

# Does Loss of Biodiversity Compromise Productivity in Intensive Agriculture?

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## ABSTRACT

*This paper explores the dynamic interrelation between biological diversity, technical change and agricultural productivity. The theoretical insights regarding these linkages are furthered by deriving the comparative dynamic solutions of an optimal control model. This provides testable hypotheses that are investigated using an output-distance parametric model to a large panel of specialised cereal farms from the UK (1989-97). The results of this paper can inform policy makers on the design of sound biodiversity conservation policies in semi-natural habitats, particularly in the context of restructuring the CAP. The results suggest that productivity is positively related to technical change and that the impact of increased biodiversity on frontier output is positive with declining marginal effects over time. It is thus suggested that the objective of enhancing biodiversity levels in semi-natural habitats is being met without impairing agricultural productivity. Further, it would appear that specialised producers are converging towards a unique best practice technology.*

**KEYWORDS:** Biodiversity, Optimal Control Model, Stochastic Production Frontier, Technical change

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## 1. Introduction

The emphasis in agricultural practice in industrialised countries has been on creating the optimum environment for a single target species (the 'crop'). This has been achieved by adjusting the environment so that growing conditions for the target species are optimised while those for competing species ('weeds' and 'pests') are deliberately worsened. This view of the agro-ecosystem as involving simple competitive relationships between species has dominated agricultural practice; the co-operative or integrative multi-cropping and agro-forestry systems are now mostly found in LDCs where low input agriculture generally reflects lack of capital and specific environmental constraints for intensification of production processes. In these cases agriculture provides a multifunctional system. By contrast, the competitive vision of agricultural production ignores interactions between species and is being questioned for not encompassing factors that may significantly contribute to short and long term agro-ecosystem productivity (Mader et al., 2002). The new thrust of measuring agricultural sustainability is indicative of this (Kirchman and Thorvaldsson, 2000).

More recently it has been pointed out that ecosystem sustainability is more likely related to maintenance of specific ecosystem functions rather than species per se. This implies that sustainability is less related to the diversity of biological species than to preserving particular species that support the necessary ecosystem functions (Myers, 1996). In the context of an agro-ecosystem, additional species might reduce agricultural productivity through competition (for nutrients, light etc.), or alternatively might increase output by supporting ecosystem functions that enhance productivity (e.g. through pollination, soil nutrient enhancement, integrated pest control etc.). Thus, there is a balance being struck between direct competition between different species including crop species, and the support provided by non-crop species for the growing crop through the ecosystem functions.

This paper, which seeks to identify the effect of biodiversity conservation on agricultural productivity using a behavioural model and an empirical application to the UK, starts from the notion that land use change ranks ahead of all other physical changes as a driver of terrestrial biodiversity enhancement and degradation (Duchme et al., 1997; Swanson, 1994). It explores the dynamic interrelations between biological diversity and crop productivity. Theoretical insights from an optimal control model of bio-economic interactions in an agro-ecological system are used both to extend our understanding of the potential linkages between biodiversity enhancement and degradation, technical change and agricultural adjustment and to construct testable hypotheses.

In particular the results from a comparative dynamic analysis provide insights about likely responses to specific exogenous changes along the optimal path of the agro-ecological

system. Hypotheses that we construct around these insights are tested by applying an output-based distance function model to a large panel of specialised cereal producers in the UK.

The key relationships between agricultural activity and biodiversity are based on measures of species diversity from the Countryside Surveys (Haines-Young et al., 2000) and indices of input use and conservation activity on panel farms derived from the Farm Business Survey. Parameters of this relationship, initially estimated for the panel as a whole, are applied to the farm level data set to generate a farm level biodiversity index for all 466 farms over the sample period.

The principal focus in this paper is on the potential impact of changes in biodiversity on agricultural productivity. Section 2 describes an optimal control model that sets out a simplified framework in which these relationships can be explored from a theoretical perspective. Section 3 investigates the dynamics of the relationship between biodiversity technical change and agricultural output in more detail, and section 4 describes the farm panel data and the construction of the biodiversity index. Section 5 then outlines the specification and testing of alternative versions of the stochastic frontier model and the key results related to technical change and output. Finally some concluding comments are in section 6.

## 2. A model of biodiversity and artificial input allocation

The present model is based on the maximisation of the discounted present value of society's utility flows to perpetuity. The direct utility function is specified as  $U=U[B(t),Y(t)]$ , where  $Y(t)$  represents the flow actual agricultural output at time  $t$ , and  $B(t)$  stands for biodiversity loss attributable to intensive use artificial inputs,  $X(t)$ , and buffered by environmental conservation expenditures,  $R(t)$ . The problem is to find the optimal trade-off in the allocation of utility yielding services: agricultural food supply,  $Y(t)$ , and the biodiversity stock,  $Z(t)$ .<sup>2</sup>, assuming that the marginal utilities are as follows  $U_Y > 0, U_{YY} < 0, ,$  and  $U_B < 0, U_{BB} < 0$ , for a strictly concave and linearly separable utility function.

As agricultural output relies on the integrity of the agro-ecosystem for its productivity and sustainability, the modelling of agricultural development over time should consider the relationship between agricultural productivity and biodiversity. Recent ecological studies suggest that the relationship is positive (Bullock et al. 2001; Richards, 2001). Hence, the stock of biodiversity,  $Z(t)$ , enters into the production function alongside  $X(t)$ , i.e.  $F[X(t),Z(t)]$  represents potential agricultural output and is assumed strictly concave with  $F_Z > 0, F_{ZZ} < 0$  and  $F_X > 0, F_{XX} < 0$ , alongside weak essentiality:  $F(0) = 0$ .

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<sup>2</sup> Note that  $Z(t)$  refers to the level (*stock*) of biodiversity in time  $t$ , while  $B(t)$  refers to biodiversity 'loss' (a *flow* variable).

In the present model, biodiversity conveys a somewhat general notion at any of three levels (species, genetic and ecosystem diversity) with each level having a set of sub-components and hence a different interaction with the production process. This implies that the effect of a change in  $Z(t)$ , on the marginal product of  $X(t)$ , is likely to be different at each level or sublevel of  $Z(t)$ . For instance, an increase in insect or micro-organism diversity would increase the marginal product of fertiliser since it enhances the soil productivity ( $F_{XZ} \geq 0$ ). Alternatively, an increase in natural vegetation diversity would decrease the marginal product of fertiliser as it increases the competition against the cultivated crops ( $F_{XZ} \leq 0$ ). Similar examples could be stated for other components of biodiversity. For simplicity,  $F[X(t), Z(t)]$ , is assumed to be linearly separable in  $Z(t)$  and  $X(t)$ . i.e.  $F_{XZ} = F_{ZX} = 0$ . In order to include the effect of technological progress, a dynamic production function is proposed in the form of  $F[X(t), Z(t), A(t)]$ , where  $A(t)$  represents the state of art, or an exogenous representation of the production possibility frontier.

The ‘biodiversity impact (or ‘loss’) function’,  $B=B[X(t), Z(t)]$ , is assumed to depend on the level of agricultural intensification through use of  $X(t)$ , and on the existing state of biodiversity,  $Z(t)$ . The latter effect is included to reflect the notion that the level of biodiversity makes a positive contribution to ecosystem resilience, in the sense that biodiversity can enhance the ability of the agro-ecosystem to tolerate and overcome the adverse effect of agricultural activities (Xu and Mage, 2001; Trenbath, 1999; Swanson, 1997). It is further assumed that, at the margin, biodiversity loss,  $B$ , increases (decreases) at an increasing (decreasing) rate due to increases in input intensification (biodiversity stock) i.e.  $B_X > 0, B_{XX} > 0$ , and  $B_Z < 0, B_{ZZ} > 0$ , and that the biodiversity impact function is linearly separable in  $X$  and  $Z$ , i.e.  $B_{XZ} = B_{ZX} = 0$ .

The problem is to choose the optimal time paths of the control variables ( $Y(t)$  and  $R(t), X(t)$ ), accounting for the evolution of  $Z(t)$ . In general this evolution ought to reflect (a) the natural growth function of  $Z$ , (b) the conservation activities undertaken,  $R$ , and (c) the intensification of artificial input use:

$$\dot{Z} = G[Z(t), X(t), R(t)] \quad (1)$$

Using an extended logistic function:

$$\dot{Z} = \alpha Z(1 - Z/K) + \delta R - \gamma X \quad (1a)$$

where  $\alpha > 0$  reflects the natural rate of growth of  $Z$  and  $K$  stands for the agro-ecosystem’s biodiversity carrying capacity. On intensified agricultural systems it is typical to find relatively low levels of  $Z$  relative to the potential carrying capacity. Hence, since the term  $Z/K$  is possibly negligible, equation (1a) simplified to yield:

$$\dot{Z} = \alpha Z + \delta R - \gamma X \quad (1b)$$

where  $\alpha$ ,  $\delta$  and  $\gamma$  are all constant parameters. According to Eq. (1b),  $Z$  is enhanced proportionally to investment in conservation,  $R$ ,  $\delta$  being the rate of induced growth<sup>3</sup>, and it is proportionally reduced due to artificial input application. It is worth noting that whilst biodiversity is considered to be natural capita, it is assumed that no depletion in biodiversity occurs as a result of its support to the production process.

