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From strategic to Price Taking Behavior

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Abstract We address the asymptotic convergence of active Nash equilibria of strategic market games to Walrasian ones for general sequences of economies whose distribution of characteristics has compact support.

Key words strategic behavior, price taking, convergence.
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1 Introduction

It is widely accepted among economists that in a system of markets where individual participants are small relative to the market size, individuals have a negligible effect on the determination of market outcomes, so they may be thought of as exhibiting a 'price taking' behavior. Of course, in order to make sense of this statement one has to attach a meaning to 'a small individual relative to market size'. In this way we would be able to distinguish when price taking is a reasonable assumption and when it is not. In other words, the centrality of the price taking hypothesis in economics calls for a formal justification for it -a 'theory of competition' so to speak.

One of the tools of economic theory to achieve this is the study of asymptotic convergence of equilibrium outcomes of finite economies, when the number of individuals increases without limit. The idea is that if we can identify conditions under which equilibrium outcomes of finite economies converge asymptotically (in some sense) to Walrasian ones, then we would have a picture of when individuals have negligible effect on market outcomes and hence when 'price taking' can be justified as a reasonable hypothesis. Another motivation for the asymptotic study is the equivalence between 'large but finite' and 'atomless' economies. In atomless economies 'negligibility' is built in the non atomicity of the measure of the space of agents.

If a large finite economy is to be thought as a reasonable substitute of the idealized continuum model, it should be the case that equilibrium outcomes of a large finite economy are close to those of the atomless limit, i.e., the equilibrium outcomes of the former should asymptotically converge (in some sense) to those of the latter as the number of individuals increases. Asymptotic studies have been performed for a variety of equilibrium notions in finite economies. The bulk of those studies focus on the asymptotic limits of the core[1] and Nash equilibria, mainly because those notions are associated with the traditional theories of Edgeworth and Cournot, which are prevalent in economic theory.

In this paper we study the asymptotic convergence of Nash equilibria of strategic market games to Walrasian ones. This issue has been addressed by several authors, Dubey and Shubik (1978), Postlewaite and Schmeidler (1981), Mas-Colell (1982), Peck and Shell (1989), Sahi and Yao (1989), Amir et. al. (1990) among many others, albeit in the fragile context of sequences of economies obtained through replication. Besides the particularity of this type of sequences (finite number of types of individual characteristics), the above results are shown only for 'type symmetric' Nash equilibria.[2] By contrast, our results apply to general sequences of economies with characteristics drawn from compact sets. In this way we address the issue of asymptotic convergence of Nash equilibria, at the same generality as the well known core convergence results.

2 The model

Let H be a finite set of agents. There are L commodity types in the economy and the consumption set of each agent is identified with $\Re^L_+$. Each individual $h \in H$ is characterized by a preference relation $\succeq_h \subset \Re^L_+ \times \Re^L_+$ and an initial endowment $e_h \in \Re^L_+$. We use the following assumption:

Assumption 1 *Preferences are convex and strictly monotone.*

Denote by $\mathcal{P}_{cm}$ the set of preferences that satisfy (1) endowed with the topology of closed convergence. Let $T \subset \mathcal{P}_{cm} \times \Re^L_+$. An economy is defined as a mapping $\mathcal{E} : H \to T$.

We now turn to describe a strategic market game, which proposes an explicit model of how exchange in the economy takes place.

2.1 Trade using inside money

We will develop our results for the strategic market game version appearing in Postlewaite and Schmeidler (1978) and in Peck et. al. (1992) which is described below.

[1] See Anderson (1992) for a survey of core equivalence results and the references therein.

[2] Note that type symmetry is a property in the case of the core, but not of Nash equilibria.

