Economics
Discussion Paper Series
EDP-1007

The regulation of a large sports league

Paul Madden

March 2010

Economics
School of Social Sciences
The University of Manchester
Manchester M13 9PL

THE REGULATION OF A LARGE SPORTS LEAGUE

Paul Madden

University of Manchester

JEL classification numbers; L10, L83

Keywords; sports leagues, revenue sharing, salary caps, ticket price caps, aggregate surplus

Abstract Literature on sports league regulation has focused on whether revenue sharing or salary caps increase competitive balance (the degree of equality in team qualities), but lacks models integrating ticket price (and price cap) analysis, welfare evaluation of policy and consistent modelling of strategic interactions, all incorporated here, the last by their absence due to large club numbers. Conclusions: whether increased competitive balance improves welfare depends on infra-marginal fan willingness-to-pay distributions; ticket price caps dominate revenue sharing and salary caps; salary (and ticket price) caps may save unsustainable leagues (which otherwise face autarchy); there is nothing to recommend revenue sharing.

Acknowledgment I am grateful to the Centre for Competition and Regulatory Policy (City University, London) for the opportunity to present the first draft of this paper (available at http://www.city.ac.uk/economics/CCRP/januaryworkshop.html) to a 2009 workshop, and to workshop participants and Francesc Trillas (discussant) for helpful comments. Similar thanks to participants in the EARIE conference, Llubljana 2009. I have also benefitted from most helpful conversations on the general subject of the economics of sports leagues with Terry Robinson, Stephen Ross and Stefan Szymanski. None of these colleagues should be assumed to concur with views expressed in the paper – the usual caveat applies.

Author Paul Madden, School of Social Sciences, University of Manchester, Manchester M13 9PL; e-mail, Paul.Madden@manchester.ac.uk

1. INTRODUCTION

The economics of competition between clubs in a professional team sports league has a number of unusual features, and has been the subject of research for over half a century now[1]. For instance, there is a benefit for any one club in seeing its rivals produce better teams, to some extent, so as to create better quality games against the rivals in the eyes of its fans (consumers), and thus greater revenue; there is a revenue externality, and driving out rival clubs from the league is clearly not desirable in this context. Indeed a major focus of the literature has been the concern that, without regulation, "bigger" clubs facing larger fan markets may produce teams that are so good as to lead to one-sided games and an economically unsuccessful league; in the jargon, the concern has been that unregulated competition between clubs may produce insufficient competitive balance (degree of equality in team qualities). The regulatory focus has been the affect on competitive balance of policies of revenue sharing (where a share of home club gate revenue goes to the visiting team, so that smaller clubs receive some of the relatively large revenue accruing when they play away against a bigger club) and salary capping (whereby there is an upper bound on a club's wage bill for players, constraining the bigger clubs from producing teams of too high a quality). The conventional textbook wisdom (using the so-called "Walras" or "fixed supply conjecture" analytical approach) is that salary caps do increase competitive balance, but revenue sharing has no affect on this balance (the so-called invariance principle). However the theoretical models behind these conclusions are fragmented, and incomplete in certain directions. The objective here is to present an integrated and complete analysis of a sports league whose structure is basically similar to that of the textbooks. The upshot is a quite different perspective on regulation.

We follow the conventional textbook model in a number of features. With the major North American team sports leagues in mind[2], the model assumes a perfectly inelastic supply of playing talent to the league, which is therefore the sole buyer of the specialised talent, reflecting more closely the situation in the North American leagues than, say, in European soccer. The league is assumed to consist of an exogenously given set of independent profit-maximizing[3] clubs (or franchises) that hire playing talent to field teams that meet each other over a season with home and away games in stadiums of given large capacity. In deciding their demand for playing talent (which becomes a measure of team quality) clubs believe that they cannot influence its wage, so the talent market is effectively perfectly competitive. And again with the North American context in mind, games are attended by home club fans only, and clubs are therefore monopoly providers of tickets for games to their fans. The resulting gate revenue is the only revenue source, and the expenditure on talent is the only cost[4].

---

[1] The earliest paper is Rottenberg (1956). Other landmark early contributions are Neale (1964), El-Hodiri and Quirk (1971) and Sloane (1971).

[2] The major leagues are the National Football League (NFL), Major League Baseball (MLB), National Basketball Association (NBA) and National Hockey League (NHL),), each with currently 30 clubs.

[3] A commonly studied alternative club objective is "win maximization", usually thought to be more relevant in the European soccer context; Kesenne has a number of papers on this subject and Kesenne (2007) provides a full account. Recently Madden (2010a) has suggested another objective, namely "fan welfare maximization", but again European soccer is the more obvious setting.

[4] Such models can be found in the major surveys by Fort and Quirk (1995) and Szymanski (2003), the handbook of Andreff and Szymanski (2006), the textbooks by Fort (2006) and Sandy et al. (2004), and in the materials for the increasing numbers of courses on Sports Economics being taught around the world.

However we go beyond existing models by integrating a precise treatment of the following four aspects. First, existing models are largely without a welfare economics foundation for the evaluation of league performance. This is rectified here via investigation of the aggregate surplus accruing to those agents who benefit from the league, namely the fans, players and club owners. Pursuit of increased aggregate surplus is taken to be the policy objective, and whether increased competitive balance produces a welfare improvement is a question for our agenda[5]. Secondly it has been usual to by-pass explicit analysis of ticket price decisions without which it is not possible to evaluate fan surplus and carry through the first objective. A detailed analysis of club decisions on ticket prices instead provides the starting point of the model here[6]. Thirdly the fact that clubs have monopoly power in their sale of tickets to fans suggests a regulatory policy of capping ticket prices, a policy that has hardly been addressed in the literature with its implicit treatment of ticket pricing, but is added here to the set of regulatory possibilities[7]. Fourthly, the assumption that the market for playing talent is perfectly competitive can be easily justified if the number of clubs in the league is sufficiently large that strategic interactions can be ignored. However ignoring strategic interactions makes little sense if there are just two clubs in the league, as is commonly assumed in many papers and textbook expositions. Instead it is assumed explicitly and consistently throughout this paper that there are sufficiently large[8] numbers of two types of club so that all strategic interactions between individual clubs are indeed negligible[9]. The analysis that follows is also complete, in that it determines equilibrium values of all endogenous variables for the league as a whole, namely ticket prices, the wage for talent and the allocations of talent to clubs (and hence competitive balance).

As a result the paper produces some quite different and novel perspectives on the working and regulation of the league. First the ticket price analysis produces a problem with existence of equilibrium in unregulated leagues, due to a novel source of talent demand discontinuity. Leagues with a large disparity between big and small fan market clubs may collapse into autarky or unsustainability (=non-existence). This provides a new potential rationale for policy, namely the rescue of an otherwise

---

[5] Welfare is discussed in existing papers by Fort and Quirk (2009) and Madden (2010). The latter is in a different (European soccer) framework. The former is similar to here but arrives at quite different conclusions, to which we return later.

[6] The main exception in the existing literature is Kesenne and Pauwels (2006) – see also Kesenne (2007) – who study individual club decisions on prices. Our focus is on prices in league equilibrium, but overlaps in part, and naturally builds on the club decision base; Kesenne and Pauwels (2006) also go beyond our current remit, analysing club pricing decisions for "win maximizers" and with stadium capacity constraints. Again we return later to more detailed comparisons.

[7] Again Kesenne and Pauwels (2006) provide analysis of the effect of ticket price caps on club decisions.

[8] In this sense the league is said to be large. The term large is used rather than perfectly competitive (or Walrasian) - although the talent market is perfectly competitive, the full model includes the monopolised ticket markets.

[9] The large league setting allows us to reach similar conclusions to those of the Walras/fixed-supply conjecture models, but without reliance on fixed-supply conjectures which are difficult to justify (see Szymanski (2004)). A discussion of "small" league models where strategic interactions do matter (including the "Nash" or "contest-Nash" approach of Szymanski (2004) and Szymanski and Kesenne (2004)) can be found in Madden (2010b). Whether the number 30 in the NFL, MLB, NBA and NHL makes the large or small setting more appropriate does not seem to me decidable a priori. In my view there is room for both types of model in the literature, the value of their insights and predictions to be the deciding factor.

unsustainable league. Secondly, the best that a league can do without regulation or with revenue sharing or with salary caps is a second best, because all three regimes lead to monopoly ticket prices and welfare losses. But if the distributions of fans' willingness to pay for tickets possess a certain invariance property, an unregulated sustainable league in fact attains the second best optimum. In this case, any policy that fails to preclude monopoly ticket pricing but increases competitive balance will *reduce* aggregate surplus – increased competitive balance is certainly not a general indicator of welfare improvement. Also in this case, neither revenue sharing nor salary caps can improve aggregate surplus, but the imposition of ticket price caps is welfare improving on the second best, at least locally, because of the elementary affect they have on consumer (fan) surplus via the lower ticket prices and increased demand (attendance). Thirdly, and now generally, revenue sharing leads to a *super-invariance principle* – not only does revenue sharing have no affect on talent allocations in a sustainable league (the invariance principle), it also cannot save leagues from unsustainability, and can never produce a welfare improvement. Salary caps continue to increase competitive balance in a sustainable league (which may well reduce aggregate surplus, as above), but may also be able to avert unsustainability in which case there is a clear welfare gain. However this welfare gain is less than could be achieved with the imposition of a ticket price cap, at least locally – ticket price caps are then the welfare dominant policy because of their impact on attendance and fan surplus.

The analysis leading to the above conclusions intersects with some general themes in the broader industry and regulation literature. The clubs make decisions on quality (talent demand) and are monopoly sellers of the product (tickets to games). There is a clear parallel to the literature on monopoly provision of quality, which initiated with Sheshinski (1976) and Spence (1976), and some of the findings that follow are related to those in that literature. However the parallel is not exact. Whereas the Sheshinski/Spence monopolists are selling a private good, our clubs are offering an excludable public good – when attendance is less than stadium capacity, an additional spectator will be non-rival. So a link to the literature on the monopoly provision of excludable public goods also appears (see Brito and Oakland (1981), Fraser (1996, 2000)). Relative to both literatures the sports league framework is more analytically challenging, as the various monopoly providers to their separate fan markets are also linked via the revenue externality and the talent market.

Section 2 presents the framework and Section 3 addresses individual club decisions on ticket prices and talent demand. Section 4 provides an existence result for unregulated league equilibrium. The positive economics of the impact of revenue sharing and salary caps are the subject of Section 5. Section 6 picks up on the normative aspects of previous sections, and Section 7 returns to the existence question and the issues around unsustainable leagues. Section 8 is devoted to all aspects of ticket price caps, and section 9 concludes.

2. THE FRAMEWORK

This section presents the basic framework of the paper. The league consists of an exogenously given set of clubs, with independent, profit-maximizing owners, and with teams that play each other over the season, with home and away games in stadiums of given, large capacity. Think of each club as located in a region, where its market of potential spectators for its home games (=its fans) is a subset of the population of the region; fans do not travel to away games. There is a continuum[10] of clubs of 2 types, with a mass of size 1 of each type $i = 1,2$; the distinction between types is described below.

On the input side the supply of playing talent to the league is assumed to be perfectly inelastic in a quantity normalised to unity, with a reservation wage normalised to zero; $w \geq 0$ denotes the price of a unit of playing talent. $t_{1j} \geq 0,\ j \in [0,1]$ denotes the allocation of playing talent to type 1 club $j \in [0,1]$ and $t_{2j} \geq 0,\ j \in [1,2]$ denote the corresponding allocations for type 2; t_{ij} is also referred to as the quality of team *ij*, and will be a choice variable for the owners of club *ij*. It will be necessary to consider only talent allocations that exhaust the available talent supply, $\int_0^1 t_{1j}dj + \int_1^2 t_{2j}dj = 1$, and so the average talent allocation across clubs is then $\bar{t} = \frac{1}{2}$.

On the output side the owners of club *ij* also makes decisions on its price (p_{ij}) for season tickets that allow entry to all subsequent home games over the season. Clubs cannot price discriminate, and because we allow only season ticket sales, attendance at all of the home games of club *ij* over the season can be taken to be the same[11]. The demand for tickets will be assumed to be the same for all clubs of the same type, as follows. When home team quality is t_{ij}, the intercepts of the ticket price-demand curve facing club *ij* are $n_i(t_{ij})$ (the demand axis intercept, referred to as the number of fans) and $v_i(t_{ij})$ (the price axis intercept, referred to as the maximum willingness to pay for a ticket amongst the fans). Generally $D_i(p_{ij}, t_{ij})$ is the demand for tickets from a club of type i at price p_{ij} when its team quality is t_{ij}; by definition, $D_i(0, t_{ij}) = n_i(t_{ij})$ and $D_i(p_{ij}, t_{ij}) = 0$ if $p_{ij} \geq v_i(t_{ij})$. It is useful to define also $V_i(t_{ij}) = v_i(t_{ij})n_i(t_{ij})$, referred to as the revenue potential of a club of type *i*, which will constitute an upper bound on the attainable revenue.

