition.

Overall, ambiguity attitudes cause more variety in the players' behaviour than the characteristics of the common prior in MZ. In the latter model, the equilibrium locations are also symmetric, with product differentiation $\Delta^*_{MZ} = 3/2 + 2\sigma^2/3$, σ^2 being the variance of the distribution of M. Because of the restriction on the support of this distribution ($L < 1/2$), the maximum possible variance is $1/4$, and hence Δ^*_{MZ} ranges from $1\frac{1}{2}$ (certainty) to $1\frac{2}{3}$ (maximum uncertainty). In contrast, in the current model we have $\Delta^* = 3/2 + L$ for $\alpha = 0$ and $\Delta^* = 3/2 - L$ for $\alpha = 1$, i.e. product differentiation ranging from 1 to 2 depending on the size of the uncertainty.

4 Extensions

We now turn to consider the possibility of the players being characterized by different degrees of pessimism. One would strongly expect the comparative statics results of the previous section to continue to hold, i.e. an increase of uncertainty should still decrease the equilibrium product differentiation for *both* duopolists not too optimistic relative to the minimum transportation costs. For this reason, let $L = 1/2$ and $t_0 = 1$, with the focus instead on the effect of a *ceteris paribus* change of attitude by a particular player on the equilibrium profits. The first-stage payoff function is:

$$\Pi_i(x_1, x_2) = \alpha_i \pi_i^* \left(x_1, x_2, -[-1]^i/2, 1 \right) + (1 - \alpha_i) \pi_i^* \left(x_1, x_2, [-1]^i/2, 1 \right)$$

The logic of the proof derives from that presented in the previous section. Firstly, the discussion of cases $(1) - (3)$ applies just as well to the present situation, i.e. the players' best-response mapping still ensures that any equilibrium locations must be such that the resulting price equilibrium is competitive for any realisation of the uncertainty. Consequently, the result is, once again, obtained as the unique solution to the first order conditions within the range of qualifying locations. However, because of the complexity of the involved algebra, only a sketch of the proof is provided here, with the details of all derivations (as well as the exact solution formulae) relegated to the online appendix.

Let $x_i^*(\alpha_i, \alpha_{-i})$ denote the unique equilibrium locations, where uniqueness holds 'up to symmetry'. For instance, there is an equilibrium in which $x_1^*(0,1) = -\frac{5}{4}$ and $x_2^*(1,0) = \frac{1}{4}$, and one in which $x_1^*(1,0) = -\frac{1}{4}$ and $x_2^*(0,1) = \frac{5}{4}$, i.e. the pessimistic player is always located closer towards the centre of the market and both receive the same payoffs regardless of which configuration is selected. This of course raises a certain coordination problem, which is, however, no different from the classic Hotelling case, since each player gets to choose between two alternative equilibrium locations. See [5] for the related discussion.

We will now show that a *ceteris paribus* increase of α_i increases the ex-post equilibrium profit of player i and decreases that of the other player, for any demand realisation $M \in [-1/2, 1/2]$. To this end, consider the ex-post competitive equilibrium profit of player 1 given a particular value of M:

$$\begin{aligned}&\pi_1\left(x_1^*(\alpha_1,\alpha_2), x_2^*(\alpha_2,\alpha_1), M\right) = \\ &\qquad = \left[x_2^*(\alpha_2,\alpha_1) - x_1^*(\alpha_1,\alpha_2)\right]\left[-3+2M-x_1^*(\alpha_1,\alpha_2)-x_2^*(\alpha_2,\alpha_1)\right]^2/18\end{aligned}$$

differentiating with respect to α_1 gives a product of:

$$\left[-3+2M-x_1^*(\alpha_1,\alpha_2)-x_2^*(\alpha_2,\alpha_1)\right]/18 < 0$$

and:

$$\begin{aligned}&\frac{\partial x_2^*(\alpha_2,\alpha_1)}{\partial \alpha_1}\left[-3+2M+x_1^*(\alpha_1,\alpha_2)-3x_2^*(\alpha_2,\alpha_1)\right] - \\ &\qquad - \frac{\partial x_1^*(\alpha_2,\alpha_1)}{\partial \alpha_1}\left[-3+2M+x_2^*(\alpha_1,\alpha_2)-3x_1^*(\alpha_2,\alpha_1)\right]\end{aligned}$$