Since the optimisation problem is specified with an infinite time horizon, to allow for inter-temporal interactions between agriculture and its impact on biodiversity, we show that the solution of the first order conditions would lead to a steady state marked as  $(\bar{Z}, \bar{Y}, \bar{X}, \bar{\varphi})$  and it is reachable from the initial state condition  $Z(0) = Z_0$ . That is, there is an implicit terminal state  $\lim_{t \rightarrow \infty} Z(t) = \bar{Z}(\phi)$  where  $\phi$  is a vector of exogenous parameters and variables including the discount rate,  $\rho$ , and technological progress,  $A$ .

The aggregate objective function is defined as follows:

$$\underset{Y, X, R}{\text{Max}} W(Y(t), B(t)) = \int_{t=0}^{\infty} e^{-\rho t} u(Y(t), B(t)) dt \quad (2)$$

where  $\rho > 0$  is the discount rate;

subject to

- (i) the equation of motion for  $Z(t)$ ,
- (ii) the non-negativity constraints, i.e.  $X \geq 0$  and  $B \geq 0$ ,
- (iii) the initial condition  $Z(0) = Z_0$ ,
- (iv) the impact function  $B(\cdot)$ ,
- (v) the environmental conservation investment function (3):

$$R(t) = F(X(t), Z(t)) - Y(t) \quad (3)$$

The current-value Hamiltonian is in turn:

$$H_c = U(Y, B) + \varphi(\alpha Z + \delta F(\cdot) - \delta Y - \gamma X) \quad (4)$$

where  $\varphi$  is the current shadow value of biodiversity or costate variable. Applying the Maximum Principle for an optimal interior solution shows that:<sup>4</sup>

$$\frac{\partial H_c}{\partial \varphi} = \dot{Z} = \alpha Z + \delta[F(\cdot) - Y] - \gamma X \quad (5a)$$

$$\frac{\partial H_c}{\partial Y} = U_Y - \delta \varphi = 0 \quad (5b)$$

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<sup>3</sup> The parameter  $\delta$  also can be interpreted as the marginal degradation in  $Z(t)$  caused by increase in  $Y(t)$  i.e. the opportunity cost of  $R(t)$ .

<sup>4</sup>See Omer et al. (2003) to verify that the current value Hamiltonian is maximised.

$$\frac{\partial H_c}{\partial X} = U_B B_X + \varphi[\delta F_X - \gamma] = 0 \quad (5c)$$

$$\dot{\varphi} = -\frac{\partial H_c}{\partial Z} + \rho\varphi = -U_B B_Z - \varphi[\alpha + \delta F_Z - \rho] \quad (5d)$$

Equation (5a) restates the state equation, (5b) establishes that the current shadow value of biodiversity ( $\varphi$ ) is positive, while (5c) states that  $X$  should be allocated such that the marginal utility and disutility of artificial input use are balanced. For an interior solution, the bracketed term ( $\delta \partial F / \partial X - \gamma$ ) is positive as  $\varphi$  is positive and the first term is unambiguously positive. From (5b-5c)  $X$  can be defined as an implicit function of  $Y$  and  $Z$  with  $X_Z > 0$  and  $X_Y < 0$ , i.e.  $X(Y, Z)$  is the level of  $X$  that solves the FOCs.

### 3. The effect on agricultural output of technological change and biodiversity

This section looks at the effect of technological change ( $A$ ) on (a) the steady state equilibrium levels for  $Y$  and  $B$  (static comparative analysis), and (b) the optimal time paths to such an equilibrium (dynamic comparative analysis).

The steady state solution of this agro-system is<sup>5</sup>

$$\bar{Z} = -\frac{1}{|J_s|} \left[ f_Y(\alpha Z + \delta F - \delta Y - \gamma X) + g_Y \frac{U_Y}{U_{YY}} \left( \alpha - \rho + \delta F_Z - [\delta F_X - \gamma] \frac{B_Z}{B_X} \right) \right] \quad (6a)$$

$$\bar{Y} = -\frac{1}{|J_s|} \left[ -f_Z(\alpha Z + \delta F - \delta Y - \gamma X) - g_Z \frac{U_Y}{U_{YY}} \left( \alpha - \rho + \delta F_Z - [\delta F_X - \gamma] \frac{B_Z}{B_X} \right) \right] \quad (6b)$$

To investigate the effect of an exogenous change in  $A$  on the steady state, we differentiate Eq. (6) with respect to  $A$ :

$$\frac{\partial \bar{Z}}{\partial A} = \frac{-\delta F_A f_Y}{|J_s|} > 0 \quad (7a)$$

$$\frac{\partial \bar{Y}}{\partial A} = \frac{\delta F_A f_Z}{|J_s|} > 0 \quad (7b)$$

According to the model, an increase in technological progress, leads to higher steady state value of both  $Z$  and  $Y$ .

The comparative dynamics analysis shows how the state and control variables change along their optimal time paths in response to changes in technological progress ( $A$ ). The time paths, defined by the definite solution of the dynamic system of the model, are given as:

$$\begin{bmatrix} Z(t; Z_0, \phi) \\ Y(t; Z_0, \phi) \end{bmatrix} = \begin{bmatrix} \bar{Z} \\ \bar{Y} \end{bmatrix} + [Z_0 - \bar{Z}] \begin{bmatrix} 1 \\ k_1 \end{bmatrix} e^{n t} \quad (8)$$

<sup>5</sup> The derivation of the comparative static solutions are shown in the appendix.

where  $k_1$  is defined as follows:

$$k_1 = \frac{-\dot{Y}_Z - (r_1 - \dot{Z}_Z)}{r_1 - \dot{Y}_Z + \dot{Z}_Y} \quad (8a)$$

To derive the local comparative dynamics, the time paths given by (8) are differentiated with respect to  $A$  to show both the short-run and long run effect:<sup>6</sup>

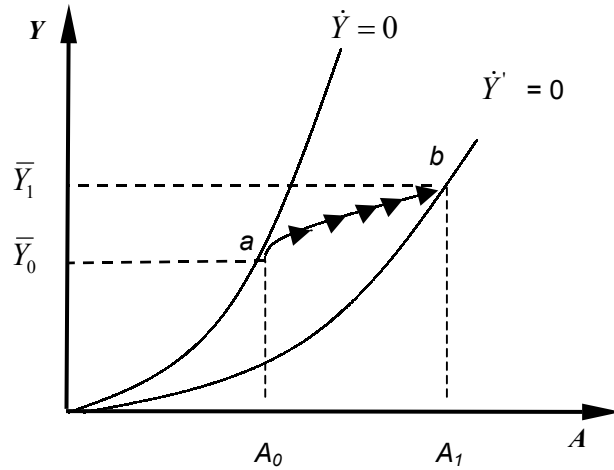
$$\left. \frac{\partial Z(t; Z_0, \phi)}{\partial A} \right|_{Z_0 = \bar{Z}} = \bar{Z}_A (1 - e^{r_1 t}) > 0 \quad (9a)$$

$$\left. \frac{\partial Y(t; Z_0, \phi)}{\partial A} \right|_{Z_0 = \bar{Z}} = \bar{Y}_A - \bar{Z}_A k_1 e^{r_1 t} > 0 \quad (9b)$$

Eq (9b) states that optimal levels of  $Y(t)$  increase with an increase in technological progress, but the rate of increase declines as the term  $\bar{Z}_A k_1 e^{r_1 t}$  approaches zero as time approaches infinity, given  $k_1 < 0$  and  $r_1 < 0$ . That is, the effect of improving technology in crop production shifts the steady state of  $Y$  to a higher level. Figure 1 depicts the effect in a  $(A, Y)$  space. The left hand side of (9b) is the slope of the  $Y$  isocline, which equals  $\bar{Y}_A$  at  $t=0$ , and begins to decline as  $t$  increases since the negative term  $\bar{Z}_A k_1 e^{r_1 t}$  becomes smaller as  $k_1 < 0$  and  $r_1 < 0$ . A positive change from  $A_0$  to  $A_1$ , (through time) is associated with a higher steady state for  $Y$ , located on a new ‘flatter’ isocline (Figure 1). The increase in crop output due to technical change is a key prediction which comes as a testable hypothesis in the next section. Similarly the model also predicts that biodiversity increases due to technological progress but while the rate of increase is initially below that at the steady state, it increases over time as  $e^{r_1 t}$  goes to zero (as time approaches infinity).