Before proceeding two comments are in order. First it is not generally credible to assume that demand depends on the single talent variable of home club quality – fans are also interested in the quality of the visiting teams. However we are implicitly thinking that demand depends not only on t_{ij} but also on the average quality of the visiting teams, which has more credibility. But given the inelastic talent supply and the continuum setting, the average quality of the visiting teams will be independent of t_{ij} and invariant at $\bar{t} = \frac{1}{2}$, and so can be suppressed, as we do here. Secondly the textbook expositions also have demand dependant on a single talent variable, usually

[10] Like other uses of "infinity" in economic modelling, the assumption of a continuum of clubs cannot of course be taken literally. The assumption legitimises the parametric treatment of the wage for talent when clubs formulate their talent demand. And like other uses of the assumption it does provide tractability here, which is less when strategic interactions of a small league are incorporated (see Madden (2010b)).

[11] The assumption that season tickets are the only product offered to fans by clubs is a simplifying assumption, obviating the need to model separately demand for games against type 1 and type 2 clubs.

the win percentage of the team, with demand starting to fall if this becomes too large. Although win percentage is not technically well-defined in the continuum setting and does not seem to be a plausible primitive single quality indicator in any setting[12], the falling off in demand can be captured here as follows.

We assume that talentless teams have no revenue potential, $V_i(0) = 0$, and it is convenient for the exposition to assume that $n_i(0) = v_i(0) = 0$; thus the price-demand curve degenerates to a point (the origin) when $t_{ij} = 0$. As t_{ij} increases from 0 the price-demand curve shifts uniformly outwards until $t_{ij} = m_i > \frac{1}{2}$, at which point home team quality is so far above the average quality of the other teams and games are on average so one-sided in favour of the home team, that the price-demand curve thereafter shifts inwards, reaching the origin again at some $M_i > m_i$. Adding natural restrictions on continuity, differentiability and slopes, and using the notation $D_{ip}^{'}(p_{ij}, t_{ij})$, $D_{ipp}^{''}(p_{ij}, t_{ij})$ for the first and second partial derivatives of D_i with respect to price, $D_{it}^{'}(p_{ij}, t_{ij})$, $D_{itt}^{''}(p_{ij}, t_{ij})$ for the corresponding talent partial derivatives, and (later) $D_{ipt}^{''}(p_{ij}, t_{ij})$ for the second cross-partial, the first demand assumptions are:

<u>Demand Assumption 1 (DA1)</u>

$n_i : \Re_+ \to \Re_+, v_i : \Re_+ \to \Re_+$ are continuous functions, and there exist $M_i > m_i > \frac{1}{2}$ such that $n_i(t_{ij}) = v(t_{ij}) = 0$ when $t_{ij} = 0$ and when $t_{ij} \geq M_i$; n_i and v_i are C^3 when $t_{ij} \in (0, M_i)$ with $n_i^{'}(t_{ij}), v_i^{'}(t_{ij}) > 0,\ 0 < t_{ij} < m_i$ and $n_i^{'}(t_{ij}), v_i^{'}(t_{ij}) \leq 0,\ M_i > t_{ij} \geq m_i$.

<u>Demand Assumption 2 (DA2)</u>

$D_i : \Re_+^2 \to \Re_+$ is a continuous function with $D_i(p_{ij}, t_{ij}) = 0$ when $t_{ij} = 0$, when $t_{ij} \geq M_i$, and when $p_{ij} \geq v_i(t_{ij})$; D_i is C^3 when $(p_{ij}, t_{ij}) \in ((0, v_i(t_{ij})), (0, M_i))$ with $D_{ip}^{'}(p_{ij}, t_{ij}) < 0$, and $D_{it}^{'}(p_{ij}, t_{ij}) > 0, 0 < t_{ij} < m_i$, $D_{it}^{'}(p_{ij}, t_{ij}) \leq 0, M_i > t_{ij} \geq m_i$

Notice that (DA1-2) imply that the revenue potential has the properties:
$V_i : \Re_+ \to \Re_+$ is continuous and $V_i(t_{ij}) = 0$ when $t_{ij} = 0$ and when $t_{ij} \geq M_i$; V_i is C^3 when $t_{ij} \in (0, M_i)$ with $V_i^{'}(t_{ij}) > 0,\ 0 < t_{ij} < m_i$ and $V_i^{'}(t_{ij}) \leq 0,\ M_i > t_{ij} \geq m_i$ (2.1)

The resulting revenue and profit functions are defined by;

$$R_i(p_{ij}, t_{ij}) = p_{ij} D_i(p_{ij}, t_{ij}), \quad \Pi_i(p_{ij}, t_{ij}) = R_i(p_{ij}, t_{ij}) - w t_{ij} \qquad (2.2)$$

3. MONOPOLY PRICING AND TALENT DEMAND

The club profit maximization problem is addressed in two stages, first solving for optimal prices with a given talent level and then solving for the talent demand. For the first stage we assume that revenue is strictly concave in price:

[12] See Madden (2010b) for further development of this point.

Demand Assumption 3 (DA3)
For all $(p_{ij}, t_{ij}) \in (0, v_i(t_{ij}) \times (0, M_i)$, $\frac{\partial^2 R_i}{\partial p_{ij}^2} = 2D_{ip}^{'}(p_{ij}, t_{ij}) + p_{ij} D_{ipp}^{''}(p_{ij}, t_{ij}) < 0$

(DA3) ensures that the revenue maximizing or monopoly price for a club of type *i* of quality t_{ij} defines a C^2 function $p_i : (0, M_i) \to \Re_{++}$ where $p_{ij} = p_i(t_{ij})$ is characterised by the usual unit elastic price-demand condition:

$$D_i(p_{ij}, t_{ij}) + p_{ij} D_{ip}^{'}(p_{ij}, t_{ij}) = 0 \qquad (3.1)$$

Moreover, from (DA1), this monopoly pricing function or rule extends continuously to $p_i : \Re_+ \to \Re_+$ with $p_i(0) = 0$ (and $p_i(t_{ij}) = 0, t_{ij} \geq M_i$). Given the rule, the revenue and profit functions in (2.2) become the following "reduced form" functions of t_{ij} only:

$$r_i(t_{ij}) = p_i(t_{ij}) D_i(p_i(t_{ij}), t_{ij}), \;\; \pi_i(t_{ij}) = \Pi_i(p_i(t_{ij}), t_{ij}) = r_i(t_{ij}) - wt_{ij} \qquad (3.2)$$

It follows that:
$r_i : \Re_+ \to \Re_+$ is continuous and $r_i(t_{ij}) = 0$ when $t_{ij} = 0$ and when $t_{ij} \geq M_i$;
r_i is C^2 when $t_{ij} \in (0, M_i)$ with $r_i^{'}(t_{ij}) \equiv p_i(t_{ij}) D_{it}^{'}(p_i(t_{ij}), t_{ij}) > 0, 0 < t_{ij} < m_i$ and
$r_i^{'}(t_{ij}) \leq 0, M_i > t_{ij} \geq m_i$ (3.3)

The reduced form marginal and average revenues (defined on $(0, M_i)$) are $mr_i(t_{ij}) \equiv r_i^{'}(t_{ij})$ and $ar_i(t_{ij}) = r_i(t_{ij}) / t_{ij}$, respectively. The second stage maximization of $\pi_i(t_{ij})$ produces the profit-maximizing choice of t_{ij} and will be the same for all clubs of the same type; the resulting talent demand for a type *i* club is denoted $t_i(w)$.
The reduced form marginal revenue curves are the starting point for the analysis of profit maximization in many papers and textbooks which tend to by-pass explicit analysis of ticket pricing decisions, and simply assume that these curves are downward sloping whenever marginal revenue is positive (so r_i is concave there). However in many cases, indeed in a sense in most cases, such an assumption cannot hold, a fact which has been overlooked in existing literature and which has important eventual consequences for the economics of sports leagues. The problem concerns $mr_i(0) \equiv \lim p_i(t_{ij}) D_{it}^{'}(p_i(t_{ij}), t_{ij})$ as $t_{ij} \to 0$. Since $p_i(0) = 0$, it follows that $mr_i(0) = 0$ if $\lim D_{it}^{'}(p_i(t_{ij}), t_{ij})$ as $t_{ij} \to 0$ is finite, a condition that must be satisfied if the demand function can be extended differentiably to (0,0). But since $mr_i(t_{ij}) > 0$ for small positive t_{ij}, marginal revenue is locally *increasing* at 0 (i.e. the reduced form revenue function is locally *convex* at the origin)[13, 14]. An assumption which admits this possibility is:

[13] Kesenne and Pauwels (2006) and Kesenne (2007) address ticket pricing in much greater detail than the rest of the literature. Their analysis is based on the original profit function in (2.2) supported by its assumed concavity (rather than the two-stage approach here of solving first for optimal prices leading to reduced form revenues), and also overlooks the problem.
[14] It is possible to avoid this conclusion, with demands that satisfy an Inada condition at the origin. However admitting this possibility has negligible effect on our results, so we leave it out.

Demand Assumption 4 (DA4)
$mr_i(0)=0$ and there exists $l_i \in (0,m_i)$ such that $mr_i'(t_{ij})>0, t_{ij} \in (0,l_i), mr_i'(l_i)=0$ and $mr_i'(t_{ij})<0, t_{ij} \in (l_i,m_i)$.

It follows from (DA4) that average revenue will be maximized at some $s_i \in (l_i,m_i)$ where $ar_i(s_i)=mr_i(s_i)$. When $w=\overline{w}_i \equiv mr_i(s_i)$, profit maximization produces zero profit with talent demand of either 0 or s_i, higher wages lead to 0 talent demand and lower wages produce demand $t_{ij} \in (s_i,m_i]$ where $mr_i(t_{ij})=w$. s_i is referred to as the minimum sustainable team quality for a club of type i and $\overline{w}_i$ is the choke wage. Talent demand is therefore characterised by:

Proposition 1 Assume (DA1-4). There exists $s_i \in (l_i,m_i)$ with $\overline{w}_i \equiv mr_i(s_i)$ such that the demand for talent of a type i club is:
$t_i(w)=0$ if $w>\overline{w}_i$; $t_i(\overline{w}_i)=\{0,s_i\}$; and if $w<\overline{w}_i$, $t_i(w)=t_{ij}$ where $mr_i(t_{ij})=w$.

When (DA1-4) are satisfied, the typical talent demand curve is as shown in Figure 1.

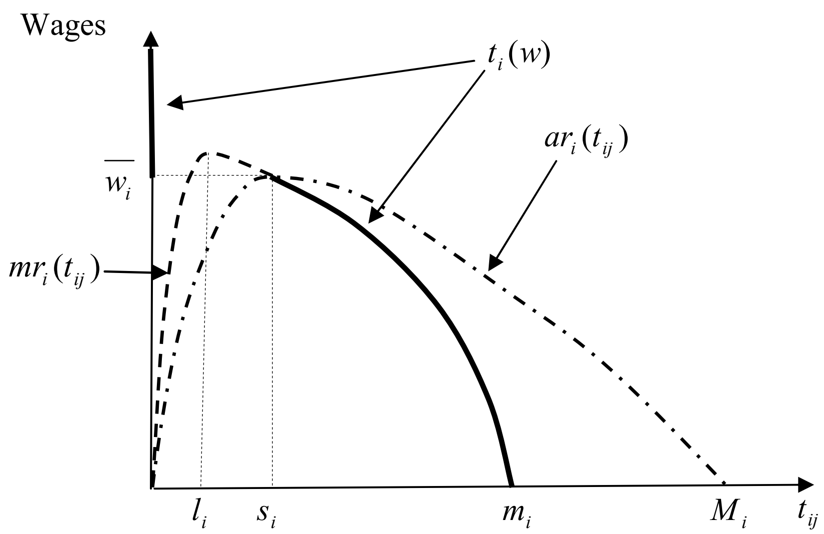


Figure 1; a talent demand curve (bold)

What generates this demand discontinuity is straightforward to see. Given monopoly pricing, marginal revenue from increasing talent (or quality) is (by the envelope theorem) $\partial R_i / \partial t_{ij} = p_{ij} \partial D_i / \partial t_{ij}$. As $p_{ij} \to 0$, $t_{ij} \to 0$ and marginal revenue goes to zero also (provided the talent demand derivative remains defined). Thus marginal revenue is locally increasing at zero talent, and there has to be a strictly positive minimum sustainable talent level for the monopolist.

