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Bigger Countries with Probably Lower Commodity Taxes

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ABSTRACT; The paper studies a spatial model of commodity tax competition between 2 countries that may differ in both population density and land mass, synthesizing two earlier contributions where pure strategy equilibria always existed, and where the "bigger" country (that with the larger population) set the higher tax. In our more general setting, pure strategy equilibria may not exist, but we can compute a mixed strategy equilibrium, which offers probabilistic alternatives to the empirically questionable "big country-high tax" correlation; the bigger country may have a lower *expected* tax rate, and may be *more likely* to be the low tax country.

JEL: H20; H71; H73; R51

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1. Introduction

In a well-known paper, Kanbur & Keen (1993) use a model of spatial competition between neighbouring countries (or jurisdictions) on a Hotelling line, to investigate the relation between commodity tax rates and country characteristics. Specifically, with 2 countries of equal land mass but different population densities, the country with the larger population density (and hence the larger population = the "bigger" country here) will set the higher commodity tax-rate. In a similar framework, but with countries having the same population density and differing land masses, Ohsawa (1999) concludes that the country with the larger land mass (and hence again the bigger country in our sense) will set the higher tax rate. Moreover this positive correlation between bigger countries and higher commodity taxes is borne out in various other theoretical models.[1] However empirical evidence is mixed. For instance, in 2 of the models in Estelle-More and Sole-Olle (2001), US state taxes are negatively related to the state population; the Netherlands, with one-fifth of the population of neighbouring Germany, has a petrol tax rate that is five percent higher than Germany; and the tax percentage in Luxembourg's petrol price is 59.9, whilst that in the bigger neighbours Belgium and France is 70.6 and 76.4, respectively.[2] So it is certainly not clear that the empirical reality offers full support for such a theoretical correlation. Our paper offers a way out.

Synthesising Kanbur & Keen (1993) and Ohsawa (1999) in a model with differing population densities and differing land masses, we confirm what is a very general "bigger country-higher tax" relation in *pure strategy equilibrium* of the tax competition game. However, as Kanbur & Keen (1993) reveals, differing population densities lead to discontinuous reaction functions in this game, and in our more

[1] For instance, Trandel (1994) argued that when the governments are maximising social welfare of the domestic residents, dependent on the provision of public goods financed by tax revenue, the more populated country sets the higher tax rate. Wang (1999) studied a Stackelberg game based on Kanbur and Keen (1993), with the big country as leader. At equilibrium, the 'bigger country higher tax rate' rule is repeated, and the tax difference is even larger than that in the simultaneous tax game. Nielsen (2001) also reproduced the 'big country higher tax rate' result. And Ohsawa and Koshizuka (2003) have a version of the result based on a 2-dimensional geography, to which we return later.

[2] See Rietveld and van Woundenberg (2004).

general setting there are cases where only a mixed strategy equilibrium exists. We are able to compute fully all such mixed strategy equilibria, and we show that, probabilistically speaking, the bigger country-higher tax link may be broken. Precisely, bigger countries may set lower *expected* tax rates, and bigger countries may be *more likely to be* (with probability exceeding ½) the lower tax country.

Section 2 describes the model and its pure strategy equilibria. Section 3 offers the mixed strategy analysis, and section 4 concludes.

2. The model and its pure strategy equilibria

The world is a 1-dimensional interval $[0, L_1 + L_2]$ with $[0, L_1]$ representing the land mass of country 1 and $[L_1, L_1 + L_2]$ that of country 2, where $L_1, L_2 > 0$. The population size of country i is $A_i > 0$, uniformly distributed over its land with density $\delta_i > 0$, so $A_i = \delta_i L_i$, $i = 1, 2$. Without loss of generality we assume throughout that $\delta_1 \geq \delta_2$. Each individual in the world wishes to buy inelastically 1 unit of the only good available, and incurs costs γx to travel a distance x. There are producers located at each point in the world, who produce the good at constant marginal cost (normalized to zero), meeting the demand of all consumers wishing to buy at their location. The government of country i levies a tax of p_i throughout i, and the competition between the continuum of producers in i ensures that this will also be the price of the good throughout i. An individual living in i will then make their purchase either from the producer at their location (cost p_i) or by traveling to the border (distance x say) to buy from country j (cost $p_j + \gamma x$). If $p_i = p_j$, there will be no "cross-border shopping", all consumers buying locally. Generally, if $p_i \geq p_j$, there will be cross-border shopping out of country i equal to

