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Alejandro Saporiti

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School of Social Sciences
The University of Manchester
Manchester M13 9PL

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Abstract

The literature on electoral competition frequently assumes that politicians are *symmetric* and *single-minded*, in the sense that their electoral motivations are identical and fully focused on either (i) being in power, or (ii) the ideological position of the winning policy. This paper relaxes both assumptions. It analyzes a one-dimensional model of electoral competition with two candidates, which care about what policy is enacted after the election, but they intrinsically value being in office too, and not necessarily in the same way. The paper provides necessary and sufficient conditions for policy convergence and policy differentiation, and it characterizes the equilibrium platforms in both cases, uncovering a new type of pure strategy equilibrium, called “one-sided policy differentiation.” This work also shows that the effect of candidates’ motivations over voters’ welfare depends on the underlying model of electoral uncertainty.

JEL Classification: C72; D72.

Key words: Spatial competition; electoral motivations; power; ideology; policy differentiation; policy convergence; electoral uncertainty.

1 Introduction

The *spatial* theory of electoral competition begins with the seminal work of Hotelling (1929). The simplest model assumes two candidates or political parties competing in

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[†]School of Social Sciences, University of Manchester, Oxford Road, Manchester M13 9PL, United Kingdom. E-mail: alejandro.saporiti@manchester.ac.uk.

a winner-take-all election, which is carried out to select a policy located on a one-dimensional scale (e.g., a proportional income tax rate). Among the alternatives available in the policy space, each candidate chooses, simultaneously and independently of the other, one proposal or *platform* that it commits to implement in the case of winning the contest. Every voter is endowed with a preferred or *ideal* policy over the space, and it votes after comparing the platforms for the alternative that provides higher individual welfare. The winner is the candidate who gets more votes or, if there is a tie, the candidate chosen by the toss of a fair coin.

Assuming some homogeneity in the electorate, so that there is an alternative in the space (called a Condorcet winner) that beats all other alternatives by pair-wise majority voting, the existence and the location of the equilibrium policies in the election game described above depends on candidates' motivations for choosing their platforms. Roughly speaking, these motivations can be winning the election and being in power, the location of the winning policy, or a mixture of both.

As is known from the earliest contributions of Hotelling (1929) and Downs (1957), when each candidate chooses its platform with the purpose of maximizing the probability of winning the contest, the election game has a unique Nash equilibrium, where both candidates announce the same platform. The equilibrium policy coincides with the median of the distribution of voters' most preferred policies. This result, with platforms converging to the "center of the space," also takes place if candidates are uncertain about voters' preferences, provided that they share a common prior about the location of the median ideal point.¹ In that case, however, platforms converge to the *estimated* median voter's ideal point (Calvert 1985).

Alternatively, when each candidate has a preferred location in the policy space, called its *ideology*, and chooses its platform according with Wittman's (1983) hypothesis, i.e., to minimize the expected loss arising from the distance between its preferred policy and the winning platform, several studies have found that the convergence of the equilibrium platforms to the median ideal policy is achieved only if there is no uncertainty about the electorate's preferences (cf. Calvert 1985, Roemer 1994, and Duggan and Fey 2005). By contrast, the game with electoral uncertainty and ideological motivations possesses a pure strategy equilibrium with differentiated platforms (Roemer 1997).²

Surprisingly, until very recently the analysis of electoral competition when candidates are interested not only in holding power, but also in policy outcomes, has not received much attention in the literature. This assumption, called sometimes *mixed* or *hybrid*

¹See Bernhardt, Duggan, and Squintani (2007, 2009a) for a model with private polling, where candidates receive private signals about voters' preferences before choosing their platforms.

²Morton (1993) have shown with experimental data that uncertainty over voters' preferences is a major determinant of platform divergence when candidates are ideological.

motivations, was first suggested by Calvert (1985), and it has been used in several studies, including Ball (1999), Groseclose (2001), Aragonés and Palfrey (2005), Duggan and Fey (2005), Saporiti (2008), Callander (2008), and Bernhardt, Duggan, and Squintani (2009b). To the best of my knowledge, however, the full characterization of Nash equilibria in the hybrid case is still an open problem in the literature.

Unquestionably, the mixed motivation assumption is more realistic than the extreme hypotheses previously considered.³ For example, a simultaneous interest in power and ideology might arise if candidates are professional politicians, as happens in modern democracies. Since it is natural to assume that politicians may be namely interested in their career and, therefore, in winning the election, whereas regular party members may focus more on policy outcomes, it seems reasonable to expect that these two objectives will enter into the candidate's payoff function with some weight.

Of course, these weights need not be the same for all candidates. They could depend, for instance, on the specific features of the political organization that the candidate represents, such as the number of regular members, the level of activism and proselytism of them, the internal process to nominate electoral candidates, etc. The intrinsic value of winning the election may also vary depending on whether the party of the candidate is the incumbent in office or a challenger in the opposition. In any case, the point to underline is that a model of electoral competition with mixed motivations and, in particular, with *asymmetric* interests among political candidates constitutes a perfectly reasonable situation, though it did not get much attention until recently.⁴

To the best of my knowledge, Ball (1999) was the first to analyze the implications of the mixed motivations for the existence of Nash equilibria in electoral competition. He showed that, due to the discontinuities of the payoff functions created by the mixed motivations, the electoral contest with hybrid motives and uncertainty about the median voter's ideal point does not always possess a Nash equilibrium in pure strategies. This stands in sharp contrast with what happens in the extreme cases studied by Wittman (1983) and Calvert (1985), where a pure strategy equilibrium always exists. Therefore, it shows that the mixed motivation assumption is not a vacuous assumption, as one could be possibly tempted to think from Calvert's continuity analysis.⁵ Apart from offering a more realistic description of the electoral objectives of the political parties, the hybrid

³In one of the few experimental studies that deals with this matter, Morton (1993) points out that subjects in the laboratory placed a weight of approximately 32 per cent on winning the election, and 68 per cent on the expected utility derived from the implemented platform.

⁴The most notable example of political parties with asymmetric motivations that I am aware of is the Radical Party and the Peronist Party in Argentina. These two parties are the main political actors of the country. The Radicalism is a strongly ideological party, whereas the Peronism is a "movement," as it has been defined by Perón, basically motivated by being in power.

⁵Calvert (1985) showed that *small* departures from the classical assumptions of electoral competition, namely, office motivation and certainty, lead to only *small* departures from convergence.

case also provides a deeper understanding of electoral competition, uncovering features of the problem that cannot be appreciated in the extreme scenarios.

Taking the problem of equilibrium existence pointed out by Ball (1999) as the starting point, a recent paper by Saporiti (2008) makes a step forward by clearing up the root of that problem. Saporiti argues that, in contrast with the usual causes behind the nonexistence of equilibria in the traditional models of electoral competition, essentially, the multi-dimensionality of the policy space and the heterogeneity of voters' preferences, the lack of pure strategy equilibria in one-dimensional contests with hybrid motives and electoral uncertainty is related with the heterogeneity or asymmetry of interests of the political candidates. To be precise, Saporiti proves the existence and uniqueness of a pure strategy equilibrium when candidates possess mixed but identical (*symmetric*) motivations, complementing in that way the results of Calvert (1985) and Roemer (1997).

An existence and uniqueness result for a similar family of election games has been provided later by Bernhardt et al. (2009b). The main difference between this paper and Saporiti (2008) relies on the source of the electoral uncertainty. While Bernhardt et al. (2009b) analyzes the case where voters' preferences are subject to an exogenous and common shock realized after the candidates have announced their platforms, Saporiti (2008) follows Roemer's (2001) "error term" model of uncertainty, by assuming that the preferences of the electorate are given, but candidates are unable to perfectly foresee them (due, for example, to the margin of error of the opinion polls, the inability to exactly predict voter participation rates of populations with different attributes, etc.).

Apart from using a different model of electoral uncertainty, Bernhardt et al. (2009b) deals exclusively with the symmetric case, focusing on the implications of electoral competition with mixed motivations on voters' welfare. On the contrary, Saporiti (2008) also examines an example with asymmetric motives in which a pure strategy equilibrium fails to exist. This example is used to motivate the analysis of equilibrium existence in mixed strategies, which relies on Reny (1999). To elaborate, Saporiti finds that, regardless of candidates' aspirations, the mixed extension of the hybrid election game satisfies the property of *better-reply security* and, consequently, that a Nash equilibrium always exists.⁶ This result is alternative (and, to some extent, complementary) to the proof of equilibrium existence given by Ball (1999), which is based on Dasgupta and Maskin's (1986) approach and, therefore, on the upper semi-continuity of the sum of the payoffs.⁷

Although the existence results pointed out above may be interesting in their own right,

⁶Following Reny (1999), we say that a game is *better-reply secure* if for every nonequilibrium profile \mathbf{y} , and every payoff vector limit \mathbf{Z} that results from a sequence of strategies approaching \mathbf{y} , some player i has a strategy yielding a payoff above Z_i even if the other players deviate slightly from \mathbf{y} .

⁷One advantage of Reny's approach is the quasi ordinality of the property of better-reply security (Reny 1999, p. 1034), which allows us to apply the existence results to linear transformations of the payoff functions.

the study of electoral competition matters mainly for the predictions it can provide about the determinants and the location of the equilibrium platforms. The central purpose of the current paper is precisely to shed some light on this matter. The results coming out from the analysis can be summarized as follows.