For subsequent price analysis the following assumption on demand second derivatives is added:

Demand Assumption 5 (DA5) For all $(p_{ij}, t_{ij}) \in (0, v_i(t_{ij}) \times (0, m_i)$ where $\frac{\partial R_i}{\partial p_{ij}} \geq 0$;

(a) $\frac{\partial^2 R_i}{\partial t_{ij}^2} = p_{ij} D_{itt}^{''}(p_{ij}, t_{ij}) < 0$ and (b) $\frac{\partial^2 R_i}{\partial p_{ij} \partial t_{ij}} = D_{it}^{'}(p_{ij}, t_{ij}) + p_{ij} D_{ipt}^{''}(p_{ij}, t_{ij}) > 0$.

An immediate consequence is for the slope of the monopoly pricing rule. Differentiating (3.1) with $p_{ij} = p_i(t_{ij})$ produces the following where all derivatives are at $(p_i(t_{ij}), t_{ij})$:

$$p_i^{'}(t_{ij}) = -[D_{it}^{'} + p_i(t_{ij}) D_{ipt}^{''}]/[2D_{ip}^{'} + p_i(t_{ij}) D_{ipp}^{''}]$$

(DA3) and (DA5b)[15] ensure that $p_i^{'}(t_{ij}) > 0$, at least for sustainable team qualities – better teams lead to higher ticket prices[16].

4. UNREGULATED LEAGUE EQUILIBRIUM

Consider now a league as a whole with the asymmetry that some clubs face bigger fan markets than others; without loss of generality the type 1 clubs will be the bigger clubs in what follows, and it is natural to assume then that, with the same quality team and ticket price, ticket demand and so revenue accruing to type 1 clubs will be larger than that for type 2. Moreover it is assumed that marginal revenues with respect to price and talent are larger for the bigger clubs in the following senses:

Demand Assumption 6 (DA6) $m_1 \geq m_2$ and for $t \in (0, m_2]$, $p \in (0, v_2(t))$:

(a) $\frac{\partial R_1}{\partial p_{1j}} > \frac{\partial R_2}{\partial p_{2j}}$, derivatives evaluated at *(p,t)*;

(b) $mr_1(t) > mr_2(t)$.

Part (b) of (DA6) is the assumption used in previous literature to define the bigger clubs. It follows that $\overline{w}_1 > \overline{w}_2$ and $t_1(w) > t_2(w)$ for all $w \in (0, \overline{w}_1)$, with weak inequality elsewhere. Thus the bigger type 1 clubs would demand more talent than type 2 at any wage. Part (a) says merely that, starting from the same team quality and ticket price, an increase in price will produce a larger revenue increase (or smaller decrease) for the bigger clubs. It then follows from (DA3) that $p_1(t) > p_2(t)$ for $t \in (0, m_2)$, so with the same team quality, bigger clubs would charge the higher ticket price. Since $p_i^{'}(t_{ij}) > 0$, a combined consequence of (DA6)(a) and (b) is that with a uniform wage the bigger type 1 clubs will demand more talent and set higher ticket prices than the smaller type 2.

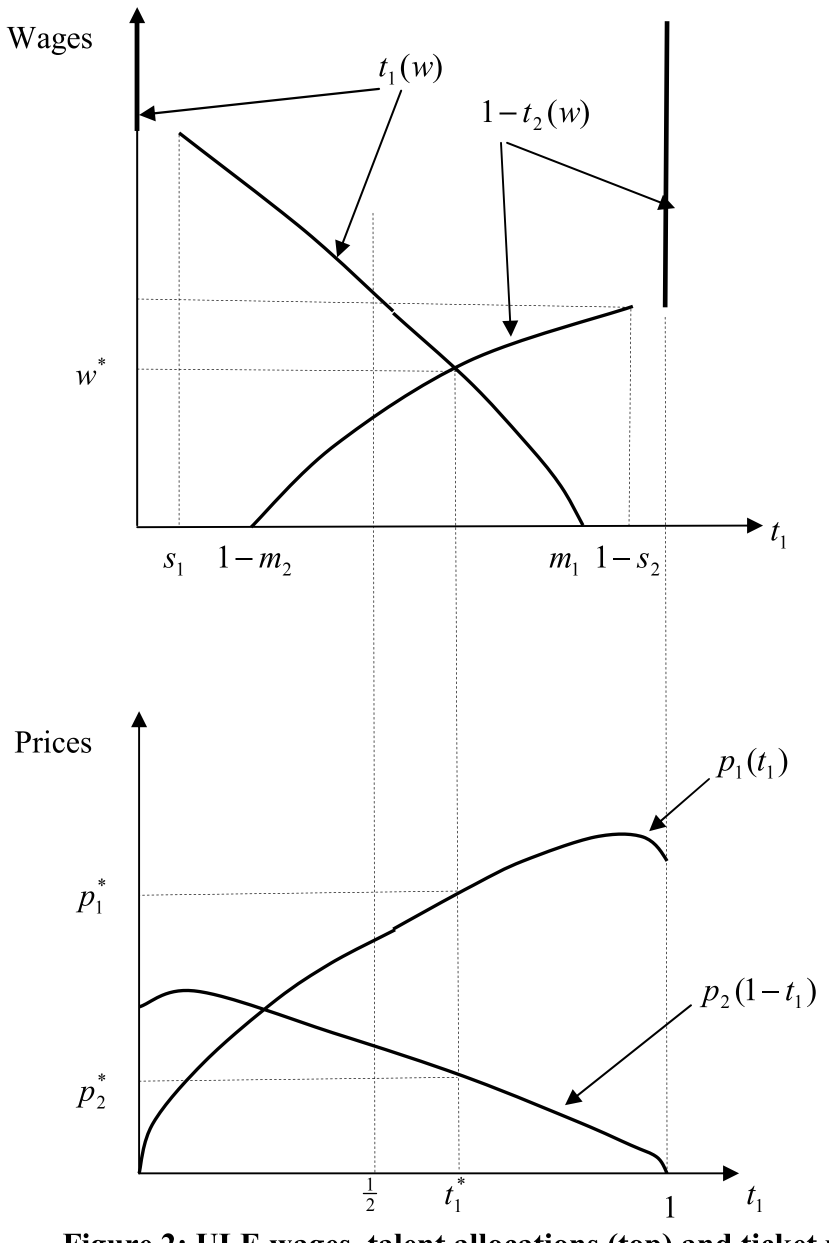
---

[15] Part (a) of (DA5) is used only in the ticket price cap Section 9.

[16] $n_i(t_{ij}) = v_i(t_{ij}) = t_{ij}(M_i - t_{ij})$ and $D_i(p_{ij}, t_{ij}) = n_i(t_{ij}) - p_{ij}$ provides a simple example satisfying (DA1-5). $p_i(t_{ij}) = \frac{1}{2} t_{ij}(M_i - t_{ij})$, $r_i(t_{ij}) = \frac{1}{4} t_{ij}^2 (M_i - t_{ij})^2$ and $mr_i(t_{ij}) = \frac{1}{2} t_{ij}(M_i - t_{ij})(m_i - t_{ij})$ where $m_i = \frac{1}{2} M_i$. Thus $mr_i(0) = 0$ and $mr_i^{'}(t_{ij}) = 2m_i^2 + 3t_{ij}^2 - 6m_i t_{ij} = 0$ when $t_{ij} = \frac{3-\sqrt{3}}{3} m_i \equiv l_i$. Also $ar_i(t_{ij}) = \frac{1}{4} t_{ij}(M_i - t_{ij})^2$ is maximized when $t_{ij} = \frac{2}{3} m_i \equiv s_i$ with $mr_i(s_i) = \frac{8}{27} m_i^3 \equiv \overline{w}_i$. With these l_i, s_i and $\overline{w}_i$, (DA1-5) are satisfied. Section 6 uses a more general family satisfying (DA1-5).

When no regulatory restrictions are imposed on clubs, equilibrium for the league simply requires that clubs make consistent profit-maximizing choices. The following assumption means attention can be restricted to "interior" equilibria where all clubs have strictly positive talent, and wages and prices are also strictly positive:
<u>Equilibrium Assumption 1 (EA1)</u> $m_1, m_2 < 1;\ m_1 + m_2 \le 1$
The first part ensures strictly positive talent allocations and so prices, the second part excess demand for talent at zero wage and strictly positive equilibrium wages. Thus:
Unregulated League Equilibrium (ULE) is a strictly positive 5-tuple $w^*, t_1^*, t_2^*, p_1^*, p_2^*$ such that $t_1^* + t_2^* = 1$ and for $i = 1,2$, $t_i^* = t_i(w^*),\ p_i^* = p_i(t_i^*)$.

The top half of Figure 2 is a standard diagram[17] found in many textbooks and papers, amended so that the horizontal axis measures the talent allocation to *each* type 1 club, and to show the discontinuity. The bottom half shows the monopoly pricing.



<u>Figure 2; ULE wages, talent allocations (top) and ticket prices (bottom)</u>

[17] The diagram is attributed to Quirk and Fort (1992).

Because of the talent demand discontinuity, existence of equilibrium is problematic. In fact Figure 2 displays a set of conditions that are sufficient to ensure existence, namely:
Equilibrium Assumption 2 (EA2) $s_1 \le 1 - m_2$, $s_2 \le 1 - m_1$.

(EA2) requires that the minimum sustainable team qualities are sufficiently small. Then the downward sloping segment of $t_1(w)$ in Figure 2 must intersect the upward sloping segment of $1 - t_2(w)$, ensuring existence, and the intersection must be unique.

Equilibrium competitive balance is defined here as $CB^* = 1 - \left| t_1^* - t_2^* \right|$, which attains the upper bound of 1 in a perfectly balanced equilibrium and 0 at the other extreme. Since $t_1^* > \frac{1}{2} > t_2^*$, $CB^* \in (0,1)$ and competitive balance is less than maximal;
Proposition 2 Assume (DA1-6) and (EA1-2). Then there exists a ULE, and this equilibrium is unique, with $t_1^* > \frac{1}{2} > t_2^*$, $CB^* \in (0,1)$ and $p_1^* > p_2^*$.

The league is then said to *sustainable*, assumed until Section 8 where the consequences of the possible non-existence are picked up again.

5. REVENUE SHARING AND SALARY CAPS

The focus now is regulation, in particular, the impact of revenue sharing and salary capping for competitive balance. The results of this section merely confirm the standard lessons reached in existing literature using the "Walras" or "fixed-supply conjecture" approach, but provide the background for later discussion of welfare, unsustainable leagues and ticket price caps.

With the revenue sharing regulatory policy, home teams retain only the fraction $\alpha \in (0,1]$ of their home gate revenue, the rest going to the away teams. A club's price decision impacts only on its home gate revenue, now $\alpha R_i(p_{ij}, t_{ij})$, and for each t_{ij} the revenue maximizing monopoly price does not vary with α. Such a policy can have no impact on monopoly ticket pricing – whatever talent allocations ensue the profit maximizing clubs will still universally adopt the same monopoly pricing rules as before. To formalise the consequences, suppose that almost all type 1 (type 2) clubs have talent t_1 (t_2), so that aggregate home gate revenue across the league from ticket sales is $r_1(t_1) + r_2(t_2)$.

Since attendance at all games at a stadium is the same, each club gets an equal share of the aggregate amount due to away teams, so the reduced form revenue and profit functions become:

$$r_i^{RS}(t_{ij}) = \alpha r_i(t_{ij}) + \tfrac{1}{2}(1-\alpha)[r_1(t_1) + r_2(t_2)], \quad \pi_i^{RS}(t_{ij}) = r_i^{RS}(t_{ij}) - wt_{ij} \qquad (5.1)$$

Because of the large league assumption, club *ij* has no influence over the second term in the revenue expression, and the profit maximizing choice of t_{ij} (denoted $t_i^{RS}(w,\alpha)$) is characterised by (5.2) which has Proposition 1 in the special case $\alpha = 1$:
There exists $s_i \in (l_i, m_i)$ and $\overline{w}_i(\alpha) \equiv \alpha mr_i(s_i)$ such that $t_i^{RS}(w,\alpha) = 0$ if $w > \overline{w}_i(\alpha)$;

$t_i^{RS}(\overline{w}_i,\alpha)=\{0,s_i\}$; if $w<\overline{w}_i(\alpha)$, $t_i^{RS}(w,\alpha)=t_{ij}$ where $\alpha mr_i(t_{ij})=w$ (5.2)

The effect of revenue sharing on Figure 1 is that m_i remains the talent demand at zero wage, the minimum sustainable team quality is also unchanged at s_i, but $\overline{w}_i$ falls and the demand curve between s_i and m_i swivels down around m_i.

