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## On the Paradigm of Loss Aversion

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# On the Paradigm of Loss Aversion

by Horst Zank\*

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**Abstract:** This paper discusses the various approaches that have been adopted to analyze loss aversion under prospect theory. Subsequently, the view is promoted that loss aversion is a property of observable choice behavior. Under prospect theory loss averse behavior is predicted if an utility based measure of loss aversion, corrected by a probability based measure of loss aversion, exceeds 1. It is shown that prominent parametric families of weighting functions, while successful in accommodating empirical findings on probabilistic risk attitudes, may not be suitable for the analysis of loss averse behavior.

Keywords: Choice Behavior, Loss Aversion, Probability Weighting, Prospect Theory, Utility.

JEL Classification Code: D81, C60.

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# 1 Introduction

The normative view in decision under risk requires that an economic agent's utility is defined over final levels of wealth. In the classical theory of expected utility risk attitudes have completely been absorbed by qualitative features of utility (see Arrow 1971 and Pratt 1964). Empirically, having only a single function to model risk attitudes has proven to be unsatisfactory. To overcome this difficulty a new measure has been invoked. Rank-dependent utility theory (Quiggin 1981, 1982, Schmeidler 1989) incorporates, alongside utility, a second function which models the sensitivity towards the event of ending up with a particular level of wealth.

While the idea of rank-dependence has proven to be successful in resolving some of the major criticism on expected utility (Allais 1953, Ellsberg 1962), the empirical and theoretical literature has uncovered an additional aspect of risk behavior —loss aversion— that seriously challenges the assumption of final wealth levels being the appropriate primitive for utility (Kahneman and Tversky 1979, Rabin 2000).

The empirical evidence supporting the view that people are sensitive to changes in wealth can be traced back at least to Markowitz (1952) and Edwards (1955) and it suggests that the outcomes over which utility is maximized are deviations relative to a reference point. These and many subsequent findings (e.g., Kahneman and Tversky 1979, Hershey and Schoemaker 1985, Budescu and Weiss 1987, Camerer 1989, Tversky and Kahneman 1992, Huber, Ariely, and Fischer 2002, Schmidt and Traub 2002, Brooks and Zank 2005) give immense support for theories where the perception of outcomes and associated probabilities is differentiated following the sign of the corresponding outcomes (Luce 1991, Luce and Fishburn 1991, Kahneman and Tversky 1991, Tversky and Kahneman 1992). The asymmetry in the perception and its effect on choice behavior is reconciled under the paradigm of loss aversion.

Among the nowadays most influential descriptive models of choice, prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992) distinguishes outcomes into gains and losses relative to a reference point and thereby provides a framework for the analysis of loss aversion. In this theory, in addition to the exhibited sensitivity towards outcomes (captured by utility) and the sensitivity towards probabilities (captured by probability weighting functions), there is a further important aspect of behavior that exhaustively complements the picture on risk attitudes. Loss aversion is the term used to describe

this latter aspect. But while the former components of risk attitude have received a good theoretical account, the intense discussions on the latter component have not yet lead to a commonly agreed upon formal underpinning.

The commonly adopted view is that loss aversion refers to the asymmetric perception of gains compared to the perception of losses. Indeed, an abundance of empirical studies have confirmed that people exhibit more sensitivity towards losses than they do towards gains, and intuitively, one would think that this asymmetry is reflected only in the component of risk attitude that refers to sensitivity towards outcomes, hence, in utility. This is the predominant interpretation found in the literature (Tversky and Kahneman 1992, Wakker and Tversky 1993, Köbberling and Wakker 2005). But even under that interpretation there is disagreement on what the correct measure of loss aversion should be (for a discussion on proposed measures of loss aversion see Abdellaoui, Bleichrodt and Paraschiv 2007). Loss aversion is sometimes viewed as the asymmetric effect on utility of absolute changes from the reference outcome (i.e., losses weigh heavier than comparable gains) while sometimes it is viewed as the asymmetric effect of marginal changes from gains and losses of equal size (i.e., utility is steeper for losses than for similar gains). In modeling loss aversion, often a single parameter has been invoked (Benartzi and Thaler 1995, Tversky and Kahneman 1992, Aizenman 1998, Bowman, Minehart, Rabin 1999, Köbberling and Wakker 2005). In dynamic settings, this parameter may vary, depending on the direction in which the reference outcome is shifted (Barberis, Huang, and Santos 2001, and Barberis and Huang 2001), but is typically seen as a component of utility.

It is, however, equally justified to think of loss aversion as capturing both the disproportionate sensitivity towards gains and losses and as well the disproportionate sensitivity towards probabilities of gains and of losses. Empirical research on probability weighting has found significant differences between the weighting function for gain probabilities and that for loss probabilities (Tversky and Kahneman 1992, Camerer and Ho 1994, Abdellaoui 2000, Abdellaoui, Vossmann and Weber 2005) and it is plausible to attribute these differences also to the sign of the associated outcomes. Earlier studies have also discussed the potential dependence of probability perception on the sign of the associated outcomes (e.g., Edwards 1962, Einhorn and Hogarth 1985). However, a connection to loss aversion has not been explored.

The study of Schmidt and Zank (2005) defined loss aversion as being revealed by spe-

cific choice behavior. Schmidt and Zank showed that under the modern version of prospect theory (Tversky and Kahneman 1992) ignoring the sensitivity towards probabilities will lead to predictions that are in contrast to the intuition of loss averse behavior (e.g., willingness to take on symmetric 50:50 prospects may hold with a utility that is steeper for losses than for gains).

The objective of this paper is to review the development of loss aversion within the development of prospect theory. The incorporation of rank-dependence into prospect theory, leading to the modern version of Tversky and Kahneman (1992) did not happen in the theoretical analysis of loss aversion. In an attempt to account for rank-dependence, new definitions of loss averse behavior emerge, which, as is shown below, are compatible with modern prospect theory.

Two particular ways of accounting for loss averse behavior are discussed in this paper. To reflect on the difference it is useful to recall the intuition on loss aversion. Kahneman and Tversky (1979, p. 279) argued that loss aversion entails two aspects of behavior. Individuals dislike symmetric 50:50 prospects and further the aversion to such prospects increases with the magnitude of outcomes. The first aspect, suggesting an account for loss attitude driven by absolute values, is the property of general aversion to symmetric prospects. The second aspect, suggesting an account for loss attitude driven by marginal values, is the property of aversion to common increases in the absolute values of the stakes of symmetric prospects. Both aspects are progressed in this paper, and a further one is added. Common increases in the likelihood of the symmetric stakes do also lead to more aversive behavior.

The structure of the subsequent discussion is as follows. In the next section the modern version of prospect theory is presented and the qualitative features of utility and the weighting functions are reviewed, with loss aversion discussed separately in Section 3. Section 4 presents the new definitions of loss averse behavior and explores their consequences for prospect theory. The paper concludes with some final remarks in Section 5.

