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Labor Market Pooling, Outsourcing and Contracts in Chamberlinian Cities

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Abstract

Economic regions, such as urban agglomerations, face external demand and price shocks that produce income risk. Workers in large and diversified agglomerations may benefit from reduced wage volatility, while firms may outsource the production of intermediate goods and realize benefits from Chamberlinian externalities. Firms may also protect workers from wage risks through fixed wage contracts. This paper explores the relationships between firms' risks, workers' contracts, and the structure of production in cities.

Keywords: Labor market, Labor contracts, Chamberlinian Externalities.

1 Introduction

As recognized long ago by Arrow and Debreu, efficient resource allocation in a world of uncertainty requires a "complete" set of markets within which risks can pooled and traded. With incomplete markets, risks may fall on agents for whom risk-bearing is costly. Economic institutions – not just financial

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institutions like banks and insurance companies, but all firms, and all contractual relationships – transmit and distribute fundamental risks. These institutional structures adapt over time to the changing fundamental structure of the economy. In this paper, we explore the transmission, pooling, relationships between industry structure, labor market contracting, and earnings risk using a model of an urban agglomeration or other spatially-integrated economic region. Industries within this region, which export products to the rest of the economy, face stochastic demands for their products, producing output and income shocks that affect the region. We investigate the simultaneous determination of industrial structure, of employment relationships, and of earnings risk, under varying assumptions about the ability of firms (or workers) to access financial markets within which risks can be traded.

More specifically, as in Ethier (1982), assume a set of firms in an urban area or economic region operate under conditions of increasing returns to scale. These firms – henceforth called "manufacturing firms", for convenience – produce tradeable goods, using non-traded intermediate goods and services called "components". These intermediate commodities may be produced within the manufacturing firms themselves (a vertically-integrated industrial structure) or by specialized component producers ("outsourcing"). Stochastic demand for traded goods implies that manufacturing outputs, prices, and profits are also random, as are derived demands for intermediate and primary inputs. In particular, external demand shocks may result in stochastic fluctuations in the wages received by risk-averse workers – depending on the nature of labor contracting, and in particular, on whether workers are hired at fixed wages in advance of the realization of external demand shocks or are instead hired (and fired) at wages that depend on these realizations. The equilibrium structure of firms and of employment contracts simultaneously determine the ultimate distribution of income risks among firms and workers within the region.

As will become apparent, a wide range of equilibrium configurations is possible, depending on the underlying fundamental economic data of the regional economy. The analysis begins, in Section 2, with the specification of a basic model of regional production and employment reflecting indivisibilities at the level of export firms and increasing returns in the production of intermediate components as well. Section 3 proceeds under the assumption that employers cannot in any way contract with workers in advance of the realization of demand shocks and shows that equilibria can emerge with fully integrated manufacturing firms, manufacturing firms that outsource all production of components, or a mix of the two, depending on the number of manufacturing firms in the region. The extent of wage income risk faced by workers in equilibrium also depends on the number of manufacturing firms and on the degree of "diversification" of the region's industrial structure. Section 4 allows for the possibility that firms may be able to absorb some of the income risk otherwise borne by workers by contracting with them *ex ante*. Such contracts impose costs on firms, in part by limiting their ability to adapt employment to realized demand. Whether such contracts are found in equilibrium depends on tradeoffs between these costs and their benefits to risk-averse workers: a range of equilibrium configurations is possible.

Although the model is, of necessity, highly stylized, it offers insights into the ways that industrial structure and the distribution of earnings, profit, and employment risk are simultaneously determined. We do not attempt to link the analysis closely to empirical findings on urban and regional production, employment, earnings, and profits, but note that different economic regions, in different parts of the world and at different historical periods, could fruitfully be compared using the structural determinants identified below. For example, the process of urbanization and industrialization in advanced and developing countries is often accompanied by increases in the number and diversity of major export-oriented industries and by the development of financial markets within which the profit risks facing firm owners can be pooled, both of which, our analysis suggests, can affect the equilibrium degree of vertical integration and of labor contracting.

Related literature: A number of previous contributions highlight the importance of "thick" labor markets skills, qualification or training (see Helsley and Strange, 1990; Brueckner, Thisse and Zenou, 2002; or Hamilton and Thisse 2000). Picard and Toulemonde (2004) show that firms and workers can increase the chance of good job matches in large cities; Ellison and Fudenberg (2003) explore this issue with integer numbers of firms. In Krugman (1991, Appendix) firms are uncertain about their costs and tend agglomerate to reduce wage variability and the risk premium paid to workers. Using a similar model but with no endogenous agglomeration, Wildasin (1995) examines the implications of integrated or pooled labor markets for the distribution of income risk and for the benefits and costs of tax/transfer policies that protect against such risks. These analyses ignore the possibility, explored in Section 4 below, that firms may explicitly or implicitly (Baily (1974), Azariadis (1975), Rosen (1985)) insure workers against earnings risk.

The analysis extends previous research on diversity and specialization.

Abdel-Rahman and Fujita (1993) analyze specialized cities (with a single industry) versus diversified cities (with two industries). Duranton and Puga (2001) discuss the migration process of firms from diversified Chamberlinian cities to specialized factory towns. Many recent studies of economic geography postulate the existence of Chamberlinian cities where final good producers outsource production of intermediate goods to small, diversified, monopolistically competitive firms (see for instance Venables 1996) but take city structure as exogenous in the sense that downstream firms never integrate vertically to incorporate upstream production. As shown in Section 3, vertically integrated industries may but need not always arise when industry structure is determined endogenously.

2 The model

Production structure. We model a city with a predetermined number M of firms, each of which is the sole producer of a commodity that is sold on external markets. These firms form the "economic base" of the city (Tiebout (1962)). Each firm j, $j = 1, \ldots M$, faces a demand for its product given by $Y_j = \theta_j P_j^{-\varepsilon}$, where P_j is the price of the commodity, $\varepsilon > 1$ is the price elasticity of demand, and $\theta_j \in [\underline{\theta}, \overline{\theta}]$, $0 < \underline{\theta} < \overline{\theta} < \infty$, is a firm-specific random variable that reflects demand shocks on external markets. Variations in θ_j may result from fluctuations in incomes, preferences, technologies, and prices of related goods in the rest of the world. With little loss of generality, we assume that $E(\theta_j) = 1 \quad \forall j$, and thus the variance and covariance of these shocks are $\mathrm{var}[\theta_j] = E\left(\theta_j - 1\right)^2$ and $\mathrm{cov}[\theta_j, \theta_k]_{j \neq k} = E\left(\theta_j - 1\right)\left(\theta_k - 1\right)$. In some instances, we assume that these random shocks are i.i.d., but we also allow for them to be correlated, including the possibility that they are perfectly correlated. A low degree of correlation of these shocks can be interpreted as a situation where the city's economic base is highly "diversified", and, as we shall see, this sometimes allows for important pooling of risks; this is not possible when the demand shocks are perfectly correlated. We let $\theta \equiv (\theta_1, \ldots, \theta_M)$ denote the vector of random shocks.

Each of the M firms uses various components in the production process; for ease of reference, we henceforth refer to each of these firms as a "manufacturing" firm. As in Ethier (1982), we can think of the production activities of these firms as an "assembly" process in which components are combined to yield a final product, according to a CES production function

$Y_j = [\int_0^{N_j} x_j(i)^\rho di]^{1/\rho}$, where $x_j(i)$ is the quantity of component i in the final product j; here, $\rho < 1$ and the elasticity of substitution between components is $1/(1-\rho)$. [1] N_j represents the total range of components used by firm j. The production of each component requires $l_j(i) = a + bx_j(i)$ units of labor, where $a > 0$ and $b > 0$ are parameters that represent fixed and variable input requirements; components are thus produced under conditions of increasing returns to scale. Letting w_j denote the wage rate paid by firm j, the production of components thus costs $c_j(i) = (a + bx_j(i))\, w_j$.

In order to insure that each firm's profit and output is bounded, we assume that

$$\delta \equiv \rho - 1 + 1/\varepsilon > 0.$$

This implies that $\rho > 0$ (components are substitutes). Note that $\delta < \rho$ and $\delta < \rho/\varepsilon < 1/\varepsilon$.

While the manufacturing firms are the locus of "assembly" activities, they can produce components internally, in which case we refer to them as "integrated" manufacturers, or they may obtain components from specialized local component producers, in which case we refer to them as "outsourcing" manufacturers. The local component producers utilize the same increasing-returns production technologies as are available to the manufacturing firms. In this case, we obtain an industrial structure characterized by Chamberlinian monopolistic competition in the components sector of the city's economy. One goal of the analysis is to determine endogenously which of these two structures emerges in equilibrium.

Workers. There is a fixed stock L of identical workers in the city, each supplying a single unit of labor. Each worker's utility is a concave function of the wage w; in some cases, we focus on the constant-elasticity form $u(w) = w^\alpha/\alpha$ for $\alpha \leq 1$. (w^α/α reduces to $ln(w)$ when $\alpha = 0$.) In this case, the coefficient of relative risk aversion is $1 - \alpha$.

Labor Markets. We assume that whereas firms have market power in the product and component markets, firms and workers are price and (expected) utility takers in the labor market. This price-taking condition naturally holds

[1] At the cost of notational complexity, it would be possible to incorporate other inputs in the production process, but this would not affect the results of the analysis in any important way.

when the number of firms is large. When the number of firms is small, we keep this assumption for the sake of consistency and tractability.