In addition to the median ideal policy, the determinants of the equilibrium platforms of the hybrid election game are three: (1) the electoral uncertainty, (2) the aggregate interest in office, and (3) its distribution between the candidates.⁸ To be more precise, when the intrinsic value of being in power is the *same* for the two candidates, both announce a platform located either over (i) the estimated median ideal point (policy convergence) if the electoral uncertainty is low compared with the aggregate interest in office, or over (ii) their own ideological side (two-sided policy differentiation) if the uncertainty is high.⁹

Alternatively, when candidates have *asymmetric* motivations, platforms still converge to the estimated median voter's ideal point for low levels of uncertainty. On the contrary, when the length of the interval over which the median is distributed increases, and so does the uncertainty, an equilibrium in pure strategies fails to exist, and candidates randomize on one side of the median. As the electoral uncertainty continues raising, a pure strategy equilibrium is eventually reestablished and each candidate assigns all of the probability mass to a different platform, which are located initially over the same ideological side (one-sided policy differentiation), and then over each candidate's political ground.

The equilibria described above are derived assuming that the electoral uncertainty follows the "error term" model proposed by Roemer (2001, pp. 45-46). However, when the uncertainty does not obey Roemer's model, but it arises from a common and exogenous shock on voters' ideal policies realized after platforms have been announced, which is the case in Bernhardt et al. (2009b), we show that the probability of winning the election coincides with the function obtained in the former case; and so the equilibrium characterization remains the same.

Although the location of the equilibrium platforms does not seem to be affected in this work by the model of electoral uncertainty adopted, the effect of candidates' motivations over voters' welfare does depend on it. We find that when the electorate is risk averse and the uncertainty behaves according with Roemer's model, at least a majority of the electorate is better off with policy convergence (i.e., with relatively more office-motivated candidates) than with policy differentiation (i.e., with relatively more policy-oriented candidates). That majority includes all voters when candidates' motivations are symmetric.

This result stands in contradiction with Bernhardt et al. (2009b), which find that

⁸Notice that candidates' ideologies do not directly affect the equilibrium platforms. As we explain in the text, this is due to the assumption of risk neutrality adopted in the model.

⁹Along this paper, candidates' preferred policies are assumed to be distributed on either side of the median ideal point, so that the ideology of one candidate lies down on the left and the other on the right.

when the electoral uncertainty is created by an exogenous shock, the differentiation of platforms arising in the symmetric motivation case has, within a certain range of values of the parameters of the model, a completely opposite effect on welfare; that is, all voters are better off with (two-sided) policy differentiation than with convergence. The asymmetric motivation case in this second scenario of uncertainty has not been analyzed yet.

To conclude, it may be worth emphasizing that the results coming out from this paper together with those coming out from Bernhardt et al. (2009b) show that the effect of policy differentiation (ideological candidates) over social welfare depends on the model of electoral uncertainty that is behind the probability of winning function. In particular, the welfare-improving effect found by Bernhardt et al. (2009) is fully reversed under Roemer's (2001) error-distribution model, where candidates possess imperfect estimates of their vote shares, but voters' preferences are not stochastic. Therefore, any advocate for policy differentiation based on the greater choice opportunities it offers to voters, which is sometimes referred to as "a call for responsible parties," must be taken with caution, since even under uncertainty that kind of platform configuration does not necessarily lead to a greater welfare of the majority of the electorate.

2 The model

Two candidates, indexed by $i = L, R$, compete in a winner-take-all electoral contest by simultaneously and independently announcing a platform $x_i \in X = [0, 1]$. The electorate is made of a continuum of voters. Every voter is endowed with a preferred policy or *ideal point* θ , which is randomly drawn from the uniform distribution U over $[0, 1]$. An individual of type θ has preferences over X represented by the utility function $u_\theta(x) = -w(|x - \theta|)$, where $|\cdot|$ denotes the absolute value on \mathbb{R} , and w is a twice differentiable real-valued function, with $w'(0) = 0$, and $w'(z) > 0$ and $w''(z) \geq 0$ for all $z > 0$.

Given any pair of proposals $(x_L, x_R) \in X^2$, each voter votes sincerely for the platform it likes the most. That is, each voter votes for the alternative closer to its ideal point. The assumption of sincere voting is not restrictive in this model because there are only two candidates, and each candidate enacts its proposed policy once elected. As is usual in the literature, candidate i wins the election if its platform x_i gets more than half of the ballots. If there is a tie, then the 'equal sharing' rule is used to break it and to determine the winner, so that in that event each candidate wins with probability $1/2$.

Apart from the uncertainty due to the possibility of a tie, candidates have uncertainty about voters' preferences. Following Roemer's (2001, pp. 45-46) error-distribution model, it is assumed that candidates perceive the fraction of types supporting their respective platforms with a noise ξ , which is uniformly distributed over the interval $[-\beta, \beta]$, with

$\beta > 0$.¹⁰ To be more precise, candidates believe that the fraction of types voting for x_L is given by $U[S(x_L, x_R)] + \xi$, where $S(x_L, x_R) = \{\theta \in [0, 1] : u_\theta(x_L) > u_\theta(x_R)\}$ is the set of types that prefer x_L to x_R .¹¹ This noise in candidates' estimates of their vote shares at the time they announce their policies is used to reflect that voters' preferences and their participation rate are hard to predict. Notice, however, that ξ has no effect over the ideal policies of the voters. It affects only the accuracy with which candidates perceive them.

The uncertainty model adopted above implies that, for any strategy profile $(x_L, x_R) \in X^2$, the probability that candidate L attaches to winning the election is given by $p(x_L, x_R) = \text{Prob}\{U[S(x_L, x_R)] + \xi > 1/2\}$. Obviously, candidate R 's probability of winning is $1 - p(x_L, x_R)$. If $x_L = x_R$, then the tie-breaking rule implies that $U[S(x_L, x_R)] = 1/2$, i.e., the electorate splits equally between the two candidates, so that $p(x_L, x_R) = 1/2$. On the contrary, if $x_L \neq x_R$, then: (i) $p(x_L, x_R) = \frac{1}{2} + \frac{U[S(x_L, x_R)] - 1/2}{2\beta}$ if $1/2 - U[S(x_L, x_R)] \in (-\beta, \beta)$; whereas (ii) $p(x_L, x_R) = 1$ (resp., $p(x_L, x_R) = 0$) if $1/2 - U[S(x_L, x_R)] \leq -\beta$ (resp., $1/2 - U[S(x_L, x_R)] \geq \beta$).

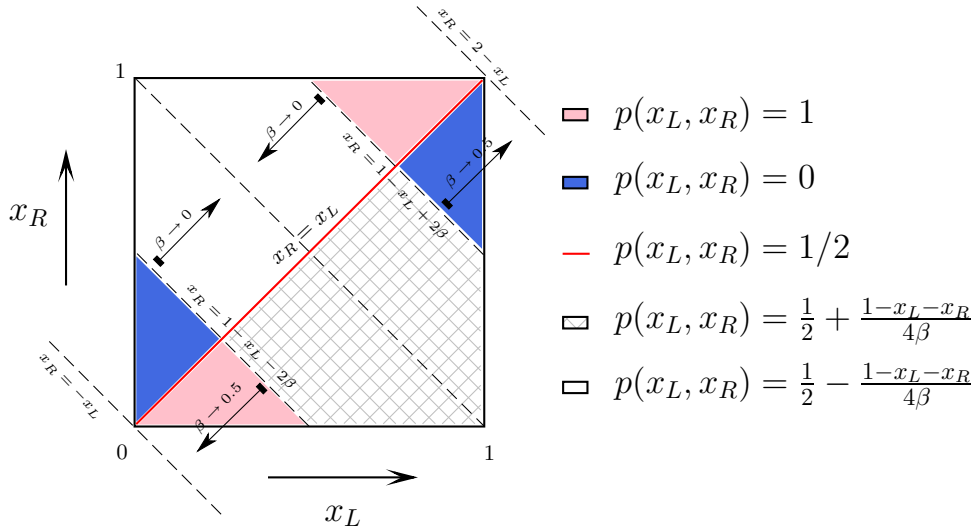


Figure 1: Probability of winning function

Lemma 1 For any two platforms $x_L < x_R$ (resp., $x_L > x_R$), $p(x_L, x_R)$ is non-decreasing (resp., non-increasing) in x_i , for all $i = L, R$.

Lemma 1, whose proof follows immediately from the definition of $p(\cdot)$ and is therefore omitted, reflects the spatial nature of electoral competition. Roughly speaking, it ensures

¹⁰See Section 5 for an alternative model of electoral uncertainty (with stochastic preferences), which leads to the same probability of winning function described in Fig. 1.

¹¹Note that, since $w'(z) > 0 \forall z > 0$, for each $x_L \neq x_R$, $u_\theta(x_L) = u_\theta(x_R)$ if and only if $|x_L - \theta| = |x_R - \theta|$. Hence, $|\{\theta \in [0, 1] : u_\theta(x_L) = u_\theta(x_R)\}| = 1$, and $U(\{\theta \in [0, 1] : u_\theta(x_L) = u_\theta(x_R)\}) = 0$.

that if one candidate moves its platform toward that of its opponent, then it does not decrease (and may increase) the probability with which it wins the election. Likewise, if it moves its platform away from its opponent's, then it does not increase (and may decrease) its probability of winning. We invoke this result several times in the paper.