## **2 Prospect Theory, Utility and Weighting Functions**

In this section the modern version of prospect theory, as presented in Tversky and Kahneman (1992), is reviewed. Particular attention is paid to the curvature of utility and of the

weighting functions proposed to accommodate the empirical evidence on risk behavior.

The paper considers the case of decision under risk. The set of outcomes is identified with the real line,  $\mathbb{R}$ , with the *reference outcome* assumed to be 0. Positive outcomes are *gains* and negative outcomes are *losses*.

A *prospect* is a finite probability distribution over the set of outcomes, e.g.,  $P = (p_1, x_1; \dots; p_n, x_n)$ , where the probabilities  $p_i$  are nonnegative and they sum to 1. With this notation it is implicitly assumed that the outcomes in the prospect  $P$  are ordered from best to worst. Therefore, there exists an integer  $0 \leq k \leq n$ , which is used to distinguish gains from losses, such that

$$x_1 \geq \dots \geq x_k > 0 \geq x_{k+1} \geq \dots \geq x_n,$$

with all outcomes being gains if  $k = n$ , and all outcomes being losses (or 0) if  $k = 0$ .

A preference relation, denoted by the symbol  $\succsim$ , is assumed over the set of prospects. As usual,  $\succ$  denotes strict preference,  $\succsim$  means weak preference, and  $\sim$  is indifference. The corresponding symbols for reversed strict and weak preference are  $\prec$  and  $\precsim$ , respectively.

In its general form, prospect theory requires a utility function to evaluate gains and losses, and two possibly different weighting functions to evaluate the probabilities associated with gain and loss outcomes, respectively. This modern version of prospect theory refers to the variant that is derived in axiomatic research: (*Cumulative*) *prospect theory* (PT) holds if any prospect  $P = (p_1, x_1; \dots; p_n, x_n)$  is evaluated by the functional

$$\begin{aligned} PT(P) = & \sum_{i=1}^k [w^+(\sum_{j=1}^i p_j) - w^+(\sum_{j=1}^{i-1} p_j)] U(x_i) \\ & + \sum_{i=k+1}^n [w^-(\sum_{j=i}^n p_j) - w^-(\sum_{j=i+1}^n p_j)] U(x_i) \end{aligned} \quad (1)$$

described next.<sup>2</sup>

The three functions involved in the PT-functional are the utility function,  $U$  (sometimes also called value function in the literature), the weighting function for gain probabilities,  $w^+$ , and the weighting function for loss probabilities,  $w^-$ .

The utility function assigns a real value to each outcome, and is strictly increasing and continuous. In general under PT, utility is unique up to scale and location, that is,  $U$  can be replaced by any  $V = aU + b$  for  $a > 0$  and a real value  $b$ . However, in most applications

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<sup>2</sup>For convenience, I apply the mathematical convention  $\sum_{j=m+1}^m p_j = 0$ , for  $m = 0, n$ .

it is convenient to set  $U(0) = 0$ , turning utility into a ratio-scale (i.e., it is unique up to multiplication by a positive constant). I follow the latter restriction as it will simplify the subsequent analysis without loss of generality.

The gain and the loss weighting functions map the probability interval  $[0, 1]$  into itself, and they are strictly increasing functions assigning 0 to 0 and 1 to 1. In Equation (1) the weighting functions apply to cumulative probabilities, and the differences in these distorted probabilities (i.e., the terms  $w^+(\sum_{j=1}^i p_j) - w^+(\sum_{j=1}^{i-1} p_j)$  and  $w^-(\sum_{j=i}^n p_j) - w^-(\sum_{j=i+1}^n p_j)$  in the notation above) are referred to as decision weights of the corresponding outcomes  $x_i$ ,  $i = 1, \dots, n$ . Under PT the weighting functions are uniquely determined.

Axiomatizations of general PT have been provided by Tversky and Kahneman (1992), Wakker and Tversky (1993), Chateauneuf and Wakker (1999), and Köbberling and Wakker (2003). Other axiomatic studies have focused on a more general account of rank and sign-dependence (Luce and Fishburn 1991, and Luce 1991). Related theories have incorporated a more general notion of reference-dependence (e.g., Sugden 2004, Schmidt 2003, Schmidt, Starmer and Sugden 2005, Bleichrodt 2006).

The interest in parametric estimations has led to specific preference foundations for prospect theory. Some derivations have imposed behavioral properties that restrict the general form of the utility function but maintain general probability weighting functions: utility is a power function in Wakker and Zank (2002), power and linear/exponential in Zank (2001), and linear in Schmidt and Zank (2006).

Popular variants of PT with general utility but restricted weighting functions have been proposed by Prelec (1998) (see also Luce 2001, Aczel and Luce 2006 and al-Nowaihi and Dhami 2006). Gonzalez and Wu (1999) identified a necessary condition for the parametric weighting function of Goldstein and Einhorn (1987) and Lattimore Baker and Witte (1992). In Diecidue, Schmidt and Zank (2006) alternative classes of weighting functions are derived. They obtained rank-dependent utility jointly with the parametric specifications by proposing behavioral preference conditions characterizing power and exponential weighting functions, and inverse- $S$  weighting functions that combine the former families.

The restrictive shapes of utility and of the weighting functions are motivated by extensive empirical research relating to risk attitudes. The qualitative features of the functions can be classified into sensitivity towards outcomes and sensitivity towards probabilities, with an additional dimension referring to the sensitivity caused by the sign of outcomes.

Further, the sensitivity can be measured in an absolute sense, by looking at the effect of absolute changes in the corresponding functions, or measured in a relative sense by looking at the effect of marginal changes. These various aspects are discussed next.

Considering outcomes, sensitivity has been modeled by a convex utility for losses and a concave utility for gains (see Kahneman and Tversky 1979, Hershey and Schoemaker 1980, Camerer 1989, Currim and Sarin 1989, Battalio, Kagel, and Jiranyakul 1990, Tversky and Kahneman 1992, Fennema and Van Assen 1998, Abdellaoui 2000, Bleichrodt and Pinto 2000, Smith, Dickhaut, McCabe, and Pardo 2000, Laury and Holt 2002, Abdellaoui, Vossman, and Weber 2005, Abdellaoui, Bleichrodt and Paraschiv 2007). However, the evidence for specific curvature of the utility function in the loss domain is mixed. The frequent finding is convex utility with less curvature compared to the utility for gains. Some studies also reported evidence for linear and concave utility (Schneider and Lopes 1986, Cohen, Jaffray, and Said 1987, Weber and Bottom 1989, Loehman 1998, Lopes and Oden 1999, Abdellaoui 2000, Barron and Erev 2003, Laury and Holt 2002, Etchart-Vincent 2004, Abdellaoui, Vossman and Weber 2005). Because curvature becomes less pronounced after repeated choices, Myagkov and Plott (1997) hypothesized that with experience convexity for losses may disappear or even turn into concavity. Because earlier studies did not adjust for probability weighting, those findings of linear and concave utility for losses are ambiguous and suggest that there may well be more convexity for losses than documented (see Abdellaoui, Bleichrodt and Paraschiv 2007 for a discussion and review of the empirical evidence).