We allow for two possible types of employment relationships. In the first, firms hire workers *ex post* (i.e., subsequent to the realization of demand shocks) in single competitive market at a wage rate of w. In the second, firms may contract with workers *ex ante* to supply their labor at a fixed (non-state-contingent) wage w'. For the sake of analytical tractability, we assume that contracts are exclusive and non-renegotiable: on the one hand, firms are not allowed to hire additional workers *ex post*, to fire contracted workers, or to reduce the contracted wage payment; on the other hand, contracted workers are obliged to offer their labor, cannot quit the firm or renegotiate higher wages. In contrast to Schöb and Wildasin (2002), wage contracts not only impose constraints on labor price but also on the size of a contracting firm. Also, in contrast to Duranton and Combes (2001) a firm is not allowed to 'poach' workers from other firms by offering higher *ex post* wages.[2] In Section 3, we restrict attention to the case where hiring only occurs *ex post*; later, we allow for *ex ante* contracting and for the endogenous determination of contracting form.

3 Chamberlinian Externalities

To understand the incentives to outsource, we compare the profits of manufacturing firms under integration and under outsourcing.

3.1 Integrated Manufacturers

Let M_I denote the set (and the number) of integrated manufacturing firms. Each firm $j \in M_I$ chooses the number of components, hires workers in a perfectly competitive labor market, sets its product price and, finally, supplies its product market.

By symmetry, firm j produces identical quantities of components: $x_j(i) \equiv x_j$ for all i. The production function then simplifies to $Y_j = x_j N_j^{1/\rho}$ and the firm's production cost is $(aN_j + bY_j N_j^{\frac{\rho-1}{\rho}})w$, a convex function of N_j. For any desired level of output Y_j, the cost-minimizing numbers of each components

[2]Benefits from larger component variety in cities are not considered in Schöb and Wildasin (2002) and Combes and Duranton (2001).

and of component varieties are given by

$$x_j = \overline{x} \equiv \frac{a}{b}\frac{\rho}{1-\rho} \quad \text{and} \quad N_j = (Y_j/\overline{x})^{\rho}. \tag{1}$$

Note that changes in output affect only the number of varieties of components, not the amount of each that is used. The optimal labor requirement and the indirect cost function are

$$l_j(Y_j) = cY_j^{\rho} \quad \text{and} \quad c_j(Y_j) = cwY_j^{\rho}. \tag{2}$$

where

$$c \equiv \left(\frac{b}{\rho}\right)^{\rho}\left(\frac{a}{1-\rho}\right)^{1-\rho} = \frac{b}{\rho}\overline{x}^{1-\rho}.$$

Cost is a sub-additive function of output. Economies of scope are internalized in the integrated firm so that average costs decrease with output and are larger than marginal cost. Furthermore, both marginal and average costs decrease with output at the (negative) rate $\rho - 1$ while inverse demand and marginal revenue functions decrease at the (negative) rate $-1/\varepsilon$. Under the assumption $\delta \equiv \rho - 1 + 1/\varepsilon > 0$, interior solutions exist and yield maxima for the profit functions $\Pi_j = (\theta_j I)^{1/\varepsilon} Y_j^{1-1/\varepsilon} - cwY_j^{\rho}$. Equilibrium output and profit is then given by

$$\begin{aligned} Y_j^{\delta} &= \theta_j^{1/\varepsilon}\frac{1}{w}\frac{\varepsilon-1}{c\rho\varepsilon} \\ \Pi_j^I &= \theta_j^{\frac{\rho}{\delta\varepsilon}}\left(w\frac{c\rho\varepsilon}{\varepsilon-1}\right)^{-(\frac{\rho}{\delta}-1)}\frac{\delta}{\rho} \end{aligned} \tag{3}$$

where $\frac{\rho}{\delta} > 1$. When wages rise, the equilibrium output Y_j decreases. When the firm faces a positive demand shock (i.e., a high value of θ_j), it increases its output and decreases its product price. Since the rise in output reduces the average fixed cost of a component, the firm is therefore able to increase the number of components. In turn, a larger variety of components raises production efficiency, which decreases marginal cost and price. Finally, profit decreases with wages.

Labor demand by all integrated firms is given by

$$L_I(w, M_I, \Psi_I) = \sum_{j \in M_I} l_j(Y_j) = c\left(\frac{1}{w}\frac{\varepsilon-1}{c\rho\varepsilon}\right)^{\frac{\rho}{\delta}} M_I(\Psi_I)^{\frac{\rho}{\delta\varepsilon}} \tag{4}$$

where

$$\Psi_I \equiv [(1/M_I) \sum_{j \in M_I} \theta_j^{\frac{\rho}{\delta\varepsilon}}]^{\frac{\delta\varepsilon}{\rho}}$$

is a measure of the impact of integrated firms' shocks on their total demand for labor. This is a increasing and jointly convex function of θ because $\delta\varepsilon < \rho$.

3.2 Outsourcing Firms

Let M_o denote the set (and number) of outsourcing manufacturers. These firms outsource their production of all components to a set of Chamberlinian firms, each producing a single component variety. Outsourcing manufacturers are then just assembly line firms that use components produced by others in the city.

The component sector is subject to Chamberlinian monopolistic competition. Each component maker $i \in N$ produces a single component and sets a profit-maximizing price $p(i)$, conditional on the realization of all demand shocks for outsourcing manufacturers. Free entry insures that profits fall to zero at the equilibrium number of component producers N.

The demand for a component is given by the sum of the outsourcing manufacturers' demands. Each outsourcing firm maximizes its profit $Y_j P_j - \int_0^N p(i) x_j(i) di$. Profits are concave under the assumption that $\delta > 0$. The first-order condition with respect to $x_j(i)$ implies that manufacturer j's demand for component i is given by

$$\frac{x_j(i)}{Y_j} = \left(\frac{p(i)}{\mathsf{p}} \right)^{\frac{1}{\rho-1}}$$

where

$$\mathsf{p} \equiv [\int_0^N p(i)^{\frac{\rho}{\rho-1}} di]^{\frac{\rho-1}{\rho}} \tag{5}$$

is the component price index in the city.

Manufacturing firms' profits can be written as function of product prices and component price index as $\Pi_j = (p_j - \mathsf{p}) Y_j$. The production and profit of each outsourcing manufacturing firm is computed as

$$Y_j = \theta_j \xi \mathsf{p}^{-\varepsilon} \quad \text{and} \quad \Pi_j = \frac{1}{\varepsilon - 1} \mathsf{p} Y_j.$$

where $\xi \equiv (\varepsilon - 1)^{\varepsilon}\varepsilon^{-\varepsilon}$. Aggregating across manufacturers, component producer i faces an iso-elastic demand function

$$x(i) \equiv \sum_{j \in M_o} x_j(i) = \left(\frac{p(i)}{\mathsf{p}}\right)^{\frac{1}{\rho-1}} \sum_{j \in M_o} Y_j = \left(\frac{p(i)}{\mathsf{p}}\right)^{\frac{1}{\rho-1}} \mathsf{p}^{-\varepsilon}\xi M_o \Psi_o \qquad (6)$$

where

$$\Psi_o \equiv \frac{1}{M_o} \sum_{j \in M_o} \theta_j$$

is a measure of the average demand shock realized by the outsourcing manufacturers.

The supply of components is derived as follows. Each component producer i manufactures a single variety of component and hires workers in the labor market, where it is price taker. It thus maximizes $p(i)x(i) - (a + bx(i))\,w$, treating the wage and the price index p as constants. Because of iso-elastic demand, the optimal price is a constant markup on marginal cost equal to $p(i) = bw/\rho$. Furthermore, free entry requires that profits of component producers are equal to zero. This implies that $x(i) = (a/b)(\rho/(1-\rho)) = \overline{x}$. Note that the optimal level of component production turns out to be equal to that chosen by integrated firms.[3]

Applying these results firstly to (6) yields

$$\mathsf{p}^{\varepsilon\delta} = \frac{wb}{\rho}\left(\frac{\xi M_o \Psi_o}{\overline{x}}\right)^{\rho-1}. \qquad (7)$$

In equilibrium, the price index of components is directly related to *ex post* wages. Using this last expression and (5), we can determine the equilibrium number of component firms:

$$N_o(w, M_o, \Psi_o) = \left(\frac{wb}{\rho}\right)^{-\frac{\rho}{\delta}} \left(\frac{M_o \Psi_o}{\overline{x}}\right)^{\frac{\rho}{\delta\varepsilon}}. \qquad (8)$$

The outsourcing manufacturer's output and profit are then

$$\begin{aligned} Y_j &= \theta_j \xi^{1/\delta\varepsilon} \left(\frac{wb}{\rho}\right)^{1/\delta} \left(\frac{M_o \Psi_o}{\overline{x}}\right)^{(\rho-1)/\delta} \\ \Pi_j^o &= \theta_j \xi^{\rho/\delta\varepsilon} \left(\frac{wb}{\rho}\right)^{1-\rho/\delta} \left(\frac{M_o \Psi_o}{\overline{x}}\right)^{\frac{1-\rho}{\delta}\frac{\varepsilon-1}{\varepsilon}} \end{aligned}$$

[3]This is a standard result of CES functions; Dixit and Stiglitz (1977) show, for CES preferences, that equilibrium production is equal to the first best production.