Trade in the economy is organized via a system of trading posts where individuals offer commodities for sale and place bids for purchases of commodities. Bids are placed in terms of a unit of account. The strategy set of each agent is $S_h = \{(b_h, q_h) \in \Re_+^{2L} : q_h^i \leq e_h^i,\ i = 1, 2, \ldots, L\}$. Given a strategy profile $(b, q) \in \prod_{h \in H} S_h$ let $B^i = \sum_{h \in H} b_h^i$ and $Q^i = \sum_{h \in H} q_h^i$ denote aggregate bids and offers for each $i = 1, 2, \ldots, L$. Also for each agent h denote $B_{-h}^i = \sum_{k \neq h} b_k^i$, $Q_{-h}^i = \sum_{k \neq h} q_k^i$. For a given a strategy profile, the consumption of consumer $h \in H$ is determined by $x_h = e_h + z_h\,(b, q)$, where for $i = 1, 2, \ldots, L$:

$$z_h^i\,(b, q) = \begin{cases} \frac{b_h^i}{B^i} Q^i - q_h^i & \text{if } \sum_{i=1}^L \frac{B^i}{Q^i} q_h^i \geq \sum_{i=1}^L b_h^i \\ -q_h^i & \text{otherwise} \end{cases} \tag{1}$$

and it is postulated that whenever the term $0/0$ appears in the expressions above it is defined to equal zero. When $B^i Q^i \neq 0$ the fraction $\pi^i(b, q) = \frac{B^i}{Q^i}$ has a natural interpretation as the (average) market clearing 'price'. The relation $\sum_{i=1}^L \pi^i(b, q) q_h^i \geq \sum_{i=1}^L b_h^i$ is a 'bookkeeping' restriction which ensures that units of account remain at zero net supply (inside money). The interpretation of this allocation mechanism is that commodities (money) is distributed among non bankrupt consumers in proportion to their bids (offers), while the purchases of bankrupt consumers are confiscated.

An equilibrium is defined as a strategy profile $(b, q) \in \prod_{h \in H} S_h$ that forms a Nash equilibrium in the ensuing game with strategic outcome function given by (1). Let $\mathbf{N}(\mathcal{E}) \subset \prod_{h \in H} S_h$ denote the set of Nash equilibrium strategy profiles of the strategic market game and $\mathcal{N}(\mathcal{E}) \subset \Re_+^{LH}$ the set of consumption allocations corresponding to the elements of $\mathbf{N}(\mathcal{E})$.

The following notation and familiar facts will be useful in the sequel. Fix $(b_{-h}, q_{-h}) \in \prod_{k \neq h} S_k$ and let[3] $g_h(y) = \sum_{i=1}^L \frac{B_{-h}^i (y^i - e_h^i)}{Q_{-h}^i + e_h^i - y^i}$. The set of allocations which an individual $h \in H$ can achieve via the strategic outcome function is given[4] by the convex set

$$c_h = \{y \in \Re_+^L : g_h(y) \leq 0, y \leq Q_{-h} + e_h\}$$

i.e., $(b_h, q_h) \in S_h \Rightarrow e_h + z_h\,(b, q) \in c_h$. Conversely, $x_h \in c_h \Rightarrow \exists (b_h, q_h) \in S_h$ *s.t.* $x_h = e_h + z_h\,(b, q)$. Therefore, due to the bankruptcy rule, at an equilibrium with nonzero bids and offers we have that $\bar{x} \in \mathcal{N}(\mathcal{E})$ if and only if:

$$\forall h \in H,\ \bar{x}_h \in c_h \text{ and } c_h \cap \{y \in \Re_+^L : y \succ_h \bar{x}_h\} = \emptyset \tag{2}$$

We say that $\bar{x} \in \mathcal{N}(\mathcal{E})$ is fully active if for the corresponding $(\bar{b}, \bar{q}) \in \mathbf{N}(\mathcal{E})$ we have $\pi(\bar{b}, \bar{q}) \gg 0$, i.e., there is trade in all commodities. In the sequel we will focus on such equilibria. [5]

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[3] In order to save on notation we omit the dependency on (b_{-h}, q_{-h}). In the results the values of those variables will be fixed so no confusion should arise.

[4] This is obtained by a straightforward manipulation of (1); see [15], [14] or [10].

[5] Alternatively we could consider the subset of commodities L' for which there is active trade.