One observes that the empirical evidence for the sensitivity in outcomes refers to the effect on utility of marginal changes in outcomes. This *S*-shaped form for the utility function, which has been adopted in many applications (Tversky and Kahneman 1992, Gneezy and Potters 1997, Bowman, Minehart, Rabin 1999, Shalev 2000, Siegmann 2002, Berkelaar, Kouwenberg and Post 2004, Gomes 2005), is sometimes confounded with “risk seeking for losses and risk aversion for gains,” however, that interpretation is ignoring the fact that risk attitudes in prospect theory are far more complex due to the perception of probabilities and due to loss aversion. Diminishing sensitivity to outcomes as the magnitude increases, which is the original interpretation, seems a much more appropriate term.

The sensitivity towards marginal changes in probabilities displays analogy to that of the



utility, leading to inverse-S shaped probability weighting functions, which have a concave region for small probabilities followed by a convex region for larger probabilities (Tversky and Fox 1995, Abdellaoui 2000, Bleichrodt and Pinto 2000, Etchart-Vincent 2004, Abdellaoui, Vossman and Weber 2005). This shape indicates that sensitivity towards changes in probabilities is intensifying the less likely a bad or a good outcome becomes, and is interpreted as optimism and pessimism regarding the extreme outcomes within prospects, respectively (see, e.g., Wakker 1994, 2001).

The two weighting functions in PT share these two features of extreme sensitivity at 1 and 0, although the intensity by which probabilities are weighted may differ depending on the sign of the associated outcomes. For gain probabilities evidence has been provided, among others, by Tversky and Kahneman (1992), Wu and Gonzalez (1996), Wu and Gonzalez (1998), Abdellaoui (2000), Jullien and Salanié (2000). Fewer studies exist for loss probabilities; evidence was given by Hogarth and Einhorn (1990), Tversky and Kahneman (1992), McClelland, Schulze, and Coursey (1993), Abdellaoui (2000), Etchart-Vincent (2004), and Abdellaoui, Vossman, and Weber (2005).

In many empirical studies only binary prospects are considered. In that case, observing decision weights which exceed the objective probabilities are interpreted as overweighting, while decision weights which are below objective likelihood are viewed as underweighting. This interpretation is focusing on the effect of absolute likelihood of outcomes on probability perception rather than the marginal effect (i.e., the decision weights). A little reflection reveals that for binary prospects overweighting and underweighting cannot be distinguished from optimistic and pessimistic behavior. The latter therefore gives a more accurate interpretation of probabilistic attitudes because it is independent of the nature of prospects.

The afore mentioned marginalistic and absolutistic interpretations of the shape of the weighting functions seem to have their origin in the distinct prospect theories. Original prospect theory (Kahneman and Tversky 1979) agrees with modern prospect theory (Tversky and Kahneman 1992) if binary prospects are considered. But the models may differ if more complex prospects are employed (for a recent discussion see Wu, Zhang and Abdellaoui 2005, and L'Haridon 2006). The former was developed only for simple prospects involving at most two non-zero outcomes, and the prospects used in its empirical foundation were binary ones. Modern PT took into account developments regarding probability

weighting initiated by Quiggin (1981, 1982) and Schmeidler (1989), and therefore applies to general prospects. So, a distinction between the marginal and absolute analysis of probabilistic risk attitude is not possible for original prospect theory, but it is for modern PT as outlined above.

The picture on risk attitudes under PT is not fully accounted for, unless one discusses loss attitude. Because of its importance, I devote the next section to this phenomenon, even though in a discussion on risk attitude under prospect theory it appears unnatural to separate loss aversion from the former aspects of risk attitude.

### 3 The Approaches to Loss Aversion

The third component of risk attitude under prospect theory is loss aversion. In my view, it is this aspect of the theory that accounts largely for the success and popularity of the model. However, while the modern prospect theory has incorporated theoretical developments regarding probability weighting (rank-dependence), for quite some time loss aversion remained a theoretically nonexamined feature. Recall that Tversky and Kahneman (1992, p. 303) adopted loss aversion as a feature of marginal utility (i.e.  $U'(x) < U'(-x)$  for all  $x > 0$ ) as was concluded in original prospect theory (Kahneman and Tversky 1979), and the subsequent literature followed this “utilitarian” interpretation of loss aversion (Wakker and Tversky 1993, Bowman, Minehart and Rabin 1999, Starmer 2000, Neilson 2002, Schmidt and Traub 2002, Köbberling and Wakker 2005, Abdellaoui, Bleichrodt and Paraschiv 2007). Therefore, loss aversion was understood as reconciling only the risk attitudes for gains and for losses that are captured by utility.

It must also be noted that Tversky and Kahneman (1992) highlighted the adoption of two weighting functions for the probabilities of gains and the probabilities of losses as a distinctive feature of modern PT. But what is it that justifies having two separate weighting functions to model sensitivity towards probabilities if it is not the sign of the associated outcomes? If the perception of probability is independent of the sign of outcomes, then having two distinct weighting functions under prospect theory seems conceptually void. So, the sign of outcomes matters for utility as well as for probability perception, and if utility captures risk attitudes towards outcomes and probability weighting captures probabilistic risk attitudes, then it seems natural to have loss aversion reconciling differences between

the two probability weighting functions in addition to the asymmetric effects captured by utility. This then also means that rank-dependence, which is an important aspect of modern PT, will also play its part for loss attitude.

Theoretically, it seems quite natural to compare the two probability weighting functions because they apply to the same domain (i.e., the probability interval), and then attribute any difference in probability perception to loss attitude. Edwards (1962) and Hogarth and Einhorn (1990) also argued that probability perception may well depend on the sign or size of outcomes (see also Einhorn and Hogarth 1985, 1986). They summarized early empirical evidence and also provided additional experimental support for this thesis. In prospect theory, the only plausible source for such differences in probability weighting is the sign of the outcomes. This argument cannot apply to utility because gains and losses are separate domains within the set of outcomes. So, when comparing the outcome sensitivity for gains with that for losses one cannot exclude that differences are, at least in part, caused by what Köbberling and Wakker (2004) coined basic utility instead of being caused by loss attitude. This makes the utilitarian interpretation of loss aversion, although simple and for many purposes practical, a problematic one.

There is also a different criterion that must be taken into account when analyzing loss attitudes under prospect theory. For a descriptive theory it is important to model observed choice behavior, and therefore and in particular for prospect theory, loss averse behavior. Assuming that probability weighting has no influence on loss attitude one can easily generate plausible weighting functions and a utility that is steeper for losses than for gains within a prospect theory model that implies the opposite, gain seeking, behavior (see Schmidt and Zank 2005). From a descriptive point of view this is not satisfactory. It should be noted that, accounting for different probability weighting functions will avoid such counter-intuitive predictions.

