A model of unionized oligopoly in general equilibrium

by

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Abstract

This paper introduces a model of unionized oligopoly in general equilibrium. The analysis shows that: (i) the industry-wide negotiated wage varies with product market features; (ii) the union effect on equilibrium employment depends upon the extent of the firm’s pass-through (from wages to prices) per unitary percentage decrease in demand; (iii) the extent of pass-through per unitary percentage decrease in demand varies with the bargaining arrangement: industry-wide centralization vs. decentralization, and with the intensity of competition: price-setting vs. quantity-setting competition (except for the case of industry-wide centralization); (iv) there is a general equilibrium relation, between union bargaining power and product market parameters, that captures the threshold value of union power at which workers would be indifferent between union wage coverage and the competitive (nonunion) outcome. The exact threshold value of union power depends upon product market features. An important aspect of the paper is the introduction of the ‘extended linear-homothetic’ preferences.

Keywords: Oligopoly, General Equilibrium, Unions, Wage Bargaining, Homothetic Preferences.

JEL Classification: D43, E24, J51.

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1. Introduction

Over the last two and a half decades it has been increasingly common to incorporate the assumption of imperfectly competitive markets in any attempt at modelling the macroeconomy. A reason for this trend is based on the observation that many industrialized economies appear to be characterized by monopoly power-endowed agents behaving strategically in the markets they are present. In order to explain macroeconomic phenomena, the models created within the literature have extensively used the monopolistically competitive general equilibrium framework developed by Blanchard and Kiyotaki (1987). A relatively unexplored path in the literature relates to the functioning of the oligopolistically competitive and unionized economy: a type of ‘theoretical’ economy, one might argue, that closely describes the actual structure of many economies in the industrial world. This vacuum in the literature would justify the search for a route to modelling the macroeconomy that is able to encompass unionized oligopoly. A further motivation for this analysis stems from the possibility of assessing, in the context of a fully microfounded general equilibrium framework, whether the predictions of the rapidly-growing literature of unionized oligopoly can be taken to the general equilibrium level; in particular, in the commonly assumed scenario of symmetric industries. In this paper, we develop a general equilibrium model of unionized oligopoly aimed at addressing these points.

One of the obvious reasons for the existence of this vacuum in the imperfectly competitive macro literature lies with the difficulty of finding suitable underlying preferences to embed oligopoly in a secure general equilibrium foundation (for a discussion, see Neary (2003b)). In an attempt to overcome such difficulty, we introduce the ‘extended linear-homothetic’ preferences to model consumer choice over specific goods. ‘Extended linear-homothetic’ preferences adopt the dual approach, more particularly, they are an extended version of the ‘linear-homothetic’ preferences developed in Datta

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2 See the survey in Dixon and Rankin (1994) and the discussion in Booth’s (1995) ch.8.
3 Blanchard and Kiyotaki’s (1987) general equilibrium model is based on monopolistic competition, à la Dixit-Stiglitz, in goods and labor markets. This implies iselastic product demand functions and constant markups. Note that their framework was not the first attempt to model imperfect competition at the macroeconomic level (see, for example, earlier work in Hart (1982)).
and Dixon (2000). The microfoundations that the ‘extended linear-homothetic’ preferences provide allow us to work with product demand functions that exhibit: (i) (perceived) linearity, (ii) variable elasticity, and (iii) cut axes. These properties open the possibility of embedding oligopoly in a tractable general equilibrium framework.4

In this paper, we integrate different aspects of oligopoly theory and the economic theory of the trade union into a general equilibrium model. Imperfect competition emerges from two sources: firms interact within differentiated oligopolies, and the labor force is unionized. Ultimately, the labor market is the recipient of the effects of these sources of imperfection, where oligopolistic competition determines the nature of labor demand schedules and unionization controls the effective supply of labor. We assume an economy composed of a large number of symmetric oligopolies. In each oligopoly, firms produce imperfect substitutes and act strategically in their market by setting prices (price-setting competition) or quantities (quantity-setting competition). Marginal costs are endogenous and entirely determined by the wage rate. Wage formation occurs in a number of independent and simultaneous right-to-manage negotiations. The model can be solved under various assumptions on the degree of centralization of the wage-setting process. We firstly focus on industry-wide centralization, where in each sector a single negotiation takes place between the sectoral union and the employers’ confederation, then we derive the solution of the model under decentralized bargaining, where in each sector there are a number of negotiations at the union-firm level. In addition, the competitive labor supply will serve as a benchmark outcome. In this theoretical framework, we explore how macroeconomic variables vary with alternative specifications governing product and labor markets.

The paper is related to the literature that analyzes unionized oligopoly in partial equilibrium (see, for example, Davidson (1988), Horn and Wolinsky (1988), Dowrick (1989), Santoni (1996) and Naylor (1998, 1999)). In particular, to the work that explores the effects of product market

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4 An alternative recent approach to formalize oligopoly in general equilibrium can be found in Neary (2003a, 2003b).
features on bargained wages under alternative bargaining structures. In a unionized homogenous $n$-firm symmetric oligopoly, Dowrick (1989) examines how exogenous shocks to product and labor market parameters affect equilibrium wages. Among a variety of results, Dowrick (1989) shows that the equilibrium bargained wage is independent of the degree of product market collusion if wage bargaining is centralized at the industry level.\(^5\) This independence property has been generalized recently in Dhillon and Petrakis (2002). Specifically, Dhillon and Petrakis (2002) provide the sufficient analytical conditions for the centralized bargained wage rate to be independent of a number of product market characteristics, including the number of firms in the symmetric oligopoly, the degree of product substitutability, and the intensity of product market competition (i.e. quantity-setting vs. price-setting). Industry-wide centralization yields an equilibrium wage that is invariant to product market features for the class of models that exhibit: (i) log-linear technology in labor input, (ii) log-linear union utility in employment and in a function of the wage, and (iii) log-linear firm’s equilibrium output and equilibrium profits in both a function of the wage and a function of the list of product market parameters. Dhillon and Petrakis (2002) illustrate that this is the case in a broad class of models widely used in both the industrial organization and the labor theory literature. From an aggregate perspective, the analysis in Dhillon and Petrakis (2002) implies that if sectors throughout the economy exhibit this constant structure - hence oligoplies may or may not share common product market features - the negotiated wage in each sector will be identical and invariant to the structure and behavior of the specific oligopoly.\(^6\) This result provides an explanation for wage rigidity in economies characterized by wage bargaining co-ordinated at the industry level: negotiated wages may be rigid to product market reform.

The analysis in this paper yields a variety of results, of which we emphasize the following. First, we find that the industry-wide negotiated wage is not invariant to product market features. In

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\(^5\) In Dowrick’s (1989) model, centralization at the industry level corresponds to fully collusive unions.

\(^6\) In addition, Dhillon and Petrakis (2002) identify the sufficient conditions upon which the centralized wage is also independent of the bargaining institution (right-to-manage bargaining vs. efficient bargaining). In decentralized (firm-level) bargaining, the independence property does not hold since the firm’s equilibrium output is not log-linear in the wage and in the vector of product market features.
particular, it varies with parameters that affect product demand elasticity and with the intensity of competition. We discuss an important factor underpinning the break down of the wage independence property established in Dhillon and Petrakis (2002): the microfounded union utility function that we derive in the model is not log-linear in employment and in a function of the wage.\footnote{The union objective function that we use is fully consistent with the consumer-worker (or household) utility function microfounded in Blanchard and Kiyotaki (1987). See, for example, Oswald (1985) and Booth (1995) for a full taxonomy of the more commonly adopted union objective functions, and see the discussion in Dhillon and Petrakis (2002) on the issue of log-linearity.} Second, equilibrium employment is determined by the real wage that the firm sets in the product market and by the effect that the union exerts on employment through the wage negotiation. We show that the latter effect is contingent upon the extent of the firm’s ‘pass-through’ (from wages to prices) per unitary percentage decrease in demand (output). We find that if wage bargaining is centralized at the industry-level, the union’s effect on equilibrium employment is invariant to the intensity of competition: the extent of pass-through per unitary percentage decrease in demand is the same whether the oligopoly competes in prices or in quantities. However, if wage bargaining is decentralized at the union-firm level, the impact of the union on equilibrium employment is larger in the presence of quantity-setting firms. The ability to pass an increase in the firm-specific negotiated wage through to price per unitary percentage decrease in demand is larger for a quantity-setting firm than for a price-setting firm. And thirdly, the model shows that there is a general equilibrium relation, between union bargaining power and product market parameters, which captures the threshold value of union power at which workers would be indifferent between the outcome delivered by industry-wide wage bargaining and the outcome delivered by the competitive labor supply. In other words, the model indicates that if union bargaining power is sufficiently low, workers are indifferent between union wage coverage and the competitive (nonunion) outcome. The exact threshold value of union power depends upon product market characteristics.