$$c_i(p_i - p_j) = \min\{\delta_i (p_i - p_j) / \gamma, A_i\} \qquad (2.1)$$

Governments choose tax rates $p_1, p_2 \geq 0$ and payoff functions (revenues) in the simultaneous move, non-cooperative tax competition game are

$$\pi_1(p_1,p_2) = \begin{cases} \pi_1^L(p_1,p_2) = [A_1 + c_2(p_2 - p_1)]p_1, & if \quad p_1 \leq p_2 \\ \pi_1^R(p_1,p_2) = [A_1 - c_1(p_2 - p_1)]p_1, & if \quad p_1 \geq p_2 \end{cases} \qquad (2.2)$$

$$\pi_2(p_1,p_2) = \begin{cases} \pi_2^L(p_1,p_2) = [A_2 + c_1(p_1 - p_2)]p_2, & if \quad p_1 \geq p_2 \\ \pi_2^R(p_1,p_2) = [A_2 - c_2(p_1 - p_2)]p_2, & if \quad p_1 \leq p_2 \end{cases} \qquad (2.3)$$

We consider first pure strategy Nash equilibrium. We can take it in (2.1) that $c_i(p_i - p_j) = \delta_i(p_i - p_j)$ since choosing p_i so that $\delta_i(p_i - p_j)/\gamma \geq A_i$ implies $\pi_i = 0$ which can never be a best response for i ($\pi_i > 0$ is attainable at any $p_j \geq 0$ from p_i positive and small enough). The resulting payoff functions π_i in (2.2) and (2.3) are continuous everywhere, differentiable except when $p_1 = p_2$, strictly concave in p_i for $p_i \geq p_j$ and for $p_i \leq p_j$, but not necessarily globally strictly concave because of the kink at $p_1 = p_2$, as follows. Recalling that $\delta_1 \geq \delta_2$, the derivatives of π_i^L and π_i^R at $p_1 = p_2 = p$ are such that;

$$\frac{\partial \pi_1^L}{\partial p_1} = A_1 - \frac{\delta_2 p}{\gamma} \geq \frac{\partial \pi_1^R}{\partial p_1} = A_1 - \frac{\delta_1 p}{\gamma} \qquad (2.4)$$

$$\frac{\partial \pi_2^L}{\partial p_2} = A_2 - \frac{\delta_1 p}{\gamma} \leq \frac{\partial \pi_2^R}{\partial p_2} = A_2 - \frac{\delta_2 p}{\gamma} \qquad (2.5)$$

If $\delta_1 = \delta_2$ both π_1 and π_2 are globally strictly concave, generating well-behaved (continuous) best response (reaction) functions. Moreover, from (2.4), π_1 "kinks down" as p_1 increases through p_2 when $\delta_1 > \delta_2$, so π_1 is again globally strictly concave. However, π_2 "kinks up" from (2.5) when $\delta_1 > \delta_2$, precluding global concavity of π_2. The reason for this difference is clear. If p_1 is a little smaller than p_2 there is cross-border shopping into country 1 and increases in p_1 reduce this at a rate proportional to δ_2; if p_1 is a little larger than p_2 there is cross-border shopping out of country 1 at a rate proportional to $\delta_1 > \delta_2$. Hence the upward kink for the country with the lower population density, and the downward kink for the high density country. The consequences for best responses are (see appendix for proof):

Lemma 1 Suppose $\delta_1 \geq \delta_2$.

