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GAME-THEORETIC ANALYSIS OF BASIC TEAM SPORTS LEAGUES

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Abstract The paper proposes a new "strategic market game" approach to modelling strategic interactions between clubs in professional team sports leagues, and generalises a basic framework used in previous literature (two clubs, fixed talent supply and club revenues that depend only on relative team qualities), to allow variable talent supply and club revenues that depend on absolute (and relative) team qualities. The new approach incorporates club talent market power (duopsony), overlooked by existing approaches; in both the basic and the more general framework Nash equilibrium competitive balance is analysed, with and without revenue sharing, and with comparisons to existing analyses.

JEL classification numbers; L10, L83

Keywords; sports leagues, duopsony, revenue sharing.

Acknowledgement; This paper extends selectively Madden (2009b), developing the suggested alternative approach to game-theoretic analysis of basic sports leagues to deal with variable talent supply and revenues that depend on absolute as well as relative team talent levels, and clarifying the relationship to existing approaches. Earlier versions were presented at the 1st European Conference on Sports Economics, Paris (September 2009), at the University of Manchester (October 2009) and at the University of Zurich (December 2009). I am grateful for comments received at the presentations, and to Richard Cornes, Roger Hartley, Leo Kaas, Leonidas Koutsougeras and Vic Tremblay for further helpful remarks and suggestions, not all of which have yet been followed - errors and shortcomings are certainly the author's responsibility.

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1. INTRODUCTION

At the heart of many papers and books on the economics of professional team sports leagues[1] lie two different Game–theoretic[2] approaches to the analysis of a very simple and basic league. The league framework for the analyses consists of the following.

(I) Two profit-maximizing clubs, $i = 1,2$. The clubs play each other twice over the season, once at home and once away, in stadiums of given, large capacities.
(II) A supply of talent function to the league. A large number of players offer their varying talent levels to the talent market, and the supply function relates the aggregate talent supply to the wage per unit of talent (w), as in standard efficiency labour models in labour economics. To reflect the fact that the major North American sports leagues face little competition for their specialised playing talent from other leagues, the talent supply is assumed perfectly inelastic in a quantity normalised to unity[3]. The allocation of talent to club i (=its team quality) is denoted t_i , and the expenditure on playing talent $(e_i = wt_i)$ is the only club cost.
(III) Club revenue functions. A club's home game is attended by its own fans (consumers) who are, to some extent, partisan towards the home team. The gate revenue generated depends in some way on both the home and away team qualities. Letting $W_i = t_i / T$ denote the "win percentage" for team *i* where $T = t_i + t_j$, revenue is usually assumed to be a function of W_i only, $r_i(W_i)$ say, which attains a maximum at some $W_i \in (\frac{1}{2},1)$ reflecting the partisan fan preferences. Home gate revenue is the only club revenue.

The primary focus of the analyses is the concern that "big" clubs (with large fan markets) may produce teams that are so good as to lead to one-sided games and an economically unsuccessful league. In the jargon, the concern has been the potential lack of competitive balance in the league (degree of equality in team qualities), and whether a regulatory policy of (in particular) revenue sharing (where away teams receive a fraction of home gate revenue) may increase this balance.

Kesenne (2007) refers to the two analytical approaches as the "Walras" approach, associated with Fort, Quirk and others (see e.g. Fort and Quirk (1995)), and the "Nash" approach, as introduced by Szymanski (2004) in his critique on Game-theoretic grounds of the earlier "Walras" approach, and as developed further in Szymanski and Kesenne (2004)[4]. The approaches provide different solutions for the allocation of talent to teams (and hence competitive balance) and the wage for talent. In an unregulated league, the "Walras" approach predicts a lower competitive balance

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[1] See for instance the books by Fort (2006), Kesenne (2007), Sandy et al. (2004) and the surveys by Fort and Quirk (1995), Szymanski (2003).
[2] Certain terms (e.g. game, player) have meanings in the sporting context which differ from those in economic modelling. Capitals are thus used to distinguish the economic modelling meaning. Also, throughout, Game theory refers to non-cooperative Game theory.
[3] The major leagues are the National Football League (NFL), Major League Baseball (MLB), National Basketball Association (NBA) and National Hockey League (NHL). In each sport there are no comparable other leagues to provide competition for the specialised playing talent, unlike the case in European soccer.
[4] Alternative terms are the Walrasian Fixed-Supply Conjecture approach and the Contest-Nash approach.

and higher wage than "Nash". Also the consequences of revenue sharing for competitive balance are quite different, with "Nash" predicting a decrease and "Walras" no change (the "invariance principle"). The two approaches continue to co-exist, sometimes uneasily, with many contributions reporting developments using one or other or sometimes both. It will be argued here that both approaches are lacking[5], and there is room for an alternative, or at least an additional approach.

A peculiarity of the (I), (II), (III) industry framework that seems to have escaped previous attention is that the two clubs potentially face "two-way" strategic interactions, via the revenue externality in (III), but also because they will have some duopsony power as the sole buyers of the specialised talent in (II). However both existing approaches ignore the duopsony power, modelling the two-club talent market as essentially a perfectly competitive market, with clubs treating wages as parametric when formulating their desired talent demand (or, equivalently with parametric wages, their desired talent expenditure) and with the wage adjusting to clear the market. The objective of this paper is to present an alternative approach, referred to as the *strategic market Game approach*, which incorporates the duopsony aspect and thus provides a full account of all relevant strategic interactions. To substantiate the case for this alternative approach, the paper traces its consequences (and those of the existing approaches) not only in the (I), (II), (III) framework, but also in a more general framework. In particular, we allow constant elastic talent supply to range from 0 as in (II), up to the perfectly elastic opposite extreme ($+\infty$) that is usually assumed when a European soccer league is the context[6]. In addition the revenue functions assumed in (III) have very little credibility as a primitive reflection of fans' willingness to pay to attend games – they are homogeneous of degree zero in team qualities, so that multiplying both qualities by a positive factor in excess of unity will leave win percentages unchanged, but also revenue. Instead we generalise so that revenue is homogeneous of some degree between zero and one. The focus of the strategic market Game analysis will be the same primary focus mentioned above, namely the extent of competitive balance in an unregulated league and the impact of revenue sharing on this balance, with comparisons to the existing "Walras" and "Nash" results.

The strategic market Game approach, as labelled in this paper, captures market power in a similar way to that of the classic Cournot model of imperfect competition, except that expenditures rather than quantities are the strategic variables. In the Cournot model of a talent market clubs would choose quantities of talent as strategic variables, anticipating the way that the wage for talent would subsequently adjust to clear the market. Since each club is non-negligible relative to the market (one of just two

[5] It certainly should not be inferred that this paper is a generic criticism of all previous Game-theoretic analysis of sports leagues. For instance, and there are many others, Falconieri et al. (2004) and Palomino and Sakovics (2004) provide important insights into issues associated with broadcasting revenues in the sports league context using Game theory. The focus of our critique is as described - the "Walras" and "Nash" approaches to the analysis of the basic framework. The reason for this focus is its importance to the sub-discipline; it is what appears from the literature, from textbooks and from courses on the subject as its theoretical core.

[6] Particularly since the Bosman agreement, there has been relatively fierce competition for soccer playing talent between the major European soccer leagues, and the literature has converged on the stylised assumption of perfectly elastic talent supply for models of a European soccer league, and the opposite perfectly inelastic supply if the context is a major North American sports league. Our generalised setting thus bridges the gap between these two extremes.

buyers here), they know that their strategy (quantity) choice will affect the wage, thus capturing their market power; and in the Nash equilibrium, given other club quantities, no club wishes to change its quantity choice (and hence the wage). The methodology is the same in the strategic market Game approach here, with expenditures on talent replacing quantities. Thus clubs choose expenditures on talent as their strategies, anticipating the way the resulting market-clearing wage depends on their strategy, which is chosen optimally given other club choices in the Nash equilibrium[7].

Three remarks on this expenditure/quantity issue are in order. First, the choice of expenditures as strategies does seem a much more realistic assumption in the sports context, where club owners typically decide on a player budget within which coaches and others directly involved with team planning acquire players. Secondly, the Cournot alternative is anyway not available for the basic league framework – the inverse talent supply function which would specify the market-clearing wage is not well-defined in the perfectly inelastic supply case. Finally, Szymanski has, at various points, suggested and used expenditures rather than quantities in expositions[8]. However the duopsony aspect has never emerged, with no indication of anything other than the "Nash" approach.