3.3 Labor Market

A city can include integrated and outsourcing manufacturing firms. Integrated manufacturing firms compete for labor with component producers. Both types of firms are price takers in the labor market. Labor market clearing condition requires that

$$L = L_I\left(w, M_I, \Psi_I\right) + N_o(w, M_o, \Psi_o)(a + b\overline{x})$$

from which it follows that the equilibrium wage satisfies

$$\left(\frac{wb}{\rho}\right)^{\frac{\rho}{\delta}} = \xi^{\rho/\delta\varepsilon}\left(\frac{1-\rho}{a}\right)^{\frac{\varepsilon-1}{\varepsilon\delta}} \overline{x}^{\frac{(\varepsilon-1)\rho}{\delta\varepsilon}} \left[\rho^{\frac{\varepsilon\rho}{\varepsilon-1-\varepsilon\rho}} M_I\left(\Psi_I\right)^{\frac{\rho}{\delta\varepsilon}} + (M_o\Psi_o)^{\frac{\rho}{\delta\varepsilon}}\right] L^{-1}. \tag{9}$$

3.4 The Deterministic Case

It is instructive to consider first the special case of the model where all demand uncertainty disappears ($\theta_j = 1 \quad \forall j$). What determines whether manufacturers choose to be integrated or to outsource?

We say that the set of manufacturing firms are in an *equilibrium configuration* when no firm can profitably switch its own organization of production, given the organizational configuration of all other manufacturers. Because of the assumption that firms act competitively in the labor market, none believe that their choices of integration versus outsourcing have an impact on wages. For any given wage w, the ratio of profits under each configuration is equal to

$$\frac{\Pi_j^o}{\Pi_j^I} = \left(\frac{M_o}{\tilde{M}}\right)^{\frac{1-\rho}{\delta}\frac{\varepsilon-1}{\varepsilon}} \quad \text{where} \quad \tilde{M} \equiv \left(\frac{\varepsilon-1}{\varepsilon}\frac{\rho^{\rho/\delta}}{\delta}\right)^{-\frac{\delta}{1-\rho}\frac{\varepsilon}{\varepsilon-1}}$$

It is apparent that $\tilde{M}$ is a critical value, such that each manufacturer has an incentive to integrate when the number of firms that outsource, M_o, is less than $\tilde{M}$, whereas the opposite is true when $M_o > \tilde{M}$.

Proposition 1 *Consider a city of M manufacturing firms with identical deterministic demands.*
(i) If $\tilde{M} < 1$, then all firms choose to outsource.
(ii) If $\tilde{M} \geq 1$ and $M \geq \tilde{M} + 1$, then all firms choose to integrate.
(iii) If $\tilde{M} \geq 1$ and $M < \tilde{M} + 1$ then there are two equilibrium configurations: either all manufacturers will be integrated, or all will outsource.

This proposition shows that multiple equilibria are possible (see also McLaren (2000)). In particular, it is possible to have an equilibrium with integrated firms in which each firm could increase its profits if it and all other firms would simultaneously switch to the alternative configuration. Firms thus face a coordination problem. For the sake of simplicity, we assume henceforth that some institutional structure exists (perhaps city planning, zoning, or collusion) that results in the more profitable configuration to emerge, whenever multiple equilibria are possible.

3.5 The Stochastic Case

Let us now reintroduce demand uncertainty into the model. We shall assume that firms must choose their organizational form prior to the realization of demand shocks. In this uncertain environment, the *ex post* wage is also stochastic, as shown in equation (9). We assume throughout that manufacturing firms are risk neutral, that is, they seek to maximize expected profits. The assumption of risk neutrality for these firms rests on a presumption that they have access to financial markets (e.g., equities markets) or are part of larger diversified corporations external to the city, such that their profit risks are effectively shared at no cost.

We say that the set of manufacturing firms are in an *equilibrium configuration* when no firm can increase its expected profits by switching its own organization of production, given the organizational configuration of all other manufacturers. As before, none believe that their choices of integration versus outsourcing have an impact on *ex post* wages. The ratio of expected profits under each configuration is given by

$$\frac{E\Pi_j^o}{E\Pi_j^I} = \left(\frac{M_o}{\tilde{M}}\right)^{\frac{1-\rho}{\delta}\frac{\varepsilon-1}{\varepsilon}} G_j(M_o, M_I) \quad \text{where} \quad G_j(M_o, M_I) = \frac{E\left(\theta_j \Psi_o^{\frac{1-\rho}{\delta}\frac{\varepsilon-1}{\varepsilon}} w^{1-\frac{\rho}{\delta}}\right)}{E\left[\theta_j^{\frac{\rho}{\delta\varepsilon}} w^{1-\frac{\rho}{\delta}}\right]}.$$

As in the deterministic case, the incentive for a firm j to choose integration or outsourcing depends on the number of outsourcing firms M_o, but now it also depends on the nature of its own demand shocks and the correlation of those shocks with the *ex post* wage. Again, the incentive to outsource is high when sufficiently many other firms do so (M_o large).

The impact of shocks is embedded in the function $G_j(M_o, M_I)$ that depends on the sets of firms that outsource and integrate component production. When there are no shocks $\theta_j = 1\ \forall j$, this function is equal to 1 and Proposition 1 obviously applies. It also equal to 1 applies when shocks are symmetric and perfectly correlated $\theta_j = \theta_0\ \forall j$. When shocks are small (i.e., variance of θ_j is small compared to its mean value 1), the function remains close to 1 and Proposition 1 applies to the stochastic case, to a first order approximation.

As an important special case, firms whose demand shocks are identically distributed are *ex ante* symmetric and the identity of the firms that outsource or integrate is thus irrelevant. The function $G_j(M_o, M_I)$ becomes a function $G(M_o)$ that depends only on the number M_o of outsourcing firms. If the function $\left(M_o/\tilde{M}\right)^{\frac{1-\rho}{\delta}\frac{\varepsilon-1}{\varepsilon}} G(M_o)$ increases in M_o, the argument of Proposition 1 applies but with a new threshold $\tilde{M}_1$. By Proposition 1, at least with small risks, multiple equilibria may occur where all firms either integrate or outsource; again, we postulate a coordination institution (city planning, zoning, or collusion) that selects the most profitable configuration among multiple equilibria.

When firms' risks are not identically distributed, Chamberlinian externalities dominate for small and large numbers of firms as in the preceding paragraphs. As before, there may exist multiple equilibrium configurations for intermediate numbers of firms. Still, there are two important differences with the previous analysis. First, because firms are not *ex ante* symmetric, they may take different decisions with respect to their structures. An equilibrium configuration may then include a *mix* of integrated and outsourcing firms. Second, in the absence of *ex ante* symmetry, firm get different *ex ante* profits and may not agree on an equilibrium configuration. We assume, however, that coordinating institutions select the most profitable equilibrium configuration and facilitate (negotiated) agreements on possible surplus splits. With this assumption we can state the following proposition.

Proposition 2 *Consider a city of M manufacturing firms with stochastic demands and institutions that facilitate coordination on the most profitable equilibrium. If risks are sufficiently small, there exist values $\tilde{M}_1$ and $\tilde{M}_2$ such that*
(i) if $M < \tilde{M}_1$, then all firms will choose to integrate;
(ii) if $M > \tilde{M}_2$, then all firms choose to outsource;

(iii) if $\tilde{M}_1 < M < \tilde{M}_2$, then some firms choose to outsource while others choose to integrate.

To conclude this section, note that the equilibrium wage is, in general, dependent on the realizations of the demand for the manufacturing firms in the city, which means that workers face wage risk. This is true, of course, when workers are employed by integrated manufacturing firms, since these firms face stochastic demand for their products. It is also true when workers are employed by component-producing firms because demand shocks in external markets are transmitted through the manufacturing firms to the upstream component producers.

Under either type of industrial structure, the magnitude of wage risk depends on the number of manufacturing firms in the city M and on the joint distribution of their demand shocks. In particular, if M is large and the demand shocks are uncorrelated (the case where the θ_j's are i.i.d.), the variance of the equilibrium wage will be small: the high demand for labor by firms with high levels of output demand will offset the low demand for labor with low demand realizations, so that the total demand for labor is relatively stable. This corresponds to a "diversified" industrial structure for the city. If M is relatively small, however, or if external demand shocks are highly correlated, then demand for labor will vary significantly across states of nature, as will the equilibrium wage; in this case, wage risk will be high. Correlated risks may arise when the city's economic base is highly specialized around just a few types of exported commodities (the automobile industry in Detroit provides one illustration). Since demand on external markets may depend on aggregate economic conditions (the economy-wide business cycle, for instance), the case of correlated risks can also arise when there are many different types of firms in the city's economic base.

If workers are indifferent to risk, then the fact that wages vary stochastically in response to fluctuations in external demand is, for them, a matter of no consequence. In practice, however, households are risk averse, and wage risk can be very costly to them. This means that workers would value employment relationships that provide some form of insurance against wage fluctuations. We therefore turn to an analysis of such contracting arrangements.

4 Ex Ante Labor Contracting

In order for workers to be protected from wage risk, they must either be able to obtain insurance that offers state-contingent payments based on wage realizations, or they must be able to contract for wages before the state of nature is known. Because of well-known issues relating to transactions costs and moral hazard, we assume that workers are unable to purchase earnings insurance individually. This means that the only opportunity to obtain protection against wage risk must come from labor contracts. In the spirit of the theory of implicit contracts (see Baily (1974), Gordon (1974), Azariadis (1975)), when firms are risk neutral, they can provide insurance to workers by offering labor contracts in which wages and employment are fixed *ex ante*. There are of course many alternative models of labor contracting, but we focus for simplicity on contracts that are exclusive and non-renegotiable. Our principal goal is to highlight the possible relationships between contracts and the industrial structure of a city.

