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Minimum quality standards with more than two firms under Cournot competition

by

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Abstract

The paper studies the effects of the adoption of a Minimum Quality Standard (MQS) in a vertically differentiated market. Three identical firms compete in qualities and quantities and face quality dependent fixed costs. After the introduction of a MQS, the average provision of quality increases in contrast to what has been shown under the assumption of Bertrand competition with more than two firms. Total domestic welfare unambiguously decreases if all firms are domestic, whereas, if foreign firms are allowed to enter the domestic market, the effects on welfare might be reversed.

Keywords: vertically differentiated products, minimum quality standards, welfare

JEL classification: L13, L15, L52

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1. Introduction

The paper analyzes the effects of the introduction of a Minimum Quality Standard (MQS) in a vertically differentiated market in which three identical firms play a two stage quality-quantity game; we assume quality dependent fixed costs, i.e. we study markets in which the development of a quality product is obtained by a process of R&D or Advertising that takes mainly the form of fixed cost related to quality. In what follows we refer in particular to the framework, which has been proposed initially by Mussa and Rosen (1978), Shaked and Sutton (1982), (1983), (1987), and analyzed in detail by Motta (1993).

The type of competition in which firms are engaged in the short run has significant effects on the strategic behaviour with respect to quality of these firms. Under unregulated Bertrand duopolistic competition[1] qualities are strategic complements for both firms, i.e. the slopes of the quality best response functions of the high and low quality firms are positive with respect to the quality chosen by the competitors. Instead, under Cournot competition[2], qualities are strategic complements for the high quality firm and strategic substitutes for the low quality firm, i.e. the slopes of the quality best response functions of the low and high quality firms are respectively negative and positive with respect to the quality chosen by the competitor. Such a difference is the key element behind the opposite effects that the introduction of a MQS produces on a vertically differentiated duopoly and it plays an important role also when a third firm is allowed to enter the market, as we are going to show in what follows.

The first important contribution that studied the effects of the adoption of a MQS[3] on a duopolistic market is offered by Ronnen (1991): the author studies a duopoly model with Bertrand competition and quality dependent fixed costs[4]. He shows that the introduction of a MQS (that marginally increases the quality offered by the low quality firm) increases the substitutability of the two products, making the competition stronger. The high quality firm increase its quality (qualities are strategic complements), in order to restore partially the product differentiation and to soften the degree of price competition. With fiercer competition (and therefore lower prices) and higher qualities, consumers' surplus increases. Profits of the high quality firm decrease since it has to incur higher fixed costs for quality, while the low quality firm's profits increase because the MQS imposes a commitment to quality and the market pays for it. Valletti (2000), assuming again quality dependent fixed costs, shows that under Cournot competition the benefits (in terms of total welfare) of the introduction of a MQS disappear. Now both firms earn lower profits and this negative effect more than offsets the increase in consumers' surplus. However, under both duopolistic Bertrand and Cournot competition average quality increases due to the introduction of the standard.

To our knowledge, the only example of a model in which three firms play a quality-pricing game is offered by Scarpa (1998). The author shows that the quality chosen by the lowest quality firm is a strategic substitute for the highest quality firm, while it is a strategic complement for the intermediate quality firm in equilibrium; such a feature produces results in terms of average quality, profits and welfare that differ from previous contributions. The intermediate firm has to face a "dilemma" once a MQS is adopted: increasing its own quality would restore the differentiation with the low quality firm, but it would decrease in turn the degree of differentiation with the high quality firm and increase in addition the production costs. Scarpa shows that the intermediate quality firm solves the dilemma increasing the provision of quality. Consequently, there are two opposing forces that affect the high quality firm's choice once a MQS is introduced. Since the quality of the intermediate firm increases, the high quality firm would tend to increase quality (and face higher costs) in order to soften

[1] See Ronnen (1991) and Lutz (2003).

[2] See for example Jinji (2000), Valletti (2000) and Herguera et al. (2002).

[3] Issues related to the process of the adoption of a MQS are not addressed in this paper. Lutz (2003) analyzes the effects of the introduction of alternative standard setting arrangements in an international system such as the European Union. In particular, the state of the EU regulations on the matter is summarized in *Standards, CE Marking and "New Approach" Directives,* see www.newapproach.org for more information on EU regulations.

[4] Crampes and Hollander (1995) study a similar problem where the quality dependent costs are variable and the market is covered.

price competition. Since the marginal revenues of the high quality firm are decreasing in the quality chosen by the low quality firm, it is less profitable to choose the highest quality. It is shown that, starting from the equilibrium, the dominant effect is generated by the price competition stage; it is less profitable to offer a high quality once a MQS is introduced. The results of this model are strongly related to the assumption of Bertrand competition and the presence of an intermediate quality firm[5]. It seems reasonable to imagine that under a softer competition (such as quantity competition) the highest and intermediate quality firms might react differently to a MQS. If that is the case, the average quality provision might even increases after the introduction of a MQS, restoring the duopoly results.

Our paper studies this particular formulation of the problem. Since Cournot competition is softer than Bertrand competition, we show that the quality chosen by the lowest quality firm is a strategic complement for the highest quality firm and a strategic substitute for the intermediate quality firm. As a result, the introduction of a MQS that does not change the market structure increases the average quality provision. The exercise is interesting for two reasons: first, we present the analysis of an unregulated vertically differentiated Cournot triopolistic market, to our knowledge missing in the literature; second, we study the effects of the introduction of a MQS in such a market, comparing the results to the Bertrand triopoly model and to the Cournot duopoly model. In addition, two other results are presented. We consider the problem faced by a welfare maximizing central planner and we show that choosing a standard that allows only one firm to be active in the market would be a first best solution. Finally, we analyze the effects on welfare of a MQS when at least one of the three firms is foreign and show that the results might completely been reversed.