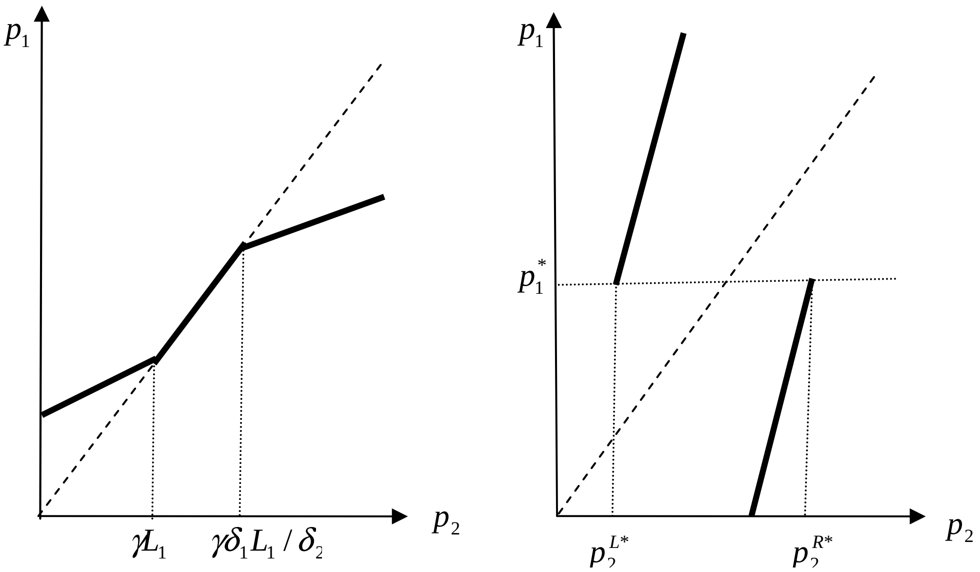
(a) Country 1's best response is:

$$p_1 = \begin{cases} \gamma\delta_1 L_1 / 2\delta_2 + p_2 / 2 & \text{if } p_2 \geq \gamma\delta_1 L_1 / \delta_2 \\ p_2 & \text{if } \gamma L_1 \leq p_2 \leq \gamma\delta_1 L_1 / \delta_2 \\ \gamma L_1 / 2 + p_2 / 2 & \text{if } p_2 \leq \gamma L_1 \end{cases}$$

(b) With $p_1^* = \gamma L_2 \sqrt{\delta_2 / \delta_1}$, country 2's best response is:

$$p_2 = \begin{cases} \gamma\delta_2 L_2 / 2\delta_1 & \text{if } p_1 \geq p_1^* \\ \gamma L_2 / 2 & \text{if } p_1 \leq p_1^* \end{cases}$$

Figure 1 illustrates the best response graphs when $\delta_1 > \delta_2$; that of country 2 "jumps" at p_1^* between the values $p_2^{R*} = \frac{1}{2}\gamma L_2 (1 + \sqrt{\delta_2 / \delta_1})$ and $p_2^{L*} = p_2^{R*}\sqrt{\delta_2 / \delta_1}$. It is clear from figure 1 that there will be at most one pure strategy Nash equilibrium when $\delta_1 > \delta_2$, either with $p_1 > p_2$ at an intersection of the best response graph branches that are above the (dashed) 45 degree line in figure 1, or with $p_1 < p_2$ when the branches below the 45 degree line intersect. When $\delta_1 = \delta_2$, both best responses are linear functions, and they intersect uniquely.



(a) Best response of country 1 (b) Best response of country 2

FIGURE 1

The resulting pure strategy Nash equilibria are now easily derived (see appendix for proof):

<u>Theorem 1</u> Suppose $\delta_1 \geq \delta_2$, and let $h = \sqrt{\delta_2 / \delta_1}$. Pure strategy Nash equilibria are as follows:

(a) $p_1 = 2\gamma L_1 / 3h^2 + \gamma L_2 / 3$, $p_2 = \gamma L_1 / 3h^2 + 2\gamma L_2 / 3$, when $L_1 / L_2 \leq (3h^3 - h^2)/2$;

(b) $p_1 = 2\gamma L_1 / 3 + \gamma h^2 L_2 / 3$, $p_2 = \gamma L_1 / 3 + 2\gamma h^2 L_2 / 3$, when $L_1 / L_2 \geq (3h - h^2)/2$;

(c) there is no pure strategy equilibrium when $(3h - h^2)/2 > L_1 / L_2 > (3h^3 - h^2)/2$;

(d) in any pure strategy equilibrium $p_1 - p_2$ has the sign of $A_1 - A_2$.

Figure 2 illustrates parameter values indicated in theorem 1. In region I, $A_1 > A_2$ and from (b) and (d) there is a pure strategy equilibrium with $p_1 > p_2$; in region II $A_1 < A_2$ and from (a) and (d) there is a pure strategy equilibrium with $p_1 < p_2$; and in regions III (where $A_1 > A_2$) and IV (where $A_1 < A_2$) there is no pure strategy equilibrium, from (c).