2.2 Strategic vs price taking behavior

Let us fix a fully active $\bar{x} \in \mathcal{N}(\mathcal{E})$ corresponding to a strategy profile $(\bar{b}, \bar{q}) \in \mathbf{N}(\mathcal{E})$. Consider one $h \in H$ and denote $\bar{z}_h = \bar{x}_h - e_h$.

The monotonicity of preferences implies that $g_h(\bar{x}_h) = 0$, i.e., $\bar{x}_h$ lies on the boundary of the convex set c_h, which is C^2. Since preferences are also convex, by the separating hyperplane theorem there is a $p_h \in \Re^L_+$, specifically $p_h = Dg_h(\hat{x}_h)$, such that and

$$w \succeq_h \bar{x}_h \Rightarrow p_h w \geq p_h \bar{x}_h \text{ and } w \in c_h \Rightarrow p_h w \leq p_h \bar{x}_h \tag{3}$$

Using the definition of c_h we have

$$p_h = Dg_h(\bar{x}_h) = \left(\frac{\bar{B}^i_{-h} \bar{Q}^i_{-h}}{(\bar{Q}^i_{-h} - \bar{z}^i_h)^2} \right)^L_{i=1} = \left(\pi^i(\bar{b}, \bar{q}) \frac{\bar{Q}^i_{-h}}{(\bar{Q}^i_{-h} - \bar{z}^i_h)} \right)^L_{i=1} \tag{4}$$

Now observe that if for some $\lambda_h > 0$, $p_h = \lambda_h \pi(\bar{b}, \bar{q})$ then the behavior of such an individual would be identical to price taking at the market clearing prices $\pi(\bar{b}, \bar{q})$. To see this notice that because $\pi(\bar{b}, \bar{q}) \gg 0$ ($\bar{x}$ is active) there is a cheaper point, i.e., $w \in \Re^L_+$ with $\pi(\bar{b}, \bar{q})w < \pi(\bar{b}, \bar{q})\bar{x}_h = \pi(\bar{b}, \bar{q})e_h$. Since furthermore preferences are convex, the first part of (3) implies $y \succ_h \bar{x}_h \Rightarrow \pi(\bar{b}, \bar{q})y > \pi(\bar{b}, \bar{q})e_h$. Finally, $\pi(\bar{b}, \bar{q})\bar{x}_h = \pi(\bar{b}, \bar{q})e_h$.

Therefore, the measurement

$$\delta_h(\bar{x}) = \max \left\{ \mid \frac{p^i_h}{p^j_h} \cdot \frac{\pi^j(\bar{b}, \bar{q})}{\pi^i(\bar{b}, \bar{q})} - 1 \mid : i, j = 1, 2, \ldots, L \right\} \tag{5}$$

evaluated at a strategic market equilibrium, serves as an indicator of 'how far' the strategic behavior of individual h falls from price taking.[6] Clearly, for each agent h we have $\delta_h(\bar{x}) \geq 0$. Obviously, we are at a Walrasian equilibrium if (and only if) $\delta_h(\bar{x}) = 0$ for each agent h. Therefore, a sequence of market game price-allocation pairs tends to become a price taking one, if (and only if) the above indicator tends to zero for all individuals.

We are ready now to proceed with the results of this paper.

3 Results

For the results that follow we consider a sequence $\{\mathcal{E}_n\}_{n \in N}$ of economies $\mathcal{E}_n : H_n \to \mathcal{P}_{cm} \times [0, r]^L$, where $\#H_n \to \infty$, $\lim \frac{1}{\#H_n} \sum_{h \in H_n} e_h \gg 0$ and associated $x_n \in \mathcal{N}(\mathcal{E}_n)$, for each $n \in N$ which are fully active. Let $(b_n, q_n) \in \mathbf{N}(\mathcal{E}_n)$, be the corresponding strategies and $z_{n,h} = x_{n,h} - e_h$ the corresponding net trades for each $h \in H$.

The following result is shown in Koutsougeras (2007) and its proof applies unchanged here.

[6] In the case of C^2 preferences, the indicator $\delta_h(\cdot)$ coincides with $\gamma_h(\cdot)$ in Koutsougeras (2007).