More generally, the model reproduces results that are standard to the literature of imperfectly competitive macroeconomics - namely, the inefficiency of the oligopolistically competitive equilibrium
and money neutrality. Note that the models developed by Hart (1982) and Blanchard and Kiyotaki (1987) capture the inefficiency of the imperfectly competitive equilibrium. In a monopolistically competitive economy, Blanchard and Kiyotaki (1987) show that such inefficiency is caused by an aggregate demand externality. Soskice (2000) outlines that an inefficient equilibrium in the labor market is the result of a prisoners’ dilemma type of outcome that may be present in unionized economies. Our modelling approach demonstrates that the standard results of inefficiency and neutrality can be extended to the case in which imperfect competition is of the oligopoly form.

The paper is organized as follows. Section 2 develops the model of unionized oligopoly in general equilibrium: it integrates the microfoundations provided by the ‘extended linear-homothetic’ preferences into a general equilibrium framework. We focus on symmetric equilibrium. In Section 3 we solve the model under two alternative scenarios regarding the intensity of product market competition: price-setting competition and quantity-setting competition. In addition, the competitive labor supply provides the benchmark outcome. Section 4 explores how macroeconomic variables are determined in the context of a wage bargaining process centralized at the industry level. Section 5 compares the macroeconomic outcomes of the industry-wide co-ordinated economy to the macroeconomic outcomes of the economy characterized by perfect competition in the labor market. In Section 6 we investigate the employment effects of decentralizing the wage-setting process. Finally, Section 7 concludes and indicates directions for further research.

2. A general equilibrium model with money

2.1. Description of the economy

The economy consists of $K$ sectors ($k = 1, \ldots, K$) with $n$ firms ($i = 1, \ldots, n$) in each, where $K$ is large and $n \geq 2$. Hence, $F_{ik}$ denotes firm $i$ of sector $k$ producing good $x_{ik}$. It is assumed that goods across sectors are independent and goods produced within a sector are substitutes. Sectors are assumed to be symmetric. Firms within a sector exhibit the following short-run technology: $x_{ik} = l_{ik}$, where $l_{ik}$ is firm $F_{ik}$’s labor input. Firms within sectors are assumed to be symmetric.
There is a large number of identical consumers in the economy. Each consumer demands goods from each sector and owns a fraction of every firm in the economy. Workers are identical and evenly distributed across sectors. Workers in each sector are fully unionized and divided evenly into $n$ unions, where $U_{ik}$ denotes union $i$ of sector $k$. Unions are firm-specific. Hence, $U_{ik}$ receives demand for labor from firm $F_{ik}$ and controls the supply of labor to $F_{ik}$. Labor is not mobile from union to union: neither to a union of the same sector nor to a union of any other sector.

Overall, we can talk about consumer-workers. Therefore, given $K$ large, the assumption that each consumer-worker consumes goods from all sectors implies that a firm cannot significantly influence the income of its consumers, hence firms take income as given. The underlying assumptions of the model imply that economic agents take aggregate income and aggregate price as given when making their sectoral strategic decisions.\footnote{The assumptions specified above follow, to a large extent, the Hartian tradition. Hence, from the point of view of consumer-workers, the beneficial effect derived from a wage increase exceeds any adverse effect in the form of a higher output price or lower profit. Correspondingly, from the point of view of shareholders, the beneficial effect of any profit increase exceeds any adverse effect yielded by a higher output price. The dominance of those beneficial effects are due to how consumption and shareholding of consumer-workers are spread among the $K$ sectors of the economy. Additionally, they justify the fact that firms maximize profits and unions care about total wage receipts.}

2.2. The model

The representative consumer-worker $s$ derives utility from consumption of goods and accumulation of real money balances, and disutility from work. We model consumer choice as occurring in two sequential steps. Firstly, individual $s$ allocates income between consumption and money holdings. Secondly, individual $s$ allocates consumption across specific goods. The individual delegates the labor supply decision to the union.

Since preferences are homothetic over consumption and real money balances, we can extrapolate and deal with the aggregated individual, who solves the following optimization problem:

\begin{align}
\max_{(X,M)} \ & \frac{1}{c^c(1-c)^{1-c}} X^c \left( \frac{M}{P} \right)^{1-c} - \rho N^c, \\
\text{s.t.} \ & PX + M = I
\end{align}

\footnote{\textit{The assumptions specified above follow, to a large extent, the Hartian tradition. Hence, from the point of view of consumer-workers, the beneficial effect derived from a wage increase exceeds any adverse effect in the form of a higher output price or lower profit. Correspondingly, from the point of view of shareholders, the beneficial effect of any profit increase exceeds any adverse effect yielded by a higher output price. The dominance of those beneficial effects are due to how consumption and shareholding of consumer-workers are spread among the $K$ sectors of the economy. Additionally, they justify the fact that firms maximize profits and unions care about total wage receipts.}}
where, in utility, \( X \) is total quantity demanded of the \( nK \) goods, \( M \) represents nominal money balances held by individuals, \( P \) is an aggregate price associated with consumption, and parameter \( c : 0 < c < 1 \) weights consumption and real money balances in utility. The term \( \rho N^c \) captures the disutility from supplying \( N \) units of labor (e.g. hours), where \( N \leq T \) and \( T \) is the total number of labor units available, \( \rho : \rho > 0 \) parameterizes marginal disutility from work, and \( e − 1 \) captures the elasticity of marginal disutility with respect to work, where \( e > 1 \). In the budget constraint, \( PX \) is aggregate nominal expenditure on the \( nK \) goods, and \( I \) is nominal income where:

\[
I = WL + \Omega + M^s, \quad \text{hence} \quad I \text{ is the sum of total rents from labor, } WL, \text{ total profits, } \Omega, \text{ and the initial level of nominal money balances, } M^s. \]

Following the assumptions of the model, the individual takes \( P \) and \( I \) as exogenously given. The solution to (1) constitutes the basic macroeconomic framework, where \( PX = Y = cI \) and \( M = (1-c)I \). Thus, the income-expenditure identities yield:

\[
Y = \left(\frac{c}{1-c}\right)M^s. \tag{9}
\]

In the second step, the aggregated individual optimally allocates the consumption budget \( cI \) across the \( nK \) goods produced in the economy, such that \( PX = Y = cI \) must hold. In order to solve for the optimal allocation of nominal expenditure, we model consumer choice by using the ‘extended linear-homothetic’ preferences. The preferences make use of the expenditure function approach. More particularly, they are an extended version of the ‘linear-homothetic’ preferences introduced in Datta and Dixon (2000).

Assume that the aggregated individual’s expenditure function is described by:

\[
E(p,u) = b(p)u,
\]

where \( p \in \mathbb{R}_{+}^{nK} \) is the price vector of the \( nK \) goods. The unit cost function \( b(p) : \mathbb{R}_{+}^{nK} \rightarrow \mathbb{R}_{+} \) takes the following form:

\[
b(p) = (1-\delta)\mu + \delta \Psi + \gamma [\mu - \pi], \tag{2}
\]

where \( \delta : \delta \in (0,1] \) and \( \gamma : \gamma > 0 \) are parameters of the model.\(^{10}\) Function \( b(p) \) is composed of the

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\(^{9}\) Notice that the utility function in (1) is similar to the one that is found in Blanchard and Kiyotaki (1987). The difference being that \( X \) is not a Constant-Elasticity-of-Substitution function of the \( nK \) goods, but total quantity demanded.

\(^{10}\) The first version of the preferences considered a unit cost function of the form: \( b(p) = \mu + \delta \Psi + \gamma [\mu - \pi] \). I am indebted to Huw Dixon for suggesting the normalization in (2).
following price indices:

\[
\mu = \frac{\sum_{k=1}^{K} \sum_{i=1}^{n} p_{ik}}{nK} \quad ; \quad \Psi = \frac{\sum_{k=1}^{K} \psi_k}{K} \quad ; \quad \psi_k = \left( \frac{2 \sum_{i=1}^{n} \sum_{j=1}^{n} p_{ik} p_{jk}}{n(n-1)} \right)^{\frac{1}{2}} \quad ; \quad \pi = \left( \frac{\sum_{k=1}^{K} \sum_{i=1}^{n} p_{ik}^2}{nK} \right)^{\frac{1}{2}} \quad ; \quad (3)
\]

\( \mu \) is the arithmetic average of individual prices, \( \Psi \) is the arithmetic average of sectoral-specific price indices, where \( \psi_k \) captures the interaction of prices within sector \( k \) or ‘within-sector effects’, and \( \pi \) is the variance of prices from zero. Notice that \( b(p) \) would produce Datta and Dixon’s (2000) ‘linear-homothetic’ preferences when \( \delta = 0 \), and Leontief preferences when both \( \delta = 0 \) and \( \gamma = 0 \).\(^{11}\)

Validity of the unit cost function defined by (2) and (3) is shown in Appendix A.1.\(^{12}\)

Applying Shephard’s lemma to (2) yields: \( p_{ik} x_{ik} / Y = (\partial b / \partial p_{ik})(p_{ik} / b) \equiv \alpha_{ik} \), that is the share of aggregate nominal expenditure going to good \( x_{ik} \) equals the elasticity of the unit cost function with respect to the individual price. Thus, we can derive the Marshallian demand function for the representative good:

\[
x_{ik} = \frac{Y}{b n K} \left( a + \frac{\delta}{n - 1} \sum_{j=1}^{n} \psi_k - \gamma \frac{p_{ik}}{\pi} \right), \quad (4)
\]

where \( a = 1 + \gamma - \delta \). A novel aspect introduced by the ‘extended linear-homothetic’ preferences is the presence of a sectoral-specific price index in the firm’s direct demand. The latter provides an additional channel through which the individual price may affect product demand. As a result, the price-setting firm may take into account the effect of its strategy on the sectoral-specific price index \( \psi \). In the current paper, we keep linearity in order to focus on endogenous wage bargaining. Thus, firm \( F_{ik} \) perceives (4) to be linear in own-price, \( p_{ik} \), and the individual prices of the goods produced

\(^{11}\) Leontief preferences imply that the minimum expenditure to be incurred in order to obtain an additional util is the arithmetic mean price, that is \( b(p) = \mu \). ‘Linear-homothetic’ preferences imply that the cost of an additional util would also encompass the deviation of the average of prices from their variance. ‘Extended linear-homothetic’ preferences suggest that sectoral-specific price indices also matter for the unit cost of utility.