Section 2 introduces the model and the results. Section 3 compares our results with those found in previous literature. Section 4 considers the possibility that at least one firm is not domestic. Section 5 concludes.

2. The model

We present a two stage model in which three firms simultaneously compete, first, in qualities in stage one and, then, in quantities in stage two. We look for Subgame Perfect Nash Equilibria by the method of backwards induction; costs are fixed (quadratic and identical for the three firms) and quality depended. It is a model slightly more general than those presented in Motta (1993) and Scarpa (1998). We show that, in contrast to Scarpa (1998), since the reaction function of the high quality firm is positively sloped with respect to the MQS, the average quality under Cournot triopolistic competition increases with the introduction of the standard. As in Scarpa (1998), consumer surplus increases and total welfare decreases.

It follows that the results presented in Valletti (2000) can be generalized for the case of three firms competition.

2.1. The unregulated equilibrium

The model presents the following assumptions:

- three identical domestic firms sell the same good, differentiated only by quality; $s_i \in \Re_+$ and $q_i \in \Re_+$ represent respectively the quality and the quantity offered by

[5] To my knowledge, in the literature there are two other examples of vertically differentiated markets with more than two firms. Donnenfeld and Weber (1992) consider a sequential entry model in which incumbents face the possibility of entry of a third firm. Frascatore (1999) presents a model in which more than three firms compete in quality and prices in a market in which the supply of quality if finite and firms obtain quality through an auction.

firm i, where $i = 1,2,3$. Let us assume without loss of generality that $s_1 > s_2 > s_3 \geq 0$;

- each consumer wants to buy one unit of the good; consumers differ only on their willingness to pay for quality, represented by the parameter $t \in [0, T]$, uniformly distributed on its support;
- the market is uncovered;
- the production cost functions[6] are given by: $F_i = b(s_i)^2$, $b > 0$;
- the surplus[7] of the generic consumer k who buys one unit of the good produced by firm i at price p_i and quality s_i is given by: $U_k^i = t_k s_i - p_i$;
- each firm's problem is to maximize the profit function: $\Pi_i(q_i, s_i) = p_i q_i - b(s_i)^2$, i=1,2,3.

Let us analyze competition in stage two first. We need to find the "marginal consumer", i.e. the consumer indifferent to buy from either of two neighbouring (in terms of quality) firms. Such consumers are given by:

$$t_1 = \frac{p_1 - p_2}{s_1 - s_2}$$
$$t_2 = \frac{p_2 - p_3}{s_2 - s_3} \qquad (1)$$
$$t_3 = \frac{p_3}{s_3}$$

where t_i, i=1,2 is the consumer indifferent to buy from firm i and i+1, and t_3 is the consumer indifferent to buy from firm 3 or not to buy at all.
Given the assumptions on consumers' preferences, demands are given by:

$$q_1 = T - t_1$$
$$q_2 = t_1 - t_2 \qquad (2)$$
$$q_3 = t_2 - t_3$$

Adding the fact that $T - t_3 = q_1 + q_2 + q_3$, following from the assumption of Cournot competition, we can find the *inverse demand functions*:

$$p_1 = s_1 T - s_1 q_1 - s_2 q_2 - s_3 q_3$$
$$p_2 = T s_2 - s_2 q_1 - s_2 q_2 - s_3 q_3 \qquad (3)$$
$$p_3 = (T - q_1 - q_2 - q_3) s_3$$

With profits given by $\Pi_i(q_i, s_i) = p_i q_i - b(s_i)^2$, i=1,2,3, the First Order Conditions, FOCs, for the simultaneous profit maximisation and resulting quantity best responses in stage two are:

[6] In Scarpa (1998) b=1/2 and T=1.
[7] The utility function is identical to the one proposed by Mussa and Rosen (1978); it presents the feature that richer consumers are more willing to pay for higher quality.

$$\frac{\partial \Pi_1}{\partial q_1} = 0 \quad \Rightarrow \quad q_1 = \frac{Ts_1 - q_2 s_2 - q_3 s_3}{2s_1}$$
$$\frac{\partial \Pi_2}{\partial q_2} = 0 \quad \Rightarrow \quad q_2 = \frac{Ts_2 - q_1 s_2 - q_3 s_3}{2s_2} \tag{4}$$
$$\frac{\partial \Pi_3}{\partial q_3} = 0 \quad \Rightarrow \quad q_3 = \frac{T - q_1 - q_2}{2}$$

It is interesting to note that only own or lower qualities are present in the expressions of the quantity best response functions in stage two. In particular, no quality variable appears in the expression of best response of the low quality firm.
The solutions of the system of the FOCs are:

$$\tilde{q}_1 = \frac{s_1 s_3 T + 2s_2^2 T - 4s_1 s_2 T}{2\left(s_1 s_3 + s_2^2 - 4s_1 s_2\right)}$$
$$\tilde{q}_2 = \frac{s_1 s_3 T - 2s_1 s_2 T}{2\left(s_1 s_3 + s_2^2 - 4s_1 s_2\right)} \tag{5}$$
$$\tilde{q}_3 = \frac{s_1 s_2 T}{2\left(-s_1 s_3 - s_2^2 + 4s_1 s_2\right)}$$

It is easy to verify that the Second Order Conditions, SOCs, are satisfied.
The equilibrium quantities are functions of all the qualities in the market, even for the low quality firm. As a result, the payoff functions in stage one are function of all qualities in the market as well.
Table 1 shows how qualities affect market shares.