The main conclusions from the study of the basic (I), (II), (III) framework and its generalisation are, first, that the "Walras" approach can be seen as an attempt to by-pass both the two-way interactions whilst the "Nash" approach omits just the duopsony power and is best viewed as a special case of the strategic market Game approach when talent supply is perfectly elastic. Secondly, across all approaches there emerges a unified revenue sharing principle, namely that the introduction of revenue sharing, at least locally, causes competitive balance to move in the direction of the level that would maximize aggregate revenue for the league.

Section 2 sets out the standard "Walras" and "Nash" solutions for the basic (I), (II), (III) league, and section 3 generalises beyond the basic league. Section 4 sets out the alternative strategic market Game approach and section 5 concludes.

2. EXISTING ANALYSES OF THE BASIC LEAGUE

The (I), (II), (III) characteristics of the league to be studied are as set out in the introduction. The club revenue functions[9] $r_i : [0,1] \rightarrow \Re_+$ are assumed to be C^2 and

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[7] The literature on strategic market Games (see Giraud (2003) for an introduction) is largely concerned with general equilibrium analysis of exchange economies, where each agent submits bids (to buy, in terms of money) and/or offers (quantities for sale) to each market. On each market a price emerges that is the ratio of aggregate bids to offers, allowing the market to clear. Here there is a single talent market, the offers come from a large number of players offering their talent, and the bids come from the clubs, the wage being the ratio of aggregate bids to offers as in the strategic market Game literature. Hence our terminology.

[8] See Szymanski (2004, footnote 8, p.125), Szymanski (2006, p.242), Szymanski (2009).

[9] We are following the vast majority of the existing literature in taking revenue functions as a primitive. Behind this is an implicit micro-foundation of clubs choosing ticket prices for games, with monopoly power over their fans. Madden (2009a) provides details, and the observation that concavity of the resulting revenue functions is problematic. This problem is assumed away here, as in all existing literature.

strictly concave, with $r_i(0)=0$ (so a completely talentless home team will earn no revenue from its fans) and with maximum[10] at $W_i = m \in (\frac{1}{2},1)$. Club marginal revenue functions are $mr_i(W_i) \equiv r_i^{'}(W_i)$, and we follow the literature in assuming that club 1 is the "bigger" club in that $mr_1(W) > mr_2(W)$, for all $W \in (0,m)$.

This section produces the solutions associated with the "Walras" and "Nash" approaches from a base which is a well-defined Game, not found in the existing literature where one of the problems is the lack of such a base[11]. The Game, solved by backward induction, is a 2-stage Game with 3 Players, the 2 clubs plus a fictitious auctioneer. At stage 1 the auctioneer sets the wage, with positive payoff if the market eventually clears and negative payoff otherwise, incentivising the choice of market-clearing wages. At stage 2, given *w* at stage 1, clubs formulate simultaneously desired talent demand (or equivalently given *w*, talent expenditure, replacing t_i by e_i / w) to maximize profit;

$$\pi_i = r_i(\tfrac{t_i}{T}) - wt_i \qquad (2.1)$$

The difference between the two approaches is in the conjectures held by clubs about the rival club's choice when formulating their stage 2 demands (or expenditures).

In the "Walras" approach clubs hold the fixed-supply conjecture that T will remain constant at stage 2. Hence, the stationary point condition characterizing a club's talent demand (or equivalently given *w*, talent expenditure, replacing t_i by e_i / w) is;

$$r_i^{'}(\tfrac{t_i}{T})\tfrac{1}{T} = w \qquad (2.2)$$

By backward induction to stage 1, the auctioneer will choose the wage which ensures $T=1$ (so $W_i = t_i$), leading to the following "Walras" solutions for the wage, the win percentages and the allocations of talent;

$$mr_i(W_i) = w \text{ , } W_i = t_i \text{, } i = 1,2 \qquad \text{(W)}$$

Since marginal revenue curves are decreasing whenever they are positive, there is a unique (W) solution, which can be pictured in Figure 1 below (by w^W, W_1^W), as seen in textbooks and many papers. In this solution, the big club has the better team, and competitive balance, $CB = 1 - |W_1 - W_2|$ say, is less than maximal.

With the revenue sharing regulatory policy, home teams retain only the fraction $\alpha \in (\frac{1}{2},1]$ of their home gate revenue, the rest going to the away team. (2.1) becomes;

$$\pi_i = \alpha r_i(\tfrac{t_i}{T}) + (1-\alpha) r_j(\tfrac{t_j}{T}) - wt_i \qquad (2.3)$$

The stationary point condition in (2.2) is now;

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[10] Nothing of substance changes if *m* is allowed to differ in value for the two clubs – the exposition is simplified without this.

[11] What follows is therefore an attempt to make more Game-theoretic sense of existing approaches by providing an exact definition of the underlying Game. For instance, the expositions of Szymanski (2004) and Szymanski and Kesenne (2004) leave hanging the way the wage is determined in the Game. Bringing in a fictitious auctioneer resolves this, and seems to be the only way to define a Game consistent with the "Nash" approach.

$$\alpha r_i^{'}(\tfrac{t_i}{T})\tfrac{1}{T}-(1-\alpha)r_j^{'}(\tfrac{t_i}{T})=w, \qquad (2.4)$$

Backward induction to the auctioneer's market-clearing wage gives the solution;

$$mr_i(W_i)-(1-\alpha)[mr_i(W_i)+mr_j(1-W_i)]=w, W_i=t_i,\ i=1,2 \qquad \text{(WRS)}$$

Thus the effect of revenue sharing is that both marginal revenue curves fall by the same amount, producing the well-known "invariance principle" whereby the talent allocation and CB do not change with $\alpha \in (\tfrac{1}{2},1]$, the wage falling from w^W to 0 as α falls from 1 to $\tfrac{1}{2}$. Again there is a unique solution for $\alpha \in (\tfrac{1}{2},1]$. As $\alpha \to \tfrac{1}{2}$ the solution approaches the cartel solution.

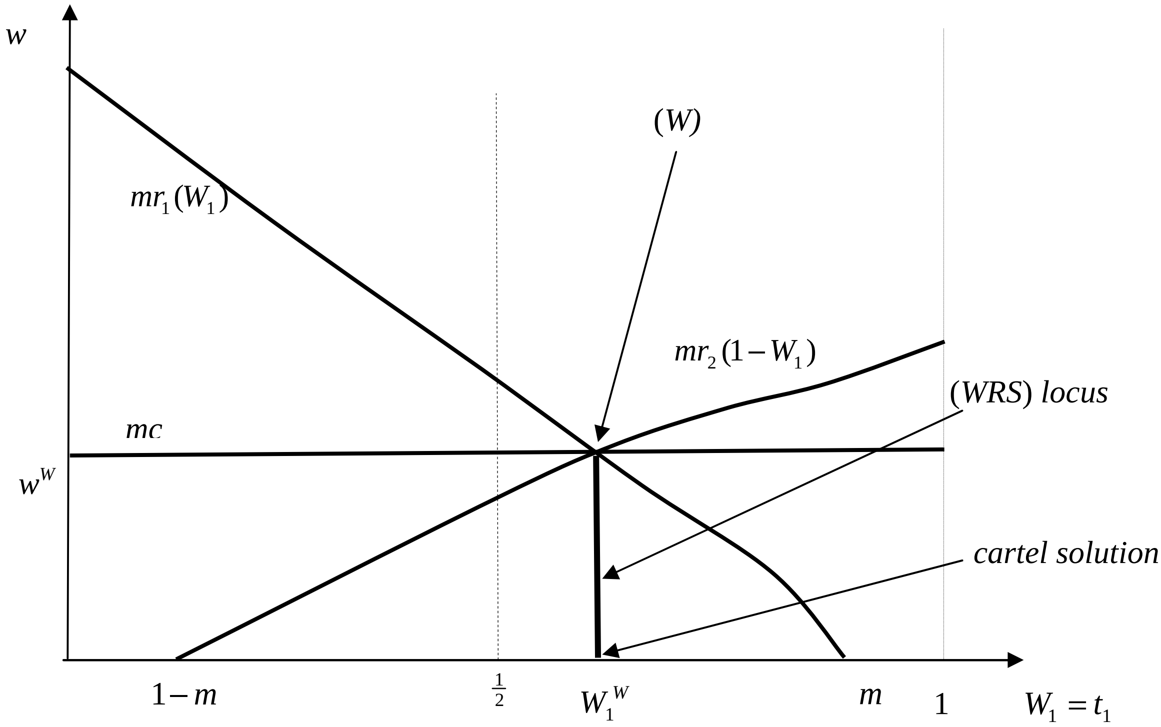

Figure 1; The Walras solution in the basic league(W) and with revenue sharing (WRS)

Define the *production efficient* win percentage (and hence CB) to be that which maximizes $r_1(W_1)+r_2(1-W_1)$, namely where $mr_1(W_1)=mr_2(1-W_1)$. The "Walras" solution leads to production efficiency, as does the cartel solution (also shown in Figure 1) where wages and win percentage are chosen to maximize aggregate profit[12].