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<sup>6</sup> The derivation of the comparative dynamics are shown in the appendix.

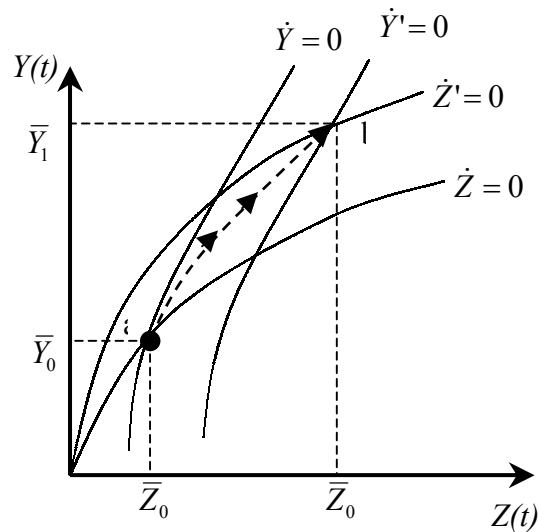


**Figure 1: Dynamic comparative analysis of a change in A on long run level of Y**

The impact of biodiversity on output can also be investigated through comparative dynamic analysis.

$$\left. \frac{\partial Y(t; Z_0, \phi)}{\partial Z} \right|_{Z_0 = \bar{Z}} = -k_1 e^{r_1 t} > 0 \quad (9c)$$

Equation (9c) states that the along the optimal path Y increases over time when biodiversity increases. It can be noted that as time increases, the optimal rate of change in Y decreases until the new steady state is reached. The change towards the new steady state value for Y is depicted graphically in Figure 2. Because the Y and Z isoclines are upward sloping (Omer et al, 2003), any change in Z will increase the steady state level of Y (figure 2). This hypothesis is also tested in the empirical model.



**Figure 2: Dynamic comparative analysis of a change in Z on long run level of Y**



The empirical analysis will therefore focus on two key predictions that relate to the dynamic behaviour of agricultural productivity:

- a) Agricultural productivity *along the optimal path* is positively related to technical change
- b) The impact of changes in biodiversity on frontier output is positive and declining over time

#### 4. The Data

Empirical analysis in this study is focused on testing the two key predictions of the theoretical model as indicated above. This is carried out using a data set constructed from the UK Farm Business Survey (FBS)<sup>7</sup> records of a panel of approximately 230 specialist cereal producers (from 1989 to 1997). In addition biodiversity measures taken from plot-level observations in the UK Countryside Survey have been used to construct a farm level biodiversity index. This dataset (described in more detail below) is used to estimate the parameters of a dynamic production frontier. It is assumed that this frontier models the optimal path of actual agricultural output corresponding to  $Y$  in the theoretical model.<sup>8</sup>

The variables included in the analysis are: (i) value of crop output per hectare (£UK), the main crop enterprise output excluding set-aside payments, (ii) total labour cost per ha, including imputed cost of family and hired labour, (iii) cost of machinery use per ha, (iv) cost of fertiliser use per ha, and (v) cost of pesticide use per ha. The data on costs and revenues have been deflated by the relevant Agricultural Price Index (API), base year 1990.<sup>9</sup> In addition, a biodiversity index measure is included in the analysis (the following section explains the data and construction of this index). Summary statistics for these variables are shown in table 1.

**Table 1: Summary statistics for selected variables on cereal farms in the East of England**

Variable	Mean	St.Dev	Minimum	Maximum
Output	877.82	214.00	261.55	2130.21
Biodiversity (index)	13.49	1.04	9.36	16.52
Fertiliser	85.93	29.35	0.68	329.59
Labour	165.03	87.52	4.41	1093.45
Machinery	211.75	85.97	12.55	873.63
Pesticide use	89.36	25.81	0.51	221.93

<sup>7</sup> The UK FBS is an annual survey undertaken by the Department of Environment, Food and Rural Affairs (DEFRA).

<sup>8</sup> Note that the model in section 2 assumes that farmers are fully efficient, and thus, they can be said to operate on the production frontier.

<sup>9</sup> The price indices which are taken from DEFRA's website, [www.defra.gov.uk](http://www.defra.gov.uk)

Area (ha)	175.64	131.65	8.90	1052.79
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Values per hectare unless otherwise specified.

### The Biodiversity Index:

The biodiversity index used in this study is based on measures of plant diversity from the major Countryside Surveys undertaken in 1978, 1990 and 1998. Data was available for individual survey plots located in six Environmental Zones (EZs) across England. This study focuses on data for Environmental Zone 1 since the boundaries of this zone correspond most closely to those of the area spanned by the panel of farms. Measures of species richness<sup>10</sup> were supplied for eight Aggregate Vegetation Classes (AVCs)<sup>11</sup> and a number of Broad Habitats in this Zone.

A single biodiversity index has been constructed from this data, following the aggregation approach used by the Centre of Agriculture and Environment of the Netherlands (Wenum et al. 1999). Since farms may cover more than one habitat type, habitat diversity has been accounted for. Further, the index is also corrected to account for the fact that farms often present different landscape features, e.g. hedges, walls and field margins, which typically host diverse vegetation classes. The biodiversity index,  $Z$  is given by:

$$Z_k = \sum_j \sum_i a_{kj} n_{kij} S_{kij} \quad (10)$$

where,  $S_{kij}$  is the mean species richness of AVC  $i$  in Broad Habitat  $j$  in  $EZ_k$ ;  $n_{kij}$  is a measure of AVC  $i$  dominance in Broad Habitat  $j$  (the proportion of the number of plots of AVC  $i$  in BH  $j$  to the total number of plots of all AVCs in BH  $j$ ), and,  $a_{kj}$  is the scalar associated with Broad Habitat dominance in  $EZ_k$  (the proportion of the area of BH  $j$  in  $EZ_k$  to the total area of all BH in  $EZ_k$ ).<sup>12</sup>

Besides the 1978, 1990 and 1998 periods for which the data from the major surveys is available<sup>13</sup> two additional observations, for 1997 and 1999, have been constructed from the national estimates on each AVC published as part of CS2000 results adjusted for EZ. This biodiversity index together with the measures on biodiversity conservation  $R$  and agricultural input use  $X$  are used to estimate by OLS the parameters (Table 2) of a discrete-time aggregate version of the state equation of biodiversity:

$$Z_{t+1} - Z_t = \alpha \ln Z_t - \gamma \ln X_t + \delta R_t, \quad (11)$$

<sup>10</sup> A measure of species richness per plot, based on counting only native and consistently identified species, is used in CS as a simple measure of plant diversity.

<sup>11</sup> The Countryside Vegetation System (CVS) describes eight aggregate vegetation types

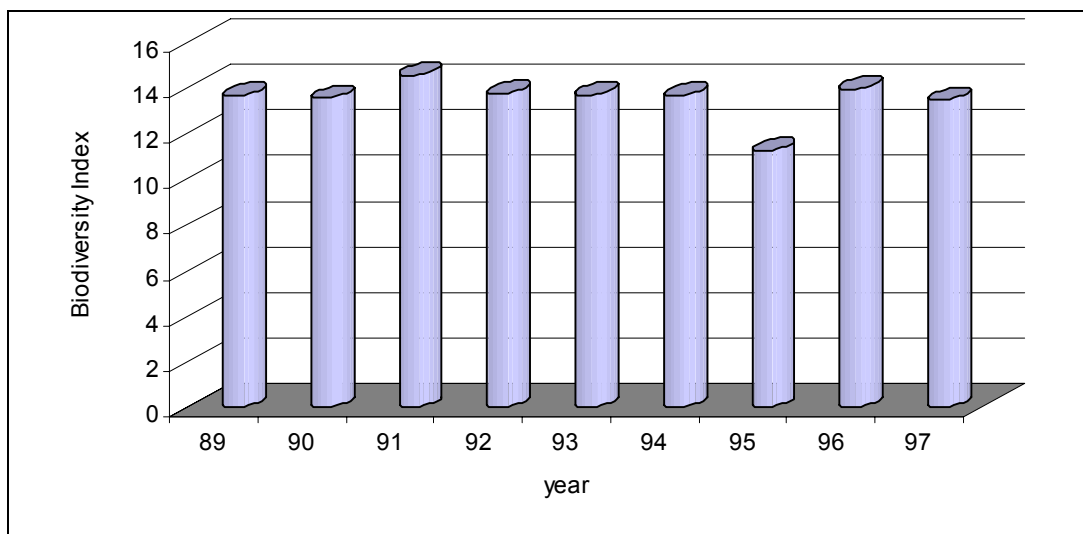
<sup>12</sup> Data on area of broad habitats is taken from the CS2000 website, [www.cs2000.org.uk](http://www.cs2000.org.uk)

<sup>13</sup> The data for 1978 is not presented by BH, so the BH breakdown from 1990 is used as a proxy for 1978 by merging the two data sets by plot id, in SAS, and then using only those plots for 1978 which are repeated in 1990 to construct the 1978 index.

