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MODELLING STRATEGIC INTERACTIONS IN SPORTS LEAGUES

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Abstract A game-theoretic sports league model is presented, with profit-maximizing clubs and perfectly inelastic supply of playing talent. Clubs interact strategically in revenues (home revenues depend on visiting teams' talents) and costs (as oligopsonists in the talent market). Existing models ignore the latter and have unclear game-theoretic foundations, both rectified here. Motivated by strategic market games, clubs make "bids" (talent expenditures) to the talent market against the inelastic supply "offers", a market-clearing wage and talent allocation emerging; club revenues depend on talent allocations. The effects of revenue sharing and salary caps on wages and competitive balance in Nash equilibrium are investigated.

JEL classification numbers; L10, L83

Keywords; sports leagues, oligopsony, revenue sharing, salary caps.

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1. INTRODUCTION

A professional team sports league constitutes a somewhat unusual industry whose peculiarities have been the subject of economics research for over half a century now[1]. For instance, driving out rival clubs from the league to attain a monopoly position is clearly not desirable in this context. Moreover there is a benefit for any one club in seeing its rivals produce better teams, to some extent, so as to create better quality games against the rivals in the eyes of its fans, and thus greater revenue. Indeed a major focus of the literature has been the concern that "big" clubs with large fan markets may produce teams that are so good as to lead to one-sided games and an economically unsuccessful league. In the jargon, the concern has been the potential lack of competitive balance in the league (degree of equality in team qualities), and whether regulatory policies of (in particular) revenue sharing or salary caps may increase this balance. This paper re-visits these well-established themes. The reason for doing so is that the theoretical, in particular Game-theoretic[2], foundations of existing models seem to be lacking; a framework is suggested which offers a complete and consistent way forward.

Competition between clubs in a professional team sports league will typically entail strategic interactions on both the revenue and cost sides of their activity, at least when the number of clubs is relatively small. On the revenue side there is the interaction suggested in the previous paragraph - as an away team a club's decisions on the hiring of playing talent will dictate its team's quality and will impact non-negligibly on other clubs' home gate revenues over the league season, since these revenues will depend on both home and away teams' qualities. But equally clubs will have some non-negligible power as oligopsonists in the market for playing talent, at least when the supply of talent to the league as a whole is less than perfectly elastic. Modelling such strategic interactions clearly calls for the methods of Game theory, and yet the existing literature seems to have failed to provide a complete and consistent specification and analysis, indeed completely overlooking the oligopsony aspect, gaps which are filled here.

In common with many previous authors and the textbooks, and with the US context in mind, clubs are profit maximizers and the supply of playing talent to the league is perfectly inelastic, the latter reflecting the relative lack of competition for specialised talent faced by the major US sports leagues. The paper joins a literature stimulated by Szymanski and Kesenne (2004) and Szymanski (2004), who criticised appropriately the reliance of the earlier literature on non-Nash conjectures; the focus here will be exclusively on Nash equilibria of a non-cooperative normal form Game. We also follow Szymanski and Kesenne (2004) and Szymanski (2004) in the modelling of the strategic interaction on the revenue side[3], and in assuming that the cost of hiring playing talent is the only club cost and gate revenue from home games is the only revenue source for clubs. However we deviate significantly from their analysis of the talent market because of the following.

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[1] The subject has grown such that courses in Sports Economics are now common, served by textbooks such as Fort (2006) and Sandy et al. (2004).

[2] Certain terms (e.g. game, player) have meanings in the sporting context which differ from those in economic modelling. Capitals are thus used to distinguish the economic modelling meaning.

[3] However we do argue that this specification is only credible if the supply of playing talent is perfectly inelastic, which is the reason for this assumption here – see Remark 1 at the end of Section 1.

The main problem with the previous models[4] as Games is that clubs formulate their profit-maximizing quantities of talent demand *taking as given the wage for talent*. To complete the model the wage needs to be endogenised[5]. The most obvious assumption, and the one that previous authors seem to have in mind, explicitly or implicitly, is that the wage adjusts to clear the talent market, equating clubs' demand for talent to the supply. But, assuming that clubs are small (in principle anything finite) in number, this wage will depend in general on each individual club's talent demand, a fact which is overlooked by clubs when formulating their demands. The outcome is thus not an equilibrium outcome in the sense that clubs would typically want to change their talent demands to influence the wage. The aim of the paper is to provide a model where clubs are aware of their talent market (oligopsony) power when formulating best responses[6]. A normal form Game is defined in which Nash equilibrium determines the allocation of the perfectly inelastic talent supply to clubs and the wage[7]. The extent of competitive balance in Nash equilibrium and the affect of revenue-sharing and salary caps on this balance are the main focus of the subsequent analysis. The findings point towards salary caps rather than revenue sharing as the appropriate policy.

The oligopsony modelling is novel, in continuing to follow the sports literature in assuming that playing talent is a homogeneous good, whereas the recent oligopsony literature has focused on labour markets with differentiated goods (jobs)[8]. The strategies chosen by clubs are taken to be their *expenditures* on talent, rather than the quantities, for two reasons. First, it does seem a more realistic assumption in the sports context, where club owners typically decide on a player budget within which coaches and others directly involved with team planning acquire players. Secondly, the quantity of talent alternative leads inevitably to a homogeneous Cournot oligopsony, a specification that is not available since the inverse talent supply is not defined when supply is perfectly inelastic – attempts to define a normal form Game with quantity of talent as the strategic variable fail. In fact the oligopsony modelling is closely related to the imperfect competition models found in the strategic market Game literature, where agents make bids (talent expenditures here) and offers (the inelastic talent supply) to the two sides of a market, and a market clearing price (wage for talent) emerges as the ratio of aggregate bids to offers[9]. The switch to expenditure strategies, inspired by strategic market Games, is essential for the purpose here[10].

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[4] Similar analysis can be found in Kesenne (2007, p.49-53, 114-20, 125-31) and in a large number of working papers.

[5] The alternative is to leave the wage as fixed exogenously. But then the model is incomplete in an important dimension, and merely begs the question as to who is doing the exogenous fixing, and how.

[6] An alternative way out of this problem is to assume that the number of clubs in the league is sufficiently large to legitimize the parametric wage assumption, and obviate the need for a Game-theoretic analysis. Consistency then demands that there are no strategic interactions on the revenue side either; see Madden (2009).

[7] It should be stressed that this paper is, to the best of my knowledge, the only one in the literature so far that can make this claim.

[8] For instance see Bhaskar and To (1999,2003), Kaas and Madden (2008a, 2008b). Note also that the homogeneity does not mean player homogeneity – players have differing amounts of talent and receive the wage for each unit of their talent.

[9] The literature on strategic market Games (see Giraud (2003) for an introduction) is largely concerned with general equilibrium analysis of exchange economies, where each agent submits bids (to buy, in terms of money) and/or offers (quantities for sale) to each market. On each market a price emerges that is the ratio of aggregate bids to offers, allowing the market to clear. Here there is a single talent market, the offers come from a large number of players offering their talent inelastically, and the bids come

To make use of various diagrams similar to those of the previous literature, and to make the main points at their clearest, Sections 2-5 are concerned with a two-club league. Section 6 generalizes to a league with a finite number of clubs exceeding two, and Section 7 concludes.

2. THE FRAMEWORK WITH TWO CLUBS

The sports league consists of two profit-maximizing clubs $i = 1,2$ whose teams play each other twice, once at home and once away. Clubs make decisions on their expenditure on playing talent e_i, $i = 1,2$, and receive gate revenue from their home game. In this section there are no regulatory policies in force, and eventually a normal form Game will be described in which the Players are the clubs and strategy sets are $e_i \in \Re_+, i = 1,2$.

On the cost side of the industry players are allocated to clubs via a market for talent. With the major US professional team sports in mind, and as is common in the literature, we assume that a perfectly inelastic supply of talent (normalised to unity) will be offered to the league at any wage in excess of the reservation wage; for simplicity we take this reservation wage to be zero, at which no talent is supplied. The two clubs are therefore duopsonists in the market for talent, and we borrow from the strategic market Game imperfect competition literature to model the duopsony interaction. Specifically, clubs' decisions on their expenditures on talent constitute the "bids" to the talent market in the market Game terminology, with the unit amount of talent delivered inelastically[11] as the "offers". With a wage equal to the ratio of aggregate bids to offers, $w = e_1 + e_2$, the talent market clears with allocations of talent $t_i = e_i / w = e_i /(e_1 + e_2), i = 1,2$ when $w > 0$; when $w = 0$ no talent is supplied and $t_i = 0, i = 1,2$. t_i is also referred to as the quality of team *i*.