It seems therefore reasonable to reconsider the intuition of loss averse behavior in original prospect theory and to extend that notion in a way that is compatible with the empirical evidence and theoretical advances on utility and probability weighting. Two potential ways to proceed emerge: an approach reconciling absolute effects under loss attitude and an approach reconciling marginal effects. At this stage it is worth noting that, in view of the discussion regarding utility and probability weighting presented in the preceding section, a clear distinction between the absolute and the marginal interpretation

of loss aversion has emerged under both versions of PT. The statements “losses weigh heavier than gains” (i.e.,  $U(x) < -U(x)$  for all  $x > 0$ , see Starmer 2000) and “utility is steeper for losses than for gains” (i.e.,  $U'(x) < U'(-x)$  for all  $x > 0$ , see Wakker and Tversky 1993) reflect precisely these two interpretations in the analysis of loss attitude. What is missing in these analyses is the incorporation of probability weighting.

Recall, that Kahneman and Tversky (1979, p. 279) motivate loss aversion with the observation that individuals dislike symmetric 50-50 prospects (the absolute account of loss aversion), and further, that the aversion to such prospects increases with the absolute size of the outcomes (the marginal account of loss aversion). So, in the component of loss aversion referring to the size of outcomes, what deters is the fact that there is a possibility (i.e., an event with positive probability) of losing an amount and this event cannot be compensated by the possibility of gaining that same amount. The component of loss aversion referring to changes in outcomes says that additional potential losses vis-a-vis additional potential gains of similar magnitude strengthen the aversive behavior. Observe that, beyond the symmetry in outcomes and the symmetry requirement for probability, such prospects do not offer the possibility of compensating losses otherwise. In contrast to choice under certainty (Kahneman and Tversky 1991), the possibility (but not certainty) of obtaining losses is relevant for the analysis of choice behavior under risk. Once this is recognized, it seems natural to include probability weighting into loss aversion. This is done formally in the next section.

## 4 Loss Averse Behavior and Prospect Theory

This section will pay attention to the implication for prospect theory of three features of behavior. First, there is the unwillingness to take on prospects in which with the same probability the same amount can be won and can also be lost. Second, increments in the symmetric stakes of such prospects results in even more aversive behavior. A third aspect of behavior is added into the debate: the aversion to symmetric prospects increases also as a result of increased likelihood of the symmetric stakes. Therefore, the intuition that underlies loss aversion (Kahneman and Tversky 1979) is extended and formalized.

Two new definitions of loss aversion are proposed and their implications for modern prospect theory are discussed. In particular, the compatibility of the new definitions with

existing evidence on probability weighting is explored.

The aversion to symmetric prospects suggests the following condition for choice behavior:

**DEFINITION 1** A preference relation  $\succsim$  exhibits *absolute loss aversion* if for all probabilities  $0 < p \leq 1/2$  and all outcomes  $x > 0$  the relation  $(1, 0) \succ (p, x; 1 - 2p, 0; p, -x)$  holds. *Absolute gain seeking behavior (insensitivity)* holds if the previous preferences are reversed (replaced by indifference,  $\sim$ ).

As a first remark, note that if  $p = 1/2$  is set in the above definition, then one obtains the condition of aversion of symmetric 50:50 prospects quoted in Kahneman and Tversky (1979, p. 279). The above definition maintains three symmetry requirements. The first refers to the magnitude of outcomes: gains and losses are of the same size; the second requirement refers to the likelihood of outcomes: gains and losses are equally probable; and the third requirement refers to the rank of outcomes: gains and losses are always the extreme (i.e., best and worst, respectively) outcomes, so, within the corresponding domain, of the same rank.

Substituting the PT function for the preference in the previous definition implies

$$w^+(p)U(x) < -w^-(p)U(-x)$$

for all  $0 < p \leq 1/2$  and all  $x > 0$ . An alternative notation is

$$-\frac{w^-(p)}{w^+(p)} \frac{U(-x)}{U(x)} > 1 \tag{2}$$

for all  $0 < p \leq 1/2$  and all  $x > 0$ , if all ratios are well defined. Observe that in this latter inequality the utility ratio depends solely on the outcome  $x > 0$ , and the ratio of weighted probabilities solely on  $0 < p \leq 1/2$ . This is natural for prospect theory because the risk component captured by utility is independent of the risk component captured by probability weighting. So, absolute loss aversion combines two measures. The first relates the absolute utility of a loss with that of an equally sized gain, and the second relates the probability weight of a loss with the probability weight of an equally likely gain.

If  $w^-$  intersects  $w^+$  at some  $0 < p^* \leq 1/2$ , then (2) implies that, in terms of utility, losses weigh heavier than corresponding gains, i.e.,  $-U(-x)/U(x) > 1$  for all  $x > 0$ . This occurs, for example, if one assumes that the inverse- $S$  weighting functions are from the

family used in Tversky and Kahneman (1992),

$$w(p) = \frac{p^\gamma}{[p^\gamma + (1-p)^\gamma]^{1/\gamma}}, 0 < \gamma < 1,$$

and one takes their estimated  $\gamma^+ = 0.61$  and  $\gamma^- = 0.69$ .

Similarly, if the functions  $-U(-(\cdot))$  and  $U(\cdot)$  intersect, as would be the case if power functions of distinct powers were assumed, then the weighting functions for losses would exceed the weighting function for gains for probabilities  $0 < p \leq 1/2$ .

In general, absolute loss averse behavior does not imply that the utility based measure of loss aversion,  $-U(-x)/U(x)$ , exceeds 1. Hence, one may observe absolute loss averse behavior with a utility such that gains weigh heavier than losses if the weighting function for losses sufficiently exceeds that for gains. Inequality (2) only requires that both the utility ratio and the ratio of weighted probabilities are bounded from below, such that their product exceeds 1. For practical reasons, having a bounded measure of loss aversion is important. As a consequence, the power utility function proposed in Tversky and Kahneman (1992) is not suitable for measuring absolute loss aversion because, unless the power for gains equals the power for losses, the ratio of utilities either approaches 0 or infinity for arbitrary small outcomes. This drawback of the power utility in measuring loss aversion has also been pointed out in Köbberling and Wakker (2005), who demonstrated that an exponential form for utility does not exhibit such difficulties.

In much analogy to the inappropriateness of power utility for loss aversion it turns out that absolute loss averse behavior puts some constraints on the parametric forms used in the estimation of the probability weighting functions. Next, I discuss these implications for some families of inverse-S shaped weighting functions under prospect theory. The functions considered are the one-parameter family of Tversky and Kahneman (1992), the two-parameter family of Goldstein and Einhorn (1987) and Lattimore, Baker and Witte (1992), the two-parameter family of Prelec (1998), and the two-parameter family of Diecidue, Schmidt and Zank (2006).