A dummy variable is specified for  $R$  based on the introduction of Agri-environmental schemes following the reforms of the CAP<sup>14</sup> in 1992, while the national average of pesticide use is used as a proxy measure for  $X$ .

The calibrated parameters (standard deviations in brackets) are:  $\alpha=0.32$  (0.18),  $\gamma=2.24$  (0.88),  $\delta=0.31$  (0.41). Using these values, in the state equation allows estimation of the value of  $Z$  (using an iterative approach) at any particular year ( $t$ ) for any of the surveyed farms given farm observed values for  $R$  and  $X$  and a starting value for  $Z$ : Average farm specific biodiversity indexes for the period are shown in Figure 3.

**Figure 3. Biodiversity Index, 1989-97**



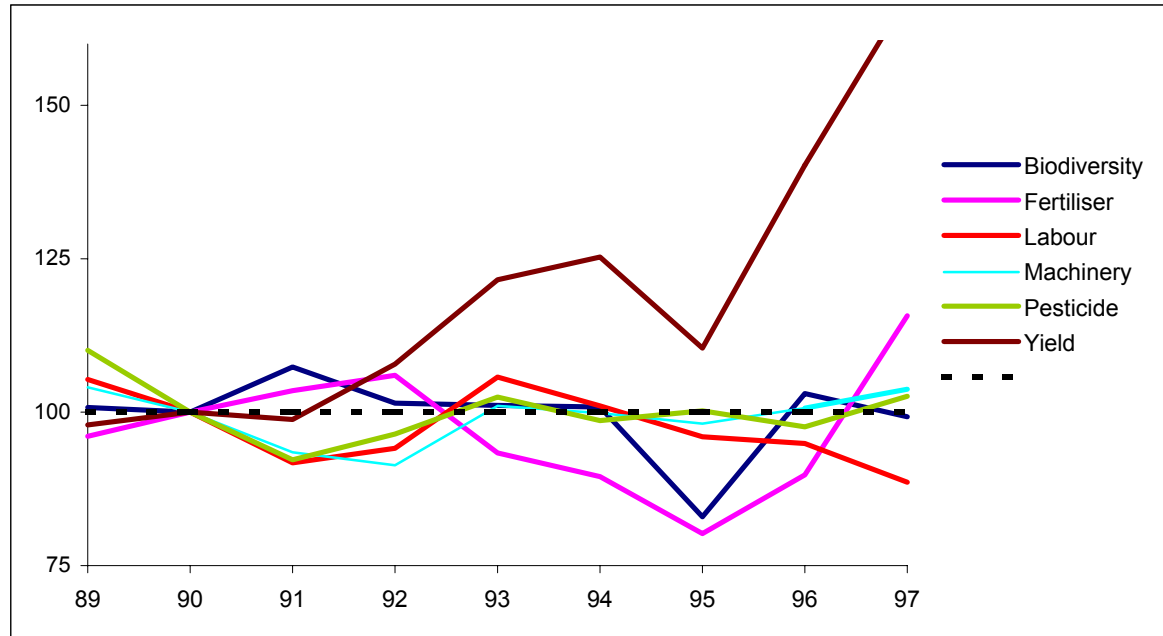
The complete per hectare data set in index form (1990 = 100), including the biodiversity index, is presented in Figure 4. This shows that biodiversity remains approximately constant over the data period as a whole, rising slightly to 1991, then declining significantly to 1995, recovering in the latter years of the data period. Fertilizer follows a similar pattern with a more substantial decline to 1995 and a more vigorous recovery towards the end of the period. Labour, machinery and pesticide use decline initially, recovering to a peak in 1993 after which labour continues to decline while machinery and pesticides remain at approximately 1990 levels. Value of output per hectare increases substantially over the period, with a significant dip below trend in 1995 and a substantial recovery towards the end of the period.

There are clearly two identifiable dynamic patterns. On the one hand labour, machinery and pesticide use follows a pattern of initial decline with recovery to peak in 1993 followed by constant or slight decline. On the other hand the behaviour of biodiversity

<sup>14</sup> The dummy values are zero for periods before 1993 and one for and after 1993.

pesticide and output is more volatile. All three data series show a significant reduction to a trough in 1995. This would point to biodiversity and fertilizer use as the key determinants in output and also to the weak relationship between labour use and output.

**Figure 4: Average values for all inputs, 1989-97**



Note: The baseline data values for 1990 are as follows: Biodiversity = 13.53 (index); Fertilizer = £88/ha; Labour=£169/ha; Machinery = £213/ha; Pesticide = £89/ha; Output = £737/ha.

### 5. The Stochastic Frontier Model:

A Cobb-Douglas stochastic frontier production model is defined for arable crop production on cereal farms in the East of England<sup>15</sup>

$$Y_{it} = \beta_0 + \sum_k \beta_k X_{kit} + V_{it} - U_{it} \quad (12)$$

where

$Y_{it}$  is the log of crop output of the  $i^{\text{th}}$  farm at time period  $t$  (hundreds pounds per ha);

$X_1$  is the log of biodiversity;

$X_2$  is the log of fertiliser use (hundreds pounds per ha);

$X_3$  is the log of labour use (hundreds pounds per ha);

$X_4$  is the log of machinery use (hundreds pounds per ha);

$X_5$  is the log of pesticide use(hundreds pounds per ha);

$X_6$  is the year of observation where  $X_6 = 1, 2, \dots, 9$ ;

<sup>15</sup> A translog was also tried but the interaction terms created significant multicollinearity.

The  $V_{it}$ s are assumed to be independently and identically  $N(0, \sigma_v^2)$  distributed random errors that are independent of the  $U_{it}$ s. The  $U_{it}$ s are non-negative random variables associated with technical inefficiency of production. Three different frontier models are considered based on different specifications for the  $U_{it}$ s.

The Cobb-Douglas stochastic frontier production function (12) is estimated<sup>16</sup>, given three different specifications of the technical inefficiency effects defined by equations (12a), (12b) and (12c). Several versions of each of these three models were estimated to test various hypotheses using the generalized likelihood ratio statistics (Table 3).

**Model 1** is a time-varying inefficiency model, as described by Battese and Coelli (1992), in which the inefficiency effects are defined as:

$$U_{it} = \{\exp[-\eta(t-T)]\}U_i \quad (12a)$$

where  $\eta$  is an unknown parameter to be estimated, and  $U_i$  are independent and identically distributed random variables obtained by the truncation at zero of a the  $N(0, \sigma_u^2)$  distribution. The parameter estimates for model 1 are given in table 4.

**Model 2** is a neutral stochastic frontier model, based on Battese and Coelli (1995), in which the inefficiency effects are defined as

$$U_{it} = \delta_0 + \sum_j \delta_j Z_{jit} + W_{it} \quad (12b)$$

where  $Z_1$  is farmer's age (years),  $Z_2$  represents 'environmental payments',  $Z_3$  is a dummy variable defined for participation in agri-environmental schemes,  $Z_4$  is a hired labour index,  $Z_5$  is a dummy variable defined for hiring labour,  $Z_6$  is the year of observation, and  $W_{it}$ s are unobservable random variables that are independent and identically distributed, obtained by the truncation at zero of a the  $N(0, \sigma_u^2)$  distribution, such that  $W_{it}$  are non-negative.

**Model 3** is a non-neutral stochastic frontier model, based on Huang and Liu (1994), in which the inefficiency effects are defined as

$$U_{it} = \delta_0 + \sum_j \delta_j Z_{jit} + \sum_j \sum_k \delta_{jk} X_{kit} Z_{jit} + W_{it} \quad (12c)$$

This model is an extended version of model 2, in which there are interactions between farm-specific variables ( $Z$ s) and the input variables ( $X$ s) in the stochastic frontier. However, it should be noted that model 2 and 3 are not a generalization of model 1, and thus cannot be tested against model 1

Table 2 shows the results of testing some interesting null hypotheses for the specification of the above models.

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<sup>16</sup> Maximum likelihood estimates of the parameters of these models were obtained using the computer program, FRONTIER version 4.1 (Coelli, 1996).