It is important to note that the "Walras" solution is arrived at by a route which by-passes *both* of the two-way strategic interactions mentioned earlier. On the one hand the absence of duopsony market power is clear – the talent market is treated as perfect competition. But the fixed-supply conjecture essentially reduces revenue to dependence only on t_i, losing, or internalising, the revenue externality also.

Szymanski (2004) provided a critique of this solution based on its non-Nash, fixed-supply conjectures, a critique which seems appropriate, and went on to suggest an

[12] See Atkinson et al. (1998) for an analysis of the concepts of this paragraph.

alternative, elaborated more fully in Szymanski and Kesenne (2004). For the same 2-stage, 3-Player Game, the only change is that Nash conjectures (t_j constant) replace the fixed-supply conjectures. (2.1) is the same, and (2.2) becomes;

$$r_i^{'}(\tfrac{t_i}{T})\tfrac{t_j}{T^2} = w \qquad (2.5)$$

By backward induction, the auctioneer again chooses the wage which ensures $T = 1$ (so $W_i = t_i$), leading to the following alternative "Nash" solutions for the wage, the win percentages and the allocations of talent[13];

$$qmr_i(W_i) \equiv r_i^{'}(W_i)(1 - W_i) = w, W_i = t_i, i = 1,2 \qquad \text{(N)}$$

For reasons discussed later, the left hand side of (N) is called here the *quasi marginal revenue function* of club *i*, $qmr_i(W_i) \equiv mr_i(W_i)(1 - W_i)$. The same diagrammatic framework (Figure 2) shows that there is a unique solution (denoted w^N, W_1^N). The big club still has the better team, so CB is less than maximal but greater than at (W).

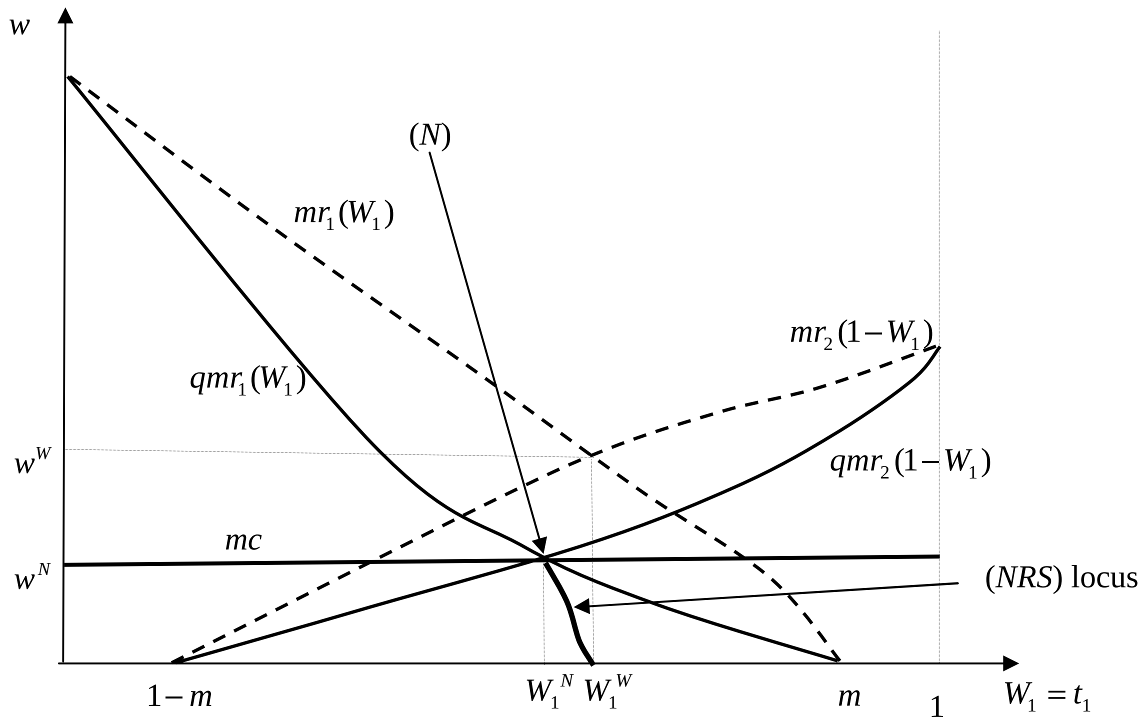

Figure 2; The Nash solution in the basic league (N) and with revenue sharing (NRS)

With revenue sharing, (2.3) is unchanged and (2.4) is[14];

$$\alpha r_i^{'}(\tfrac{t_i}{T})\tfrac{t_j}{T^2} - (1-\alpha) r_j^{'}(\tfrac{t_j}{T})\tfrac{t_j}{T^2} = w \qquad (2.6)$$

The usual backward induction produces the solution;

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13 The solution is now the subgame perfect Nash equilibrium of the 2-stage Game.

14 In the "Nash" approach, with and without revenue sharing, π_i need not be a globally concave function of t_i. However $\frac{\partial \pi_i}{\partial t_i}$ always has the sign of $\alpha r_i^{'}(W_i) - (1-\alpha) r_j^{'}(1 - W_i) - w\frac{T}{1-W_i}$, which is monotonically decreasing in t_i, so the stationary point is a global maximum.

$$qmr_i(W_i)-(1-\alpha)(1-t_i)[mr_i(W_i)+mr_j(1-W_i)]=w, W_i=t_i, i=1,2, j\neq i \quad \text{(NRS)}$$

The effect in Figure 2 is a unique solution for $\alpha \in (\frac{1}{2},1]$, with falling quasi marginal revenue curves, but that of the big club falling less, producing a larger talent allocation for the big club, reduced CB and a fall in the wage. As $\alpha \to \frac{1}{2}$ the solution again converges to the cartel solution, as shown in Figure 2.

Notice that both results on revenue sharing are consistent with the following unifying statement, which will be returned to in a later section; the affect of revenue sharing is to move the win percentages and CB towards their production efficient levels.

Generally, compared to the "Walras" solution, the "Nash" alternative again by-passes the duopsony power (the talent market is still perfectly competitive), but does now capture the revenue externality by the switch to Nash conjectures. The new strategic market Game approach in section 4 picks up both the strategic interactions. First we generalise the existing approaches beyond the basic framework so far.

3. A GENERALISED LEAGUE: EXISTING APPROACHES

(I) remains a league characteristic, but (II) and (III) generalise as follows.

(IIA) Talent supply $S(w)$ is constant elastic, with elasticity $\varepsilon \in [0,\infty]$: that is, $S(w)=w^{\varepsilon}$ if $\varepsilon \in [0,\infty)$, and w=1 if $\varepsilon=\infty$.

(IIIA) Revenue functions $\rho_i(W_i,T)$ now depend on the aggregate quality of the teams, $T=t_i+t_j$, as well as the home club win percentage. $\rho_i(W_i,T)$ is assumed to be homogeneous of degree $\sigma \in [0,1]$ in (t_i,t_j): that is, $\rho_i(W_i,T)=T^{\sigma}\rho_i(W_i,1)=T^{\sigma}r_i(W_i)$. σ measures the constant elasticity of revenue with respect to changes in the aggregate team quality, win percentage held constant, or the quality elasticity of revenue for short.

The basic league is now the special case where $\varepsilon=\sigma=0$.

The motivations behind the generalisations are as follows. The basic talent supply assumption may serve well the major North American sports leagues $(\varepsilon=0)$ but the opposite extreme $(\varepsilon=\infty)$ is usually taken as the appropriate simplifying assumption for European soccer leagues. The motivation for (IIA) is to encompass both these extremes, and all intermediate cases, in the same model. The extremes are the previous perfectly inelastic supply in a quantity normalised to unity, and the perfectly elastic case with a wage normalised to unity.