By inspecting the behavior of the probability ratio in (2) for  $p$  approaching 0, we observe, for the Tversky and Kahneman weighting functions, that

$$\lim_{p \rightarrow 0, p > 0} \frac{w^-(p)}{w^+(p)} = \lim_{p \rightarrow 0, p > 0} \left\{ p^{(\gamma^- - \gamma^+)} \frac{[p^{\gamma^-} + (1-p)^{\gamma^-}]^{1/\gamma^-}}{[p^{\gamma^+} + (1-p)^{\gamma^+}]^{1/\gamma^+}} \right\} = 0, \text{ if } \gamma^- > \gamma^+.$$

Hence, the relationship  $\gamma^- \leq \gamma^+$  follows if (2) is assumed.

For the Goldstein and Einhorn parametrization,

$$w(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}, \gamma \in (0, 1), \delta > 0,$$

it follows similarly that  $\gamma^- \leq \gamma^+$  if (2) is assumed.

For the parametric family of Prelec,

$$w(p) = \exp(-\delta(-\ln p)^\gamma), \gamma \in (0, 1), \delta > 0,<sup>3</sup>$$

assuming (2) the relationship  $\gamma^- < \gamma^+$  (or  $\gamma^- = \gamma^+, \delta^- \leq \delta^+$ ) is implied.

For the Diecidue, Schmidt and Zank switch-power weighting functions,

$$w(p) = \begin{cases} \delta^{1-\gamma} p^\gamma, & \text{if } 0 \leq p \leq \delta \\ 1 - (1-\delta)^{1-\gamma} (1-p)^\gamma, & \text{if } \delta < p \leq 1 \end{cases}, \delta, \gamma \in (0, 1),$$

which are concave up to the fix point  $\delta$  and then convex, it follows that  $\gamma^- \leq \gamma^+$  holds if (2) is assumed.

For all the above families of weighting functions, the restriction  $\gamma^- < \gamma^+$  implies that  $w^-$  is above  $w^+$  for small probabilities. One could interpret this as “the same likelihood if associated with a loss weighs more heavily than if it is associated with a gain of the same size.” This statement, however, may not be valid for all probabilities if the above parametric forms are used.

At first glance, focusing only on estimated parameters, it appears that the requirement  $\gamma^- \leq \gamma^+$  for the above families of weighting functions is not supported by empirical findings. Taking the parameter estimates of Tversky and Kahneman (1992),  $\gamma^+ = 0.61$  and  $\gamma^- = 0.69$ , one observes that the relationship  $\gamma^- \leq \gamma^+$  is violated, suggesting that (2) is too demanding. Note, however, that for moderate probabilities these parameters still imply that  $w^-$  is above  $w^+$ . But a closer examination of related empirical evidence reveals that the Tversky and Kahneman weighting functions may not be appropriate for an analysis of the difference in the weighting of probabilities of gains and the weighting of probabilities of losses. For example, Abdellaoui (2000) found similar parameter estimates for the Tversky and Kahneman weighting function ( $\gamma^+ = 0.6$  and  $\gamma^- = 0.7$ ) but estimated also the parameters of the Goldstein and Einhorn weighting function ( $\gamma^+ = 0.6, \delta^+ = 0.65$  and  $\gamma^- = 0.65, \delta^- = 0.84$ ). The extra flexibility resulting from the additional parameter

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<sup>3</sup>A further upper bound constraint applies to  $\delta$  to ensure concavity for small probabilities:  $\delta \leq (\gamma - \ln p - 1)/[\gamma(-\ln p)^\gamma]$  for all  $p \in (0, p^*)$ ,  $p^* > 0$  arbitrary small.

shows that, although  $\gamma^- \leq \gamma^+$  is violated, only for very small probabilities (e.g.,  $0 < p < 0.006$ ) is the gain weighting function above the loss weighting function. Further, the data in Abdellaoui (2000) suggests that the curvature parameters  $\gamma^-$  and  $\gamma^+$  do not differ significantly, and that the difference in the weighting functions is captured mainly by the elevation parameters  $\delta^+$  and  $\delta^-$ . Beyond their role for the analysis of probabilistic risk attitudes (see Wu and Gonzalez (1996) and Gonzalez and Wu (1999) for a discussion), this finding highlights the importance of the afore mentioned weighting functions which involve two parameters for the analysis of loss averse behavior. In particular, clarifying on the potential role of the elevation parameter for loss averse behavior calls for further empirical and theoretical investigation.

Abdellaoui (2000) is, to my knowledge, the only study that looked specifically at the difference between the gain and the loss weighting function without invoking parametric specifications for the latter. Abdellaoui (p. 1507) notes that the loss weighting function is significantly above the gain weighting function, supporting the view that people are more sensitive to probabilities of losses relative to probabilities of gains.<sup>4</sup> Combined with inequality (2) this means that the utility ratio between a loss and a comparable gain, typically used to measure loss aversion, may underestimate the true degree of loss aversion. Rewriting inequality (2) as

$$-\frac{U(-x)}{U(x)} \frac{w^-(p)}{w^+(p)} > 1, \text{ for all } 0 < p \leq 1/2 \text{ and all } x > 0, \quad (3)$$

indicates that  $w^-$  being above  $w^+$  leads to an upward adjustment of the utility based measure of loss aversion by the “probability based measure of loss aversion”  $w^-(p)/w^+(p)$ . How large this adjustment should be needs to be determined by further empirical studies. A rough estimate using the ratio of the elevation parameters estimated in Abdellaoui (2000) gives an adjustment factor of approximately 1.29. As I argue below, this pragmatic measure results by combining absolute loss aversion with the Goldstein and Einhorn parametrization under prospect theory.

The left term in inequality (3) can be used to derive an index of absolute loss aversion. Following Köbberling and Wakker (2005) one may specify the utility index of loss aversion as the ratio of marginal utility at the reference point,  $U'(0_-)/U'(0_+)$ , whenever this ratio is

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<sup>4</sup>In a related study, Etchart-Vincent (2004) provides evidence for the probability weighting function elicited using large losses being significantly above that elicited using small losses under prospect theory.



well defined.<sup>5</sup> One may then define the probability index of loss aversion as the maximum of the ratio  $w^-(p)/w^+(p)$  for  $0 < p \leq 1/2$ , if an upper bound for  $w^-(p)/w^+(p)$  exists. Or, in analogy to the utility index of loss aversion, one can define  $w^{-'}(0_+)/w^{+'}(0_+)$  as the probability index of loss aversion, if this ratio is well defined.