$\frac{\partial t_1}{\partial s_1} = \frac{s_2^2(s_3 - 4s_2)T}{2\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} < 0$	$\frac{\partial t_1}{\partial s_2} = \frac{s_1 s_2(2s_2 - s_3)T}{\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} > 0$	$\frac{\partial t_1}{\partial s_3} = \frac{s_1 s_2^2 T}{2\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} > 0$
$\frac{\partial t_2}{\partial s_1} = -\frac{s_2^3 T}{\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} < 0$	$\frac{\partial t_2}{\partial s_2} = \frac{s_1\left(s_2^2 - s_1 s_3\right)T}{\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} > 0 \; if \; s_2 > \frac{s_1 s_3}{s_2}$	$\frac{\partial t_2}{\partial s_3} = \frac{s_1^2 s_2 T}{\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} > 0$
$\frac{\partial t_3}{\partial s_1} = \frac{s_2^3 T}{2\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} < 0$	$\frac{\partial t_3}{\partial s_2} = \frac{s_1\left(s_2^2 - s_1 s_3\right)T}{2\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} > 0 \; if \; s_2 > \frac{s_1 s_3}{s_2}$	$\frac{\partial t_3}{\partial s_3} = \frac{s_1^2 s_2 T}{2\left(s_2^2 + s_1(s_3 - 4s_2)\right)^2} > 0$

Table 1: Derivatives of marginal consumers' willingness to pay w.r.t. quality

t_3 represents also the hedonic price of the low quality firm and it increases when s_3 increases; it happens because an increase in s_3 does not affect the quantity reaction function of firm 3, but decreases the quantity of firm 2 and has negative effects on the quantity of firm 1. Firm 1, on the other hand, would increase its quantity to respond to the lower quantity of firm 2. The aggregate effect is that the quantity offered in the market is lower, implying that p_3 increases relatively[8] more than s_3.
It can be shown that the *direct* effect of an increase of s_3 on the hedonic price of firm 1 and firm 2 is negative $\left(\partial\left(p_i / s_i\right)/\partial s_3 < 0, i = 1,2\right)$.
We can now move to the analysis of first stage of the game.
Substituting the equilibrium quantities into the expression of profits, we obtain the payoff functions for the first stage of the game: quality choice. In fact, now profits are expressed simply as a function of (all) the qualities in the market.

[8] Remember that $p_3 = s_3\left(T - q_1 - q_2 - q_3\right)$.

$$
\begin{aligned}
\widetilde{\Pi}_1 &= \frac{s_1\left(2s_2\left(s_2-2s_1\right)+s_1s_3\right)^2T^2}{4\left(s_2^2+s_1\left(s_3-4s_2\right)\right)^2}-bs_1^2 \\
\widetilde{\Pi}_2 &= \frac{s_1^2s_2\left(s_3-2s_2\right)^2T^2}{4\left(s_2^2+s_1\left(s_3-4s_2\right)\right)^2}-bs_2^2 \\
\widetilde{\Pi}_3 &= \frac{s_1^2s_2^2s_3T^2}{4\left(s_2^2+s_1\left(s_3-4s_2\right)\right)^2}-bs_3^2
\end{aligned}
\tag{6}
$$

The FOCs for the profit maximisation in stage 1 are:

$$
\begin{aligned}
\frac{\partial\widetilde{\Pi}_1}{\partial s_1} = 0 \Rightarrow s_1 &= \frac{1}{2b}\left(-\frac{\left(4s_1s_2-2s_2^2-s_1s_3\right)\left(2s_s^4+s_1s_2^2\left(s_3-4s_2\right)+s_1^2\left(s_3-4s_2\right)^2\right)T^2}{4\left(s_2^2+s_1\left(s_3-4s_2\right)\right)^3}\right) \\
\frac{\partial\widetilde{\Pi}_2}{\partial s_2} = 0 \Rightarrow s_2 &= \frac{1}{2b}\left(-\frac{s_1^2\left(2s_2-s_3\right)\left(2s_2^2\left(4s_1+s_2\right)-s_2s_3\left(2s_1+3s_2\right)+s_1s_3^2\right)T^2}{4\left(s_2^2+s_1\left(s_3-4s_2\right)\right)^3}\right) \\
\frac{\partial\widetilde{\Pi}_3}{\partial s_3} = 0 \Rightarrow s_3 &= \frac{1}{8b}\left(-\frac{s_1^2s_2^2\left(s_1\left(4s_2+s_3\right)-s_2^2\right)T^2}{4\left(s_2^2+s_1\left(s_3-4s_2\right)\right)^3}\right)
\end{aligned}
\tag{7}
$$

In order to find the solutions of the system of simultaneous equations, we have to manipulate the expressions in (7). The aim is to transform the system of three equations and three unknowns into a more tractable system of two equations and two unknowns. To do so, let us define: $s_1 / s_2 \equiv \lambda > 1$ and $s_2 / s_3 \equiv \mu > 1$. Dividing the first equation in (7) by the second and the second by the third, and inserting the definitions of λ and μ, after some simplifications we obtain:

$$
\begin{aligned}
\lambda &= \frac{\left(-2\mu+\lambda\left(4\mu-1\right)\right)\left(\lambda^2\left(4\mu-1\right)^2+2\mu^2+\lambda\left(\mu-4\mu^2\right)\right)}{\lambda^2\left(2\mu-1\right)\left[\lambda-\mu\left(3+2\lambda\right)+2\mu^2\left(1+4\lambda\right)\right]} \\
\mu &= \frac{\left(2\mu-1\right)\left[\lambda-\mu\left(3+2\lambda\right)+2\mu^2\left(1+4\lambda\right)\right]}{\mu^2\left(\lambda-\mu+4\lambda\mu\right)}
\end{aligned}
\tag{8}
$$

Solving the system[9] in (8) is relatively easier and the only real solutions that are greater than 1 are $\lambda = 2.81908$ and $\mu = 3.42824$.