which is negative for all $M \in [-1/2, 1/2]$ if and only if it is negative for $M = -1/2$, because it can be shown that $\partial x_1^*(\alpha_2,\alpha_1)/\partial\alpha_1 > \partial x_2^*(\alpha_2,\alpha_1)/\partial\alpha_1 > 0$, i.e. when a player becomes more pessimistic, both shift towards the other player's end of the market, with the player who changed her attitude shifting more than the counterpart. This immediately implies that the player whose attitude remains the same becomes worse off for all values of M. For the other player to become better off, we need:

$$\begin{aligned}&\frac{\partial x_2^*(\alpha_2,\alpha_1)}{\partial \alpha_1}\left[-4+x_1^*(\alpha_1,\alpha_2)-3x_2^*(\alpha_2,\alpha_1)\right] < \\ &\qquad < \frac{\partial x_1^*(\alpha_2,\alpha_1)}{\partial \alpha_1}\left[-4+x_2^*(\alpha_1,\alpha_2)-3x_1^*(\alpha_2,\alpha_1)\right]\end{aligned}$$

which can be shown to be the case for all $\alpha_1, \alpha_2 \in [0,1]$. A converse of this argument holds for a change in the attitude of player 2 (by symmetry of the game) and we may summarize these findings as follows:

Proposition 2 *Consider a variant of the Hotelling duopoly game in which the distribution of the median of the (uniform) consumer preferences M is unknown with support on $[-1/2, 1/2]$. Suppose the firms choose locations so as to maximize the α-maxmin value of the ex-post second-stage equilibrium profits, based on their respective degrees of pessimism $\alpha_1, \alpha_2 \in [0, 1]$. Then the equilibrium locations $x_i^*(\alpha_i, \alpha_{-i})$ are unique up to symmetry and such that for any $i, j \in \{1, 2\}$:*

$$(1) \quad : \quad (-1)^i \left[\partial x_i^*(\alpha_i, \alpha_{-i}) / \partial \alpha_i\right] < (-1)^i \left[\partial x_{-i}^*(\alpha_{-i}, \alpha_i) / \partial \alpha_i\right] < 0$$

$$(2) \quad : \quad \forall M \in [-1/2, 1/2] : (-1)^{i-j} \, \partial \pi_i \left(x_1^*(\alpha_1, \alpha_2), x_2^*(\alpha_2, \alpha_1), M\right) / \partial \alpha_j > 0$$

This result is particularly interesting if the firms have some way of committing to a pessimistic policy, for instance, by appointing cautious CEO's or by putting themselves in a position where losing customers due to a sudden change in preferences could mean bankruptcy, thereby making it necessary to take the necessary precautions. In such case, pessimism could serve as a way of strategic deterrence, discouraging the competitor from targeting one's market niche and instead making him withdraw into his own hinterland. However, a similar motive on behalf of the rival generates a Prisoner's Dilemma situation, as it was shown in the previous section that when both players become more pessimistic, they locate closer together and so earn less for all demand realizations. In this way, the firms' self-imposed pessimism becomes a self-fulfilling prophecy.

Alternatively, rather than through conscious commitment, the approach based on concentrating fully on the worst-case scenario could become prevalent *via* gradual elimination of underperforming, overly optimistic firms Either way, assuming the duopolists are eventually characterized by $\alpha_1 = \alpha_2 = 1$, the resulting product differentiation is equal to 1, less than the one resulting from the players following *any* common prior. Since the associated prices are also lower, the presence of ambiguity seems to benefit the consumers, although the average transport costs they pay (and hence, the socially-optimal locations) will depend on the actual distribution of M. On the other hand, the presence of ambiguously distributed demand fluctuations adversely affects the firms (compared with the certainty case), the opposite of what happens when the demand variations follow a commonly known pattern (as evidenced by the MZ model).

Interestingly, the equilibrium locations associated with $\alpha_1 = \alpha_2 = 1$ coincide with the ones which are socially-optimal, given customers uniformly distributed on the $[-1, 1]$ interval (see, for example [7] or [10]). In the current framework, all that is known to the firms is that the consumer preferences are contained in $[-1, 1]$. Hence, the equilibrium locations seem like a sensible choice for an equally uninformed social-planner.