League Equilibrium with Revenue Sharing (LERS) and home team share $\alpha \in (0,1]$ is a strictly positive 5-tuple $w^{RS}, t_1^{RS}, t_2^{RS}, p_1^{RS}, p_2^{RS}$ such that $t_1^{RS}+t_2^{RS}=1$ and for $i=1,2$, $t_i^{RS}=t_i^{RS}(w,\alpha)$ and $p_i^{RS}=p_i(t_i^{RS})$.

The effect of revenue sharing on Figure 2 is that, as α falls from 1, m_i, s_i remain unchanged and the positive segments of the talent demand curves fall, intersecting where $mr_1(t_1)=mr_2(1-t_1)=w/\alpha$. But the last condition implies that $t_i^{RS}=t_i^*, i=1,2$, so the LERS exists and is unique with the same talent allocation (and competitive balance and prices) as the ULE, a result known in the literature as the invariance principle. The only difference created by the revenue sharing is a lowering of the wage ($w^{RS}=\alpha mr_1(t_1^*)$).

Proposition 3 Assume (DA1-6) and (EA1-2). Revenue sharing with any $\alpha \in (0,1]$ as the revenue share of home clubs leaves invariant at their ULE levels the talent allocation, competitive balance and ticket prices. As α decreases the only effect is a reduction in the wage.

The second policy is an upper bound, *S* say, on the wage bills of clubs, a so-called salary cap. Once again this cannot affect monopoly ticket pricing. The typical reduced form profit maximization problem becomes;

$\max \pi_i(t_{ij})$ subject to $wt_{ij} \le S$. (5.3)

Talent demand, $t_i^{SC}(w,S)$, is now described by:

$t_i^{SC}(w,S)=0$ if $w>\overline{w}_i$; $t_i^{SC}(\overline{w}_i,S)=0$ if $s_i>S/w$ and $t_i^{SC}(\overline{w}_i,S)=\{0,s_i\}$ if $s_i \le S/w$; for $w<\overline{w}_i$, $t_i^{SC}(w,S)=t_i(w)$ if $wt_i(w)\le S$, $t_i^{SC}(w,S)=S/w$ if $wt_i(w)>S$ and $ar_i(S/w)>w$, $t_i^{SC}(w,S)=\{0,S/w\}$ if $wt_i(w)>S$ and $ar_i(S/w)=w$, $t_i^{SC}(w,S)=0$ if $wt_i(w)>S$ and $ar_i(S/w)<w$ (5.4)

The effect of a salary cap on Figure 1 is that talent demand at *w* becomes the lower envelope of $t_i(w)$ and the rectangular hyperbola $wt_{ij}=S$ up to $\overline{w}_i$ if intersections of mr_i and the hyperbola are to the right of s_i, or, otherwise up to a new lower choke wage defined by the leftmost intersection of mr_i and the hyperbola.

League Equilibrium with a Salary Cap (LESC) at level *S* is a strictly positive 5-tuple $w^{SC}, t_1^{SC}, t_2^{SC}, p_1^{SC}, p_2^{SC}$ such that $t_1^{SC}+t_2^{SC}=1$, and for $i=1,2$, $t_i^{SC}=t_i^{SC}(w,S)$ and $p_i^{SC}=p_i(t_i^{SC})$.

With (DA1-6) and (EA1-2), $t_1^*>\frac{1}{2}>t_2^*$, and a salary cap with $S \ge w^*t_1^*$ has no effect at all. As *S* falls from $w^*t_1^*$ it first produces LESC where the cap binds only on big clubs, leaving type 2 clubs on their original talent demand schedule to the left of

(w^*, t_1^*) in Figure 2, with t_1 decreasing (so p_1 decreasing) and t_2 increasing (so p_2 increasing) until they reach $\frac{1}{2}$ when $S = \frac{1}{2} mr_2(\frac{1}{2})$. Thereafter $(S < \frac{1}{2} mr_2(\frac{1}{2}))$ the cap binds on all clubs, t_1^{SC}, t_2^{SC}, p_1^{SC} and p_2^{SC} remain constant, and the wage falls to 0 as S falls to 0. Figure 3 indicates in bold the locus of LESC as S falls from $w^* t_1^*$ to 0, summarised in Proposition 4.

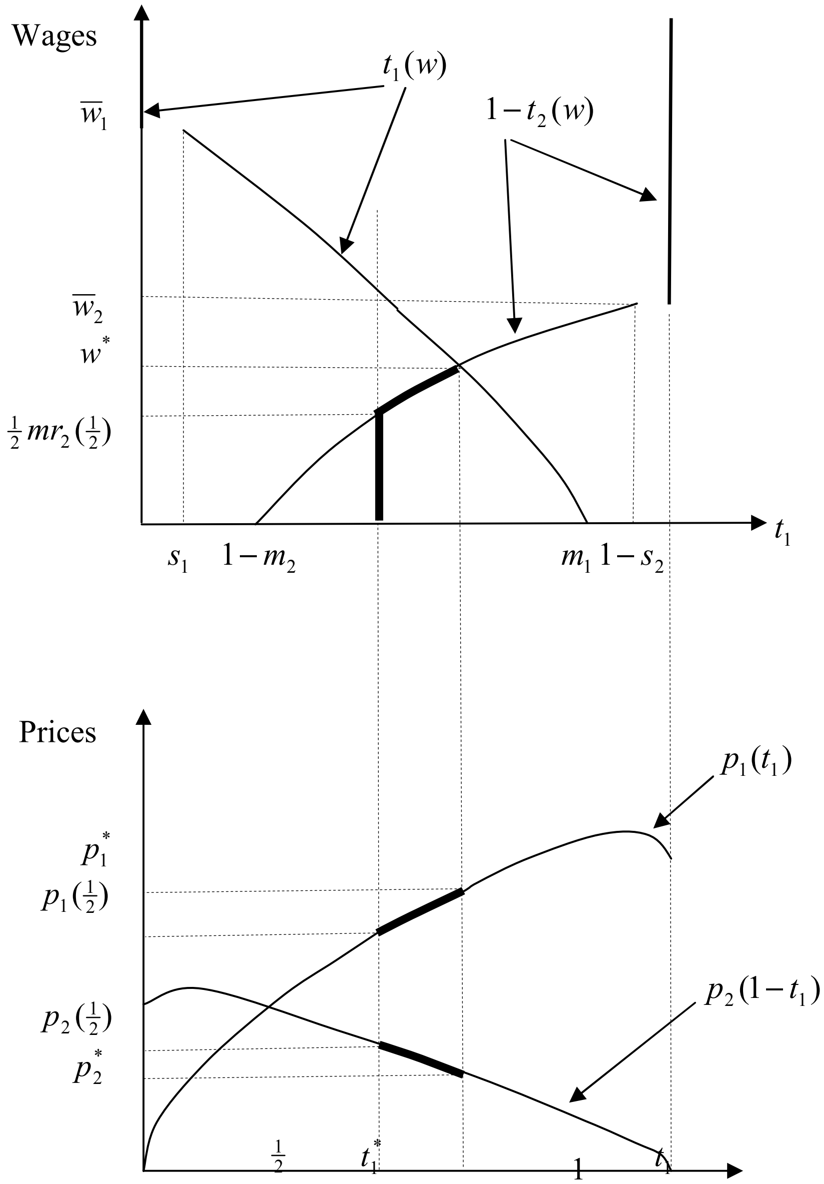


Figure 3; LESC wages, talent allocations (top, bold) and ticket prices (bottom, bold)

Proposition 4 Assume (DA1-6), (EA1-2) and a salary cap *S*. As *S* falls from $w^* t_1^*$ to $\frac{1}{2} mr_2(\frac{1}{2})$, t_1^{SC} falls from t_1^* to $\frac{1}{2}$, CB^{SC} increases from $CB^* \in (0,1)$ to 1, w^{SC} falls from w^* to $mr_2(\frac{1}{2})$, p_1^{SC} falls from p_1^* to $p_1(\frac{1}{2})$ and p_2^{SC} increases from p_2^* to $p_2(\frac{1}{2})$. As S falls further, from $\frac{1}{2} mr_2(\frac{1}{2})$ to 0, $t_1^{SC} = \frac{1}{2}$, $CB^{SC} = 1$, $p_1^{SC} = p_1(\frac{1}{2})$ and $p_2^{SC} = p_2(\frac{1}{2})$ are invariant, w^{SC} continuing to fall from $mr_2(\frac{1}{2})$ to 0.

6. AGGREGATE SURPLUS

6.1 Definitions

The parties who derive surplus from the league activity are the fans, club owners and players. Under the usual quasi-linear utilities assumption, Pareto efficiency equates to maximization of the aggregate surplus accruing to all parties, and so evaluation of the performance of the unregulated league and the impact of regulatory policies will be based on this aggregate surplus[18].

For the welfare analysis attention can be restricted to situations where all big (small) clubs offer the same ticket price and the same quality team, and we use the notation FS_i, OS_i, PS_i for the surplus accruing to, respectively, fans, owners and players of a club of type $i, i = 1,2$, and $AS = FS_1 + FS_2 + OS_1 + OS_2 + PS_1 + PS_2$ is aggregate surplus. Figure 4 illustrates the typical ticket price-demand curve and surpluses. The surplus formulae are $FS_i = \int_{p_i}^{v_i(t_i)} D_i(p,t_i)dp$, $OS_i = \Pi_i(p_i,t_i)$ and $PS_i = wt_i$.

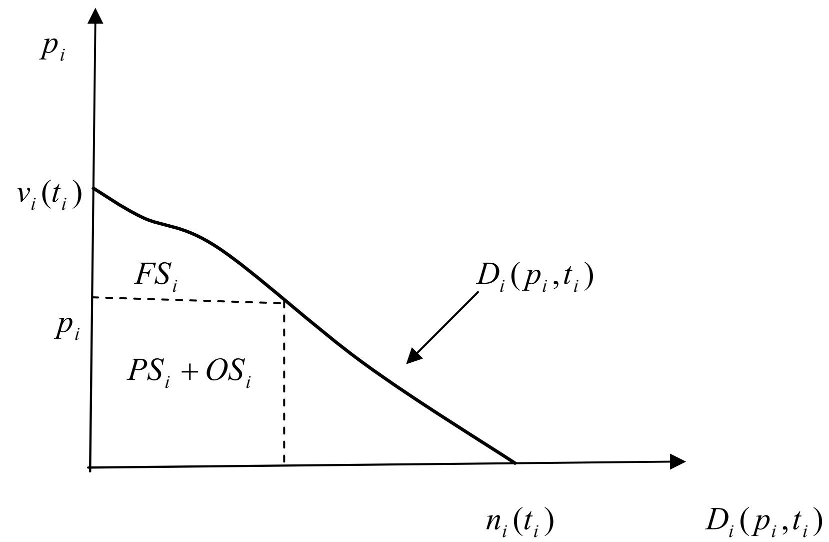


Figure 4; Surpluses

A peculiarity of the industrial structure is that, once talent has been allocated, the provision of tickets to fans is a problem of the allocation of excludable public goods – given that stadiums are large, and the absence of congestion or other stadium costs, attendance by an extra fan is purely non-rival in its nature. To maximize aggregate surplus, and so attain first-best Pareto efficiency, ticket prices should be zero with all fans receiving tickets, and talent should be allocated (with $t_2 = 1 - t_1$) to maximize:

$$S(t_1) = \int_0^{v_1(t_1)} D_1(p,t_1)dp + \int_0^{v_2(1-t_1)} D_2(p,1-t_1)dp \tag{6.1}$$

[18] Little attention has been given to welfare issues in the sports literature. Madden (2010a) studies the welfare consequences of alternative owner objectives in small (two club) leagues with perfectly elastic talent supply. Then players earn no surplus from the league and the relevant aggregate surplus is the sum of fan and owner surpluses. Recently Fort and Quirk (2010) study a league with perfectly inelastic talent supply, but also set out to use fan plus owner surplus as the welfare measure, for reasons which are not clear; Fort and Quirk (2010) is the subject of a further remark later here (p. 18).