On the revenue side we follow much of the literature by abstracting from modelling game ticket price decisions but take as the primitive of the model club revenues as functions of the strategies, as follows. If e_1 or $e_2 > 0$ the revenue of club *i* depends on the resulting home game quality, reflected in the participating teams' qualities[12] $t_i = e_i /(e_1 + e_2)$ and $t_j = e_j /(e_1 + e_2)$; but since $t_j = 1 - t_i$ always we can write revenue

from the clubs, the wage being the ratio of aggregate bids to offers as in the strategic market Game setting.

[10] In models where clubs treat wages as parametric in making decisions, the switch between quantity and expenditure is inconsequential to the decision analysis, and authors in the previous literature have occasionally switched to talk of expenditure rather than quantity decisions. Recently, with a view to providing a framework for classroom sports league experimentation, Szymanski (2009) specifies a model exclusively in terms of expenditure; although wage determination and talent supply are not discussed, the resulting best response analysis and Nash equilibrium allocations of talent are entirely consistent with the presentation here.

[11] Implicitly there are a large number of players who consequently have no market power, and supply inelastically their available talent.

[12] The implicit micro foundations for game ticket demand and prices are as detailed in Madden (2009) – clubs are monopolist sellers of tickets to their fans, the resulting monopoly prices and sales producing "reduced form" revenue functions dependant only on talent allocations, taken as the primitive object here.

concisely as a function of a single variable, $r_i(t_i)$ or $r_i(\frac{e_i}{e_i+e_j})$. We assume $r_i(0)=0$ which is also assumed to be the outcome if $e_1=e_2=0$. In addition, $r_i:[0,1]\rightarrow\Re_+$ is continuous, C^2 on $(0,1)$, strictly concave[13] with a global maximum at some $m\in(\frac{1}{2},1)$; the last of these assumptions is nearly universal in the literature, whereby the fans attending games and providing the revenue are assumed to be home fans and to have a preference for the home team to be of higher quality than the opposition, but only to an extent ($t_i=m$) after which games become so one-sided in favour of the home team that demand and revenue start to fall[14].

The normal form Game specification is completed with the following club payoffs:

$$\pi_i(e_i,e_j)=r_i(\tfrac{e_i}{e_i+e_j})-e_i,\ i=1,2 \tag{2.1}$$

<u>Remark 1</u> Although implicit, it should be stressed that the primitive revenue specification in mind here is a function of two variables, the talents of the two teams. It is because of the inelastic talent supply that one can credibly reduce this to a function of one variable, the home team talent; alternatively one can take that single variable to be what is usually referred to as the home team win percentage, $t_i/(t_i+t_j)=W_i$ say. However it is common in the literature to assume also that revenue is a function of only W_i when the supply of talent is less than perfectly inelastic, and so variable. This seems implausible. It implies that revenue is homogeneous of degree zero in team qualities, so multiplying the talent levels of both teams by a large positive factor will have no affect on revenue. So it seems that the revenue specification here and in most of the literature can apply credibly only to the case of perfectly inelastic supply[15], which is why attention is restricted to this case here.

3. BEST RESPONSES AND NASH EQUILIBRIA

The best response problem faced by club *i* is to choose $e_i\geq 0$ to maximize the payoff in (2.1). Under the assumptions made, the result is a best response function, $e_i=b_i(e_j)$ or BR_i for short, with the following features and illustrated in Figure 1.

<u>BR1</u> $b_i(0)=0$, since $\pi_i(0,0)=0$ and deviation by i to $e_i>0$ produces $\pi_i(e_i,0)=0-e_i<0$. It follows that $(e_1,e_2)=(0,0)$ is always a (degenerate) NE.

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[13] The micro foundation referred to in footnote 11 shows that the strict concavity assumption may be easily violated near the origin. The resulting complications are ignored here.

[14] Nothing of substance changes if *m* is allowed to differ in value for the two clubs – the exposition is simplified without this.

[15] Some revenue specification other than as a function of W_i only is needed to address less than perfectly inelastic talent supply. For instance with perfectly *elastic* talent supply, Falconieri et al. (2004) use a function of aggregate league talent and its variance, which can reduce to a Cobb-Douglas form, also used in Madden (2008). However it is not yet clear what is the best assumption here.

BR2 $b_i(e_j) = 0$ also if $e_j \geq r_i^{'}(0)(>0)$, since $\pi_i(e_i, e_j)$ is concave in e_i and $\frac{\partial \pi_i(0,e_j)}{\partial e_i} = r_i^{'}(0)/e_j - 1 \leq 0$.

BR3 For $e_j \in (0, r_i^{'}(0))$, $e_i = b_i(e_j)$ is characterised by the first order condition $\frac{\partial \pi_i(e_i,e_j)}{\partial e_i} = 0$, or;

$$r_i^{'}(\tfrac{e_i}{e_i+e_j})\frac{e_j}{(e_i+e_j)^2} = 1 \qquad (3.1)$$

BR4 From (3.1) with $e_i = e_j$, $b_i(e_j) = e_j$ if $e_j = \frac{1}{4}r_i^{'}(\frac{1}{2})$.

BR5 When $e_j \in (0, r_i^{'}(0))$ so $e_i > 0$;

(i) The slope of the line from the origin to BR_i, $b_i(e_j)/e_j$, declines monotonically from $\frac{m}{1-m}$ as $e_j \to 0$, to 0 as $e_j \to r_i^{'}(0)$.

(ii) The slope of the line from the origin to BR_i always exceeds the slope of BR_i itself, i.e. $b_i(e_j)/e_j > b_i^{'}(e_j)$.

(iii) The implicit wage along BR_i, $w = b_i(e_j) + e_j$, increases monotonically from 0 as $e_j \to 0$, to $r_i^{'}(0)$ as $e_j \to r_i^{'}(0)$.

(iv) The implicit talent allocation to *i*, $t_i = b_i(e_j)/(b_i(e_j) + e_j)$, decreases monotonically from m as $e_j \to 0$, to 0 as $e_j \to r_i^{'}(0)$.

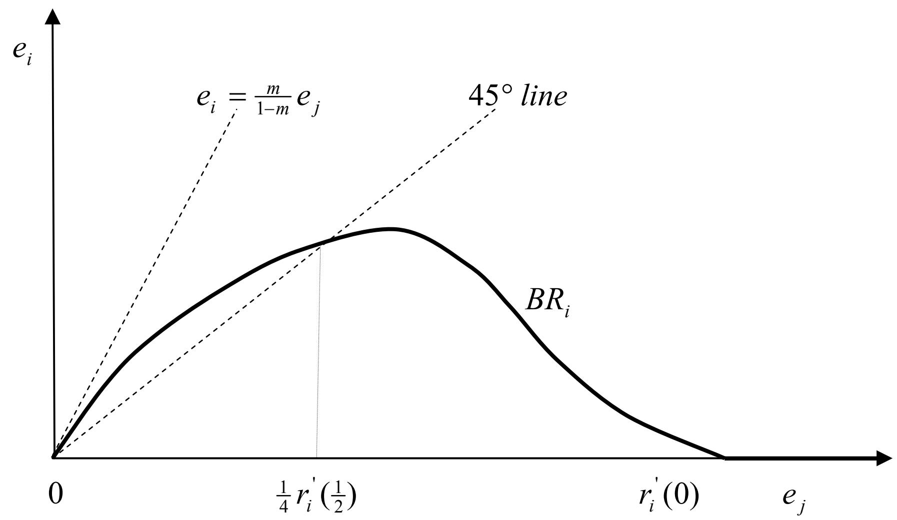


Figure 1; A typical best-response function

NE are $e_1, e_2 \geq 0$ where $e_1 = b_1(e_2)$ and $e_2 = b_2(e_1)$. There is always a degenerate NE, $(e_1, e_2) = (0,0)$, and the focus is now on any non-degenerate NE[16].
Consider first the special case where the revenue markets facing both clubs are identical:

(A1) For all $t \in [0,1]$, $r_1(t) = r_2(t) \equiv r(t)$.

Define competitive balance as $CB(t_1, t_2) = 1 - [\max(t_1, t_2) - \min(t_1, t_2)]$, which ranges from 0 (when one club has all the talent) up to 1 (when the allocation is equal). The following is clear from BR4, BR5 and Figure 1:

<u>Proposition 1</u> Assume (A1). Then there is a unique non-degenerate NE, $e_1^* = e_2^* = \frac{1}{4} r'(\frac{1}{2})$, with $t_1^* = t_2^* = \frac{1}{2}$ and $CB^* = 1$.