Finally, we may follow MZ in considering the possibility of a change of timing, so that *both* stages of the game are played before uncertainty is resolved. For the firms uncertain only about the demand's location (but not the transport costs) the model is then relatively straightforward. Since the price is the same for all demand realizations, the payoff associated with a particular set of locations and prices is equal to a firm's own price multiplied by the α-maxmin value of the corresponding demand, i.e. of the consumer mass located on the relevant side of the 'indifferent consumer'. Hence, each player acts 'as if' being involved in a certainty Hotelling game in which the customer distribution is given by a weighted average of the demand realisation located as far as possible towards the rival and the one at the opposite extreme, with α and $1-\alpha$ being the respective weights. Consequently, a pessimistic player will consider an increase of uncertainty in similar terms as a shift of a small mass of consumers into his own hinterland matched by an opposite shift of a larger mass of customers into the hinterland of the competitor. Naturally, such a change would persuade a sufficiently pessimistic firm to move towards the rival, i.e. more uncertainty would decrease product differentiation and prices, as in the model discussed in the previous section.

The situation is somewhat complicated with the introduction of uncertainty about transport costs. Since lower costs (i.e. higher price elasticity of the demand for a firm's product) are better for the lower priced firm, while higher costs are better for the firm with a higher price, we have:

$$\pi_i\left(x_1,x_2,p_1,p_2\right)=\alpha\left[-L+1/2-(-1)^i\,\widetilde{x}\right]p_i+(1-\alpha)\left[L+1/2-(-1)^i\,\widehat{x}\right]p_i$$

where $\widetilde{x}$ is the location of the indifferent consumer in the worst case scenario, i.e. the value of x that solves:

$$p_1+t\left(x_1-x\right)^2=p_2+t\left(x_2-x\right)^2$$

with $t=1$ if $p_i<p_{-i}$ and $t=t_0$ otherwise. Similarly, $\widehat{x}$ is the best possible indifferent consumer location, obtained for $t=t_0$ if $p_i<p_{-i}$ and $t=1$ otherwise. The profit function is concave in p_i for $\alpha>1/2$, but its first derivative is discontinuous at $p_i=p_{-i}$, so that the best-response functions are:

$$BR_i=\begin{cases}\left(p_{-i}+p_i^0\right)/2 & \text{for } p_{-i}>p_i^0\\ \left(p_{-i}+p_i^1\right)/2 & \text{for } p_{-i}<p_i^1\\ p_{-i} & \text{otherwise}\end{cases}$$

where:

$$p_i^j = \frac{t_0 \left(x_2 - x_1\right) \left[1 + \left(2 - 4\alpha\right) L - \left(-1\right)^{i+j} \left(x_1 + x_2\right)\right]}{\left(t_0\right)^j + \left(-1\right)^{j+1} \alpha \left(1 - t_0\right)}$$

In other words, for a range of the counterpart's prices $[p_i^1, p_i^0]$ each firm would choose to respond with an identical price. If this coincides with a similar range of prices on the competitor's behalf, the set of Nash Equilibria of the price-subgame associated with locations x_1, x_2 is:

$$\left\{(p_1, p_2) : \left(\exists p^* \in \left[p_1^1, p_1^0\right] \cap \left[p_2^1, p_2^0\right]\right) (p_1 = p_2 = p^*)\right\}$$

It is easy to check that this happens when the firm locations are not 'too asymmetric', so that they satisfy:

$$x_1 + x_2 \in \left[-x_T, x_T\right], \;\; x_T = \left(1 - 2\alpha\right) \left[(4\alpha - 2)L - 1\right] \left(1 - t_0\right) / \left(1 + t_0\right) \qquad (2)$$

More specifically, the range of equilibrium prices is:

$$\begin{cases} [p_1^1, p_2^0] & \text{for } x_1 + x_2 \in [-x_T, 0] \\ [p_2^1, p_1^0] & \text{for } x_1 + x_2 \in [0, x_T] \end{cases}$$

while for $x_1 + x_2 \notin [-x_T, x_T]$ there is a unique asymmetric equilibrium.