**Table 2: Generalized Likelihood-Ratio Tests for Parameters of the Stochastic Frontier Production Models for Cereal Farmers in East of England**

Null Hypothesis	log likelihood	LR	CV (10%)	C V(5%)
<b>Model 1</b>	1126.68			
$H_0: \gamma = \mu = \eta = 0$	669.80	913.75	6.25	7.81473*
$H_0: \beta_6 = 0$	875.95	501.45	2.71	3.84
$H_0: \eta = 0$	1109.70	33.96	2.71	3.84
$H_0: \mu = 0$	1125.00	3.36	2.71	3.84
<b>Model 2</b>	855.02			
$H_0: \gamma = \delta_0 = \delta_j = 0$	669.80	370.43	14.07	16.274*
$H_0: \beta_6 = 0$	714.51	281.01	2.71	3.84
$H_0: \delta_1 = \dots = \delta_6 = 0$	798.82	112.39	10.64	12.59
<b>Model 3'</b>	954.92			
$H_0: \gamma = \delta_0 = \delta_j = \delta_{jk} = 0$	669.80	570.24	51.25	55.19*
$H_0: \beta_6 = 0$	812.99	283.87	2.71	3.84
$H_0: \delta_{jk} = 0, k, j = 1, \dots, 6$	855.02	199.81	40.25	43.77
$H_0: \delta_{6k} = \delta_{k6} = 0, k = 1, \dots, 6$	901.27	107.31	9.24	11.07
$H_0: \delta_{3k} = \delta_{4k} = 0, k = 1, \dots, 6$	935.66	38.53	17.28	19.92

\*This CV (critical value) is obtained from Kodde and Palm (1986).

Given model 1 specification, the null hypothesis that technical inefficiency is not present, i.e.  $H_0: \gamma = \mu = \eta = 0$ , is rejected by the data. The null hypothesis of no technical change,  $H_0: \beta_6=0$ , is also rejected and the hypothesis that the technical inefficiency effects are time invariant,  $H_0: \eta = 0$ , is rejected as well. Further, the half-normal distribution seems an adequate representation for the distribution of the farm technical inefficiency effects, i.e.  $H_0: \mu = 0$ , is not rejected at the 5% level of significance. These hypotheses tests suggest that the preferred model is Model 1, with a half normal distribution and time-varying farm inefficiency effects. For this model, it can be noted that  $\eta > 0$ , implying that technical inefficiency decrease over time.

Given model 2, the null hypothesis that inefficiency is not present,  $H_0: \gamma = \delta_0 = \delta_j = \delta_{jk} = 0$ , is strongly rejected, and the null hypothesis of no technical change,  $H_0: \beta_6 = 0$ , can be also rejected. The hypothesis that the neutral model (Model 2) is an adequate representation of the data,  $H_0: \delta_{jk} = 0$ , is also rejected by the data, similarly to the null for no year interaction with the explanatory variables in the inefficiency sub-model,  $H_0: \delta_{6k} = 0$ . Lastly, the null of

no interaction between the dummy variables in the inefficiency sub-model and the input variables,  $H_0: \delta_{3k} = \delta_{4k} = 0$ , is also rejected. These tests indicate that Model 3 is preferred to Model 2. The parameter estimates are given in table 3.

**Table 3: MLE parameter estimates of the generalized Cobb-Douglas stochastic frontier production models 1 and 3**

Variable	Parameter	Model 1		Model 3	
		Coefficient	T-ratio	Coefficient	T-ratio
Constant	$\beta_0$	0.69	7.73	0.91	4.56
Biodiversity	$\beta_1$	0.51	15.35	0.41	5.31
Fertilizer	$\beta_2$	0.08	8.41	0.09	7.41
Labour	$\beta_3$	0.01	1.15	0.02	1.73
Machinery	$\beta_4$	0.08	6.60	0.13	7.59
Pesticides	$\beta_5$	0.12	10.93	0.10	7.10
Time	$\beta_6$	0.06	30.53	0.05	29.32
<b><u>Inefficiency model</u></b>					
Constant	$\delta_0$			-0.38	-2.10
Age	$\delta_1$			-0.08	-3.82
Environmental pay	$\delta_2$			0.08	1.90
D1	$\delta_3$			0.24	0.25
Hired labour Index	$\delta_4$			0.24	0.26
D2	$\delta_5$			-0.39	-0.42
Time	$\delta_6$			1.05	5.43
Biodiversity-Age	$\delta_{11}$			0.03	4.10
Biodiversity-Env. Pay	$\delta_{12}$			-0.03	-2.01
Biodiversity-D1	$\delta_{13}$			0.14	0.40
Biodiversity-Hired lab.	$\delta_{14}$			-0.20	-0.57
Biodiversity-D2	$\delta_{15}$			0.13	0.36
Biodiversity-Time	$\delta_{16}$			-0.40	-5.36
Fertilizer-Age	$\delta_{21}$			0.00	2.61
Fertilizer-Env. Pay	$\delta_{22}$			0.02	1.40
Fertilizer-D1	$\delta_{23}$			0.32	1.07
Fertilizer-Hired lab.	$\delta_{24}$			0.24	2.47
Fertilizer-D2	$\delta_{25}$			-0.01	-0.06
Fertilizer-Time	$\delta_{26}$			-0.06	-4.05
Labour-Age	$\delta_{31}$			0.00	2.10
Labour-Env. Pay	$\delta_{32}$			0.00	-0.64
Labour-D1	$\delta_{33}$			-0.15	-1.10
Labour-Hired lab.	$\delta_{34}$			-0.09	-1.26
Labour-D2	$\delta_{35}$			-0.01	-0.14
Labour-Time	$\delta_{36}$			0.00	-0.01
Machinery-Age	$\delta_{41}$			0.00	-0.05
Machinery-Env. Pay	$\delta_{42}$			-0.01	-0.91
Machinery-D1	$\delta_{43}$			-0.02	-0.11
Machinery-Hired lab.	$\delta_{44}$			0.15	1.83
Machinery-D2	$\delta_{45}$			0.08	0.99
Machinery-Time	$\delta_{46}$			0.04	4.05
Pesticides-Age	$\delta_{41}$			0.00	2.80
Pesticides-Env. Pay	$\delta_{42}$			0.00	-0.03

Pesticides-D1	$\delta_{43}$		0.14	0.78
Pesticides-Hired Lab.	$\delta_{44}$		0.15	1.47
Pesticides-D2	$\delta_{45}$		-0.46	-5.05
Pesticides-Time	$\delta_{46}$		-0.04	-3.06
Time-Age	$\delta_{41}$		0.00	2.32
Time-Env. Pay	$\delta_{42}$		0.00	3.98
Time-D1	$\delta_{43}$		-0.08	-3.26
Time-Hired Lab.	$\delta_{44}$		0.00	-0.30
Time-D2	$\delta_{45}$		0.00	-0.10
Time-Time	$\delta_{46}$		-0.02	-5.93
$\sigma^2$		0.05	11.08	0.07
$\gamma$		0.72	27.43	0.85
$\eta$		0.05	5.83	
Log-likelihood		1125.00		954.92

D1: Dummy variable for environmental payments received (1 if received, 0 otherwise); D2 dummy variable for hired labour (1, if positive expenditures in hired labour, 0 otherwise)

### Elasticities of mean production for cereal producers:

The elasticity of mean production with respect to  $k^{\text{th}}$  input variable for a non-neutral stochastic frontier production function is

$$\frac{\partial \ln E(Y_{it})}{\partial X_k} = \frac{\partial \beta X}{\partial X_k} - C_{it} \left( \frac{\partial \mu_{it}}{\partial X_k} \right) \quad (13)$$

where

$$\mu_{it} = \delta_0 + \sum_j \delta_j Z_{jit} + \sum_j \sum_k \delta_{jk} X_{kit} Z_{jit} \quad (13a)$$

$$C_{it} = 1 - \frac{1}{\sigma} \left\{ \frac{\phi\left(\frac{\mu_{it}}{\sigma} - \sigma\right)}{\varphi\left(\frac{\mu_{it}}{\sigma} - \sigma\right)} - \frac{\phi\left(\frac{\mu_{it}}{\sigma}\right)}{\varphi\left(\frac{\mu_{it}}{\sigma}\right)} \right\}_{it} \quad (13b)$$

and  $\phi$  and  $\varphi$  represent the density and distribution functions of the standard normal random variable, respectively.

The elasticity of mean output with respect to  $k^{\text{th}}$  input variable in (14) has two components. The first one is the traditional elasticity of the output with respect to the  $k^{\text{th}}$  input,  $\frac{\partial \beta X}{\partial X_k}$ , which is referred to as the ‘*elasticity of frontier output*’. For the Cobb-Douglas non-neutral stochastic frontier production function, the coefficients of the logarithm of the inputs,  $\beta_k$ s, are the elasticities of the frontier crop output with respect to the corresponding input. The estimated *frontier elasticities*. are shown in table 4.



The second component of the elasticity of mean output,  $-C_{it}\left(\frac{\partial\mu_{it}}{\partial X_k}\right)$ , is the ‘technical efficiency elasticity’ with respect to the  $k^{\text{th}}$  input. According to the non-neutral SPF model (Model 3), the frontier, efficiency and mean output elasticities for each of the inputs are presented in table 4. Mean output elasticities by input, for each year, are presented in table 5 and illustrated in figure 5.