We take up first the case where all manufacturing is undertaken by integrated firms ($M < \hat{M}_1$), then the case where all manufacturers outsource ($M > \hat{M}_2$). We then examine the situation where both types of organizational structures exist. Throughout this section, we continue to assume that manufacturing firms have access to financial markets or are otherwise sufficiently diversified that they are indifferent to risk.

4.1 Labor Contracting By Integrated Manufacturers

Suppose again that L workers and M manufacturing firms have settled in a city and suppose that $M_I > 0$ firms have chosen to integrate. Consider an integrated manufacturing firm $j \in M_I$. We first determine the properties of its wage contract and then compare the expected profits of this firm with and without such contracts. Firms can potentially hire workers *ex post*, in which case firms pay workers the stochastic equilibrium wage as described previously. Alternatively, they can offer *ex ante* contracts with a fixed wage. Firms and workers are assumed to act as price-takers both in the *ex ante* and *ex post* labor markets. This means that no firm can pay a wage lower than the prevailing wage in the *ex ante* market, w', or in the *ex post* market, w. Equilibrium is attained when labor demand is equal to labor supply both *ex ante* and *ex post*.

Let M_I' denote the number (and the set) of integrated manufacturers that choose to hire *ex ante*, and let L'_I denote the amount of labor that they employ. There are thus $M_I - M'_I$ firms that hire the remaining $L - L'_I$ workers at a wage rate of w, subsequent to the realization of demand shocks. There are three possible types of equilibrium: one in which all manufacturers hire in the *ex ante* market ($M'_I = M_I$), one in which none hire in the *ex ante* market ($M'_I = 0$), and one in which some hire in both ($0 < M'_I < M_I$). If $u(w') < Eu(w)$, L'_I must be zero because all workers prefer *ex post* employment, whereas $L'_I = L$ if $u(w') > Eu(w)$. Thus, to determine which type of equilibrium emerges, we need only to compare the expected profits of integrated firms when $u(w') = Eu(w)$.

Suppose that firm j offers an *ex ante* contract with a wage w' and chooses its production level to maximize $E\Pi_j = E\left(\theta_j\right)^{1/\varepsilon} Y_j^{1-1/\varepsilon} - cw'Y_j^{\rho}$. The expected levels of output and profits for integrated manufacturers will be

$$
\begin{aligned}
Y_j^{I'\delta} &= \left(E\theta_j^{1/\varepsilon}\right) \frac{1}{w'}\frac{\varepsilon - 1}{c\rho\varepsilon} \\
\Pi_j^{I'} &= \left(E\theta_j^{1/\varepsilon}\right)^{\frac{\rho}{\delta}} \left[w'\frac{c\rho\varepsilon}{\varepsilon - 1}\right]^{-\left(\frac{\rho}{\delta}-1\right)} \frac{\delta}{\rho}
\end{aligned}
$$

where I' denotes the levels obtained under *ex ante* contracting by integrated firms and where $\frac{\rho}{\delta} - 1 > 0$. An integrated firm's labor demand is $l_j^{I'} = cY_j^{I'\rho}$. We now compare the expected profits with and without wage contracts:

$$
\frac{\Pi_j^{I'}}{E\Pi_j^{I}} = \frac{\left(E\theta_j^{1/\varepsilon}\right)^{\frac{\rho}{\delta}} (w')^{-\left(\frac{\rho}{\delta}-1\right)}}{E\left(\theta_j^{\frac{\rho}{\delta\varepsilon}} w^{-\left(\frac{\rho}{\delta}-1\right)}\right)}
$$

where $E\Pi_j^I$ denotes expected profits for an integrated manufacturer who hires labor in the *ex post* market.

Assuming constant relative risk aversion, the condition that $u(w') = Eu(w)$ means that $w'^{\alpha}/\alpha = Ew^{\alpha}/\alpha$, in which case

$$
\frac{\Pi_j^{I'}}{E\Pi_j^{I}} = F_j(M_I \backslash M'_I, M_o) \equiv \frac{\left(E\theta_j^{1/\varepsilon}\right)^{\frac{\rho}{\delta}} \left(Ew^{\alpha}\right)^{-\frac{1}{\alpha}\left(\frac{\rho}{\delta}-1\right)}}{E\left(\theta_j^{\frac{\rho}{\delta\varepsilon}} w^{-\left(\frac{\rho}{\delta}-1\right)}\right)}.
$$

Observe from this condition that the incentive to hire *ex ante* depends on the set of firms that are active in the *ex post* labor market $M_I \backslash M'_I$ and M_o, but it is independent of the size of the *ex post* labor market (as measured by $L_I^{'}$). An integrated firm j does not offer *ex ante* contracts iff F_j is below one. Thus, an equilibrium occurs when the integrated manufacturers can be partitioned in such a way that $F_j(M_I \backslash M_I', M_o) < 1$ for all $j \in M_I \backslash M_I'$ and $F_j(M_I \backslash M_I', M_o) \geq 1$ for all $j \in M_I'$. There need not be a unique equilibrium since several partitions of integrated firms may satisfy these equilibrium conditions.

Obviously, at a given configuration $(M_I \backslash M_I', M_o)$, a decrease in workers' risk aversion (larger α) increases workers' certainty equivalent of wages $(Ew^\alpha)/\alpha$ which reduces F_j and reduces the profitability of an *ex ante* contract. When workers are risk-neutral ($\alpha = 1$), the ratio F_j is below 1 provided that firm j's shocks are not correlated too much with wages. Indeed, one can successively write

$$\begin{aligned}
\frac{\left(E\theta_j^{1/\varepsilon}\right)^{\frac{\rho}{\delta}} (Ew)^{-\left(\frac{\rho}{\delta}-1\right)}}{E\left(\theta_j^{\frac{\rho}{\delta\varepsilon}} w^{-\left(\frac{\rho}{\delta}-1\right)}\right)} &< \frac{\left(E\theta_j^{\frac{\rho}{\varepsilon\delta}}\right)\left(Ew^{-\left(\frac{\rho}{\delta}-1\right)}\right)}{E\left(\theta_j^{\frac{\rho}{\delta\varepsilon}} w^{-\left(\frac{\rho}{\delta}-1\right)}\right)} \\
&= \frac{1}{E\left[\left(\theta_j^{\frac{\rho}{\delta\varepsilon}}/E\theta_j^{\frac{\rho}{\varepsilon\delta}}\right) * \left(w^{-\left(\frac{\rho}{\delta}-1\right)}/Ew^{-\left(\frac{\rho}{\delta}-1\right)}\right)\right]} \\
&= \frac{1}{1+\mathrm{cov}\left[\left(\theta_j^{\frac{\rho}{\delta\varepsilon}}/E\theta_j^{\frac{\rho}{\varepsilon\delta}}-1\right), \left(w^{-\left(\frac{\rho}{\delta}-1\right)}/Ew^{-\left(\frac{\rho}{\delta}-1\right)}-1\right)\right]}
\end{aligned}$$

where the inequality in the first line results from Jensen's inequality and the fact that $(z)^{\frac{\rho}{\delta}}$ and $z^{-\left(\frac{\rho}{\delta}-1\right)}$, $\frac{\rho}{\delta} > 1$, are convex functions of z, where the second line presents a simple algebraic rearrangement in the denominator and where the last line uses the definition of covariance for variables with means equal to one. When firm j's shocks are negatively correlated or completely uncorrelated with wages, the covariance in the last line is non-negative and the ratio is below 1. In this case, firm j does not offer *ex ante* contracts: *ex ante* contracting constrains the ability of the firm to adjust output and employment in response to demand shocks, a cost that is not offset by the willingness of workers to accept a lower wage *ex ante.*

Some city configurations are easy to characterize. First, if the number of manufacturing firms is large and demand shocks are i.i.d., the *ex post* wage w is almost constant. We get $F_j < 1$ because $E(\theta_j^{\frac{\rho}{\delta\varepsilon}}) > (E\theta_j^{1/\varepsilon})^{\frac{\rho}{\delta}}$, $\frac{\rho}{\delta} > 1$. Hence, integrated firms do not offer *ex ante* contracts in the presence of many other firms with uncorrelated demands.

Second, if all firms have perfectly correlated demand shocks, so that $\theta_j = \theta_0$ for all j, where θ_0 is a common demand shock, then the *ex post* wage w becomes proportional to $\theta_0^{\frac{1}{\varepsilon}}$ and we get that $F_j = [\frac{(E\theta_0^{\frac{\alpha}{\varepsilon}})^{\frac{1}{\alpha}}}{\alpha}/E\theta_0^{\frac{1}{\varepsilon}}]^{-(\frac{\rho}{\delta}-1)} > 1$ because $\theta_0^{\frac{\alpha}{\varepsilon}}/\alpha$ is a more concave function than $\theta_0^{\frac{1}{\varepsilon}}$ provided that $\alpha \in (0,1)$. Thus, integrated firms with perfectly correlated demands offer *ex ante* contracts. The case of a single integrated firm may be viewed as a particular instance of the case of perfectly correlated shocks. Hence, we should expect to see *ex ante* labor contracting in "factory towns".[4] Note that when workers are risk neutral (i.e., $\alpha = 1$) and risks are perfectly correlated, firms are indifferent between hiring *ex ante* or *ex post*: on the on hand, workers place no value on wage stability, and, on the other hand, firms gain nothing from *ex post* employment flexibility. To summarize:

Proposition 3 *(i) In diversified cities with many manufacturers, firms do not contract with workers* ex ante.