As we said in the Introduction, candidates possess *mixed* or *hybrid* motives for running for office. That means that they are *office-motivated*, because they intrinsically value winning the election; but, at the same time, they are *policy-motivated* too, because they care about what policy is enacted after the election. Formally, the payoffs for candidate L and candidate R associated to any pair $(x_L, x_R) \in X^2$ are given by, respectively, $\pi_L(x_L, x_R) = p(x_L, x_R) \cdot [\psi_{\theta_L}(x_L, x_R) + \chi_L]$ and $\pi_R(x_L, x_R) = (1 - p(x_L, x_R)) \cdot [\psi_{\theta_R}(x_R, x_L) + \chi_R]$, where $\chi_i > 0$ denotes candidate i 's intrinsic value (rents) for being in power, θ_i stands for i 's ideological position on X , and for any $(x, y) \in X^2$, $\psi_{\theta}(x, y) = u_{\theta}(x) - u_{\theta}(y)$.¹² Notice that Hotelling (1929)-Downs' (1957) office motivation hypothesis, according to which candidates maximize the probability of winning the election, is obtained from the previous specification of the payoffs by letting χ_i be arbitrarily large for all i . Likewise, Wittman's (1983) entirely ideological candidates follow from the same particular by setting out the rents χ_i of both candidates equal to zero.

In the sequel, it is assumed that candidates' ideological positions are distributed on either side of the median voter's ideal policy (i.e., $\theta_L < 1/2 < \theta_R$). In addition, to rule out uninteresting equilibria with large electoral uncertainty and no trade-off between power and ideology, the essence of this investigation, we suppose that $\beta < \bar{\beta} \equiv \min\{1/2 - \theta_L + \chi_L/2, \theta_R - 1/2 + \chi_R/2\}$. If that were not the case, then in an equilibrium with differentiated policies at least one candidate would maximize its payoff at its preferred location θ_i , independently of the position chosen by the other. Finally, for the sake of simplicity, except where otherwise noted candidates are assumed to be risk neutral (with respect to the distance $|x - \theta_i|$). This provides closed form solutions in pure strategies, and it considerably facilitates the analysis.

It is worth mentioning, however, that risk neutrality entails a loss of generality because, in spite of being ideologically different, risk averse candidates tend to move closer to each other and toward to the center. Indeed, given the position of one candidate, the other candidate chooses a less differentiated platform when it is risk averse than when it is risk neutral because it must compensate a higher utility loss due to the risk aversion with a rise in the probability of winning the contest (see the discussion and the example following Lemma 3). We fail to spot this effect under risk neutrality. Yet the

¹²Strictly speaking, candidate L 's payoff function is given by the convex combination $\Pi_L(x_L, x_R) = p(x_L, x_R) \cdot (u_{\theta_L}(x_L) + \chi_L) + [1 - p(x_L, x_R)] \cdot u_{\theta_L}(x_R)$. However, since the independent term $u_{\theta_L}(x_R)$ does not affect L 's optimal choices, we work with the linear transformation $\pi_L(x_L, x_R) \equiv \Pi_L(x_L, x_R) - u_{\theta_L}(x_R)$. The same comment applies to candidate R 's payoff function.

model considered in here is rich enough to pick up several interesting features of electoral competition that have been overlooked by the literature.

As usual, a pure strategy equilibrium (PSE) for the *hybrid election game* $\mathcal{G} = (X, \pi_i)_{i=L,R}$ described before is a strategy profile $(x_L^*, x_R^*) \in X^2$ such that for all $(x_L, x_R) \in X^2$, $\pi_L(x_L^*, x_R^*) \geq \pi_L(x_L, x_R^*)$ and $\pi_R(x_L^*, x_R^*) \geq \pi_R(x_L^*, x_R)$.

3 Equilibrium

We begin the equilibrium analysis noting that \mathcal{G} possesses neither (i) a PSE where the left-wing candidate chooses a platform further to the right than the right-wing candidate's proposal, nor (ii) a PSE where one of the candidates wins the election for sure.

Lemma 2 *If the strategy profile $(x_L^*, x_R^*) \in X^2$ is a pure strategy equilibrium for the election game $\mathcal{G} = (X, \pi_i)_{i=L,R}$, then $\theta_L < x_L^* \leq x_R^* < \theta_R$ and $p(x_L^*, x_R^*) \in (0, 1)$.*

The previous lemma, whose proof (as well as all other proofs of this section) is given in the Appendix, allows us to focus the equilibrium analysis on the white and the red regions of Fig. 1. In particular, it is used below to characterize each candidate's platform in a PSE with policy differentiation, and to provide a necessary condition for such equilibrium to exist.

Lemma 3 *The election game $\mathcal{G} = (X, \pi_i)_{i=L,R}$ has a pure strategy equilibrium with $x_L^* < x_R^*$ only if $\chi_L + \chi_R < 4\beta$, $x_L^* = 1/2 - \beta + \chi_L/2$, and $x_R^* = 1/2 + \beta - \chi_R/2$.*

The platforms characterized in Lemma 3 are a function of the electoral uncertainty β and the office rents χ_i , and with the expected sign. All the rest equal, as candidates become less certain about how moderate is the median voter (higher β), they also become more polarized. On the contrary, a reduction of the uncertainty (resp., an increase in the office rents) moves both platforms toward to the center of the political space.

These platforms however are independent of candidates' ideologies. Moreover, under the conditions of Lemma 3, they are independent of each other too, in the sense that a change in candidate i 's equilibrium policy x_i^* (due, for example, to a change in χ_i) does not affect x_j^* . These are mainly consequences of risk neutrality. If candidates are risk averse, equilibrium platforms are interdependent and sensible (directly or indirectly) to the ideology of each candidate. Here is one example.

Example Assume first risk neutrality, i.e., $u_{\theta_i}(x) = -|x - \theta_i|$. Let $\beta = 0.25$, $\theta_L = 0.2$, $\theta_R = 0.8$, and $\chi_L = \chi_R = 0.1$. It is immediate from Lemma 3 that $x_L^* = 0.3$ and $x_R^* = 0.7$.

Next, suppose w is a quadratic function, so that $u_{\theta_i}(x) = -(|x - \theta_i|)^2$. Solving the first-order conditions, it can be shown that the left-wing and right-wing equilibrium platforms move to the center, to a position of 0.455 and 0.545, respectively, reflecting that each candidate's desire for policy differentiation diminishes under risk aversion. Let's finally move upwards the right-wing candidate's ideology, from 0.8 to 0.9. As we would expect, the equilibrium platforms remain the same under risk neutrality; i.e. $x_L^* = 0.3$ and $x_R^* = 0.7$. On the contrary, they shift under risk aversion (with $u_{\theta_i}(x) = -(|x - \theta_i|)^2$) to $x_L^* \approx 0.46$ and $x_R^* \approx 0.58$. Notice that, with the second utility function, in going from $\theta_R = 0.8$ to $\theta_R = 0.9$ candidate R moves its platform to the right and away from $1/2$, whereas L reacts to that change by shifting its own platform toward to the center.

The equilibrium platforms characterized above are obtained from the first-order conditions; that is, they are stationary points of the conditional payoffs. To ensure that they belong to the best response correspondence, these functions must exhibit some form of concavity. Unfortunately, situations where the conditions of Lemma 3 hold but a PSE does not exist are easy to find. A case in point takes place when $\chi_R = 0.2$, $\chi_L = 0.6$, and the rest of the parameters are $\beta = 0.25$, $\theta_L = 0.2$, and $\theta_R = 0.9$. For this case, the conditional payoffs associated with the profile of platforms of Lemma 3, namely, $(x_L^*, x_R^*) = (0.55, 0.65)$, are illustrated in Figs. 2 and 3. Clearly, this profile cannot be a PSE, since $x_R^* = 0.65$ does not maximize $\pi_R(0.55, x_R)$ over $x_R \in [0, 1]$. A bit of extra work confirms that for these values of the parameters, any other profile of pure strategies fails to be an equilibrium too.¹³

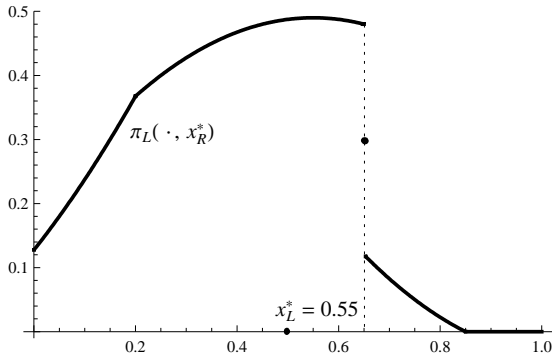


Figure 2: Left-wing candidate's conditional payoff given $x_R^* = 0.65$.

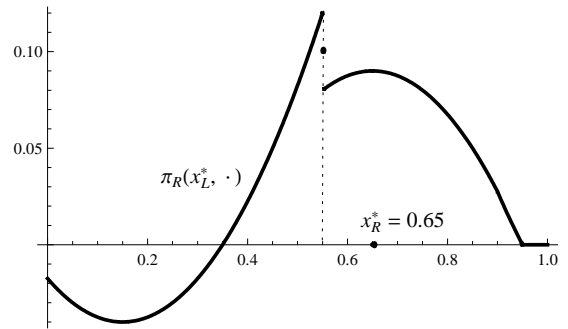


Figure 3: Right-wing candidate's conditional payoff given $x_L^* = 0.55$.

¹³First, by Lemma 2, if a PSE is going to exist, it must be that $x_L \leq x_R$. As we said, the example doesn't have a PSE with $x_L < x_R$ because Lemma 3 requires $x_L = 0.55$ and $x_R = 0.65$, but 0.65 is not R 's best reply to 0.55 (see Fig. 3). By Proposition 1, the only remaining possibility is the profile $(1/2, 1/2)$. As is shown in the proof to Proposition 1, a deviation $x'_i = 1/2 \pm \delta$ from $x_i = 1/2$ is profitable if $\delta < 2\beta - \chi_i$. Hence, $(1/2, 1/2)$ is not a PSE either, because $\pi_R(1/2, x'_R) > \pi_R(1/2, 1/2)$ for any $\delta < 0.3$.