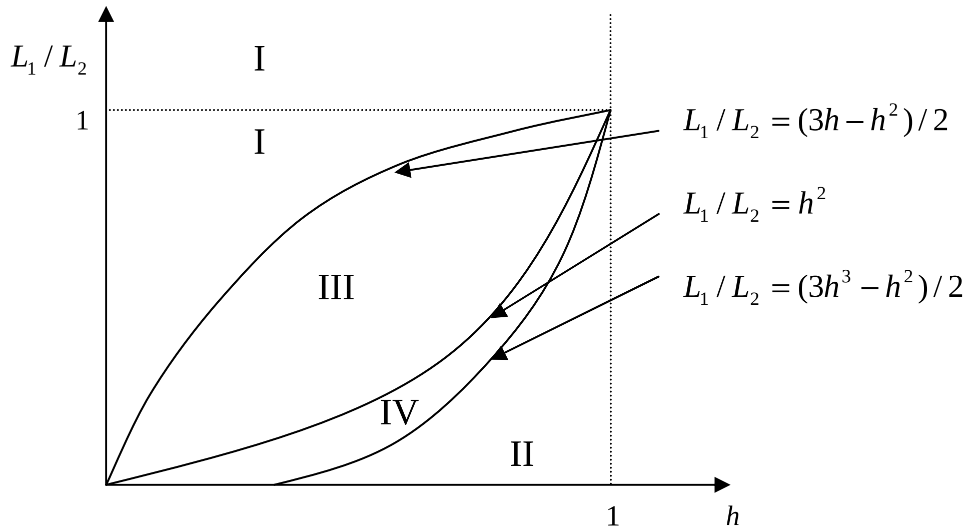


FIGURE 2

The earlier results of Kanbur and Keen (1993) and Ohsawa (1999) emerge as special cases of Theorem 1. Ohsawa (1999) takes the case where $\delta_1 = \delta_2$ (h=1 in figure 2), so the kinks/jumps in best response of figure 1 disappear and there is always a unique pure strategy equilibrium in which the country with the greater land mass sets the higher tax rate ($L_1 / L_2 \geq 1, h = 1$ implies region I and $L_1 / L_2 \leq 1, h = 1$ implies region II). Kanbur and Keen (1993) assume $L_1 = L_2$ but allow differing δ_i 's. Here the kinks/jumps of figure 1 do emerge, but they do not preclude existence of a unique pure strategy equilibrium which has the property that now the country with the greater population density sets the higher tax rate ($L_1 / L_2 = 1, h \leq 1$ implies region I). The unifying result of theorem 1 (d) shows that in any pure strategy equilibrium the country with the larger population (our "bigger" country) sets the higher tax rate.

In fact theorem 1 (d) is true much more generally than in our specific model. It requires only minimal assumptions in the cross-border shopping functions, and remains true also in the two-dimensional worlds studied by Ohsawa and Koshizuka (2003). Indeed they have a version of the following theorem 2. We offer the following alternative, which applies to both the one- and two- dimensional worlds, but which assumes more general cross-border shopping functions $c_i : R_+ \to [0, A_i]$, where $c_i\left(p_i - p_j\right)$ is the cross-border shopping out of country i when $p_i (p_j)$ is the tax rate in country $i(j)$. These functions need only be continuous with a weak monotonicity in $p_i - p_j$. The appendix proves:

<u>Theorem 2</u> In any 2-country model where $A_i > A_j$ and where the cross-border shopping functions are continuous with $c_i(0) = 0$ and $c_i(p_i - p_j) > 0$ when $p_i > p_j$, a necessary condition for a pure strategy Nash equilibrium is $p_i > p_j$.

3. Mixed strategy equilibria

Returning to our specific model, we now compute mixed strategy equilibrium for region III and IV in figure 2, using a method found usually in the product differentiation literature (see, e.g. Krishna (1989)).[3] The procedure derives from the fact that if country 1 has a strictly concave payoff function and country 2's (pure strategy) best response jumps between 2 values (p_2^{L*} and p_2^{R*}) at some particular (pure) strategy of country 1 (p_1^*) in a way which precludes a pure strategy equilibrium, there will be a mixed strategy equilibrium in which country 1 chooses p_1^* purely and country 2 randomises, choosing p_2^{L*} with some probability $\alpha \in (0,1)$ and p_2^{R*} with probability $1-\alpha$. The reason is that any $\alpha \in [0,1]$ will be a best response by country 2 to p_1^*, since they will be indifferent (at the jump) to