Inserting these two values in the expression of the qualities as functions of λ and μ, we obtain the equilibrium quality choice for the three firms:

$$
\begin{aligned}
s_1^* &= \frac{0.12610\ T^2}{b} \\
s_2^* &= \frac{0.04473\ T^2}{b} \\
s_3^* &= \frac{0.01305\ T^2}{b}
\end{aligned}
\tag{9}
$$

[9] Calculations have been performed with the software Mathematica.

The second order conditions hold locally. As noted by Lutz (2000) and Scarpa (1998) under Bertrand competition, within such a framework "leapfrogging" is not a profit improving strategy[10].
Table 2 shows the values of the unregulated equilibrium profits, demands and prices for the high, intermediate and low quality firm.

$\Pi_1^* = \dfrac{0.00931T^4}{b}$	$\Pi_2^* = \dfrac{0.00090T^4}{b}$	$\Pi_3^* = \dfrac{0.00012T^4}{b}$
$q_1^* = 0.44711\text{T}$	$q_2^* = 0.25469\ \text{T}$	$q_3^* = 0.14909\text{T}$
$p_1^* = \dfrac{0.05638\,T^3}{b}$	$p_2^* = \dfrac{0.01139\,T^3}{b}$	$p_3^* = \dfrac{0.00194\,T^3}{b}$

Table 2: Equilibrium values under Cournot triopoly

2.2. The effects of the introduction of a standard

In order to study the effects on different variables of the model of the adoption of a marginal MQS that does not change the structure of the market, we need to perform some comparative static analysis. First of all, we study the sign of the quality best responses of the three firms with respect to the quality chosen by the competitors, focusing in particular on the effects of the marginal increase in s_3 on the highest and intermediate qualities provided in the market.

The Focs in stage one can be expressed as $\partial\widetilde{\Pi}_i / \partial s_i = \partial\widetilde{R}_i / \partial s_i - 2bs_i = 0$, with $\widetilde{R}_i = \widetilde{p}_i\,\widetilde{q}_i$, revenues for firm i, i=1,2,3.
We can then derive an expression for s_i:

$$s_i = \frac{1}{2b}\left(\frac{\partial\widetilde{R}_i}{\partial s_i}\right), i = 1,2,3 \tag{10}$$

Revenues for each firm are a function of all qualities in the market and the total differential of the above expression produces therefore:

$$\frac{ds_i}{ds_j} = \frac{\dfrac{1}{2b}\left(\dfrac{\partial^2\widetilde{R}_i}{\partial s_i \partial s_z}\dfrac{ds_z}{ds_j} + \dfrac{\partial^2\widetilde{R}_i}{\partial s_i \partial s_j}\right)}{1 - \dfrac{1}{2b}\dfrac{\partial^2\widetilde{R}_i}{\partial s_i^2}}; i \neq j \neq z \tag{11}$$

If the Socs are satisfied, the denominator will be always positive and the sign of each slope will depend on the sum $\partial^2\widetilde{R}_i / \partial s_i \partial s_z ds_z / ds_j + \partial^2\widetilde{R}_i / \partial s_i \partial s_j$. Inserting into the above expressions the equilibrium qualities and solving the systems of simultaneous equations by pairs, we obtain the values of the slopes of the quality best response functions at equilibrium[11], listed in table 3.

---

[10] It can be easily proven that no firm has the incentive to leapfrog the competitors under Cournot competition with three firms. A mathematical proof can be provided upon request.
[11] Note that the slopes of the quality best response functions depends neither on b nor on T.

$\frac{ds_1}{ds_2} = 0.04216$	$\frac{ds_1}{ds_3} = 0.01209$
$\frac{ds_2}{ds_1} = \text{-}0.20666$	$\frac{ds_2}{ds_3} = \text{-}0.01012$
$\frac{ds_3}{ds_1} = \text{-}0.02558$	$\frac{ds_3}{ds_2} = -0.04878$

Table 3: Slopes of quality best response functions in equilibrium

In contrast to what Scarpa (1998) shows under Bertrand competition, in equilibrium ds_1 / ds_3 is positive, while ds_2 / ds_3 is negative, i.e. the lowest quality is treated as a strategic complement by firm 1 and as a strategic substitute by firms 2. The reason has to be found in the Cournot competition assumption: under quantity competition in stage two, the profitability to be the highest quality firm in the market still justifies an increase in the quality (and costs) after the introduction of a MQS[12] in order to partially restore the degree of differentiation. The intermediate quality firm, instead, reacts lowering its quality: the competition is not so fierce to justify an increase in quality (and costs) in order to restore the degree of differentiation between firm 2 and 3.
We can now analyze the effects of the introduction of a MQS on variable such as average quality offered, consumer surplus, profits and total welfare (defined as the summation of profits and consumer surplus).
The effects on demands and profits are summarized in table 4.

$\frac{dq_1}{ds_3} = -\frac{0.33477b}{T}$	$\frac{dq_2}{ds_3} = -\frac{1.64871b}{T}$	$\frac{dq_3}{ds_3} = \frac{0.99174b}{T}$
$\frac{d\Pi_1}{ds_3} = -0.03832\,T^2$	$\frac{d\Pi_2}{ds_3} = -0.03732T^2$	$\frac{d\Pi_3}{ds_3} = -8.34680\ 10^{-6}\ T^2$

Table 4: Effects of the introduction of a MQS on demands and profits

The demand of the low quality firm, in response to an increase in its quality, increases, even if the profits decrease: since, in equilibrium, $ds_1 / ds_3 > 0$ and $ds_2 / ds_3 < 0$ and in general $\partial R_3 / \partial s_1 < 0$, $\partial R_3 / \partial s_2 < 0$ *if* $s_2 < s_1 s_3 / s_2$, $\partial R_3 / \partial s_3 > 0$, the aggregate effect is negative. As a matter of fact, even if the differential presents a negative sign, the change in profits for the low quality firm is very close to zero, as a percentage of T^2. The decrease in profits for firm 1 and 2 is more noticeable, being about 4% of T^2 .
Average quality can be defined as:

$$\bar{s} = \frac{\sum_i s_i q_i}{\sum_i q_i}\text{, i=1,2,3} \tag{12}$$

Under unregulated Cournot competition the equilibrium average quality in the market is $\bar{s}^* = 0.08194\,T^2 / b$, slightly smaller than under Cournot duopoly, but greater than under Bertrand triopoly. In addition, the effect of the adoption of a MQS on the average quality in the market is positive, in fact:

[12] Instead, under Bertrand competition after the introduction of the standard and the consequent decrease in the degree of differentiation, the competition is so strong that the high quality firm prefers to incur fewer costs and to decrease its quality.

$$\frac{d\bar{s}}{ds_3} = 0.15297 \tag{13}$$

This result is in contrast to Scarpa (1998) and it is generated by the sign of the slope of the quality best response functions: the highest quality provided in the market increases while the intermediate quality decreases. The "perverse" effect generated by the MQS under Bertrand triopolistic competition disappears under Cournot competition .

The formula for the calculation of the consumer surplus is given by:

$$\begin{aligned} CS &= \int_{t_3}^{t_2}(ts_3 - p_3)dt + \int_{t_2}^{t_1}(ts_2 - p_2)dt + \int_{t_1}^{T}(ts_1 - p_1)dt = \\ &= \frac{t_2^2}{2}(s_3 - s_2) + \frac{t_1^2}{2}(s_2 - s_1) + t_2(p_2 - p_3) - \frac{t_3^2 s_3}{2} + p_3 t_3 + t_1(p_1 - p_2) + \frac{T^2 s_1}{2} - p_1 T \end{aligned} \tag{14}$$

Substituting the unregulated equilibrium quality values into (14), we obtain: $CS^* = 0.02066\, T^4 / b$, higher than under Cournot duopoly, but smaller than the CS under Bertrand competition with two or three firms.

Substituting the equilibrium values of qualities[13], the effect of a MQS on consumer surplus is given by:

$$\frac{dCS}{ds_3} = 0.04819\, T^2 \tag{15}$$

The introduction of the MQS increases the value of t_1, t_2, t_3; a smaller number of consumers is served in aggregate and a smaller number of consumers buys the highest quality product. However, consumers buying from each firm are better off. Some consumers that in the unregulated case bought from the intermediate quality firm, after the adoption of a MQS now buy from the low quality firm (that has increased its hedonic price): the aggregate effect is that consumer surplus for the consumers buying from the low quality firm increases. The hedonic prices of the intermediate and high quality firms decrease and their consumers are better off.

The total welfare is easily calculated as the summation of consumer surplus and profits. In equilibrium: $TW^* = 0.03099\, T^4 / b$, lower than under Bertrand competition and under Cournot duopoly and the effect of a MQS on TW is clearly negative (as in Scarpa (1998) and Valletti (2000) and in contrast to Ronnen (1991)) and given by:

$$\frac{dTW}{ds_3} = -0.02753 T^2 \tag{16}$$

In particular, the negative effects on TW are milder under Cournot triopoly than under Cournot duopoly. Under Bertrand triopoly the decrease in TW is the lowest.

While consumers are better off in all sections of the market (high, intermediate and low quality), profits decrease and more than off set the positive effects over consumers.

The results are summarized in proposition 1.

Proposition 1

if three firms choose simultaneously first qualities and then quantities, fixed costs are quality dependent and variable costs are equal to zero, the adoption of a MQS that leaves the market structure unchanged produces:

(a) low and high quality firms increases quality offered, while the intermediate quality firm decreases its quality;

(b) the average quality in the market increases;

---

[13] Note that $\frac{\partial CS}{\partial s_i} > 0, \, for \, s_1 > s_2 > s_3 \geq 0.$

(c) all firms' profits decrease;
(d) consumer surplus increases;
(e) total welfare decreases.

Proof
(a) is proved according to equation (11) and table 3; (b) is proved according to equation (13); (c) is proved according to table 4; (d) is proved according to equation (15) and (e) is proved according to equation (16).
Q.E.D.

3. Comparisons

Lemma 1 summarizes the results of the analysis under Bertrand duopolistic competition and MQS and the proof can be found in Ronnen (1991)[14]. Lemma 2 shows instead the results of the analysis under duopolistic Cournot competition as proved by Valletti (2000).

Lemma 1
if two duopolists choose simultaneously first qualities and then prices, and fixed costs are quality dependent and variable costs are equal to zero,, the adoption of a MQS that leaves the market structure unchanged produces:
-both firms increase quality offered and the average quality provided in the market increases;
-low quality firm's profits increase, while high quality firm's profits decrease;
-consumer surplus increases;
-total welfare increases.

Lemma 2
if two duopolists choose simultaneously first qualities and then quantities, and fixed costs are quality dependent and variable costs are equal to zero,, the adoption of a MQS that leaves the market structure unchanged produces:
-both firms increase quality offered and the average quality provided in the market increases;
-both firms' profits decrease;
-consumer surplus increases;
-total welfare decreases.

The results found in Scarpa (1998) are listed in table 5 and summarized in Lemma 3.

Lemma 3
if three firms choose simultaneously first qualities and then prices, and quality dependent costs are only fixed, the adoption of a MQS that leaves the market structure unchanged produces:
-low and intermediate quality firms increases quality offered, while the high quality firm decreases its quality; the average quality provided in the market decreases;
-all firms' profits decrease;
-consumer surplus increases;
-total welfare decreases.

[14] See also Lutz (2000) and Lutz et al. (2000) for, respectively, an application of the model in an international trade model and for an analysis of the effects of different timing in the setting of the standard.