Both definitions for the probability index of loss aversion will hinge upon the behavior of the weighting functions at 0. For the parametric forms discussed before, these indices are not well defined, except if  $\gamma^- = \gamma^+ =: \gamma$  ( $\gamma^- = \gamma^+$  and  $\delta^- = \delta^+$  in the case of the Prelec weighting functions). If the latter holds, then the Tversky and Kahneman and the Prelec weighting functions give a probability index of loss aversion equal to 1 as a consequence of  $w^+(p) = w^-(p)$  for all  $p$ . The Goldstein and Einhorn weighting functions lead to  $w^{-'}(0_+)/w^{+'}(0_+) = \delta^-/\delta^+$ , and therefore also the  $\max_{0 \leq p \leq 1/2}[w^-(p)/w^+(p)]$  on the closed interval  $[0, 1/2]$  is well defined. Similarly, the Diecidue, Schmidt and Zank weighting functions lead to  $w^{-'}(0_+)/w^{+'}(0_+) = (\delta^-/\delta^+)^{1-\gamma}$ , and note that in this case the  $\max_{0 \leq p \leq 1/2}[w^-(p)/w^+(p)]$  is also equal to  $(\delta^-/\delta^+)^{1-\gamma}$ .

Summarizing the analysis thus far, it can be concluded that absolute loss averse behavior is compatible with general PT preferences and with the empirical evidence on probability weighting. The overall degree of absolute loss aversion is the product of two indices, one utility based measure of absolute loss aversion and a new probability based measure of absolute loss aversion. For some parametric specifications of utility and weighting functions the individual indices of loss aversion are not well defined. As pointed out in Köbberling and Wakker (2005), this suggests that new parametric forms for utility are warranted. As indicated above, this conclusion can be extended to the probability weighting functions.

I now turn to the impact on absolute loss aversion caused by joint increments in the equally likely symmetric outcomes or by joint increments in their equal likelihood.

**DEFINITION 2** A preference relation  $\succsim$  exhibits *marginal loss averse behavior* if

$$(q, y; 1 - 2q, 0; q, -y) \succsim (q + p, y + x; 1 - 2q - 2p, 0; q + p, -y - x)$$

holds for all probabilities  $q > 0$ , and  $p$  such that  $0 < q + p \leq 1/2$  and all outcomes  $y > 0, x \geq 0$ , with a strict preference whenever  $p$  or  $x$  are positive. *Marginal gain seeking behavior (insensitivity)* holds if the previous preferences are reversed (replaced by indifference,  $\sim$ ).

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<sup>5</sup>The subscripts “+” and “-” indicate left and right derivatives, respectively.

One can observe that marginal loss averse behavior does not require absolute loss averse behavior. But notice that the three symmetry requirements regarding size, rank and likelihood of the symmetric outcomes, mentioned after Definition 1, are maintained. Under prospect theory, which satisfies comonotonic independence, one could replace in both prospects the common  $(1 - 2q - 2p)$ -chance of retaining the reference point with a common  $(1 - 2q - 2p)$ -chance of getting any outcome between  $-y$  and  $y$ . It seems, however, natural to keep the reference point as the common outcome in the definition above due to its special role in demarcating gains and losses.

Next, two implications for prospect theory of marginal loss averse behavior are analyzed. The first results by setting  $p = 0$  in the previous definition, when one obtains increased aversion due to equal marginal increments in the symmetric stakes. This is precisely the second aspect of loss aversion pointed out by Kahneman and Tversky (1979). Substitution of prospect theory implies

$$w^+(q)[U(y+x) - U(y)] < w^-(q)[U(-y) - U(-y-x)], \quad (4)$$

for all  $0 < q \leq 1/2$  and all  $y, x > 0$ . One observes that the discussion presented prior to Definition 2 concerning the weighting functions applies here as well, but note that the resulting consequences now apply to the ratio of marginal utilities at similar sized losses and gains, respectively.

Recall that the empirical evidence regarding the curvature of utility under prospect theory can be summarized as concavity for gains and convexity for losses (see Abdellaoui, Bleichrodt and Paraschiv 2007) with a kink at 0. Although the picture is less clear-cut for losses, it appears that utility is closer to linear for losses than it is for gains, suggesting that utility diminishes slower for losses than for gains, hence, that utility is steeper for losses than for gains over a significant domain of outcomes. Inequality (4) qualifies this asymmetric diminishing sensitivity property by requiring that the weighted marginal utility for losses exceeds the corresponding weighted marginal utility for gains, whereby the weights used are the ones generated by the corresponding weighting function.

If in addition to the marginal utility for losses exceeding the marginal utility for gains we have  $w^-$  is above  $w^+$ , as discussed following Definition 1, then inequality (4) is trivially satisfied. This suggests that the existing evidence on probability weighting and utility curvature drawn from separate sources may well support the aspect of marginal loss averse behavior that refers to changes in symmetric outcomes. In a recent study Brooks and

Zank (2005) fixed  $q$  at  $1/3$  and  $1/4$ , respectively, and found that more than half of their participants exhibited more aversive behavior when the symmetric stakes within a prospect increased, while roughly a quarter of the participants were exhibiting the opposite behavior.

The second implication of marginal loss averse behavior concerns the new aspect referring to the effect on preferences of equal marginal changes in the probabilities of the symmetric outcomes. This results by setting  $x = 0$  in Definition 2, in which case one obtains that the absolute loss averse behavior is aggravated if the symmetric outcomes of a prospect become more likely. Substitution of prospect theory implies that

$$[w^+(q+p) - w^+(q)]U(y) < -[w^-(q+p) - w^-(q)]U(-y), \quad (5)$$

for all probabilities  $q, p > 0$  such that  $0 < q + p \leq 1/2$  and all  $y > 0$ .

Suppose that for some small  $y > 0$  and small  $q > 0$  one observes

$$(1, 0) \succ (q, y; 1 - 2q, 0; q, -y),$$

hence, ‘local’ absolute loss aversion. Then, substitution of PT and inequality (4) implies

$$(1, 0) \succ (q, y + x; 1 - 2q, 0; q, -y - x),$$

for all  $x > 0$ , hence absolute loss aversion for fixed  $q$ . A subsequent substitution of PT together with inequality (5) then implies that absolute loss aversion holds.

So, absolute loss averse behavior is obtained in the presence of marginal loss averse behavior under prospect theory. This observation shows that if the index of absolute loss aversion  $[w^-(0_+)/U'(0_-)]/U'(0_+)w^+(0_+)$  exceeds 1, then marginal loss aversion is sufficient to imply absolute loss averse behavior, indicating that the former index is a plausible measure of loss aversion.

## 5 Conclusion

Modern prospect theory incorporates several features that have contributed to the popularity of the theory. It has received sound preference foundations and it can explain many relevant phenomena where other theories struggle to convince (Starmer 2000, Kahneman and Tversky 2000, Barberis and Thaler 2003, Camerer, Loewenstein and Rabin 2004). The distinction of outcomes into gains and losses has been a crucial step in understanding and formally modeling risk attitudes. The decomposition of risk attitude into utility and

probability weighting through rank-dependence has been another important development. The modern version of prospect theory combines the distinction of outcomes into gains and losses elegantly with rank-dependence, and thereby offers a compelling account of human choice behavior. The debate on loss aversion has not yet lead to a commonly agreed upon formal definition for this concept. Earlier studies regarded utility as the sole carrier of loss attitude. The recent study of Köbberling and Wakker (2005) argued that loss attitude is a third independent aspect of risk attitude alongside utility and probability weighting. In this paper, I have argued that loss aversion can be observed from choice behavior, and that the afore mentioned developments regarding the sign and the rank of outcomes are relevant for loss attitude. For prospect theory this means that risk attitudes for gains are reflected for losses through the magnifying glass of loss aversion which enlarges all sensitivity, the one captured by utility and the one captured by probability weighting. While the empirical results on utility curvature have received a good account, there seem too few studies on probability weighting for losses in order to verify the proposed measures of loss aversion. As such the claim that probabilities of losses are weighted heavier than similar probabilities of gains, which has been uncovered as a plausible and potentially meaningful aspect of loss aversion, remains a hypothesis rather than a definite conclusion.