Therefore, a sensible question to ask is what conditions prevent this from happening. The next three propositions are meant to shed some light into this inquiry. We begin by offering necessary and sufficient conditions for the classical result of electoral competition, that is, policy convergence.

Proposition 1 (convergence) *The election game $\mathcal{G} = (X, \pi_i)_{i=L,R}$ has a pure strategy equilibrium with $x_L^* = x_R^* \equiv x^*$ if and only if $x^* = 1/2$ and $\chi_i \geq 2\beta$ for all $i = L, R$.*

The statement of the last proposition bears some similarity with Calvert’s (1985) assertion that *small* departures from “office motivation and certainty” lead to only *small* departures from convergence. In line with that result, Proposition 1 asserts that both candidates will choose in equilibrium a platform located over the expected median ideal point if and only if the relative value of holding office $\chi_i/2\beta$ is high enough for all i .¹⁴

An immediate consequence of Proposition 1 and Lemma 3 is as follows.

Corollary 1 (uniqueness) *If the election game $\mathcal{G} = (X, \pi_i)_{i=L,R}$ possesses a pure strategy equilibrium, then the equilibrium is unique.*

The uniqueness result expressed in Corollary 1 is to some extent more general than the related results found in Saporiti (2008) and Bernhardt et al. (2009b), because the latter only cover the homogeneous motivation case. It is worth reminding, however, that the three models are different and, therefore, that the results are not directly comparable.

Our second proposition provides a necessary and sufficient condition for another famous configuration of equilibrium platforms, due to Wittman (1983) and Roemer (1997), where each candidate chooses a policy on its own ideological side.

Proposition 2 (two-sided differentiation) *The election game $\mathcal{G} = (X, \pi_i)_{i=L,R}$ has a pure strategy equilibrium with $x_L^* < 1/2 < x_R^*$ if and only if $\chi_i < 2\beta$ for all $i = L, R$.*

Thus, combining Propositions 1 and 2, the first conclusion that can be drawn here is that, when candidates possess identical motivations, these two results offer a full description of the equilibrium outcomes of the hybrid election game. To illustrate, Fig. 4 displays the equilibrium platforms as a function of the electoral uncertainty β , and for a particular level of office rents $\chi \equiv \chi_L = \chi_R$. As Proposition 1 points out, both policies are located at the estimated median voter’s ideal point for any level of uncertainty lower than or equal to $\chi/2$. Above that threshold, Lemma 3 and Proposition 2 indicates that

¹⁴In this paper, the winner candidate enjoys an absolute payoff for being elected equal to χ_i . From the candidates’ viewpoint, however, hitting the median ideal point with a particular policy platform and actually winning the election has a chance of $(2\beta)^{-1}$ (the inverse of the length of the support of the error term ξ). Therefore, the term $\chi_i/2\beta$ can be interpreted as a kind of adjusted or relative value of office.

the equilibrium platforms lie down on each candidate's ideological ground, in accordance with the expressions $x_L^* = 1/2 - \beta + \chi_L/2$ and $x_R^* = 1/2 + \beta - \chi_R/2$. That gives rise to a region of two-sided policy differentiation as is shown in the graph.

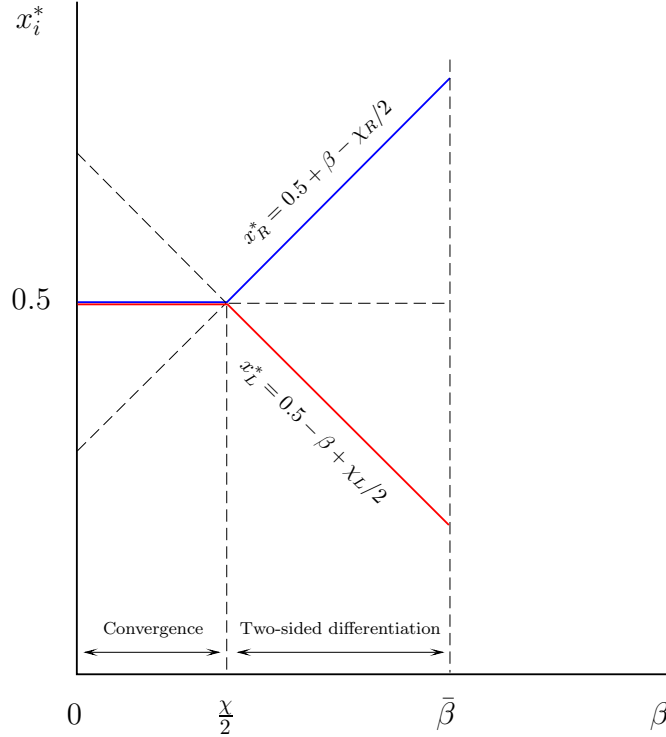


Figure 4: Symmetric case: $\chi_L = \chi_R \equiv \chi$.

Interestingly, when candidates hold asymmetric interests, Propositions 1 and 2 do not cover the whole spectrum of possibilities. The main contribution of this paper is precisely to analyze what may happen in that case. To help the reader gain more insight about the equilibrium configurations that might arise in the asymmetric scenario, assume that $\chi_L = 0.6$ and $\chi_R = 0.05$, and suppose that $\beta = 0.25$, $\theta_L = 0.2$, and $\theta_R = 0.9$. For these values of the parameters, Lemma 3 says that $x_L^* = 0.55$ and $x_R^* = 0.725$. Figs. 5 and 6 confirm that these policies form in fact a pure strategy equilibrium.

Fig. 5 displays the left-wing candidate's conditional payoff given $x_R^* = 0.725$. Likewise, Fig. 6 exhibits the right-wing candidate's payoff given that the other candidate's chosen policy is $x_L^* = 0.55$. A simple inspection of the graphs shows that these platforms are best responses to each other, proving that $(x_L^*, x_R^*) = (0.55, 0.725)$ is a PSE. Interestingly, in this equilibrium candidates locate not only on a different platform, but also over the same side of the median voter's ideal point (i.e., $1/2 < x_L^* < x_R^*$). In particular, being the most opportunistic of the two, candidate L 's proposal lies down on the other candidate's ideological ground. We refer to this kind of equilibria as pure strategy equi-

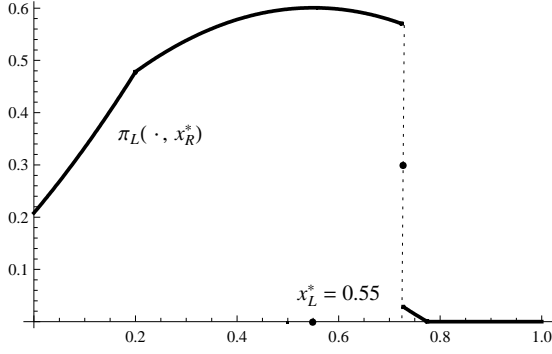


Figure 5: Left-wing candidate's conditional payoff given $x_R^* = 0.725$.

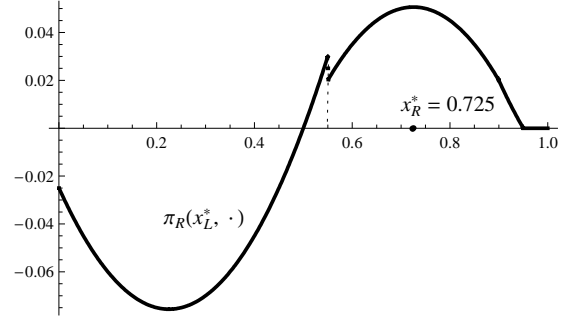


Figure 6: Right-wing candidate's conditional payoff given $x_L^* = 0.55$.

libria with *one-sided policy differentiation*. The next proposition provides a necessary and sufficient condition for that equilibrium to occur.

Proposition 3 (one-sided differentiation) *The election game $\mathcal{G} = (X, \pi_i)_{i=L,R}$ has a pure strategy equilibrium with $1/2 < x_L^* < x_R^*$ (resp., $x_L^* < x_R^* < 1/2$) if and only if $(\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2} \leq 2\beta < \chi_L$ (resp., $(\chi_R - \chi_L)/2 + (\chi_R \cdot \chi_L)^{1/2} \leq 2\beta < \chi_R$).*

As a matter of illustration, let us point out that evidence of one-sided policy differentiation over the center-right of the political spectrum appears to be found in the British election of the year 1997. That election was very much a two-party contest, with the Conservative (C) and the Labour (L) party getting together a massive 88.7 per cent of the 659 parliamentary seats contested (data source: www.historylearningsite.co.uk). Moreover, using a survey conducted among 117 political scientists from British universities, Laver (1998) showed that all of the policy dimensions with high salience in the 1997's election were highly correlated with the economic dimension. This implies, in his own words, that “the British policy space [was indeed] very one-dimensional [in 1997].”

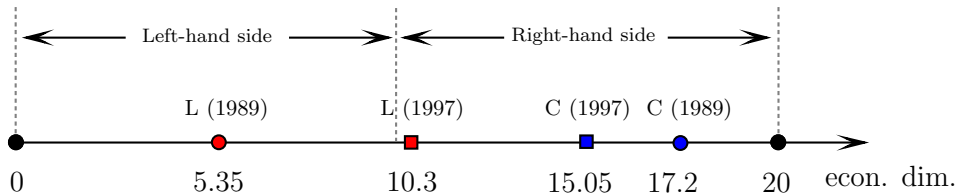


Figure 7: Parties' policy positions in 1989 and 1997.