[3] Theorem 2 of Krishna (1989) is an analogous result.

playing p_2^{L*} or p_2^{R*} purely. Given nonexistence of pure strategy equilibrium, country 1's best response to p_2^{L*} would be a pure strategy (given the payoff strict concavity) lower than p_1^* (see figure 1) whilst that to p_2^{R*} would be above p_1^*. By continuity there will exist $\alpha \in (0,1)$ where 1's best response is exactly p_1^*, completing the Nash properties. The calculations here produce:

Theorem 3 Suppose $\delta_1 > \delta_2$, $h = \sqrt{\delta_2 / \delta_1}$ and $(3h - h^2)/2 > L_1 / L_2 > (3h^3 - h^2)/2$. Then there exists a mixed strategy equilibrium in which:

(a) country 1 chooses the pure strategy $p_1 = p_1^*$;

(b) country 2 chooses the mixed strategy $p_2 = p_2^{L*}$ with probability α, and $p_2 = p_2^{R*}$ with probability $(1-\alpha)$, where $\alpha = \dfrac{2L_1 + h^2 L_2 - 3h^3 L_2}{3hL_2\left(1-h^2\right)} \in (0,1)$;

(c) country 1's tax rate, p_1^*, is lower than the expected tax rate in country 2, $Ep_2 = \alpha p_2^{L*} + (1-\alpha) p_2^{R*}$, if and only if $L_1 / L_2 < h(3 + 3h^2 - 4h)/2$;

(d) country 1 is more likely to have the lower tax rate (i.e. $\alpha < 1/2$), if and only if $L_1 / L_2 < h(3 + 3h^2 - 2h)/4$.

Proof Since π_1 is strictly concave, country 1's expected payoff will be strictly concave and we now find conditions for $p_1^* \in (p_2^{L*}, p_2^{R*})$ to be a stationary point of this expected payoff. For $p_1 \in (p_2^{L*}, p_2^{R*})$, the expected payoff is: $E\pi_1 = \alpha[\delta_1 L_1 + \delta_1 (p_2^{L*} - p_1)/\gamma] p_1 + (1-\alpha)[\delta_1 L_1 + \delta_2 (p_2^{R*} - p_1)/\gamma] p_1$.

$\dfrac{dE\pi_1}{dp_1} = 0$ implies:

$$\delta_1 L_1 + \delta_2 (p_2^{R*} - p_1)/\gamma - \delta_2 p_1 / \gamma = \alpha[\delta_1 (p_1 - p_2^{L*}) + \delta_1 p_1 + \delta_2 (p_2^{R*} - p_1) - \delta_2 p_1]/\gamma$$

Noting $p_2^{R*} - p_1^* = \gamma L_2 (1-h)/2$, $p_1^* - p_2^{L*} = \gamma L_2 h(1-h)/2$, with $p_1 = p_1^* = \gamma h L_2$ this gives;

$$2\delta_1 L_1 + \delta_2 L_2 - 3h\delta_2 L_2 = \alpha L_2 (3\delta_1 h - 3\delta_2 h + \delta_2 - \delta_1 h^2)$$

This rearranges to provide the α formula claimed in (b). And $\alpha \in (0,1)$ under the parameter restrictions assumed in the theorem statement, completing the proof of (a) and (b).

For (c), $Ep_2 - p_1^* = \alpha p_2^{L*} + (1-\alpha) p_2^{R*} - p_1^*$

$$= \frac{3hL_2 + 3h^3 L_2 - 2L_1 - 4h^2 L_2}{6h} \geq 0 \quad \text{if} \quad \frac{L_1}{L_2} \leq \frac{1}{2} h(3 - 4h + 3h^2)$$

For (d), $\frac{1}{2} - \alpha = \frac{3hL_2 - 2h^2 L_2 + 3h^3 L_2 - 4L_1}{6hL_2\left(1-h^2\right)} \geq 0$, if $\frac{L_1}{L_2} \leq \frac{1}{4} h(3 - 2h + 3h^2)$. Q.E.D.