$\frac{dp_3}{ds_3} = 0.077$	$\frac{dp_2}{ds_3} = -0.114$	$\frac{dp_1}{ds_3} = -0.200$
$\frac{d\Pi_3}{ds_3} = -0.081$	$\frac{d\Pi_2}{ds_3} = -0.033$	$\frac{d\Pi_1}{ds_3} = -0.116$
$\frac{dCS}{ds_3} = 0.141$	$\frac{dTW}{ds_3} = -0.007$	

Table 5: Total differentials Bertrand triopoly

As we mentioned and stressed already, average quality decreases with the standard, since the high quality firm decides to offer a lower quality. It means that a central planner would achieve under this framework a result opposite to the one he was targeting: instead of increasing the provision of quality, on average, quality decreases. This is a very serious "perverse" effect of the adoption of a MQS under triopolistic Bertrand competition. When the standard is supposed to increase the quality provided for special goods such as safety products (for example fire alarms) or to protect the environment, price competition with more than two firms generates negative results.

The following tables summarize our results and compare them with the main results of literature on MQS so far, with quality fixed costs equal to $F_i = bs_i^2$.

$s_1^* = \frac{0.12665T^2}{b}$	$s_2^* = \frac{0.02411T^2}{b}$	$q_1^* = 0.52499T$	$q_2^* = 0.2638T$
$p_1^* = \frac{0.05383T^3}{b}$	$p_2^* = \frac{0.00513T^3}{b}$	$\Pi_1^* = \frac{0.01221T^4}{b}$	$\Pi_2^* = \frac{0.00076T^4}{b}$
$\frac{d\Pi_1}{ds_2} = -0.15847T^2$	$\frac{d\Pi_2}{ds_2} = 0.0002T^2$		
$CS = \frac{0.02169T^4}{b}$	$\frac{dCS}{ds_2} = 0.2792T^2$	$TW = \frac{0.03459T^4}{b}$	$\frac{dTW}{ds_2} = 0.12096T^2$
$t_1 = 0.4754T$	$t_2 = 0.2116T$	$\bar{s} = \frac{0.09247T^2}{b}$	$\frac{d\bar{s}}{ds_2} = 0.4294$
$\frac{ds_1}{ds_2} = 0.1441$	$\frac{ds_2}{ds_1} = 0.0494$		

Table 6: Bertrand duopoly

$s_1^* = \frac{0.12597T^2}{b}$	$s_2^* = \frac{0.04511T^2}{b}$	$q_1^* = 0.4508T$	$q_2^* = 0.2747T$
$p_1^* = \frac{0.05679T^3}{b}$	$p_2^* = \frac{0.01239T^3}{b}$	$\Pi_1^* = \frac{0.00973T^4}{b}$	$\Pi_2^* = \frac{0.00136T^4}{b}$
$\frac{d\Pi_1}{ds_2} = -0.13596T^2$	$\frac{d\Pi_2}{ds_2} = -0.00021T^2$		
$CS = \frac{0.02008T^4}{b}$	$\frac{dCS}{ds_2} = 0.10335T^2$	$TW = \frac{0.03118T^4}{b}$	$\frac{dTW}{ds_2} = -0.03283T^2$
$t_1 = 0.5492T$	$t_2 = 0.2745T$	$\bar{s} = \frac{0.09536T^2}{b}$	$\frac{d\bar{s}}{ds2} = 0.31331$
$\frac{ds_1}{ds_2} = 0.0409$	$\frac{ds_2}{ds_1} = -0.2168$		

Table 7: Cournot duopoly

$s_1^* = \frac{0.12633T^2}{b}$	$s_2^* = \frac{0.02485T^2}{b}$	$s_3^* = \frac{0.00476T^2}{b}$	
$\frac{ds_1}{ds_3} = -0.0605$	$\frac{ds_2}{ds_3} = 0.2263$	$\frac{ds_1}{ds_2} = 0.133674$	
$\frac{ds_3}{ds_2} = -0.006249$	$\frac{ds_2}{ds_1} = 0.047972$	$\frac{ds_3}{ds_1} = 0.008132$	
$p_1^* = \frac{0.05301T^3}{b}$	$p_2^* = \frac{0.00456T^3}{b}$	$p_3^* = \frac{0.00044T^3}{b}$	
$q_1^* = 0.5225T$	$q_2^* = 0.2721T$	$q_3^* = 0.1136T$	
$\Pi_1^* = \frac{0.01174T^4}{b}$	$\Pi_2^* = \frac{0.00062T^4}{b}$	$\Pi_3^* = \frac{0.00003T^4}{b}$	
$\frac{d\Pi_1}{ds_3} = -0.115767T^2$	$\frac{d\Pi_2}{ds_3} = -0.032726T^2$	$\frac{d\Pi_3}{ds_3} = -0.000121T^2$	
$CS = \frac{0.02216T^4}{b}$	$\frac{dCS}{ds_3} = 0.14148T^2$	$TW = \frac{0.03455T^4}{b}$	$\frac{dTW}{ds_3} = -0.00713T^2$
$t_1 = 0.4775T$	$t_2 = 0.2054T$	$t_3 = 0.0918T$	
$\bar{s} = \frac{0.08072T^2}{b}$	$\frac{d\bar{s}}{ds_3} = -0.12387$		