(ii) If manufacturers have perfectly correlated demand shocks, they do contract with workers ex ante*, providing workers with actuarially fair insurance ($w' = Ew$).*

(iii) When manufacturers face demand shocks manufacturers that are partially correlated, equilibria are possible in which firms with demand shocks that are highly positively correlated with ex post *wages contract for labor* ex ante*, while firms with demand shocks that are highly negatively correlated with* ex post *wages contract for labor* ex post.

More intuition can be obtained if we study the case of small demand uncertainty (i.e., variance of θ_j is small compared to its mean value 1).

Small risks. For small demand uncertainty (see Appendix), we can compute that $F_j(M_I, M_o) < 1$ iff

$$2\mathrm{cov}(\theta_j, w/w_0) < \frac{1}{\varepsilon}\mathrm{var}\,(\theta_j) + \varepsilon \left[1 + (\alpha - 1)\frac{\delta}{\rho}\right] \mathrm{var}\,(w/w_0)$$

where w is the endogenously-determined *ex post* wage.

[4]Of course the assumption of price-taking behavior is not really justifiable in the case of a sole manufacturing firm, but this result emphasizes the importance of lack of risk diversification as the underlying reason for ex ante labor contracting in this model.

This formula confirms the two previous results: firms do not offer *ex ante* labor contracts when there are many firms and wages are almost non-stochastic, whereas they do offer such contracts when shocks are highly positively correlated. Furthermore, if workers are sufficiently risk averse, α is very negative and the inequality is not satisfied, which is to say that *ex ante* contracting dominates in this case.

This formula also shows how *ex ante* and *ex post* employment contracts can co-exist: a firm is more likely to hire *ex ante* if its demand shock has low variance or if it is positively correlated with the *ex post* wage. Indeed, if it has low variance, the firm does not really need production flexibility and contracting with workers is beneficial. If its shock is positively correlated with *ex post* wages, the firm expects high product demand when wages are high and and low product demand when wages are low; as a consequence, its production and demand for labor is stable and *ex ante* contracts imposes little cost while offering wage stability to workers.

To make things even clearer, suppose that there are only integrated manufacturers in the city ($M_o = 0$). Then, the hiring choice can be expressed in terms of the demand shocks (see Appendix):

$$F_j\left(M_I \backslash M_I', 0\right) < 1 \iff 2\mathrm{cov}\left(\theta_j, m_I\right) < \mathrm{var}\left(\theta_j\right) + \left(1 + (\alpha - 1)\frac{\delta}{\rho}\right)\mathrm{var}\left(m_I\right)$$

where m_I is the mean of shocks of integrated firms that do not contract *ex ante*: $m_I = \sum_{k \in M_I \backslash M_I'} \theta_k / (M_I - M_I')$. A first result is that no integrated manufacturer hires *ex ante* if $\alpha = 1$ because $2\mathrm{cov}(\theta_j, m_I) \leq \mathrm{var}(\theta_j) + \mathrm{var}(m_I)$. A second result is that each integrated firm wants to hire *ex ante* for large risk aversion provided that there remains an integrated firm that does not contract *ex ante* ($m_I > 0$).

We summarize these results in the following proposition.

Proposition 4 *Suppose that there are only integrated manufacturers in the city and that risks are small. For low values of risk aversion by workers, no firm hires* ex ante*. If workers are highly risk-averse, all firms but one hire* ex ante*. For intermediate values of risk aversion, some firms may hire* ex ante *while others hire* ex post*.*

4.2 Labor Contracting and Outsourcing

Let us now consider the nature of labor contracting when some ($M_o = M - M_I > 0$) manufacturing firms outsource the provision of components. The makers of components are assumed to be free to enter both before the realization of demand shocks and subsequent to their realization. In the latter case, entry occurs until *ex post* profits are zero. Entry prior to the realization of the shocks occurs until expected profits are zero.

Smallness is a main feature of monopolistic competition in which component makers engage. Firms are so small that they do not anticipate having an impact on aggregate prices and outputs. However, smallness also has implications for the financial management of firms because small firms usually have less access to financial markets. In many cities, the intermediate sector often includes small family-run firms funded with limited and unsophisticated capital structures, perhaps built mainly around family assets. Also, it is more costly for investors to monitor smaller firms; the lack of collateral in small businesses usually restricts credit (See Audretsch and Elston, 2002). Hence it seems natural to assume that small firms pay a larger risk premium on financial investment than large firms do. This financial issue becomes critical when firms consider offering labor contracts that commit them to specific wage and employment levels. Therefore, small component producers are less likely to propose *ex ante* labor contracts than large manufacturing firms that, as we have supposed above, are able to absorb and manage risk with minimal cost.

To capture the differential costs that small firms incur in offering wage contracts, we assume that each component producer pays a premium as an additional fixed cost $(\tau - 1)aw > 0$ $(\tau > 1)$ that is proportional to the production fixed cost a and to the realization of the wage. The total fixed cost then becomes τaw. The parameter τ could represent the transactions costs absorbed by small firms when dealing in financial markets, including the extra staffing and other costs that a firm must incur when meeting auditing and control requirements for outside financial counterparties.

Suppose again that L workers and M manufacturing firms have settled in a city and suppose that $M_o > 0$ manufacturing firms have chosen to outsource. As before, firms can hire workers *ex post*, paying them the stochastic equilibrium *ex post* wage, or can hire them *ex ante*, offering a fixed wage. Competition in both markets means that all firms take as given the market-determined wages, w for the *ex post* stochastic wage and w' for the *ex ante* fixed wage. Equilibrium is attained when labor demand is equal to labor supply both *ex ante* and *ex post*.

Again, there are three possible types of equilibrium: one in which all workers are hired *ex ante*, one in which all are hired *ex post*, and one in which some are hired in both periods. Let N' denote the number of component producers that hire in the *ex ante* market; $N - N'$ is then the number of component firms hiring workers *ex post*.

In the *ex ante* stage, a component producer i chooses its production $x'(i)$ to maximize its *ex ante* profit given contractual wage w'. That is, the component producer maximizes $\pi'(i) = E\left[p'(i)x'(i) - w'\left(\tau a + bx'(i)\right)\right]$ where, by (5),

$$p'(i) = x'(i)^{\rho-1}\mathsf{p}^{\varepsilon\delta}(M_o\Psi_o)^{1-\rho}. \tag{10}$$

The optimal output and profit are computed as

$$x'(i) = M_o\left[\frac{\rho E\left(\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}\right)}{bw'}\right]^{\frac{1}{1-\rho}} \tag{11}$$

and

$$\pi'(i) = \frac{aw'}{\overline{x}}\left[x'(i) - \tau\overline{x}\right]$$

Component producers enter in the *ex ante* stage as long as they make non-negative profits, which is equivalent to the condition: $x'(i) > \tau\overline{x}$. This condition implies that wage offers remain low enough for entry to occur. Therefore, component producers enter *ex ante* if they are able to offer a wage w' such that

$$Eu(w) \leq u(w') \leq u\left[\frac{\rho}{b}\left(\frac{M_o}{\tau\overline{x}}\right)^{1-\rho}E\left(\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}\right)\right].$$

The first inequality requires that the *ex ante* wage offers workers a level of utility as great as the expected utility that they obtain in the *ex post* market; the second inequality requires that the *ex ante* wage is low enough for the component makers to break even. If the first inequality is not satisfied, workers are unwilling to contract *ex ante*, and if the second inequality is not satisfied, then no firms wish to enter *ex ante*; in either case, $N' = 0$.

For expository purposes, we distinguish the configurations of cities in which integrated firms offer *ex ante* labor contracts and those in which they do not. Let M_I' denote the number (and set) of the former, so that $M_I - M_I'$ integrated manufacturers hire workers in the *ex post* market.

4.2.1 Integrated Manufacturers Hire only ex ante ($M_I' = M_I$)

When component producers hire workers *ex post*, free entry insures that the price index for components will satisfy (7). Given this price index, the *ex ante* production of component producers will be

$$x'(i) = \left(\frac{Ew}{w'(i)}\right)^{\frac{1}{1-\rho}} \overline{x}.$$

Therefore, the profitability condition $x'(i) > \overline{x}$ is equivalent to $w' \leq \tau^{\rho-1} Ew$ and it follows that labor will be hired in the *ex ante* labor market if

$$Eu(w) \leq u(w') \leq u\left(\tau^{\rho-1} Ew\right).$$

We analyze the case of perfect and imperfect financial markets in the city.

Perfect Financial Market. When $\tau = 1$, the above condition is always satisfied. When financial markets are perfect, free entry implies that the right-hand relationship is satisfied as an equality (i.e., $w' = Ew$) while the fact that $u(Ew) > Eu(w)$ implies that no workers are willing to be hired *ex post*. At the same time, since $w' = Ew$, $x'(i) = \bar{x}$.

We therefore get the following proposition.

Proposition 5 *Suppose that integrated firms (if any) only contract with workers* ex ante *($M_I' = M_I$), and suppose that financial markets in cities are perfect ($\tau = 1$). Then, component producers always contract with workers* ex ante. *The* ex ante *level of output of each component is equal to its* ex post *level. Workers receive actuarially fair insurance against shocks ($w' = Ew$) and they have strictly higher utility than if component firms were not able to offer* ex ante *contracts.*

When $\tau = 1$, *ex ante* entry does not have any impact on *ex post* wages because the labor resources required by firms contracting *ex ante*, $a + bx'(i)$, are exactly equal to the labor demand of the same firms if they hired workers *ex post*, $a + b\overline{x}$. As a result, the expected utility of workers increases only through *better insurance*.