The First-Best Talent Allocation is t_1^{FB} ($t_2^{FB} = 1 - t_1^{FB}$) where $t_1^{FB} = \arg\max S(t_1)$.
The First-Best Aggregate Surplus is $S^{FB} = S_1(t_1^{FB})$.

Clearly monopoly pricing will preclude attainment of the first-best and we focus instead on the second best optimum where prices are restricted to follow the monopoly pricing rules $p_i = p_i(t_i), i = 1,2$. The surplus accruing to the fans, owners and players of club i is then:

$$S_{Mi}(t_i) = \int_{p_i(t_i)}^{v_i(t_i)} D_i(p, t_i)dp + r_i(t_i) \qquad (6.2)$$

The corresponding aggregate surplus is:

$$S_M(t_1) = S_{M1}(t_1) + S_{M2}(1 - t_1)$$

The Second-Best Talent Allocation is t_1^{SB} ($t_2^{SB} = 1 - t_1^{SB}$) where $t_1^{SB} = \arg\max S_M(t_1)$.
The Second-Best Aggregate Surplus is $S^{SB} = S_M(t_1^{SB})$.

6.2 ULE and second-best optimality

Assuming monopoly pricing, the sum of all owner and player surpluses is the reduced form revenue sum $r_1(t_1) + r_2(1 - t_1) \equiv r(t_1)$. Under (DA1-6) and (EA1-2), it follows that $r(t_1)$ is increasing on $(0, 1 - m_2)$, decreasing on $(m_1, 1)$ and strictly concave on $[1 - m_2, m_1]$, hence with a global maximum at the ULE t_1^* where $mr_1(t_1^*) = mr_2(1 - t_1^*)$. Thus, generally and is well-known in the literature, ULE does maximize the second-best sub-aggregate surplus accruing to owners and players. But does ULE ever attain the full second-best optimum? A positive answer follows in a special case, as follows.

The special case is described by two assumptions, which together imply an invariance in fans' distributions of willingness to pay for tickets:
<u>Special Case Demand Assumption 1 (SC1)</u>
(a) The functions $n_i : \Re_+ \to \Re_+$ and $v_i : \Re_+ \to \Re_+$ satisfy (DA1)
(b) $n_i^{'}(0) > 0, v_i^{'}(0) > 0$ and for all $t_{ij} \in (0, m_i)$, $n_i^{''}(t_{ij}), v_i^{''}(t_{ij}) < 0$, $n_i^{'''}(t_{ij}), v_i^{'''}(t_{ij}) \leq 0$
(c) $m_1 \geq m_2$ and for all $t \in [0, m_2)$, $n_1^{'}(t) > n_2^{'}(t) > 0$, $v_1^{'}(t) > v_2^{'}(t) > 0$

In particular, as t_{ij} increases from 0 to m_i both the price-demand curve intercepts increase in a strictly concave fashion and with non-positive third derivatives; also part (c) implies that the intercepts for the big type 1 clubs are (for the same team quality) larger than those for type 2 ($n_1(t) > n_2(t)$, $v_1(t) > v_2(t)$ is implied by (c) and (a)) and increase more quickly. Between the intercepts there is the following invariance to the demand structure:
<u>Special Case Demand Assumption 2 (SC2)</u>
(a) $D_i(p_{ij}, t_{ij}) = 0$ if $t_{ij} = 0$, $t_{ij} \geq M_i$ or $p_{ij} \geq v_i(t_{ij})$
(b) $D_i(p_{ij}, t_{ij}) = n_i(t_{ij}) G(\frac{p_{ij}}{v_i(t_{ij})})$ if $(p_{ij}, t_{ij}) \in [0, v_i(t_{ij})] \times (0, M_i)$

(c) $G:[0,1]\to[0,1]$ is a continuous (decumulative distribution) function, C^3 and strictly decreasing on $(0,1)$, with $G(0)=1$, $G(1)=0$.
(d) $xG(x)$ is strictly concave, i.e. $2G'(x)+xG''(x)<0,\ x\in(0,1)$

Here (b) is the critical specialisation. With $x=\frac{p_{ij}}{v_i(t_{ij})}$, $G(x)$ is the proportion of fans of club *ij* (of team quality t_{ij}) who are willing to pay for a ticket at least the fraction x of the maximum willingness to pay. The formulation is special in that this proportion depends only on x, and not on team quality or club type - the willingness to pay distributions are invariant in this sense[19].

Revenue can be written $xG(x)V_i(t_{ij})$ and the properties of G in (SC2) ensure that there is a unique maximum for $xG(x)$ at some $x^*\in(0,1)$ with $G(x^*)\equiv G^*\in(0,1)$. Thus the monopoly pricing rule is $p_i(t_{ij})=x^*v_i(t_{ij})$, and the monopoly price is always the same fraction of the maximum willingness to pay; also ticket demand at the monopoly price (=attendance) is $G^*n_i(t_{ij})$ and is always the same fraction of the number of fans. The reduced form revenue functions are $r_i(t_{ij})=x^*G^*V_i(t_{ij})$. These new assumptions are always sufficient to ensure all the previous demand assumptions (see appendix for proof):
Lemma 1 (SC1-2) imply (DA1-6).

Thus if (SC1-2) and (EA1-2) are satisfied, Proposition 2 holds and there is a unique ULE with $t_1^*>\frac{1}{2}>t_2^*$. For the welfare result note that the earlier, monopoly pricing fan surplus formula may now be re-written;

$$FS_i=\int_{p_i(t_i)}^{v_i(t_i)}D_i(p,t_i)dp=V_i(t_i)\int_{x^*}^{1}G(x)dx=\frac{\int_{x^*}^{1}G(x)dx}{x^*G^*}r_i(t_i)$$

Thus, $FS_i/(OS_i+PS_i)$ is invariant to team quality and club type, and t_1^* also globally maximizes FS_1+FS_2, and hence $S_M(t_1)$[20]:
Proposition 6 Assume (SC1-2) and (EA1-2). Then the level of aggregate surplus in ULE is S^{SB}, and $t_1^*=t_1^{SB}$.

In general in the ULE, talent is unequally allocated towards the big clubs, and competitive balance is less than maximal. In the special case, however, the allocation of talent and the aggregate surplus in ULE are at their second-best values. Thus there can be no welfare gain from policies that fail to preclude monopoly price behaviour, like revenue sharing and salary caps, but unlike ticket price caps in Section 8 later.

There is a sense in which the special case demand assumptions characterise the circumstances that lead to second-best optimality of ULE, namely that certain small

---

19 In the familiar uniform distribution case where $G(x)=1-x$, $x^*=G^*=\frac{1}{2}$. The idea for the demand specification of this section came from the micro-foundation in Falconieri et al. (2004), who use the uniform distribution. Another uniform distribution specification is used by Madden (2010a).

20 Moreover the first-best fan surplus formula is $r_i(t_i)\int_0^1 G(x)dx$, and similarly t_1^* maximizes $S(t_1)$ also. Thus $t_1^*=t_1^{FB}$ in the special case, but the ULE surplus is below S^{FB} because of the monopoly pricing.

perturbations of demand destroy the optimality, as follows. Start from a ULE with (SC1-2) satisfied and where $xG(x)$ is as indicated in Figure 5. Suppose that type 2 markets remain the same but perturb type 1 markets either "up" to $xG_U(x)$ or "down" to $xG_D(x)$. Details are easily provided to support the statements that small such perturbations are possible with the consequences that: (i) (DA1-6) continue to be satisfied, (ii) x^* (with the same G^*) continues to maximize $xG_U(x)$ and $xG_D(x)$, so (iii) the ULE does not change, and (iv) all surpluses are as they were before perturbation except FS_1 which increases after "up" and decreases after "down". Hence after up (resp., down) aggregate surplus will be increased compared to the ULE level by allocating marginally more (resp., less) talent to the big clubs, or by decreasing (resp., increasing CB).

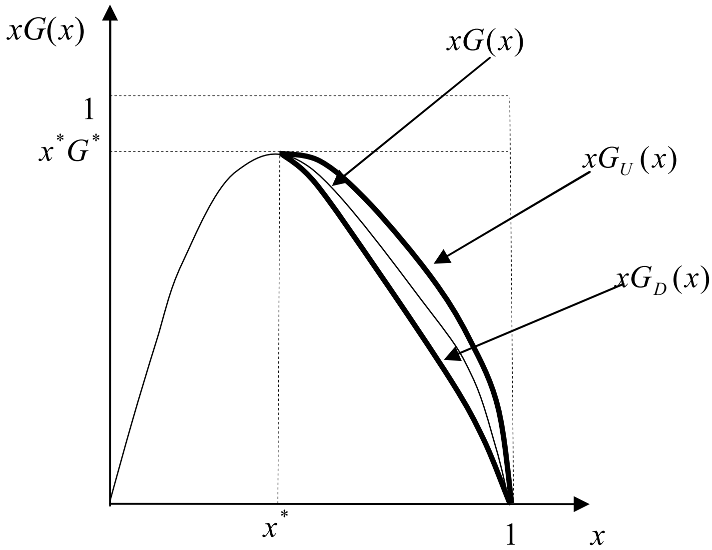


Figure 5; Perturbing the special case demand assumptions

Behind this argument is the observation that ULE is essentially determined by the marginal spectator (where $G(x^*) + x^*G'(x^*) = 0$), in the sense that changes in the willingness to pay distribution only for infra-marginal spectators will not change the equilibrium. However such a infra-marginal upward shift for type 1 clubs (say) increases their fans' surplus and the social desirability of increasing the talent allocated to the big clubs, and vice versa for a downward shift. Thus whether one should be aiming to increase or decrease the ULE competitive balance depends on the fine detail of the infra-marginal fan willingness to pay distributions. A point to note is that market data on ULE cannot reveal this fine detail. The problem is essentially the same as that faced by the planner attempting to regulate monopoly provision of quality in Spence (1976), and that faced in order to optimise provision of excludable public goods and typically addressed by collection of survey data on consumers' willingness to pay, also suggested by Spence (1976, p.425).

6.3 Competitive balance and aggregate surplus

If increasing competitive balance is to be a policy objective with a welfare justification, a positive answer to at least the following question is needed: starting from ULE, and given monopoly pricing, would a small increase in competitive balance increase aggregate surplus? It is clear from Section 6.2 that a universal positive response is not available – under the special case demand assumptions increases in competitive balance from the ULE value in fact reduce aggregate surplus. Whether there are any other useful restrictions on demand that lead to a positive answer is also pretty clear. Since ULE maximizes the sub-aggregate of player plus owner surpluses, increased competitive balance will increase aggregate surplus if and only if it increases the sum of fan surpluses from its ULE level. But, similar to Section 6.2, the demand circumstances where this is true depend on the fine detail of the infra-marginal fan willingness to pay distributions, which is not observable from market data. In the absence of such information, it will not be clear from ULE observation whether increasing competitive balance is a good or bad thing. A precise formula and statement is as follows.

Reverting to the general demand specification, the derivative of aggregate surplus at the ULE, using the fact that ULE equalises marginal revenues (or maximizes the owner plus player sub-aggregate surplus) is just the following difference between the derivatives of fan surpluses;

$$S_M'(t_1^*) = \int_{p_1(t_1^*)}^{v_1(t_1^*)} D_{1t}(p,t_1^*)dp - D_1[p_1(t_1^*),t_1^*]p_1'(t_1^*) - \{\int_{p_2(1-t_1^*)}^{v_2(1-t_1^*)} D_{2t}(p,1-t_1^*)dp - D_2[p_2(1-t_1^*),1-t_1^*]p_2'(1-t_1^*)\}$$

Hence, since increasing competitive balance requires a reduction in t_1 from t_1^*;

<u>Proposition 7</u> Assume (DA1-6) and (EA1-2). Then increasing competitive balance from its ULE level locally increases aggregate surplus if and only if;

$$\int_{p_1(t_1^*)}^{v_1(t_1^*)} D_{1t}(p,t_1^*)dp - D_1[p_1(t_1^*),t_1^*]p_1'(t_1^*) < \int_{p_2(1-t_1^*)}^{v_2(1-t_1^*)} D_{2t}(p,1-t_1^*)dp - D_2[p_2(1-t_1^*),1-t_1^*]p_2'(1-t_1^*)$$

The condition in this proposition requires that, as t_1 falls from t_1^* and monopoly prices adjust accordingly, the change in the usual fan surplus area under the demand curve is larger for the smaller clubs. Once again, this depends (via the integral terms) on the nature of the willingness to pay distributions for infra-marginal fans and how they change with team quality, information that will not be revealed merely by ULE observations but could in principle emerge from surveys on willingness to pay for tickets[21].