For the non-identical case, piecing together both best response graphs using BR4 and BR5 leads to (A2), Proposition 2 and Figure 2:

(A2) $r_1'(\frac{1}{2}) > r_2'(\frac{1}{2})$

So BR_1 crosses the $45°$ line at a value of expenditure larger than that for BR_2.

<u>Proposition 2</u> There is a unique non-degenerate NE in which $e_1^* > e_2^* > 0$, $m > t_1^* > t_2^* > 0$ and $CB^* \in (0,1)$ if and only if (A2) holds.

Figure 2 illustrates a NE with (A2) and $w^* > 1$.

A theme in the literature is how "big" clubs facing "big" markets may emerge with better quality teams. One fairly weak sense of "big" is: for all $t \in (0,1)$, $r_1(t) > r_2(t)$. However this does not imply (A2). A stronger version which does imply (A2) is the following global ranking of marginal revenues, usually assumed in the literature:

(A3) For all $t \in (0, m)$, $r_1'(t) > r_2'(t)$.

<u>Proposition 3</u> Assume (A3). Then there is a unique non-degenerate NE in which $e_1^* > e_2^* > 0$, $m > t_1^* > t_2^* > 0$ and $CB^* \in (0,1)$.

[16] Degenerate, autarky equilibria also occur in the strategic market Game general equilibrium models. Alternative specifications of what happens with zero bids/offers can avoid them. Here it is neater to leave them in. They will always be unstable in the standard best response dynamics (see Figure 2), and the subsequent focus on non-degenerate equilibria is natural.

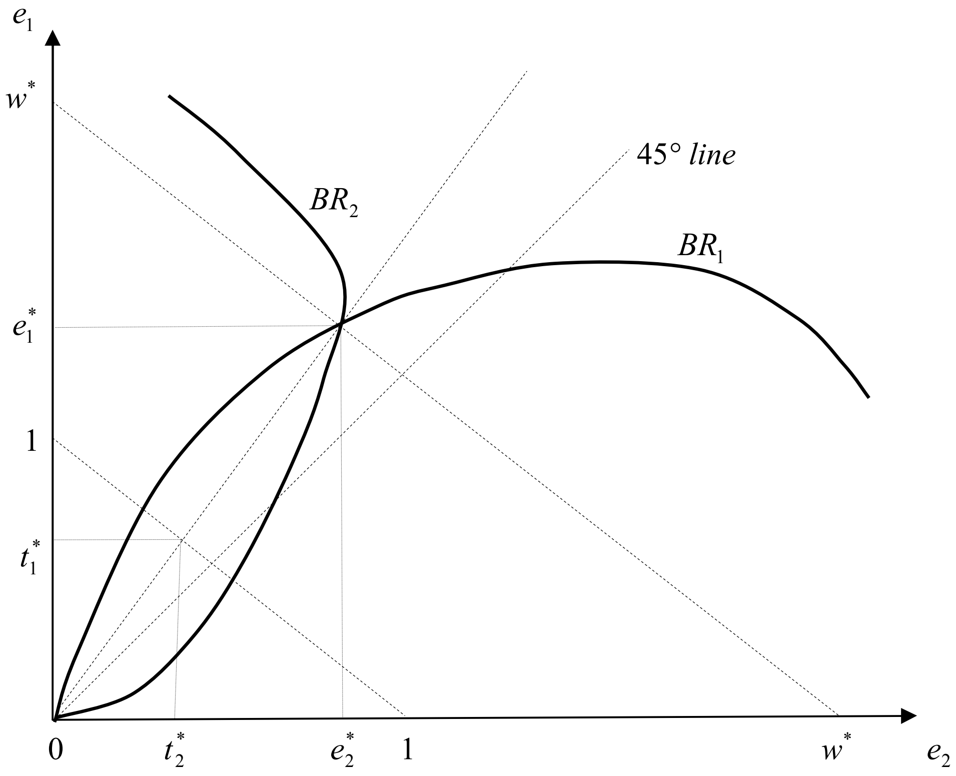


Figure 2; Best responses and NE

The best response problem of club *i* can be written with t_i as the choice variable. For a given e_j club *i* can be equivalently viewed as choosing t_i to:

$$\max r_i(t_i) - w(t_i, e_j)t_i \text{ where } w(t_i, e_j) = e_j/(1-t_i)$$

Here $w(t_i, e_j)$ is the wage that would emerge, given e_j, if *i* is to end up with t_i. When $t_i = 0$, $w(0, e_j) = e_j$, and as t_i increases $w(t_i, e_j)$ increases indicating the duopsony affect on wages – attempts by club *i* to increase its talent lead to an increase in the wage. Defining $mr_i = r_i'(t)$, $c_i(t_i, e_j) = w(t_i, e_j)t_i$ and $mc_i = \partial c_i / \partial t_i = e_j/(1-t_i)^2$, the best response condition $mr_i = mc_i$ leads to Figure 3, showing the talent allocation t_i and wage *w* that would result from *i*'s best response to e_j.

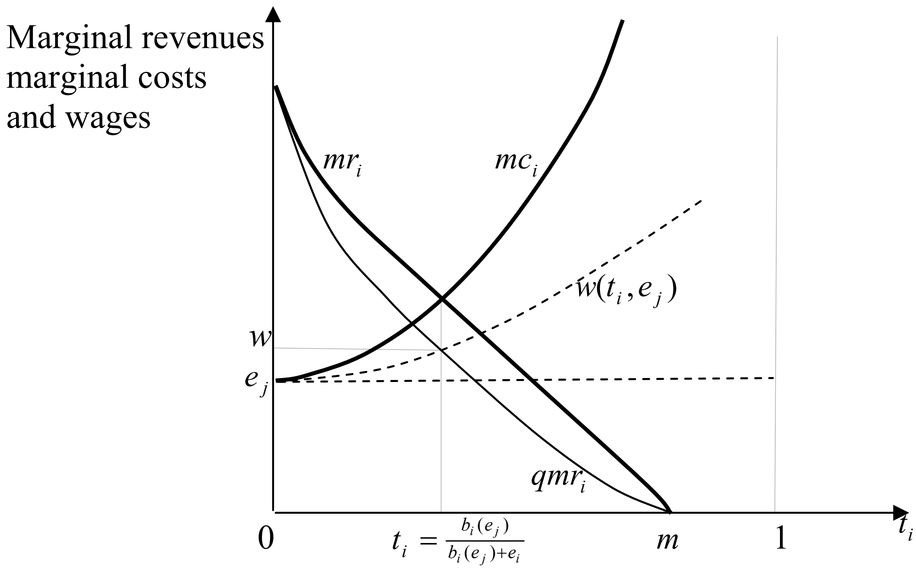


Figure 3; Marginal revenue, marginal cost and quasi-marginal revenue

As e_j varies the locus of the t_i and w pairs corresponding to i's best responses in Figure 3 is referred to as i's *quasi marginal revenue.* Rearranging $mr_i = mc_i$ with $w = e_j/(1-t_i)$ this is defined by:

$$qmr_i(t_i) \equiv r_i^{'}(t_i)(1-t_i) = w \tag{3.2}$$

A typical qmr_i curve is also shown in Figure 3. To repeat, qmr_i is the locus of t_i and w that would ensue from i's best response to e_j as e_j varies in the specified Game; it is not the best quantity of talent response to a parametric wage *w*. When $e_j = 0$, the corresponding point is *w*=0, $t_i = m$, and we move up the qmr_i curve as e_j increases. The terminology stems from the fact that a wage-taking (perfectly competitive), profit-maximizing club choosing t_i and facing the marginal revenue curve qmr_i would generate the same t_i and w pairs (demand curve for talent) as the expenditure choosing imperfect competitor. The effect of the imperfect competition is then *as if* a perfect competitor faced a fall in marginal revenue from mr_i to qmr_i, with the natural and intuitive fall in talent demand.

Non-degenerate NE can now be alternatively characterised by:

$$qmr_1(t_1) \equiv qmr_2(1-t_1) = w \tag{3.3}$$

From (A3), $qmr_1(t) > qmr_2(t)$ for all $t \in (0,1)$, and the existence of a unique non-degenerate NE with $m > t_1^* > t_2^* > 0$ (and so $e_1^* > e_2^* > 0$ and $CB^* \in (0,1)$) can now be alternatively seen in Figure 4.