Imperfect Financial Markets. When financial markets are not perfect, each component producer i must produce no less than $\tau\overline{x}$ to break even. Because production of components is inversely related to the *ex ante* wage w', component producers tend to offer lower wages as τ rises. However, when *ex ante* wages are too small, workers are unwilling to contract *ex ante*.

Ex ante wage depends on the anticipation of the level and variance of *ex post* wages. In contrast to the case of perfect financial markets in the city, we here need to make the wages explicit. Ex post wages are subject to two effects: a competition effect in the component market and a crowding out effect in the labor market. On the one hand, when a set of firms swaps from *ex post* to *ex ante* contracting, they depress their own component price and, through the component market competition, they also depress the price of *ex post* contracting firms. As result the *ex post* wage drops. To see this, note that *ex post* wages depend on the component prices $p'(i)$ set by firms that offer *ex ante* contracts (see (10) and apply $x'(i) = \tau\overline{x}$), and on the component prices p set by the firms that enter and contract labor *ex post* (see (6) and apply $x(i) = \overline{x}$). One readily checks that these prices satisfy the relationship, $p'(i)/p = \tau^{\rho-1} < 1$. Firms contracting *ex ante* have higher fixed costs to cover and they must then set higher outputs and lower prices. Combining (5) and (6), one can eliminate the price index and get the price set by *ex post* contracting firms as it follows:

$$p^{\frac{\rho}{\delta}} = (M_o\Psi_o/\overline{x})^{\frac{\rho}{\varepsilon\delta}} \left[N'(\tau^{\rho} - 1) + N\right]^{-1}. \tag{12}$$

which, because $\tau > 1$, decreases with N'. Hence, since $p = bw/\rho$, when firms swap from *ex post* to *ex ante* contracting, they harshen competition and depress the *ex post* wages.

On the other hand, when some firms swap from *ex post* to *ex ante* contracts, they crowd the *ex post* labor supply out and they force *ex post* wages to rise. Indeed, at the equilibrium, the *ex post* labor clears as it follows:

$$(a + b\overline{x})\,(N - N') = L - L'_I - \int_0^{N'} (\tau a + bx'(i))di,$$

where the LHS is the labor demand by component makers that hire *ex post* and where the RHS includes the total labor supply minus the *ex ante* labor demands by integrated manufacturing firms and by component makers. Ex ante component makers require additional labor to pay the financial transaction cost τ. Hence, when these firms swap from *ex post* to *ex ante* contracts, they crowd the *ex post* labor supply out and they push *ex post* wages up.

Using the last two expressions and the equalities $p = bw/\rho$ and $x'(i) = \tau\overline{x}$ we get the following *ex post* wage:

$$\left(\frac{wb}{\rho}\right)^{\frac{\rho}{\delta}} = \left(\frac{IM_o\Psi_o}{\overline{x}}\right)^{\frac{\rho}{\varepsilon\delta}} \frac{(a+b\overline{x})}{L - L_I' - (a+b\overline{x})N'(\tau - \tau^\rho)}. \tag{13}$$

which increases in N' because $\tau > \tau^\rho$. When manufacturing firms hire only *ex ante* ($M_I' = M_I$), *ex post wages increase with the number* N' *of component producers that hire ex ante.* The crowding out effect in the labor market dominates the competition effect in the component market. Also, *ex post* wages obviously increase with the number M_o of outsourcing firms and vary with their aggregate shocks Ψ_o, which impact falls when N' increases.

Let us now study the situation in which component producers are just able to break even when they offer an *ex ante* wage that makes workers indifferent between *ex ante* and *ex post* contracting. That is, we look at the situation in which $x'(i) = \tau\overline{x}$ and $u(w') = Eu(w) = u(\tau^{\rho-1}Ew)$. Two values of the financial cost τ are of particular interest. Let first $\underline{\tau}$ be that value of τ at which component makers respectively exhaust the full labor supply *ex ante:* $(\tau a + \tau b\overline{x})N' = L - L_I'$. Let also denote by $\overline{w}_0$ the value of the *ex post* wage under the same condition but when $\Psi_o = 1$. The *ex post* wage is thus equal to $w = \overline{w}_0\Psi_o^{1/\varepsilon}$. Then, we get

$$\underline{\tau}^{1-\rho} = \frac{\overline{w}_0 E\left(\Psi_o^{1/\varepsilon}\right)}{u^{-1}\left[Eu\left(\overline{w}_0\Psi_o^{1/\varepsilon}\right)\right]} > 1.$$

When financial costs lie below this value, all component makers hire workers *ex ante* and exhaust *ex ante* the labor supply in the city.

Let then $\overline{\tau}$ be the value of τ at which component makers respectively hire no worker *ex ante* ($N' = 0$). As previously, let also denote by $\underline{w}_0$ the value of the *ex post* wage under the same condition but when $\Psi_o = 1$. Note that because *ex post* wages increases in N', we have that $\overline{w}_0 > \underline{w}_0$. We then get

$$\overline{\tau}^{1-\rho} = \frac{\underline{w}_0 E\left(\Psi_o^{1/\varepsilon}\right)}{u^{-1}\left[Eu\left(\underline{w}_0\Psi_o^{1/\varepsilon}\right)\right]} > 1.$$

When financial costs lie above this value, no component makers hire workers *ex ante*. When they lie between $\underline{\tau}$ and $\overline{\tau}$, component makers hire *ex ante* only a part of the labor supply.

Obviously the values of $\underline{\tau}$ and $\overline{\tau}$ tend to 1 when workers' risk aversion tends to zero. In that case, workers put a low value on insurance and any small financial cost impedes component producers to provide insurance. By contrast when workers are very risk averse, firms provide insurance even at large financial costs.

Note that workers gain better insurance as well as higher average wages. Insurance by component producers occurs at a cost and that this cost increases the demand for workers, which raises their wages.

Proposition 6 *Suppose that integrated firms (if any) only contract with workers* ex ante *($M_I' = M_I$), and suppose that financial markets in cities are imperfect ($\tau > 1$). There exist two values of financial cost ($\underline{\tau}, \overline{\tau}$) such that*
(i) for $\tau \leq \underline{\tau}$, all component producers offer ex ante *wage contracts,* ex ante *exhausting the labor supply. These contracts make workers better off ($w' > Eu(w)$);*
(ii) for $\underline{\tau} < \tau \leq \overline{\tau}$, some component producers offer ex ante *wage contracts, hiring a portion of the labor force. Workers are indifferent between taking* ex ante *wage contracts ($u(w') = Eu(w)$) or not but equilibrium expected utility for workers is higher than if no* ex ante *contracts were offered; and*
(iii) for $\tau > \overline{\tau}$, no component producer offers ex ante *an wage contract ($u(w') < Eu(w)$).*

The ranking of the thresholds ($\underline{\tau}, \overline{\tau}$) depend on the properties of the utility function. For constant relative risk aversion ($u(w) = w^{\alpha}/\alpha$), the thresholds are independent of the wage level and are equal to $\underline{\tau} = \overline{\tau} = \tau_0$ where

$$\tau_0^{1-\rho} = \frac{E\Psi_o^{1/\varepsilon}}{\left[E\Psi_o^{\alpha/\varepsilon}\right]^{1/\alpha}}.$$

Hence only two industrial configurations occur: either all or none component firms offer *ex ante* wage contract to workers.

Corollary 7 *Suppose that workers have constant relative risk aversion. Then, there exists a τ_0 such that all component firms hire workers* ex ante *if $\tau < \tau_0$ and none of them offer hire workers* ex ante *otherwise.*

It is here readily verified that workers gain better insurance as well as higher average wages. Indeed, when all component firms hire *ex ante,* workers get the utility level $u(w')$ which is above or equal to $Eu\left(\overline{w}_0\Psi_o^{1/\varepsilon}\right) = \overline{w}_0 E\left(\Psi_o^{\alpha/\varepsilon}\right)^{1/\alpha}$. By contrast, when none of them hire *ex ante*, workers get a lower utility level $Eu\left(\underline{w}_0\Psi_o^{1/\varepsilon}\right) = \underline{w}_0 E\left(\Psi_o^{\alpha/\varepsilon}\right)^{1/\alpha}$ where $\underline{w}_0 < \overline{w}_0$.

From the corollary, we also infer that $\underline{\tau} < \overline{\tau}$ under decreasing relative risk aversion. Then, the three configurations of the last proposition exist. That means that when more component makers offer *ex ante* contract, *ex post* wages increase, which rises the *ex ante* wage and reduces the attractiveness of the component markets. In contrast, for increasing relative risk aversion, we have $\underline{\tau} > \overline{\tau}$. Then, only the industrial configurations (i) and (iii) occur.

We now turn to cities including integrated firms without wage contracts $(M_I' < M_I)$.

4.2.2 Integrated Manufacturers Hire ex ante and ex post ($M_I - M_{I'} > 0$)

When $M_I - M_I' > 0$, some integrated manufacturers have positive *ex post* demands for labor. When the *ex post* supply of labor becomes small, these firms are willing to offer high *ex post* wages. This impedes component makers to hire the whole set of workers *ex ante* because they are unable to offer comparably high wages *ex ante*. Let financial markets in cities be perfect $(\tau = 1)$ or imperfect $(\tau > 1)$. We get the following proposition.