Table 8: Bertrand triopoly

$s_1^* = \frac{0.12610T^2}{b}$	$s_2^* = \frac{0.04473T^2}{b}$	$s_3^* = \frac{0.01305T^2}{b}$	
$\frac{ds_1}{ds_3} = 0.0121$	$\frac{ds_2}{ds_3} = -0.0101$	$\frac{ds_1}{ds_2} = -0.042163$	
$\frac{ds_3}{ds_2} = -0.048781$	$\frac{ds_2}{ds_1} = -0.20666$	$\frac{ds_3}{ds_1} = -0.025583$	
$p_1^* = \frac{0.05638T^3}{b}$	$p_2^* = \frac{0.01139T^3}{b}$	$p_3^* = \frac{0.00194T^3}{b}$	
$q_1^* = 0.4471T$	$q_2^* = 0.2547T$	$q_3^* = 0.1491T$	
$\Pi_1^* = \frac{0.00930T^4}{b}$	$\Pi_2^* = \frac{0.00090T^4}{b}$	$\Pi_3^* = \frac{0.00012T^4}{b}$	
$\frac{d\Pi_1}{ds_3} = -0.038381T^2$	$\frac{d\Pi_2}{ds_3} = -0.037319T^2$	$\frac{d\Pi_3}{ds_3} = -8.34 \times 10^{-6}T^2$	
$CS = \frac{0.02065T^4}{b}$	$\frac{dCS}{ds_3} = 0.0482T^2$	$TW = \frac{0.03098T^4}{b}$	$\frac{dTW}{ds_3} = -0.0275T^2$
$t_1 = 0.5529T$	$t_2 = 0.2982T$	$t_3 = 0.1491T$	
$\bar{s} = \frac{0.08194T^2}{b}$	$\frac{d\bar{s}}{ds_3} = 0.90149$		

Table 9: Cournot triopoly

First, let us compare the Cournot duopoly with the Cournot triopoly. In equilibrium, when three firms compete in the market, the highest quality is higher, even if the average quality offered in the market is lower than in the duopoly case. As we would expect, profits are lower in the case with three firms because of a higher degree of competition. At the same time, the number of consumers served increases when a third firm enters the market. As a result the aggregate consumer surplus is higher under triopoly, even if the total welfare is lower, because of the negative effect on profits. What are the effects of the adoption of a MQS that does not affect the market structure? Qualitatively, Valletti (2000)'s results are robust. In both cases, consumer surplus increases and total welfare decreases; we have to notice, however, that under triopoly the consumer surplus increases marginally less than under duopoly, while the negative effect on total welfare is weaker under triopoly.

Now let us compare the Bertrand triopoly, as in Scarpa (1998), and the Cournot triopoly proposed in this paper. Under Cournot competition, in equilibrium, the highest quality offered in the market is lower, while the intermediate and low qualities are higher. Under Cournot, the high quality firm earns lower equilibrium profits than under Bertrand competition, while the other two firms' profits are higher under quantity competition. Consumer surplus and total welfare are smaller under Cournot competition; moreover, the number of consumers served decreases (t_3 is higher under Cournot competition). However, under quantity competition the average quality is higher than under price competition and the introduction of the MQS as a positive effect on the average provision only under Cournot competition. Not just under unregulated Cournot competition the average quality is higher, but also the perverse effect of the MQS showed in Scarpa (1998) disappears. Even if the sign of the effects of the MQS on consumer surplus and total welfare are identical in both kinds of competition (consumer surplus increases and welfare decreases), the size of the change differs: consumer surplus increases less and total welfare decreases more under Cournot competition. As in the duopoly case, also with the presence of an intermediate quality firm the introduction of a MQS should be avoided when firms compete in quantities; in addition, as in the duopoly case, the Bertrand competition is a better framework for the introduction of a MQS, when the main objective of a policy maker is the maximization of the total welfare.

From what we said so far, it seems that a policy maker (who does not want to affect market structure) should never introduce a MQS when more than two firms compete in the market, since total welfare decreases regardless to the kind of competition in the short run. However, there are more variable to be taken into account: total welfare decreases because of a general decrease in profits; consumers are unambiguously better off; in addition, the reason behind the introduction of a MQS is to increase the quality offered in the market. Under triopolistic Bertrand competition average quality decreases, while under Cournot competition increases. From this observation it seems that when more than two firms compete in the market, Bertrand desirability is no longer arguable. Thinking of the sector of safety goods or car makers who have to meet some fuel-economy standards for environmental reasons, a MQS under Bertrand competition would produce results opposite to those policy makers would aim to. Such a problem will not happen under Cournot competition.

In this paper we mostly considered the case of the adoption of a MQS that would not change market structure. If the central planner, instead, chooses a MQS that can drive the low quality firm completely out of the market, welfare and market structure change. We want to identify explicitly the MQS that, once adopted, creates an "innocent entry barrier" for the low quality firm. To do so, we need to substitute into the expression of the profits of the low quality firm the equilibrium values of the high and intermediate quality and, then, set the expression equal to zero and solve the equation for the quality of firm 3. Doing so[15], we obtain[16]: $s_3 = 0.02704\, T^2 / b$.

[15] There are four roots (with an imaginary part tending to zero); only one of them is greater than zero and smaller than the quality that the intermediate firm would choose in equilibrium.

[16] Scarpa (1998) under Bertrand competition with T=1 and b=1/2 obtains $s_3 = 0.01865$.

Lutz (2000) points out in a duopolistic framework under Bertrand competition that, if the MQS is set equal to level of quality that drives the low quality firm's profits to zero, the high quality firm has an incentive to take advantage of a "strategic" entry barrier: it can slightly lower its quality in order to drive the low quality firm completely out of the market[17].
While Scarpa (1998) shows that a welfare maximizing central planner would not set a standard that would change market structure with more than two firms, under Cournot competition the situation is much different.
Under Cournot monopoly, the market generates the highest welfare and quality, as shown in the following table. However, consumers are clearly worse off.

$s_m^* = \dfrac{T^2}{8b}$	$p_m = \dfrac{T^3}{16b}$	$\Pi_m = \dfrac{T^4}{64b}$
$CS = \dfrac{T^4}{64b}$	$TW = \dfrac{T^4}{32b}$	

Table 10: Cournot monopoly

Therefore, in a market with an efficient transfer system, monopoly would be the first best; otherwise, it would be necessary to estimate and weight the incidence of consumer's surplus and equilibrium profits in the definition of total welfare.
Figure 1 shows the total welfare as a function of the MQS.

