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A Theory of Infrastructure-led Development

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Abstract

This paper proposes a theory of long-run development based on public infrastructure as the main engine of growth. The government, in addition to investing in infrastructure, spends on health services, which in turn raise labor productivity and lower the rate of time preference. Infrastructure affects the production of both commodities and health services. As a result of network effects, the degree of efficiency of infrastructure is nonlinearly related to the stock of public capital itself. This in turn may cause multiplicity of equilibrium growth paths. Provided that governance is adequate enough to ensure a sufficient degree of efficiency of public investment outlays, an increase in the share of spending on infrastructure (financed by a cut in unproductive expenditure or foreign grants) may facilitate the shift from a low growth equilibrium, characterized by low productivity and low savings, to a high growth steady state.

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Contents

> Means of communication were not constructed in the colonial period... to facilitate internal trade in African commodities. There were no roads connecting different colonies and different parts of the same colony in a manner that made sense with regard to Africa's needs and development. All roads and railways led down to the sea. They were built to extract gold or manganese or coffee or cotton. They were built to make business possible for the timber companies, trading companies and agricultural concession firms, and for white settlers.
>
> Walter Rodney, *How Europe Underdeveloped Africa* (1973, Chapter 6).

1 Introduction

Lack of infrastructure continues to be a key obstacle to growth and development in many low-income countries. In Sub-Saharan Africa in particular, only 16 percent of roads are paved, and less than one in five Africans has access to electricity. Transport costs are the highest in the world. A study by the African Development Bank (1999) found that freight charges (as a proportion of cif value) are on average 20 percent higher for African exports to the United States than for comparable products from other low-income countries. Based on econometric regressions, the study also found that poor infrastructure (measured by an index combining road, rail, and telecom density) accounts for 40 percent of predicted transport costs for coastal countries and up to 60 percent for landlocked countries.

To alleviate these constraints to growth and poverty reduction, several observers have advocated a large increase in public investment in infrastructure, in line with the "Big Push" view of Rosenstein Rodan (1943).[1] A common argument for doing so is that infrastructure services have a strong growth-promoting effect through their impact on production costs, the productivity of private inputs, and the rate of return on capital—particularly when, to begin with, stocks of infrastructure assets are relatively low. More recent research, however, has emphasized that infrastructure may also affect growth indirectly through a variety of channels, most notably by affecting health out-

[1] Actually, most of Rosenstein Rodan's article is dedicated to the role of complemetarities between industries, not to a large increase in public investment. However, in keeping with tradition, we will credit the "Big Push" view to him.

comes.[2] Access to clean water and sanitation helps to improve health and thereby productivity. By reducing the cost of boiling water, and reducing the need to rely on smoky traditional fuels (such as wood, crop residues, and charcoal) for cooking, access to electricity also helps to improve hygiene and health—in the latter case by reducing indoor air pollution and the incidence of respiratory illnesses. Availability of electricity is essential as well for the functioning of hospitals (storing some types of vaccines requires continuous and reliable refrigeration, for instance) and the delivery of health services. Better transportation networks (particularly in rural areas) make it easier to access health care and to attract (or retain) qualified medical workers. Indeed, Wagstaff and Claeson (2004) found that road infrastructure (as measured by the length of the paved road network) had a significant effect on a number of health indicators, such as infant and female mortality rates.

Dwelling in part on this evidence, this paper presents a theory of development based on public infrastructure as the main engine of growth. In doing so, I deliberately leave aside two other powerful forces that have been identified in the literature: human capital accumulation and endogenous technological progress. In the model, the growth rate depends on the interactions between infrastructure, health, and savings. Infrastructure raises the economy's ability to produce health services; in turn, greater access to health services enhances workers' productivity, and thus output. Thus, the accumulation of human capital results not from the acquisition of knowledge, but from better quality of effective labor. Unlike endogenous growth models in the Uzawa-Lucas tradition, this requires thinking of knowledge as embodied in workers, as opposed to books and libraries. In addition, improvements in health raise incentives to save. This effect occurs not through its impact not on life expectancy—given that the representative household in the model is infinitely-lived—but by reducing the degree of impatience, or equivalently, the rate of preference for the present. In turn, a lower time-preference rate raises the marginal utility of future consumption and stimulates savings, which in turn promotes physical capital accumulation and growth.

I also assume, crucially, that the degree of efficiency of public infrastructure is positively (and nonlinearly) related to the stock of public capital itself. There is some empirical support for this specification; Arestoff and Hurlin (2005*b*) and Hurlin (2006), using a production function approach,

[2] See Agénor and Moreno-Dodson (2006) and Agénor and Neanidis (2006) for a more detailed discussion of these channels.

found strong evidence of threshold effects in the productivity of public capital in developing countries. The threshold variable is the stock of capital per worker, as suggested by Fernald (1999).[3] The introduction of this external effect leads to multiple equilibria. The realization of a specific steady-growth equilibrium depends therefore on expectations of private agents and the initial position of the economy—as well as public policy itself.

The existence of a nonlinear relationship between the efficiency of public capital and its level can be motivated in two ways. The first is based on the view that infrastructure investment is lumpy, that is, a certain quantity of infrastructure assets must be accumulated before it begins to contribute at all to the production activities of the private sector. However, lumpiness can explain piecewise-linear threshold effects, but not necessarily nonconvexities. The second is based on network effects. Economies of scale due to network externalities are a widely recognized imperfection in infrastructure services (see World Bank (1994)). An important characteristic of modern infrastructure is indeed the fact that services are often supplied through a networked delivery system designed to serve a multitude of users. This interconnectedness means that the benefits from investment at one point in the network will generally depend on capacities at other points. Put differently, inherent to the structure of a network is that many components are required for the provision of a service; these components are thus complementary to each other. Having electricity to produce commodities in rural areas but no roads to carry them to urban markets limits the productivity effects of a program designed to increase access to energy. In that sense, electricity and roads are complementary components of the infrastructure network, and only joint availability or operation will generate efficiency gains, that is, positive externalities. Similarly, to operate an airport requires not only building runways but also adequate access to telecommunications.

The network character of infrastructure capital may therefore induce a strong nonlinearity in its productivity.[4] Until the network is built, public

[3] What Arestoff and Hurlin consider is nonlinearities in the effect of public capital on the elasticity of output with respect to public capital (α, in my notation). By contrast, in the model all technological parameters are taken as given, and the efficieny parameter is entered multiplicatively in the production function.

[4] This discussion also suggests that there are two different issues, when it comes to network effects associated with infrastructure: one is the *level* of public capital (taken as a whole), and the second the *composition* of the public capital stock. Here I focus only on the first.

capital has a low (or even null) marginal productivity. Once the basic parts of a network are established, and a critical mass has been reached, strong gains are associated with small additional increases in infrastructure investment.[5] But beyond that, the productivity gains induced by additional investments tend to slow down. Thus, in contrast to much of the literature on network externalities, which is static in nature (as noted by Economides (1996)), the perspective adopted here is explicitly dynamic.

This second interpretation probably carries some weight for understanding the current plight of many poor countries. As noted earlier, inadequate transport networks remain a key characteristic of Sub-Saharan Africa today. Moreover, a number of observers have argued that the current state of affairs is intimately linked to the history of the continent. Colonization and a focus on resource extraction dramatically affected the use of space in the region, shifting growth and urbanization from inland to costal areas. Many African capitals today are ports that were built at the end of railways designed to carry flows of raw materials and labor from the inland. Historians like Cooper (1993), as well as geographers and political scientists—particularly those in the Marxist tradition, like Walter Rodney (1973), in his classic book *How Europe Underdeveloped Africa*, as quoted above—have long pointed out that transport networks inherited from that growth were located perpendicularly to the seashores, and were not built to occupy space widely.[6] As indicated earlier, however, I go beyond a "pure" infrastructure (or physical capital) view of development by accounting for the impact of infrastructure on the production of health services and endogenizing the impact of health on labor productivity and savings. The model illustrates therefore the crucial links between public infrastructure accumulation and the joint evolution of private physical capital, productivity, and savings.

The remainder of the paper is organized as follows. Section 2 presents the basic framework, which takes as given the degree of efficiency of public capital. Section 3 derives the balanced growth path and establishes uniqueness

[5] In line with this argument, Fernald (1999) reports that once the highway system in the US was roughly completed, after 1973, the hypothesis that the marginal productivity of roads is zero cannot be rejected. In other words, road building gave a boost to productivity growth in the years before 1973, but post-1973 investment did not yield the same benefits at the margin.

[6] By and large, however, mainstream economists have not followed this lead. In their analysis of the historical causes of African underdevelopment, Bertocchi and Canova (2002) for instance barely mention this aspect of colonization.

and stability conditions. Section 4 endogenizes the efficiency of public capital in infrastructure, by relating it to the stock of infrastructure itself measured in proportion of the private capital stock. Section 5 discusses their policy implications of the results and compares them with those obtained in some recent studies. I examine, in particular, how a shift in government spending allocation toward infrastructure, financed by either a cut in unproductive expenditure or an increase in foreign aid, affects the equilibrium dynamics. This exercise is therefore consistent with the Big Push view referred to earlier. In addition, I consider as well the case where the budget-neutral shift in outlays is toward health expenditure. The final section offers some concluding remarks.

2 The Basic Framework

The economy that I consider is populated by a single, infinitely-lived household-producer (or household, for short). It produces a single traded commodity, which can be used for either consumption or investment. The government invests in infrastructure and spends on commodities, which are used to produce (when combined with public capital in infrastructure) health services. It also spends on "unproductive" activities—that is, activities that have no direct effect on the supply of infrastructure or the production of health services. Both categories of services are assumed to be nonrival and nonexcludable.

2.1 Production of Commodities

Commodities, in quantity Y, are produced with private capital, K_P, public capital in infrastructure capital, K_I, and "effective" labor, defined as the product of the quantity of labor and a productivity index. In turn, productivity depends solely on the available supply of health services, H, with a strict proportionality relationship.[7] Because the growth rate of the population is zero and population size is normalized to unity, effective labor is simply H.

[7] It could be assumed that productivity depends not only on access to health services but also directly on infrastructure as well. Better roads, for instance, allow easier access to work and reduces stress associated with traffic jams. This effect of infrastructure on productivity could also be subject to nonlinearities similar to those related to the production of commodities, as discussed later.

Assuming a Cobb-Douglas technology yields[8]

$$Y = (\theta K_I)^\alpha H^\beta K_P^{1-\alpha-\beta} = (\frac{\theta K_I}{K_P})^\alpha (\frac{H}{K_P})^\beta K_P, \tag{1}$$

where $\alpha, \beta \in (0,1)$ and $\theta > 0$ is an efficiency parameter which I take as constant for the moment. Thus, production of commodities exhibits constant returns to scale in all factors.[9] For simplicity, the flow of services provided by each capital stock is taken to be directly proportional to the available stock.

2.2 Production of Health Services

As discussed in the introduction, public infrastructure in this economy improves health outcomes, by providing for instance greater access to drinking water or by facilitating garbage collection and disposal (through sewage systems). Specifically, I assume that production of health services requires combining government spending on health, G_H, and public capital in infrastructure. Assuming also a Cobb-Douglas technology yields

$$H = K_I^\mu G_H^{1-\mu}, \tag{2}$$

where $\mu \in (0,1)$. The provision of health services takes place therefore under constant returns to scale in K_I and G_H.

2.3 Household

The household maximizes the discounted present value of utility:

$$\max_C V = \int_0^\infty \ln C \exp(-\rho t) dt, \tag{3}$$

where C is consumption and $\rho > 0$ the subjective discount rate. For simplicity, I assume that the instantaneous utility function is logarithmic in consumption only. A more general specification would be to assume, as in

[8]In what folllows, time subscripts are omitted for simplicity. Also, $\dot{x} \equiv dx/dt$ is used to denote the time derivative of any variable x.