Theorem 1 *For each $\epsilon > 0$, there is an $n_\epsilon \in N$ so that for all $n > n_\epsilon$*

$$\frac{1}{\#H_n} \cdot \#\{h \in H_n : \delta_h(x_n) > \epsilon\} < \epsilon$$

or equivalently

$$\frac{1}{\#H_n} \cdot \#\{h \in H_n : \delta_h(x_n) > \epsilon\} \to 0$$

Lemma 1 *Define $A(k) = \Big\{h \in H_n : \mid z^i_{n,h} \mid \leq kr \text{ for } i = 1, 2, \ldots, L\Big\}$, where $k \geq L - 1$. Then $\frac{1}{\#H_n} \#A(k) \geq 1 - \frac{L}{k+1}$.*

Proof :
Define $T_i(k) = \Big\{h \in H_n : \mid z^i_{n,h} \mid > k \cdot r\Big\}$, for $i = 1, 2, \ldots, L$. Notice that $T_i(k) = \Big\{h \in H_n : z^i_{n,h} > k \cdot r\Big\} \cup \Big\{h \in H_n : z^i_{n,h} < -k \cdot r\Big\}$. But, the second term is empty so $T_i(k) = \Big\{h \in H_n : z^i_{n,h} > kr\Big\}$.
Also notice that $\#A(k) = \#H_n - \#\left(\bigcup_{i=1}^L T_i(k)\right)$.
From the definition of $T_i(k)$ it follows that:

$$\begin{aligned}
\#T_i(k) \cdot k \cdot r &< \sum_{h \in T_i(k)} z^i_{n,h} \\
&= Q^i_n \cdot \sum_{h \in T_i(k)} \frac{b^i_{n,h}}{B^i_n} - \sum_{h \in T_i(k)} q^i_{n,h} \\
&\leq Q^i_n - \sum_{h \in T_i(k)} q^i_{n,h} \\
&= \sum_{h \neq T_i(k)} q^i_{n,h} \\
&\leq (\#H_n - \#T_i(k)) \cdot r
\end{aligned}$$

Therefore, we conclude that $\#T_i(k) < \#H_n \cdot \frac{1}{k+1}$
from which it follows that $\#\left(\bigcup_{i=1}^L T_i(k)\right) \leq \sum_{i=1}^L T_i(k) < \#H_n \cdot \frac{L}{k+1}$.
Hence, we have that $\#A(k) = \#H_n - \#\left(\bigcup_{i=1}^L T_i(k)\right) \geq \#H_n \cdot \left(1 - \frac{L}{k+1}\right)$
□

Lemma 2 *Consider $1 \leq i \leq L$ and suppose that $\lim_{n\to\infty} \frac{1}{\#H_n} \sum_{h \in H_n} e^i_h = a > 0$. There is a subsequence (still indexed by n), and $\epsilon > 0$ such that $\#\{h \in H_n : x^i_{n,h} \geq \epsilon\} \geq \#H_n \epsilon$.*

Proof :
Suppose not. Then for each $\epsilon > 0$ we have $\frac{1}{\#H_n} \#\{h \in H_n : x^i_{n,h} \geq \epsilon\} < \epsilon$ for n large enough. For each $M > 0$ consider a truncation $\{x^{i,M}_n\}_{M \in N}$ of the original sequence:

$$x_{n,h}^{i,M} = \begin{cases} x_{n,h}^{i} & \text{if } x_{n,h}^{i} < M \\ M & \text{otherwise} \end{cases} \tag{6}$$

This sequence is non decreasing, $x_n^{i,M} \leq x_n^i \ \forall M \in N$ and $x_n^{i,M} \to x_n^i$ as $M \to \infty$.