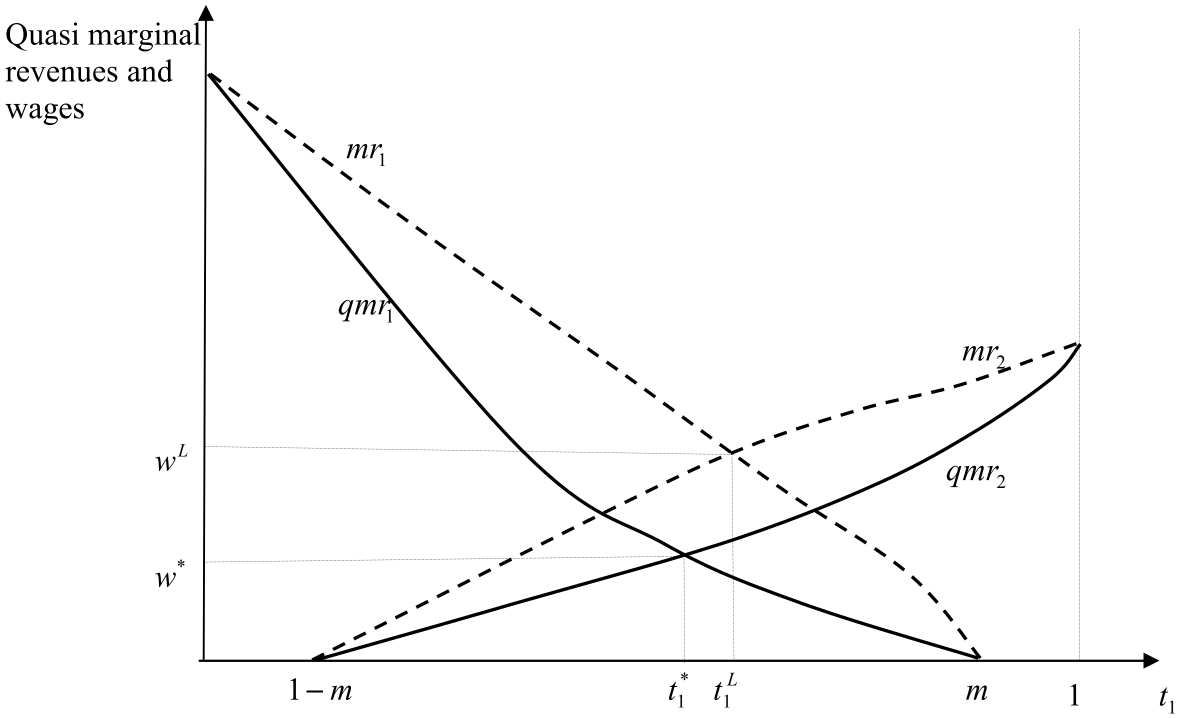


Figure 4; Quasi marginal revenues and NE

The framework of Figure 4 was also used in Madden (2009) to depict equilibria in a large sports league model where the number of clubs is sufficiently large that all strategic interactions between individual clubs disappear, and it is of interest to compare the outcomes. Define the equivalent large league model to consist of a unit mass of two types of clubs, each type 1 club with revenue function r_1 and type 2 with r_2, the rest of the model as here. Then, from Madden (2009) and as in textbook expositions, the equilibrium will be where $mr_1 = mr_2 = w$. Assuming type 1 clubs are bigger in the (A3) sense, the equilibrium will have $t_1 > t_2$, where t_i is the talent allocation to each type *i* club, as shown at w^L, t_1^L in Figure 4. At this talent allocation qmr_1 will be more below mr_1 than qmr_2 is below mr_2 (since $t_1 > t_2$), leading to the NE w^* and t_1^* shown[17]. Thus the equivalent large league model will have a smaller competitive balance and a higher wage than the current duopsony model.

Another useful benchmark for future comparisons is the cartel solution. If the league allocated talent and set wages to maximize aggregate profit, the talent allocation would be the same as in the equivalent large league but with a wage of zero[18].

Remark 2 Szymanski and Kesenne (2004), Szymanski (2004) and Kesenne (2007) arrive at condition (3.2) and the qmr_i curves via the following mathematics. Let

[17] In Figure 4, $mr_1 = mr_1(t_1)$, $mr_2 = mr_2(1-t_1)$, $qmr_1 = qmr_1(t_1)$ and $qmr_2 = qmr_2(1-t_1)$.

[18] Strictly speaking, an arbitrarily small positive wage is needed to induce the unit talent supply.

$\pi_i = r_i(\frac{t_i}{t_i+t_j}) - wt_i$; treating t_j (Nash conjectures) and w as parameters, $\partial\pi_i / \partial t_i = r_i^{'}(\frac{t_i}{t_i+t_j})t_j/(t_i+t_j)^2 - w$, and equating to zero in the inelastic case where $t_i + t_j = 1$ (see Remark 1 on other cases), produces the qmr_i curve $r_i^{'}(t_i)(1-t_i) = w$ and equilibrium diagrams similar to Figure 4. But this "equilibrium" is certainly not the Nash equilibrium of a normal form Game with strategies t_1 and t_2 and equilibrium wages $w = r_i^{'}(t_i)(1-t_i)$, for the reasons explained in the introduction. What has been shown is that it does however depict the Nash equilibrium of the normal form Game specified here. Aside from providing a coherent Game-theoretic foundation of the model, the change also leads to quite different interpretations. Consider for instance the above comparison of the equilibrium with that of the equivalent large league model. In the large league clubs face marginal revenue curves mr_1 and mr_2 and correctly perceive that they cannot influence the wage, leading to the large league equilibrium at the intersection of mr_1 and mr_2 in Figure 4. The actual situation with just two clubs is that they face the same mr_1 and mr_2 but with upward sloping marginal cost curves, because of their talent market power. This causes talent demand to fall and it is this duopsony effect that causes the lower equilibrium wage here compared to the large league. The equation of marginal revenue (mr_i) to marginal cost $(mc_i = w/(1-t_i)$ in equilibrium) produces equilibrium in the two-club model where both are higher for the big club, and so a talent allocation with greater competitive balance than in the large league. Thus whilst the quasi marginal revenue curves produce a convenient "as if" story, the actual difference between the large league and the two club league outcomes is because of the change on the cost side due to the talent market power of clubs in the latter (in contrast to Szymanski (2004)).

4. REVENUE SHARING

With the revenue sharing regulatory policy, home teams retain only the fraction $\alpha \in [\frac{1}{2},1]$ of their home gate revenue, the rest going to the away team. Payoffs are:

$$\pi_i(e_i, e_j; \alpha) = \alpha r_i(\tfrac{e_i}{e_i+e_j}) + (1-\alpha) r_j(\tfrac{e_j}{e_i+e_j}) - e_i,\ i = 1,2$$

The best response of club i is characterized by the first–order condition[19], generalising (3.1):

$$\alpha r_i^{'}(\tfrac{e_i}{e_i+e_j})\frac{e_j}{(e_i+e_j)^2} - (1-\alpha) r_j^{'}(\tfrac{e_j}{e_i+e_j})\frac{e_j}{(e_i+e_j)^2} = 1 \qquad (4.1)$$

The alternative quasi marginal revenue characterisation of (3.2) of the resulting (non-degenerate) NE becomes:

$$qmr_i(t_i;\alpha) \equiv \alpha r_i^{'}(t_i)(1-t_i) - (1-\alpha)(1-t_i) r_j^{'}(1-t_i) = w \qquad (4.2)$$

[19] As a function of the single variable e_i, π_i has a strictly negative second derivative at any stationary point, which must therefore be a global maximum.

As α falls from 1 the quasi marginal revenue curves also fall from their original positions in Figure 4. Below their original intersection qmr_2 falls by more than qmr_1 (since $t_1^* > t_2^*$) leading to a new (non-degenerate) NE with higher t_1^{RS} and lower wage w^{RS}, typically as shown in Figure 5. Writing (4.2) for $i = 1, j = 2$ and for $i = 2, j = 1$ with $t_2 = 1 - t_1$, and eliminating α produces the locus of revenue sharing equilibria:

$$w^{RS} = (1 - t_1)[\alpha r_1'(t_1^{RS}) - (1 - \alpha) r_2'(1 - t_1^{RS})]$$

The locus (shown in bold in Figure 5) starts at the original equilibrium when $\alpha = 1$, and slopes down to the right converging to the cartel solution as $\alpha \to \frac{1}{2}$. Thus revenue sharing leads to reductions in both wages and competitive balance.

Proposition 4 Assume (A3) and revenue sharing with home team share $\alpha \in [\frac{1}{2}, 1]$. As α falls from 1 to $\frac{1}{2}$ the (non-degenerate) Nash equilibrium competitive balance and wage decrease monotonically, converging to the cartel solution as $\alpha \to \frac{1}{2}$.

