Agricultural Intensification and Biodiversity Loss: Is There an Agri-EKC?

by*

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ABSTRACT

This paper explores the dynamic interactions between biodiversity loss and conservation on the one hand, and agricultural development on the other. An optimal control model is used to show that the level of artificial input application to boost agricultural production and its impact on biodiversity loss could follow a reverse-EKC relationship. While increasing input use at low levels of output leads to an optimal reduction in biodiversity degradation, this reduction can also be achieved at higher levels of output by reducing input use, thus describing a U-shaped relationship between levels of agricultural input use and output, and ecological damage. The implication is that social welfare would be maximised when biodiversity conservation activities are segregated from economic activity (agriculture) in areas with high biodiversity, while these activities are integrated in areas that are poor in biodiversity

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1. Introduction

The precise impacts of biodiversity loss in agro-ecological systems are not well known but are generally understood to encompass a range of widely diffused indirect impacts on biological systems generated by productive activities. It is the contribution of agricultural activities and other anthropogenic activities that raise serious concerns. In fact, modern intensive agriculture is recognised as one of the main driving factors in the loss of biodiversity (Fry, 1989; Barrett, 1991; Ehrlich, 1995). This paper looks into the inter-connectedness between agricultural `growth' and changes in biodiversity using an optimal control bio-economic model. This provides an analytical framework that can be used to explore the dynamic interactions between biodiversity loss and conservation on the one hand, and agricultural development on the other. The model here is focused on input-intensive agro-ecological systems typical of those used in developed economies, and highly modernised areas in less developed countries.

The model shows that biodiversity loss and the level of activity in the agricultural sector could follow a U-shape relationship, quite different to that being investigated for pollution levels and aggregate economic activity commonly referred to as the Environmental Kuznets Curve (EKC) (Grossman and Krueger, 1996; Ansuategi, 2000; Anderson and Cavendish, 2001; Andreoni and Levinson, 2001). The present analysis differs from standard EKC models in that, by considering biodiversity, we analyse the problem of under-provision of a positive externality, rather than the over-supply of a negative externality (pollution). In addition, the economic variable in this study is related to sector level economic activity rather than that at aggregate national or even international level. It also differs from models that hypothesize about the EKC in traditional agricultural systems in developing economies and where the household is the unit of analysis (Bulte and van Soest, 2001; Pascual and Barbier, 2001). Further, the model applies optimal control theory and solves the *comparative dynamic* analysis to shed light on the non-linear dynamic effects of input intensification on biodiversity loss.

While consistent with some recent ecological studies suggesting that there may be a negative relationship between agricultural productivity and biodiversity loss (Bullock et al. 2001; Richards, 2001), our analysis points to a more complex agro-ecological link arising partly as a result of economic decisions.

The paper proceeds as follows: Section 2 outlines the optimal control model and explains the assumed interactions between the variables. Section 3 provides a qualitative analysis of the dynamics of the system and section 4 focuses on the comparatives dynamics to reflect the effect of input intensity on biodiversity loss. Section 5 explains the reverse agri-EKC result and the last section concludes.

2. Model Preliminaries

The present model is based on the maximisation of the discounted present value of society's utility flows to perpetuity. The utility function is assumed to depend on a sustained flow agricultural output supply, *Y(t)*. In addition, it is assumed that disutility arises from biodiversity loss, *B(t),* attributable to negative impacts of agricultural production through the intensive use artificial inputs, *X(t)* . The problem is how to optimally allocate at any period *t*, the economy's resources, between the two competing utility yielding services:: agricultural food supply, *Y(t),* and environmental conservation expenditures, *R(t),* to enhance the current biodiversity stock, *Z(t).*[1] The direct utility function, *U=U(B(t),Y(t),* is assumed strictly concave, linearly separable in *Y* and *B,* and with positive but diminishing marginal utility with respect to *Y(t)*. In addition, the marginal utility of *B(t)* is specified to be negative and decreasing, i.e. $U_Y > 0, U_{YY} < 0, \lim_{Y \to 0} U_Y = \infty$, and $U_B < 0, U_{BB} < 0, \lim_{B \to 0} U_B = 0$.