Figure 3 focuses on the regions (III and IV in figure 2) of non-existence of pure strategy equilibrium. Theorem 3(c) holds in regions IIIb, IIIc and IV, and Theorem 3(d) holds in regions IIIc and IV. Recalling that country 1 has the larger population throughout region III, it follows that the bigger country is more likely to set the lower tax rate in region IIIc and, in addition, the tax rate in the bigger country is, in expectation, lower in region IIIb.[4]

[4] A numerical example may help illustrate the picture. When $\delta_1 = 4$, $\delta_2 = 1$, $L_1 = 10$, $L_2 = 32$, country 1 is bigger than country 2, where $\delta_1 L_1 = 40 > \delta_2 L_2 = 32$. However, $p_1 = 5\gamma$, $Ep_2 = \frac{56}{3}\gamma \Rightarrow p_1 < Ep_2$. On the other hand, $\alpha = \frac{4}{9}$, below 50%. In this case, the bigger country is setting a lower expected tax rate, and is more likely to set a lower tax rate.

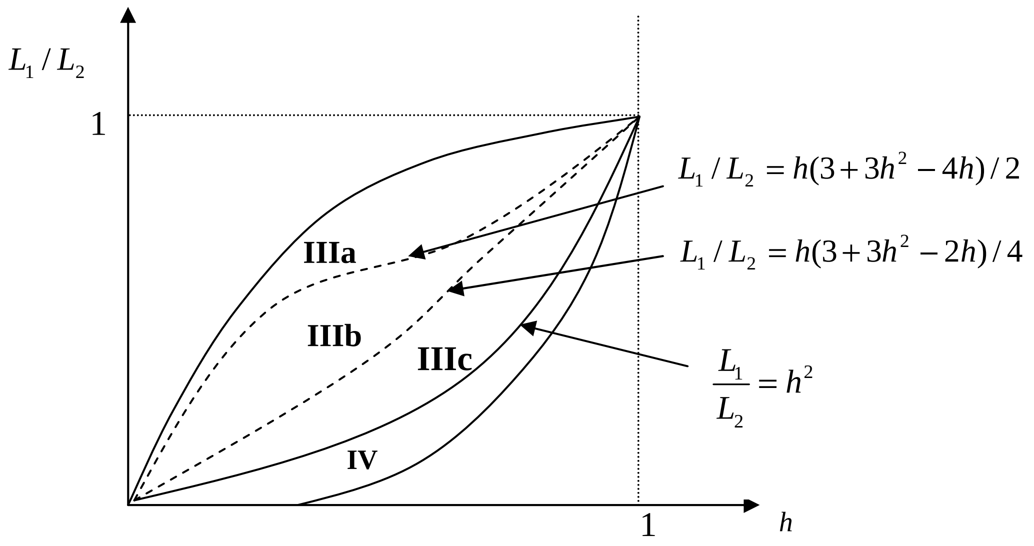


FIGURE 3

CONCLUSIONS

In price-setting games amongst firms, a common criticism of mixed strategy equilibrium is that, once prices are randomly selected, firms would typically want to choose differently when they see the rivals' choices, and in many real contexts such ex post information is available and such ex post price adjustment is easily feasible, undermining to some extent the appropriateness of the equilibrium concept. In our context we feel the ex post tax adjustment is less straightforward, and the criticism less appropriate. Countries and jurisdictions typically plan to make decisions on tax rates only at well-defined points in time (e.g. the start of a fiscal year), and immediate adjustments otherwise in the light of neighbour choices are less feasible than in (say) the case of 2 neighbouring high street shops. Synthesizing models of Kanbur and Keen (1993) and Ohsawa (1999), the paper has shown how such mixed strategy commodity tax equilibria can provide explanations of why, probabilistically speaking, bigger countries may set lower tax rates, when only the reverse and empirically questionable correlation can be found with pure strategies.

APPENDIX

Proof of Lemma 1

a) $\partial \pi_1^L / \partial p_1 = A_1 + \delta_2 \left(p_2 - p_1 \right) / \gamma - \delta_2 p_1 / \gamma = 0$ when $p_1 = \gamma \delta_1 / (2\delta_2 L_1) + p_2 / 2 \leq p_2$ if $p_2 \geq \gamma L_1 \delta_1 / \delta_2$. Given the strict concavity of π_1, this ensures the top branch of the p_1 statement in (a).

Similarly $\partial \pi_1^R / \partial p_1 = A_1 - \delta_1 \left(p_1 - p_2 \right) / \gamma - p_1 / \gamma = 0$ when $p_1 = \gamma L_1 / 2 + p_2 / 2 \geq p_2$ if $p_2 \leq \gamma L_1$, ensuring the bottom branch. When $p_2 \in \left[\gamma L_1, \gamma L_1 \delta_1 / \delta_2 \right]$, the maximum of π_1 is at the kink $p_1 = p_2$, ensuring the middle branch.