When $\alpha = \frac{1}{2}$, the clubs share the common objective of maximizing joint profits, and naturally the cartel solution appears as the NE; as α falls from 1 to $\frac{1}{2}$, the change in NE is monotonic, wages and competitive balance falling.

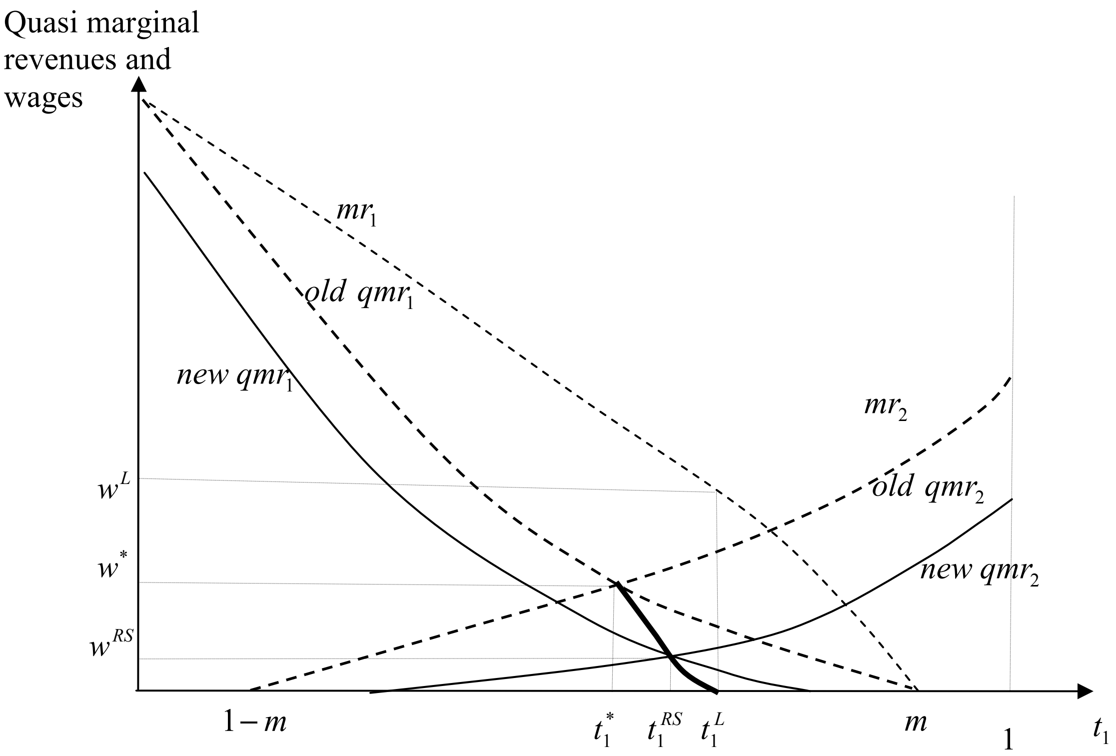


Figure 5; Quasi marginal revenues and NE with revenue sharing (equilibrium locus in bold)

Remark 3 Proposition 4 offers a similar conclusion to Szymanski and Kesenne (2004) in the inelastic supply case (see Remark 1 for other cases), now from a coherent Game-theoretic foundation. Again the interpretation is quite different. In the equivalent large league model the effect of revenue sharing is to cause the mr_i curves to fall by the same amount, so they continue to intersect at t_1^L, but now with a lower wage. This is the textbook invariance principle – revenue sharing has no affect on competitive balance but leads to a lower wage (see Madden (2009)). In the two club model here revenue sharing again causes the mr_i curves to fall by the same amount, δ say, so $mr_i < mc_i$ for both clubs at the original, pre-revenue sharing equilibrium w^*, t_1^*, leading to reductions in talent demand and again a lower wage in the equilibrium with revenue sharing. But now competitive balance is affected by revenue sharing. Reducing the wage so that it is consistent with a small club best response at the talent allocation t_1^*, i.e. so that $mr_2 - \delta = mc_2 = w/t_1^*$, implies that $mr_1 - \delta - mc_1 = mr_1 - mr_2 + w/t_1^* - w/(1-t_1^*) = (w^* - w)(\frac{1}{1-t_1^*} - \frac{1}{t_1^*}) > 0$. Thus the big club would then want to increase its talent allocation which leads to the reduced competitive balance at the revenue sharing equilibrium. Again the difference between the large and small league is because of the difference in their treatment of marginal costs, not marginal revenues as suggested in Szymanski and Kesenne (2004).

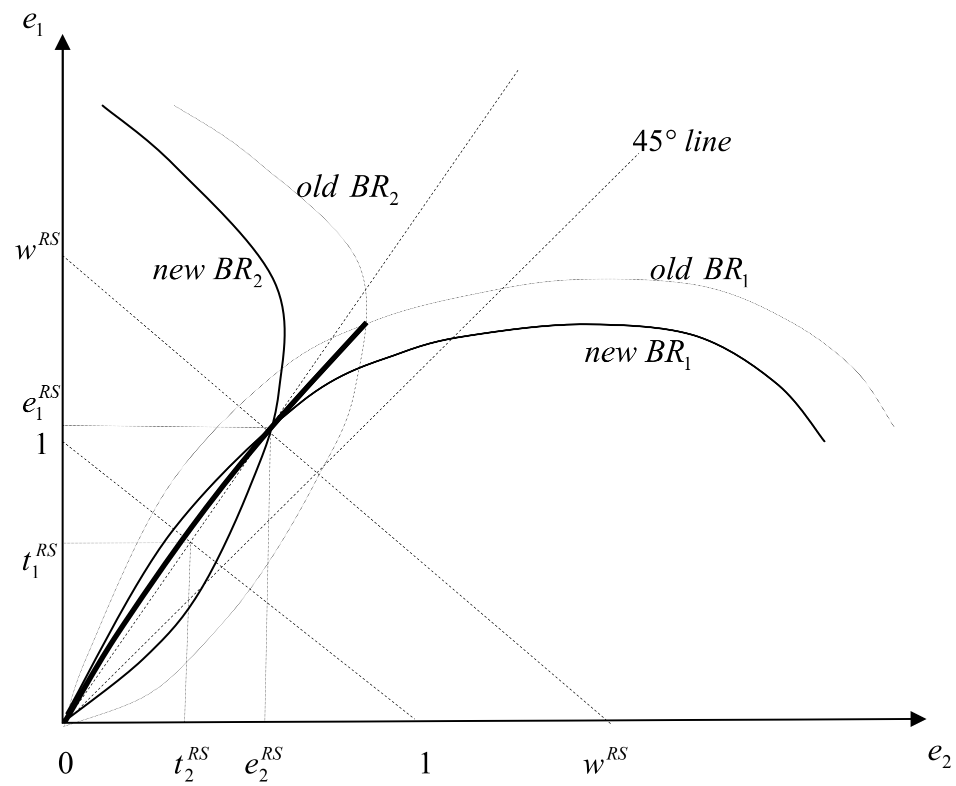


Figure 6; Best responses and NE with revenue sharing (equilibrium locus in bold)

Alternatively and equivalently to Figure 5, Figure 6 shows the effect of revenue sharing on actual best responses and NE. Revenue sharing causes the best response graphs to fall, with a smaller fall for the big club, leading to a new equilibrium with lower wages and lower competitive balance, the equilibrium locus again shown in bold.

5. SALARY CAPS

The second policy is an upper bound, *S* say, on the wage bills of clubs, a so-called salary cap. Payoffs remain as in (3.1) and the typical best response problem becomes:

$$\max \pi_i(e_i, e_j) = r_i\left(\tfrac{e_i}{e_i+e_j}\right) - e_i \text{ subject to } e_i \le S \qquad (5.1)$$

As $e_i = b_i(e_j)$ is the unique stationary point and global maximum of $\pi_i(e_i, e_j)$, the solution to (5.1) is min[$b_i(e_j)$,*S*]. If $S \ge e_1^*$ in Figure 2 the salary cap has no effect. For $S \in [\frac{1}{4}r_2'(\frac{1}{2}), e_1^*)$, there will be a new (non-degenerate) equilibrium in which the big club (only) is constrained by the cap, as shown in Figure 7.