Given $0 < \epsilon < M$ we have that:

$$\begin{aligned}
\frac{1}{\#H_n} \sum_{h \in H_n} x_{n,h}^{i,M} &= \frac{1}{\#H_n} \Big(\sum_{\{h \in H_n : x_{n,h}^{i,M} \geq \epsilon\}} x_{n,h}^{i,M} + \sum_{\{h \in H_n : x_{n,h}^{i,M} < \epsilon\}} x_{n,h}^{i,M} \Big) \\
&< \frac{1}{\#H_n} (M \#\{h \in H_n : x_{n,h}^{i,M} \geq \epsilon\} + \epsilon \#\{h \in H_n : x_{n,h}^{i,M} < \epsilon\}) \\
&= \frac{1}{\#H_n} (M \#\{h \in H_n : x_{n,h}^{i} \geq \epsilon\} + \epsilon \#\{h \in H_n : x_{n,h}^{i} < \epsilon\}) \\
&< M\epsilon + \epsilon
\end{aligned}$$

Therefore, for each M we have $\lim_{n\to\infty} \frac{1}{\#H_n} \sum_{h \in H_n} x_{n,h}^{i,M} = 0$.

Fix $0 < \delta < a$. We have that:

$$\forall M, \exists n_M \in N \ s.t. \ \frac{1}{\#H_n} \sum_{h \in H_n} x_{n,h}^{i,M} < \delta, \forall n \geq n_M$$

In particular, $\frac{1}{\#H_{n_M}} \sum_{h \in H_{n_M}} x_{n_M,h}^{i,M} < \delta$, for all M. Since $x_n^{i,M} \to x_n^i$ we have that for each index n_M, $\frac{1}{\#H_{n_M}} \sum_{h \in H_{n_M}} x_{n_M,h}^{i} \leq \delta$.

But $\lim_{n\to\infty} \frac{1}{\#H_n} \sum_{h \in H_n} x_{n,h}^i = a > \delta$, which implies that for n_M large enough $\frac{1}{\#H_{n_M}} \sum_{h \in H_{n_M}} x_{n_M,h}^{i} > \delta$, contradicting the preceeding statement. This contradiction establishes the claim of the lemma. □

We now turn to develop an asymptotic convergence theorem, by introducing appropriate assumptions on the distribution of characteristics along a sequence of economies. In particular, consider a sequence of economies $\mathcal{E}_n : H_n \to T$ where $T \subset \mathcal{P}_{cm} \times [0, r]^L$ is compact. We can now extend a result of Mas-Colell (1982) without any reference to types.

Proposition 1 *Let $\{\mathcal{E}_n\}_{n \in N}$ be a sequence of economies, $\mathcal{E}_n : H_n \to T$ where $\#H_n \to \infty$ and let $x_n \in \mathcal{N}(\mathcal{E}_n)$, for each $n \in N$ be fully active. There is $B \subset \Re_+^L$, which is bounded and depends only on T, such that for all $n \in N$ $x_{n,h} \in B$ for each $h \in H_n$, i.e., the set of Nash equilibrium allocations remains uniformly bounded along a sequence of economies with characteristics drawn from T.*

Proof:
Step I Let $\pi_n = \pi(b_n, q_n)$ and normalize prices so that $\sum_{i=1}^L \pi_n^i = 1$.

Suppose that $sup\{x_{n,h}^j : h \in H_n\} \to \infty$ for some $j = 1, 2, \ldots, L$. Then it must be $sup\{\frac{b_{n,h}^j}{\pi_n^j} - q_{n,h}^j : h \in H_n\} \to \infty$. It follows that $\pi_n^j \to 0$. Hence, there must be $\pi_n^i > 1/L$ for some $i \neq j$ along a subsequence, so $\pi_n^i / \pi_n^j \to \infty$.

Step II By lemma (2), passing to a subsequence if necessary, we may assume that for some $1 > \epsilon > 0$,

$$\#\{h \in H_n : x^i_{n,h} \geq \epsilon\} \geq \#H_n \epsilon \tag{7}$$

Also by lemma 1, setting $k \geq 2L\epsilon^{-1} - 1$, we have that for all $n \in N$,

$$\frac{1}{\#H_n} \# \left\{ h \in H_n : \mid z^i_{n,h} \mid \leq (2L\epsilon^{-1} - 1)r \;\forall i = 1, 2, \ldots, L \right\} > 1 - \frac{\epsilon}{2} \tag{8}$$