[9]See Eicher and Turnovsky (1999) for a discussion of the relation between the existence of a balanced growth path and the assumption of constant returns to scale in endogenous growth models.

Agénor (2005) and Agénor and Neanidis (2006), that utility is non-separable in consumption and health services, that is

$$u(C, H) = \frac{(CH^{\kappa})^{1-1/\sigma}}{1 - 1/\sigma}, \quad \sigma \neq 1, \ \kappa > 0,$$

where σ is the intertemporal elasticity of intertemporal substitution and κ measures the relative contribution of health to utility. However, as can be inferred from the results in the papers cited above, this specification would simply complicate derivations and would not add much to the discussion as long as $\sigma < 1$—the empirically relevant case for developing countries, as documented in a variety of studies (see Agénor and Montiel (2006)). For the purpose at hand—and given my focus on growth, rather than welfare—the logarithmic specification is simpler and I shall retain it throughout.[10]

The rate of time preference is assumed to depend negatively on consumption of health services, and positively on the stock of private capital:

$$\rho = \rho(H, K_P), \tag{4}$$

with $\rho_H < 0$, and $\rho_{K_P} > 0$.

This specification differs in significant ways from the formulation first proposed by Uzawa (1968), who introduced the idea of a relation between the rate of time preference and consumption.[11] In the present case, it is health services, not consumption of goods or commodities, that affect the degree of impatience. The essential motivation here is that healthier individuals are less myopic and tend to value the future more. This can be viewed as a way to capture, in a representative agent model with infinite horizon, the "life expectancy" effect typically emphasized in OLG models with endogenous lifetimes (or mortality rates), as for instance in Aísa and Pueyo (2006), where the flow of government expenditure is taken to affect the instantaneous

[10] Note also that, as shown by Cazzavillan (1996), if government spending exerts positive externalities on household preferences (in the form of increasing returns to public expenditure in utility), multiple equilibria and indeterminacy may also emerge.

[11] More specifically, Uzawa (1968, p. 489) assumed that the change in the rate of time preference at time t is a positive function of the level of utility at time t. Subsequently, in their taxonomy of intertemporal utility functions, Shi and Epstein (1993, p. 62) characterized Uzawa preferences as consisting of a relation between the rate of time preference and (current and past) consumption levels.

probability of death.[12] This specification is also consistent with the evidence that poor people tend to have a relatively high rate of time preference, as documented for instance by Lawrance (1991), if one takes a step further and assumes that consumption of health services is positively related to income.[13]

Moreover, I also assume that the rate of time preference is increasing in wealth—or equivalently, here, given that there are no other (private) stores of value in this economy, the stock of private physical capital. As discussed by Mohsin (2004) and Kam (2005) in a different context, this link avoids some of the difficulties posed by Uzawa preferences.[14] For tractability, I will assume that ρ is homogenous in H and K_P, so that

$$\rho = \rho(\frac{H}{K_P}), \tag{5}$$

with $\rho' < 0$ and $\rho'' > 0$.

The household's resource constraint takes the simple form

$$\dot{K}_P = (1-\tau)Y - C, \tag{6}$$

where $\tau \in (0,1)$ is the tax rate on income. For simplicity, I assume that private capital does not depreciate.

The household takes the rate of time preference, the tax rate, and the efficiency parameter as given when choosing the optimal time profile of consumption. Using (1) and (6), the current-value Hamiltonian for problem (3) can be written as

$$L = \ln C + \lambda[(1-\tau)Y - C],$$

[12]See also Chakraborty (2004) and Hashimoto and Tabata (2005), where the survival probability of individuals depends on per capita public health capital. Blackburn and Cipriani (2002), Kalemli-Ozcan (2002), and Ehrlich and Kim (2005), all endogenize the mortality rate by assuming instead that the survival probability of individuals depends on either per capita income or consumption.

[13]See Frederick, Loewenstein, and O'Donoghue (2002) for a review of the microeconomic evidence linking discount rates and health outcomes.

[14]In a standard model of optimal saving, Uzawa preferences require increasing marginal impatience for stability; see for instance Epstein and Hynes (1983), and Obstfeld (1990) for an intuitive diagrammatic exposition. The plausibility of increasing marginal impatience has been questioned by many; Blanchard and Fischer (1989, p. 75) went as far as to recommend avoiding the Uzawa specification because "...with its assumption [that the richer are more impatient than the poor, it] is not particularly attractive as a description of preferences."

where λ is the co-state variable associated with constraint (6). From the first-order condition $dL/dC = 0$ and the co-state condition $\dot{\lambda} = -dL/dK_P + \rho\lambda$, optimality conditions for this problem take the familiar form

$$1/C = \lambda, \tag{7}$$

$$\dot{\lambda} = \lambda[\rho(\frac{H}{K_P}) - \frac{sY}{K_P}], \tag{8}$$

where $s \equiv (1-\tau)\eta$, with $\eta \equiv 1 - \alpha - \beta$, together with the budget constraint (6) and the transversality condition

$$\lim_{t \to \infty} \lambda K_P \exp(-\rho t) = 0. \tag{9}$$

2.4 Government

The government invests in infrastructure capital, G_I, spends on health, G_H, and unproductive activities, G_U. It also collects a proportional tax τ on output. If fiscal deficits are precluded, the government budget constraint is given by

$$G_I + G_H + G_U = \tau Y. \tag{10}$$

Spending components are all taken to be fixed fractions of tax revenues:

$$G_h = \upsilon_h \tau Y, \qquad h = I, H, U \tag{11}$$

where $\upsilon_h \in (0, 1)$.

Combining (10) and (11), the government budget constraint can be rewritten as

$$\upsilon_I + \upsilon_H + \upsilon_U = 1. \tag{12}$$

The stock of public capital in infrastructure evolves over time according to

$$\dot{K}_I = \varphi G_I, \tag{13}$$

where $\varphi \in (0, 1)$ is an efficiency parameter that measures the extent to which investment flows translate into actual accumulation of public capital. The case $\varphi < 1$ reflects the view that investment spending on infrastructure is subject to inefficiencies, which tend to limit their positive impact on the public capital stock. Arestoff and Hurlin (2005*b*) and Hurlin (2006), for instance, estimate the value of φ to vary between 0.4 and 0.6 for developing countries. As it turns out, this parameter (which can broadly be thought of as an indicator of the quality of public sector management, or governance, or as a measure of absorptive capacity) plays an important role in discussing the role of public policies in escaping from a low-growth trap.[15] For simplicity, public capital is assumed not to depreciate.

Combining (6), (10), and (13) yields the equilibrium condition of the market for commodities:

$$Y = C + (\dot{K}_P + \dot{K}_I) + G_H + G_U. \tag{14}$$

3 Constant Efficiency

I first study the properties of the model with a constant efficiency parameter, θ. In the present setting, a competitive equilibrium under a balanced budget can be defined as follows:

Definition 1. *A competitive equilibrium is a set of infinite sequences for the quantities* $\{C, K_P, K_I\}_{t=0}^{\infty}$*, such that* $\{C, K_P\}_{t=0}^{\infty}$ *satisfy equations (7), (8), and (9), the path* $\{H, K_P, K_I\}_{t=0}^{\infty}$ *satisfies equations (2), (6), and (14), for given values of the tax rate,* τ*, the spending shares,* υ_I*,* υ_H*, and* υ_U*, and the efficiency parameter,* θ*.*

From equation (7),

$$\dot{C}/C = -\dot{\lambda}/\lambda, \tag{15}$$

whereas, from (11) and (13),

$$\frac{\dot{K}_I}{K_I} = \varphi \upsilon_I \tau (\frac{Y}{K_I}) = \varphi \upsilon_I \tau k_I^{-1} (\frac{Y}{K_P}), \tag{16}$$

where $k_I = K_I/K_P$. From (1), (2), and (11),

$$Y = (\theta k_I)^{\alpha} (\frac{K_I^{\mu} G_H^{1-\mu}}{K_P})^{\beta} K_P = \theta^{\alpha} k_I^{\alpha+\mu\beta} (\upsilon_H \tau)^{(1-\mu)\beta} (\frac{Y}{K_P})^{(1-\mu)\beta} K_P.$$

This expression can be rewritten as

$$\frac{Y}{K_P} = (\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} k_I^{(\alpha+\mu\beta)/\Omega}, \tag{17}$$

[15] An interesting avenue to explore would be to endogenize φ, by linking it to the degree of corruption or political incentives (see, for instance, Robinson and Torvik (2005)). However, this is well beyond the scope of this paper.

where $\Omega \equiv 1-(1-\mu)\beta > 0$, so that $\alpha/\Omega < 1$.

Combining this result with (16) yields

$$\frac{\dot{K}_I}{K_I} = \varphi \upsilon_I \tau^{1/\Omega} \upsilon_H{}^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} k_I^{-\eta/\Omega}. \tag{18}$$

Substituting (1) and (17) in the household budget constraint (6) yields

$$\frac{\dot{K}_P}{K_P} = (1-\tau)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} - c, \tag{19}$$

where $c = C/K_P$.

Using (2), the health-private capital ratio, denoted h, is given by

$$h = (\frac{H}{K_I})(\frac{K_I}{K_P}) = (\frac{G_H}{K_I})^{1-\mu} k_I = (\frac{G_H}{K_P})^{1-\mu} k_I^{\mu},$$

that is, using (11) and (17),

$$h = (\upsilon_H \tau)^{1-\mu} [(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} k_I^{(\alpha+\mu\beta)/\Omega}]^{1-\mu} k_I^{\mu},$$

so that

$$h = (\upsilon_H \tau)^{(1-\mu)/\Omega} \theta^{\alpha(1-\mu)/\Omega} k_I^{\Phi/\Omega}, \tag{20}$$

where $\Phi \equiv \mu + \alpha(1-\mu) > 0$.

Substituting this result in (5) yields

$$\rho = \chi(k_I; \theta), \tag{21}$$

with $\chi_{k_I} \equiv \rho' \Phi \tilde{h}/\Omega \tilde{k}_I < 0$ and $\chi_\theta \equiv \rho' \alpha(1-\mu)\tilde{h}/\theta\Omega < 0$.

Substituting (8) in (15) yields $\dot{C}/C = s(Y/K_P) - \rho$, that is, using (17) and (21),

$$\frac{\dot{C}}{C} = s(\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} - \chi(k_I; \theta). \tag{22}$$

Equations (18), (19), and (22) can be further condensed into a first-order nonlinear differential equation system in $c = C/K_P$ and $k_I = K_I/K_P$:

$$\frac{\dot{c}}{c} = \xi k_I^{(\alpha+\mu\beta)/\Omega} - \chi(k_I; \theta) + c, \tag{23}$$

$$\frac{\dot{k}_I}{k_I} = \upsilon_H{}^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} \left\{ \varphi \upsilon_I \tau^{1/\Omega} k_I^{-\eta/\Omega} - (1-\tau)\tau^{(1-\mu)\beta/\Omega} k_I^{(\alpha+\mu\beta)/\Omega} \right\} + c, \tag{24}$$

where

$$\xi \equiv -(1-\tau)(\alpha+\beta)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega} < 0. \tag{25}$$

These two equations, together with an initial condition on k_I at $t = 0$, and the transversality condition (9), characterize the dynamics of the economy.

The balanced-growth path (BGP) can therefore be defined as follows:

Definition 2. *The BGP is a set of sequences $\{c, k_I\}_{t=0}^{\infty}$, constant spending shares, tax rate, and efficiency parameter, satisfying Definition 1, such that for an initial condition $k_I = k_I^0$, equations (23) and (24) and the transversality condition (9) are satisfied, the government budget constraint (12) holds, and consumption, the stocks of private and public capital, all grow at the same constant rate γ.*

By implication, γ is also the rate of growth of output of commodities and supply of health services. Using (7), the transversality condition (9) can be rewritten as

$$\lim_{t\to\infty} c^{-1}\exp(-\rho t) = 0, \tag{26}$$

which is also satisfied, because c is constant along the BGP.