(b) Note fist the following 3 observations:

(i) $\partial \pi_2^L / \partial p_2 = A_2 + \delta_1 \left(p_1 - p_2 \right) / \gamma - \delta_1 p_2 / \gamma = 0$ when $p_2 = \gamma \delta_2 L_2 / \left(2\delta_1 \right) + p_1 / 2$ and $\pi_2^L = \delta_1 \left(\gamma L_2 \delta_2 / \delta_1 + p_1 \right)^2 / (4\gamma)$ when $p_1 \geq p_2$, iff $p_1 \geq \gamma L_2 \delta_2 / \delta_1$.

(ii) $\partial \pi_2^R / \partial p_2 = A_2 - \delta_2 \left(p_2 - p_1 \right) / \gamma - \delta_2 p_2 / \gamma = 0$ when $p_2 = \left(\gamma L_2 + p_1 \right) / 2$ and $\pi_2^R = \delta_2 \left(\gamma L_2 + p_1 \right)^2 / (4\gamma)$ when $p_1 \leq p_2$ iff $p_1 \leq \gamma L_2$.

(iii) at the values in (i) and (ii), $\pi_2^L \geq \pi_2^R$ iff $p_1 \geq \gamma \sqrt{\delta_2 / \delta_1} L_2$. Given the concavity of π_2^L and π_2^R, and the downward kink in π_2 when $p_1 = p_2$, it follows that the p_2 value in (i) provides the best response if either, (1) $p_1 \geq \gamma L_2 \delta_2 / \delta_1$, $p_1 \leq \gamma L_2$ and $p_1 \geq \gamma \sqrt{\delta_2 / \delta_1} L_2$ ($\pi_2^L \geq \pi_2^R$), or, (2) $p_1 \geq \gamma L_2 \delta_2 / \delta_1$ and $p_1 \geq \gamma L_2$. Since $\delta_1 \geq \delta_2$, the union of the conditions in (1) and (2) is simply $p_1 \geq \gamma \sqrt{\delta_2 / \delta_1} L_2$, establishing the upper branch of the p_2 formula in (b). Similarly the p_2 value in (ii) provides the best response if, either, (3), $p_1 \geq \gamma L_2 \delta_2 / \delta_1$, $p_1 \leq \gamma L_2$ and $p_1 \leq \gamma \sqrt{\delta_2 / \delta_1} L_2$ ($\pi_2^L \leq \pi_2^R$), or (4), $p_1 \leq \gamma \delta_2 L_2 / \delta_1$ and $p_1 \leq \gamma L_2$. Now (3) and (4) are equivalent to $p_1 \leq \gamma \sqrt{\delta_2 / \delta_1} L_2$, completing the proof. Q.E.D.

Proof of Theorem 1 If $\delta_1 = \delta_2$, so h=1, best responses are $p_1 = \left(\gamma L_1 + p_2 \right) / 2$ and

$p_2 = (\gamma L_2 + p_1)/2$, intersecting uniquely at

$p_1 = (2\gamma L_1 + \gamma L_2)/3, p_2 = (\gamma L_1 + 2\gamma L_2)/3$. When $\delta_1 > \delta_2$ there is a unique equilibrium with $p_1 > p_2$, iff there is a solution with $p_1 \geq p_2$ (see figure 1) to the equations: $p_1 = (\gamma L_1 + p_2)/2$ and $p_2 = (\gamma L_2 \delta_2 / \delta_1 + p_1)/2$.

Remember $p_1{*} = \gamma \sqrt{\delta_2 / \delta_1} L_2$. At the solution, $p_1 = (2\gamma L_1 + \gamma h^2 L_2)/3, \; p_2 = (\gamma L_1 + 2\gamma h^2 L_2)/3$ and $p_1 \geq p_1{*}$, iff $L_1 / L_2 \geq (3h - h^2)/2$, completing (a). Conversely there is a unique equilibrium, with $p_1 < p_2$, iff there is a solution with $p_1 \leq p_1{*}$ (again see figure 1) to $p_1 = (\gamma L_1 \delta_1 / \delta_2 + p_2)/2$ and $p_2 = (\gamma L_2 + p_1)/2$.