The need for (IIIA) is more pressing. The problem with (III) is that revenue depends only on W_i, which seems totally implausible as a primitive reflection of the underlying fan preferences and their willingness to pay to attend games. It implies that it is solely the quality of a fan's team relative to the rival that dictates willingness to pay to watch games and revenue; the absolute quality of teams and games has no impact. Alternatively phrased, revenue is homogeneous of degree zero in team qualities. But one would surely expect that, for instance, multiplying both team qualities by the same positive factor exceeding unity, thus keeping win percentages unchanged, would lead to an increase in the willingness to pay from fans and an increase in revenue. This problem undermines the "Nash" solution of the last section where the partial differentiation with Nash conjectures leading to (2.5) implies some variation in aggregate talent, but one which is assumed to have no affect on revenue. Hence we generalise in the simplest way, allowing revenue to become homogeneous of positive degree whilst retaining the nature of the dependence of revenue on win percentage. The specification assumes that the quality elasticity of revenue is the same for both clubs, a simplification that will be seen to have the big advantage of allowing continued use of the same diagrammatic framework as in section 2.

Although it does seem to be marginalised by the Szymanski (2004) critique, for completeness and because not everyone is convinced by this critique, we first look at the generalised model from the perspective of the "Walras" approach. Without revenue sharing the 2-stage Game solution procedure of the last section now produces the following generalisations of (2.1) and (2.2), where the fixed-supply conjecture again equates to constant *T*;

$$\pi_i = T^{\sigma} r_i(\tfrac{t_i}{T}) - wt_i \qquad (3.1)$$

$$r_i^{'}(\tfrac{t_i}{T}) = wT^{1-\sigma}$$

Hence the generalised "Walras" solution is;

$$\text{For } i = 1,2,\ mr_i(W_i) = \begin{cases} w^{1+\varepsilon(1-\sigma)},\ T = w^{\varepsilon},\ if\ \varepsilon < \infty \\ T^{1-\sigma},\ w = 1,\ if\ \varepsilon = \infty \end{cases}$$

Equilibrium win percentages (and so CB) do not vary with ε or σ, continuing to equate marginal revenues as defined earlier, and as shown in Figure 1, now with $w^{1+\varepsilon(1-\sigma)}$ or $T^{1-\sigma}$ on the vertical axis. And in the generalised setting, for any *T*, the production efficient win percentage is also unchanged since it is the W_1 that maximizes $T^{\sigma}[r_1(W_1) + r_2(1-W_1)]$, thus requiring the same marginal revenue equalisation.

With revenue sharing we have;

$$\pi_i = \alpha T^{\sigma} r_i(\tfrac{t_i}{T}) + (1-\alpha) T^{\sigma} r_j(\tfrac{t_j}{T}) - wt_i \qquad (3.2)$$

$$\alpha r_i^{'}(\tfrac{t_i}{T})\tfrac{1}{T} - (1-\alpha) r_j^{'}(\tfrac{t_i}{T}) = wT^{1-\sigma}$$

$$\text{For } i = 1,2,\ mr_i(W_i) - (1-\alpha)[mr_i(W_i) + mr_j(1-W_i)] = \begin{cases} w^{1+\varepsilon(1-\sigma)},\ T = w^{\varepsilon},\ if\ \varepsilon < \infty \\ T^{1-\sigma},\ w = 1,\ if\ \varepsilon = \infty \end{cases}$$

Again there is the invariance principle – revenue sharing has no effect on win percentages or CB, the falling marginal revenue curves leading to a reduced wage and aggregate talent.

Taking the "Nash" approach to the generalised framework, the case where $\sigma = 0$ with any $\varepsilon \in [0,\infty]$ produces the same solution values for win percentage as in the last section, with the same consequences of revenue sharing for the win percentage, since (3.1) and (3.2) are then unchanged from section 2. Thus we assume $\sigma > 0$, which is anyway necessary for the "Nash" approach to be credible, as already remarked. (3.1) leads to the following stationary point condition for stage 2 of the Game without revenue sharing;

$$\tfrac{\partial \pi_i}{\partial t_i} = T^{\sigma-1}\{r_i^{'}(W_i)(1-W_i) + \sigma r_i(W_i)\} - w = 0 \qquad (3.3)$$

A first issue is whether this implies a global payoff maximum. Define;

$$f_i(W_i,\sigma) = r_i^{'}(W_i)(1-W_i) + \sigma r_i(W_i)$$

These functions will be very useful, playing eventually a similar role to the quasi-marginal revenue functions earlier. Some initial properties are;

(i) $f_i(m,\sigma) = \sigma r_i(m) > 0$ and, for some $\eta > 0$, $f_i^{'}(W_i,\sigma) < 0$ on $[0, m+\eta]$;

(ii) $f_i(W_i,\sigma) < f_i(m,\sigma)$ if $W_i \in [m+\eta, 1]$;

(iii) $f_1(W_1^N,\sigma) > f_2(1-W_1^N,\sigma) > 0$.

The following is a resolution of the global maximum question[15];

Lemma 1 The stationary point condition (3.3) defines a global payoff maximum for *i* if $W_i \in [0,m]$.

Proof See appendix.

It also follows from (3.3) that a necessary condition for the "Nash" solution win percentages is, incorporating the auctioneer's stage 1 market-clearing;

$$\text{For } i = 1,2,\ f_i(W_i,\sigma) = \begin{cases} w^{1+\varepsilon(1-\sigma)},\ T = w^{\varepsilon},\ if\ \varepsilon < \infty \\ T^{1-\sigma},\ w = 1,\ if\ \varepsilon = \infty \end{cases} \qquad (3.4)$$

Thus we require an intersection of the graphs of $f_1(W_1,\sigma)$ and $f_2(1-W_1,\sigma)$, and it follows that the solution values for win percentages will in general depend on σ but not on ε; both these properties stem from the assumed equality of the clubs' quality elasticity of revenue. From (i)-(iii), there will be a unique intersection with $W_1 \in (W_1^N, m)$ if $f_1(m) < f_2(1-m)$. And since Lemma 1 applies, the intersection is indeed the unique "Nash" solution. A primitive assumption which ensures that $f_1(m,\sigma) < f_2(1-m,\sigma)$ is[16];

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[15] The new difficulty for the global maximum analysis is that (unlike $qmr_i(W_i)$), $f_i(W_i,\sigma)$ may be positive and increasing for large W_i. For instance this does happen with a quadratic $r_i(W_i)$ if $\sigma \in (0,1)$ is large enough.

Assumption 1 (A1) $r_2^{'}(1-m)m + r_2(1-m) > r_1(m)$.

By strict concavity $r_2^{'}(1-m)m + r_2(1-m) > r_2(m)$, so (A1) is non-vacuous and requires that, for any given *T*, the maximum revenue attainable by the big club is not too much bigger than that for the smaller club.

Proposition 1 In a generalised league without revenue sharing, with $\sigma > 0$, and where (A1) is satisfied, the "Nash" approach produces a unique solution characterised by (3.4) with a win percentage $W_1^*(\sigma) \in (W_1^N, m)$.

Figure 3 illustrates. As already remarked, if $\sigma = 0$ the solution is W_1^N, as in Section 2. As σ increases from 0, the $f_1(W_1,\sigma)$ and $f_2(1-W_1,\sigma)$ graphs move upwards from qmr_1 and qmr_2, respectively, with $f_1(W_1,\sigma)$ moving further than $f_2(1-W_1,\sigma)$, ensuring a higher equilibrium value of W_1 (shown as $W_1^*(\sigma)$), with a decrease in CB. The reason is that increases in talent demand by a club not only increase its win percentage with the same qmr_i effect as when $\sigma = 0$, but also now the increase in aggregate talent leads to extra revenue which is larger for the big club, causing the big club to increase its talent demand by more. Thus increases in σ (ceteris paribus) lead to increases in W_1 and reductions in CB.