Based on the results in the Appendix, the following proposition can be established:

Proposition 1. *With a constant degree of efficiency of public infrastructure, the BGP is unique and locally determinate. A high degree of sensitivity of the rate of time preference with respect to the health-private capital ratio enhances stability.*

From (18) and (22), the steady-state growth rate γ is given by the equivalent forms

$$\gamma = \varphi \upsilon_I \tau^{1/\Omega} {\upsilon_H}^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}\tilde{k}_I^{-\eta/\Omega}, \tag{27}$$

$$\gamma = s(\upsilon_H \tau)^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \chi(\tilde{k}_I;\theta), \tag{28}$$

where $\tilde{x}$ denotes the stationary value of x.

Adjustment toward the long-run equilibrium is illustrated in Figure 1. Curve KK corresponds to combinations of (c, k_I) for which $\dot{k}_I = 0$, whereas curve CC corresponds to combinations of (c, k_I) for which $\dot{c} = 0$. Both curves are concave, and saddlepath stability requires KK to cut CC from below. The saddlepath, SS, also has a positive slope. The long-run equilibrium obtains at point E.

To briefly illustrate the functioning of the model—and with an eye to the Big Push policy recommendation discussed later on—consider a budget-neutral increase in the share of spending on infrastructure, υ_I, offset by a reduction in unproductive expenditure (that is, $d\upsilon_I = -d\upsilon_U$).[16] Based on the results in the Appendix, as well as equations (27) and (28), the following proposition can be established:

Proposition 2. *A permanent increase in the share of investment in infrastructure, financed by a cut in unproductive expenditure, raises the steady-state growth rate as well as the steady-state values of the consumption-private capital ratio and the public-private capital ratio.*

A diagrammatic analysis of the adjustment path associated with a budget-neutral shift in spending toward infrastructure is also shown in Figure 1. Such a shift involves only a rightward shift in KK and no change in CC. As a result, both the public-private capital ratio and the consumption-private capital ratio are unambiguously higher in the new equilibrium. The adjustment path corresponds to the sequence EAE'. Consumption (which is here a perfect substitute to physical capital formation) jumps upward on impact and continues to rise over time.

Intuitively, the positive long-run effect on permanent income associated with the increase in government investment in infrastructure leads to an immediate increase in consumption. However, output is given on impact, because neither one of the capital stocks can change instantaneoulsy. The economy's resource constraint (14) implies therefore that the jump in consumption and the rise in public investment (that is, $\dot{K}_I(0) > 0$) must be accompanied by a fall private investment on impact, that is, $\dot{K}_P(0) < 0$. Consequently, the public-private capital ratio rises unambiguously on impact.

Over time, the transitional dynamics of the public-private capital ratio and the growth rate of output are driven by the marginal physical product of private capital, which in turn depends on the stock of public capital in infrastructure. This dependence is both direct and indirect—given the assumed shape of the production function for health services, the effective supply of

[16] See Agénor (2005*d*) and Agénor and Neanidis (2006) for an analysis of budget-neutral shifts in the spending shares υ_H and υ_I. Growth- and welfare-maximizing spending allocations are also discussed in those papers. Note that here, increases in productive spending shares would not involve a trade-off *per se*, if they are offset by cuts in unproductive spending. I will return to this issue later.

labor depends on the stock of infrastructure capital as well. When the share of government investment (and thus the level of investment itself) increases, and the stock of public capital expands, that product rises at first and growth will tend to be high. Agents will be investing more, causing physical capital to be accumulated more rapidly and the growth rate to rise initially. Thus, the rate of private capital accumulation follows a nonmonotonic process: after falling at first (to accommodate the initial increase in consumption and public investment), it begins to rise, to reflect a greater rate of return on physical assets.

But because the marginal product of private capital is negatively related to the stock of private capital itself (given diminishing marginal returns to all inputs), private investment will tend to fall over time as more of that type of capital is accumulated. The transition to the steady-state growth rate will be therefore characterized by a relatively high growth rate of private capital and output initially, followed by a slowdown in both variables that may vary in speed— depending on the relative strength of decreasing returns. The increase in private capital falls nevertheless short of the rate of accumulation of public infrastructure assets, implying that the public-private capital ratio rises continuously over time. In turn, the increase in that ratio tends to raise production of health services (measured in proportion to the private capital stock) and therefore to reduce preference for the present. The greater the sensitivity of the discount rate to health, the greater will be the incentive to shift resources toward the future, and the higher will be the rates of private capital accumulation and output growth. During the transition, nonetheless, the consumption-private capital ratio increases continuously, implying that consumption rises at a faster rate than private capital. The increase in savings needed to finance private capital accumulation is brought about by an increase in output, whose growth rate must therefore exceed the growth rate of consumption and public investment.

Although one could perform a detailed investigation of how stability is affected by the various structural parameters of the model, I now turn to the main focus of the paper—the existence of network externalities associated with public infrastructure, and the extent to which the nonlinearities that they entail with respect to the degree of efficiency of public capital may lead to multiple equilibria.

4 Network Externalities and Efficiency

I now consider the case where the efficiency of public capital, θ, is endogenously related to the stock of public infrastructure itself through a convex-concave function. This specification aims to capture the existence of network effects, discussed in the introduction—the idea that the stock of public infrastructure must be sufficiently large for efficiency effects to kick in. At the same time, while it may be highly beneficial to *build up* a network of, say, roads, the benefits from *expanding* or *improving* the network may not be as high in terms of efficiency gains. Moreover, congestion effects may kick in beyond a certain point, even if the stock of public capital continues to expand. Indeed, if some portion of public capital is rival, the critical mass that generates network externalities would depend in part on the level of private production. To capture these effects, as in Futagami and Mino (1995), I assume that threshold levels depend on the *ratio* of public infrastructure assets to private capital.[17]

Formally, the efficiency function is taken to be given by

$$\theta = \begin{cases} \theta_L \text{ for } k_I < k_I^L, \\ \theta_M(k_I) \text{ for } k_I^L \leq k_I < k_I^H, \\ \theta_H(k_I) \text{ for } k_I \geq k_I^H, \end{cases} \tag{29}$$

where $\theta'_M, \theta''_M > 0$, whereas $\theta'_H > 0$ and $\theta''_H < 0$. I also assume that $\lim_{k_I \to \infty} \theta_H(k_I) = \theta_H$, where $\theta_H > \theta_L$.[18] Thus, the efficiency function is constant over the interval $(0, k_I^L)$, convex over the interval (k_I^L, k_I^H), and concave over the interval $(k_I^H, \infty[$. This is depicted in Figure 2, where point B corresponds to $\theta''_M(k_I^H) = \theta''_H(k_I^H) = 0$. Put differently, over the interval $(k_I^L, \infty[$, the efficiency function has a logistic shape.

I assume that, as before, the household takes θ as given when optimizing. Proceeding as before, and using the results in the Appendix, the following proposition can be established:

Proposition 3. *Suppose that the degree of efficiency of public infrastructure is subject to threshold effects, as described in (29). Depending on the*

[17] Note that in Futagami and Mino (1995), it is the efficiency of *private* capital that takes a convex-concave shape, whereas the focus here is on the efficiency of public capital.

[18] Although I do not elaborate in this paper on the determinants of θ_L, it should be noted that inadequate and weak institutions may translate into poor efficiency of public capital in infrastructure, just like it may affect the efficiency of public investment, φ (as discussed again later).

strength of the efficiency externality, and other model parameters, there may be either no equilibrium, one equilibrium, or multiple equilibria.

Figures 3 and 4 illustrate the cases of zero and one equilibrium, under the assumption that the efficiency externality is strong enough to ensure that both CC and KK have a convex portion (see the Appendix). In both figures, G and G' correspond to the points at which externalities kick in. In Figure 3, point E corresponds to the point where CC and KK would have intersected with θ constant; but because this point is located to the right of the threshold level of the public-private capital ratio k_I^L, it cannot be reached. After point G, both CC and KK turn convex, because both depend on θ; they retain a convex shape during the whole range of increasing returns, that is, during the interval (k_I^L, k_I^H). Once the public-private capital stock reaches k_I^H, both curves turn concave again. But they never intersect—although the slope of KK may be steeper than CC during the range of increasing returns (as shown in the figure), they would have intersected at a value higher than k_I^H; decreasing returns in efficiency beyond that threshold value prevents this from happening. Beyond point G', curve CC is shown as steeper than KK, but no equilibrium can be achieved: as long as the slope of KK is flatter than (or, at most, equal to) the slope of CC, the two curves cannot intersect.

Figure 4 considers, instead, the case where a unique equilibrium may occur. There are two cases to consider. In the upper panel, curves CC and KK intersect before the threshold level k_I^L is reached; the steady-equilibrium obtained at point E is unique, and any initial capital ratio k_I^0 that differs from $\tilde{k}_I$ will converge toward that value along SS. In a sense, network externalities are powerless to lead the economy away from point E, which will be referred to as the "low growth" equilibrium. Put differently, because this is the only stable equilibrium, the economy stagnates. Moreover, because poor infrastructure leads to low output of health services, which in turn hampers labor productivity and reduces incentives to save, this low-growth equilibrium can be equally characterized as a "low productivity-low savings" trap.[19] In the lower panel, curves CC and KK intersect beyond k_I^L, that is, after increasing returns in efficiency start kicking in. As shown in the figure, although both curves turn convex, curve KK cuts CC from below—as required for saddlepath stability (see Figure 1). Moreover, the curves

[19] This view is not inconsistent with Dasgupta's (2004) emphasis on poor health generating a low-growth trap through the mechanism of poor health making workers unproductive, and the resulting low incomes reinforcing poor health.

intersect only once, because the range of increasing returns in efficiency is sufficiently large to ensure that, once decreasing returns set in (beyond G'), and both curves turn concave again, they cannot intersect once more (in the figure, KK remains steeper than CC beyond k_I^H as well). In what follows, the equilibrium at point E will be referred to as the "moderate growth" equilibrium.

Consider now the case where multiple equlbria exist. From the results of the Appendix, the following proposition can be established:

Proposition 4. *Suppose that the degree of efficiency of public infrastructure is subject to threshold effects, as described in (29). If the efficiency externality is sufficiently strong, there may be three local equilibria. The two extreme steady states are saddlepoint stable, whereas the intermediate steady state is locally unstable.*

Figures 5 and 6 illustrate the dynamics in this case.[20] The three equilibria are labeled E^1, E^2, and E^3, with corresponding notation for the steady-state values of c and k_I. As shown in Figure 5, for an economy starting to the left of E^1, or to the right of E^1 before point G (that is, for $\tilde{k}_I^1 < k_I^0 < k_I^L$), as well as to the right of E^3, or to the left of E^3 after point G' (that is, for $k_I^H < k_I^0 < \tilde{k}_I^3$) the realization of a particular equilibrium depends entirely upon history—that is, the initial value k_I^0. For instance, an economy whose initial level of public infrastructure is relatively low, $k_I^0 < \tilde{k}_I^1$, only the low-growth equilibrium can be attained, whereas an economy whose initial stock of capital is $k_I^0 > \tilde{k}_I^3$, but with otherwise identical characteristics, the high-growth equilibrium E^3 will eventually be reached. In both cases, curve KK cuts CC from below, ensuring saddlepath stability.