Because of the possible multiplicity of price equilibria, the firms' location decisions will not be examined here. Nevertheless, assuming that locations are indeed not 'too asymmetric' (in the sense explained above), it may be observed that:

$$\forall i, j : \frac{\partial p_i^j}{\partial L} < 0 \Leftrightarrow \alpha \in (1/2, 1] \Leftrightarrow \forall i, j : \frac{\partial p_i^j}{\partial L \partial t_0} < 0$$

leading to the following statement:

Proposition 3 *Consider a variant of the Hotelling duopoly game in which the joint distribution of transport costs and the median of the (uniform) consumer preferences is unknown with support on $[t_0, 1] \times [-L, L]$, where $t_0 \in (0, 1]$ and $L \in \left[0, \frac{1}{2}\right]$. Suppose the firms' locations satisfy condition (2) and that the firms choose prices so as to maximize the α-maxmin value of the resulting profits. In any Nash Equilibrium, the firms set identical prices, where the lowest and highest possible equilibrium prices both decrease as L increases, provided that $\alpha \in (1/2, 1]$. However, the effect is dampened by a fall in t_0.*

Apart from the complication associated with the existence of multiple equilibria, the results are in line with those in the previous section. For pricing decisions made upon learning the consumer preferences, an increase of uncertainty led pessimistic firms to reduce product differentiation, thereby indirectly decreasing the resulting equilibrium prices. In the present case, uncertainty causes a similar change in prices by affecting the pricing decisions directly. Either way, the increase in the intensity of price competition can be dampened by a possibility of a more price elastic consumer demand.

Furthermore, we observe a tendency of the firms to mimic the competitor, where in the present case this takes an extreme form of what is effectively a coordination problem, even when each firm's initial situation ahead of the pricing stage is different. Once again, this may be related to the sports betting example, where the odds on offer converge shortly before the start of a race. Interestingly, some of the major bookmarkers offer a so called 'best odds guarantee'[7] within a given time before the start of the biggest races, pledging to at least match the competitors' prices when paying out the winnings. This could be seen as an attempt to solve the coordination problem, because the effective prices are all equal to the maximum of the ones actually offered by the bookmakers participating in the scheme.

5 Concluding Remarks

The paper examined a variation of the "Product differentiation and location decisions under demand uncertainty" model by Meagher and Zauner, in which the firms are unaware of the exact distribution of demand fluctuations, but resolve the resulting ambiguity using the Arrow-Hurwicz α-maxmin criterion. The change made it possible to accommodate a scenario in which firms mimic the competitors' behaviour in response to an increase of demand uncertainty, as illustrated by the sports betting industry.

Intuitively, a large range of possible demand variations is potentially more harmful to a player when product differentiation is bigger. Hence, a pessimistic entrepreneur may respond to an increase of uncertainty by making his offer more similar to that of the other firm, thereby leaving less room for his product being disadvantaged. This mechanism was analyzed in the context of the current model, where it turns out that, contrary to the existing literature, an increase of uncertainty about the location of consumer preferences makes the equilibrium more competitive when the firms are sufficiently pessimistic. In particular, product differentiation decreases,

[7] see, for example, `http://tinyurl.com/2g65uyy`

and with it the second-stage equilibrium prices and profits for any realisation of the uncertainty. This effect is moderated by uncertainty about the transport cost parameter (reflecting the price elasticity of demand), which has no effect on location decisions under certainty, but interacts with uncertainty about the placement of consumer demand.

When price-competition takes place before the realisation of uncertainty, it is affected in a similar way. In particular, despite the existence of multiple equilibria for 'not too asymmetric' locations, the highest and lowest possible equilibrium prices both decrease when demand location uncertainty increases, as long as the firms are pessimistic. Once again, the positive effect of this type of uncertainty on the degree of competition between pessimistic firms is moderated by a possibility of higher price elasticity of demand.

Finally, whenever a particular firm adapts a more pessimistic approach, it becomes better off at the competitor's expense, suggesting that 'being prepared for the worst' could serve as a form of strategic deterrence. Whether by conscious changes in approach, or *via* elimination of underperforming firms, pessimistic attitudes towards ambiguity seem destined to become prevalent. This means considerably less strategic product differentiation and lower prices than under certainty, suggesting that ambiguously distributed demand variations make the market more competitive, as opposed to ones that can be characterized by a common prior.

6 Appendix

Click on the link `http://tinyurl.com/36pnrl4` to download (Wolfram Mathematica file) or enter directly into browser.

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