\(^{12}\) Note that the assumed expenditure and unit cost functions may not lead to a straightforward derivation of primal utility (for referencing, see G.A. Jehle and P.J. Reny (2001)). In the case of ‘linear-homothetic’ preferences Datta and Dixon (2001) acknowledge that ‘to our best knowledge there is no explicit functional form for primal utility’. Primal utility is not necessary to the analysis. In fact, the paper finds that the adoption of the dual approach may tell us something new about, for example, product demands.
by other firms in the sector, \( \sum_{j \neq i} p_{jk} \). In other words, each firm takes \( b, \pi \) and the sectoral-specific price index \( \psi \) as given when making optimal production decisions.\(^{13}\) In (4), it is straightforward to check that the own-price elasticity and markup vary along the linear product demand schedule, where the absolute value of price elasticity is increasing in own-price. Overall, aggregate-price-taking is in the spirit of the monopolistic competition model of Dixit and Stiglitz (1977).

Due to the symmetric nature of the economy, we can focus on product demand functions that anticipate symmetry in price indices. Simplifying (4) yields:

\[
x_{ik} = \frac{y}{nK} \left( a + \frac{\delta}{(n-1)} \sum_{j \neq i}^{n} \frac{p_{jk}}{P} - \gamma \frac{p_{ik}}{P} \right),
\]

(5)

where \( \mu = \psi_1 = \psi_2 = \ldots = \psi_K = \Psi = \pi = P, b = P, \) and \( y = Y/P \) is real aggregate expenditure.

The symmetric own-price elasticity of direct product demand is given by:

\[ |b \varepsilon_{ik}| = \gamma, \] such that \(|b \varepsilon_{ik}| > 1 \leftrightarrow \gamma > 1\). Hence, \( \gamma \) characterizes elasticity in symmetric equilibrium. The symmetric cross-price elasticity is given by:

\[ b \phi_{jk}^{i} = \left( \frac{\partial x_{ik}}{\partial p_{jk}} \right) \left( \frac{p_{jk}}{x_{ik}} \right) = \frac{\delta}{(n-1)}, \] where parameter \( \delta \) captures the degree of substitutability of goods within a sector in the sense that the cross-price elasticity increases in \( \delta \). A greater \( \delta \) implies closer substitutes.\(^{14}\)

The inverse demand function for good \( x_{ik} \) is derived from (4):

\[
p_{ik} = \frac{nK}{(\gamma - \delta)} \left( a + \frac{\delta}{nK} \sum_{j \neq i}^{n} \frac{x_{jk}}{y} - \frac{(\gamma - \delta)(n-1) + \delta x_{ik}}{\gamma(n-1) + \delta y} \right),
\]

(6)

where the simplifying assumption of symmetry in price indices is introduced. Function (6) is perceived linear in own output, \( x_{ik} \), and competitors’ outputs, \( \sum_{j \neq i}^{n} x_{jk} \), by the representative firm.

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\(^{13}\) There could be various reasons why a firm would abstract from the effect of its strategy on the sectoral-specific price index. For example, the number of firms in the oligopoly might not be too small, such that the effect of an individual price on the sectoral-specific price index can be ignored.

\(^{14}\) Symmetric elasticity refers to elasticity evaluated in the symmetric solution, where \( p_{ik} = p_{jk} \forall i, j, \forall k \). Notice that an increase in \( n \) would simply replicate the price-setting (or Bertrand) economy, hence elasticity in symmetric equilibrium is invariant to \( n \). We could introduce \( n \) in elasticity by assuming: \( \gamma = \theta f(n) \), where \( \theta \) is an exogenous constant and \( f(\cdot) > 0 \). In symmetric equilibrium, the unit cost of utility would be still invariant to \( n \). However, in this paper, we leave aside the issue of the appearance of new capacity - in the form of entry in a sector (\( n \) endogenous) or, for example, an increase in the number of sectors. That is, we assume throughout the analysis \( n \) and \( K \) fixed. Finally, note that \( \gamma > \delta \), which obtains from the elasticity condition, ensures that in absolute terms the own-price effect on product demand is greater than the sum of the cross-price effects.
The symmetric own-price elasticity of (inverse) product demand is given by: 
\[ |\varepsilon_{ik}| = \frac{(\gamma - \delta)(\gamma(n - 1) + \delta)}{((\gamma - \delta)(n - 1) + \delta)^2}. \]

Having derived the product demand functions that follow from the aggregated individual’s optimal behavior, we turn to the analysis of how price and wage formation takes place in the economy. It occurs in two stages. In the first stage, nominal wages are agreed between bargaining parties, unions and firms, in a number of simultaneous and independent negotiations carried out throughout the economy. In the second stage, firms simultaneously set product market variables by optimizing profits - thus, firms set employment. The model is solved by backward induction.

The objective function of the representative firm is given by:
\[ \Omega_{ik} = (p_{ik} - w_{ik})x_{ik}. \]  

The objective function of a typical union, say \( U_{ik} \), is given by:
\[ U_{ik} = \frac{w_{ik}}{P}l_{ik} - \rho l_{ik}. \]  

In (8), we assume that the union cares about the total surplus of its representative worker, that is the (expected) real wage bill minus the disutility from supplying \( l_{ik} \) units (e.g. hours) of labor. There is equal rationing of employment among workers, hence the model is one of underemployment rather than unemployment. Expression (8) is derived from the aggregated individual’s utility function in (1), once the individual has allocated wealth between consumption and real money balances. The individual’s utility is linear in labor income, where marginal utility of real wealth equals one. Notice that expectations are rational since the expected price level is equal to the actual one. For analytical convenience the model is solved for \( e = 2 \), this implies a constant unitary elasticity of marginal disutility with respect to work.

The model can be solved under various assumptions on the degree of centralization of the wage-setting process. In the interest of brevity, the core of the paper develops the solution under the

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15 Accordingly, we obtain an implicit assumption on \( \gamma \) for inverse demand to be elastic, \(|\varepsilon_{ik}| > 1 \leftrightarrow \gamma > \gamma^C \). It is applicable when product market competition is Cournot (i.e. in quantities).
assumption of industry-wide centralization. That is, a single negotiation takes place in each sector over the industry-wide wage. The solution of the model under decentralized (firm-union) bargaining will be reported in Section 6.

In sector \( k \), the bargained nominal wage, \( w_k \), is the outcome of the maximization of the sub-game perfect Nash bargaining maximand \( B_k \):

\[
B_k = \left( \sum_{i=1}^{n} U_{ik} \right)^{\beta} \left( \sum_{i=1}^{n} \Omega_{ik} \right)^{1-\beta}.
\]

(9)

In (9), parameter \( \beta \in [0,1] \) reflects the bargaining strength of each party. The disagreement payoffs for the sectoral union and the employers’ confederation are set equal to zero. Expression (9) shows that centralization is modelled following the approach in Horn and Wolinsky (1988): the ‘centralized’ or ‘co-ordinated’ agent’s utility is the sum of utilities of its members. Note that throughout the analysis we impose, when necessary, the restriction of employment being the minimum of the labor demand and the competitive labor supply.

Next, Section 3 solves the general equilibrium model of unionized oligopoly. Two scenarios are considered regarding the intensity of product market competition that may characterize each oligopoly: price-setting and quantity-setting competition. We first solve for partial equilibrium in the representative sector \( k \), then we derive the imperfectly competitive equilibrium for the aggregate economy. The solution of the model is straightforward. Some partial equilibrium results are reported in Appendix A.2.