If the initial public-private capital ratio lies between k_I^L and k_I^H, as shown in Figure 6, several ranges can be distinguished. Let k_C^1 and k_C^2 be defined as threshold levels that are such that $k_I^L < k_C^1 < k_C^2 < k_I^H$, with $\left|k_I^L - k_C^1\right|$ and $\left|k_I^H - k_C^2\right|$ arbitrarily small. Standard dynamic analysis suggests therefore that the interval (k_C^1, k_C^2), which corresponds to points J and J' on the saddlepaths leading to E^1 and E^3 and which includes the unstable equilibrium E^2, defines a zone of indeterminacy. Paths originating in the interval (k_I^L, k_C^1) would tend to converge to the low productivity-low savings trap, E^1,

[20]In principle, it is possible for the model to generate also two equilibria, one stable and one unstable. However, this is rather unlikely, as suggested by the discussion in the Appendix.

whereas paths originating in the interval (k_C^2, k_I^H) would tend to converge to the high-growth equilibrium, E^3.

By contrast, for any path originating within the interval (k_C^1, k_C^2), the economy could go either way: the initial value of c is indeterminate for k_I^0 given, so there exists an infinite number of perfect foresight equilibrium paths—all of which legitimate in the sense that they do not violate the transversality condition for household's optimization (equation (26)). In that interval, "optimistic" expectations (that is, the belief that the economy can reach the high-growth steady state) could prove self-fulfilling and stir the economy to the high-growth equilibrium; but because nothing ensures that expectations can be coordinated in such a way, "pessimistic" expectations could do as well, and the economy may end up in the low-growth equilibrium. This is illustrated by the two paths originating from point E^2, under the assumption that $k_I^0 = \tilde{k}_I^2$.[21] Thus, for some initial values k_I^0 of the (predetermined) public-private capital ratio, there could be two equilibrium trajectories—one leading to stagnation, the other one to high growth. There is therefore a coordination problem, which creates a possible role for public policy—an issue to which I now turn.

5 The Role of Public Policy

To illustrate the role of public policy, I first examine the case where, starting from a position of a unique equilibrium (as illustrated in Figure 4) the government implements a budget-neutral increase in the share of public investment in total infrastructure, financed by a cut in unproductive expenditure (that is, $d\upsilon_I + d\upsilon_U = 0$). As shown in Figure 7, there are two cases to consider. In the upper panel, as in Figure 4, the initial equilibrium occurs at a value of the public-private capital ratio that is less than the lower threshold level k_I^L. The increase in υ_I, as in Figure 1, shifts curve KK only to the right. But if this shift is relatively small, the new curve KK will still intersect CC to the left of k_I^L, at a point like E'; although the consumption-capital ratio and the steady-state growth rate eventually increase as a result of the policy

[21] As for instance in Futagami and Mino (1995), it is possible for closed orbits (or limit cycles) to exist around E^2. Indeed, the conditions $\text{tr}J = 0$ and $\det J > 0$, where J is the Jacobian matrix evaluated at E^2, imply that the system is unstable and has two purely imaginary eigenvalues; this result cannot be excluded *a priori*. However, this is of little interest in the present context.

shift, the effect is relatively limited because externalities do no kick in. By contrast, if the increase in υ_I is large enough to shift KK in such a way that the point of intersection with CC occurs to the right of k_I^L, at a point like E'', the economy will be in the zone of increasing returns associated with the efficiency of public capital, and the increase in the consumption-capital ratio will exceed by an order of magnitude (due to the convexity of θ in that range, which implies that both curves are also convex) what would have been achieved in the absence of network effects. In a manner similar to Figure 1, if the initial equilibrium is at E, the adjustment process will take the economy through a point such as D to point E''.

A similar result obtains in the lower panel of Figure 7, where the initial steady-state position of the economy is at the "moderate growth" equilibrium point located to the right of the lower threshold level k_I^L. If the increase in the share of public investment in infrastructure is not large, the rightward shift in KK will lead to a new intersection point with CC located to the left of k_I^H, such as point E'. A larger shift in KK, however, would lead to a steady-state equilibrium such as E'', located to the right of k_I^H. At that point, the consumption-capital ratio is unambiguously higher than what would obtains at E', because θ remains increasing in k_I (albeit at a decreasing rate after k_I^H). The adjustment process will again follow the sequence EDE''. These results can be summarized in the following proposition:

Proposition 5. *If the economy's equilibrium is unique, a large budget-neutral shift toward infrastructure investment and away from unproductive spending will move the economy to a position where it can benefit from the externalities associated with public capital.*

Consider now the case where externalities associated with public capital are strong, and multiple equilibria exist. As noted in the previous section, a coordination problem then arises—which of the several paths is taken will depend on the ability of private agents to coordinate their actions on the good equilibrium. As usual, the problem can be thought of in terms of the tendency for optimistic or pessimistic expectations to become self-fulfilling. In some sense, the problem is one of coordinating beliefs.

The role of public policy in coordinating beliefs is indeed crucial in the present setting because of the interplay between private and public capital accumulation. Suppose that initially the economy is at the low growth steady state. The household takes the stock of public infrastructure as given. Then, there is no incentive to reduce consumption, and engage in capital accumulation, in order to switch to a higher growth path. Stagnation results from the lack of public capital relative to private capital, which itself translates into a low degree of efficiency of public capital, given the existence of network effects. The low growth equilibrium is stable in the sense that small increases in private capital stock will not lead to a jump to the higher growth steady state. The reason is that,because public investment is financed by a tax on private output, the supply of public infrastructure is an increasing function of the private capital stock. Moreover, with limited supply of public capital, production of health services is low, and the rate of time preference remains high. Thus, as note earlier, low savings and labor productivity also characterize the stagnating equilibrium.

Suppose that, in an attempt to coordinate beliefs, the government implements the same policy shift as above, starting from the low growth equilibrium point E^1. As shown in Figure 8, there are two possible outcomes. If the shift in spending translates into a shift in KK that is relatively mild, the two curves will intersect first at a point such as A, located to the left of the initial unstable equilibrium E^2, and again at a point such as A' located to the right of E^3. Essentially, the unstable steady state is now "lower" whereas the high growth stable steady state is higher. Put differently, the policy shifts tends to reduce the zone of indeterminacy (k_C^1, k_C^2) while expanding the interval (k_C^2, k_I^H).[22] Although this is not sufficient to ensure that the policy shift will lead necessarily to the high-growth equilibrium (the realization of a particular steady state will continue to depend on history as well as on expectations of private agents in the indeterminacy zones), the reduction in the indeterminacy range increases the likelihood that the economy will select a trajectory that will lead eventually to E^3.

If the shift in spending translates into a large shift in KK (so much so that the the new curve passes below point B) the unstable equilibrium point E^2 disappears entirely, given that CC is steeper than KK in the interval (k_I^H, k_I^L); only the high growth equilibrium, point A'', remains. In that case, if the economy's position is E^1, the adjustment path is the same as in Figure 1—on impact the consumption-capital ratio will jump from $\tilde{c}^1$ to a point such as D located on the saddlepath leading to A''. These results can be summarized in the following proposition:

Proposition 6. *If there are three steady-state equilibria, a budget-neutral shift toward infrastructure investment and away from unproductive*

[22] The change in the interval (k_I^L, k_C^1) is unclear.

spending may either reduce the zone of indeterminacy around the unstable steady state E^2 or entirely eliminate it, thereby increasing the likelihood that the economy will achieve the high growth steady state.

In the foregoing discussion, the increase in the share of public expenditure on infrastructure was assumed to be offset by a cut in unproductive spending. Given the amount of waste that often characterizes public spending in developing countries, this is not an unreasonable assumption. But a more general interpretation is also possible, by assumption $G_U < 0$ and by defining $-G_U$ as foreign aid (that is, grants). Thus, the analysis provides a theoretical rationale for those who have advocated development strategies based on large increases in public investment in infrastructure financed by foreign aid, as for instance in some recent international reports on external assistance to low-income countries, such as the Millennium Project (2005) of the United Nations.

My analysis, however, offers a note of caution. In the model, the ability of a shift in the share of public spending on infrastructure to guide the economy toward a high growth path is predicated on two critical parameters—the elasticity of output with respect to public infrastructure, α, which determines the effect of the public capital stock on the marginal product of private capital; and the degree of efficiency of public investment, φ, which, as noted earlier, can be viewed as a broad indicator of the quality of governance (or absorptive capacity). The lower the values of these two coefficients, the larger the increase in public investment in infrastructure will need to be to generate desirable effects. The role of φ is particularly important in my view; weak governance is often viewed as a principal reason for inefficiency and why public expenditures often fail to achieve outcomes.[23] Moreover, a *negative* correlation could exist between aid and the efficiency parameter φ, if indeed, as found by Svensson (2000), aid increases corruption in ethnically divided societies. This can be stated in the form of the following corollary to the above propositions:

Corollary to Propositions 5 and 6. *A large shift toward spending on infrastructure will generate desirable effects only if the degree of efficiency of public investment, φ, is sufficiently high.*

[23]There is no shortage of anecdotal evidence for this in the development literature. It is also confirmed by a number of recent studies, which show that corruption distorts incentives to allocate public investment to its initial purpose.

A Big Push policy may therefore require concomitant measures to improve governance. Of course, despite past evidence, aid programs themselves could be structured so as to bring about an *increase* in efficiency of public investment—by changing for instance the nature of conditionality in assistance programs and making it performance-based, rather than policy-based, and by allocating a sufficient fraction of aid to capacity building and institutional reform. If efficiency and governance can indeed be made to depend on aid itself, the argument for a Big Push in infrastructure investment financed by external assistance would be strengthened.

Finally, one may ask if results similar to those associated with a shift in investment in infrastructure could not be achieved by a shift instead toward health expenditure, again financed by a cut in unproductive outlays or foreign aid. After all, spending on health is also directly productive in this economy, and health services have a positive effect on savings. If the degree of efficiency of investment, φ, is low, and the direct effect of public infrastructure on output is somewhat limited, the economy may well be better off by spending more on health. However, health services do not generate externalities in the present framework; moreover, their production requires also infrastructure services. If the parameter μ (the elasticity of output of health services with respect to infrastructure) is relatively large, as suggested by some of the recent studies reviewed in Agénor and Moreno-Dodson (2006), a strategy based exclusively on the expansion of health services may not be sufficient to help an economy move to a higher growth path. Of course, this conjecture is predicated in part on the way health services are modeled in the present setting. If, for instance, health services generate large positive externalities on labor productivity (with strong convex effects initially) a case for a "health first" strategy could in principle be viable; but whether the empirical evidence would support this view would still remain an open question.

6 Concluding Remarks

The purpose of this paper has been to propose a theory of long-run development based on public infrastructure as the main engine of growth. In addition to investing in infrastructure, the government spends on health services, which raise labor productivity and lower the rate of time preference. The rate of time preference is modeled as a decreasing function of health services (relative to the stock of private capital). Agents become less impatient as their health improves. Consumption of health services (as opposed to consumption of commodities) induces patience and lowers the rate of time preference. This, in turn, raises savings and stimulates growth. Thus, although I abstract from the issue of human capital accumulation *per se* as in Uzawa-Lucas type models, "effective" labor is considered. In addition, infrastructure affects the production of both commodities and health services, and therefore labor efficiency.

The first part of the paper described the model and illustrated its functioning by considering a budget-neutral shift in the share of public investment in infrastructure. This policy shift was shown to raise the steady-state value of health production; this lowers the rate of time preference and raises savings. This additional savings translates into higher private capital and consumption in the steady state. Indeed, in the model, it is not only increases on the rate of return on physical capital that leads households to save more, but rather improvements in the consumption of health services—the supply of which depends on the availability of public capital in infrastructure. At the same time, the rate of time preference depends on a wealth effect. Although the model does not explicitly account for demographic factors, its prediction that low growth tends to be associated with low consumption of health services and poor productivity is consistent with several studies suggesting that health improvements tend to have a large impact on growth. Fogel (1994, 1997) for instance, argued that a significant fraction of economic growth in Britain during the period 1780-1980 (about 0.33 percent per annum) was due to an increase in effective labor inputs that resulted from workers' better nutrition and improved health.[24] More recently, Sohn (2000) found that improved nutrition increased available labor inputs in South Korea by 1 percent a year or more during 1962–95. A number of other studies have shown that initial levels of life expectancy tend to have a significant effect on subsequent growth rates (see Agénor and Moreno-Dodson (2006)). Lorentzen, McMillan and Wacziarg (2005) found that countries with a high rate of adult mortality also tend to experience low rates of growth—possibly because when people expect to die relatively young, they have less incentives to save and invest in the acquisition of skills.[25] What the model adds to these studies is that

[24] Boucekkine et al. (2003) estimate that a steady decline in adult mortality (while child mortality stayed level) accounts for 70 percent of the growth acceleration that modern Europe experienced between 1700 and 1820.