<u>Remark</u> As noted in footnote 18, Fort and Quirk (2009) set out (in their abstract and introduction) to investigate the welfare consequences of changes in competitive balance using the sum of only fan and owner surpluses. However they revert (on page 5) to the same fan, owner plus player surpluses as here[22]. Their Proposition 2 (page 6)

[21] Surveys of willingness to pay for new stadia have been conducted in various regions of the US – see Owen (2006), for example. But they do not seem to convey the information needed here. I am grateful to Pamela Winker for bringing this literature to my attention.

[22] The reason given for the reversion is: "We use revenues for team surpluses since …decentralized profit maximization by owners…leads to maximization of the league's total revenue anyway" (Fort and Quirk (2010, p.5)). Here the reason for it is that, with perfectly inelastic talent supply, players will also

should then contain an equivalent inequality condition to that in Proposition 7 above, but instead has only the integral terms – the difference stems from their equation 8 on page 5, where a partial derivative of demand is taken with respect to win percentage where a total derivative is needed, thus losing the terms which would create a completely analogous condition to that in Proposition 7 above[23]. Either way we differ here also from their apparent conclusion (pages 7 and 8) that market data can decide the inequality[24].

6.4 Revenue sharing, salary caps and aggregate surplus

Although the answers are more or less immediate corollaries of earlier results, for completeness pose the question: does the introduction of revenue sharing or salary caps increase aggregate surplus? For revenue sharing the invariance principle reveals that its introduction has no affect on aggregate surplus at all, merely leading to some redistribution from players to owners. For salary caps we know that if the special case demand assumptions are satisfied, the increase in competitive balance will actually reduce aggregate surplus. In general salary caps can be welfare improving only if the condition in Proposition 7 is satisfied, information that cannot be revealed by market data on ULE.

In the sustainable league case there is therefore nothing from the welfare viewpoint to recommend revenue sharing and little that is easily implementable for salary caps.

7. UNSUSTAINABLE LEAGUES

An Unsustainable League is one for which no ULE exists.

In an unsustainable league, attempts by the market adjustment of wages to find an equilibrium alternative to the status quo initial allocation fail – wage tatonnement processes will never converge to a steady-state, for instance. In such a case the most plausible assumption is that the outcome is the status quo – the league remains inactive, with no talent hired, no games played, and no surplus earned by any party. Thus autarky is assumed to be the unsustainable league outcome.

Leagues may be unsustainable because of the potential discontinuity of talent demand. To explore this, continue to assume (DA1-6) and (EA1) but replace (EA2) with:
Equilibrium Assumption 3 (EA3) $s_1, s_2 < \frac{1}{2}$, $s_1 > 1 - m_2$, $s_2 > 1 - m_1$

---

earn surpluses from the league, and player surplus should be added to fan plus owner surpluses to give the appropriate welfare measure from the outset.

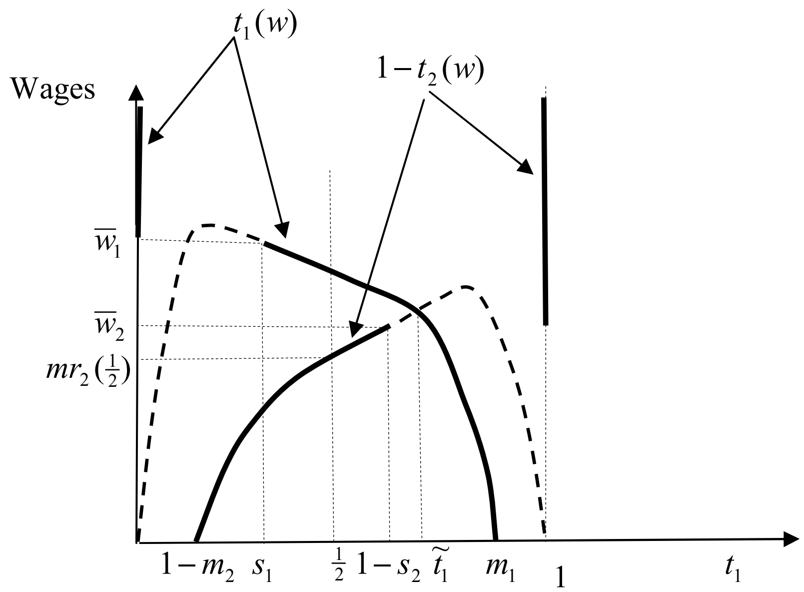
[23] As indicated earlier what is needed for an increase in competitive balance (via an increase in the small club talent allocations) to be welfare improving is that the change in the usual fan surplus area under the demand curve is larger for the smaller clubs. The inequality of Fort and Quirk (2010) overlooks the fact that that the changes in talent allocations will affect price, and hence the fan surplus.

[24] "Marginal impacts of winning percent can be obtained from estimating attendance demand" and "Careful estimates of impacts of talent choice on attendance demand for all teams are required in order to choose intervention mechanisms that effectively hit the optimal level of competitive balance" (Fort and Quirk (2010, p. 7,8)) suggest a belief that market data can provide the answers. There is no mention of infra-marginal fans or the earlier related literature.

Thus neither type of club needs more than half the talent to be sustainable, but they each need more than the upper bounds assumed in (EA2). The horizontal axis in Figure 6 shows the new rankings of (EA3). Suppose the marginal revenue and talent demand curves of type 2 clubs are also as shown in Figure 6. It is clear that the league is unsustainable if and only if the type 1 talent demand is such that $t_1(\overline{w}_2) > 1 - s_2$, as shown. Hence:

<u>Proposition 8</u> Assume (DA1-6), (EA1) and (EA3). Then the league is unsustainable if and only if $t_1(\overline{w}_2) > 1 - s_2$.

What causes the unsustainability is, in a sense, the "size" differential between clubs. Type 1 clubs are bigger in that talent demand is higher for them than for type 2 ((DA5)), and $\Delta = t_1(\overline{w}_2) - s_2$ is a measure of the size differential at the type 2 choke wage. Proposition 6 then says that the league is unsustainable if and only if the size differential is sufficiently large, $\Delta > 1 - 2s_2$. Thus leagues with large asymmetries in fan markets are susceptible to unsustainability[25].



<u>Figure 6; an unsustainable league</u>

The interesting question is then; can revenue sharing or salary capping make sustainable an otherwise unsustainable league? Perhaps surprisingly the answers are quite different, a definite no for revenue sharing, and a typical yes for salary capping, as follows.

From earlier discussions the effects of revenue sharing (with home team share $\alpha \in [0,1]$) are reductions in choke wages, downward movements of marginal revenue and talent demand curves maintaining m_1 and $1 - m_2$ at wage 0, and invariance of the talent level ($\tilde{t}_1$ in Figure 6) where marginal revenues are equal. And we also know that revenue sharing has no effect on the minimum sustainable team qualities, so s_1

[25] Similar non-existence of equilibrium could emerge alternatively if avoidable fixed costs were added to a model without the reduced form revenue function non-concavity here. The unsustainability would then depend on the distribution of the fixed costs relative to the fan market asymmetries.

and $1-s_2$ are also unchanged. It follows that type 1 talent demand at the new $\overline{w}_2$ continues to exceed $1-s_2$, and the non-existence remains.

Proposition 9 Assume (DA1-6), (EA1), (EA3) and $t_1(\overline{w}_2) > 1-s_2$. Then the unsustainable league will remain unsustainable with revenue sharing where home team share is $\alpha \in [0,1]$, for any $\alpha \in [0,1]$.

Consider now a salary cap in Figure 6 with $S = \overline{w}_2(1-s_2)$.With this constraint type 1 clubs have a constrained talent demand of $(1-s_2)$ at $\overline{w}_2$, and type 2 clubs are not constrained ($S = \overline{w}_2(1-s_2) > \overline{w}_2 s_2$ since $s_2 > \frac{1}{2}$) and demand s_2; LESC exists. Moreover as S falls from $\overline{w}_2(1-s_2)$ to $\frac{1}{2}mr_2(\frac{1}{2})$ the LESC exist again with a wage and talent allocation that moves left in Figure 6 along the type 2 demand curve, analogous to Figure 3. Thereafter, exactly as in Figure 3, further reductions in S leave the talent allocation unchanged, with wages falling to 0 as $S \to 0$.

Proposition 10 Assume (DA1-5), (EA1), (EA3) and $t_1(\overline{w}_2) > 1-s_2$. Then the introduction of a salary cap $S \in (0, \overline{w}_2(1-s_2)]$ makes sustainable the otherwise unsustainable league.

Thus salary caps can save leagues from unsustainability, unlike revenue sharing.

8. TICKET PRICE CAPS

The final policy considered is a ticket price cap whereby prices cannot exceed some upper bound, P say. Assume first the general assumptions (DA1-6) plus (EA1-2), so Proposition 2 holds again and the league is sustainable.

Under a price cap, the full profit maximization problem faced by a club is to choose ticket prices and talent demand to solve; $\max \Pi_i(p_{ij}, t_{ij})$ subject to $p_{ij} \le P$. Given (DA3) the optimal pricing rule is;

$$p_{ij} = p_i^{PC}(t_{ij}, P) \equiv \begin{cases} p_i(t_{ij}), & p_i(t_{ij}) \le P \\ P, & p_i(t_{ij}) > P \end{cases}$$

Thus clubs choose the monopoly price rule unless the price cap precludes it.
The reduced form profit problem becomes;

$$\max \pi_i^{PC}(t_{ij}) = \begin{cases} \pi_i(t_{ij}), & p_i(t_{ij}) \le P \\ \Pi_i(P, t_{ij}), & p_i(t_{ij}) > P \end{cases} \tag{8.1}$$

Denoting talent demand by $t_i^{PC}(w, P)$, equilibrium is:

League Equilibrium with a Price Cap (LEPC) is a strictly positive 5-tuple $w^{PC}, t_1^{PC}, t_2^{PC}, p_1^{PC}, p_2^{PC}$ such that $t_1^{PC} + t_2^{PC} = 1$, and for $i = 1,2$, $t_i^{PC} = t_i^{PC}(w, P)$ and $p_i^{PC} = p_i^{PC}(t_i^{SC}, P)$.

(DA1-6) and (EA1-2) produce ULE with $t_1^* > \frac{1}{2} > t_2^*$ and $p_1^* > p_2^*$, shown as usual in Figure 7. Clearly a price cap with $P \geq p_1^*$ will have no effect at all on the ULE, so consider P just below p_1^*, as shown. This will not affect type 2 clubs, whose talent demand curve is unchanged. For $t_1 \leq \bar{t}_1$ shown the price cap does not constrain type 1 clubs either, and their talent demand curve is unchanged for $w \geq mr_1(\bar{t}_1)$. For $t_1 > \bar{t}_1$, type 1 clubs will be price constrained to charge the cap *P*. At such price and talent combinations, $\frac{\partial R_i}{\partial p_{ij}} > 0$ and from the concavity of (DA6)(a), the optimal talent demand is where $mr_1^{PC}(t_1, P) \equiv PD_{1t}^{'}(P, t_1) = w$. For $w < mr_1(\bar{t}_1)$ the new type 1 talent demand curve lies to the left of the original (since $\frac{\partial}{\partial P}[PD_{1t}^{'}(P, t_1)] > 0$ from (DA6)(b)), and is downward sloping (from (DA6)(a) again), as shown in Figure 7.