3. Unionized oligopoly in general equilibrium

3.1. Equilibrium with price-setting competition

Consider the scenario where firms throughout the economy compete in prices. The price elasticity
of demand for the representative good is derived from (5):  

$$|\varepsilon_{ik}^B| = \frac{\gamma |a|}{a + \frac{\delta}{(n-1)} \sum_{j \neq i} p_{jk} P - \gamma |a|^\frac{p_{ik}}{P}}.$$  

(10)

The first order condition of profit optimization, that is $p_{ik}(1 - (1/|\varepsilon_{ik}^B|)) = w_{ik}$, yields the following Bertrand-Nash price best-reply function:  

$$\frac{p_{ik}}{P} = \frac{1}{2\gamma} \left( a + \frac{\delta}{(n-1)} \sum_{j=1, j \neq i}^n p_{jk} + \frac{\gamma w_{ik}}{P} \right).$$  

(11)

The Bertrand-Nash equilibrium, where no party has an incentive to change its strategy taking into account the (re)actions of the other parties, is achieved at the following price:  

$$\frac{p_{ik}}{P} = \frac{\gamma}{(2\gamma - \delta)} \left( a + \frac{\delta}{2\gamma(n-1) + \delta} \sum_{j=1, j \neq i}^n w_{jk} + \frac{2\gamma(n-1) - \delta(n-2) w_{ik}}{2\gamma(n-1) + \delta} \right).$$  

(12)

From (5) and (12), we derive the expression for firm $F_{ik}$'s labor demand:  

$$x_{ik} = \frac{y\gamma}{nK(2\gamma - \delta)} \left( a + \frac{\delta\gamma}{2\gamma(n-1) + \delta} \sum_{j=1, j \neq i}^n w_{jk} - \frac{\gamma(2\gamma(n-1) - \delta(n-2)) - \delta^2 w_{ik}}{2\gamma(n-1) + \delta} \right).$$  

(13)

Under industry-wide centralization, the firm's labor demand simplifies to:  

$$x_{ik} = \frac{y\gamma}{nK(2\gamma - \delta)} \left( a - (\gamma - \delta) \frac{w_k}{P} \right),$$  

(14)

since $w_{ik} = w_{jk} = w_k \forall i, j \in k$.

The first order condition of the right-to-manage negotiation is obtained from the maximization of (9) with respect to the nominal wage. The representative bargaining parties anticipate expression (14). The outcome of the optimization yields the firm’s wage rule, which depends upon aggregate variables and product and labor market parameters. The expression of the wage rule is reported in Appendix A.2.

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16 Throughout the analysis, superscripts $B$ and $C$ stand for, respectively, Bertrand (or price-setting) and Cournot (or quantity-setting) outcomes; subscripts $D$ and $IW$ stand for, respectively, decentralized and industry-wide bargaining.

17 Firm $F_{ik}$’s choice variable is the nominal price $p_{ik}$, it is easy to write the outcome of the optimization in terms of the relative price $p_{ik}/P$. Note that firm $F_{ik}$’s price best-reply function is upward-sloping, this indicates that the price-setting game is played in strategic complements.
Macroeconomic equilibrium under symmetry implies that the relative price equals one. Introducing \( \frac{p_{ik}}{P} = 1 \), \( \forall i \in k \), in the first order condition of profit optimization yields the equilibrium real wage:

\[
\left( \frac{w}{P} \right)^{B^*} = 1 - \frac{1}{\gamma} \equiv 1 - \lambda^B. \tag{15}
\]

Expression (15) indicates that in macroeconomic equilibrium the bargained nominal wage \( w^{B^*} \) is below the price level \( P^{B^*} \), hence the price-setting firm keeps a constant markup given by: \( \lambda^B = 1/\left| \varepsilon^B_{ik} \right| = 1/\gamma \). From (15) and the expression of the wage rule under industry-wide centralization, we obtain the equilibrium level of output and employment in aggregate:\(^{18}\)

\[
y_{IW}^{B^*} = \frac{nK}{\rho} \left( 1 - \lambda^B - \frac{\beta}{2} \left( \frac{a}{\gamma - \delta} \right) \right). \tag{16}
\]

The equilibrium price level is derived from expression (16) and the macroeconomic income-expenditure identity \( y = \left( c/(1 - c) \right)(M^*/P) \). Similarly, by introducing the macroeconomic income-expenditure identity and the equilibrium price level in the wage rule, we obtain the equilibrium bargained nominal wage for every firm and union in the economy:

\[
w_{IW}^{B^*} = \frac{2pcM^S}{nK(1-c)} \left( \frac{(1 - \lambda^B)(\gamma - \delta)}{2(1 - \lambda^B)(\gamma - \delta) - \beta a} \right). \tag{17}
\]

Finally, by imposing symmetry of wages in (12), and by introducing the resulting expression, together with (14) and (15), in expression (7), we derive the equilibrium level of nominal profits with price-setting competition:

\[
\Omega_{ik}^{B^*} = \frac{cM^S}{nK(1-c)} \left( \frac{1}{\gamma} \right) \equiv \frac{Y}{nK}\lambda^B. \tag{18}
\]

\(^{18}\) Note that, when necessary, we apply a lower-bound limit on \( \gamma \) - i.e. \( \gamma > \gamma^B \) where \( \gamma^B > 1 \) - to exclude negative solutions. In particular, \( \gamma^B \) is obtained by setting \( y_{IW}^{B^*} = 0 \) when \( \beta = 1 \). We check whether \( \gamma^B \), when applicable, is overly restrictive. It is straightforward to show that for \( \gamma = \gamma^B \) symmetric elasticity \( \left| \varepsilon^B_{ik} \right| \in (3.341) \) and markup \( \lambda^B_{ik} = 1/\left| \varepsilon^B_{ik} \right| \in (0.33,0.29) \) when \( \delta = \{0,1\} \), respectively. In other words, when applying \( \gamma^B \), we implicitly ask product demand elasticity to be above 3 - 3.41% and the markup not to exceed 29 – 33%. Given that product demand elasticities are expected high when competition is oligopolistic, this does not seem overly restrictive.
3.2. Equilibrium with quantity-setting competition

Next, we consider the scenario where firms compete in quantities. Profit optimization yields the following Cournot-Nash quantity best-reply function:19

\[ x_{ik} = \frac{y(\gamma(n-1)+\delta)}{2((\gamma-\delta)(n-1)+\delta)} \left( \frac{a}{nK} - \frac{\delta}{\gamma(n-1)+\delta} \sum_{j=1}^{n} \frac{x_{jk}}{y} - \frac{\gamma-\delta}{nK} \frac{w_{ik}}{P} \right). \]  

(19)

The labor demand schedule with quantity-setting competition and industry-wide centralization is given by:

\[ x_{ik} = \frac{y(\gamma(n-1)+\delta)}{nK(2\gamma(n-1)-\delta(n-3))} \left( a - (\gamma-\delta) \frac{w_{ik}}{P} \right). \]  

(20)

As in the price-setting case, we subsequently derive the wage rule under industry-wide centralization. Its expression is reported in Appendix A.2.

The equilibrium real wage with quantity-setting competition takes the form:

\[ (\frac{w}{P}) C^* = 1 - \frac{(\gamma-\delta)(n-1)+\delta}{(\gamma-\delta)(\gamma(n-1)+\delta)} \equiv 1 - \lambda^C, \]  

(21)

and the quantity-setting firm keeps the following constant markup: \( \lambda^C = 1/\left| \frac{w_{ik}}{P} \right| = ((\gamma-\delta)(n-1)+\delta)/(\gamma(n-1)+\delta)).20 \) The equilibrium level of output and employment is obtained by introducing (21) in the corresponding wage rule:21

\[ y_{IW} C^* = nK \rho \left( 1 - \lambda^C - \frac{\beta}{2} \left( \frac{a}{\gamma-\delta} \right) \right). \]  

(22)

The equilibrium price level is evaluated from expression (22) and the macroeconomic income-expenditure identity. Subsequently, by introducing the macroeconomic income-expenditure identity and the equilibrium price level in the wage rule, we derive the following expression for the equilibrium bargained nominal wage with quantity-setting competition and industry-wide centralization:

\[ w_{IW} C^* = \frac{2pcMS}{nK(1-c)} \left( \frac{(1-\lambda^C)(\gamma-\delta)}{2(1-\lambda^C)(\gamma-\delta) - \beta a} \right). \]  

(23)

---

19 The quantity best-reply function is downward-sloping, this indicates that the quantity-setting game is played in strategic substitutes.

20 In symmetric equilibrium \( x_{ik} = x_{jk} \forall i, j \in k, \forall k \). The total quantity of output produced equals: \( y = nKx_{ik} \).