[25] They also found that the estimated effect of high adult mortality on growth is large enough to explain Africa's poor economic performance between 1960 and 2000. Indeed, in

infrastructure may well be one of the main engines behind the improvement in health outcomes.

In the second part of the paper, it was argued that public infrastructure generates strong nonconvexity of the economy's production technology. The network character of public infrastructure has important consequences for the relationship between public capital and economic growth. As a result of network effects, the degree of efficiency of public infrastructure is nonlinearly related to the stock of public capital (relative to the private capital stock) itself. It was shown that, as a result of these nonlinearities, there may be no equilibrium, a unique equilibrium, or multiple equilibria.

The third part of the paper focused on the role of public policy. It was shown that if there are three steady-state equilibria, a budget-neutral shift toward infrastructure investment and away from unproductive spending (or financed by foreign aid) may either reduce the zone of indeterminacy around the unstable steady state or entirely eliminate it—if it is large enough. However, the analysis suggests some caution in the Big Push view recently revived by Sachs (2005), among others. Sachs emphasizes the lack of savings at low levels of income as the main cause of a poverty trap.[26] This paper suggests that his analysis is incomplete. In particular, in the present paper a large shift toward spending on infrastructure will generate desirable effects only if the degree of efficiency of public investment is sufficiently high. A Big Push policy may therefore require concomitant measures to improve governance–which may itself be enhanced through aid conditionality.

The analysis can be extended in several directions. First, with higher growth induced by higher public investment in infrastructure, and thus higher income, the capacity to pay for health and education services will increase, enabling the state to deliver more in those areas as well. Second, greater access to health services enhances not only workers' productivity, but also the ability to learn and accumulate human capital—a significant constraint to growth in many low-income countries.[27] It would therefore be useful to introduce human capital accumulation and consider its interactions with health.

the 40 countries with the highest adult mortality rates in their sample of 98 countries, all are in Sub-Saharan Africa, except three.

[26]He also discussed increasing returns (or threshold effects) associated with the capital stock, but without an explicit formal analysis focusing on public infrastructure, as was done in the present paper.

[27]Oketch (2006) found strong evidence that physical capital investment is critical to human capital accumulation and growth in Sub-Saharan Africa.

Much recent evidence suggests that causality goes both ways, as documented for instance by Agénor and Moreno-Dodson (2006) and Agénor and Neanidis (2006). As noted in the introduction, higher life expectancy—or a lower rate of time preference—also increases the payoff from investment in education and thereby raises incentives to invest in the acquisition of skills. Conversely, higher levels of education tend to improve health outcomes, in part because it increases awareness of diseases, both to the individual and their family members (such as mothers teaching their children to wash their hands before preparing and eating food). In Tamura (2006) for instance, human capital accumulation lowers mortality, which in turn reduces fertility, thereby inducing a demographic transition and economic growth—lower fertility reduces the cost of human capital investment, inducing parents to invest more in the education of their children. It is not difficult to see in that setting how the lack of infrastructure can create another source of low-growth trap—poor transportation increases the time needed to get to school.[28] And because the time spent to get to school raises the total amount of time that must be allocated to acquire skills, it increases the opportunity cost of education. Given the inability to borrow for many (poor) households, parents tend to keep children out of school, because they are unable to cover the upfront cost of schooling (broadly defined to include the opportunity cost of not working), in return for future (and uncertain) benefits.[29] Of course, adding education could lead to additional sources of nonlinearities; in Mayer-Foulkes (2003, 2005) for instance, the acquisition of human capital is subject to threshold effects, with threshold levels depending endogenously on technological change and credit constraints.

Finally, it would be worth exploring another possible externality associated with infrastructure—regarding not the efficiency of public capital for a given technology (as was done here), but rather the choice of technology itself. At low levels of infrastructure, producers may have no choice but to adopt (or continue to use) a "subsistence" (or inefficient) technology. In the absence of a reliable power grid, for instance, firms may not be able to switch to more advanced machines and sophisticated equipment—even though it would be profitable to do so. With no roads to transport commodities between rural and urban areas in a timely fashion, the adoption

[28] The nonlinearity in the "learning curve" defined in Kejak (2003) for instance could be related to infrastructure.

[29] Better roads would also make it easier to attract more qualified teachers to rural areas, and to deliver learning materials.

of new production techniques in agriculture may not be feasible either. But once infrastructure provision has reached a certain threshold, producers may find it easier to adopt a "modern" (or highly productive) technology and reap the benefits from doing so. This, in turn, would lead to a faster pace of growth in output and sustained improvements in productivity. Endogenizing the switch in technology in this way would shed additional light on the development process while bringing to the fore the critical role of the state in fostering private sector growth.

Appendix

The dynamic system consists of equations (23) and (24), which are repeated here for convenience:

$$\frac{\dot{c}}{c} = \xi k_I^{(\alpha+\mu\beta)/\Omega} - \chi(k_I;\theta) + c, \tag{A1}$$

$$\frac{\dot{k}_I}{k_I} = \upsilon_H^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}\left\{\varphi\upsilon_I\tau^{1/\Omega}k_I^{-\eta/\Omega} - (1-\tau)\tau^{(1-\mu)\beta/\Omega}k_I^{(\alpha+\mu\beta)/\Omega}\right\} + c, \tag{A2}$$

where ξ is defined in (25).

Consider first the case where θ is exogenous. In the vicinity of the steady state, equations (A1) and (A2) can be linearized to give

$$\begin{bmatrix} \dot{c} \\ \dot{k}_I \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} c-\tilde{c} \\ k_I - \tilde{k}_I \end{bmatrix} + \begin{bmatrix} a_{13} & 0 \\ a_{23} & a_{24} \end{bmatrix}\begin{bmatrix} \theta \\ \upsilon_I \end{bmatrix}, \tag{A3}$$

where $\tilde{c}$ and $\tilde{k}_I$ are interior stationary values of c and k_I and the a_{ij} are given by

$$a_{11} = \tilde{c} > 0, \qquad a_{21} = \tilde{k}_I > 0, \tag{A4}$$

$$a_{12} = \tilde{c}(\frac{\alpha+\mu\beta}{\Omega\tilde{k}_I})\xi\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \tilde{c}\chi_{k_I}, \tag{A5}$$

$$a_{13} = \tilde{c}(\frac{\alpha}{\Omega\theta})\xi\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \tilde{c}\chi_\theta,$$

$$a_{22} = \frac{\upsilon_H^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}}{\Omega}\left\{-\eta\varphi\upsilon_I\tau^{1/\Omega}\tilde{k}_I^{-\eta/\Omega} - \Gamma\tau^{(1-\mu)\beta/\Omega}\tilde{k}_I^{(\alpha+\mu\beta)/\Omega}\right\} < 0, \tag{A6}$$

$$a_{23} = -(\frac{\alpha}{\Omega\theta})\tilde{c} < 0,$$

$$a_{24} = \upsilon_H^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}\varphi\tau^{1/\Omega}\tilde{k}_I^{-\eta/\Omega} > 0,$$

where $\Gamma \equiv (1-\tau)(\alpha+\mu\beta) > 0$.[30] If ρ' is not too large, $a_{12}, a_{13} < 0$.

Because c is a jump variable, whereas k_I is predetermined, saddlepath stability requires one unstable (positive) root. To ensure that this condition

[30]Note that, because in (A2) k_I is raised to a negative power, $\tilde{k}_I = 0$ cannot be a steady state. The economically meaningless case $\tilde{c} = 0$ can also be ruled out.

holds, the determinant of the Jacobian matrix of partial derivatives of the dynamic system (A3), J, must be negative:

$$\det J = a_{11}a_{22} - a_{12}a_{21} < 0.$$

To examine if this condition holds, consider first the case where ρ is constant (implying that the second term in the expression for a_{12} is zero) and note that, from (A5) and (25), we have

$$a_{12} = -(\frac{\tilde{c}}{\Omega\tilde{k}_I})(\alpha+\beta)(\alpha+\mu\beta)\left\{(1-\tau)(\upsilon_H\tau)^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}\tilde{k}_I^{(\alpha+\mu\beta)/\Omega}\right\},$$

or, using (28), and noting that $s/\eta = 1-\tau$ and $-(\alpha+\beta)/\eta = 1-\eta^{-1}$,

$$a_{12} = (\frac{\tilde{c}}{\Omega\tilde{k}_I})(1-\eta^{-1})(\alpha+\mu\beta)(\gamma+\rho) < 0. \tag{A7}$$

Similarly, from (A6),

$$a_{22} = -\frac{\eta\theta^{\alpha/\Omega}}{\Omega}\left\{\varphi\upsilon_I\tau^{1/\Omega}\upsilon_H^{(1-\mu)\beta/\Omega}\tilde{k}_I^{-\eta/\Omega}\right\} - \frac{\Gamma}{s\Omega}\left\{s(\upsilon_H\tau)^{(1-\mu)\beta/\Omega}\tilde{k}_I^{(\alpha+\mu\beta)/\Omega}\right\},$$

or, using (27) and (28),

$$a_{22} = -\frac{\eta}{\Omega}\gamma - \frac{\Gamma}{s\Omega}(\gamma+\rho). \tag{A8}$$

From (A4), (A7), and (A8),

$$\Delta = -\frac{\tilde{c}\eta}{\Omega}\gamma - \frac{\tilde{c}\Gamma}{s\Omega}(\gamma+\rho) - (\frac{\tilde{c}}{\Omega})(1-\eta^{-1})(\alpha+\mu\beta)(\gamma+\rho),$$

or equivalently, $\Delta = \Delta'\tilde{c}/\Omega$ where

$$\Delta' = -\eta\gamma - \left\{\frac{\Gamma}{s} + (1-\eta^{-1})(\alpha+\mu\beta)\right\}(\gamma+\rho),$$

so that $\mathrm{sg}(\Delta) = \mathrm{sg}(\Delta')$.

From the definition of Γ, we have $\Gamma/s = (\alpha+\mu\beta)\eta^{-1}$. Thus

$$\Delta' = -\eta\gamma - (\alpha+\mu\beta)\left\{\eta^{-1} + (1-\eta^{-1})\right\}(\gamma+\rho),$$

that is

$$\Delta' = -\eta\gamma - (\alpha + \mu\beta)(\gamma + \rho) < 0.$$

Thus, the saddlepath condition is always satisfied for ρ constant. If ρ is endogenous and ρ' is sufficiently large, a_{12} may be positive, in which case (given that $a_{21} > 0$), the condition $\det J < 0$ always holds. In what follows, it is assumed that ρ' is small enough to ensure that a_{12} remains negative.