In the context of intensified agricultural systems, the assumption here is that economic activity is strictly more constrained by biodiversity than by man-made capital. Furthermore, given the complex ecological services provided by biological diversity, substitution between man-made capital and biodiversity services are ruled out in the setting of the model. Regarding the capital stock, in order to focus on the basic problem of this agro-ecological system we are assuming away capital accumulation and population growth (Forster, 1977).

As agricultural output relies on the integrity of the agro-ecosystem and natural environment for its productivity and sustainability, the modelling of agricultural development over time should consider this interaction (Tisdell, 1999). Recent ecological studies suggest that there may be a positive relationship between agricultural productivity and biodiversity (Bullock et al. 2001; Richards, 2001). Our model reflects this result by assuming that a sustainable stock of biodiversity, *Z(t),* ensures ecological integrity, and hence should enter into the production function alongside a vector of artificial inputs, *X(t).* Accordingly, the agricultural production function is *F[X(t),Z(t)]* and is assumed strictly concave and twice differentiable,

[1] Note that *Z(t)* refers to the level (*stock*) of biodiversity in time *t*, while *B(t)* refers to biodiversity 'loss' (a *flow* variable)

with. $F_Z > 0, F_{ZZ} < 0$ and $F_X > 0, F_{XX} < 0$. It is also assumed to exhibit weak essentiality i.e. $F(0) = 0$.

In the present model, the notion of biodiversity is somewhat general since it can be understood at any of the three levels (species, genetic and ecosystem diversity) even though each level has a set of sub-components and hence a different interaction with production process. However, the main effect of a change in stock of biodiversity, *Z(t)*, on the marginal product of artificial input, *X(t)*, is likely to be different at each level or sublevel of *Z(t)*. For instant, an increase in insect or micro-organism diversity would increase the marginal product of fertiliser since it enhances the soil productivity i.e. $F_{XZ} \geq 0$. Alternatively, an increase in natural vegetation diversity would decrease the marginal product of fertiliser as it increases the competition against the cultivated crops i.e. $F_{XZ} \leq 0$. Similar examples could be stated for other components of biodiversity. Generally to determine the effect of an increase in *Z(t)* on marginal product of *X(t)*, requires detailed and specific information on which component(s) of biodiversity is changed. Hence, for simplicity, *F[X(t), Z(t)]*, is assumed to be linearly separable in *Z(t)* and *X(t).* i.e. $F_{XZ} = F_{ZX} = 0$. In addition, specifying the production function in a static way implies that no technological or productivity progress can take place, thus, a dynamic production function is proposed in the form of *F[X(t), Z(t), A(t)]*, where *A(t)* represents the state of art. Hence, under technological progress over time: $\partial F / \partial A > 0$. Here *A(t)* is treated as exogenous variable for simplicity, although this dynamic production function would also allow to treat *A* endogenously.

The 'biodiversity impact (or 'loss') function', *B=B[X(t), Z(t)]*, shows the effect on the biodiversity stock of intensified agricultural production. But, the extent of the induced impact, through use of *X(t)*, depends on the existing state of biodiversity and its ability to tolerate and overcome the adverse effect of agricultural activities, reflecting the notion that the level of biodiversity makes a positive contribution to ecosystem resilience (Xu and Mage, 2001; Trenbath, 1999; Swanson, 1997). It is assumed that the marginal biodiversity 'loss' due to application of *X* is increasing at an increasing rate. That is, additional units of *X* would contribute at least as much as the preceding units to biodiversity loss arising from agricultural production, i.e. $B_X > 0, B_{XX} > 0$. Equivalently, the marginal biodiversity 'gain' of X is negative and decreasing. It is also assumed that at low stocks of biodiversity, the marginal effect of *Z* is more effective in overcoming the adverse impact of agriculture than at higher stocks, hence $B_Z < 0, B_{ZZ} > 0$. In addition, for simplicity, the impact function is made linearly separable in *X(t)* and *Z(t)*, i.e. $B_{XZ} = B_{ZX} = 0$.