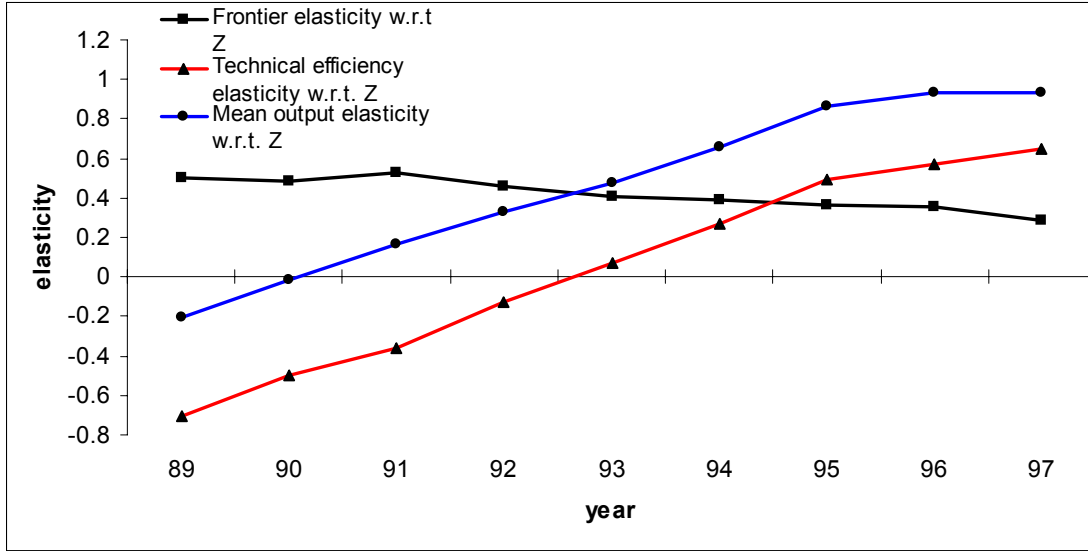
**Table 4: Elasticities of crop output with respect to all the inputs**

Variable	Frontier output	Technical efficiency	Mean output
Biodiversity	0.41	0.07	0.48
Fertilizer	0.09	-0.07	0.02
Labour	0.02	-0.02	-0.01
Machinery	0.13	-0.17	-0.04
Pesticides	0.10	0.11	0.21
Time	0.05	0.06	0.11
Return to Scale			0.67

**Table 5: The elasticities of mean crop output with respect to the different inputs for each year (1989-98)**

Year	Biodiversity	Fertilizer	Labour	Machinery	Pesticides	Technical change
1989	-0.21	-0.07	-0.01	0.07	0.18	0.00
1990	-0.01	-0.04	0.00	0.04	0.20	0.03
1991	0.17	-0.02	-0.01	-0.02	0.22	0.07
1992	0.33	0.01	-0.01	-0.03	0.21	0.09
1993	0.48	0.02	0.00	-0.04	0.21	0.11
1994	0.66	0.03	0.00	-0.06	0.21	0.13
1995	0.86	0.06	-0.01	-0.10	0.26	0.18
1996	0.93	0.07	-0.01	-0.10	0.23	0.17
1997	0.94	0.10	-0.01	-0.11	0.22	0.17
Avg. 89-97	0.48	0.02	-0.01	-0.04	0.21	0.11

**Figure 5: Change in elasticity of crop output with respect to Biodiversity)**



The results depicted in Figure 5 are consistent with the prediction generated in (9c): the impact of biodiversity on *frontier output* is positive and declining. This implies that, for frontier output among the sampled farms, returns to increases in biodiversity are declining. In addition, as also shown in figure 5, increased biodiversity has been associated with increased technical efficiency after 1992, when broad based environmental payments were introduced. Thus, up to 1992 higher biodiversity was associated with producers that were furthest below the production frontier, while after this time, increased levels of biodiversity seem to have increased efficiency levels. This also implies that the net impact of biodiversity on mean output has been positive over most of the data period.

**Technical progress and productivity change:**

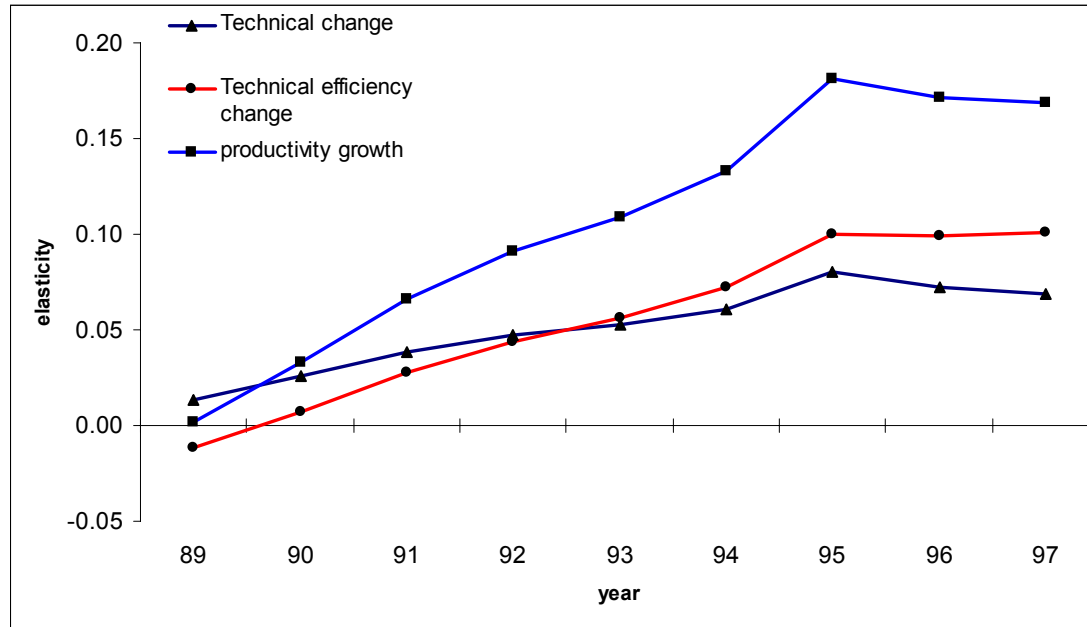
The SPF model 3 also allows to investigate productivity growth by obtaining estimates of the time derivative of the mean crop output. The estimated time coefficient is significantly different from zero, and being positive ( $\hat{\beta}_6 = 0.06$ ), indicates there is positive annual technical progress in mean frontier crop output of about 6%. This supports the positive predicted by the theoretical model, equation (9b).

As with the impact of biodiversity the impact of exogenous technical progress (the rate of the productivity growth over time) is decomposed into two components associated with technical change and efficiency change (Battese et al. 2000). This decomposition of the rate of change of mean crop output with respect to time is given by

$$\frac{\partial \ln E(Y)}{\partial t} = \frac{\partial X\beta}{\partial t} - C\left(\frac{\partial \mu}{\partial t}\right) \tag{14}$$

where  $\frac{\partial X\beta}{\partial t}$  represents the impact of exogenous technical change and  $-C\left(\frac{\partial\mu}{\partial t}\right)$  shows the impact of this change in efficiency levels. Figure 6 illustrates these values for the current analysis.

**Figure 6: The rate of change of mean crop production with respect to time**



### Technical Efficiency and Convergence to best practice:

It has been emphasised here that while the theoretical model assumes that all farmers are fully efficient (i.e. they operate on the frontier using best practice technology), the empirical model allows that some farmers may be more successful than others at achieving best practice. The systematic part of deviations in output and input use from frontier levels are represented by the measures of technical efficiency that are provided by this model. Though not encompassed by the theoretical framework adopted here, we believe that additional analysis of these measures can provide useful insights for the problems being investigated. In particular, these measures and how they change over time can illustrate convergence (or non-convergence) to best practice among our sample of farms.