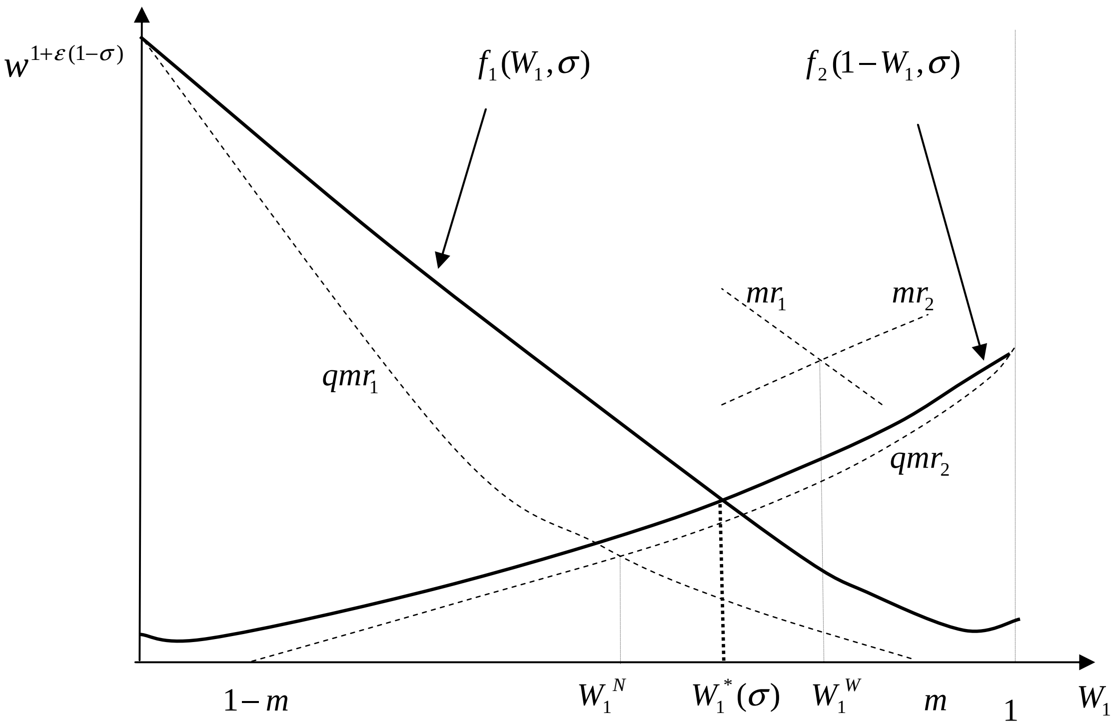

Figure 3; The Nash solution in the generalised league

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$r_2^{'}(1-m)m + r_2(1-m) > r_1(m) \Rightarrow r_2^{'}(1-m)m > r_1(m) - r_2(1-m) \Rightarrow r_2^{'}(1-m)m > \sigma[r_1(m) - r_2(1-m)]$
which is the same as $f_1(m) < f_2(1-m)$.

An important further question for the revenue sharing discussion is how far does $W_1^*(\sigma)$ increase as σ increases towards 1? In particular, unlike the case shown in Figure 3, does it ever exceed the production efficient level W_1^W? This will happen if (and only if) $f_1(W_1^W,\sigma) > f_2(1-W_1^W,\sigma)$, which rearranges to;

$$\sigma > \frac{r_1'(W_1^W)(2W_1^W-1)}{r_1(W_1^W)-r_2(1-W_1^W)} \equiv K \tag{3.5}$$

The strict concavity of revenues ensures that K<1 and (3.5) is non-vacuous.

<u>Corollary to Proposition 1</u> The unique “Nash” solution value $W_1^*(\sigma)$ increases as σ increases (ceteris paribus), and exceeds W_1^W if and only if $\sigma > K$.

Turning to the revenue sharing analysis, (3.2) leads to the stationary point condition for stage 2 of the Game;

$$\frac{\partial \pi_i}{\partial t_i} = T^{\sigma-1}\{\alpha f_i(W_i,\sigma) + (1-\alpha) f_j(1-W_i,\sigma) - (1-\alpha) r_j'(1-W_i)\} - w = 0 \tag{3.6}$$

Again there is the global payoff maximum issue.

<u>Lemma 2</u> There exists $\delta > 0$ such that for $\alpha \in (1-\delta,1]$, the stationary point condition (3.6) defines a global payoff maximum for *i* if $W_i \in [0,m]$.
<u>Proof</u> See appendix.

Define, as in the proof of Lemma 2;
$\phi_i(W_i,\alpha,\sigma) = f_i(W_i,\sigma) - (1-\alpha)[f_i(W_i,\sigma) - f_j(1-W_i,\sigma) + r_j'(1-W_i)]$

Then a necessary condition for the “Nash” solution win percentage is, from (3.6) and analogous to (3.4);

$$\text{For } i = 1,2, \ \phi_i(W_i,\alpha,\sigma) = \begin{cases} w^{1+\varepsilon(1-\sigma)}, T = w^{\varepsilon}, \text{ if } \varepsilon < \infty \\ T^{1-\sigma}, w = 1, \text{ if } \varepsilon = \infty \end{cases} \tag{3.7}$$

<u>Proposition 2</u> In a generalised league with revenue sharing, with $\sigma > 0$, and where (A1) is satisfied, there exists $\delta \in (0,\frac{1}{2})$ such that, for all $\alpha \in (1-\delta,1]$, the “Nash” approach produces a unique solution characterised by (3.7) with a win percentage $W_1^*(\alpha,\sigma) \in (W_1^N, m)$.
<u>Proof</u> See appendix.

As α falls from 1, the ϕ_i curves fall, producing the usual reduction in the wage and aggregate talent, at least locally. The effect of the introduction of revenue sharing on the win percentage, and hence on CB, is more ambiguous, following from;

<u>Corollary to Proposition 2</u> $\partial W_1^*(\alpha,\sigma)/\partial\alpha$ has the sign of $W_1^W - W_1^*(\alpha,\sigma)$.
<u>Proof</u> See appendix.

As in the earlier analysis without revenue sharing, $W_1^*(\alpha,\sigma) > W_1^W$ if (and only if) $\phi_1(W_1^W,\alpha,\sigma) > \phi_2(1-W_1^W,\alpha,\sigma)$ which reduces to the same condition as earlier, namely $f_1(W_1^W,\sigma) > f_2(1-W_1^W,\sigma)$, or $\sigma > K$. Thus if the quality elasticity of revenue is large enough ($\sigma > K$), the CB predicted by the "Nash" approach will be *lower* than that at the production efficient talent allocation (i.e. at W_1^W), and the introduction of revenue sharing (i.e. for $\alpha \in (1-\delta,1]$) will *increase* the CB back towards its production efficient level, reversing the basic league conclusion[17]. And vice versa if $\sigma < K$.

4. THE STRATEGIC MARKET GAME APPROACH

In the previous 2 sections we have used a 2-stage, 3-Player Game specification as the vehicle for exposition of the existing "Walras" and "Nash" approaches to the analysis of a basic and a more general sports league with and without revenue sharing regulation. In this section we present our alternative and preferred *strategic market Game approach*.

A problem with the existing approaches is that they treat the two-club talent market as perfectly competitive, overlooking the market power clubs might be expected to have in such a setting. This is seen in the Game specification, whereby clubs perceive that their decisions (at stage 2) will have no impact on the wage, set (at stage 1 by the auctioneer) to clear the talent market in the usual competitive market fashion. The crucial change to bring in club talent market power is to reverse the sequencing of the 2 stages in the previous Game. In the resulting backward induction (or subgame perfect Nash) solution of the new Game, the two clubs will anticipate correctly the (non-negligible) impact that their stage 1 decisions will have on the stage 2 market-clearing wage, thus capturing their talent market power. If the stage 1 decision variables were quantities of talent, the new model would simply and exactly be a Cournot duopsony model. However in the most basic league, with its perfectly inelastic supply of talent, the stage 2 market-clearing wages are not then well-defined. So in addition to reversing the 2 stages, we insist now on talent expenditures as the club strategic choice variables at stage 1, an assumption that is anyway probably more realistic for the context of a sports league, as already remarked in the introduction.

So given stage 1 club expenditure decisions, e_i, the market-clearing wage at stage 2 will be w such that $S(w) = E/w$, where $S(w) = w^{\varepsilon}$ is the general talent supply function and $E = e_i + e_j$. In terms of the expenditure strategies, the wages, talent allocations, aggregate talent and win percentages will be $w = E^{\frac{1}{1+\varepsilon}}$, $t_i = e_i / E^{\frac{1}{1+\varepsilon}}$,

[17] Marburger (1997) also reverses the basic league conclusion, in a model with $\varepsilon = 0$ and with a different approach to modelling the dependence of revenue on relative and absolute team qualities.