21 We apply a lower-bound limit on \( \gamma - i.e. \gamma > \gamma^C \) where \( \gamma^C > 3 \) to exclude negative solutions. It is straightforward to show that this lower-bound limit, when applicable, is not overly restrictive (product demand elasticity is asked to be above 3 – 3.41% and the markup not to exceed 29 – 33%).
Finally, the equilibrium level of nominal profits is obtained from expressions (6), (20), (21), and (7):

\[ \Omega^C_{ik} = \frac{cM^S}{nK(1-c)} \left( \frac{(\gamma - \delta)(n-1) + \delta}{(\gamma - \delta)(\gamma(n-1) + \delta)} \right) = \frac{Y}{nK} \lambda^C. \]  

(24)

Note that, in the absence of other distortions, money is neutral in the oligopolistically competitive and unionized economy. In other words, nominal variables are homogenous of degree one in \( M^S \) under both price-setting and quantity-setting competition. Before we analyze the macroeconomic implications of unionized oligopoly, we obtain the solution of the model in the benchmark case of a competitive labor market.

3.3. Oligopolistic competition and the competitive labor market

The benchmark scenario that we use later in the paper is established by the competitive labor supply, or labor supply in short. The representative worker maximizes \( \left( \frac{w}{P} \right) l - \rho l^e \), where \( e = 2 \), in order to choose \( l \). Thus, labor supply is given by: \( l^S = (1/2\rho)(w/P) \). Substituting the expression of the real wage in general equilibrium into the expression of the labor supply yields equilibrium employment when the labor market operates in perfect competition. This is given, with price-setting firms, by the following expression:

\[ y^B = \frac{nK}{2\rho} \left( 1 - \frac{1}{\gamma} \right) \equiv \frac{nK}{2\rho} (1 - \lambda^B). \]  

(25)

Accordingly, equilibrium employment with quantity-setting firms and a competitive labor market is given by:

\[ y^C = \frac{nK}{2\rho} \left( 1 - \frac{(\gamma - \delta)(n-1) + \delta}{(\gamma - \delta)(\gamma(n-1) + \delta)} \right) \equiv \frac{nK}{2\rho} (1 - \lambda^C). \]  

(26)

4. Aggregate effects of product and labor market features with centralized negotiations

The interesting economic question to address is how macroeconomic variables are determined in the model, whether they are solely determined by product market parameters or by the interaction of product and labor market features. We are also interested in identifying any difference that may
emerge between the predictions of this modelling approach and the predictions of the approach more commonly used in the economic theory literature of the trade union (see, for example, Booth (1995)). In this Section, we address these issues in the context of industry-wide centralization. Section 5 compares the macroeconomic outcomes of the industry-wide co-ordinated economy to the macroeconomic outcomes of the economy characterized by perfect competition in the labor market. In Section 6 we explore the employment effects of decentralizing the wage-setting process. Results are summarized in a number of Propositions; graphical representations of macroeconomic equilibrium are included.

**Proposition 1.** In general equilibrium and with industry-wide bargaining: (i) the equilibrium nominal wage decreases in the elasticity of product demand and increases in the bargaining strength of the union - this result holds with both price-setting and quantity-setting competition; (ii) the equilibrium nominal wage is higher with quantity-setting competition than with price-setting competition; (iii) the absolute values of product and labor demand elasticities are greater with price-setting competition.

**Proof.** See Appendix A.3.

In Proposition 1 we show that the negotiated wage depends upon product market features: in particular, it depends upon the parameters that may affect product demand elasticity, and upon the intensity of competition. In our setting, the wage independence property established in Dhillon and Petrakis (2002) breaks down. A key explanation rests with the microfounded union utility function used in the model. Unlike many of the union objective functions used in the labor theory literature (see Oswald (1985) and Carruth and Oswald (1987) for a full taxonomy), expression (8) is not log-linear in employment and in a function of the wage. Furthermore, the firm’s equilibrium

\footnote{Let us recall that $\gamma$ parameterizes product demand elasticity. A functional specification of the determinants of $\gamma$ (for example, of the form $\gamma = \theta f(\cdot)$) would allow an accurate exploration of the effects of certain product market characteristics, such as the number of firms in the oligopoly and the degree of product substitutability.}

\footnote{Notice that (8) is identical to the microfounded households utility function that we find in Blanchard and Kiyotaki (1987). Log-linearity only emerges for the special case of $e = 1$, which implies a zero elasticity of marginal disutility of labor. As the discussion in Blanchard and Kiyotaki (1987) suggests, the assumption of constant marginal utility of leisure is not particularly appealing. In our model, $e = 1$ would yield a flat wage}
output is not log-linear in both a function of the sectoral wage and a function of product market parameters, as expressions (14) and (20) show. The analysis seems to suggest that co-ordination of wage bargaining at the industry level may not be a source of explanation of inflexible wages, in contrast to the argument in Dhillon and Petrakis (2002). Notice that the negotiated wage varies with product and labor market features with the expected sign. The result in part (ii) of Proposition 1 is easily explained by the higher wage elasticity of labor demand under price-setting competition: it is more costly in terms of employment to negotiate higher wages in the context of a price-setting oligopoly; the latter produces a lower bargained nominal wage in equilibrium.

**Proposition 2.** In general equilibrium and with industry-wide bargaining: (i) employment and the real wage increase in the elasticity of product demand, while the price level, markup and profits decrease; (ii) employment decreases and the price level increases in the bargaining strength of the union, while the real wage, markup and profits are not altered - results (i) and (ii) hold with both price-setting and quantity-setting competition; (iii) quantity-setting competition yields a lower level of output, employment and the real wage and a higher price level, markup and profits than price-setting competition.

**Proof.** See Appendix A.3.

At a higher $\gamma$ - hence with a more elastic product demand - the individual firm perceives that a one percent increase in its price reduces its share of aggregate expenditure by a larger percentage. As a result, firms pursue a lower price (higher quantity) strategy in the product market. In the model, the aggregate real wage is determined by the product market markup that oligopolistic firms keep in equilibrium, hence the real wage is independent of labor market parameters such as $\beta$.25

Before we turn to a graphical explanation of how union power affects equilibrium employment and...
the price level, we indicate a further implication of the analysis. Notice that the size of the rents that unions can extract is contingent upon the effect that unions exert on employment. With right-to-manage bargaining, the union affects employment through the wage negotiation. The comparison of the far right-hand-side of expressions (16) and (22) show that if wage bargaining is centralized at the industry-level the union’s effect on equilibrium employment is invariant to the intensity of competition. Thus, the firm’s markup is the only factor that gauges the difference in equilibrium employment between price-setting and quantity-setting competition. Given the profit function (7), where technology is described by $x_{ik} = l_{ik}$, and given the union’s objective function (8), where $e = 2$, we can derive the expression for aggregate employment in symmetric equilibrium from the first order condition of the right-to-manage negotiation:

$$y = \frac{nK}{p} \left( \frac{w}{P} - \frac{\beta}{2} \left( 1 + \frac{\eta_p}{|\eta_l|} \right) \right),$$

(27)

where $\eta_l$ is the wage elasticity of labor demand, $\eta_p$ is the wage elasticity of the price (or the extent of pass-through from wages to prices), and the following equilibrium relation that emerges from the first order condition of product market optimization holds: $\eta_l = 1 - |\varepsilon| (1 - \eta_p)$, where $\varepsilon$ represents product demand elasticity. Equilibrium employment is determined by the real wage that the firm sets in the product market and by the effect that the union exerts on employment. Note that equilibrium employment decreases linearly in union bargaining power. As expected, a higher labor demand elasticity, which produces a lower bargained nominal wage, yields a higher employment outcome. Furthermore, in (27) we show that the impact of the union on employment can be written in terms of the ratio $\eta_p / |\eta_l|$. In equilibrium, a one percent increase in the negotiated wage induces both a certain percentage decrease in demand (output) and a certain percentage increase in the price (pass-through). The ratio $\eta_p / |\eta_l|$ captures the extent of pass-through per unitary percentage decrease in demand (output). Thus, expression (27) indicates that the larger the extent of pass-through per unitary percentage decrease in demand, the larger the total negative effect on employment induced by an increase in the negotiated wage. This is because more pass-through from wages to prices
implies that less of the wage increase is accommodated by a reduction in the firm’s markup. As a result, a higher price increase reduces output and employment by a larger quantity. When the negotiation is centralized at the industry level all competing firms in the oligopoly face identical negotiated wages and identical wage shocks, and the extent of pass-through per unitary percentage decrease in demand is the same whether the oligopoly competes in prices or in quantities.

Fig. 1 represents general equilibrium with unionized oligopoly in the \{l, w/P\}-space. The product-market-determined real wage (PRW) schedules arise from expressions (15) and (21). The labor demand schedules with price-setting (l_B) and quantity-setting (l_C) competition follow from (14) and (20), respectively. Notice that, other things equal, an exogenous increase in real aggregate expenditure pivots out and flattens labor demand under both types of product market competition. As real aggregate expenditure increases, both the bargained nominal wage and the level of employment rise, since the union faces a trade-off between employment and the wage. Thus, the relationship among real aggregate expenditure, the real wage, and employment is captured by the bargained-real-wage (BRW) schedule drawn in the \{l, w/P\}-space.26 Equilibrium employment occurs at the intersection of the product-market-determined real wage schedule and the bargained-real-wage schedule, specifically l^{B*} with price-setting competition and l^{C*} with quantity-setting competition. Any deviation from the unique equilibrium employment rate would capture a scenario of either accelerating or decelerating inflation, depending upon whether the economy is to the right or to the left of equilibrium. In this analytical framework, rational expectations ensure that no deviations are possible, hence actual employment equals equilibrium employment. Or, in other words, real aggregate demand is always at the equilibrium employment rate.