Proposition 8 *Suppose that integrated firms contract with workers* ex post *$(M_I > M_I')$ and assume constant relative risk aversion. Then there exists a value of financial cost $\overline{\tau} \geq 1$ such that*
(i) for $\tau < \overline{\tau}$, some component producers ex ante *hire a portion of the labor supply and workers are indifferent between taking wage contracts or not ($w' = (Ew^{\alpha})^{1/\alpha}$), and*
(ii) for $\tau > \overline{\tau}$, no component producer ex ante *offers wage contracts.*

Proof: see Appendix.

High wages offered by integrated manufacturers refrain component producers to hire the total labor supply as it can be the case in the previous section. In particular component producers are never able to exhaust the labor supply *ex ante*, even at $\tau = 1$. Loosely speaking, this amounts to set $\underline{\tau} = 1$ and forbid item (i) in Proposition 6. Still, *ex ante* entry of component producers again pushes workers utility up through *better insurance*, and, through *higher level of wages,* as it is shown in expression (15) in the Appendix.

5 Conclusion

In this paper we have explored the interactions between labor market pooling, structure of production and contracts to workers. By pooling in large cities, workers benefit from a larger labor demands and less volatile wages. By locating in large cities, firms benefit from Chamberlinian externalities which entice them to outsource the production of components. Firms also benefit from offering insurance to worker through fixed wage contracts.

We develop a model *a la* Ethier (1982) in which firms have increasing returns to scale and produce their output assembling a set of components. Firms have CES production function and consumers iso-elastic demands. Firms firstly locate in a city, they secondly decide to outsource or integrate the production of their components, they thirdly choose to offer wage contracts to workers and they finally produce and sell their output. The model offers several interesting results.

Firms' outsourcing decisions may suffer from a coordination problem as, for a same set of economic parameters, equilibria with outsourcing, integration or both structures may co-exist. In particular, when a city develops by hosting new manufacturing firms, firms may be inefficiently locked in the integration structure. One has to assume the existence of institutional structures (city planning, zoning or collusion) to restore the efficient structure. Furthermore, when firms are not ex ante symmetric, firms may even not agree on the optimal structure and institutions must also arrange transfers between firms.

The nature of contract to workers strongly depends on the structure of manufacturing firms in a city. First, integrated manufacturers have access to internal or global financial markets that allows them to insure against both demand risks and wage risks. Integrated manufacturers are therefore enticed to offer insurance to workers through fixed wage contracts. Yet workers' benefit from insurance is smaller in larger cities with more diversified risks. Therefore, it turns out that integrated manufacturers tend not to contract with workers *ex ante* in large cities hosting many manufacturers with uncorrelated shocks but they offer *ex ante* contracts when shocks are strongly correlated. Manufacturers with shock sufficiently correlated with the *ex post* wage hire workers *ex ante,* whereas manufacturers with own shocks sufficiently negatively correlated to *ex post* wages do not hire *ex ante.* Workers receive actuarially fair insurance. Of course, integrated firms are more likely to offer wage contracts when workers are more risk averse.

Second, when manufacturing firms outsource, labor contracts are offered by small component producers. On the one hand, the ability of small firms to insure workers depends on their access to financial markets. Many times small firms incur proportionally much higher financial transaction costs than large manufacturing firms. Too high financial transactions costs impede component firms to offer insurance to workers. On the other hand, component firms capture workers by providing insurance to them through wage contracts. Competition drastically pushes those firms to offer wage contracts.

Consider firstly a perfect access to financial markets. When the city includes only outsourcing manufacturing firms and component makers, component makers offer wage contracts to all workers, irrespectively of the risks in the city. This result is at odds with the contracting decision of integrated firms that are less likely to offer wage contracts in larger cities with diversified demand risks. When the city includes both outsourcing and integrated manufacturing firms, component firms are not able to offer ex ante wage contracts to all workers. Some workers prefer to be hired ex post and to face the risks of uncertain wages as they anticipate that integrated firms may face an ex post shortage of labor supply and may then push ex post wages up.

Consider finally an imperfect access to financial markets. On the one hand, when financial transaction costs are too high, no component firm is able to offer wage contracts. In particular, there will be no insurance to workers in a large city where Chamberlinian externalities prevail and thus where all manufacturing firms outsource. On the other hand, for intermediate values of financial transaction costs, the equilibrium includes only some component firms offering wage contracts.

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Appendix

Labor Contracting by Integrated Manufacturers

Under small risk, we use Taylor approximations around $\theta_j = 1$ and the fact that $E\theta_j = 1$. Hence, $E\theta_j^{1/\varepsilon} \simeq 1 + \frac{1}{\varepsilon}E(\theta_j - 1) - \frac{\varepsilon-1}{2\varepsilon^2}E(\theta_j - 1)^2 = 1 - \frac{\varepsilon-1}{2\varepsilon^2}\text{var}(\theta_j)$. Note then that $F_j(M_I \backslash M_I', M_o)$ can be expressed in terms of w/w_0 where w_0 is the wage w under $\theta_j = 1$ for all j. Note that w_0 is a function of $M_I \backslash M_I'$ and M_o. Then $E(w/w_0)^\alpha \simeq 1 + (\alpha - 1)\alpha\text{var}(w/w_0)/2$. Also, using a Taylor approximation around $\theta_j = 1$ and $w = w_0$, we get

$$\begin{aligned} E[\theta_j^{\frac{\rho}{\delta\varepsilon}}(w/w_0)^{-(\frac{\rho}{\delta}-1)}] \simeq\ & 1 - \rho\frac{\varepsilon-1}{\delta^2\varepsilon^2}\text{cov}(\theta_j, w/w_0) \\ & + \frac{(1-\rho)\rho(\varepsilon-1)}{2\delta^2\varepsilon^2}\text{var}(\theta_j) \\ & + \rho\frac{\varepsilon-1}{2\delta^2\varepsilon}\text{var}(w/w_0) \end{aligned}$$

Using, the Taylor approximation $(1+x)^y \simeq 1+xy$, we get

$$\begin{aligned} F_j(M_I, M_o) &\simeq 1 - \frac{\rho}{2}\frac{\varepsilon-1}{\delta^2\varepsilon^3}\mathrm{var}\,(\theta_j) - \frac{1}{2}(\varepsilon-1)\frac{\alpha\varepsilon\delta+\varepsilon-1}{\delta^2\varepsilon^2}\mathrm{var}\,(w/w_0) \\ &\quad +\rho\frac{\varepsilon-1}{\delta^2\varepsilon^2}\mathrm{cov}(\theta_j, w/w_0) \end{aligned}$$

Therefore, $F_j < 1$ iff

$$2\mathrm{cov}(\theta_j, w/w_0) < \frac{1}{\varepsilon}\mathrm{var}\,(\theta_j) + \varepsilon\left(1+(\alpha-1)\frac{\delta}{\rho}\right)\mathrm{var}\,(w/w_0)$$

which is the result in the text.

Suppose now that $M_o = 0$. So that $w/w_0 = \Psi_{I-I'}^{1/\varepsilon}$ where $\Psi_{I-I'} \equiv [(1/(M_I - M_I')) \sum_{j\in M_I\setminus M_I'} \theta_j^{\frac{\rho}{\delta\varepsilon}}]^{\frac{\delta\varepsilon}{\rho}}$. Using Taylor approximation for θ_j around 1, we get $\Psi_I^{1/\varepsilon} \simeq 1 + \sum_{k\in M_I\setminus M_I'} \left[\partial\Psi_{I-I'}^{1/\varepsilon}/\partial\theta_k\right]_{\theta=(1,\ldots,1)}(\theta_k - 1) = 1 + \frac{1}{(M_I-M_I')\varepsilon}\sum_{k\in M_I\setminus M_I'}(\theta_k-1)$. So,

$$\begin{aligned} \mathrm{var}\,(w/w_0) &= \mathrm{var}\left(\Psi_I^{1/\varepsilon}\right) \simeq \frac{1}{\varepsilon^2(M_I-M_I')^2}\sum_{k\in M_I\setminus M_I'}\mathrm{var}\,(\theta_k) \\ \mathrm{cov}(\theta_j, w/w_0) &= \mathrm{cov}(\theta_j, \Psi_I^{1/\varepsilon}) \simeq \frac{1}{\varepsilon(M_I-M_I')}\sum_{k\in M_I\setminus M_I'}\mathrm{cov}\,(\theta_j,\theta_k) \end{aligned}$$

Hence the previous condition becomes

$$\frac{2}{M_I-M_I'}\sum_{k\in M_I\setminus M_I'}\mathrm{cov}\,(\theta_j,\theta_k) < \mathrm{var}\,(\theta_j) + \frac{1+(\alpha-1)\frac{\delta}{\rho}}{(M_I-M_I')^2}\sum_{k\in M_I\setminus M_I'}\mathrm{var}\,(\theta_k)$$

Letting $m_I = \frac{1}{M_I-M_I'}\sum_{k\in M_I\setminus M_I'}\theta_k$, we get the following simple equivalence:

$$F_j(M_I, 0) < 1 \iff 2\mathrm{cov}\,(\theta_j, m_I) < \mathrm{var}\,(\theta_j) + \left(1+(\alpha-1)\frac{\delta}{\rho}\right)\mathrm{var}\,(m_I)$$

Proof of Proposition 8

In Proposition 8, component makers hire ex ante if this option gives them higher expected profit and gives workers higher expected utility. Expected profits and utility depend on ex post prices and wages which expression depends on the existence of ex post entry of component firms. In this Appendix we first derive and compare ex post prices and wages with and without entry. Then we characterize the equilibrium by comparing the condition for positive expected profit of component makers with the condition for positive expected utility of workers.