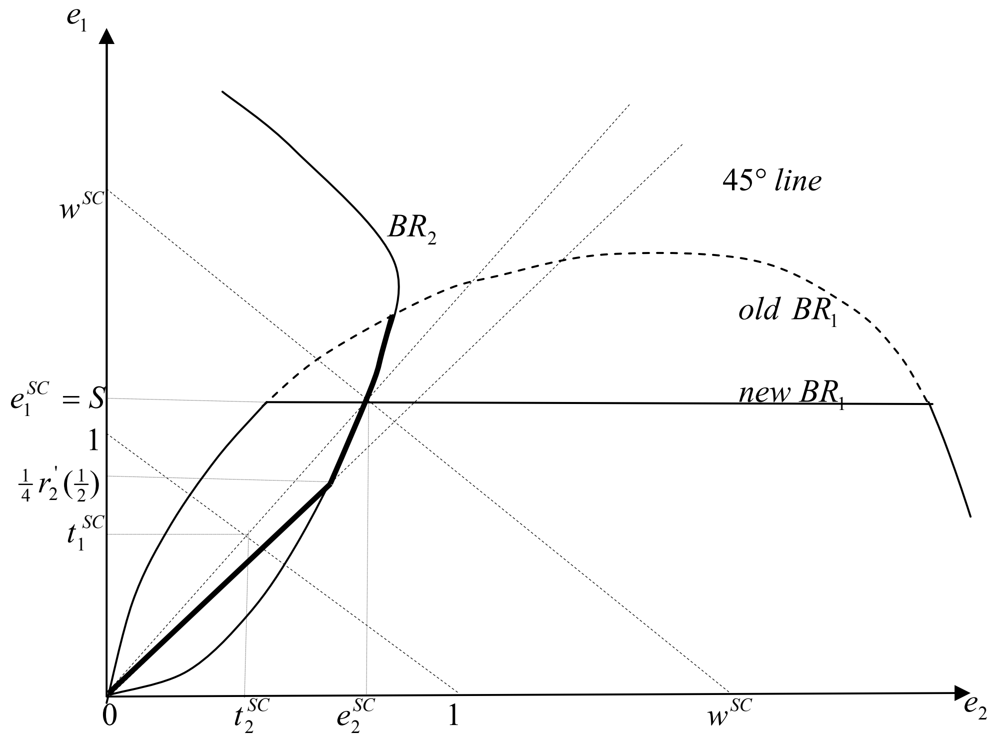
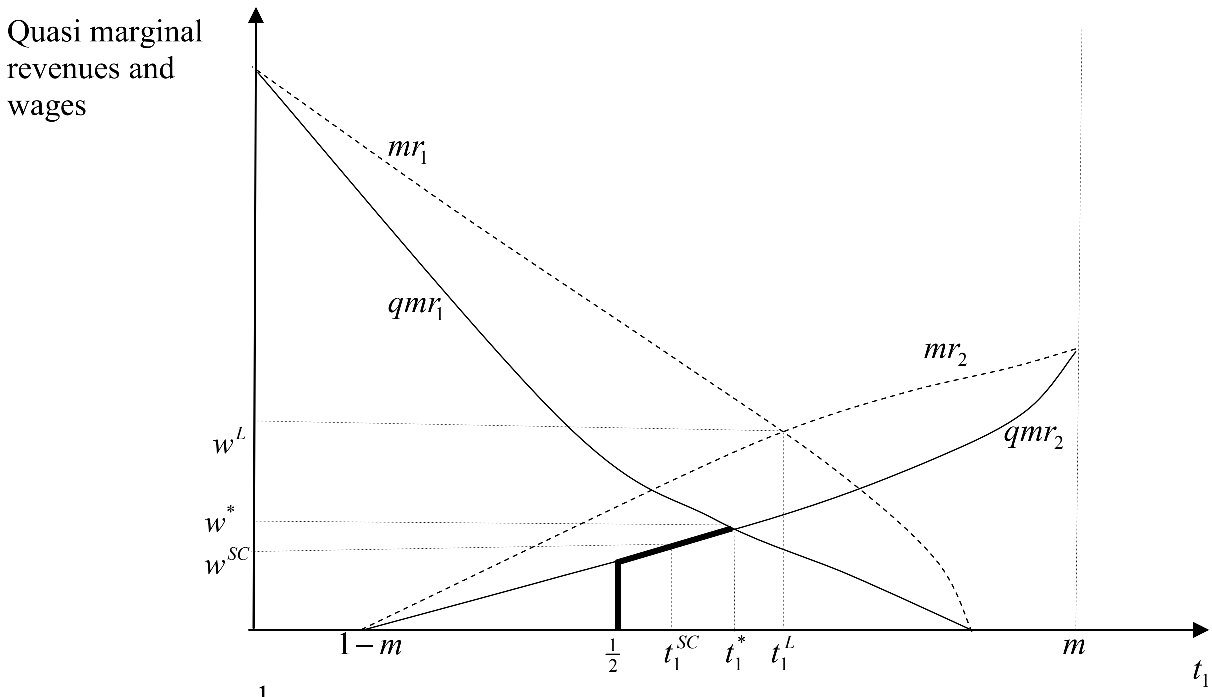


Figure 7; Best responses and NE with a salary cap (equilibrium locus in bold)

The new equilibrium follows the path of the original BR_2 down from the original equilibrium as *S* falls, until it reaches the 45° line when $S = \frac{1}{4}r_2'(\frac{1}{2})$. It follows from BR5(i) that the competitive balance *increases* now as *S* falls in this range, reaching its maximal level when $S = \frac{1}{4}r_2'(\frac{1}{2})$; it also follows that the wage is falling. Thereafter further reductions in S now bind on both clubs, causing the new equilibrium to follow the 45° line down to the origin when $S = 0$; in this range competitive balance remains at its maximal level, with continuing wage reduction.

<u>Proposition 5</u> Assume (A3). A salary cap $S \geq e_1^*$ will have no effect on Nash equilibria. As S falls in the range $[\frac{1}{4}r_2'(\frac{1}{2}), e_1^*)$, the (non-degenerate) equilibrium competitive balance increases reaching its maximal level when $S = \frac{1}{4}r_2'(\frac{1}{2})$, and the wage falls. Further reductions in S in the range $[0, \frac{1}{4}r_2'(\frac{1}{2}))$ further reduce the wage in the new equilibrium, leaving competitive balance at its maximum.

Alternatively and equivalently, Figure 8 provides the same information in terms of quasi marginal revenue curves, the locus of the equilibria with salary cap again shown in bold.



<u>Figure 8; Quasi marginal revenues and NE with a salary cap (equilibrium locus in bold)</u>

<u>Remark 4</u> The effect of a salary cap in the equivalent large league is qualitatively similar. In both cases a falling cap initially binds on only big clubs, effectively constraining downwards their talent demand, with talent demand from small clubs unaffected. In the large league the analogue of the bold locus in Figure 8 starts at w^L, t_1^L and follows mr_2 (instead of qmr_2) left until $t_1 = \frac{1}{2}$ after which further reductions in S merely reduce the wage. Szymanski and Kesenne (2004) and Szymanski (2004) do not address the salary cap issue. The conventional wisdom of the effect of a salary cap found in the textbooks is the above large league result (see Madden (2009))

6. A LEAGUE WITH MORE THAN TWO CLUBS

The previous model is now generalised so that the league consists of $n > 2$ profit-maximizing clubs who play each twice in a season, once at home and once away. Strategy sets are $e_i \in \Re_+, i = 1,...,n$, and there is a unit perfectly inelastic supply of playing talent as before. Letting $E \equiv \sum_{i=1}^{n} e_i$, the market Game ratio of bids to offers leads to the wage $w = E$ and talent allocations $t_i = e_i / w = e_i / E, i = 1,...,n$ when w>0; when w=0, $t_i = 0, i = 1,...,n$.

On the revenue side we think of clubs selling season tickets which allow entry to all home games over the season, and whose value to fans depends on the home team quality (t_i) and on the average quality[20] of the visiting teams ($\sum_{j \neq i}^{n} t_j /(n-1)$). If $E > 0$ the second variable is always $(1-t_i)/(n-1)$, and we can again write revenue as a function of a single variable, $r_i(t_i)$ or $r_i(\frac{e_i}{E})$. We assume $r_i(0) = 0$ which is also assumed to be the outcome if $E = 0$. As before, $r_i : [0,1] \to \Re_+$ is continuous, C^2 on $(0,1)$, strictly concave with a global maximum at some home team quality $m \in (\frac{1}{n},1)$ in excess of the league average.