26 In the model, bargained-real-wage schedules are derived from (32) under price-setting (BRW^B) and (33) under quantity-setting (BRW^C). Their relative location in Figs. 1 and 2 is orientative. The proofs on their slopes and shapes are reported in Appendix A.4.
A feature of the bargained-real-wage schedules depicted in Fig. 1 is their shape: as real aggregate expenditure goes up, the opportunity cost of wage increases rises. The latter has a restraining effect in unions’ demands on wages, yielding the shape of the schedules. From a partial equilibrium perspective, the shape indicates that for lower levels of economic activity a negative shock to real aggregate expenditure would fall more on the bargained wage than on employment, whereas for higher levels of economic activity a negative shock to real aggregate expenditure would fall more on employment than on the bargained wage.

An increase in $\beta$ can be graphically represented as a shift upwards, for all levels of employment, of the bargained-real-wage schedules depicted in Fig. 1.\textsuperscript{27} For a given level of real aggregate expenditure - that is for a certain labor demand schedule - higher union bargaining power increases the bargained real wage. Since the bargained real wage is above the product-market-determined real wage, that is marginal cost is above marginal revenue from the firm’s point of view, firms adjust by reducing output and increasing prices. The latter is subsequently translated into an increase in the price level, a reduction in real aggregate expenditure and, hence, an inward-shifted labor demand

\textsuperscript{27} Analytically, this is derived from expressions (32) and (33).
schedule. The process continues until the new equilibrium is reached, where the new bargained-real-wage schedule intersects the unchanged product-market-determined real wage schedule. Notice that rational expectations imply an instantaneous adjustment to the new equilibrium. In dynamic terms, one could argue that in the event of a permanent shock that increases $\beta$, real aggregate expenditure falls in order to stabilize inflation at a higher rate, where equilibrium unemployment has also increased. Thus, the predictions of the model are consistent with those in earlier literature (see Layard et al. (1991)). Finally, note that an increase in worker’s marginal disutility from work (parameter $\rho$ increases) reduces equilibrium employment and increases the price level under both price-setting and quantity-setting competition.

5. Centralized negotiations and the perfectly competitive labor market

This Section explores whether the general equilibrium model of unionized oligopoly with industry-wide negotiations might reproduce the equilibrium employment outcome of the competitive labor market. Fig. 2 incorporates the competitive labor market solution: the intersection of the product-market-determined real wage schedule and the labor supply schedule yields equilibrium employment when the labor market operates in perfect competition.

Fig. 2. Competitive labor supply and the union employment outcome
From expressions (25) and (26), and from the comparison of equilibrium real wages carried out in Proposition 2, it is straightforward to conclude that $y^B > y^C$.\footnote{Note that we impose the restriction that employment is the minimum of labor supply and labor demand. In other words, an equilibrium employment outcome occurring to the right of the labor supply, as point $B''$ in Fig. 2 indicates, is not compatible with either the existence of unions in the labor markets or with the representative worker’s choice over employment. In this scenario, the restriction applies and the labor supply binds.} The analysis yields some results that we summarize next.

**Proposition 3.** In a model of unionized oligopoly in general equilibrium with industry-wide bargaining, there are combinations of parameter values that reproduce the competitive equilibrium employment outcome in the labor market. This holds with both price-setting and quantity-setting competition.

**Proof.** See Appendix A.5.

The result established in Proposition 3 obtains when product and labor market parameters meet the necessary condition for the bargained-real-wage schedule to intersect the product-market-determined real wage schedule where the labor supply does - that is, at point $B'$ with price-setting competition and at point $C'$ with quantity-setting competition in Fig. 2. We might interpret this result by stating that: in general equilibrium and given certain product market characteristics, workers are ‘indifferent’ to union wage coverage for sufficiently low values of $\beta$ - that is where the labor supply binds. Algebraically, the combinations of parameter values identified in Proposition 3 can be written as:

$$\beta^B_{IW} = \frac{(1 - \lambda^B)(\gamma - \delta)}{a};$$
$$\beta^C_{IW} = \frac{(1 - \lambda^C)(\gamma - \delta)}{a}.$$

For given product market features and for each type of product market competition, $\{\beta^B_{IW}, \beta^C_{IW}\}$ represent the threshold values of union bargaining power at which workers are indifferent, in general equilibrium, between the outcome delivered by industry-wide wage bargaining and the outcome delivered by the competitive labor supply.
Proposition 4. For certain combinations of parameter values, it holds that equilibrium employment with price-setting competition and industry-wide bargaining would be higher than equilibrium employment with quantity-setting competition and a perfectly competitive labor market. This applies for combinations of parameter values where the labor supply does not bind.

Proof. See Appendix A.5.

Graphically, Proposition 4 points out that, for certain combinations of parameter values, point \( B \) in Fig. 2 may lie in between points \( C' \) and \( B' \).

6. Centralized vs. decentralized wage negotiations

The modelling approach introduced in this paper captures labor market reform through two direct channels: the distribution of bargaining power, and the degree of centralization of the wage-setting process. Next, we evaluate the employment effects of decentralizing the wage negotiations. Under decentralized (union-firm) bargaining, the representative Nash bargaining maximand is written as: \( B_{ik} = (U_{ik})^\beta (\Omega_{ik})^{1-\beta} \). In each sector, there are \( n \) independent and simultaneous negotiations over the firm-specific wage. We derive the following expression for equilibrium employment with price-setting competition and decentralized bargaining:

\[
y_{B^*D} = \frac{nK}{\rho} \left( 1 - \lambda^B \frac{\beta}{2} \left( 1 + \frac{2\gamma(n-1) - \delta(n-2)}{\gamma(2\gamma(n-1) - \delta(n-2)) - \delta^2} \right) \right). \tag{28}
\]

Correspondingly, equilibrium employment with quantity-setting competition and firm-level bargaining is given by:

\[
y_{C^*D} = \frac{nK}{\rho} \left( 1 - \lambda^C - \frac{\beta}{2} \left( 1 + \frac{(2\gamma(n-1) + \delta)(\gamma(n-1) - \delta(n-2)) - \delta(n-1)(\gamma(n-1) - \delta(n-3))}{(\gamma - \delta)(\gamma(n-1) + \delta)(2\gamma(n-1) - \delta(n-2))} \right) \right). \tag{29}
\]

Proposition 5. A decentralized wage bargaining system yields higher equilibrium employment than industry-wide centralization. This result holds with both price-setting and quantity-setting competition.

Proof. See Appendix A.6.
A wage bargaining system centralized at the sectoral level, which emerges from the co-ordinating activities of unions and firms, reduces equilibrium employment. The labor demand elasticity effect is smaller when the negotiation occurs over the sectoral wage.\textsuperscript{29} This has the effect of increasing the bargained real wage for all levels of employment, hence reducing equilibrium employment. Thus, the model captures one of the aspects of the Calmfors and Driffill’s (1988) argument - namely, an intermediately centralized wage bargaining system produces a higher equilibrium rate of unemployment compared to a decentralized system. Finally, from the comparison of the far right-hand-side of expressions (28) and (29), we notice that the impact of the union on equilibrium employment is larger in the context of quantity-setting oligopolies. The ability to pass an increase in the firm-specific negotiated wage through to price per unitary percentage decrease in demand is larger for a quantity-setting firm than for a price-setting firm.\textsuperscript{30}

7. Concluding remarks

This paper has introduced a model of unionized oligopoly in general equilibrium. By adopting the dual approach to model consumer choice, we have developed a theoretical framework able to embed in general equilibrium the strategic interactions that may take place among firms in oligopolistic markets and firms and unions in labor markets. The aim was to explore the macroeconomic implications of the oligopolistically competitive and unionized economy.

The model produced some of the standard results that are found in the literature of imperfectly competitive macroeconomics. In particular, the inefficiency of the oligopolistically competitive equilibrium and money neutrality. It also yielded a variety of other results. Specifically, the analysis showed that: (i) the industry-wide negotiated wage varies with product market features; (ii) the union effect on equilibrium employment depends upon the extent of pass-through per unitary percentage decrease in demand (output); (iii) the extent of pass-through per unitary percentage decrease in demand (output); (iv) the extent of pass-through per unitary percentage decrease in demand (output).

\textsuperscript{29} The labor demand elasticity effect captures the extent of the employment loss induced by a wage increase. It is perceived as a (proportional) marginal cost by the union in the right-to-manage negotiation. See Booth (1995) for a detailed exposition of the first order condition of the right-to-manage model.