With $a_{12} < 0$, the slopes of curves CC and KK in Figure 1 are given by

$$\left.\frac{dc}{dk_I}\right|_{\dot{c}=0} = -\frac{a_{12}}{a_{11}} = -(\frac{\xi}{\Omega \tilde{k}_I})(\alpha + \mu\beta)\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} + \chi_{k_I} > 0,$$

$$\left.\frac{dc}{dk_I}\right|_{\dot{k}_I=0} = -\frac{a_{22}}{a_{21}} = \frac{\upsilon_H^{(1-\mu)\beta/\Omega}}{\theta^{-\alpha/\Omega}\Omega\tilde{k}_I}\left\{\eta\varphi\upsilon_I\tau^{1/\Omega}\tilde{k}_I^{-\eta/\Omega} + \Gamma\tau^{(1-\mu)\beta/\Omega}\tilde{k}_I^{(\alpha+\mu\beta)/\Omega}\right\} > 0.$$

Saddlepath stability ($\det A < 0$) implies that curve KK in Figure 1 must be steeper than CC. The slope of the saddlepath SS, which is given by $-a_{12}/(\tilde{c} - \zeta)$, where ζ is the negative root of the system, is also positive.

To show that both curves are concave, consider the first slope defined above; differentiation yields

$$\left.\frac{d^2c}{dk_I^2}\right|_{\dot{c}=0} = -(\frac{\xi}{\Omega\tilde{k}_I^2})(\alpha + \mu\beta)[\frac{(\alpha + \mu\beta)}{\Omega} - 1]\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} + \chi_{k_I k_I}, \tag{A9}$$

Consider the first term; its sign depends on the sign of $(\alpha + \mu\beta)/\Omega - 1$. From the definition of Ω,

$$\frac{(\alpha + \mu\beta)}{\Omega} - 1 = \frac{\alpha + \mu\beta - 1 + (1-\mu)\beta}{\Omega} = -\frac{\eta}{\Omega} < 0.$$

Given that, by assumption, $\rho'' > 0$, the second term in (A9), $\chi_{k_I k_I}$, is also negative. Thus, $d^2c/dk_I^2|_{\dot{c}=0} < 0$.

Similarly,

$$\left.\frac{d^2c}{dk_I^2}\right|_{\dot{k}_I=0} = \frac{\upsilon_H^{(1-\mu)\beta/\Omega}\theta^{\alpha/\Omega}}{\Omega\tilde{k}_I^2}\left\{-\eta\varphi\upsilon_I\tau^{1/\Omega}(\frac{\eta}{\Omega} + 1)\tilde{k}_I^{-\eta/\Omega}\right. \tag{A10}$$

$$\left. + \Gamma\tau^{(1-\mu)\beta/\Omega}[\frac{(\alpha + \mu\beta)}{\Omega} - 1]\tilde{k}_I^{(\alpha+\mu\beta)/\Omega}\right\}.$$

From the previous result on $(\alpha+\mu\beta)/\Omega - 1$, both terms in the expression in brackets are negative. Thus, $d^2c/dk_I^2|_{\dot{k}_I=0} < 0$.

To see whether the equilibrium is unique, note that from (A1), setting $\dot{c} = 0$ yields

$$\tilde{c} = -\xi \tilde{k}_I^{(\alpha+\mu\beta)/\Omega} + \chi(\tilde{k}_I; \theta). \tag{A11}$$

Substituting (A11) in (A2) with $\dot{\tilde{k}}_I = 0$ yields the implicit function

$$F(\tilde{k}_I) = \chi(\tilde{k}_I; \theta) + (\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \theta^{\alpha/\Omega} \left\{ \varphi \upsilon_I \tau \tilde{k}_I^{-\eta/\Omega} - s \tilde{k}_I^{(\alpha+\mu\beta)/\Omega} \right\} = 0, \tag{A12}$$

from which it can be established that $F_{\tilde{k}_I} < 0$. Thus, $F(\tilde{k}_I)$ cannot cross the horizontal axis from below. Now, we also have $\lim_{\tilde{k}_I \to 0} F(\tilde{k}_I) = +\infty$ and $\lim_{\tilde{k}_I \to +\infty} F(\tilde{k}_I) = -\infty$. Given that $F(\tilde{k}_I)$ is a continuous, monotonically decreasing function of $\tilde{k}_I$, there is a unique positive value of $\tilde{k}_I$ that satisfies $F(\tilde{k}_I) = 0$. From (A11), there is also a unique positive value of $\tilde{c}$. Thus, the BGP is unique, and the equilibrium is locally determinate.

To establish the effect of an increase in υ_I, note that from (A3), we have $d\tilde{c}/d\upsilon_I|_{CC}^{\tilde{k}_I \text{ given}} = 0$ and $d\tilde{c}/d\upsilon_I|_{KK}^{\tilde{k}_I \text{ given}} = -a_{24}/a_{21} < 0$. Thus, an increase in υ_I has no effect on curve CC and shifts curve KK downward and to the right in Figure 1. The steady-state effects on $\tilde{c}$ and $\tilde{k}_I$ are given by

$$\frac{d\tilde{c}}{d\upsilon_I} = \frac{a_{12}a_{24}}{\Delta} > 0, \quad \frac{d\tilde{k}_I}{d\upsilon_I} = \frac{-a_{11}a_{24}}{\Delta} > 0.$$

which indicate that both $\tilde{c}$ and $\tilde{k}_I$ increase.[31]

Consider now the case where efficiency is subject to threshold effects. For $k_I < k_I^L$, and thus θ constant, stability conditions are those discussed previously. For $k_I > k_I^L$, by contrast, $\theta = \theta(k_I)$; to fix ideas, let $\theta = \theta_0 k_I^\kappa$, where $\kappa > 1$ ($\kappa < 1$) for the convex (concave) portion of the curve in Figure 2. The case of constant θ corresponds therefore to $\kappa = 0$. For simplicity, I normalize θ_0 to unity in what follows.

In this case, using (A1) and (25), the steady-state values of c and k_I must satisfy

$$\tilde{c} = \chi(\tilde{k}_I; \theta) - \xi' \tilde{k}_I^{[\alpha\kappa + (\alpha+\mu\beta)]/\Omega}, \tag{A13}$$

where $\xi' \equiv -(1-\tau)(\alpha+\beta)(\upsilon_H \tau)^{(1-\mu)\beta/\Omega}$. The properties of the slope of curve CC are therefore determined from

$$\left.\frac{dc}{dk_I}\right|_{\dot{c}=0} = \chi_{k_I} + \chi_\theta \kappa (\frac{\theta}{\tilde{k}_I}) + \xi' [\frac{\alpha\kappa + (\alpha+\mu\beta)}{\Omega \tilde{k}_I}] \tilde{k}_I^{[\alpha\kappa + (\alpha+\mu\beta)]/\Omega} > 0,$$

[31] Equation (A3) can also be used to study the effects of an increase in θ. Given the signs of a_{13} and a_{23}, it can readily be established that the impact on $\tilde{c}$ and $\tilde{k}_I$ is in general ambiguous.

$$\left.\frac{d^2c}{dk_I^2}\right|_{\dot{c}=0} = \chi_{k_Ik_I} + \chi_{\theta\theta}[\frac{\kappa(\kappa-1)\theta}{\tilde{k}_I^2}] \tag{A14}$$

$$+\frac{\xi'}{\tilde{k}_I^2}[\frac{\alpha\kappa+(\alpha+\mu\beta)}{\Omega}][\frac{\alpha\kappa+(\alpha+\mu\beta)}{\Omega}-1]\tilde{k}_I^{[\alpha\kappa+(\alpha+\mu\beta)]/\Omega},$$

With θ constant, $\kappa = 0$, and equation (A14) boils down to an expression similar to (A9), so that $d^2c/dk_I^2|_{\dot{c}=0} < 0$. However, with θ endogenous, equation (A14) is more complex; depending on the strength of the externality (the value of κ), the second term may be either positive or negative. Similarly, although, as shown earlier, $(\alpha+\mu\beta)\Omega^{-1} - 1 < 0$, the sign of the expression $[\alpha\kappa + (\alpha+\mu\beta)]\Omega^{-1} - 1$ is ambiguous and depends on the value of κ. If κ is sufficiently large, both of these terms become positive and the CC curve may turn convex. If the externality is not strong enough, the curve may remain concave.

Similarly, using (A2) and (25), the steady-state values of c and k_I must satisfy

$$\tilde{c} = \tilde{k}_I^{\alpha\kappa/\Omega}[\varepsilon_1\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \varepsilon_2\tilde{k}_I^{-\eta/\Omega}], \tag{A15}$$

where $\varepsilon_1 \equiv (1-\tau)(\upsilon_H\tau)^{(1-\mu)\beta/\Omega} > 0$, and $\varepsilon_2 \equiv \upsilon_H^{(1-\mu)\beta/\Omega}\varphi\upsilon_I\tau^{1/\Omega} > 0$. To ensure that the consumption-private capital ratio is positive requires imposing $\varepsilon_1\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \varepsilon_2\tilde{k}_I^{-\eta/\Omega} > 0$, or equivalently (after simplification) $\tilde{k}_I > \varphi\upsilon_I\tau^{1/\Omega}/(1-\tau)$.

The properties of the slope of curve KK are therefore determined from

$$\left.\frac{dc}{dk_I}\right|_{\dot{k}_I=0} = \frac{\alpha\kappa\theta^{\alpha/\Omega}}{\Omega\tilde{k}_I}[\varepsilon_1\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \varepsilon_2\tilde{k}_I^{-\eta/\Omega}],$$

$$+\frac{\theta^{\alpha/\Omega}}{\Omega\tilde{k}_I}[\varepsilon_1(\alpha+\mu\beta)\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} + \varepsilon_2\eta\tilde{k}_I^{-\eta/\Omega}] > 0$$

$$\left.\frac{d^2c}{dk_I^2}\right|_{\dot{k}_I=0} = 2\frac{\alpha\kappa\theta^{\alpha/\Omega}}{\Omega^2\tilde{k}_I^2}[\varepsilon_1(\alpha+\mu\beta)\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} + \varepsilon_2\eta\tilde{k}_I^{-\eta/\Omega}] \tag{A16}$$

$$\varepsilon_1\frac{\alpha\kappa(\kappa-1)\theta^{\alpha/\Omega}}{\Omega\tilde{k}_I^2}[\varepsilon_1\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \varepsilon_2\tilde{k}_I^{-\eta/\Omega}]$$

$$+\frac{\theta^{\alpha/\Omega}}{\Omega\tilde{k}_I^2}\left\{\varepsilon_1(\alpha+\mu\beta)(\frac{\alpha+\mu\beta}{\Omega}-1)\tilde{k}_I^{(\alpha+\mu\beta)/\Omega} - \varepsilon_4\eta(\eta+1)\tilde{k}_I^{-\eta/\Omega}\right\}.$$

Again, with θ constant, $\kappa = 0$, the first two terms on the right-hand side of (A14) disappear and (given that $(\alpha + \mu\beta)\Omega^{-1} - 1 < 0$) equation (A16) boils down to an expression similar to (A10), so that $d^2c/dk_I^2|_{\dot{k}_I=0} < 0$. However, with θ endogenous, equation (A14) is more complex. The first term is unambiguously positive, but the sign of the second depends on the strength of the externality (the value of κ). In particular, given the restriction on $\tilde{k}_I$ needed to ensure that $\tilde{c} > 0$, a high value of κ makes the second term positive and tends to make curve KK convex, as before.