The normal form of the n-Player Game is completed with the payoffs:

$$\pi_i(e_1,...,e_n) = r_i(\tfrac{e_i}{E}) - e_i, \; i = 1,...,n$$

The two-club best response characterization of (3.1) becomes (6.1), whilst that of (3.2) is unchanged as (6.2):

$$r_i'(\tfrac{e_i}{E})\tfrac{E-e_i}{E^2} = 1 \qquad (6.1)$$

$$qmr_i(t_i) \equiv r_i'(t_i)(1-t_i) = w \qquad (6.2)$$

Once again there will always be a degenerate NE, and a symmetric NE if all revenue functions are the same (as in Proposition 1). We jump to the asymmetric case and assume that club market sizes are as follows, generalising (A3):

(A4) For all $t \in (0,m), r_1'(t) > r_2'(t) > r_3'(t) > ... > r_n'(t)$.

---

[20] This assumption allows us to generalise the earlier two club analysis in a tractable way, with revenue as a primitive function of just two variables again. It is not the most general assumption that could be made, of course - the variance in other club talent levels might be a plausible third variable, or, most generally, revenue might be a function of the entire n-dimensional vector of league talent levels. I have not found any decisive results on the effects of revenue sharing or salary caps for a league with more than two clubs in the previous literature.

Thus club 1 is the biggest club in the usual marginal revenue sense, and club n the smallest. To simplify exposition and avoid equilibria where some small clubs have zero talent, we assume also that revenue functions satisfy an Inada condition:

(A5) $r_i^{'}(0) = +\infty$.

With more than 2 clubs there are numerous ways of defining competitive balance, based on the variance, range or other measures of the spread of the talent distribution. Here we use the range-based measure $CB(t_1,...,t_n) = 1 - [\max(t_1,...,t_n) - \min(t_1,...,t_n)]$, which, as earlier, has value zero when one club has all the talent and a maximum of 1 when talent is equally allocated.

Because of (A5), $t_i^* > 0, 1 = 1,..,n$ in any non-degenerate NE, which will therefore be $(t_1^*, t_2^*,..,t_n^*) \in \Re_{++}^n$ where $\sum_{i=1}^{n} t_i^* = 1$ and $w^* > 0$ such that (6.2) is satisfied for $i = 1,..,n$.
Proposition 3 has an exact analogue:

<u>Proposition 6</u> Assume (A4) and (A5). Then there is a unique non-degenerate NE in which $e_1^* > e_2^* > ... > e_n^* > 0$, $m > t_1^* > t_2^* > ... > t_n^* > 0$ and $CB^* \in (0,1)$.

<u>Proof</u> See appendix.

With revenue sharing where α ($\in [\frac{1}{n},1]$ now) continues to denote the share of revenue retained by the home team, the assumption of season ticket sales (only) means that attendance at all of a club's home games can be taken to be the same. Thus, as an away team each club will receive a fraction $(1-\alpha)/(n-1)$ of each other club's gate revenue[21], producing:

$$\pi_i(e_1,..,e_n;\alpha) = \alpha r_i(\tfrac{e_i}{E}) + \tfrac{1-\alpha}{n-1}\sum_{j \neq i} r_j(\tfrac{e_j}{E}) - e_i,\ i = 1,..,n$$

The earlier characterizations of best responses[22] in (4.1) becomes:

$$\alpha r_i^{'}(\tfrac{e_i}{E})\tfrac{E-e_i}{E^2} - \tfrac{1-\alpha}{n-1}\sum_{j \neq i} r_j^{'}(\tfrac{e_j}{E})\tfrac{e_j}{E^2} = 1 \qquad (6.3)$$

In terms of quasi marginal revenues, non-degenerate NE are $(t_1,..,t_n) \in \Re_{++}^n$ where $\sum_{i=1}^{n} t_i = 1$, $w > 0$ and for $i = 1,..,n$:

$$qmr_i(t_1,..,t_n;\alpha) \equiv \alpha r_i^{'}(t_i)(1-t_i) - \tfrac{1-\alpha}{n-1}\sum_{j \neq i} r_j^{'}(t_j)t_j = w \qquad (6.4)$$

---

[21] Because of the season ticket assumption, there is no difference here between pool sharing and gate sharing of revenues; see Kesenne (2007, chapter 6).

[22] As before, as a function of the single variable e_i, π_i has a strictly negative second derivative at any stationary point; this must therefore be a global maximum.

Notice first that from (6.4) revenue sharing continues to generate the cartel solution (zero wage and talent allocations that equalise marginal revenue) when $\alpha = \frac{1}{n}$. An almost complete generalisation of Proposition 4 now follows[23]:

<u>Proposition 7</u> Assume (A4), (A5) and revenue sharing with home team share $\alpha \in (\frac{1}{n}, 1]$.

(a) There exists a unique, non-degenerate NE in which $m > t_1^{RS} > t_2^{RS} > \ldots > t_n^{RS} > 0$ and $w^{RS} > 0$, converging to the cartel solution as $\alpha \to \frac{1}{n}$.

(b) As α falls from 1 to $\frac{1}{n}$ the (non-degenerate) Nash equilibrium competitive balance decreases monotonically.

(c) As α falls from 1 to $\frac{1}{n}$ the (non-degenerate) Nash equilibrium wage decreases monotonically at least for $1 \geq \alpha \geq 1 - \varepsilon$ for some $\varepsilon \in (0, 1 - \frac{1}{n}]$.

<u>Proof</u> See appendix

The two club results on salary caps fully generalise in a natural way:

<u>Proposition 8</u> Assume (A4), (A5) and a salary cap *S*. There exist critical values of the cap $S_1 > \ldots > S_n > 0$ such that, if $S \in [S_{c+1}, S_c)$ then there exists a unique (non-degenerate) NE in which the salary cap is binding on the biggest *c* clubs. As *S* falls from S_1 to S_n the equilibrium wage decreases monotonically, and the equilibrium competitive balance increases monotonically, reaching its maximal level at S_n. As *S* falls from S_n to 0, competitive balance remains at the maximal level and the wage continues to decrease to 0 at $S = 0$.

<u>Proof</u> See appendix

[23] The incompleteness, in part (c), is that although wages fall as α falls from 1 locally, it is not clear that this wage fall is global over the entire range $\alpha \in [\frac{1}{n}, 1]$, although the wage has declined to zero at $\alpha = \frac{1}{n}$.

7. CONCLUSIONS

The paper provides a model of a sports league with a finite number of profit-maximizing clubs where the supply of playing talent to the league is perfectly inelastic. A normal form Game is defined where, plausibly in the context and derived from the strategic market Game literature, club expenditures on playing talent are the strategies; clubs have oligopsony power in the talent market, argued to be a necessary feature of such a model but one that is missing from the previous literature. In a two-club league and in a generalisation to more than two clubs, the results show that revenue sharing and salary caps are completely opposite in their effects on competitive balance, revenue sharing creating reductions in competitive balance whilst salary caps increase it; both policies tend to reduce the wage for playing talent.

The conclusions offer support for the use of salary caps as a way of increasing competitive balance, but raise serious questions about the use of revenue sharing as a regulatory device in a sports league, the latter similar to Szymanski and Kesenne (2004) and Szymanski (2004). The difference here is that all conclusions have been reached via the study of Nash equilibrium in a fully specified normal form game that determines the allocation of playing talent and its wage. They cannot be reached satisfactorily if clubs treat wages as parametric in formulating decisions, or if quantities of talent rather than talent expenditures are the strategies. It is hoped that the paper will provide a framework for the investigation of further issues in sports leagues involving strategic interactions between clubs[24].

[24] Some of the issues that come to mind for analysis in the framework of this paper with its US (inelastic talent supply) focus are: the effects of alternative owner objectives, such as win-maximization as opposed to profit-maximization, following Kesenne (2007); the impact of bargaining between club owners and player trade unions, endowing the player side with some market power, absent here and in the previous literature but relevant to the major US sports leagues; competition between two (or more) leagues for the inelastic talent supply, which would allow applications to European soccer with its inter-league competition for players. Another agenda would follow from relaxation of the inelastic talent supply assumption itself. The paper provides a first step that opens up a rich array of possibilities for further analysis of the peculiar sports league industry.

APPENDIX

Proof of Proposition 6 Let $d_i : \Re_+ \to [0,m]$ denote the inverse function to $qmr_i : [0,m] \to \Re_+$. d_i has the properties (i) $d_i^{'}(w) < 0,\ w \in \Re_{++}$, (ii) $d_i(\infty) = 0$ and (iii) $d_i(0) = m$. Hence $d(w) = \sum_{i=1}^{n} d_i(w)$ has the properties (i) $d^{'}(w) < 0,\ w \in \Re_{++}$, (ii) $d(\infty) = 0$ and (iii) $d(0) = mn > 1$. It follows that there is a unique $w^* \in \Re_{++}$ where $d(w^*) = 1$. Defining $t_i^* = d_i(w^*),\ i = 1,..,n$ ensures that $w^*, t_1^*,..,t_n^*$ is the unique non-degenerate NE.