\textsuperscript{30} The far right-hand-side of expression (28) is smaller than the far right-hand-side of expression (29) if and only if \(-\delta^3(n - 1) < 0\), which holds for the assumed range of parameter values.
in demand varies with the bargaining arrangement, and with the intensity of competition (except for the case of industry-wide centralization); \(iv\) there is a general equilibrium relation, between union bargaining power and product market parameters, which captures the threshold value of union power at which workers would be indifferent between union wage coverage and the competitive (nonunion) outcome. The exact threshold value of union power depends upon product market features.

Regarding product and labor market deregulation, it is worth pointing out that the model captures one of the aspects of the Calmfors and Driffl’s (1988) argument: an intermediately centralized wage bargaining system produces a higher equilibrium rate of unemployment compared to a decentralized system. Furthermore, in line with the conclusion in Dhillon and Petrakis (2002), the analysis suggests that the results already obtained in the unionized oligopoly theory literature may be sensitive to the assumption of log-linear union utility (which implies constant elasticity of substitution between employment and the wage).

We hope that this framework is useful to the researcher interested in the theoretical analysis of the unionized economy. There are potential directions for further research - these include: \(i\) to analyze the impact of entry in each oligopoly; \(ii\) to evaluate the macroeconomic implications of alternative bargaining structures, such as efficient bargaining, and of ‘within-sector’ asymmetries, such as partial unionization; and \(iii\) to investigate trade-related issues, where strategic interactions among domestic and foreign economic agents occur.

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Appendix

A.1. Validity of the expenditure and unit cost functions

The domain of function \( b(p) \) is defined by \( S \equiv \{ p \in \mathbb{R}_+^{nk} : p_{ik} > 0, i = 1, ..., n, k = 1, ..., K \} \). We check that \( b(p) \) exhibits the sufficient properties: (i) Non-negative and non-decreasing in prices; (ii) Homogeneity of degree one and concavity in \( p \); (iii) Continuous differentiability. As Datta and Dixon (2000) emphasize, property (iii) is not necessary for validity but for the application of Shephard’s lemma. Given \( K \) large, \( n \geq 2, \gamma > 0 \) and \( \delta \in (0, 1] \), it is straightforward to conclude that \( b(p) \) is continuously differentiable and homogeneous of degree one.

Concavity in \( p \) is proven by checking that \( b_1(p) \) and \( b_2(p) \) are concave, where \( b_1(p) = (1 - \delta)\mu + \gamma [\mu - \pi] \) and \( b_2(p) = \delta \Psi \). Specifically, concavity of \( b_1(p) \) implies that \( \varphi b_1(\tilde{p}) + (1 - \varphi)b_1(\bar{p}) \leq b_1(\varphi \tilde{p} + (1 - \varphi)\bar{p}) \) where \( 0 < \varphi < 1 \) and \( \{\tilde{p}, \bar{p}\} \in S \). In order to assess the concavity of \( b_2(p) \) we start by checking the concavity of the representative sectoral price index \( \psi_k \), whose domain \( s_k : s_k \subset S \) is defined by \( s_k \equiv \{ p_k \in \mathbb{R}_+^n : p_{ik} > 0, i = 1, ..., n \} \). Denote \( H^{\psi_k} \) as the Hessian matrix associated to \( \psi_k \). Hence, given a \( n \)-size sector \( k \), where \( n \geq 2 \), it is straightforward to check that all principal minors of \( H^{\psi_k} \) exhibit the following signs: \( |H^{\psi_k}_m| \leq 0 \) for \( m \) odd and \( m < n \), \( |H^{\psi_k}_m| > 0 \) for \( m \) even and \( m < n \) and \( |H^{\psi_k}_n| = 0 \) for \( m = n \) (i.e. when \( H^{\psi_k}_m = |H^{\psi_k}| \)). More specifically, for \( m < n \), the leading principal minors can be expressed as follows:

\[
|H^{\psi_k}_m| = \frac{(-1)^m}{(n-1)^{m+1}n^{m+1}}\psi_k^2\left[ -n(n-1)(n-1)\psi_k^2 - \sum_{j=1}^{n} p_{jk}^2 \right]
\]

\[
(m-2) \left( \sum_{j=1}^{n} p_{jk} \right)^2 + \left( \sum_{j=1}^{n} p_{jk} \right)^2 + \left( \sum_{j=1}^{n} p_{jk} \right)^2
\]

27
\[ 2 \left( \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} + \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} + \ldots + \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} + \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} \right) \]

\[ \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} + \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} + \ldots + \sum_{j=1}^{n} p_{jk} \sum_{j=1}^{n} p_{jk} \right) \]

such that the long term in brackets in (30) equals zero for \( m = n \). Overall, we conclude that the Hessian matrix associated to \( \psi_k \) is negative semidefinite, hence \( \psi_k \) is concave. Note that the sectoral price indices - \( \{ \psi_1, \psi_2, \ldots, \psi_K \} \) - have the same functional form as \( \psi_k \) in their corresponding subset of \( S \). Thus, they are also characterized by negative semidefinite Hessian matrices whose principal minors exhibit the pattern of signs described above.

Finally, given \( \Psi = (\sum_{k=1}^{K} \psi_k) / K \), the Hessian matrix associated to \( \Psi \) is given by:

\[
H^\Psi = \begin{pmatrix}
H^{\psi_1} & 0 & 0 & \ldots & 0 \\
0 & H^{\psi_2} & 0 & \ldots & 0 \\
0 & 0 & H^{\psi_3} & \ldots & 0 \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
0 & 0 & 0 & \ldots & H^{\psi_K}
\end{pmatrix},
\]  

(31)

where the positive constant term \((1/K)\) is omitted for simplicity. The Hessian matrix \( H^\Psi \) is \( nK \times nK \), where 0 stands for a \( n \times n \) null matrix. From the structure of (31) and the analysis of \( H^{\psi_k} \) it can be shown that \( H^\Psi \) is negative semidefinite, more particularly, all its principal minors exhibit the following signs: \( |H^\Psi_m| \leq 0 \) for \( m \) odd and \( m < nk \), \( |H^\Psi_m| \geq 0 \) for \( m \) even and \( m < nk \) and \( |H^\Psi_m| = 0 \) for \( m = nk \) (i.e. where \( |H^\Psi_m| = |H^\Psi| \)). Overall, we can conclude that \( \Psi \) and, hence, \( b_2(p) \) are concave.

Given the domain defined by \( S \), property (i) is re-written such that \( b(p) \) has to be positive and non-decreasing in prices. Property (i) implies that demands are non-negative; it also implies that an additional unit of utility is costly. This property is met given the nature of firms operating in imperfectly competitive markets. Oligopolistic firms competing in substitutes will not, in general, find profitable to set a price above the choke-off price such that demands become negative. Finally, from the analysis derived above we note that the expenditure function is homothetic.
A.2. Partial equilibrium outcomes

With price-setting competition and industry-wide centralization, the wage rule is given by:

\[
\left( \frac{w_k}{P} \right)^B = \frac{(1 + \gamma - \delta)}{2(\gamma - \delta)} \left( \frac{2\rho(\gamma - \delta)(\gamma(n - 1) + \delta) y + \beta nK(2\gamma - \delta)}{\rho(\gamma - \delta)(\gamma(n - 1) + \delta) y + nK(2\gamma - \delta)} \right).
\]  

(32)

Accordingly, the wage rule when competition is in quantities is given by:

\[
\left( \frac{w_k}{P} \right)^C = \frac{(1 + \gamma - \delta)}{2(\gamma - \delta)} \left( \frac{2\rho(\gamma - \delta)(\gamma(n - 1) + \delta) y + \beta nK(2\gamma(n - 1) - \delta(n - 3))}{\rho(\gamma - \delta)(\gamma(n - 1) + \delta) y + nK(2\gamma(n - 1) - \delta(n - 3))} \right).
\]  

(33)

A.3. Comparison of aggregate variables

Equilibrium nominal wage: from the comparison of (17) and (23) we obtain that \( w^C_{IW} > w^B_{IW} \leftrightarrow \beta^2(1 + \gamma - \delta) > 0 \), which holds for \( \gamma > 1, \delta \in (0,1] \) and \( \beta \in (0,1] \). Note that \( \beta = 0 \) yields identical nominal wages - however, the labor supply binds. Real wage: from the comparison of (15) and (21) we obtain that \( (w/P)^B > (w/P)^C \leftrightarrow -\delta^2 < 0 \), which holds \( \forall \delta : \delta \in (0,1] \). The latter, together with the first order condition of profit optimization, yields the conclusion on the markups - such that \( \lambda^C > \lambda^B \) - and the conclusion on the product (and labor) demand elasticities. Output and employment: we compare (16) and (22). Since \( \lambda^C > \lambda^B \) it holds that \( y^B_{IW} > y^C_{IW} \). The result on the price level is derived from the output comparison together with the macroeconomic income-expenditure identity: \( y = (c/(1-c))(M^*/P) \). Nominal profits: we compare (18) and (24). Since \( \lambda^C > \lambda^B \) it holds that \( \Omega^C_{ik} > \Omega^B_{ik} \).