Ex post wages: Let $\Psi_{I-I'}$ be the aggregate shock of the $M_I - M_I'$ integrated manufacturers that hire *ex post*. Ex post wages depend on whether component makers enter *ex post* or not. They enter *ex post* if the *ex post* wage is attractively low. Using (12) and $p = bw/\rho$, one can check that

$$N - N' = \left(\frac{wb}{\rho}\right)^{-\frac{\rho}{\delta}} \left(\frac{M_o \Psi_o}{\overline{x}}\right)^{\frac{\rho}{\delta\varepsilon}} - N'\tau^{\rho}$$

so that, for a given N', the number of *ex post* component makers, $N - N'$, increases when the wage falls. So, setting $N = N'$, firms enter *ex post* when

$$\left(\frac{wb}{\rho}\right)^{\frac{\rho}{\delta}} < N'^{-1}\tau^{-\rho} \left(\frac{M_o \Psi_o}{\overline{x}}\right)^{\frac{\rho}{\delta\varepsilon}} \tag{14}$$

We now need to determine the *ex post* wage. First, if there is *ex post* entry, *ex post* wage is a simple extension of (13) where the aggregate shock of integrated manufacturers that hire *ex post* adds to the aggregate shock of outsourcing manufacturers:

$$\left(\frac{wb}{\rho}\right)^{\frac{\rho}{\delta}} = \frac{(a + b\overline{x})\left(\frac{M_o \Psi_o}{\overline{x}}\right)^{\frac{\rho}{\varepsilon\delta}} + d\,(M_I - M_I')\,\Psi_{I-I'}^{\frac{\rho}{\varepsilon\delta}}}{L - L_I' - (a + b\overline{x})N'\,(\tau - \tau^{\rho})}. \tag{15}$$

where $d \equiv c\left(\frac{\varepsilon-1}{c\rho\varepsilon}\frac{b}{\rho}\right)^{\frac{\rho}{\delta}}$. This is an increasing and convex function of N'. This expression is valid under (14), that is, using the last expression, *ex post* entry occurs if

$$\frac{L - L_I' - \tau(a + b\overline{x})N'}{N'\tau^{\rho}} > \frac{d\,(M_I - M_I')\,\Psi_{I-I'}^{\frac{\rho}{\varepsilon\delta}}}{\left(\frac{M_o \Psi_o}{\overline{x}}\right)^{\frac{\rho}{\varepsilon\delta}}} \tag{16}$$

Therefore, for each realization of state θ, there exists a $\hat{N}'(\theta)$ such that component makers enter *ex post* if and only if $N' < \hat{N}'(\theta)$.

Second, if there is no *ex post* entry, then the wage is given by the demand of integrated manufacturers that hire *ex post*. The labor market clears when $L = L_I' + \tau(a + b\overline{x})N' + L_I(w, M_I - M_I', \Psi_{I-I'})$. Using (4) we get the *ex post* wage

$$\left(\frac{wb}{\rho}\right)^{\frac{\rho}{\delta}} = \frac{d\,(M_I - M_I')\,\Psi_{I-I'}^{\frac{\rho}{\varepsilon\delta}}}{L - L_I' - \tau(a + b\overline{x})N'} \tag{17}$$

which is also an increasing and convex function of N'. Ex post wage tends to infinity as N' approaches $(L - L_I')\,/\tau(a + b\overline{x})$. As a result the *ex post* wage is a combination increasing and convex functions of N'. The workers' expected utility is then also an increasing and convex function of N'; so is the certainty equivalent $w' = u^{-1}[Eu(w)]$ that *ex ante* firms should offer *ex ante* to attract workers. When N' approaches $(L - L_I')\,/\,[\tau(a + b\overline{x})]$, the certainty equivalent w' tends to infinity.

Ex post prices: Component firms that hire ex ante commit to a constant production level and thus face price uncertainty. By (11), these firms decide to hire *ex ante*. if they make positive expected profits, that is, iff $bw'/\rho \leq (\tau\overline{x}/M_o)^{\rho-1} E\left[\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}\right]$. The difficulty in computing the expectation term in this condition comes from the fact that the price index p takes different functional forms whenever component makers enter *ex post* or not. First, when $N' < \hat{N}'(\theta)$, component makers enter *ex post* and the price index p follows the *ex post* wage according to (7). Then, the function

$$\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta} = \frac{wb}{\rho}\left(\frac{M_o}{\overline{x}}\right)^{\rho-1} \tag{18}$$

has the same properties as the *ex post* wage: it is increasing and convex in N'. Second, when $N' \geq \hat{N}'(\theta)$, component makers do not enter *ex post*, $N' = N$, and the price index is defined by (5) and (10). This gives the function

$$\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta} = \Psi_o\left(\frac{M_o}{\overline{x}}\right)^{\rho}(N'\tau^{\rho})^{-1} \tag{19}$$

which is decreasing and convex function of N'.

To sum up, for each realization of state θ, the functions w and $\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}$ evolve parallely as increasing and convex functions of N' for small N'. These functions are still convex functions of N' but diverge for large N'. In particular, when N' approaches $(L - L_I') / [\tau(a + b\overline{x})]$, the function w increases to infinity and the function $\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}$ decreases to a finite value. The function w intersect the function $\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}$ either zero times or one time from below.

Ex ante utility and profits: We now characterize the ex ante utility and profits under constant relative risk aversion. Let $U(N') \equiv bw'/\rho = (Ew^{\alpha})^{1/\alpha} b/\rho$ and let $V(N') \equiv (\tau\overline{x}/M_o)^{\rho-1} E\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}$. Because w and $\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}$ are continuous functions of N', $U(N')$ and $V(N')$ are also continuous.

Let also $\underline{N}' = \min_\theta \hat{N}(\theta)$ be the number of component firms below which those firms enter *ex post* for any realization of state θ. Let $\overline{N}' = \max_\theta \hat{N}(\theta)$ be the number of component firms above which no firm enters ex post for any realization of state θ. From (16), we can verify that there is *ex post* entry if no component enter *ex ante* ($N' = 0 \Rightarrow N > 0$) and that there is no *ex post* entry if *ex ante* entrants hire all workers ($N' \to (L - L_I') / [\tau(a + b\overline{x})] \Rightarrow N = N'$). Thus, we have that $0 < \underline{N}' \leq \overline{N}' < (L - L_I') / [\tau(a + b\overline{x})]$.

Equilibrium: Consider first $N' \in [0, \underline{N}']$. Component producers enter *ex post* in any realization of state; by (18), $V(N')$ is a multiple of $U(N')$. We readily obtain that $V(N') \geq U(N')$ iff $\tau^{\rho-1}Ew > (Ew^{\alpha})^{1/\alpha}$ where the *ex post* wage is given by (15). We can compute a threshold

$$\overline{\tau}^{1-\rho} = \frac{E\left[(a+b\overline{x})\left(\frac{M_o\Psi_o}{\overline{x}}\right)^{\frac{\rho}{\varepsilon\delta}} + d\left(M_I - M_I'\right)\Psi_{I-I'}^{\frac{\rho}{\varepsilon\delta}}\right]^{\frac{\delta}{\rho}}}{\left\{E\left[(a+b\overline{x})\left(\frac{M_o\Psi_o}{\overline{x}}\right)^{\frac{\rho}{\varepsilon\delta}} + d\left(M_I - M_I'\right)\Psi_{I-I'}^{\frac{\rho}{\varepsilon\delta}}\right]^{\frac{\delta}{\rho}\alpha}\right\}^{1/\alpha}} > 1$$

such that $V(N') > U(N')$ iff $\tau < \overline{\tau}$. Note that this threshold is independent of N' (because of the constant relative risk aversion). Therefore, firms *ex ante* enter ($N' = \underline{N}'$) if $\tau < \overline{\tau}$, no firm *ex ante* enter ($N' = 0$) if $\tau > \overline{\tau}$, firms enter for any $N' \in [0, \underline{N}']$ if $\tau = \overline{\tau}$.

Note then that if $V(N') \leq U(N')$ for $N' \in [0, \underline{N}']$, then $V(N') \leq U(N')$ for all $N' \in [0, (L - L_I') / [\tau(a + b\overline{x})]]$. Indeed for $N' > \underline{N}'$, the function $\Psi_o^{1-\rho}\mathsf{p}^{\varepsilon\delta}$ decreases in N' whereas w increases in N'. So, $V(N') \leq U(N')$ for all $N' \in [0, (L - L_I') / [\tau(a + b\overline{x})]]$ if $\tau \geq \overline{\tau}$. Hence, no component firms hire workers *ex ante* if $\tau \geq \overline{\tau}$.

We now consider the case where $\tau < \overline{\tau}$. We know that $V(N') > U(N')$ for $N' \in [0, \underline{N}']$; component firms make profits when they hire *ex ante*. For $N' \in [\overline{N}', (L - L_I') / (\tau(a + b\overline{x}))]$, by (17) and (19) we have that $V(N') < U(N')$ for large enough N' so that no component firm is enticed to hire *ex* *ante*. By continuity of $V(N')$ and $U(N')$, there exists (at least) a number N' such that $V(N') = U(N')$. The offered wage is then such that $bw'/\rho = V(N') = U(N')$.

Finally observe that component firms hire only a portion of the labor supply. Suppose the contrary: component firms exhaust the ex post labor supply so that $\overline{N}' = (L - L'_I) / [\tau(a + b\overline{x})]$. Then the ex post wage tends to infinity which pushes to the ex ante wage to infinity too. No component firm can hire ex ante: $\overline{N}'$ must be nil, a contradiction.