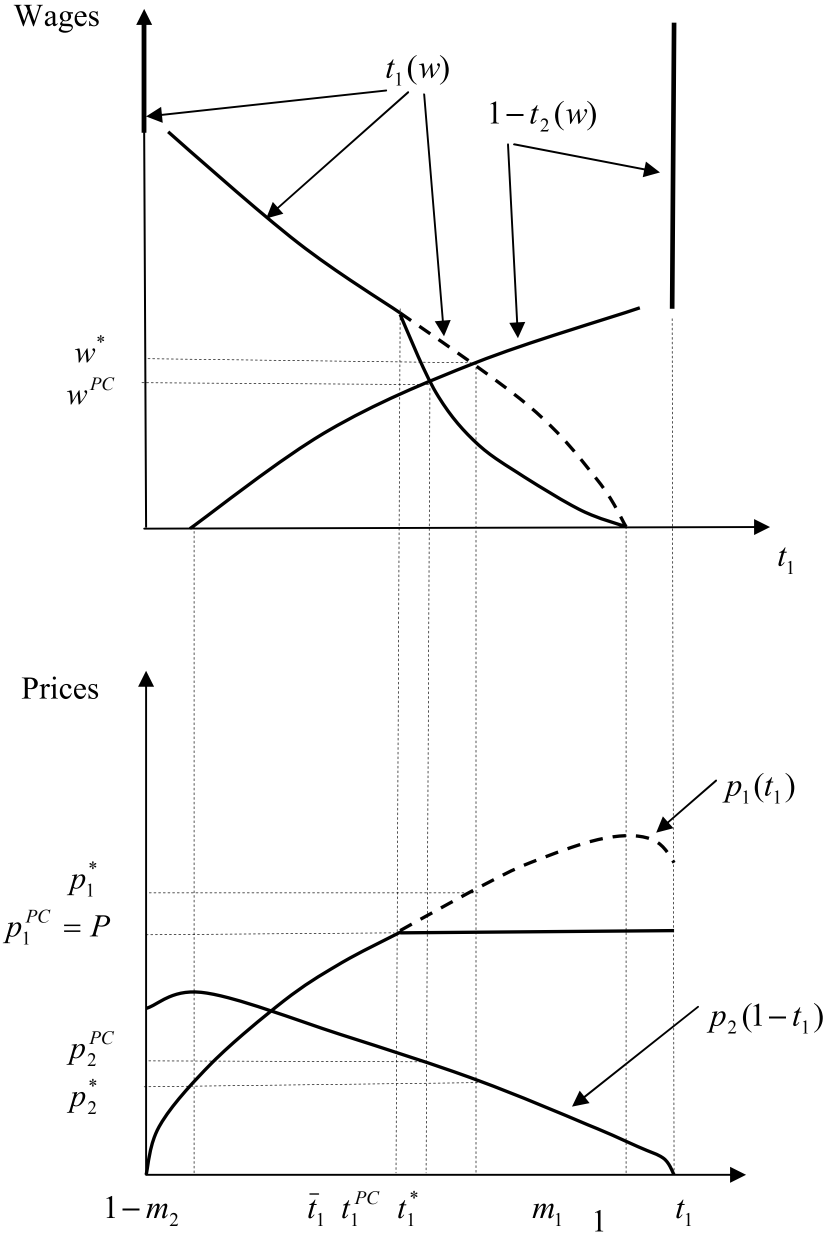


Figure 7; LEPC wages, talent allocations (top) and ticket prices (bottom)

Thus there is LEPC in Figure 7 at t_1^{PC}, with the indicated wages and prices. This kind of LEPC where P is binding only on big clubs is defined by $mr_2(1-t_1) = PD'_{1t}(P,t_1) = w$, and it is easy to check that under the current assumptions, t_1^{PC} falls as P falls, i.e. $dt_1^{PC}/dP > 0$. As P falls in Figure 7 t_1^{PC} falls until (P,t_1^{PC}) reaches $p_2(1-t_1)$, at $\widetilde{P}$ say. A definite conclusion from this is;

Proposition 11 Assume (DA1-7) and (EA1-2). Then there exists a price cap value $\widetilde{P} < p_1^*$ such that as P falls from p_1^* to $\widetilde{P}$, t_1^{PC} falls, CB^{PC} increases and w^{PC} falls.

Since both price caps and salary caps (Figure 3) have the initial effect (as they become binding) of moving the equilibrium from ULE left along the type 2 demand schedule, the following statement is true: there exists a value $\hat{S} < w^* t_1^*$ such that if t_1 is the type 1 talent allocation in LESC with $S \in (\hat{S}, w^* t_1^*)$, then there exists $P < p_1^*$ such that t_1 is also the type 1 talent allocation in LEPC with price cap *P*. Since t_1 is the same in both equilibria, the price-demand curve facing both types of clubs is the same in both equilibria. Since the small type 2 clubs monopoly price in both equilibria, $p_2^{SC} = p_2^{PC}$, and FS_2, OS_2 and PS_2 are also the same in both equilibria – see the right hand diagram in Figure 9. But for the big type 1 clubs $p_1^{PC} = P < p_1^{SC}$ and their attendances will be higher in the LEPC than in the LESC, increasing $FS_1 + OS_1 + PS_1$, as shown in the left hand diagram in Figure 9.

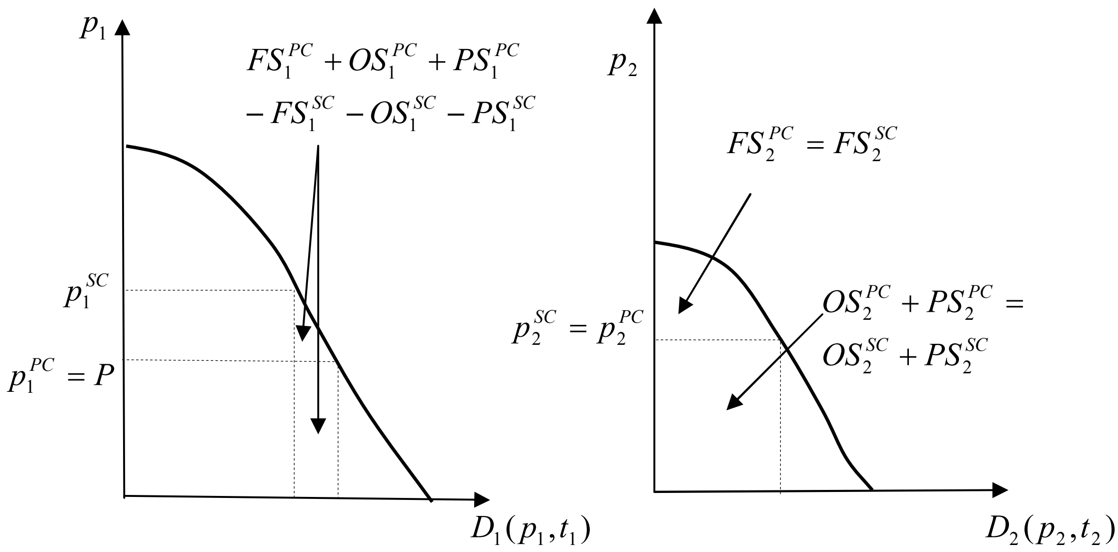


Figure 8; LEPC gain in surplus over LESC

Proposition 12 Assume (DA1-7) and (EA1-2). Then there exists $\hat{S} < w^* t_1^*$ such that, for any $S \in (\hat{S}, w^* t_1^*)$ and its LESC with $t_1 = t_1^{SC}$, there exists $P < p_1^*$ such that $t_1^{PC} = t_1^{SC}$ in the LEPC, and aggregate surplus is higher in the LEPC than in the LESC.

Proposition 12 is strongly negative towards the use of salary caps as a regulatory device in sustainable leagues. In terms of aggregate surplus, there is a clear dominance of price capping over salary capping, at least locally in the neighbourhood of the ULE – anything a salary cap can do a price cap can do socially better.

Even though the price cap does better in terms of aggregate surplus than the salary cap, this does not necessarily mean that the price cap increases aggregate surplus. The next result provides a positive recommendation for price capping, again local, and reverting to the invariant distribution special case (see appendix for a proof).

Proposition 13 Assume (SC1-2) and (EA1-2). Then there exists $\hat{P} < p_1^*$ such that the LEPC for any $P \in (\hat{P}, p_1^*)$ produces greater aggregate surplus than the ULE.

Thus in a sustainable league which attains the second-best optimum without regulation, ticket price caps are socially beneficial (at least locally), unlike revenue sharing and salary caps.

Finally in the unsustainable league case we have immediately the local analogue of Proposition 10 for salary caps:

Proposition 14 Assume (DA1-5), (EA1), (EA3) and $t_1(\overline{w}_2) > 1 - s_2$. Then there exists $\varepsilon > 0$ and $\hat{P} < p_1^*$ such that the introduction of a price cap $P \in (\hat{P}, p_1^*)$ makes sustainable the otherwise unsustainable league if $t_1(\overline{w}_2) - (1 - s_2) < \varepsilon$.

At least locally, anything that salary caps can do to rescue an unsustainable league can also be done by ticket price caps.

9. CONCLUSIONS

The paper has augmented a textbook sports league model in three directions, providing an integrated account of game ticket pricing by clubs, a central focus on aggregate surplus and the welfare evaluation of policy, and the addition of ticket price caps to the list (revenue sharing and salary caps) of regulatory policies studied in previous literature. The model has made explicit the assumption of a league with a large number of clubs to legitimise the parametric treatment of wages by clubs when formulating demands for talent, as is assumed in most of the literature.

The upshot is a number of novel insights and conclusions compared to the conventional wisdom. The ticket pricing analysis uncovered a fundamental talent demand discontinuity at high wages which made problematic the existence of unregulated league equilibrium (ULE) when the size differential between club fan markets is large. This led to the distinction between sustainable leagues, where the size differential is small and equilibrium exists, and unsustainable leagues where a large size differential causes the unregulated league to collapse into autarky. A new role for policy is that of saving from collapse an otherwise unsustainable league. Overall conclusions regarding the efficacy of the regulatory policies are that revenue sharing seems to have nothing to recommend it in the context. In a sustainable league, the invariance principle continues to imply that revenue sharing has no impact on competitive balance or aggregate surplus. And in an unsustainable league, revenue sharing again has no effect, failing to save the collapse, a kind of "super-invariance" principle. Salary caps do increase competitive balance in a sustainable league, but in a special case with invariant willingness to pay distributions, the unregulated league in fact attains the (second-best) optimum, and a salary cap is a disimprovement. There is certainly no general welfare base for adopting increases in competitive balance as an objective in a sustainable league; in general whether increasing or decreasing competitive balance is a good thing depends on the infra-marginal fan willingness to pay distribution, which cannot be revealed by market data. Whilst there is therefore no general welfare recommendation for salary caps, it is the case that they do have the potential to rescue an unsustainable league from autarky, unlike revenue sharing. This recommendation for salary caps is tempered by comparisons with ticket price capping. Locally (near the ULE), salary and ticket price caps have similar effects on talent allocations, but because they act directly to lower the high monopoly prices, the ticket price caps generate an extra, positive and surplus enhancing attendance effect - anything a salary cap can do a ticket price cap can do socially better, at least locally.

In a nutshell, the main novel lessons that this paper suggests for regulatory policy are, first, that salary capping looks to be a good device for securing leagues where large market size differentials between clubs may lead to unsustainability without regulation, secondly the welfare value of increasing competitive balance depends on the fine detail of the infra-marginal fan willingness to pay distributions which cannot be provided by observations on ULE, and thirdly there is certainly prima facie evidence that ticket price capping is worthy of at least the attention currently given almost exclusively to revenue sharing and salary caps.

Generally it is hoped also that the paper will provide an integrated and tractable framework for the study of further issues relating to sports league competition and regulation. One such issue suggested by the findings here is why revenue sharing and salary caps are both used in varying ways in the major North American leagues; and the obvious follow-up question is why ticket price caps are not used currently. Answers might come from a study of a model which builds on the current base to incorporate bargaining over the regulatory mechanisms between club representatives and player unions, as seems to be the recent North American norm. Inter alia, this should be a topic for future research.

APPENDIX

Proof of Lemma 1

(SC1)(a) is (DA1); (DA2) follows from (SC1)(a),(b); (DA3) follows from (SC2)(d). For (DA4), $\lim D_{it}^{'}(p_i(t_{ij}),t_{ij})$ as $t_{ij}\to 0$ is given by the limit as $t_{ij}\to 0$ of $G(x^*)[n^{'}(t_{ij})+v^{'}(t_{ij})\frac{n(t_{ij})}{v(t_{ij})}]>0$, since $\frac{n(t_{ij})}{v(t_{ij})}\to\frac{n^{'}(0)}{v^{'}(0)}$ as $t_{ij}\to 0$, from l'Hopital's rule. Thus $mr_i(0)=0$. For $t_{ij}\in(0,m_i)$;

$$mr_i(t_{ij})=x^*G^*[n_i^{'}(t_{ij})v_i(t_{ij})+n_i(t_{ij})v_i^{'}(t_{ij})],$$

$$mr_i^{'}(t_{ij})=x^*G^*[n_i^{''}(t_{ij})v_i(t_{ij})+n_i(t_{ij})v_i^{''}(t_{ij})+2n_i^{'}(t_{ij})v_i^{'}(t_{ij})] \text{ and}$$

$$mr_i^{''}(t_{ij})=x^*G^*[n_i^{'''}(t_{ij})v_i(t_{ij})+n_i(t_{ij})v_i^{'''}(t_{ij})+3n_i^{''}(t_{ij})v_i^{'}(t_{ij})+3n_i^{'}(t_{ij})v_i^{''}(t_{ij})]$$

<0, from (SC1)(b).