The expert survey also indicates that the position of Labour on the economic policy dimension moved very sharply to the right in 1997, from a position of 5.35 on a 1 to

20 scale in 1989 to a position of 10.3. In the same period, the Conservatives made a statistically significant shift to the left, from 17.2 to 15.05 (see Figure 7). Applying alternative techniques to estimate party policy positions, namely hand-coded, computer-coded, and word-scoring content analysis of political texts, Laver and Garry (2000) and Laver et al. (2003) arrived to similar conclusions.

Before leaving the equilibrium analysis two final comments are in order. The first one is concerned with the equilibrium value of the probability of winning the election. Recall that when candidates have the same intrinsic value of holding office, say χ , the equilibrium policies are either, symmetrically located around the estimated median if $\chi < 2\beta$, or they are equal to $1/2$ otherwise. Therefore, under the assumption of symmetric motives, both candidates have the same probability of winning the election (i.e., $p(x_L^*, x_R^*) = 0.5$). Obviously, this is also the case under the conditions of Propositions 1, i.e., if $\chi_i \geq 2\beta$ for all i , since in that situation platforms converge to $1/2$.

By contrast, under the conditions of Proposition 3, one-sided policy differentiation implies that the relatively more ideological candidate faces a lower probability of winning the contest. The same happens under the conditions of Proposition 2 if platforms are asymmetrically located on either side of the median ideal point. Hence, we could say that in this model having a relatively greater interest in the location of the implemented platform constitutes a *disadvantage* to win the election.

Finally, as we did in Fig. 4 for the symmetric case, Figure 8 displays the equilibrium platforms when candidates have *asymmetric* motivations, let's say $\chi_L > \chi_R$. As the graph shows, the platforms as a function of the electoral uncertainty still converge to the estimated median for any $\beta \leq \chi_R/2$. Instead, when $\chi_R/2 < \beta < \beta^C$, with $\beta^C \equiv \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_L \chi_R}}{2}$, the necessary conditions for the existence of an equilibrium in pure strategies stated in Lemma 3 and Propositions 1 and 3 are all violated. As a result, a PSE does not longer exist for this range of values of β , and candidates randomize over the policy space until the electoral uncertainty achieves the critical value β^C . Beyond that threshold, each candidate assigns all of the probability mass to a different platform, which are located initially (for $\beta < \chi_L/2$) over the same ideological side. As a matter of comparison, notice that when $\chi_L = \chi_R$, all of the critical values of β indicated in Fig. 8 coincide, i.e. $\beta^C = \chi_R/2 = \chi_L/2$. That explains why Fig. 4 exhibits neither a region with mixed strategy equilibria, nor one with one-sided policy differentiation.

4 Welfare

In this section, we use the equilibria characterized above to provide some insights about the welfare properties of policy differentiation (convergence). To do that, for any profile

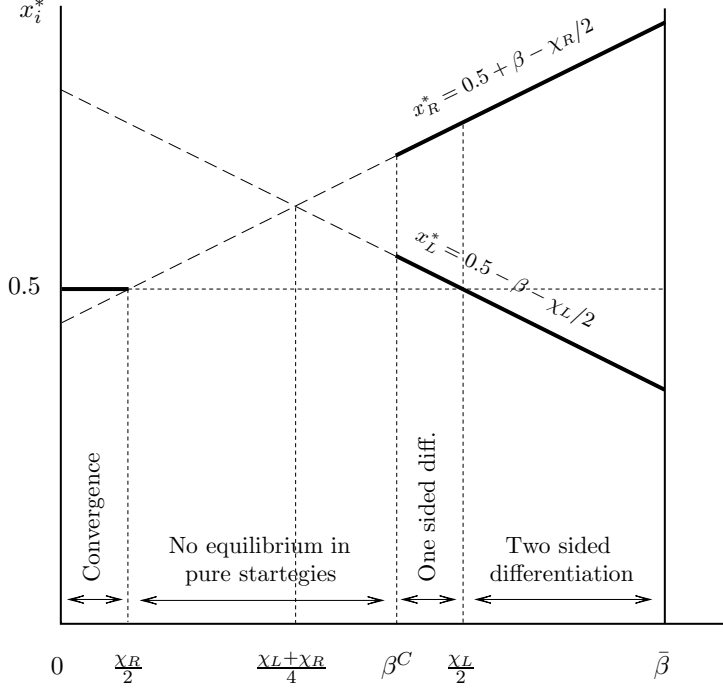


Figure 8: Asymmetric case: $\chi_L > \chi_R$.

$(x_L, x_R) \in X^2$, we define by $V_\theta(x_L, x_R) = p(x_L, x_R) \cdot u_\theta(x_L) + [1 - p(x_L, x_R)] \cdot u_\theta(x_R)$ the expected welfare of a voter of type $\theta \in [0, 1]$. Implicit in this definition is the assumption that voters have the same beliefs that candidates about their chances of winning the contest. That would be the case, for example, if any relevant information to determine the outcome of the election is in the public domain, as it is the assumption in this work.

Proposition 4 (social welfare) *Suppose voters are risk averse ($w'' > 0$) and $\chi_L + \chi_R < 4\beta$. Let $x_L^* = 1/2 - \beta + \chi_L/2$ and $x_R^* = 1/2 + \beta - \chi_R/2$ be equilibrium policies. Then: (i) If $\chi_L = \chi_R$, for all $\theta \in [0, 1]$, $V_\theta(x_L^*, x_R^*) < V_\theta(1/2, 1/2)$; and (ii) If $\chi_L > \chi_R$ ($\chi_L < \chi_R$), there exists $\hat{\theta} > 1/2$ ($\hat{\theta} < 1/2$) such that for all $\theta \leq \hat{\theta}$ ($\theta \geq \hat{\theta}$), $V_\theta(x_L^*, x_R^*) < V_\theta(1/2, 1/2)$.*

Proof Assume $(x_L^*, x_R^*) = (1/2 - \beta + \chi_L/2, 1/2 + \beta - \chi_R/2)$ is a PSE for \mathcal{G} . By hypothesis, $\chi_L + \chi_R < 4\beta$. Hence, $x_L^* < x_R^*$ and $p(x_L^*, x_R^*) = 1/2 + (\chi_L - \chi_R)/8\beta$, with $|(\chi_L - \chi_R)/8\beta| < 1/2$. Let $x_\lambda^* = p(x_L^*, x_R^*) \cdot x_L^* + (1 - p(x_L^*, x_R^*)) \cdot x_R^*$ be the convex combination of x_L^* and x_R^* , where the weights are given by each candidate's probability of winning the election. Using simple algebra, it is easy to show that

$$x_\lambda^* = \frac{1}{2} + \frac{\chi_L^2 - \chi_R^2}{16\beta}. \quad (1)$$

By risk aversion, for any voter of type $\theta \in [0, 1]$

$$V_\theta(x_L^*, x_R^*) < u_\theta(x_\lambda^*). \quad (2)$$

(i) If $\chi_L = \chi_R$, then by (1), $u_\theta(x_\lambda^*) = u_\theta(1/2)$; and combining the latter with (2) and the fact that $u_\theta(1/2) = V_\theta(1/2, 1/2)$, we get the desired result, namely, $V_\theta(x_L^*, x_R^*) < V_\theta(1/2, 1/2)$ for all $\theta \in [0, 1]$. Alternatively, (ii) if $\chi_L > \chi_R$, then $x_\lambda^* > 1/2$. Let $\hat{\theta} = (x_\lambda^* + 1/2)/2$. For all $\theta \leq \hat{\theta}$, $|x_\lambda^* - \theta| \geq |1/2 - \theta|$, which in turn implies that $u_\theta(x_\lambda^*) \leq u_\theta(1/2)$. Therefore, by (2), it follows that $V_\theta(x_L^*, x_R^*) < u_\theta(1/2) = V_\theta(1/2, 1/2)$. The remaining case where $\chi_R > \chi_L$ is proved in a similar fashion. ■

In contrast with Bernhardt et al. (2009b), Proposition 4 shows that, when candidates possess identical motivations and the source of the electoral uncertainty is the lack of perfectly accurate estimates of voters' preferences, rather than an exogenous and common shock over their ideal policies, a risk averse electorate is strictly better off with policy convergence than with policy differentiation. Moreover, if candidates' electoral goals are different, there still exists a strict majority that prefer convergent platforms. However, given that in that case platforms are asymmetrically distributed over the political space, some voters may be actually hurt by policy convergence.

5 Stochastic preferences

To help the reader gain more insight about the stochastic preference case, consider the following alternative to the error-distribution model of electoral uncertainty. Suppose the distribution of ideal policies within the electorate is known up to a shift parameter ξ , which is a common shock to the environment (e.g., a terrorist attack, a financial crisis, etc.). To be more precise, let every voter v be initially endowed with a preferred policy θ_v , which is commonly known and randomly drawn from the uniform distribution over $[\beta, 1 - \beta]$, with $0 < \beta < 0.5$. In addition to that, assume voters' ideal policies are subject to a common and random shock ξ , which is distributed uniformly over $[-\beta, \beta]$, and is unobserved by the candidates when they choose their platforms.