While the problem is to choose the optimal time paths of the control variables, these are constrained by the rate of change of the state variable, *Z(t)*, which evolves according to a process reflecting (a) the natural growth function, (b) conservation activities, and (c) agricultural activities carried out in the agro-ecosystem:

$$\dot{Z} = G[Z(t), X(t), R(t)] \tag{1}$$

which can be written as an extended logistic function:

$$\dot{Z} = \alpha Z(1 - Z/K) + \delta R - \gamma X \tag{1a}$$

where α > 0 is the natural rate of growth of *Z* and *K* is the system's carrying capacity. On intensified agricultural systems it is typical to find relatively low levels of biodiversity compared to the system natural carrying capacity. Hence, the term *Z/K* is expected to be negligible, and without much loss of generality, Equation (1a) is approximated as:

$$\dot{Z} = \alpha Z + \delta R - \gamma X \tag{1b}$$

where α, δ and γ are constants. Equation (1b) shows that biodiversity is enhanced proportionally to investment in conservation, *R*, δ being the rate of induced growth[2], and that biodiversity is degraded proportional to input use. In addition, it is worth noting that even though biodiversity is considered to be an input factor, it is assumed that none of it is consumed in the production process. In other words, no depletion in biodiversity occurs as a result of its support to the production process.

Since the optimisation problem is specified with infinite time horizon, to allow for intertemporal interactions between agriculture and its impact on biodiversity, we show that the solution of the first order conditions would lead to a steady state marked as $(Z^{\infty}, Y^{\infty}, X^{\infty}, \varphi^{\infty})$ and it is reachable from the initial state condition $Z(0) = Z_0$. That is, there is an implicit terminal state $\lim_{t \to \infty} Z(t) = Z^{\infty}(\phi)$ where ϕ is a vector of exogenous parameters and variables including the discount rate, ρ, and technological progress, *A*.

[2] The parameter δ also can be interpreted as the marginal degradation in *Z(t)* caused by increase in *Y(t)* i.e. the opportunity cost of *R(t)*.

The objective of the social planner is to choose the time paths of the control variables which maximise the total value of the objective function, *W*, considering the instantaneous effect on the rate of utility gained, and the inter-temporal impact on the state of biodiversity. Therefore, the objective of the social planner is to optimise the trade off between consumption and conservation as well as optimising the production process by controlling the scale of intensification of the agricultural activity at any time, *X (t)*. The social welfare function is therefore defined as:

$$\underset{Y,X,R}{Max}\, W(Y(t),B(t)) = \int_{t=0}^{\infty} e^{-\rho t} u(Y(t),B(t))dt \qquad (2)$$

where $\rho > 0$ is the social rate of time preference.

subject to

(i) the equation of motion for *Z(t)*,

(ii) the non-negativity constraints, i.e. $X \geq 0$ and $B \geq 0$,

(iii) the initial condition $Z(0) = Z_0$,

(iv) the impact function *B(.)*,

(v) the environmental conservation investment function (3):

$$R(t) = F(X(t), Z(t)) - Y(t) \qquad (3)$$

To seek the optimal solutions for the optimisation problem, the current-value Hamiltonian function is defined as follows:

$$\underset{Y,X}{Max}\, H_C = U(Y,B) + \varphi(\alpha Z + \delta F(.) - \delta Y - \gamma X) \qquad (4)$$

Where φ is the current-value costate variable associated with the state equation. The Maximum Principle for an optimal interior solution shows that:

$$\frac{\partial H_C}{\partial \varphi} = \dot{Z} = \alpha Z + \delta[F(.) - Y] - \gamma X \qquad (5a)$$

$$\frac{\partial H_C}{\partial Y} = U_Y - \delta\varphi = 0 \qquad (5b)$$

$$\frac{\partial H_C}{\partial X} = U_B B_X + \varphi[\delta F_X - \gamma] = 0 \qquad \textit{(5c)}$$

$$\dot{\varphi} = -\frac{\partial H_C}{\partial Z} + \rho\varphi = -U_B B_Z - \varphi[\alpha + \delta F_Z - \rho] \qquad \textit{(5d)}$$