## References

- Abdellaoui, Mohammed (2000), “Parameter-free Elicitation of Utility and Probability Weighting Functions,” *Management Science* 46, 1497–1512.
- Abdellaoui, Mohammed, Frank Vossman, and Martin Weber (2005), “Choice-based Elicitation and Decomposition of Decision Weights for Gains and Losses under Uncertainty,” *Management Science* 51, 1384–1399.
- Abdellaoui, Mohammed, Han Bleichrodt, and Corina Paraschiv (2007), “Loss aversion under Prospect Theory: A Parameter-free Measurement,” forthcoming in *Management Science*.
- Aczél, János and R. Duncan Luce (2006), “A Behavioral Condition For Prelec’s Weighting Function On The Positive Line Without Assuming  $W(1) = 1$ ,” forthcoming in *Journal of Mathematical Psychology*.

- Aizenman, Joshua (1998), "Buffer Stocks and Precautionary Savings with Loss Aversion," *Journal of International Money and Finance* 17, 931–947.
- al-Nowaihi, Ali and Sanjit Dharami (2006), "A Simple Derivation of Prelec's Probability Weighting Function," *Journal of Mathematical Psychology* 50, 521–524.
- Arrow, Kenneth J. (1971), "Essays in the Theory of Risk Bearing." North-Holland, Amsterdam.
- Barberis, Nicholas and Ming Huang (2001), "Mental Accounting, Loss Aversion, and Individual Stock Returns," *Journal of Finance* 56, 1247–1292.
- Barberis, Nicholas, Ming Huang, and Tano Santos (2001), "Prospect Theory and Asset Prices," *The Quarterly Journal of Economics* 56, 1247–1292.
- Barberis, Nicholas, and Richard H. Thaler (2003), "A Survey of Behavioral Finance," in George M. Constantinides, Milt Harris, and R. Stulz (Eds) *Handbook of the Economics of Finance*, Elsevier, Amsterdam.
- Barron, Greg and Ido Erev (2003), "Small Feedback-based Decisions and Their Limited Correspondence to Description-based Decisions," *Journal of Behavioral Decision Making* 16, 215–233.
- Battalio, Raymond C., John H. Kagel, and Komain Jiranyakul (1990) "Testing Between Alternative Models of Choice Under Uncertainty: Some Initial Results," *Journal of Risk and Uncertainty* 3, 25–50.
- Berkelaar, Arjan B., Roy Kouwenberg and Thierry Post (2004), "Optimal Portfolio Choice under Loss Aversion," *Review of Economics and Statistics* 86, 973–987.
- Bleichrodt, Han (2006), "Reference-Dependent Utility with Shifting Reference Points and Incomplete Preferences," Erasmus University Rotterdam, the Netherlands.
- Camerer, Colin F. (1989), "An Experimental Test of Several Generalized Utility Theories," *Journal of Risk and Uncertainty* 2, 61–104.
- Camerer, Colin F. (1998), "Bounded Rationality in Individual Decision Making," *Experimental Economics* 1, 163–183.

- Camerer, Colin F. and Teck-Hua Ho (1994), “Violations of the Betweenness Axiom and Nonlinearity in Probability,” *Journal of Risk and Uncertainty* 8, 167–196.
- Camerer, Colin F., George Loewenstein and Matthew Rabin (2004), “Advances in Behavioral Economics,” Princeton University Press, Princeton, NJ.
- Camerer, Colin F. and Martin Weber (1992), “Recent Developments in Modelling Preferences: Uncertainty and Ambiguity,” *Journal of Risk and Uncertainty* 5, 325–370.
- Chateauneuf, Alain and Peter P. Wakker (1999), “An Axiomatization of Cumulative Prospect Theory for Decision under Risk,” *Journal of Risk and Uncertainty* 18, 137–145.
- Cohen, Michèle and Jean-Yves Jaffray (1988) “Certainty Effect Versus Probability Distortion: An Experimental Analysis of Decision Making under Risk,” *Journal of Experimental Psychology: Human Perception and Performance* 14, 554–560.
- Cohen, Michèle, Jean-Yves Jaffray, and Tanius Said (1987), “Experimental Comparisons of Individual Behavior under Risk and under Uncertainty for Gains and for Losses,” *Organizational Behavior and Human Decision Processes* 39, 1–22.
- Currim, Imran S. and Rakesh K. Sarin (1989), “Prospect Versus Utility,” *Management Science* 35, 22–41.
- Diecidue, Enrico, Ulrich Schmidt and Horst Zank (2006), “Parametric Weighting Functions,” *Economics Discussion Paper 0622*, University of Manchester, UK.
- Edwards, Ward (1955), “The Prediction of Decisions Among Bets,” *Journal of Experimental Psychology* 50, 201–214.
- Edwards, Ward (1962), “Utility, Subjective Probability, Their Interaction, and Variance Preferences,” *The Journal of Conflict Resolution* 6, 42f02d–51.
- Einhorn, Hillel J. and Robin M. Hogarth (1985), “Ambiguity and Uncertainty in Probabilistic Inference,” *Psychological Review* 92, 433f02d–461.
- Einhorn, Hillel J. and Robin M. Hogarth (1986), “Decision Making under Ambiguity,” *Journal of Business* 59, S225–f02dS250.

- Etchart-Vincent, Nathalie (2004), "Is Probability Weighting Sensitive to the Magnitude of Consequences? An Experimental Investigation on Losses," *Journal of Risk and Uncertainty* 28, 217–235.
- Gomes, Francisco J. (2005), "Portfolio Choice and Trading Volume with Loss-Averse Investors," *Journal of Business* 78, 675–706.
- Gonzalez, Richard and George Wu (1999), "On the Shape of the Probability Weighting Function," *Cognitive Psychology* 38, 129–166.
- Hershey, John C. and Paul J.H. Schoemaker (1980), "Prospect Theory's Reflection Hypothesis: A Critical Examination," *Organizational Behavior and Human Performance* 25, 395–418.
- Hershey, John C. and Paul J.H. Schoemaker (1985), "Probability versus Certainty Equivalence Methods in Utility Measurement: Are They Equivalent?," *Management Science* 31, 1213–1231.
- Hogarth, Robin M. and Hillel J. Einhorn (1990), "Venture Theory: A Model of Decision Weights," *Management Science* 36, 780–803.
- Holt, Charles A. and Susan K. Laury (2002), "Risk Aversion and Incentive Effects," *American Economic Review* 92, 1644–1655.
- Huber, Joel, Dan Ariely, and Gregory Fischer (2002), "Expressing Preference in a Principal-Agent Task: A Comparison of Choice, Rating, and Matching," *Organizational Behavior and Human Decision Processes* 87, 66–90.
- Jullien, Bruno and Bernard Salanié (2000), "Estimating Preferences under Risk: The Case of Racetrack Bettors," *Journal of Political Economy* 108, 503–530.
- Kahneman, Daniel, Jack L. Knetsch, and Richard H. Thaler (1990), "Experimental Tests of the Endowment Effect and the Coase Theorem," *Journal of Political Economy* 98, 1325–1348.
- Kahneman, Daniel and Amos Tversky (1979), "Prospect Theory: An Analysis of Decision under Risk," *Econometrica* 47, 263–291.