Let $\tilde{\theta}_v = \theta_v + \xi$ denote voter v 's *actual* ideal policy, who has the same utility function over the policy space $X = [0, 1]$ that before; i.e., $u_v(x) = -w(|x - \tilde{\theta}_v|)$. It is easy to see that the median voter's ideal policy $\tilde{\theta}_m$ is actually a random variable; specifically,

$$\tilde{\theta}_m \sim U \left[\frac{1}{2} - \beta, \frac{1}{2} + \beta \right].$$

Hence, for any pair of platforms $(x_L, x_R) \in X^2$, the left-wing candidate's probability of winning the election is given by

$$\tilde{p}(x_L, x_R) = \begin{cases} \text{Prob} \left(\tilde{\theta}_m \in \left[0, \frac{x_L + x_R}{2} \right] \right) & \text{if } x_L \leq x_R, \\ \text{Prob} \left(\tilde{\theta}_m \in \left[\frac{x_L + x_R}{2}, 1 \right] \right) & \text{if } x_L > x_R. \end{cases}$$

Routine calculations shows that the probability of winning function $\tilde{p}(\cdot)$ of the model with stochastic preferences coincides with the probability of winning function $p(\cdot)$ of the previous model (with fixed preferences, but a noisy vote share). Therefore, since the two frameworks are the same in all other respects, the equilibrium analysis carried out before also applies to the current scenario.

The models differ, however, with respect to the welfare effect of policy differentiation. To see this, suppose candidates have symmetric motivations (i.e., $\chi_L = \chi_R$). Define voter v 's ex-ante welfare associated with an equilibrium profile (x_L, x_R) as:

$$\tilde{V}_v(x_L, x_R) = -\frac{1}{2\beta} \cdot \left[\int_{-\beta}^0 w(|x_L - \theta_v - \xi|) d\xi + \int_0^{\beta} w(|x_R - \theta_v - \xi|) d\xi \right].$$

Then, if the pair of platforms (x_L^*, x_R^*) represents the equilibrium policies characterized in Proposition 2, it follows that for all $\theta_v \in [\beta, 1 - \beta]$, $\tilde{V}_v(x_L^*, x_R^*) \geq \tilde{V}_v(1/2, 1/2)$. In words, that means that in the election game with stochastic preferences and mixed but identical motivations, all voters prefer two-sided policy differentiation to convergence, reversing the welfare result obtained in Proposition 4. That shows that the micro foundations of the probability of winning function matters in electoral competition, in the sense that policy differentiation (convergence) may be good or bad from society's viewpoint depending on the model of electoral uncertainty behind that function.

6 Discussion

It is known from Saporiti (2008) that the mixed extension of the hybrid election game is better-reply secure and, therefore, that the game always possesses a Nash equilibrium. The trouble is, however, that it doesn't need to be one in pure strategies. As Figure 8 illustrates, there is a whole set of values of β for which a PSE fails to exist.

Indeed, as happens in spatial competition when candidates differ along some valence dimension (such as quality), in the election game with mixed motivations the relatively more office-motivated candidate (which plays the role of the "advantaged" candidate) experiences an incentive to *copy* the platform of the relatively more policy-motivated

(“disadvantaged”) candidate. To avoid this undercutting behavior, if both candidates have complete information and symmetric beliefs about voters, the equilibrium has to be sometimes (i.e., for certain configurations of the parameters) in mixed strategies.¹⁵

Surely, the most important question that remains to be addressed is what are the properties of the equilibrium platforms in the absence of a pure strategy equilibrium. The answer to this question is by no means a trivial one, because of the continuity of the strategy space and the discontinuity of the payoff functions over the main diagonal.

A first attempt to tackle this complex issue is carried out by Saporiti (2010). The strategy of that paper consists in approximating the election game with continuous strategy space by a sequence of successively finer finite games, and investigating the properties of the equilibria of the limit game by studying those of the equilibria of the finite games. However, the argument is a bit involved because the sum of the payoffs $\pi_L + \pi_R$ is not upper semi-continuous, implying that we cannot be sure that the sequence of mixed equilibria of the finite games converges to an equilibrium of the limit game.

To circumvent this difficulty, the paper proceeds as follows. First, it shows that the payoffs exhibit *complementary discontinuities*, in the sense that whenever the payoff of candidate $i \in \{L, R\}$ jumps down, the payoff of candidate $j \neq i$ jumps up. Second, using the previous result, it defines an auxiliary game, denoted by $\mathcal{G}_\alpha = (X, \pi_i^\alpha)_{i=L,R}$, and it proves that: (a) for every $y \in X$, $\pi_L^\alpha + \pi_R^\alpha$ is upper semi-continuous at (y, y) ; and (b) for all i , the payoff function π_i^α is weakly lower semi-continuous in x_i . Thus, by invoking Theorem 5 in Dasgupta and Maskin (1986), the paper concludes that \mathcal{G}_α has a mixed equilibrium. Moreover, it shows that a profile of probability measures (μ_L^*, μ_R^*) is a mixed strategy equilibrium (MSE) for $\mathcal{G}_\alpha = (X, \pi_i^\alpha)_{i=L,R}$ if and only if (μ_L^*, μ_R^*) is a MSE for the original game $\mathcal{G} = (X, \pi_i)_{i=L,R}$.¹⁶

Finally, Saporiti (2010) approximates $\mathcal{G}_\alpha = (X, \pi_i^\alpha)_{i=L,R}$ by a sequence of successively finer finite games $\{\mathcal{G}_\alpha^n\}_{n>0}$, with $\mathcal{G}_\alpha^n = (X_n, \pi_i^\alpha)_{i=L,R}$, where X_n denotes the n -th finite approximation of X , and π_i^α in \mathcal{G}_α^n is the restriction on $X_n \times X_n$. By the Nash’s theorem, each \mathcal{G}_α^n possesses a MSE, let’s say $\mu^n = (\mu_L^n, \mu_R^n)$. Moreover, the properties of \mathcal{G}_α ensures that a subsequence of $\{\mu^n\}_n$ converges to a profile $\mu^* = (\mu_L^*, \mu_R^*)$, which is a MSE for \mathcal{G} . The properties of the limit equilibrium μ^* are lastly investigating by analyzing the properties of the sequence $\{\mu^n\}_n$.

In that regard, Saporiti (2010) studies a couple of examples which reveal the existence

¹⁵When candidates differ in quality, Aragones and Palfrey (2005) show that the mixed equilibria of Aragones and Palfrey (2002) can be approximated by a sequence of pure strategy equilibria, each of which represents a PSE of a game where players have private information about their policy preferences. This shows that the mixed equilibria are an artifact of the symmetric information that candidates have about each other.

¹⁶As a by-product, this result offers an alternative proof of equilibrium existence in the original hybrid election game via Dasgupta and Maskin’s approach.

of at least two different types of mixed equilibria in \mathcal{G} . For a case like Fig. 8, with $\chi_L > \chi_R$, one MSE takes place for values of β between $\chi_R/2$ and $\chi_L + \chi_R/4$, whereas the other holds for β between $\chi_L + \chi_R/4$ and β^C . (Notice that the latter conditions are precisely the conditions behind Fig. 3.) In both cases, the examples show that, although candidates have distinct ideologies distributed on either side of the median ideal point, the equilibrium platforms exhibit *one-sided probabilistic differentiation*, in the sense that the support of the probability distributions of both candidates lie down on the same side of the political ground.

A Appendix

Proof of Lemma 2. Let (x_L^*, x_R^*) be a PSE for \mathcal{G} . To see that $p(x_L^*, x_R^*) \in (0, 1)$, assume without loss of generality that $p(x_L^*, x_R^*) = 1$. Then, candidate R 's equilibrium payoff is $\pi_R(x_L^*, x_R^*) = 0$; and it would be possible for R to increase its payoff by deviating to x_L^* (which would result in a payoff equal to $\chi_R/2 > 0$), a contradiction.

Next, suppose that $x_L^* < \theta_L$. If $x_R^* \geq \theta_L$, it would be possible for L to increase its payoff by choosing θ_L , because $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [x_L^* + x_R^* - 2\theta_L + \chi_L] < p(\theta_L, x_R^*) \cdot [x_R^* - \theta_L + \chi_L] = \pi_L(\theta_L, x_R^*)$ (recall that, by Lemma 1, $p(\theta_L, x_R^*) \geq p(x_L^*, x_R^*)$). Alternatively, if $x_R^* < \theta_L$, then: (i) L would profitably deviate to x_R^* if $x_L^* < x_R^*$, because $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [x_L^* - x_R^* + \chi_L] < \chi_L/2$; (ii) R would find it beneficial to move to x_L^* if $x_R^* < x_L^*$, because $\pi_R(x_L^*, x_R^*) = [1 - p(x_L^*, x_R^*)] \cdot [x_R^* - x_L^* + \chi_R] < \chi_R/2$; and (iii) L would do better by playing θ_L if $x_R^* = x_L^*$, because $\chi_L/2 < p(\theta_L, x_R^*) \cdot [\theta_L - x_R^* + \chi_L] = \pi_L(\theta_L, x_R^*)$. Therefore, $x_L^* \geq \theta_L$.

Assume, by way of contradiction, that $x_L^* = \theta_L$. Then: (i) if $x_R^* = \theta_L$, candidate R can benefit by moving its proposal to $x_R = \theta_L + \delta$, with $\delta > 0$ small, because $\pi_R(x_L^*, x_R) = [1 - p(x_L^*, x_R)] \cdot (\chi_R + \delta) > \chi_R/2 = \pi_R(x_L^*, x_R^*)$; (ii) if $x_R^* > \theta_L$, candidate L would be able to increase its payoff by selecting $x_L = \theta_L + \epsilon$, which would result, given the assumption on β and for $\epsilon > 0$ small enough, in a positive payoff change $[p(x_L, x_R^*) - p(\theta_L, x_R^*)] \cdot [x_R^* - \theta_L + \chi_L] - \epsilon \cdot p(x_L, x_R^*)$; ¹⁷ finally (iii) if $x_R^* < \theta_L$, R would find it profitable to deviate to θ_L because $\pi_R(x_L^*, x_R^*) = [1 - p(x_L^*, x_R^*)] \cdot (x_R^* - x_L^* + \chi_R) < \chi_R/2$. Hence, from (i)-(iii), we conclude that $x_L^* > \theta_L$. A similar argument establishes that $x_R^* < \theta_R$.