To examine the issue of multiplicity of equilibria, combine (A13) with (A15) to give

$$F(\tilde{k}_I) = \chi(\tilde{k}_I; \theta) + (\upsilon_H \tau)^{(1-\mu)\beta/\Omega} \left\{ \varphi \upsilon_I \tau \tilde{k}_I^{(\alpha\kappa - \eta)/\Omega} - s\tilde{k}_I^{[\alpha\kappa + (\alpha+\mu\beta)]/\Omega} \right\} = 0, \tag{A17}$$

so that

$$F_{\tilde{k}_I} = \chi_{k_I} + \chi_\theta \kappa (\frac{\theta}{k_I}) + \frac{(\upsilon_H \tau)^{(1-\mu)\beta/\Omega}}{\Omega \tilde{k}_I} \times$$
$$\times \left\{ (\alpha\kappa - \eta)\varphi \upsilon_I \tau \tilde{k}_I^{(\alpha\kappa-\eta)/\Omega} - [\alpha\kappa + (\alpha + \mu\beta)] s \tilde{k}_I^{[\alpha\kappa + (\alpha+\mu\beta)]/\Omega} \right\}.$$

With θ constant, $\kappa = 0$, and expression (A17) boils down to (A12). Now, although the second term on the right-hand side is unambiguously negative, and $\alpha\kappa + (\alpha + \mu\beta) > 0$, $F_{\tilde{k}_I}$ is no longer necessarily negative because the first term in the parentheses can be positive, depending on the value of $\alpha\kappa - \eta$. If the elasticity of time preference with respect to θ is not too high, (so that χ_θ is not large) and the expression in parentheses remains positive, $F_{\tilde{k}_I}$ can be positive for some interval if externalities are strong (κ is high).

In that case, because the limit conditions given above still hold with endogenous θ, (that is, $\lim_{\tilde{k}_I \to 0} F(\tilde{k}_I) = +\infty$, and $\lim_{\tilde{k}_I \to +\infty} F(\tilde{k}_I) = -\infty$), $F(\tilde{k}_I)$ may intersect the horizontal axis more than once. Assuming that the value of k_I at which the derivative changes sign is larger than the threshold level k_I^L at which the efficiency externality kicks in, and that the externality is very strong, the curve might intersect the horizontal axis from above, and take negative values for some interval before the derivative turns positive and starts increasing to cross the horizontal axis once more (this time from below), and then fall again after the upper threshold value k_I^H is exceeded. In that situation, we will observe three equilibria. Or, if the externalities are strong but not excessively so, the curve might cut the horizontal axis from above and take negative values for some interval, as before, but this time around it may increase until it becomes tangent to the horizontal axis at exactly k_I^H and start falling to $-\infty$, as k_I rises further. In such conditions, we will observe 2 equilibria, the first of which only being stable. Finally, with weak externality effects, the curve might increase as before but not enough to cross the horizontal axis from below—so that $F(\tilde{k}_I)$ remains strictly negative—and a unique equilibrium may emerge.

Thus, in general, the curvature of the CC and KK will now vary between intervals $k_I < k_I^L$, $k_I \in (k_I^L, k_I^H)$, and $k_I > k_I^H$, depending on the value of κ. As shown in Figures 3, 4, and 5, which all assume that the efficiency externality (as measured by κ) is large, various cases are possible; curves CC and KK may not intersect at all (Figure 3), they may intersect once (Figure 4), twice (not shown), or they may intersect three times (Figure 5). In the latter case, stability depends now on the eigenvalues corresponding to the Jacobian matrix calculated around each steady state. As noted earlier, saddlepath stability requires $\det J < 0$; curve KK must cut curve CC from below. This is the case at points E^1 and E^3 in Figure 5. Point E^2 is therefore an unstable equilibrium.

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Figure 1

Increase in the Share of Spending on Infrastructure
Financed by a Cut in Unproductive Spending