Step III We now show the following claim: for some subsequence (still indexed by n) there exists $M > 0$ so that:

$$\#\{h \in H_n : \frac{p^i_h}{p^j_h} \leq M\} > \#H_n \frac{\epsilon}{2} \tag{9}$$

Suppose not. Then for every $M > 0$ we have $\#\{h \in H_n : \frac{p^i_h}{p^j_h} \leq M\} \leq \#H_n \frac{\epsilon}{2}$ or equivalently $\#\{h \in H_n : \frac{p^i_h}{p^j_h} > M\} > \#H_n(1 - \frac{\epsilon}{2})$. In conclusion we have for every $M > 0$

$$\#\{h \in H_n : \frac{p^j_h}{p^i_h} < M^{-1}\} > \#H_n(1 - \frac{\epsilon}{2}) \tag{10}$$

In this case, (10) along with (7) and (8), imply that for each $n \in N$ there is $h_n \in H_n$ so that the following are true: $z_{h_n} \leq (2L\epsilon^{-1} - 1)r \cdot 1_L$, so that along some subsequence (still indexed by n) $z_{h_n} \to z$, $z^i_{h_n} + e^i_{h_n} \geq \epsilon$ and $\frac{p^j_{h_n}}{p^i_{h_n}} \to 0$.

The compactness of T implies that, by passing to a subsequence if necessary we may assume that $(\succeq_{h_n}, e_{h_n}) \to (\succeq, e) \in T$.

Consider for each $n \in N$ the vectors $t_n \in \Re^L_+$ where $t^i_n = -\frac{p^j_{h_n}}{p^i_{h_n}}$, $t^j_n = 1$ and $t^l_n = 0$ for $l \neq i, j$. For these vectors we have that $p_{h_n} t_n = 0$, $|t^i_n| < \epsilon$ for n large enough, $t_n \to t \geq 0$ and $t \neq 0$. By the convexity of preferences it must be that $z_{h_n} + e_{h_n} \succeq_{h_n} z_{h_n} + e_{h_n} + t_n$. Taking limits we conclude that $z + e \succeq z + e + t$ which contradicts the monotonicity of $\succeq$. This contradiction establishes our claim, so in this step we conclude that:

$$\frac{1}{\#H_n} \#\{h \in H_n : \frac{p^i_h}{p^j_h} > M\} < \frac{\epsilon}{2} \tag{11}$$

Step IV Since $\pi^i_n / \pi^j_n \to \infty$ we have that $\pi^i_n / \pi^j_n (1 - \epsilon) > M$ for n large enough. Furthermore, by theorem (1) we have that for n large enough:

$$\frac{1}{\#H_n} \# \left\{ h \in H_n : \delta_h(x_n) \leq \epsilon \right\} \geq \frac{\epsilon}{2}$$

But then for n large enough we have the following string of inequalities

$$\begin{aligned}\frac{1}{\#H_n}\#\left\{h \in H_n : \frac{p_h^i}{p_h^j} > M\right\} &\geq \frac{1}{\#H_n}\#\left\{h \in H_n : \frac{p_h^i}{p_h^j} \geq \frac{\pi_n^i}{\pi_n^j}(1-\epsilon)\right\} \\ &\geq \frac{1}{\#H_n}\#\left\{h \in H_n : \delta_h(x_n) \leq \epsilon\right\} \\ &\geq \frac{\epsilon}{2}\end{aligned}$$

which contradicts (11). □

Since $\{x_n\}_{n\in N}$ is uniformly bounded, it follows that we can extract a subsequence (still indexed by n) which converges in distribution, i.e., by defining for each $B \in \Re^L$ $\lambda_n(B) = \frac{1}{\#H_n}\#\{h \in H_n : x_{n,h} \in B\}$ we have that $\lambda_n \to \lambda$ weakly.

Let $T \subset \mathcal{P}_{cm} \times \Re_+^L$ be compact. Consider a 'purely competitive' (see Hildenbrand (1974) p.138) sequence of economies $\{\mathcal{E}_n\}_{n\in N}$, i.e., $\mathcal{E}_n : H_n \to T$ where :

(i) $\#H_n \to \infty$.
(ii) The sequence of distributions of characteristics (μ_n) converges weakly on T.
(iii) If $\mu = \lim \mu_n$ then $\int e\ d\mu_n \to \int e\ d\mu$.
(iv) $\int e\ d\mu > 0$.