Equation (5a) restates the state equation. Equation (5b) establishes that the current shadow value of biodiversity (φ) is positive. Equation (5c) states that *X* should be allocated such that the marginal utility and disutility of artificial input use are balanced. For an interior solution, the bracketed term $(\delta\, \partial F/\partial X - \gamma)$ is positive as the φ is positive and the first term is unambiguously positive according to the assumptions made on the component derivatives.[3] From equations (5b-5c) *X* can be defined as an implicit function of *Y* and *Z* with $X_Z > 0$ and $X_Y < 0$, i.e. *X (Y, Z)* is the level of *X* that solves the first order conditions.[4]

$$U_Y[\delta F_X - \gamma] + \delta U_B B_X = 0 \qquad (6)$$

As the signs of both derivatives are negative, the current value Hamiltonian is indeed maximised with respect to both control variables.[5] The solution to the problem is expected to lead to an optimal unique equilibrium as the concavity conditions required for optimality are met, i.e. both the utility and production functions are strictly concave.[6]

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[3] If the bracketed term $(\delta F_X - \gamma)$ <0, a corner solution, (X = 0), will occur as $\partial H_C / \partial X < 0$.

[4] See Appendix A for full illustration of the two partial derivatives.

[5] To verify that the current value Hamiltonian is maximised, the signs of $\partial^2 H_C / \partial Y^2$ and $\partial^2 H_C / \partial X^2$ can be checked:

$\frac{\partial^2 H_C}{\partial Y^2} = U_{YY} < 0$ by assumption, and

$\frac{\partial^2 H_C}{\partial X^2} = U_{BB}(B_X)^2 + U_B B_{XX} + \varphi\delta F_{XX} < 0$ by following the assumptions made on the signs of these derivatives, this expression is unambiguously negative as both φ and δ are positive.

[6] Based on the concavity properties that have been assumed at the outset of the model, we have verified that the sufficiency conditions are satisfied in appendix B.

3. Qualitative Analysis

The aim of this section is to gain analytical insights into dynamic behaviour of the relationship between agriculture and its impact on biodiversity. Therefore, the analysis is focused on *ZY* space for this agro-ecological system and a differential equation for Y is derived from the basic solution as follows:

$$\dot{Y} = -\frac{U_Y}{U_{YY}}\left[\alpha - \rho + \delta F_Z - [\delta F_X - \gamma]\frac{B_Z}{B_X}\right] \qquad (7)$$

This equation, together with the original state equation, gives a new non-linear dynamic system for this agro-ecological system, with *Y(0)* left free. To examine the dynamic behaviour of the system in (*Z*, *Y*) space a phase diagram is constructed (Figure 1a, 1b).

The dynamic system, at equilibrium is denoted as:

$$\dot{Z} = g(Z,Y)$$

$$\dot{Y} = f(Z,Y)$$

This system is expected to possess a unique solution that satisfies the initial conditions $Z(0) = Z_0$. Two demarcation curves ($\dot{Z} = 0$ and $\dot{Y} = 0$) are drawn, that divide the phase space into four regions, with a different mix of time derivatives for *Y(t)* and *Z(t)*. The characteristics of these boundaries are discussed in more detail below.

The Loci of the $\dot{Z} = 0$ isocline:

At $\dot{Z} = 0$: $g(Z,Y) = \alpha Z(t) + \delta F(.) - \delta Y - \gamma X$, and the slope of $\dot{Z} = 0$ isocline is given by the implicit function:

$$\left.\frac{dY}{dZ}\right|_{\dot{Z}=0} = -\frac{g_Z}{g_Y} > 0 \qquad (g_Y \neq 0) \qquad (8)$$

where:

$$g_Z = \alpha + \delta F_Z + (\delta F_X - \gamma)X_Z > 0 \qquad (8a)$$

$$g_Y = -\delta + [\delta F_X - \gamma]X_Y < 0 \qquad (8b)$$

and $(\delta F_X - \gamma)$>0 from Eq (5b).[7] Also, since $g_Y < 0$, as Y increases, $\dot{Z}$ undergoes a steady decrease.