Given the specification of the stochastic frontier production function defined by equation 12, the technical efficiency of a given farm at a given time period is calculated as (Battese and Coelli 1992):

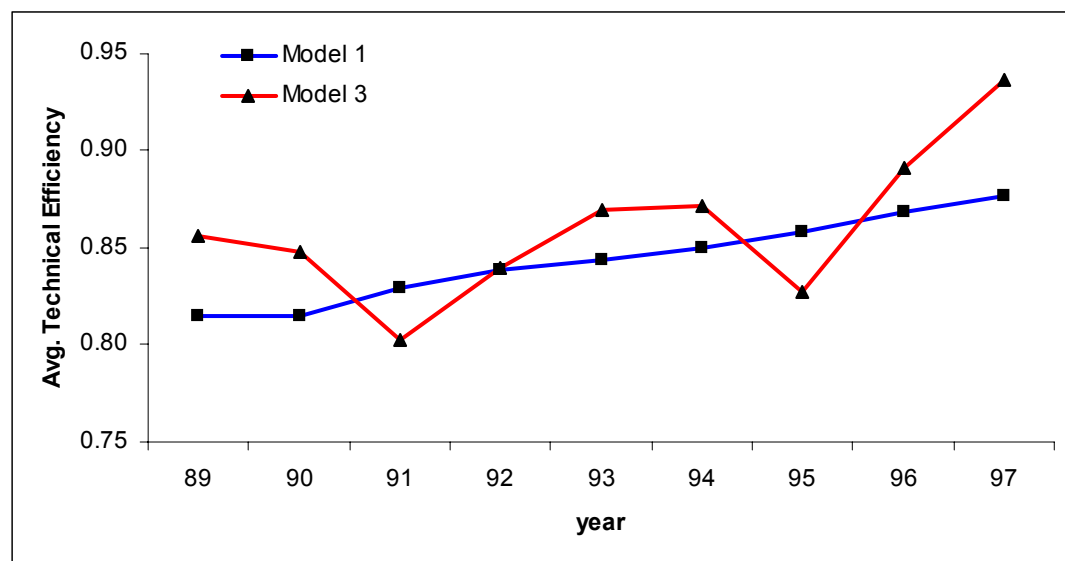
$$TE_{it} = \exp(-U_{it}) \quad (15)$$

The mean technical efficiency of cereal farms in the East of England does not differ significantly for the selected models. It is 0.86 for the non-neutral model (model 3) while it is 0.84 for the neutral model 1. However, it can be observed that whilst technical efficiency

appears to increase consistently over time according to the results from model 1, model 3 shows a cycle pattern in efficiency, even though the trend on average is increasing. Figure 7 shows how the mean technical efficiency of cereal farms in the East of England varies over time for both these models.

Table 6 reports the sample descriptive statistics of efficiency scores (from model 3) for the nine-year period. It can be noted that the standard deviation of scores is being reduced consistently from 0.11 0.12 in during 1989-91 to the measured lowest in 1997 (0.04). This implies that besides an increasing trend in mean efficiency levels, the dispersion is also being reduced over time. Note also that the minimum efficiency levels have been consistently rising. Table 6 also reports the results of a z test for the equality of means of efficiency scores for subsequent periods. The tests suggest that only between 1989-90 and 1993-94, the average efficiency scores can be considered to be equal. Histograms of the technical efficiency scores across farms for the time period 1989-97 are presented in Figure 8.

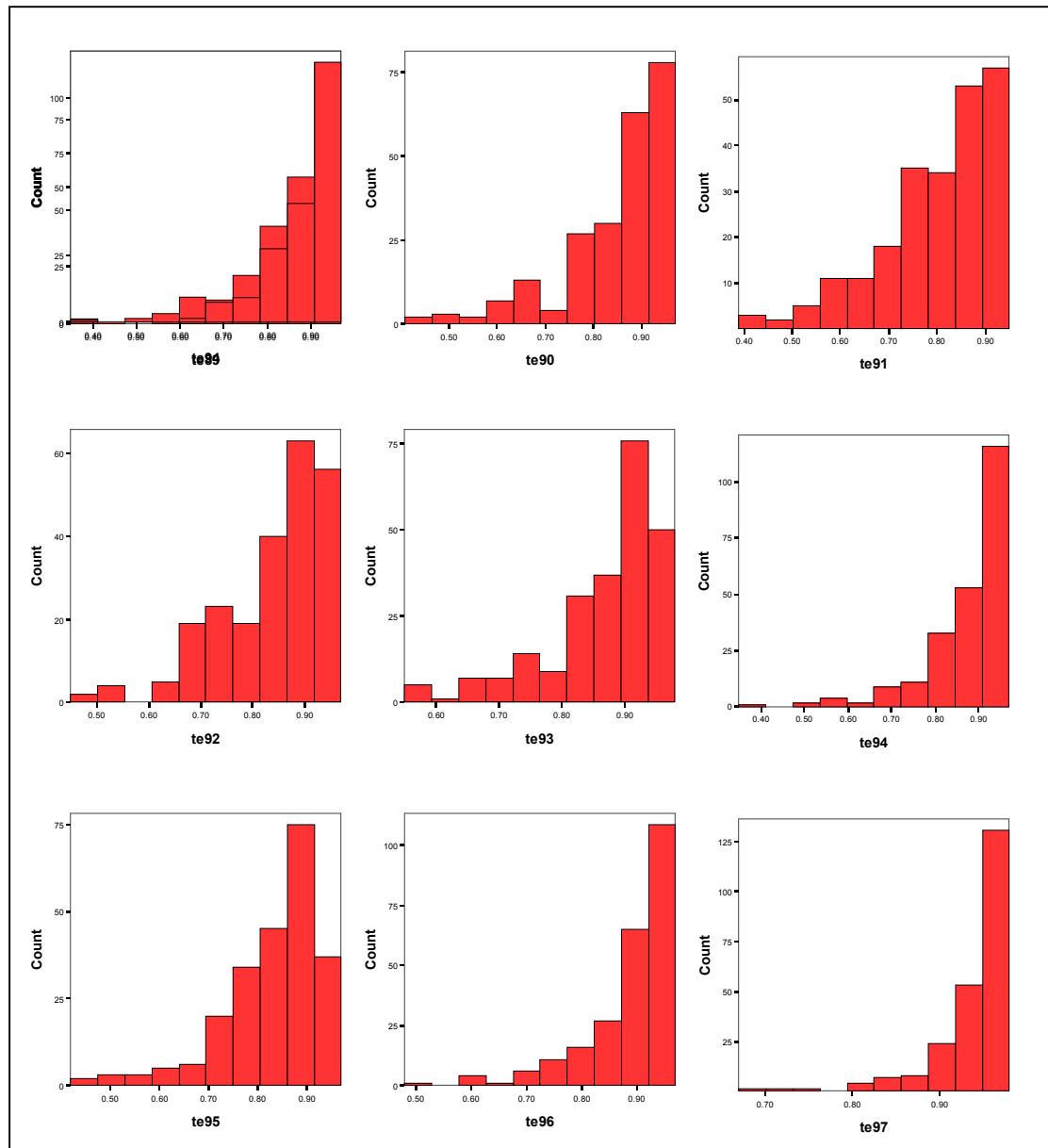
**Figure 7: Change in Mean Technical Efficiency over time (given Models 1 and 3)**



**Table 6 Sample descriptive statistics of technical efficiency scores given Model 3**

Year	Mean	Std Dev	Minimum	Maximum	N of farms	T stat Diff. means	Sig level Diff means
1989	0.86	0.11	0.35	0.97	229		
1990	0.85	0.11	0.41	0.97	229	0.86	0.39
1991	0.80	0.12	0.39	0.95	229	4.16	0.00
1992	0.84	0.10	0.45	0.97	231	-3.53	0.00
1993	0.87	0.09	0.55	0.98	237	-3.41	0.00
1994	0.87	0.10	0.35	0.97	231	-0.25	0.81
1995	0.83	0.10	0.42	0.97	230	4.84	0.00
1996	0.89	0.08	0.48	0.97	240	-7.55	0.00
1997	0.94	0.04	0.67	0.98	230	-7.78	0.00

**Figure 8: Efficiency score histograms, 1989-97**



## 6. Conclusions

This paper has explored the dynamic interconnectedness between biological diversity technological change and crop productivity in the context of specialised intensive agriculture. A theoretical optimal control model has provided two hypotheses that have been tested using an econometric model applied to a panel of specialised cereal farms in the East of England for the period 1989-97.

The testable hypotheses are that (a) productivity *along the optimal path* is positively related to technical change and that (b) the impact of changes in biodiversity on frontier output is positive and declining over time.

As regards the first hypothesis, it has not been rejected by the data. Furthermore, departing from the notion that farmers operate on the frontier, technical inefficiencies have been measured. A remarkable finding is that while both technical efficiency and productivity are increasing, the dispersion of efficiency levels across farms is being systematically reduced. This implies that specialised producers are converging towards a unique best practice technology, used by fewer farms at the beginning of the 1990s.

Similarly, the data has been unable to reject the second null hypothesis of the positive effect of biodiversity on productivity. This has important implications for environmental policy. It suggests that the introduction of CAP based biodiversity conservation policies in semi-natural habitats, represents a win-win scenario. That is, the policies are consistent with their environmental objective to enhance biodiversity levels without impairing agricultural productivity. The latter effect has been found to arise through the positive effect of biodiversity on both frontier output and also on resource use efficiency levels.

## Appendix:

To run the comparative analysis we need to find the steady state solution and the time paths of  $Z(t)$  and  $Y(t)$ .