- Kahneman, Daniel and Amos Tversky (2000), "Choices, Values, and Frames." Cambridge University Press, New York.
- Köbberling, Veronika and Peter P. Wakker (2005), "An Index of Loss Aversion," *Journal of Economic Theory* 122, 119–131.
- Köbberling, Veronika and Peter P. Wakker (2003), "Preference Foundations for Nonexpected Utility: A Generalized and Simplified Technique," *Mathematics of Operations Research* 28, 395–423.
- Lattimore, Pamela M., Joanna R. Baker, Ann D. Witte (1992), "The Influence of Probability on Risky Choice," *Journal of Behavioral Economics and Organizations* 17, 377–400.
- Loehman, Edna (1998), "Testing Risk Aversion and Nonexpected Utility Theories," *Journal of Economic Behavior and Organization* 33, 285–302.
- Lopes, Lola L. and Gregg C. Oden (1999), "The Role of Aspiration Level in Risky Choice: A Comparison of Cumulative Prospect Theory and SP/A Theory," *Journal of Mathematical Psychology* 43, 286–313.
- Luce, R. Duncan (1991), "Rank- and-Sign Dependent Linear Utility Models for Binary Gambles," *Journal of Economic Theory* 53, 75–100.
- Luce, R. Duncan and Peter C. Fishburn (1991), "Rank- and Sign-Dependent Linear Utility Models for Finite First-Order Gambles," *Journal of Risk and Uncertainty* 4, 29–59.
- Markowitz, Harry (1952), "The Utility of Wealth," *The Journal of Political Economy* 60, 151–158.
- McClelland, Gary H., William D. Schulze, and Don L. Coursey (1993) "Insurance for Low-Probability Hazards: A Bimodal Response to Unlikely Events," *Journal of Risk and Uncertainty* 7, 95–116.
- Neilson, William S. (2002), "Comparative Risk Sensitivity with Reference-Dependent Preferences," *Journal of Risk and Uncertainty* 24, 131–142.



- Pratt, John W. (1964), "Risk Aversion in the Small and in the Large," *Econometrica* 32, 122–136.
- Prelec, Drazen (1998), "The Probability Weighting Function," *Econometrica* 66, 497–527.
- Rabin, Matthew (2000), "Risk Aversion and Expected-utility Theory: A Calibration Theorem," *Econometrica* 68, 1281–1292.
- Schmidt, Ulrich (2003), "Reference-Dependence in Cumulative Prospect Theory," *Journal of Mathematical Psychology* 47, 122–131.
- Schmidt, Ulrich and Stefan Traub (2002), "An Experimental Test of Loss Aversion," *Journal of Risk and Uncertainty* 25, 233–249.
- Schmidt, Ulrich and Horst Zank (2005), "What is Loss Aversion?," *Journal of Risk and Uncertainty* 30, 157–167.
- Schmidt, Ulrich and Horst Zank (2006), "Linear Cumulative Prospect Theory with Applications to Portfolio Selection and Insurance Demand," forthcoming in *Decisions in Economics and Finance*.
- Schneider, Sandra L. and Lola L. Lopes (1986), "Reflection in Preferences under Risk: Who and When May Suggest Why," *Journal of Experimental Psychology: Human Perception and Performance* 12, 535–548.
- Schoemaker, Paul J.H. (1982), "The Expected Utility Model: Its Variations, Purposes, Evidence and Limitations," *Journal of Economic Literature* 20, 529–563.
- Siegmann, Arjen H. (2002), "Optimal Saving Rules for Loss-Averse Agents under Uncertainty," *Economics Letters* 77, 27–34.
- Smith, Kip, John W. Dickhaut, Kevin McCabe, and José V. Pardo (2002), "Neuronal Substrates for Choice under Ambiguity, Risk, Gains, and Losses," *Management Science* 48, 711–718.
- Starmer, Chris (2000), "Developments in Non- Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk," *Journal of Economic Literature* 38, 332–382.

- Tversky, Amos and Craig Fox (1995), “Weighing Risk and Uncertainty,” *Psychological Review* 102, 269–283.
- Tversky, Amos and Daniel Kahneman (1991), “Loss Aversion in Riskless Choice: A Reference Dependent Model,” *Quarterly Journal of Economics* 106, 1039–1061.
- Tversky, Amos and Daniel Kahneman (1992), “Advances in Prospect Theory: Cumulative Representation of Uncertainty,” *Journal of Risk and Uncertainty* 5, 297–323.
- Wakker, Peter P. (1994), “Separating Marginal Utility and Probabilistic Risk Aversion,” *Theory and Decision* 36, 1–44.
- Wakker, Peter P. (2001), “Testing and Characterizing Properties of Nonadditive Measures through Violations of the Sure-Thing Principle,” *Econometrica* 69, 1039–1059.
- Wakker, Peter P., Richard H. Thaler, and Amos Tversky (1997), “Probabilistic Insurance,” *Journal of Risk and Uncertainty* 15, 7–28.
- Wakker, Peter P. and Amos Tversky (1993), “An Axiomatization of Cumulative Prospect Theory,” *Journal of Risk and Uncertainty* 7, 147–176.
- Wakker, Peter P. and Horst Zank (2002), “A Simple Preference-Foundation of Cumulative Prospect Theory with Power Utility,” *European Economic Review* 46, 1253–1271.
- Weber, Elke U. and William P. Bottom (1989), “Axiomatic Measures of Perceived Risk: Some Tests and Extensions,” *Journal of Behavioral Decision Making* 2, 113–131.
- Wu, George and Richard Gonzalez (1996), “Curvature of the Probability Weighting Function,” *Management Science* 42, 1676–1690.
- Wu, George and Richard Gonzalez (1998), “Common Consequence Conditions in Decision Making under Risk,” *Journal of Risk and Uncertainty* 16, 115–139.
- Zank, Horst (2001), “Cumulative Prospect Theory for Parametric and Multiattribute Utilities,” *Mathematics of Operations Research* 26, 67–81.