The loci of the $\dot{Y} = 0$ isocline:

At $\dot{Y} = 0$: $f(Z,Y) = \alpha - \rho + \delta F_Z - [\delta F_X - \gamma]\dfrac{B_Z}{B_X}$ *(9)*

As $(-U_Y / U_{YY}) > 0$:

$$f_Z = \delta F_{ZZ} - \left[\frac{[\delta F_X - \gamma]B_X B_{ZZ} + \delta B_X B_Z F_{XX} X_Z - [\delta F_X - \gamma]B_Z B_{XX}}{(B_X)^2} \right] < 0 \qquad (9a)$$

$$f_Y = -\frac{[\delta B_X B_Z F_{XX} - [\delta F_X - \gamma]B_Z B_{XX}]X_Y}{(B_X)^2} > 0 \qquad (9b)$$

Hence, the $\dot{Y} = 0$ isocline is also upward sloping to the right. Also, $f_Y > 0$ implies that as Y increases, $\dot{Y}$ undergoes a steady increase.

As the two isoclines are positively sloped, if ($-f_Z / f_Y > -g_Z / g_Y$) then the relevant phase diagram is described by Figure 1a, where the output isocline is steeper than the biodiversity isocline in the neighbourhood of the steady state. Alternatively, Figure 1b describes the situation when ($-f_Z / f_Y < -g_Z / g_Y$). The equilibrium point, marked as E, divides the phase space into four distinct regions (labelled I to IV) where the variables increase or decrease over time. At E both variables (Z and Y) are stationary, but at any other point either Z or Y (or both) would be changing over time as shown by the directional arrows. It can be concluded from the pattern of streamlines in these phase diagrams that the equilibrium is a saddle point only when $-f_Z / f_Y > -g_Z / g_Y$. In the opposite case, the system would show no local stability.[8]

4. Biodiversity and Agricultural Intensification

At early stages of agricultural development, as consumption and output are expected to be low, none or a very low amount of artificial inputs is used. Therefore, we might need to consider the possibility of a corner solution of $X=0$, prior to X increasing as agricultural production is intensified. Drawing on the theoretical analysis by Selden and Song (1995) we also argue that a

[7] Concavity arises from the fact that $\dot{Z} = 0$ leads to $Y = F(.) + \frac{\alpha}{\delta} Z - \frac{\gamma}{\delta} X$), therefore the locus of $\dot{Z} = 0$ curve exhibits the same shape as that of the production function.

[8] A formal derivation of the local stability property of the dynamic system is derived in Appendix C.

non-linear relationship between intensification (increase in *X*) and environmental impact (i.e. biodiversity loss) can be obtained in the present model. Indeed, Eq. (6) solves for *X* as an implicit function of *Y* and *Z*, i.e. *X (Y, Z)* is the optimal level of (*X > 0*).[9] Thus, as long as $[\delta F_X - \gamma]U_Y < -\delta B_X U_B$ at the implicit function $(Y, Z, X) = 0$, there exists a nonempty set of points in the (Z, Y) space, such that $X(Z, Y) = 0$. We denote as $\breve{X}(Z, Y)$ to the locus of points on the boundary of this set, that is, the boundary curve separating the phase space between *X>0* and *X=0*. This function can be implicitly defined as the combination of *Z* and *Y*, yielding Eq (6'):

$$\left[\delta \frac{\partial F(Z,0)}{\partial X} - \frac{\partial B(Z,0)}{\partial X}\right]\frac{\partial U}{\partial Y} + \delta \frac{\partial B(Z,0)}{\partial X}\frac{\partial U}{\partial B(Z,0)} = 0 \qquad (6')$$

The slope of $\breve{X}(Z, Y)$ is given by $\frac{dY}{dZ} = -\frac{X_Z}{X_Y} > 0$, implying that the $\breve{X}(Z, Y)$ curve is upward sloping in the (Z, Y) space (Figures 2a, 2b). The phase area $X(Z, Y) = 0$ is to the left of $\breve{X}(Z, Y)$ assuming that the marginal disutility $(-U_B)$ is increasing in *B*. As *B* is also assumed to decline as *Z* increases, then for any level of *X* $(-\delta B_X U_B)$ would decline. Therefore, $[\delta F_X - \gamma]U_Y < -\delta B_X U_B$ is expected to hold at low levels of *Z*.