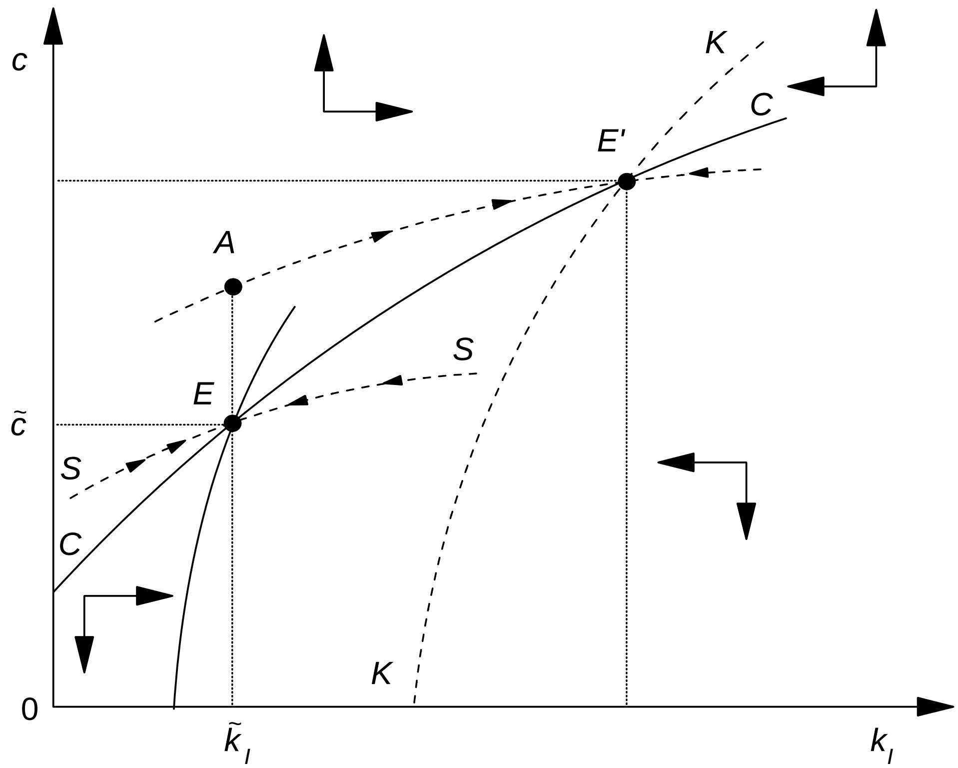

Figure 2

Network Externalities and Endogenous Efficiency

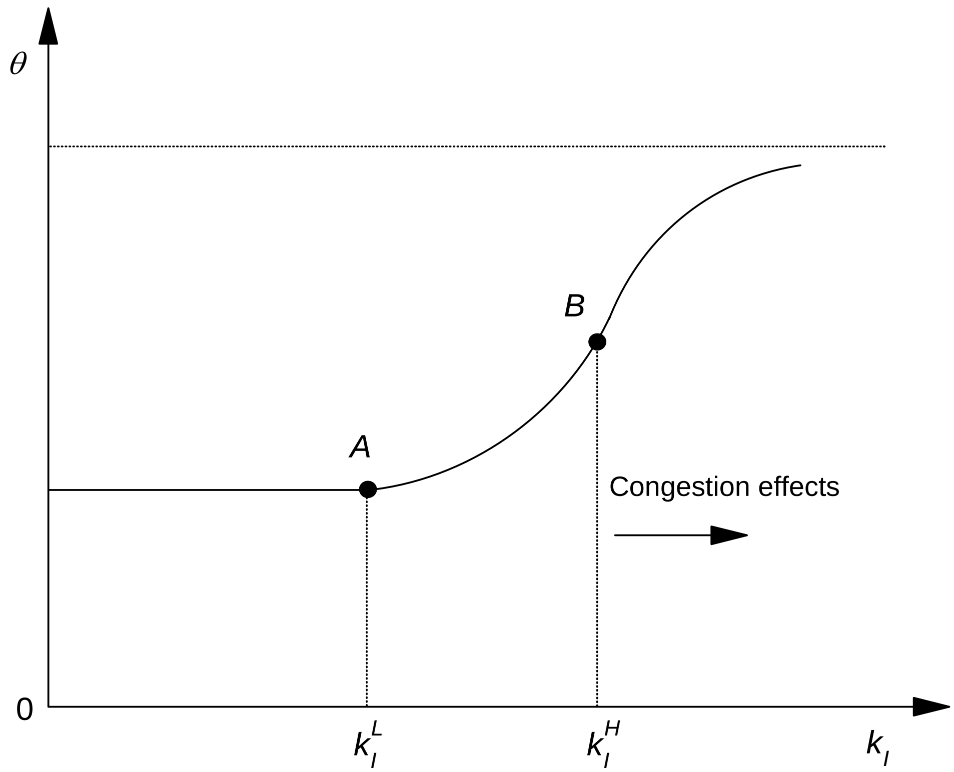

Figure 3

Network Externalities: No Equilibrium

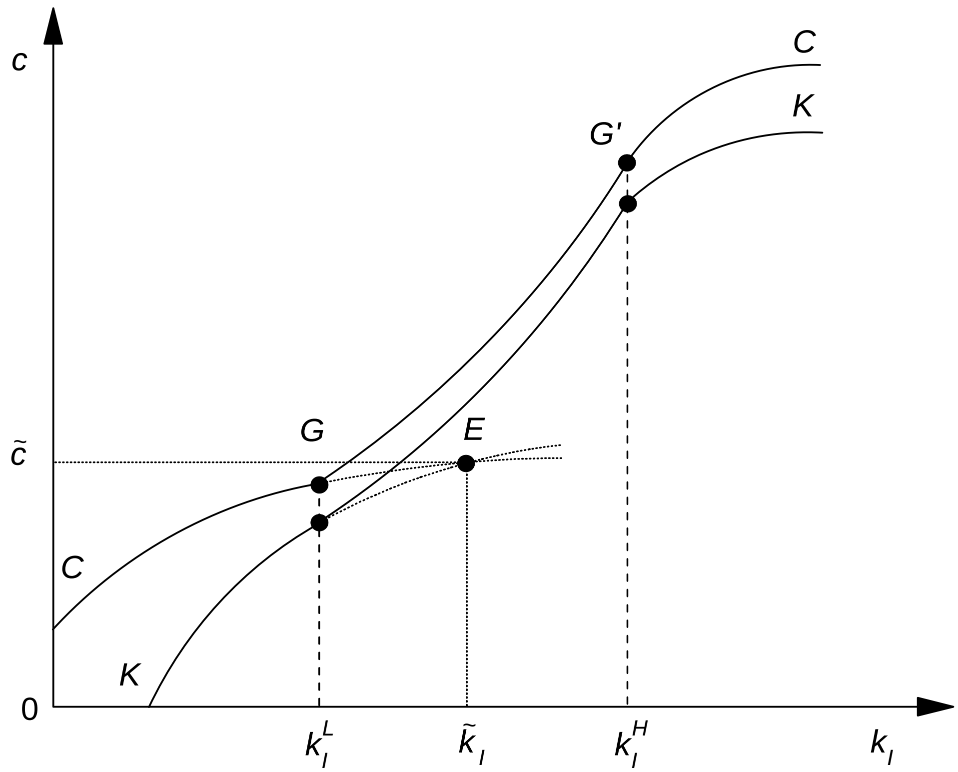

Figure 4

Network Externalities: Unique Equilibrium

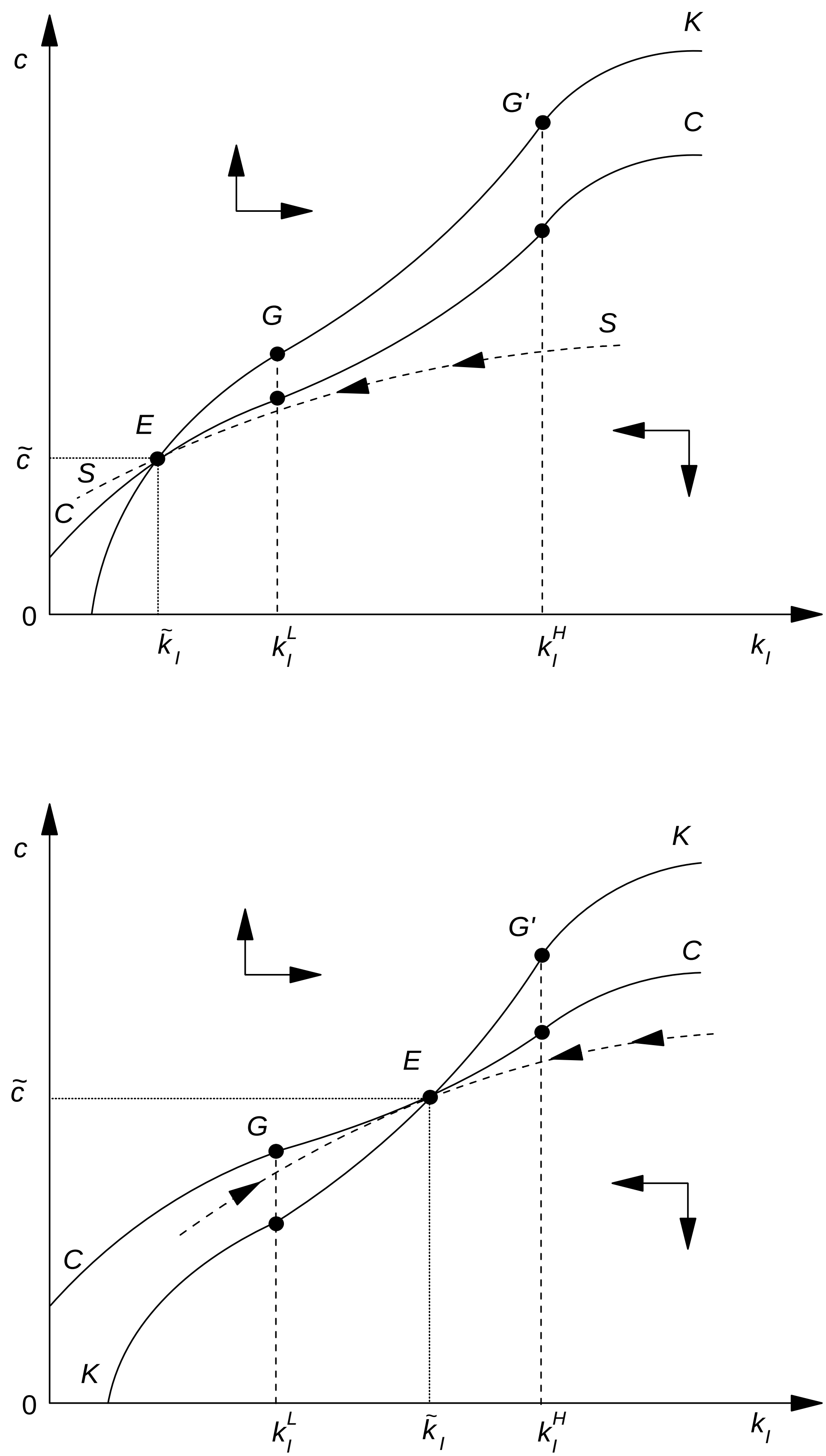

Figure 5

Network Externalities: Multiple Equilibria

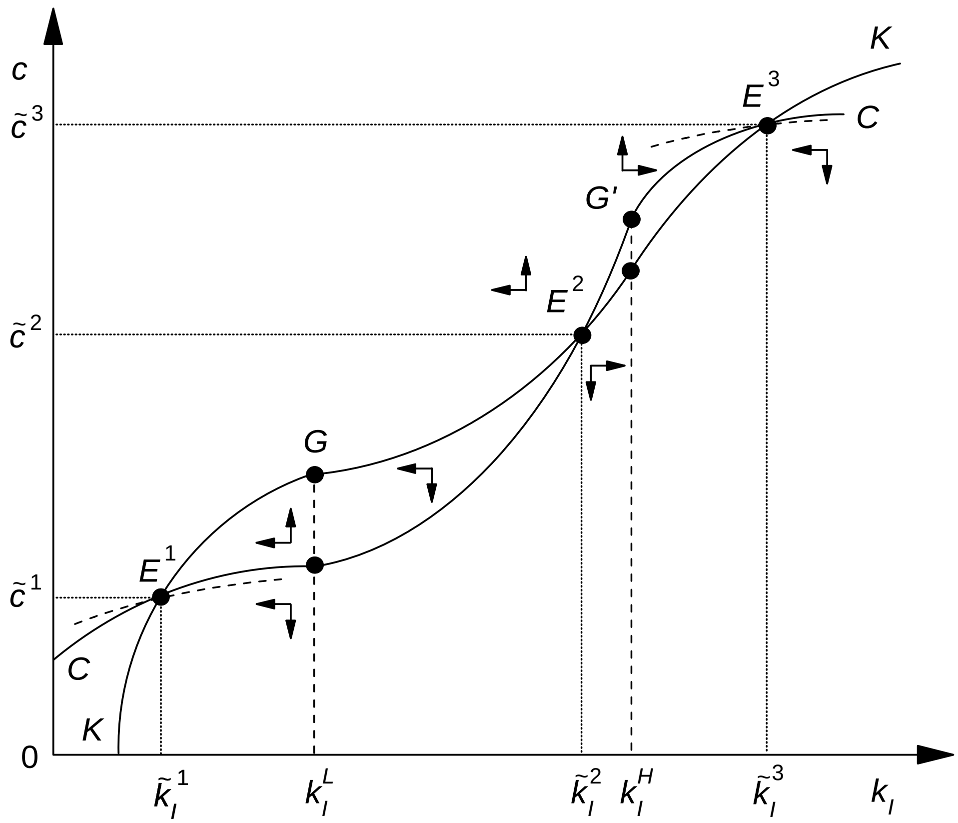

Figure 6

Trajectories around the Unstable Equilibrium

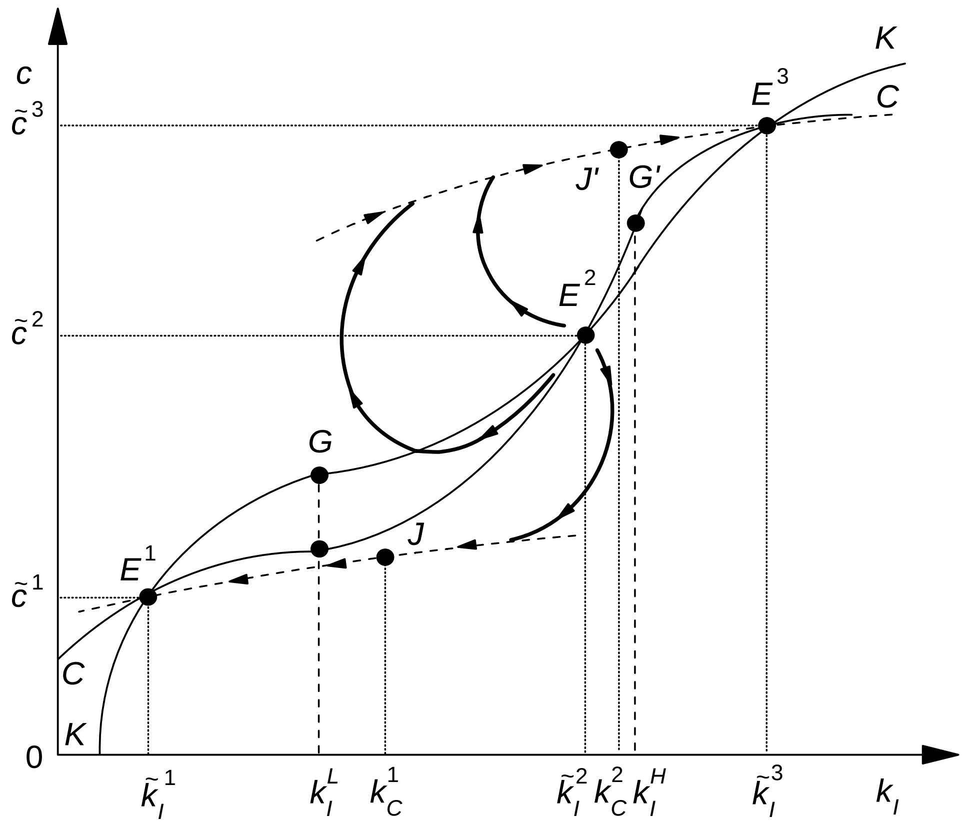

Figure 7

Role of Policy: Shift in Spending toward Infrastructure
(Unique Equilibrium)

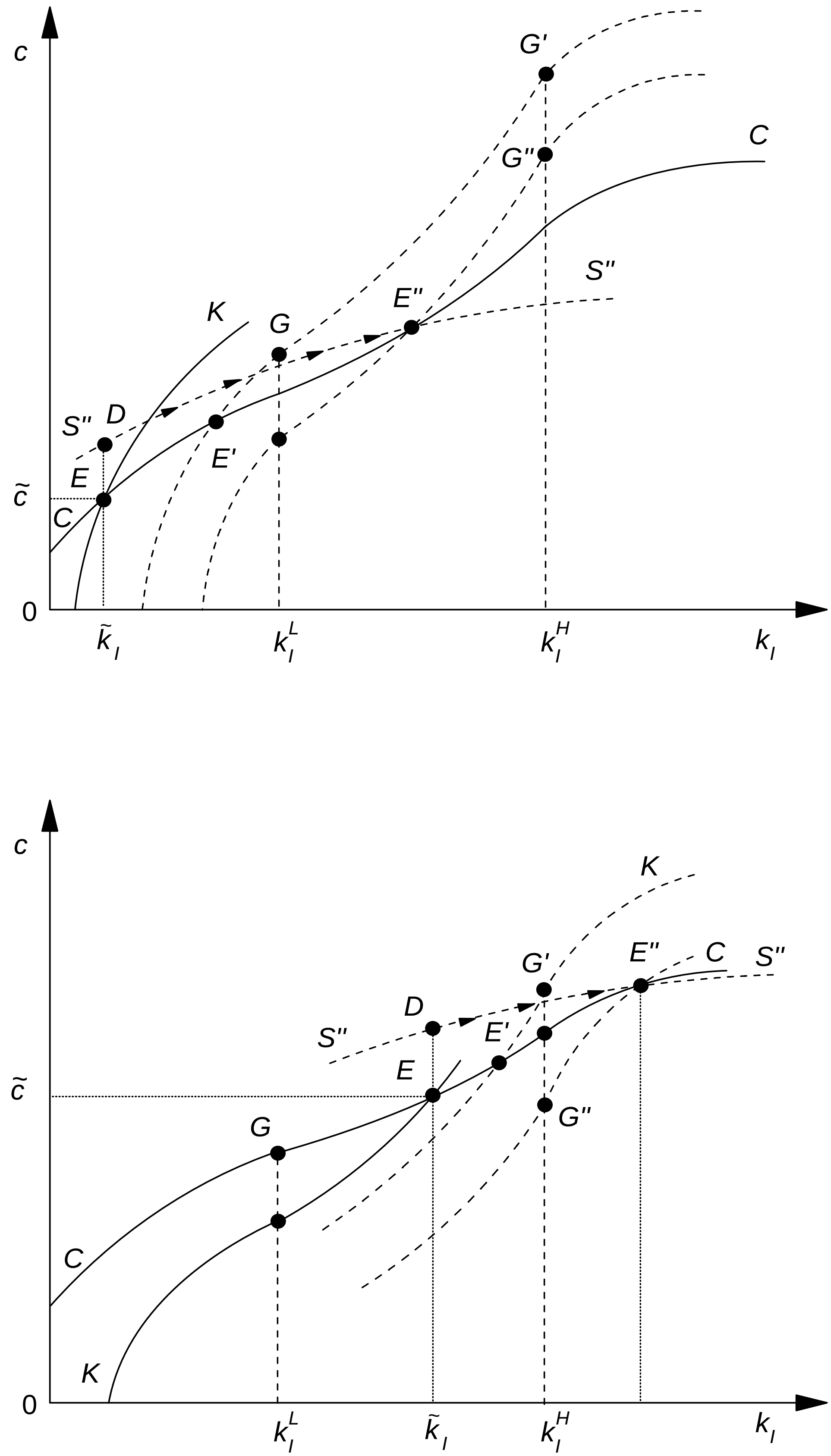

Figure 8

Role of Policy: Shift in Spending toward Infrastructure (Multiple Equilibria)