To complete the proof, it remains to be shown that $x_L^* \leq x_R^*$. Assume, by way of contradiction, that $x_L^* > x_R^*$. There are three cases to consider.

Case 1. If $x_R^* \in [0, \theta_L)$, candidate L can deviate to θ_L (recall that $x_L^* > \theta_L$), which results in a payoff change equal to $\pi_L(\theta_L, x_R^*) - \pi_L(x_L^*, x_R^*) = [p(\theta_L, x_R^*) - p(x_L^*, x_R^*)] \cdot [\theta_L -$

¹⁷Note that $p(\theta_L, x_R^*) \in (0, 1)$ because by hypothesis $x_L^* = \theta_L$. Hence, by Lemma 1, $p(x_L, x_R^*) - p(\theta_L, x_R^*) > 0$.

$x_R^* + \chi_L] + p(x_L^*, x_R^*) \cdot (x_L^* - \theta_L) > 0$, contradicting that x_L^* is candidate L 's best response to x_R^* (again $p(\theta_L, x_R^*) - p(x_L^*, x_R^*) > 0$ because of Lemma 1).

Case 2. If $x_R^* \in [\theta_L, 1/2)$, then L can deviate to $x_L = x_R^* + \epsilon$, $\epsilon > 0$, which results in a payoff change equal to $\pi_L(x_L, x_R^*) - \pi_L(x_L^*, x_R^*) = p(x_L, x_R^*) \cdot (\chi_L - \epsilon) - p(x_L^*, x_R^*) \cdot [\chi_L - (x_L^* - x_R^*)]$. By Lemma 1, $p(x_L, x_R^*) \geq p(x_L^*, x_R^*)$. Thus, for ϵ small enough, $\pi_L(x_L, x_R^*) > \pi_L(x_L^*, x_R^*)$, implying that L 's deviation is profitable and, consequently, that (x_L^*, x_R^*) is not a PSE; a contradiction.

Case 3. Finally, if $x_R^* \in [1/2, \theta_R)$, then $p(x_L^*, x_R^*) < 1/2$; and L can achieve a payoff greater than $\pi_L(x_L^*, x_R^*) = p(x_L^*, x_R^*) \cdot [\chi_L - (x_L^* - x_R^*)]$ by choosing x_R^* (which actually offers a payoff of $\chi_L/2$), contradicting the initial hypothesis that (x_L^*, x_R^*) is a PSE.

Therefore, from Cases 1-3, we conclude that $x_L^* \leq x_R^*$, as required. \blacksquare

Proof of Lemma 3. Let the profile $(x_L^*, x_R^*) \in X^2$, with $x_L^* < x_R^*$, be a PSE for \mathcal{G} . By Lemma 2, $\theta_L < x_L^* < x_R^* < \theta_R$ and $p(x_L^*, x_R^*) \in (0, 1)$. Since the probability function $p(\cdot)$ is continuous at (x_L^*, x_R^*) , there must exist $\epsilon > 0$ sufficiently small such that, for all $(x_L, x_R) \in R_\epsilon(x_L^*) \times R_\epsilon(x_R^*)$, $\theta_L < x_L < x_R < \theta_R$ and $p(x_L, x_R) \in (0, 1)$, where $R_\epsilon(x_i^*) \equiv (x_i^* - \epsilon, x_i^* + \epsilon)$, with $i = L, R$. Thus, for any profile $(x_L, x_R) \in R_\epsilon(x_L^*) \times R_\epsilon(x_R^*)$, the left-wing candidate's payoff function can be written as $\pi_L(x_L, x_R) = p(x_L, x_R) \cdot (x_R - x_L + \chi_L)$, where $p(x_L, x_R) = 1/2 + (x_L + x_R - 1)/4\beta$.

Fix $x_R^* \in R_\epsilon(x_R^*)$ and consider candidate L 's best response to x_R^* over $R_\epsilon(x_L^*)$, which is obtained by solving the problem $\max_{x_L \in R_\epsilon(x_L^*)} \pi_L(x_L, x_R^*)$. The first-order condition for this problem provides a stationary point $1/2 - \beta + \chi_L/2$. Note that this point actually maximizes $\pi_L(\cdot, x_R^*)$ over $R_\epsilon(x_L^*)$ because by hypothesis, for all $x_L \in R_\epsilon(x_L^*)$, $\pi_L(x_L^*, x_R^*) \geq \pi_L(x_L, x_R^*)$; i.e., $\pi_L(\cdot, x_R^*)$ has an interior maximum on $R_\epsilon(x_L^*)$. Moreover, since $\pi_L(\cdot, x_R^*)$ is *strictly* concave on $R_\epsilon(x_L^*)$, with $\partial^2 \pi_L(x_L, x_R^*) / \partial x_L^2 = -1/2\beta < 0$, we have that $x_L^* = 1/2 - \beta + \chi_L/2$, as required. A similar argument shows that $x_R^* = 1/2 + \beta - \chi_R/2$.

Finally, the condition $x_L^* > \theta_L$ (resp., $x_R^* < \theta_R$) is obtained from the early assumption about β , (namely, $0 < \beta < \min\{1/2 - \theta_L + \chi_L/2, \theta_R - 1/2 + \chi_R/2\}$), whereas the condition $\chi_L + \chi_R < 4\beta$ follows from the initial hypothesis, according to which $x_L^* < x_R^*$. Routine calculations also show that $\chi_L + \chi_R < 4\beta$ implies that $1/2 - U[S(x_L^*, x_R^*)] \in (-\beta, \beta)$, so that $p(x_L^*, x_R^*) \in (0, 1)$ as needed. \blacksquare

Proof of Proposition 1. To show sufficiency, fix the strategy profile $(x_L^*, x_R^*) = (1/2, 1/2)$, where both candidates propose the median voter's ideal point and receive a payoff of $\pi_i(x_L^*, x_R^*) = \chi_i/2$. Consider first a deviation for the left-wing candidate to any platform $x'_L \in (\theta_L, 1/2)$. For convenience, let's write $x'_L = 1/2 - \delta$, with $\delta > 0$.

Routine calculations show that $\pi_L(x'_L, x_R^*) \equiv \frac{\chi_L}{2} - \frac{\delta^2}{4\beta} + \left(\frac{1}{2} - \frac{\chi_L}{4\beta}\right)\delta > \chi_L/2$ if and only if $\delta < 2\beta - \chi_L$. However, the last inequality requires $\delta < 0$ because by hypothesis $\chi_L \geq 2\beta$. Hence, $\pi_L(x'_L, x_R^*) \leq \pi_L(x_L^*, x_R^*)$. A similar argument proves that for any $x'_R \in (1/2, \theta_R)$, $\pi_R(x_L^*, x'_R) \leq \pi_R(x_L^*, x_R^*)$. The careful reader should also check at this point that any deviation above $1/2$ or below θ_L (resp., below $1/2$ or above θ_R) cannot raise candidate L 's (resp., R 's) conditional payoff any further, proving in that way that the profile $(x_L^*, x_R^*) = (1/2, 1/2)$ is a PSE for \mathcal{G} .

To show necessity, fix a PSE for \mathcal{G} with the property that $x_L^* = x_R^* \equiv x^*$ for some $x^* \in X$. If $x^* > 1/2$, then candidate L can profitably deviate to $1/2$, because $p(1/2, x^*) \in (1/2, 1]$ and $\psi_{\theta_L}(1/2, x^*) = x^* - 1/2 > 0$, so that $\pi_L(1/2, x^*) = p(1/2, x^*) \cdot [\psi_{\theta_L}(1/2, x^*) + \chi_L] > 1/2 \cdot \chi_L = \pi_L(x^*, x^*)$. A similar reasoning shows that candidate R can profitably deviate to $1/2$ if $x^* < 1/2$. Therefore, $x^* = 1/2$.

Next, suppose that $\chi_L < 2\beta$, which in turn implies that $1/2 + \chi_L/2 - \beta < 1/2$. Since $p(\cdot)$ is continuous at $(1/2, 1/2)$ and strictly positive, there must exist $\delta > 0$ such that for all $x_L \in (1/2 - \delta, 1/2]$, $p(x_L, 1/2) > 0$ and $\pi_L(x_L, 1/2) = \left(\frac{1}{2} + \frac{x_L - 1/2}{4\beta}\right) \cdot (1/2 - x_L + \chi_L)$. Simple calculations show that $\pi_L(\cdot, 1/2)$ achieves a unique maximum over $(1/2 - \delta, 1/2]$ at $\hat{x}_L = 1/2 + \chi_L/2 - \beta$, implying in particular that $\pi_L(\hat{x}_L, 1/2) > \pi_L(1/2, 1/2)$, a contradiction. Hence, $\chi_L \geq 2\beta$. A similar argument proves that $\chi_R \geq 2\beta$. ■

Proof of Proposition 2. To prove necessity, suppose \mathcal{G} has a PSE with the property that $x_L^* < 1/2 < x_R^*$. By Lemma 3, $x_L^* = \frac{1}{2} - \beta + \frac{\chi_L}{2}$ and $x_R^* = \frac{1}{2} + \beta - \frac{\chi_R}{2}$. Therefore, using the initial hypothesis, it follows that $\chi_i < 2\beta$ for all $i = L, R$.