For the agro-ecological system to approach the steady state equilibrium, given Z_0, the initial output supply, Y_0, should be chosen such that (Z_0, Y_0) is located in a converging isosector. Having selected Y_0, X_0 is fixed by Eq. (6) for a given technology. Accordingly, we define $\breve{Z}$ as the biodiversity level at the point where the zero-input use boundary curve, $\breve{X}(Z, Y)$, intersects the stable branch in isosector I, where both $\dot{Z} > 0$ and $\dot{Y} > 0$ (Figure 2a). As long as $Z_0 \le \breve{Z} < Z^\infty$, $X(t) = 0$ for $Z(t) \le \breve{Z}$ along the optimal path. On the other hand, if $\breve{Z} < Z(t) < Z^\infty$, totally differentiating Eq (6) with respect to time and rearranging gives the following expression for $\dot{X}$:

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[9] Eq. (6) becomes $[\delta F_X - \gamma]U_Y + \delta B_X U_B < 0$ for a corner solution, i.e. *X = 0*, which occurs when $[\delta F_X - \gamma]U_Y < -\delta B_X U_B$. The possibility of $[\delta F_X - \gamma]U_Y > -\delta B_X U_B$, which corresponds to $\partial H_C / \partial X > 0$, is ruled out since the curve is upward-sloping and hence there cannot be a maximum.

$$\dot{X} = \frac{\left\{\left[\delta F_X - \gamma\right]U_{YY}\dot{Y} + \delta U_{BB} B_X B_Z \dot{Z}\right\}}{\left\{-\delta U_Y F_{XX} - \delta U_B B_{XX} - \delta U_{BB}\left(B_X\right)^2\right\}} \qquad (10)$$

It can be easily proved that while the denominator in (10) is positive, the sign of the numerator is ambiguous. The term $\dot{Z} \cdot B_Z$ in the numerator stands for the effect of biodiversity growth on the biodiversity loss function. In addition, $U_{BB}B_X$ represents the marginal effect of input use through its contribution to the decline of the marginal utility of increased biodiversity levels, or alternatively, the rise in the marginal concern over biodiversity loss. Hence, the second term in the numerator shows the 'weighted' effect of a change in *X* on utility. The first term in the numerator $(\delta F_X - \gamma)U_{YY}$ represents the marginal effect of a change in *X*, through its contribution to the rate of change of the marginal utility of consumption. Thus, the whole of the expression $(\delta F_X - \gamma)U_{YY}\dot{Y}$ shows the 'weighted' effect of a change in *X* on utility, where the weight is the rate of output growth $\dot{Y}$. The second part of the numerator reflects the effect of inter-temporal concern on the changes on current welfare levels. So in region I, where $\dot{Z} > 0$ and $\dot{Y} > 0$, $\dot{X} > 0$ as long as the second term (which is positive) outweighs the first negative term. This is summarised in Proposition 1.

Proposition 1: *Input intensification increases optimally over time when (a) the marginal decline in the disutility due to additional biodiversity loss, dominates (b) the marginal effect on the increased decline in utility due to increased agricultural output levels.*

The rate of change in agricultural activity is stipulated to be positive at early stages (*X* increases at first); This situation is identified with (a) a large direct effect of biodiversity growth on biodiversity outcome of agricultural production ($\dot{Z} \cdot B_Z$); (b) a low marginal effect of *X* on *B*, and (c) a high decline in the marginal disutility due to biodiversity loss with the decrease in *B*.