Proof of Proposition 7

(a) Define $s_i : [0,m] \to \Re_+$ by $s_i(t_i) \equiv r_i^{'}(t_i)[\alpha + \frac{1-\alpha n}{n-1} t_i]$, and let $f_i : \Re_+ \to [0,m]$ denote its inverse and $f(\omega) = \sum_{i=1}^{n} f_i(\omega)$. Then (i) $f^{'}(\omega) < 0,\ \omega \in \Re_{++}$, (ii) $f(\infty) = 0$, (iii) $f(0) = nm > 1$ and there exists a unique $\omega^{RS} \in \Re_{++}$ where $f(\omega^{RS}) = 1$. Defining $t_i^{RS} = f_i(\omega^{RS}),\ i = 1,..,n$ and $w^{RS} \equiv r_i^{'}(t_i^{RS})[\alpha + \frac{1-\alpha n}{n-1} t_i^{RS}] - \frac{1-\alpha}{n-1} \sum_{j=1}^{n} r_j^{'}(t_j^{RS}) t_j^{RS}$, it follows that (6.4) is satisfied for all $i = 1,..,n$ and we have the claimed equilibrium provided $w^{RS} > 0$. Using $s_1(t_1^{RS}) = s_j(t_j^{RS}),\ j = 1,..,n$ in the formula for w^{RS} leads to the alternative expression;

$$w^{RS} = (\alpha n - 1) \sum_{j \neq 1} t_j^{RS} r_j^{'}(t_j^{RS}) \left[\frac{\alpha(n-1)(1-t_j^{RS}) + (1-\alpha) t_1^{RS}}{\alpha(n-1) + (1-\alpha n) t_1^{RS}} \right]$$

When $\alpha > \frac{1}{n}$, $w^{RS} > 0$ and as $\alpha \to \frac{1}{n}$, $w^{RS} \to 0$ with $r_i^{'}(t_i^{RS}) = r_j^{'}(t_j^{RS}),\ i, j = 1,..,n$.

(b) With derivatives evaluated at the revenue sharing equilibrium for some $\alpha \in (\frac{1}{n}, 1]$, define;

$a_i = r_i^{''}(t_i)(\alpha + \frac{1-\alpha n}{n-1} t_i) + \frac{1-\alpha n}{n-1} r_i^{'}(t_i) < 0$ and $b_i = (1 - \frac{n}{n-1} t_i) r_i^{'}(t_i),\ i = 1,..,n$

Differentiating the equilibrium conditions $s_1(t_1^{RS}) = s_j(t_j^{RS}),\ j = 1,..,n$, treating talent allocations as functions of α, produces; $a_1 \frac{dt_1^{RS}}{d\alpha} + b_1 = a_j \frac{dt_j^{RS}}{d\alpha} + b_j,\ j = 2,..,n$. Using $\sum_{i=1}^{n} \frac{dt_i^{RS}}{d\alpha} = 0$ then gives;

$$\frac{dt_1^{RS}}{d\alpha}\left(1 + \frac{a_1}{a_2} + \frac{a_1}{a_3} + ... + \frac{a_1}{a_n}\right) + \frac{b_1 - b_2}{a_2} + \frac{b_1 - b_3}{a_3} + ... + \frac{b_1 - b_n}{a_n} = 0$$

Hence $\frac{dt_1^{RS}}{d\alpha}$ has the sign of $-\left(\frac{b_1 - b_2}{a_2} + \frac{b_1 - b_3}{a_3} + ... + \frac{b_1 - b_n}{a_n}\right)$. Using again the equilibrium conditions $s_1(t_1^{RS}) = s_j(t_j^{RS}),\ j = 1,..,n$, shows that for $j = 2,..,n$;

$$b_j - b_1 = r_1^{'}(t_1^{RS}) \left\{ \frac{\alpha(n-1) + (1-\alpha n) t_1^{RS}}{\alpha(n-1) + (1-\alpha n) t_j^{RS}} \left(1 - \frac{n}{n-1} t_j^{RS}\right) - \left(1 - \frac{n}{n-1} t_1^{RS}\right) \right\}$$

$$>0 \text{ if } \phi(t_j^{RS}) > \phi(t_1^{RS}) \text{ where } \phi(t) \equiv (1 - \tfrac{n}{n-1} t) / [\alpha(n-1) + (1-\alpha n) t]$$

But $\phi^{'}(t) < 0$, and so $b_j - b_1 > 0$ for $j = 2,..,n$ since $t_j - t_1 < 0$, $j = 2,..,n$. Hence $\frac{dt_1^{RS}}{d\alpha} < 0$. But the argument can be repeated interchanging 1 and n, to show $\frac{dt_n^{RS}}{d\alpha} > 0$ since $t_j - t_n > 0$, $j = 1,..,n-1$. Hence competitive balance decreases as α decreases.

(c) Differentiating (6.4) with $i=1$, treating talent allocations as functions of α, produces;

$$\tfrac{dw^{RS}}{d\alpha} = r_1'(t_1^{RS})(1-t_1^{RS}) + \alpha \tfrac{dt_1^{RS}}{d\alpha}[r_1''(t_1^{RS})(1-t_1^{RS}) - r_1'(t_1^{RS})]$$
$$+\tfrac{1}{n-1}\sum_{j\neq 1} r_j'(t_j^{RS})t_j^{RS} - \tfrac{1-\alpha}{n-1}[\sum_{j\neq i}(r_j''(t_j^{RS})t_j^{RS} + r_j'(t_j^{RS}))\tfrac{dt_j^{RS}}{d\alpha}]$$

From (b) above the first three terms on the right hand side are strictly positive, and the last term is zero at $\alpha = 1$. Hence for $\alpha \in [1-\varepsilon, 1]$ with some $\varepsilon \in (0, \tfrac{1}{n}]$, $\tfrac{dw^{RS}}{d\alpha} > 0$.

Proof of Proposition 8 With d_i as defined in the proof of Proposition 6, the critical values $S_c, c = 1,..,n$ are defined by $1-\sum_{i=c+1}^{n} d_i(w) = cd_c(w)$ which implies a unique $w = w_c$ say, and then $S_c = w_c d_c(w_c)$; for instance, when $c=1$, $w_1 = w^*$ and $S = e_1^*$, and when $c = n$, w_n is defined by $d_n(w_n) = \tfrac{1}{n}$ so $w_n = qmr_n(\tfrac{1}{n})$ and $S_n = \tfrac{1}{n}qmr_n(\tfrac{1}{n})$. It follows from (A4) that $S_c > S_{c+1}, c = 1,..,n-1$, and hence $S_1 > ... > S_n > 0$. (A4) also ensures that the only equilibria that are possible for $S \in [S_n, S_1)$ are equilibria where the biggest *c* clubs are salary cap constrained, for some $c = 1,..,n-1$, which requires $t_i = [1-\sum_{i=c+1}^{n} d_i(w)]/c, i = 1,..,c,$ and $t_i = d_i(w)$, $i = c+1,..,n$ with:

(1) $[1-\sum_{i=c+1}^{n} d_i(w)]/c < d_i(w), i = 1,..,c$

(2) $d_i(w) \leq [1-\sum_{i=c+1}^{n} d_i(w)]/c, i = c+1,..,n$

(3) $S = w[1-\sum_{i=c+1}^{n} d_i(w)]/c$

At $S = S_c$ and $w = w_c$, (1)-(3) are satisfied except that (1) holds with equality for $i = c$. But reducing *S* will reduce *w* from (3), make all inequalities in (1) strict and so produce an equilibrium with biggest *c* clubs constrained provided (4) holds which ensures (2);

(4) $d_{c+1}(w) \leq [1-\sum_{i=c+1}^{n} d_i(w)]/c$

Rearranging, (4) becomes $d_{c+1}(w) \leq [1-\sum_{i=c+2}^{n} d_i(w)]/(c+1)$, which holds as long as $w \geq w_{c+1}$, or $S \geq S_{c+1}$. It follows that equilibrium with the biggest *c* clubs constrained exists if and only if $S \in [S_{c+1}, S_c)$, and that as *S* falls in this range so does *w*. Moreover competitive balance is $1-(t_1 - t_n) = 1 + d_n(w) - [1-\sum_{i=c+1}^{n} d_i(w)]/c$ which increases as *w* and *S* fall. Finally, for $S \in [0, S_n)$ the unique equilibrium has equal talent allocations and $w = nS$.

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