$T = E^{\frac{\varepsilon}{1+\varepsilon}}$, and $W_i = e_i / E$. Anticipating these stage 2 consequences, the payoffs to the clubs, assuming the general form for revenues, are[18];

$$\pi_i(e_i, e_j) = E^{\lambda} r_i(\tfrac{e_i}{E}) - e_i, \text{ where } \lambda = \tfrac{\sigma\varepsilon}{1+\varepsilon} \qquad (4.1)$$

Thus the backward induction (or subgame perfect Nash) solution of the new 2 stage Game reduces to the Nash equilibrium of the 2-Player (club) simultaneous move Game where strategy sets are $e_i \in \Re_+, i = 1,2$ and payoffs are given by (4.1). As already remarked, the reduced Game is a variation on a theme found in the strategic market Game (SMG) literature, where Players (agents in an exchange economy) make bids (in money, to buy) and offers (in quantities, to sell) to each side of each market (one market per good), a market-clearing price emerging on each market as the ratio of aggregate bids to offers. Agents then choose simultaneously bids and offers for each market, the Nash equilibrium of the resulting Game, and its properties, being the object of study. In our context, there is just one (talent) market, clubs make the bids (talent expenditures), players supply (non-strategically – implicitly they are large in number) the offers $S(w)$, and the market-clearing wage is the ratio of aggregate bids to offers, $w = E / S(w)$; Nash equilibrium is again the solution concept.

The duopsony power of clubs in the SMG model[19] can be seen explicitly from $t_i = e_i / E^{\frac{1}{1+\varepsilon}}$. Differentiating partially with respect to t_i and using $w = E^{\frac{1}{1+\varepsilon}}$ gives $\frac{\partial e_i}{\partial t_i} = w[(e_i + e_j)/(\frac{\varepsilon}{1+\varepsilon} e_i + e_j)]$. This measures the extra expenditure, or marginal cost, incurred by club *i* in increasing its talent allocation. The marginal cost exceeds the wage, and is larger for the club with the larger expenditure. However as $\varepsilon \to \infty$ the marginal costs decrease monotonically towards *w*. Thus, naturally, the duopsony power disappears completely in the limit where the supply of talent to the league is perfectly elastic.

Consider first the SMG model when $\lambda = 0$, so either $\varepsilon = 0$ or $\sigma = 0$ or both, the latter being the basic league special case. With the changes of t_i to e_i, *T* to *E*, and *w* to 1, (2.1) is identical to (4.1). With Nash conjectures the stationary point condition for SMG produces a global maximum as in section 2, and is the corresponding translation of (2.5), namely $r_i'(\frac{e_i}{E})\frac{e_j}{E^2} = 1$. Since $W_i = e_i / E$ and $w = E$, the SMG equilibrium that results is exactly (N). A conclusion from this is that for the inelastic talent supply case $\varepsilon = 0$, the SMG approach provides a better rationalisation of (N) as a plausible solution than the "Nash" approach since it is consistent with $\sigma > 0$, whereas "Nash" requires the implausible $\sigma = 0$. The reason is the stage 1/2 order reversal. With SMG and $\varepsilon = 0$, clubs anticipate that there will anyway be no variation in talent supply at

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[18] This derivation has assumed implicitly that *E*>0. The easiest way to complete the specification is to assume $\pi_i(e_i, e_j) = 0$ if *E*=0. Trivial equilibria where *E*=0 may occur, but are Pareto dominated by the non-trivial equilibria on which we focus exclusively.

[19] The modelling of duopsony in this paper is novel. Existing literature on oligopsony has focused on models where jobs are differentiated in the eyes of workers, with Bertrand wage-setting firms (Bhaskar and To (1999, 2003), Kaas and Madden (2008, 2009)). Here jobs are undifferentiated in this sense, and firms choose expenditures on labour – players receive the same salary (=the product of their talent level and the wage for talent) wherever they play, and are indifferent as to which club they play for.

stage 2, so changing $\sigma = 0$ to $\sigma > 0$ has no effect on decisions at stage 1. With "Nash", as was seen in section 3, this is not so.

Remark Apart from the horizontal marginal cost curve, the intersection of the quasi marginal revenue curves in Figure 2 provides a useful depiction[20] of the SMG solution values of win percentages and the wage in the perfectly inelastic talent supply case, more so (for the reasons discussed above) than as a diagrammatic device to support the "Nash" approach. A club's quasi marginal revenue curve defines the proportion of total expenditure for club *i* $(W_i = e_i / E)$ which would be a best response in the SMG to the total expenditure $(E = w)$, and is well-known in the theory of aggregative Games as the Player's share function (see Cornes and Hartley(2005)); thus the quasi marginal revenue graphs in Figure 2 are graphs of share functions. The suggested "quasi" terminology used here is simply by analogy with the "Walras" approach, where the marginal revenue curves depict the relation between optimal win percentage and the wage; the quasi marginal revenue curves do the same thing for the SMG Players.

For $\lambda > 0$ the stationary point condition from (4.1) is;

$$\begin{aligned} \tfrac{\partial \pi_i}{\partial e_i} &= E^{\lambda-1}\{r_i'(W_i)(1-W_i) + \lambda r_i(W_i)\} - 1 \\ &= E^{\lambda-1} f_i(W_i, \lambda) - 1 = 0 \end{aligned} \tag{4.2}$$

Notice the similarity to (3.3) in the earlier "Nash" analysis; the re-appearance of the f_i functions will be very helpful in analysing the SMG, and comparing it to "Nash".

The arguments of Lemma 1 earlier are easily amended to produce the statement; the stationary point condition (4.2) defines a global payoff maximum for *i* if $W_i \in [0, m]$.

It also follows from (4.2) that a necessary condition for the SMG equilibrium is;

$$\text{For } i = 1,2,\ f_i(W_i, \lambda) = \begin{cases} w^{1+\varepsilon(1-\sigma)}, T = w^{\varepsilon}, if\ \varepsilon < \infty \\ T^{1-\sigma},\ w = 1, if\ \varepsilon = \infty \end{cases} \tag{4.3}$$

This is now exactly the same as (3.4), simply replacing σ by λ. The previous results for the "Nash" solution subsequent to (3.4) can now be exactly paralleled for the SMG equilibrium[21].

1. In a generalised league without revenue sharing, with $\lambda > 0$, and where (A1) is satisfied, the SMG approach produces a unique solution characterised by (4.3) with a win percentage $W_1^*(\lambda) \in (W_1^N, m)$. (Proposition 1)
2. The unique SMG solution value $W_1^*(\lambda)$ increases as λ increases (ceteris paribus), and exceeds W_1^W if and only if $\lambda > K$. (Corollary to Proposition 1)

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[20] The traditional diagrams for a 2-Player Game would involve graphs of best responses e_i as functions of e_j; these can be found in the previous paper Madden (2009b). However the quasi marginal revenue curves seem more useful in the context here.

[21] All proofs are as for section 3, replacing σ by λ.

With revenue sharing, the stationary point condition (3.6) becomes;

$$\tfrac{\partial \pi_i}{\partial e_i} = E^{\lambda-1}\{\alpha f_i(W_i,\lambda) + (1-\alpha) f_j(1-W_i,\lambda) - (1-\alpha) r_j^{'}(1-W_i)\} - 1 = 0 \qquad (4.4)$$

3. There exists $\delta > 0$ such that for $\alpha \in (1-\delta, 1]$, the stationary point condition (4.4) defines a global payoff maximum for *i* if $W_i \in [0, m]$. (Lemma 2)

(3.7) becomes;

$$\text{For } i = 1,2,\ \phi_i(W_i,\alpha,\lambda) = \begin{cases} w^{1+\varepsilon(1-\sigma)}, T = w^{\varepsilon}, \textit{ if } \varepsilon < \infty \\ T^{1-\sigma}, w = 1, \textit{ if } \varepsilon = \infty \end{cases} \qquad (4.5)$$

4. In a generalised league with revenue sharing, with $\lambda > 0$, and where (A1) is satisfied, there exists $\delta \in (0, \tfrac{1}{2})$ such that, for all $\alpha \in (1-\delta, 1]$, the SMG approach produces a unique solution characterised by (4.5) with a win percentage $W_1^*(\alpha,\lambda) \in (W_1^N, m)$. (Proposition 2)

5. $\partial W_1^*(\alpha,\lambda)/\partial\alpha$ has the sign of $W_1^W - W_1^*(\alpha,\lambda)$. (Corollary to Proposition 2)

Again analogous to section 3, $W_1^*(\alpha,\lambda) > W_1^W$ if (and only if) $\phi_1(W_1^W,\alpha,\lambda) > \phi_2(1-W_1^W,\alpha,\lambda)$ which is $f_1(W_1^W,\lambda) > f_2(1-W_1^W,\lambda)$, or $\lambda > K$. Thus if $\lambda > K$, the CB predicted by the "Nash" approach will be lower than that at the production efficient talent allocation (i.e. at W_1^W), and the introduction of revenue sharing ($\alpha \in (1-\delta, 1]$) will increase the CB towards its production efficient level. And vice versa if $\lambda < K$. Figure 4 translates these statements in terms of σ and ε.