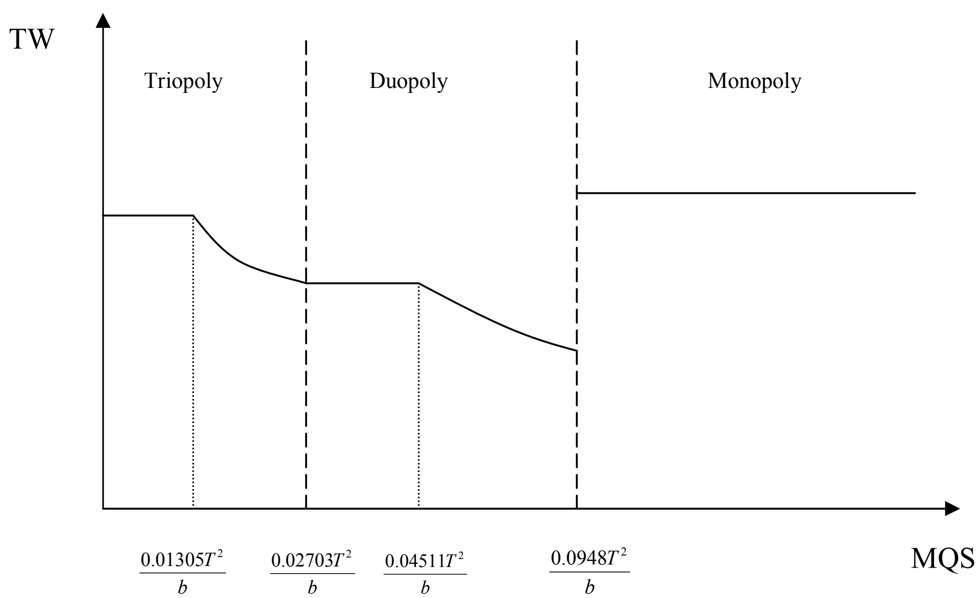

Figure 1: Total Welfare as a function of the MQS.

4. Comparisons with at least one foreign firm

When three firms compete in the market, the introduction of a MQS decreases total welfare as long as we do not consider the possibility that one or two of such firms are not domestic. The profits of the foreign firms are not considered by the central planner of the domestic country. As a matter of fact, if one or two firms are foreign, results on welfare analysis are mostly

[17] The analysis of this problem is beyond the scope of this paper. See Lutz (2000) for the analysis for the Bertrand duopoly case.

reversed: only when the low quality firm is the foreign firm the adoption of a MQS produces negative effects on welfare; otherwise, the introduction of the standard increases total welfare, both under Bertrand and Cournot competition. Clearly, the positive effects are greater under Bertrand competition: consumers surplus is unchanged (greater under Bertrand competition), but part of the negative effects on foreign firms' profits (greater under Bertrand competition) are not taken into consideration by the domestic central planner. Table 11 shows the welfare when one or two foreign firms compete in the domestic market, where $s_i^j, i = 1,2,3; j = d, f; d = domestic, f = foreign$.

One foreign firm under Bertrand competition		One foreign firm under Cournot competition	
$TW(s_1^f, s_2^d, s_3^d) = 0.0228\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.108T^2$	$TW(s_1^f, s_2^d, s_3^d) = 0.0216\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.011T^2$
$TW(s_1^d, s_2^f, s_3^d) = 0.0339\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.026T^2$	$TW(s_1^d, s_2^f, s_3^d) = 0.0300\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.010T^2$
$TW(s_1^d, s_2^d, s_3^f) = 0.0345\frac{T^4}{b}$	$\frac{dTW}{ds_3} = -0.007T^2$	$TW(s_1^d, s_2^d, s_3^f) = 0.0308\frac{T^4}{b}$	$\frac{dTW}{ds_3} = -0.027T^2$
Two foreign firms under Bertrand competition		**Two foreign firms under Cournot competition**	
$TW(s_1^d, s_2^f, s_3^f) = 0.0339\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.025T^2$	$TW(s_1^d, s_2^f, s_3^f) = 0.0299\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.010T^2$
$TW(s_1^f, s_2^d, s_3^f) = 0.0227\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.108T^2$	$TW(s_1^f, s_2^d, s_3^f) = 0.0215\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.011T^2$
$TW(s_1^f, s_2^f, s_3^d) = 0.0221\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.141T^2$	$TW(s_1^f, s_2^f, s_3^d) = 0.0207\frac{T^4}{b}$	$\frac{dTW}{ds_3} = 0.048T^2$

Table 11: Welfare when one or two foreign firms compete in the market. $s_i^j, i = 1,2,3; j = d, f; d = domestic, f = foreign$

5. Conclusions

The main objective of the paper has been to show how in a vertically differentiated market the effects of the introduction of a MQS vary depending on the market structure and on the kind of competition in the short run. While under duopolistic Bertrand competition total welfare increases after the introduction of a MQS and decreases under Cournot competition, when more than two firms enter the market Bertrand's desirability is less defendable, since total welfare and average quality provision decrease under price competition. Quantity competition, instead, is less fierce and, consequently, the highest quality firm and the intermediate quality firm react to a MQS respectively increasing and lowering the quality of their products. As a result, average quality increases after the introduction of the standard. This result is particularly important if the planner is more concerned about the provision of quality rather than total welfare, as it might be in the case of safety goods, such as fire alarms, and environmental standards. In addition, the paper has shown that that under Cournot competition a central planner would prefer a market with a monopolistic producer, while under Bertrand competition triopoly generates the highest welfare. To conclude, we also proved that when at least one of the three firms is not domestic, the results concerning welfare are mostly reversed both under Bertrand and Cournot competition.

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