Such sequences admit a 'continuous representation', i.e., for such sequences there exist: an atomless measure space $(H, \mathcal{H}, \nu)$, an economy $\mathcal{E} : H \to T$ and $a_n : H \to H_n$ measurable so that:

(i) For every $S \subset H_n$, $\nu\left(a_n^{-1}(S)\right) = \frac{\#S}{\#H_n}$.
(ii) $\mathcal{E}_n(a_n) \to \mathcal{E}$, *ae* in H.

Consider now $x_n \in \mathcal{N}(\mathcal{E}_n)$. Define $f_n : H \to \Re_+^L$ by $f_n(h) = x_{n,a_n(h)}$. In this way we can extend our indicator on H by $\hat{\delta}_h(f_n) = \delta_{a_n(h)}(x_n)$. The meaning of theorem (1) can be made more transparent as follows:

Lemma 3 $\hat{\delta}_h(f_n) \to 0$ *in measure.*

Proof :
By definition of a continuous representation of the sequence of economies:

$$\begin{aligned}\nu\left(\left\{h \in H : \hat{\delta}_h(f_n) > \epsilon\right\}\right) &= \nu\left(\left\{h \in H : \delta_{a_n(h)}(x_n) > \epsilon\right\}\right) \\ &= \nu\left(a_n^{-1}\left(\{h \in H_n : \delta_h(x_n) > \epsilon\}\right)\right) \\ &= \frac{1}{\#H_n}\#\left\{h \in H_n : \delta_h(x_n) > \epsilon\right\}\end{aligned}$$

By Theorem (1) the righthand side converges to zero. □

Note that, as a consequence, there is a subsequence (still indexed by n) so that $\hat{\delta}_h(f_n) \to 0$, *ae* in H. However, the sequence $\{x_n\}_{n\in N}$ (and consequently $\{f_n\}_{n\in N}$) need not converge in any sense.
Denote by τ_n the joint distribution of $(\mathcal{E}_n, x_n) : H_n \to T \times \Re^L$. The sequence $(\tau_n)_{n\in N}$ is tight since the sequences of its marginal distributions are tight, so we may assume, by passing to a subsequence if necessary, that $\tau_n \to \tau$ weakly. Now there is (see Hildenbrand (1974) proposition 2 p. 139) an atomless measure space $(H, \mathcal{H}, \nu)$, $(\mathcal{E}, x) : H \to T \times \Re^L$ and measurable functions $a_n : H \to H_n$, so that $(\mathcal{E}_n(a_n), f_n) \to (\mathcal{E}, x)$ *ae* in H and the distributions of $(\mathcal{E}_n(a_n), f_n)$ and $(\mathcal{E}, x)$ are τ_n and τ respectively. The following claim establishes that the allocation x is Walrasian for the economy $\mathcal{E}$, provided that the associated sequence of strategic prices does not converge to the boundary of $\Re^L_+$.

Claim Let $x_n \in \mathbf{E}(\mathcal{E}_n)$, for each $n \in N$ be fully active and suppose that the sequence of associated strategic market game prices $\{\pi_n\}_{n\in N}$ are such that no subsequence converges to the boundary of $\Re^L_+$. Then $\delta_h(x) = 0$, *ae* in H.

Proof :
Normalizing prices so that $\sum_{i=1}^{L} \pi_n^i = 1$, for each $n \in N$, we may assume, by passing to a subsequence if necessary, that $\pi_n \to p > 0$. Thus, $\delta_h(x)$ is well defined and since $f_n \to x$ *ae* in H, it follows by continuity of $\hat{\delta}_h(\cdot)$ that $\delta_h(x) = \lim \hat{\delta}_h(f_n)$, *ae* in H. Since, by lemma 3 above, $\hat{\delta}_h(f_n) \to 0$ in measure it follows that it must be $\delta_h(x) = 0$, *ae* in H. □

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