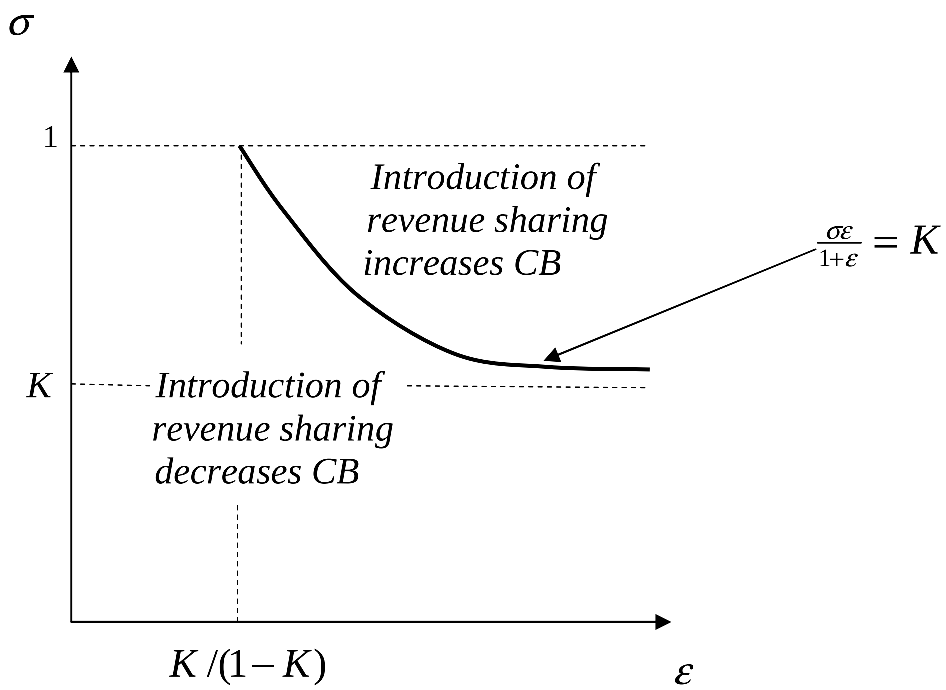

Figure 4: The effect of the introduction of revenue sharing on competitive balance in the SMG equilibrium

We conclude this section with some systematic comparisons of the "Nash" and SMG approaches. First when $\varepsilon = \infty$, $\lambda = \sigma$ and (3.4) is the same as (4.3). Hence;

Proposition 3 In a generalised league with perfectly elastic talent supply ($\varepsilon = \infty$), the "Nash" and SMG approaches produce identical conclusions, with and without revenue sharing.

The reason is clear. In the SMG approach clubs generally have duopsony market power, with marginal costs in excess of the wage. However when $\varepsilon = \infty$ this market power disappears, as noted earlier, and marginal costs equal the wage, which is the general assumption behind the "Nash" approach. So the two approaches coincide at the perfectly elastic talent supply limit.

However when $\varepsilon < \infty$ there are differences, illustrated in Figure 5. Since $\lambda < \sigma$, $W_1^*(\lambda) < W_1^*(\sigma)$, as shown, and the "Nash" approach predicts a lower CB than SMG. The reason is as follows. Starting from the "Nash" solution at $W_1^*(\sigma)$, the duopsony power of the SMG approach would lead to increased marginal costs for both clubs, more so for the big club (with its larger expenditure), thus causing a greater reduction in talent for the big club, and increased competitive balance at $W_1^*(\lambda)$.

Proposition 4 In a generalised league with imperfectly elastic talent supply ($\varepsilon < \infty$) and without revenue sharing, the "Nash" approach predicts a lower competitive balance than the SMG approach.

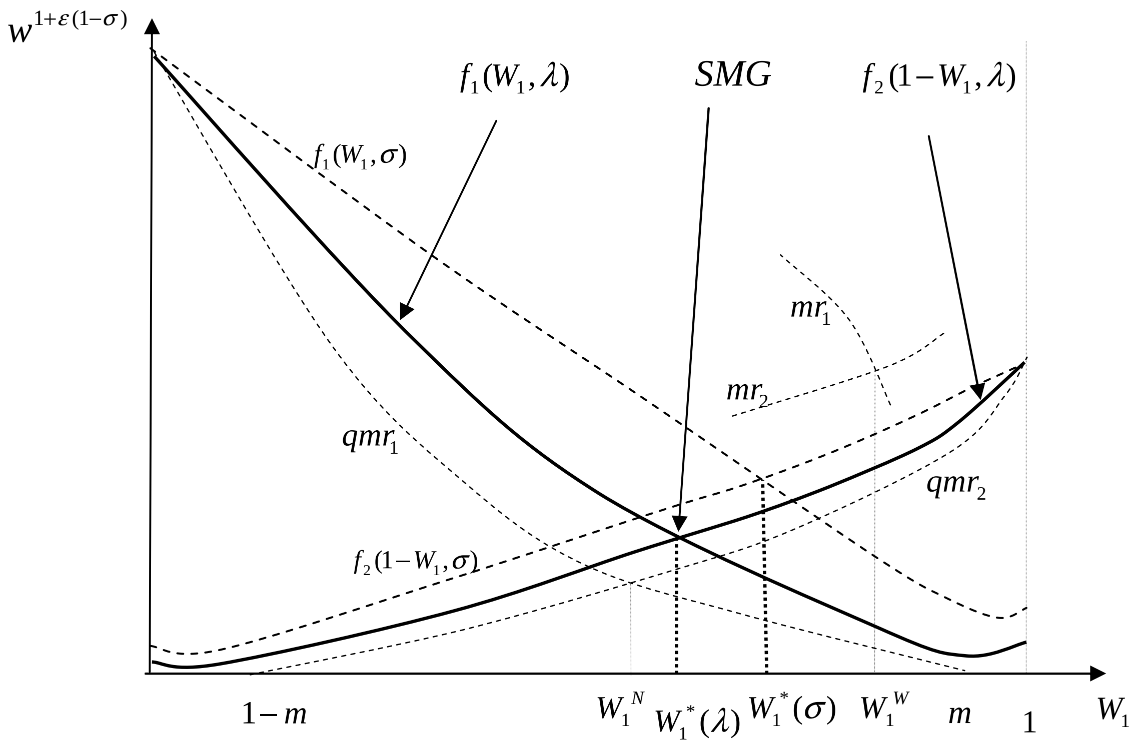

Figure 5; The SMG solution in the generalised league

With revenue sharing there are also differences, even in the direction that the introduction of revenue sharing moves CB – when $\lambda < K < \sigma$ the introduction of revenue sharing increases CB according to the “Nash” approach but decreases CB in the SMG model. More interesting perhaps is the following commonality regarding the effects of revenue sharing not only with the “Nash” and SMG approaches, but also “Walras”;

Proposition 5 In a basic league or in a generalised league, and with the “Walras”, “Nash” or SMG approaches, the following generalised revenue sharing principle holds; the introduction of revenue sharing always causes win percentages and competitive balance to move towards their production efficient levels.

There is a simple intuition. Equal revenue sharing would lead to the production efficient win percentages and CB. The introduction of revenue sharing starts the movement in this direction.

5. CONCLUSIONS

The paper has introduced a new strategic market Game (SMG) approach to modelling strategic interactions between clubs in professional team sports leagues, and generalised the basic league framework used in much of the previous literature (two clubs, perfectly inelastic talent supply and club revenues that depend only on relative team qualities) to allow variable talent supply to the league, and club revenues that depend on absolute (as well as relative) team quality levels. The consequences of the new approach for competitive balance with and without revenue sharing have been identified and compared to those of the existing (“Walras” and “Nash”) approaches in both the basic and more general frameworks.

One should expect to see “two-way” strategic interactions in a two-club context, on the revenue side since a club’s home gate revenue depends on the away team quality, and on the cost side since the clubs have duopsony power as the only clubs competing for the talent supply to the league. The “Walras” approach essentially avoids both of these, whilst the “Nash” approach picks up only on the revenue externality. The SMG approach incorporates both.