First, we need to identify the steady state solution at  $\dot{Z} = \dot{Y} = 0$ . At equilibrium, the dynamic system is

$$g(Z, Y) = \alpha Z + \delta F(.) - \delta Y - \gamma X = 0 \quad (A1)$$

$$f(Z, Y) = \alpha - \rho + \delta F_Z - [\delta F_X - \gamma] \frac{B_Z}{B_X} = 0 \quad (A2)$$

The Jacobian matrix evaluated at the steady state  $(\bar{Z}, \bar{Y})$  is:

$$J_S = \begin{bmatrix} g_Z & g_Y \\ f_Z & f_Y \end{bmatrix} \quad (A3)$$

where:

$$g_Z = \alpha + \delta F_Z + (\delta F_X - \gamma) X_Z > 0 \quad (A3a)$$

$$g_Y = -\delta + [\delta F_X - \gamma] X_Y < 0 \quad (A3b)$$

$$f_Z = \delta F_{ZZ} - \left[ \frac{[\delta F_X - \gamma] B_X B_{ZZ} + \delta B_X B_Z F_{XX} X_Z - [\delta F_X - \gamma] B_Z B_{XX}}{(B_X)^2} \right] < 0 \quad (A3c)$$

$$f_Y = - \frac{[\delta B_X B_Z F_{XX} - [\delta F_X - \gamma] B_Z B_{XX}] X_Y}{(B_X)^2} > 0 \quad (A3d)$$

Hence:

$$|J_S| = g_Z f_Y - g_Y f_Z \begin{matrix} > \\ < \end{matrix} 0 \quad (A4)$$

As it is established that  $|J_S|$  is negative for the steady state to be a saddle point (Omer et al, 2003), the following steady state values of  $Z$  and  $Y$  are derived from the linearised system of equations (A1) and (A2) using Cramer's rule.

$$\bar{Z} = -\frac{1}{|J_S|} \left[ f_Y (\alpha Z + \delta F - \delta Y - \gamma X) + g_Y \frac{U_Y}{U_{YY}} \left( \alpha - \rho + \delta F_Z - [\delta F_X - \gamma] \frac{B_Z}{B_X} \right) \right] \quad (A5a)$$

$$\bar{Y} = -\frac{1}{|J_S|} \left[ -f_Z (\alpha Z + \delta F - \delta Y - \gamma X) - g_Z \frac{U_Y}{U_{YY}} \left( \alpha - \rho + \delta F_Z - [\delta F_X - \gamma] \frac{B_Z}{B_X} \right) \right] \quad (A5b)$$

Secondly the time paths of  $Z(t)$  and  $Y(t)$  were identified by solving the model system using the linearised system of the model:

$$\begin{bmatrix} \dot{Z} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} \frac{\partial \dot{Z}}{\partial Z} & \frac{\partial \dot{Z}}{\partial Y} \\ \frac{\partial \dot{Y}}{\partial Z} & \frac{\partial \dot{Y}}{\partial Y} \end{bmatrix} \begin{bmatrix} Z - \bar{Z} \\ Y - \bar{Y} \end{bmatrix} = J_d \begin{bmatrix} Z - \bar{Z} \\ Y - \bar{Y} \end{bmatrix} \quad (\text{A6})$$

where

$$\frac{\partial \dot{Z}}{\partial Z} = \alpha + \delta F_z + [\delta F_x - \gamma] X_z > 0$$

$$\frac{\partial \dot{Z}}{\partial Y} = -\delta + [\delta F_x - \gamma] X_y < 0$$

$$\frac{\partial \dot{Y}}{\partial Z} = -\frac{U_y}{U_{yy}} \left\{ \delta F_{zz} - \left[ \frac{[\delta F_x - \gamma] \delta B_x B_{zz} + \delta B_x B_z F_{xx} X_z - [\delta F - \gamma] B_z B_{xx} X_z}{(B_x)^2} \right] \right\} < 0$$

$$\frac{\partial \dot{Y}}{\partial Y} = -\frac{U_y}{U_{yy}} \frac{[-\delta B_x B_z F_{xx} + [\delta F_x - \gamma] B_z B_{xx}] X_y}{(B_x)^2} > 0$$

to find the general solution, which is given as

$$\begin{bmatrix} Z(t) \\ Y(t) \end{bmatrix} = \begin{bmatrix} \bar{Z} \\ \bar{Y} \end{bmatrix} + \begin{bmatrix} Z_c \\ Y_c \end{bmatrix} \quad (\text{A7})$$

$\begin{bmatrix} \bar{Z} \\ \bar{Y} \end{bmatrix}$  represents the steady state equilibrium, and  $\begin{bmatrix} Z_c \\ Y_c \end{bmatrix}$  represents the complementary

functions based on the reduced equations of the model system,

$$\begin{bmatrix} \dot{Z} \\ \dot{Y} \end{bmatrix} - \begin{bmatrix} \frac{\partial \dot{Z}}{\partial Z} & \frac{\partial \dot{Z}}{\partial Y} \\ \frac{\partial \dot{Y}}{\partial Z} & \frac{\partial \dot{Y}}{\partial Y} \end{bmatrix} \begin{bmatrix} Z \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

which are found by using trial solution, such as that  $Z(t) = me^{rt}$  and  $Y(t) = ne^{rt}$ , in turn implying that  $\dot{Z} = rme^{rt}$  and  $\dot{Y} = rne^{rt}$ , where  $m$  and  $n$  are arbitrary constants and  $r$  is the characteristic root.

By substituting the trial solution into the reduced equations and multiplying by the scalar  $e^{-rt}$ :

$$\begin{bmatrix} r - \frac{\partial \dot{Z}}{\partial Z} & -\frac{\partial \dot{Z}}{\partial Y} \\ -\frac{\partial \dot{Y}}{\partial Z} & r - \frac{\partial \dot{Y}}{\partial Y} \end{bmatrix} \begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (\text{A8})$$



To find nontrivial solution of  $m$  and  $n$ , the characteristic equation of the system that gives the characteristic roots  $r_i$  of the system. can be solved:

$$\begin{vmatrix} r - \frac{\partial \dot{Z}}{\partial Z} & -\frac{\partial \dot{Z}}{\partial Y} \\ -\frac{\partial \dot{Y}}{\partial Z} & r - \frac{\partial \dot{Y}}{\partial Y} \end{vmatrix} = 0 \quad (\text{A9})$$

Each root  $r_i$  will draw out from (A8) a particular set of infinite number of solution values of  $m$  and  $n$  that tied to together by  $n_i = k_i m_i$  where  $k_i$  is a constant, which is defined from (A8) as follows:

$$k_1 = \frac{-\dot{Y}_Z - (r_1 - \dot{Z}_Z)}{r_1 - \dot{Y}_Z + \dot{Z}_Y} \quad (\text{A10})$$

Then we can define  $m_i = c_i$  and  $n_i = k_i c_i$  where  $c_i$  is an arbitrary constant, which would be determined from the initial condition. Then substituting these expressions of  $m_i$  and  $n_i$  along with the values of  $r_i$  into the trial solution gives the complementary functions,

$$\begin{bmatrix} Z_c \\ Y_c \end{bmatrix} = \begin{bmatrix} \sum_i m_i e^{r_i t} \\ \sum_i n_i e^{r_i t} \end{bmatrix} = \begin{bmatrix} \sum_i c_i e^{r_i t} \\ \sum_i k_i c_i e^{r_i t} \end{bmatrix}, \text{ and hence the optimal paths as:}$$

$$\begin{bmatrix} Z(t; Z_0, \phi) \\ Y(t; Z_0, \phi) \end{bmatrix} = \begin{bmatrix} \bar{Z} \\ \bar{Y} \end{bmatrix} + c_1 \begin{bmatrix} 1 \\ k \end{bmatrix} e^{r_1 t} + c_2 \begin{bmatrix} 1 \\ k \end{bmatrix} e^{r_2 t} \quad (\text{A11})$$

For a dynamically stable equilibrium, the complementary function should converge i.e.  $\begin{bmatrix} Z_c \\ Y_c \end{bmatrix} \rightarrow 0$  as  $t \rightarrow \infty$ . Since a saddle point has one negative root and one positive (say  $r_1 < 0$  and  $r_2 > 0$ ) then the convergence of the complementary functions requires  $c_2 = 0$  i.e. the positive root should drop out leaving only the negative root to make the equilibrium stable.

Using the initial condition  $Z(0) = Z_0$  to definitise  $c_1$  from (A11): At  $t = 0$ , ( $e^{r_1 t} = e^0 = 1$ ),  $Z(0) = \bar{Z} + c_1$ , and  $c_1 = Z(0) - \bar{Z}$ . Therefore, the definite solution of the dynamic system of the model, is given as:

$$\begin{bmatrix} Z(t; Z_0, \phi) \\ Y(t; Z_0, \phi) \end{bmatrix} = \begin{bmatrix} \bar{Z} \\ \bar{Y} \end{bmatrix} + [Z_0 - \bar{Z}] \begin{bmatrix} 1 \\ k_1 \end{bmatrix} e^{r_1 t} \quad (\text{A12})$$

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