At the solution, $p_1 = 2\gamma L_1 / (3h^2) + \gamma L_2 / 3, \; p_2 = \gamma L_1 / (3h^2) + 2\gamma L_2 / 3$ and $p_1 \leq p_1{*}$ iff $L_1 / L_2 \geq (3h^3 - h^2)/2$, completing (b), and hence (c). In case (a), $p_1 - p_2 = \gamma(A_1 - A_2)/(3\delta_2)$, and in (b), $p_1 - p_2 = \gamma(A_1 - A_2)/(3\delta_1)$, completing (d). Q.E.D.

<u>Proof of Theorem 2</u> Without loss of generality, assume $A_1 > A_2$. Let $p_1{*}$ and $p_2{*}$ be a pure strategy Nash equilibrium with payoffs $\pi_1{*}$ and $\pi_2{*}$. Here we can assume that $\pi_1{*}, \pi_2{*} > 0$, and $p_1{*}, p_2{*} > 0$. Otherwise some $\pi_i{*} = 0$ and

(i) if $p_j{*} = 0$, choosing p_i sufficiently small and positive ensures $\pi_i > 0$,

(ii) if $p_j{*} > 0$, $p_i = p_j{*}$, ensures $\pi_i > 0$.

Now suppose $p_2{*} > p_1{*} > 0$. It must be the case that country 1 or 2 does not want to Nash deviate to a slightly higher/lower tax $p_1{*} + \varepsilon$ / $(p_2{*} - \varepsilon)$, for $\varepsilon > 0$ and small such that $0 < \varepsilon < -p_1{*} + p_2{*}$.

From (2.2) and (2.3) these 2 requirements become:

$$p_1{*}[A_1 + c_2(-p_1{*} + p_2{*})] \geq (p_1{*} + \varepsilon)[A_1 + c_2(-p_1{*} + p_2{*} - \varepsilon)] \quad (1)$$

or, $p_2*\left[A_2 - c_2(-p_1*+p_2*)\right] \geq (p_2*-\varepsilon)\left[A_2 - c_2(-p_1*+p_2*-\varepsilon)\right]$ (2)

Rearranging, we have, writing $c_2* = c_2\left(p_2*-p_1*\right)$:

$$c_2(-p_1*+p_2*-\varepsilon) \leq \frac{p_1*c_2*-\varepsilon A_1}{p_1*+\varepsilon} \quad (3)$$

$$\frac{p_2*c_2*-\varepsilon A_2}{p_2*-\varepsilon} \leq c_2\left(p_2*-p_1*-\varepsilon\right) \quad (4)$$

(3) and (4) imply that the right side of (3) is at least as large as the left side of (4), which becomes:

$$\left(p_1*+p_2*\right)c_2* \leq \left(p_1*+\varepsilon\right)A_2 - \left(p_2*-\varepsilon\right)A_1 \quad (5)$$

The left side of (5) is strictly positive under our assumptions. Hence

$\frac{A_2}{A_1} > \frac{p_2*-\varepsilon}{p_1*+\varepsilon}$ for all $\varepsilon \in [0, -p_1*+p_2*]$.

For ε sufficiently small, there is a contradiction, since $A_2 < A_1$ and $p_1* > p_2*$.

Suppose finally that there is a pure strategy equilibrium where $p* = p_2* = p_1* > 0$, say. Then there is no equilibrium cross-border shopping ($c_1(0) = c_2(0) = 0$). Again it must be the case that country 1 (2) does not want to Nash deviate to a higher (lower) tax $p*+\varepsilon$ $(p*-\varepsilon)$, where $\varepsilon \in \left(0, p*\right)$, which requires, since country 1 is the high tax country in each case:

$$p*A_1 \geq (p*+\varepsilon)[A_1 - c_1(\varepsilon)] \quad (6)$$

$$p*A_2 \geq (p*-\varepsilon)[A_2 + c_1(\varepsilon)] \quad (7)$$

Hence $\frac{\varepsilon A_1}{p*+\varepsilon} \leq c_1(\varepsilon) \leq \frac{\varepsilon A_2}{p*-\varepsilon}$

And $\frac{A_2}{A_1} \geq \frac{p*-\varepsilon}{p*+\varepsilon}$, for all $\varepsilon \in \left(0, p*\right)$, which again provides a contradiction for ε sufficiently small, as $\frac{A_2}{A_1} < 1$.

Q.E.D.

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