As argued by Szymanski (2004), the “Walras” approach is limited by its reliance on non-Nash fixed-supply conjectures. On the other hand, it has been argued by Madden (2009a), that the “Walras” solution can be justified if there are large numbers of two types of clubs, rather than just two clubs. However it is difficult to see any such justification of the “Nash” approach – club numbers would need to be large to justify the absence of talent market power, but small so that the revenue externality between individual clubs remains relevant. The “Nash” approach thus seems to be satisfactorily rationalised only as the special limiting case of SMG where the supply of talent to the league is perfectly elastic, as seen in section 4 here. The view emerges that if club numbers are sufficiently small that strategic interactions between clubs are relevant, then the two-way strategic interactions need to be modelled, calling for the SMG rather than the previous approaches. If club numbers are relatively large however, “Walras” solutions may be appropriate.

Whether the reader agrees with the view above or not, it is hoped that the generalised framework of this paper will prove attractive and tractable, whichever approach is taken. On the one hand this framework leaves behind the surely embarrassing basic league assumption that revenues depend only on relative team qualities. In addition it embodies a full range of talent supply elasticities, from the basic fixed talent supply (as usually assumed for the major North American sports leagues) to the perfectly elastic opposite extreme (the stylised European soccer league assumption).

The SMG analysis of the unregulated league competitive balance in our general framework shows that it will be lower for European soccer than for the North American leagues (ceteris paribus), and revenue sharing will lower the North American competitive balance, but possibly increase the European outcome. A general, unifying revenue sharing principle emerges, across all approaches and both frameworks; the introduction of revenue sharing always causes win percentages and competitive balance to move towards their production efficient levels.

The paper is intended as only a first word on its themes, not the last. Hopefully enough has been done to convince the reader that the SMG approach and the generalised framework of the paper are worthy of attention and further development[22].

[22] Madden (2009b) does provide a SMG analysis of some further developments in the basic league framework, namely a generalisation to leagues with more than 2 clubs, and discussion of salary caps.

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APPENDIX

Note; σ is omitted from the arguments of functions for brevity.

Proof of Lemma 1 If T and $W_i \in [0, m]$ satisfy (3.2) the implied values for t_i and t_j are $t_i = TW_i$ and $t_j = T(1-W_i)$, which satisfy $f_i(\frac{t_i}{t_i+t_j}) = w(t_i + t_j)^{1-\sigma}$. Generally, if *i* deviates to $\hat{t}_i$, $\frac{\partial \pi_i}{\partial t_i}$ has the sign of $f_i(\frac{\hat{t}_i}{\hat{t}_i+t_j}) - w(\hat{t}_i + t_j)^{1-\sigma}$. If $\hat{t}_i > t_i$ then $f_i(\frac{t_i}{t_i+t_j}) > f_i(\frac{\hat{t}_i}{\hat{t}_i+t_j})$, from property (i) of $f_i(W_i)$ if $\frac{\hat{t}_i}{\hat{t}_i+t_j} \le m+\eta$, and from property (ii) of $f_i(W_i)$ if $\frac{\hat{t}_i}{\hat{t}_i+t_j} > m+\eta$; either way, $\frac{\partial \pi_i}{\partial t_i} < 0$ at $\hat{t}_i, t_j$. If $\hat{t}_i < t_i$ then $\frac{\partial \pi_i}{\partial t_i} > 0$ at $\hat{t}_i, t_j$, from property (i) again.

Proof of Lemma 2 Define $\phi_i(W_i, \alpha) = \alpha f_i(W_i) + (1-\alpha) f_j(1-W_i) - (1-\alpha) r_j^{'}(1-W_i)$. Notice $\phi_i(W_i, 1) = f_i(W_i)$. Denoting partial derivatives of $\phi_i(W_i, \alpha)$ with respect to its ath argument as $\phi_{ia}^{'}$, it follows from property (i) of $f_i(W_i)$ that $\phi_{i1}^{'}(W_i, 1) < 0$ for $W_i \in [0, m+\eta]$. By continuity it also follows that there exists $\delta_1 \in (0, \frac{1}{2})$ such that for $\alpha \in (1-\delta_1, 1]$; (a) $\phi_{i1}^{'}(W_i, \alpha) < 0$ for $W_i \in [0, m+\eta]$. In addition we know from property (ii) of $f_i(W_i)$ that $\phi_i(m,1) - \phi_i(W_i, 1) > 0$ for $W_i \in [m+\eta, 1]$, and by a similar continuity argument there exists $\delta_2 \in (0, \frac{1}{2})$ such that for $\alpha \in (1-\delta_2, 1]$; (b) $\phi_i(m, \alpha) - \phi_i(W_i, \alpha) > 0$ for for $W_i \in [m+\eta, 1]$. Then with $\delta = \min(\delta_1, \delta_2)$ both (a) and (b) are true for $\alpha \in (1-\delta, 1]$.

If T and $W_i \in [0, m]$ satisfy (3.6) the implied values for t_i and t_j are $t_i = TW_i$ and $t_j = T(1-W_i)$, which satisfy $\phi_i(\frac{t_i}{t_i+t_j}) = w(t_i + t_j)^{1-\sigma}$. Generally, if *i* deviates to $\hat{t}_i$, $\frac{\partial \pi_i}{\partial t_i}$ has the sign of $\phi_i(\frac{\hat{t}_i}{\hat{t}_i+t_j}) - w(\hat{t}_i + t_j)^{1-\sigma}$. If $\hat{t}_i > t_i$ then $\phi_i(\frac{t_i}{t_i+t_j}) > \phi_i(\frac{\hat{t}_i}{\hat{t}_i+t_j})$, from (a) above if $\frac{\hat{t}_i}{\hat{t}_i+t_j} \le m+\eta$, and from (b) above if $\frac{\hat{t}_i}{\hat{t}_i+t_j} > m+\eta$; either way, $\frac{\partial \pi_i}{\partial t_i} < 0$ at $\hat{t}_i, t_j$. If $\hat{t}_i < t_i$ then $\frac{\partial \pi_i}{\partial t_i} > 0$ at $\hat{t}_i, t_j$, from (a) again.

Proof of Proposition 2 (3.7) requires that $\phi_1(W_1, \alpha) = \phi_2(1-W_1, \alpha)$. First note the following 2 facts;

1. $\phi_1(W_1^N, \alpha) > \phi_2(1-W_1^N, \alpha)$ if $\alpha \in (\frac{1}{2}, 1]$, since $\phi_1(W_1^N, \alpha) - \phi_2(1-W_1^N, \alpha) = (2\alpha - 1)[r_1(W_1^N) - r_2(1-W_1^N)] + (1-\alpha)[r_1^{'}(W_1^N) - r_2^{'}(1-W_1^N)] > 0$.
2. $\phi_1(m, \alpha) < \phi_2(1-m, \alpha)$ if $\alpha \in (\frac{1}{2}, 1]$ and (A1) holds, since $\phi_1(m, \alpha) - \phi_2(1-m, \alpha) = (2\alpha - 1)[f_1(m) - f_2(1-m)] - (1-\alpha) r_2^{'}(1-m) < 0$.

From (a) and (b) in the proof of Lemma 2, it follows that there is a unique solution $W_1^*(\alpha) \in (W_1^N, m)$ to $\phi_1(W_1, \alpha) = \phi_2(1-W_1, \alpha)$, for any $\alpha \in (\frac{1}{2}, 1]$. Restricting α to the interval $(1-\delta, 1]$ defined by Lemma 2 ensures that $W_1^*(\alpha)$ is the unique "Nash" solution.

Proof of Corollary to Proposition 2 Define $\Psi(W_1, \alpha) = \phi_1(W_1, \alpha) - \phi_2(1-W_1, \alpha)$. From Proposition 2, for $\alpha \in (1-\delta, 1]$, there is a unique solution to $\Psi(W_1, \alpha) = 0$, namely

$W_1^*(\alpha)$. Since $\Psi_1^{'}(W_1^*(\alpha),\alpha) < 0$ (where $\Psi_a^{'}$ denotes the partial derivative with respect to the ath argument), it follows from the implicit function theorem that $\partial W_1^*(\alpha)/\partial\alpha$ has the sign of $\Psi_2^{'}(W_1^*(\alpha),\alpha)$.

Generally $\Psi_2^{'}(W_1,\alpha) = 2[f_1(W_1) - f_2(1-W_1)] + r_2^{'}(1-W_1) - r_1^{'}(W_1)$. Hence;

$\Psi_2^{'}(W_1^*(\alpha),\alpha) = [r_2^{'}(1-W_1^*(\alpha)) - r_1^{'}(W_1^*(\alpha))]/(2\alpha-1)$, and the sign of the square bracket is that of $W_1^W - W_1^*(\alpha)$.