Sensitivity of aggregate variables to product demand elasticity

With price-setting competition: in expression (15) we note that the real wage increases in \( \gamma \). Hence, the markup decreases in \( \gamma \). The derivative of equilibrium output with respect to \( \gamma \) is given by:

\[
\frac{\partial y^B_{IW}}{\partial \gamma} = \frac{nK}{\rho} \left( \frac{1}{\gamma^2} + \frac{\beta}{2(\gamma - \delta)^2} \right),
\]

which is positive for \( n \geq 2, \gamma > 1, \delta \in (0,1], \rho > 0 \) and \( \beta \in [0,1] \). The result on equilibrium output, together with the macroeconomic income-expenditure identity, yields the sign of the effect of \( \gamma \) on
the price level. Nominal profits fall as $\gamma$ increases, since $\lambda^B$ decreases in $\gamma$. The derivative of the equilibrium bargained wage with respect to $\gamma$ is given by:

$$\frac{\partial w_B^*}{\partial \gamma} = \frac{2\rho c M^S}{nK(1-c)} \left( \frac{-\beta (2\gamma(\gamma - \delta) - \delta(1-\delta))}{(2\gamma(1 - \lambda^B)(\gamma - \delta) - \beta \gamma a)^2} \right),$$

which is negative for $n \geq 2$, $\gamma > 1$, $\delta \in (0, 1]$, $\rho > 0$ and $\beta \in [0, 1]$.

With quantity-setting competition: from expression (21) we obtain that the markup decreases in $\gamma$ if and only if

$$\frac{\partial \lambda^C}{\partial \gamma} = -\frac{1}{n} \left( \frac{1}{(\gamma - \delta)^2} + \frac{(n-1)^3}{(\gamma(n-1)+\delta)^2} \right) < 0,$$

which holds for $n \geq 2$, $\gamma > 1$ and $\delta \in (0, 1]$. Accordingly, the real wage increases in $\gamma$. The derivative of equilibrium output with respect to $\gamma$ is given by:

$$\frac{\partial y_C^*}{\partial \gamma} = \frac{nK}{\rho} \left( \frac{-\beta F}{2(\gamma - \delta)^2} + \frac{\beta}{2(\gamma - \delta)^2} \right),$$

which is positive for $n \geq 2$, $\gamma > 1$, $\delta \in (0, 1]$, $\rho > 0$ and $\beta \in [0, 1]$. Once again, the result on equilibrium output together with the macroeconomic income-expenditure identity yields the sign of the effect of $\gamma$ on the price level. Nominal profits fall as $\gamma$ increases, since $\lambda^C$ decreases in $\gamma$. The equilibrium bargained wage decreases in $\gamma$ if and only if

$$\frac{\partial w^*_C}{\partial \gamma} = \frac{2\rho c M^S}{nK(1-c)} \left( \frac{-\beta F}{((\gamma(n-1)+\delta)(2(1 - \lambda^C)(\gamma - \delta) - \beta a)^2} \right) < 0,$$

which holds since $F \equiv (2\gamma^2 - \delta)(n-1)^2 - \delta(2\gamma(n-1)(n-3) - \delta(n^2 - 3n + 4)) > 0$ for $n \geq 2$, $\gamma > 1$ and $\delta \in (0, 1]$. Finally, note that the sensitivity of aggregate variables to union bargaining strength is easily inferred from the expressions in general equilibrium.

**A.4. The slope and shape of the bargained-real-wage schedules**

From expressions (32) and (33), we evaluate the derivative of the corresponding wage rule with respect to real aggregate expenditure:

$$\frac{\partial \left( \frac{w_B^*}{P} \right)}{\partial y} = \frac{nK \rho \gamma(1 + \gamma - \delta)(2\gamma - \delta)(2\beta - \beta)}{2(\rho \gamma(\gamma - \delta)y + nK(2\gamma - \delta))^2};$$

$$\frac{\partial \left( \frac{w_C^*}{P} \right)}{\partial y} = \frac{nK \rho(1 + \gamma - \delta)(\gamma(n-1) + \delta)(2\gamma(n-1) - \delta(n-3))(2\beta - \beta)}{2(\rho \gamma(\gamma - \delta)y + nK(2\gamma(n-1) - \delta(n-3)))^2};$$

30
which are strictly positive for \( n \geq 2, \gamma > 1, \delta \in (0,1], \rho > 0 \) and \( \beta \in [0,1] \). It is straightforward to conclude that the second order derivatives with respect to \( y \) are strictly negative, yielding the shape of the \( BRW \) schedules.

**A.5. Showing that \( l^B = l^B_{IW} \) for certain combinations of parameter values**

We compare (16) and (25) - divided by \( nK \): \( l^B = l^B_{IW} \) requires \( (1 - \lambda B)(\gamma - \delta) - \beta(1 + \gamma - \delta) = 0 \). This holds for the following combinations of \( \{\gamma, \delta, \beta\} \) values: \( \beta_{IW}^B = ((1 - \lambda B)(\gamma - \delta))/(1 + \gamma - \delta) \), where \( 0 < \beta_{IW}^B < 1 \) for \( \gamma > 1 \) and \( \delta \in (0,1] \). Thus, it is straightforward to conclude that, given \( \{\gamma, \delta\} \), \( \beta = \beta_{IW}^B \) yields \( l^B = l^B_{IW} \), any \( \beta > \beta_{IW}^B \) yields \( l^B > l^B_{IW} \), and any \( \beta < \beta_{IW}^B \) yields \( l^B < l^B_{IW} \).

In the latter scenario, the restriction applies and the labor supply binds.

**Showing that \( l^C = l^C_{IW} \) for certain combinations of parameter values**

We compare (22) and (26) - divided by \( nK \): \( l^C = l^C_{IW} \) requires \( (1 - \lambda C)(\gamma - \delta) - \beta(1 + \gamma - \delta) = 0 \). This holds for the following combinations of \( \{\gamma, \delta, n, \beta\} \) values: \( \beta_{IW}^C = ((1 - \lambda C)(\gamma - \delta))/(1 + \gamma - \delta) \), where \( 0 < \beta_{IW}^C < 1 \) for \( n \geq 2, \gamma > 1 \) and \( \delta \in (0,1] \). Thus, it is straightforward to conclude that, given \( \{\gamma, \delta, n\} \), \( \beta = \beta_{IW}^C \) yields \( l^C = l^C_{IW} \), any \( \beta > \beta_{IW}^C \) yields \( l^C > l^C_{IW} \), and any \( \beta < \beta_{IW}^C \) yields \( l^C < l^C_{IW} \). In the latter scenario, the restriction applies and the labor supply binds.

**Showing that \( l^B_{IW} > l^C \) for certain combinations of parameter values**

We compare (16) and (26) - divided by \( nK \): \( l^B_{IW} = l^C \) implies that \( (1 + \lambda C - 2\lambda B)(\gamma - \delta) - \beta(1 + \gamma - \delta) = 0 \), which holds for the following combinations of \( \{\gamma, \delta, n, \beta\} \) values:

\[
\beta_{IW}^C = \frac{(1 + \lambda C - 2\lambda B)(\gamma - \delta)}{1 + \gamma - \delta},
\]

where \( 0 < \beta_{IW}^C < 1 \) for \( n \geq 2, \gamma > 1 \) and \( \delta \in (0,1] \). This implies that, given \( \{\gamma, \delta, n\} \), \( \beta = \beta_{IW}^C \) yields \( l^B_{IW} = l^C \). We check that for those combinations of parameter values captured by \( \beta = \beta_{IW}^C \), where \( l^B_{IW} = l^C \), the labor supply does not bind. This requires to show that \( \beta_{IW}^C > \beta_{IW}^B \). Specifically, we find that \( \beta_{IW}^C > \beta_{IW}^B \iff \delta^2 > 0 \), which holds \( \forall \delta \). Overall, it is straightforward to conclude that, given \( \{\gamma, \delta, n, \beta\} \), \( \beta_{IW}^C < \beta \leq 1 \) yields \( l^B_{IW} < l^C \), \( \beta = \beta_{IW}^C \) yields \( l^B_{IW} = l^C \), and \( \beta_{IW}^B < \beta < \beta_{IW}^C \) yields \( l^B_{IW} > l^C \).
A.6. Proof of Proposition 5

With price-setting competition: from the comparison of the far-right-hand-side of expressions (16) and (28), we obtain that $y_{B^*}^D > y_{IW}^B$ if and only if $\delta(n-1)(2\gamma - \delta) > 0$, which holds for $n \geq 2$, $\gamma > 1$ and $\delta \in (0, 1]$. With quantity-setting competition: from the comparison of the far-right-hand-side of expressions (22) and (29), we obtain that $y_{C^*}^D > y_{IW}^C$ if and only if $\delta(n-1)(2\gamma(n-1) - \delta(n-3)) > 0$, which holds for $n \geq 2$, $\gamma > 1$ and $\delta \in (0, 1]$.

References