Thus mr_i is strictly concave on $(0,m_i)$, and (DA4) follows.

(DA5) follows from (SC1)(c) and the above formula for $mr_i(t_{ij})$.

$$R_i(p_{ij},t_{ij})=p_{ij}n_i(t_{ij})G(\tfrac{p_{ij}}{v_i(t_{ij})}),\quad \frac{\partial R_i}{\partial p_{ij}}=\frac{n_i(t_{ij})}{v_i(t_{ij})}[G(\tfrac{p_{ij}}{v_i(t_{ij})})+\tfrac{p_{ij}}{v_i(t_{ij})}G^{'}(\tfrac{p_{ij}}{v_i(t_{ij})})] \text{ and}$$

$$\frac{\partial^2 R_i}{\partial p_{ij}\partial t_{ij}}=n_i^{'}(t_{ij})[G(\tfrac{p_{ij}}{v_i(t_{ij})})+\tfrac{p_{ij}}{v_i(t_{ij})}G^{'}(\tfrac{p_{ij}}{v_i(t_{ij})})]-\frac{p_{ij}n_i(t_{ij})v_i^{'}(t_{ij})}{v_i(t_{ij})^2}[2G^{'}(\tfrac{p_{ij}}{v_i(t_{ij})})+p_{ij}G^{''}(\tfrac{p_{ij}}{v_i(t_{ij})})],$$

which is positive when $\frac{\partial R_i}{\partial p_{ij}}\geq 0$ from (SC2)(d), ensuring (DA6)(b).

$$\frac{\partial R_i}{\partial t_{ij}}=p_{ij}n_i^{'}(t_{ij})G(\tfrac{p_{ij}}{v_i(t_{ij})})-\frac{p_{ij}^2 n_i(t_{ij})v_i^{'}(t_{ij})}{v_i(t_{ij})}G^{'}(\tfrac{p_{ij}}{v_i(t_{ij})})], \text{ and}$$

$$\frac{\partial^2 R_i}{\partial t_{ij}^2}=\frac{p_{ij}n_i(t_{ij})v_i^{'}(t_{ij})}{v_i(t_{ij})^2}[2G^{'}(\tfrac{p_{ij}}{v_i(t_{ij})})+p_{ij}G^{''}(\tfrac{p_{ij}}{v_i(t_{ij})})]$$

$$+p_{ij}n_i^{''}(t_{ij})G(\tfrac{p_{ij}}{v_i(t_{ij})})-\frac{p_{ij}^2}{v_i(t_{ij})^2}G^{'}(\tfrac{p_{ij}}{v_i(t_{ij})})[2n_i^{'}(t_{ij})v_i^{'}(t_{ij})+n_i(t_{ij})v_i^{''}(t_{ij})]$$

The first line is negative from (SC2)(d). Using $\frac{\partial R_i}{\partial p_{ij}}\geq 0$, the second line is less than $-\frac{p_{ij}^2}{v_i(t_{ij})^2}G^{'}(\frac{p_{ij}}{v_i(t_{ij})})V_i^{''}(t_{ij})$, which is also negative since $V_i^{''}(t_{ij})<0$ for $t_{ij}\in(s_i,m_i)$; (DA6)(a) follows.

Since the pricing rule is $p_i(t_{ij})=x^*v_i(t_{ij})$, (DA7) follows from (SC1)(c).

Proof of Proposition 11

LEPC in which *P* binds only on big clubs (with talent allocation t_1) is characterised by the conditions $mr_1^{PC}(t_1)\equiv PD_{1t}^{'}(P,t_1)=mr_2(1-t_1)$ which with (SC1-2) is:

$$n_1^{'}(t_1)v_1(t_1)\tfrac{P}{v_1(t_1)}G(\tfrac{P}{v_1(t_1)})-n_1(t_1)v_1^{'}(t_1)(\tfrac{P}{v_1(t_1)})^2G^{'}(\tfrac{P}{v_1(t_1)})=x^*G^*V_2^{'}(1-t_1)$$

Differentiating with respect to *P*, treating t_1 as a function of *P*, and writing $T=(v_1(t_1)-Pv_1^{'}(t_1)\frac{dt_1}{dP})/v_1(t_1)^2$ gives:

$$Tn_1^{'}(t_1)v_1(t_1)[G(\tfrac{P}{v_1(t_1)})+\tfrac{P}{v_1(t_1)}G^{'}(\tfrac{P}{v_1(t_1)})]+\tfrac{dt_1}{dP}\tfrac{P}{v_1(t_1)}G(\tfrac{p}{v_1(t_1)})[n_1^{''}(t_1)v_1(t_1)+n_1^{'}(t_1)v_1^{'}(t_1)]$$

$$-T\tfrac{P}{v_1(t_1)}n_1(t_1)v_1^{'}(t_1)[2G^{'}(\tfrac{P}{v_1(t_1)})+(\tfrac{P}{v_1(t_1)})G^{''}(\tfrac{P}{v_1(t_1)})]$$

$$-\tfrac{dt_1}{dP}(\tfrac{P}{v_1(t_1)})^2G^{'}(\tfrac{P}{v_1(t_1)})[n_1^{'}(t_1)v_1^{'}(t_1)+n_1(t_1)v_1^{''}(t_1)]=-\tfrac{dt_1}{dP}x^*G^*V_2^{''}(1-t_1)$$

Evaluating at the ULE where $t_1=t_1^*, \frac{P}{v_1(t_1)}=x^*, G(x^*)=G^*$ and $G^*+x^*G^{'}(x^*)=0$, substituting for *T* and rearranging gives:

$$\tfrac{dt_1}{dP}[G^*(V_1''(t_1^*)+V_2''(1-t_1^*))+\tfrac{Pn\ (t_1^*)v_1'(t_1^*)^2}{v_1(t_1^*)^2}(2G'(x^*)+G''(x^*))] \\ =\tfrac{n_1(t_1^*)v_1'(t_1^*)}{v_1(t_1^*)}[2G'(x^*)+G''(x^*)] \tag{A.1}$$

The formula confirms that, under the assumptions made, $\frac{dt_1}{dP}>0$; we return to (A.1) later.

In LEPC where the price cap P binds only on big clubs, t_1 is the talent allocation and (SC1-2) hold, aggregate surplus is:

$$S(P,t_1)=n_1(t_1)\int_P^{v_1(t_1)}G(\tfrac{p_1}{v_1(t_1)})dp_1+n_2(1-t_1)\int_{x^*v_2(1-t_1)}^{v_2(1-t_1)}G(\tfrac{p_2}{v_2(1-t_1)})dp_2 \\ +Pn_1(t_1)G(\tfrac{P}{v_1(t_1)})+x^*G^*V_2(1-t_1)$$

With $x_i = p_i / v_i(t_i)$ this becomes:

$$S(P,t_1)=n_1(t_1)v_1(t_1)\int_{P/v_1(t_1)}^1 G(x_1)dx_1+n_2(1-t_1)v_2(1-t_1)\int_{x^*}^1 G(x_2)dx_2 \\ +Pn_1(t_1)G(\tfrac{P}{v_1(t_1)})+x^*G^*V_2(1-t_1)$$

Using $V_i(t_i)=n_i(t_i)v_i(t_i)$ gives:

$$S(P,t_1)=V_1(t_1)[\int_{P/v_1(t_1)}^1 G(x_1)dx_1+\tfrac{P}{v_1(t_1)}G(\tfrac{P}{v_1(t_1)})] \\ +V_2(1-t_1)[\int_{x^*}^1 G(x_2)dx_2+x^*G^*]$$

Differentiating S with respect to P, treating t_1 as a function of P and using the same T abbreviation as above produces:

$$\tfrac{dS}{dP}=\tfrac{dt_1}{dP}V_1'(t_1)[\int_{\frac{P}{v_1(t_1)}}^1 G(x_1)dx_1+\tfrac{P}{v_1(t_1)}G(\tfrac{P}{v_1(t_1)})]-\tfrac{dt_1}{dP}V_2'(1-t_1)[\int_{x^*}^1 G(x_2)dx_2+x^*G^*] \\ +TV_1(t_1)\tfrac{P}{v_1(t_1)}G'(\tfrac{P}{v_1(t_1)})$$

Evaluating at the ULE, the first two terms cancel and so $\frac{dS}{dP}<0$ if and only if $\frac{dt_1}{dP}<\frac{v_1(t_1^*)}{p_1^*v_1'(t_1^*)}$ which, using (A.1) is:

$$\frac{v_1(t_1^*)}{p_1^*v_1'(t_1^*)}>\frac{n_1(t_1^*)v_1'(t_1^*)[2G'(x^*)+G''(x^*)]}{G^*[V_1''(t_1^*)+V_2''(1-t_1^*)]+\frac{p_1^*n_1(t_1^*)v_1'(t_1^*)^2}{v_1(t_1^*)^2}[2G'(x^*)+G''(x^*)]}$$

Cross-multiplying and re-arranging this is $v_1(t_1^*)^2G^*[V_1''(t_1^*)+V_2''(1-t_1^*)]<0$, which is always true under the assumptions made. The result follows.

REFERENCES

Andreff, W. and Szymanski, S., (2006), Handbook on the Economics of Professional Sport: Edward Elgar.

Brito, D.L. and Oakland, W.H., (1981), "On the monopolistic provision of excludable public goods", *American Economic Review*, vol. 70, p. 691-704.

El-Hodiri, M. and Quirk, J., (1971), "An economic model of a professional sports league", *Journal of Political Economy*, vol. 79, p. 1302-1319.

Falconieri, S., Palomino, F. and Sakovics, J., (2004), "Collective versus individual sale of television rights in league sports", *Journal of the European Economic Association*, vol. 2(5), p. 833-862.

Fort, R., (2006), Sports Economics: Pearson Prentice Hall (2nd ed.).

Fort, R. and Quirk, J., (1995), "Cross-subsidisation, incentives, and outcomes in professional sports leagues", *Journal of Economic Literature,* vol. 33, p. 1265-1299.

Fort, R. and Quirk, J., (2009), "Optimal competitive balance in a season ticket league", available at http://www.rodneyfort.com/Academic/Academic.html, (forthcoming in *Economic Inquiry*).

Fraser, C.D., (1996), "On the provision of excludable public goods", *Journal of Public Economics,* vol. 60, p. 111-130.

Fraser, C.D., (2000), "When is efficiency separable from distribution in the provision of club goods?", *Journal of Economic Theory,* vol. 90, p. 204-221.

Kesenne, S., (2007), The Economic Theory of Professional Team Sports; an Analytical Treatment: Edward Elgar.

Kesenne, S. and Pauwels, W., (2006), "Club objectives and ticket pricing in professional team sports", *Eastern Economic Journal*, Vol. 32(3), p. 549-560.

Madden, P., (2010a), "Fan welfare maximization as a club objective in a professional sports league", University of Manchester Economics Discussion Paper EDP-1002.

Madden, P., (2010b), "Game-theoretic models of basic sports leagues", University of Manchester Economics Discussion Paper EDP-1006.

Neale, W., (1964), "The peculiar economics of professional sports", *Quarterly Journal of Economics*, Vol. 78(1), p. 1-14.

Owen, J.G., (2006), "The intangible benefits of sports teams", *Public Finance and Management*, vol. 6(3), p. 321-345.

Quirk, J. and Fort, R., (1992), Pay Dirt; the Business of Professional Team Sports: Princeton University Press.

Rottenberg, S., (1956), "The baseball players' labour market", *Journal of Politial Economy*, vol. 64(3), p. 242-258.

Sandy, R., Sloane, P. and Rosentraub, M., (2004), The Economics of Sport: Palgrave MacMillan.

Sheshinski, E., (1976) Price, quality and quantity regulation in monopoly situations", *Economica*, vol. 43' p.127-137.

Sloane, P., (1971), "The economics of professional football: the football club as a utility maximiser", *Scottish Journal of Political Economy*, vol. 17(2), p. 121-146.

Spence, A.M. (1975), Monopoly, quality and regulation", *Bell Journal of Economics*, vol. 6, p. 417-429.

Szymanski, S., (2003), "The economic design of sporting contests", *Journal of Economic Literature*, vol. 41, p. 1137-1187.

Szymanski, S., (2004), "Professional team sports are only a game; the Walrasian fixed-supply conjecture model, contest-Nash equilibrium, and the invariance principle", *Journal of Sports Economics*, vol. 5(2). p.111-126.

Szymanski, S. and Kesenne, S., (2004), "Competitive balance and gate revenue sharing in team sports", *Journal of Industrial Economics,* vol. 52(1), p.165-177.