To show sufficiency, fix the equilibrium candidate $(x_L^*, x_R^*) = (\frac{1}{2} - \beta + \frac{\chi_L}{2}, \frac{1}{2} + \beta - \frac{\chi_R}{2})$. By the initial hypothesis, i.e., $\chi_i < 2\beta$ for all $i = L, R$, it follows that $x_L^* < 1/2 < x_R^*$, $\chi_L + \chi_R < 4\beta$, and $p(x_L^*, x_R^*) \in (0, 1)$. By the assumption on β , $\theta_L < x_L^*$ and $x_R^* < \theta_R$. Applying the reasoning of the proof to Lemma 3, for some $\epsilon > 0$ such that $R_\epsilon(x_L^*) \equiv (x_L^* - \epsilon, x_L^* + \epsilon) \subset (\theta_L, x_R^*)$, we have that $x_L^* = \arg \max_{x_L \in R_\epsilon(x_L^*)} \pi_L(x_L, x_R^*)$, with $\pi_L(x_L^*, x_R^*) = \frac{\chi_L}{2} + (\beta - \frac{\chi_R}{2}) + \frac{(\chi_L - \chi_R)^2}{16\beta}$. Thus, $\pi_L(x_L^*, x_R^*) > \chi_L/2 = \pi_L(x_R^*, x_R^*)$.

Consider a deviation for the left-wing candidate to any platform $x'_L \in [0, 1]$ different from x_L^* and x_R^* . On one hand, if $p(x'_L, x_R^*) = 0$, then $\pi_L(x'_L, x_R^*) = 0 < \pi_L(x_L^*, x_R^*)$, implying that the alternative policy does not raise L 's payoff. On the other hand, if $p(x'_L, x_R^*) \in (0, 1]$, two cases are in order:

Case 1. Assume $x'_L \in (x_R^*, 1]$. Then: (i) if $p(x'_L, x_R^*) = 1$, it must be the case that $1/2 - U[S(x'_L, x_R^*)] \leq -\beta$, which leads to the contradiction $(x'_L - 1/2) + (\beta - \chi_R/2) \leq -2\beta$, since the left-hand side of the previous inequality is strictly positive and the right-hand side is smaller than zero; alternatively (ii) if $p(x'_L, x_R^*) \in (0, 1)$, then $\pi_L(x'_L, x_R^*) = \left(\frac{1}{2} + \frac{1 - x'_L - x_R^*}{4\beta}\right) \cdot (x_R^* - x'_L + \chi_L)$. Recall that $1 - x'_L - x_R^* < 0$ and $x_R^* - x'_L < 0$, because

$x'_L > x_R^* > 1/2$. Therefore, $\pi_L(x'_L, x_R^*) < 1/2 \cdot \chi_L < \pi_L(x_L^*, x_R^*)$, implying once again that candidate L 's deviation to x'_L is not beneficial.

Case 2. Suppose $x'_L \in [0, x_R^*)$. Then: (i) if $p(x'_L, x_R^*) = 1$, we get as before that $1/2 - U[S(x'_L, x_R^*)] \leq -\beta$ and, consequently, that $x'_L \geq 1/2 + \beta + \chi_R/2 > x_R^*$, which supplies the desired contradiction (because by hypothesis $x'_L < x_R^*$); alternatively (ii) if $p(x'_L, x_R^*) \in (0, 1)$, then: (ii.a) if $\theta_L \leq x'_L < x_R^*$, candidate L 's deviation payoff is $\pi_L(x'_L, x_R^*) = \left(\frac{1}{2} + \frac{x'_L + x_R^* - 1}{4\beta}\right) \cdot (x_R^* - x'_L + \chi_L)$; and, given that the function $f(x_L) = \left(\frac{1}{2} + \frac{x_L + x_R^* - 1}{4\beta}\right) \cdot (x_R^* - x_L + \chi_L)$ is strictly concave on $x_L \in [\theta_L, x_R^*)$ and has a maximum at $1/2 - \beta + \chi_L/2$, we conclude that $\pi_L(x'_L, x_R^*) < \pi_L(x_L^*, x_R^*)$; finally (ii.b) if $0 \leq x'_L < \theta_L$, it is easy to show that $\pi_L(x'_L, x_R^*) < \pi_L(\theta_L, x_R^*) < \pi_L(x_L^*, x_R^*)$, where the last inequality follows from the argument in (ii.a).

Summing up, Case 1 and Case 2 above, together with the fact that $\pi_L(x_L^*, x_R^*) > \pi_L(x_R^*, x_R^*)$, prove that $x_L^* = \arg \max_{x_L \in [0,1]} \pi_L(x_L, x_R^*)$. A similar reasoning also shows that $x_R^* = \arg \max_{x_R \in [0,1]} \pi_R(x_L^*, x_R)$. Therefore, the profile (x_L^*, x_R^*) is a PSE for \mathcal{G} . ■

Proof of Proposition 3. We prove the proposition for $1/2 < x_L^* < x_R^*$. The argument for $x_L^* < x_R^* < 1/2$ is similar. First, assume the election game \mathcal{G} has a PSE with the property that $1/2 < x_L^* < x_R^*$. By Lemma 3, $x_L^* = 1/2 - \beta + \chi_L/2$ and $\chi_L + \chi_R < 4\beta$. That implies that $\frac{\chi_L}{2} > \beta > \frac{\chi_L + \chi_R}{4}$ and, therefore, that $\chi_R < \chi_L$. Using simple algebraic manipulation, it also follows that

$$\frac{\chi_L + \chi_R}{4} < \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_R \cdot \chi_L}}{2} < \frac{\chi_L}{2}. \quad (3)$$

Suppose, by way of contradiction, that $2\beta < (\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2}$. By definition, $\pi_R(x_L^*, x_R^*) = \beta - (\chi_L - \chi_R)/2 + (\chi_L - \chi_R)^2/16\beta$. Fix any $x_R \in [1/2, x_L^*)$. Candidate R 's payoff at (x_L^*, x_R) is $\pi_R(x_L^*, x_R) = \left(\frac{1}{2} + \frac{x_L^* + x_R - 1}{4\beta}\right) (x_R - x_L^* + \chi_R)$. Therefore, $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R) = \frac{\chi_L \chi_R}{4\beta}$. Notice that the difference between $\pi_R(x_L^*, x_R^*)$ and $\lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R)$ gives rise to a second-order polynomial equation in β , namely, $4\beta^2 - 2\beta(\chi_L - \chi_R) + (\chi_L - \chi_R)^2/4 - \chi_L \cdot \chi_R$, which has the following two roots: $\frac{\chi_L - \chi_R}{4} \pm \frac{\sqrt{\chi_R \cdot \chi_L}}{2}$. Therefore, for any $\beta \in \left(\frac{\chi_L + \chi_R}{4}, \frac{\chi_L - \chi_R}{4} + \frac{\sqrt{\chi_R \cdot \chi_L}}{2}\right)$, we have that $\pi_R(x_L^*, x_R^*) < \lim_{x_R \rightarrow x_L^*} \pi_R(x_L^*, x_R)$, contradicting that the strategy profile (x_L^*, x_R^*) is by hypothesis a PSE of \mathcal{G} . Hence, $2\beta \geq (\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2}$.

To carry out the second part of the proof, suppose $(\chi_L - \chi_R)/2 + (\chi_R \cdot \chi_L)^{1/2} \leq 2\beta < \chi_L$, and consider the equilibrium candidate $(x_L^*, x_R^*) = \left(\frac{1}{2} - \beta + \frac{\chi_L}{2}, \frac{1}{2} + \beta - \frac{\chi_R}{2}\right)$. By the initial hypothesis and (3), we have that $\chi_L + \chi_R < 4\beta$. Therefore, since by assumption $2\beta < \chi_L$, it follows that $\chi_R < 2\beta$ and, consequently, that $1/2 < x_L^* < x_R^*$ and $p(x_L^*, x_R^*) \in (0, 1)$. Moreover, using the argument of the proof to Proposition 2,

$x_L^* = \arg \max_{x_L \in [0,1]} \pi_L(x_L, x_R^*)$. To show that $x_R^* = \arg \max_{x_R \in [0,1]} \pi_R(x_L^*, x_R)$ we proceed as follows. Firstly notice that, by applying the reasoning of the proof to Lemma 3, it can be shown that for some $\epsilon > 0$ with the property that $R_\epsilon(x_R^*) \equiv (x_R^* - \epsilon, x_R^* + \epsilon) \subset (x_L^*, \theta_R)$, $\frac{1}{2} + \beta - \frac{\chi_R}{2} = \arg \max_{x_R \in R_\epsilon(x_R^*)} \pi_R(x_L^*, x_R)$, with $\pi_R(x_L^*, x_R^*) = \frac{\chi_R}{2} + (\beta - \frac{\chi_L}{2}) + \frac{(\chi_R - \chi_L)^2}{16\beta}$. Secondly, to prove that $\pi_R(x_L^*, x_R^*) > \frac{\chi_R}{2}$, observe that $\frac{\chi_R}{2} < \frac{\chi_L \chi_R}{4\beta}$ because $\chi_L/2\beta > 1$. Moreover, since $\lim_{x_R \rightarrow -x_L^*} \pi_R(x_L^*, x_R) = \frac{\chi_L \chi_R}{4\beta}$, it also follows that $\lim_{x_R \rightarrow -x_L^*} \pi_R(x_L^*, x_R) > \frac{\chi_R}{2}$. Thus, the desired result, i.e., $\pi_R(x_L^*, x_R^*) > \frac{\chi_R}{2}$ is obtained using the fact that, by hypothesis, $\lim_{x_R \rightarrow -x_L^*} \pi_R(x_L^*, x_R) \leq \pi_R(x_L^*, x_R^*)$. The rest of the proof follows the argument of the proof to Prop. 2 and is left to the readers.¹⁸ ■

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¹⁸A complete version of it is available from the author upon request.

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