Moreover, it is possible to establish that *X* is increasing over time along the optimal development path in region 1 from the steady state condition: At $\dot{Y} = 0$, the following relationship should hold (Eq. 9):

$$\frac{[\delta F_X - \gamma]}{B_X} = \frac{\alpha - \rho + \delta F_Z}{B_Z}$$

Based on the assumptions made on these partial derivatives, it expected that at low levels of *X* and *Z*, $\frac{[\delta F_X - \gamma]}{B_X} > \frac{\alpha - \rho + \delta F_Z}{B_Z}$. The r.h.s. becomes smaller as *Z* increases to toward its steady state level, so that *X* must be increasing toward the steady state (since the l.h.s. of the expression will decline only if *X* is increasing).

5. The reverse EKC for biodiversity loss and input intensification:

We investigate the possibility of an U-shaped curve for biodiversity loss due to agricultural intensification by considering the impact of $\dot{X}$ on $\dot{B}$. Totally differentiating the biodiversity loss (impact) function, *B (X, Z),* with respect to time: $\dot{B} = B_X \dot{X} + B_Z \dot{Z}$. If $Z_0 \le Z(t) < \breve{Z}$, $\dot{X} = 0$. In other words, when no change occurs in the level of artificial input use, then $\dot{B} = B_Z \dot{Z}$, that is to say, the change in *B* at any instant is only induced by the change in stock of biodiversity, *Z*, caused by changes in the natural conditions of the agro-ecosystem (by a change in *Z*). This implies that biodiversity loss is decreasing and unaffected by agricultural activities at the stationary level of modern agricultural inputs, X. The effect of the change in these inputs on *B* along the optimal development path (when $\dot{X} > 0$), can be derived by substituting for $\dot{X}$ from *(5b).* After simplification the following equation of motion for *B* is obtained:

$$\dot{B} = \frac{\left\{ \left[\delta U_B B_{XX} + \delta U_Y F_{XX} \right] B_Z \dot{Z} - \left[\delta F_X - \gamma \right] B_X U_{YY} \dot{Y} \right\}}{\left\{ \delta U_Y F_{XX} + \delta U_B B_{XX} + \delta U_{BB} \left(B_X \right)^2 \right\}} \qquad (11)$$

From equation (5c), $\dot{B}$ is negative as numerator is positive and denominator is negative. This corresponds to isosector I, where both $\dot{Z} > 0$ and $\dot{Y} > 0$ (and $\dot{X}$ >0). That is, when $\breve{Z} \le Z(t) < Z^{\infty}$ then $\dot{B} < 0$, implying that biodiversity loss declines when modern input use increases at lower levels of Y. However, in isosector III (where both $\dot{Z} < 0$ and $\dot{Y} < 0$) with higher levels of Y and $\breve{Z} \le Z^{\infty} < Z(t)$, $\dot{B} > 0$ along the optimal path, i.e. biodiversity degradation is aggravated (Figure 2a). This result is summarised by proposition 2:

Proposition 2: *Along the optimal path, increasing input use at low levels of output leads to reductions in biodiversity degradation. These reductions can be achieved at higher levels of output by reducing input use, thus describing an U-shaped relationship between level s of agricultural input use and output and ecological damage.*

Similarly, when the $\breve{X}(Z,Y)$ curve intersects the other stable branch of the saddle point *to the right of the steady state equilibrium* (point $\breve{Z}$ in Figure 2b) the following is expected to happen: If $Z(t) \leq Z^{\infty} < \breve{Z}$, $\dot{B} < 0$ (isosector I, where both $\dot{Z} > 0$ and $\dot{Y} > 0$). This means that biodiversity loss declines as a result of the direct effect of biodiversity growth only, $\dot{Z} \cdot B_Z$, since the optimal level of agricultural input use is zero, i.e. for $Z(t) \leq \breve{Z}$: $X = 0$. While in case that $\breve{Z} < Z(t)$, (isosector III, where $\dot{Z} < 0$ and $\dot{Y} < 0$), $\dot{B} > 0$ i.e. biodiversity loss is aggravated if the level of agricultural activity is greater than zero ($X > 0$). This might imply that optimal level of agricultural production is one with zero level of input use i.e. using only production practices that exploit the natural fertility of the soil. This situation arises when the zero-input boundary intersects the stable branch of the saddle point to the right of the steady state equilibrium at high level of *Z*. That is, in systems that are rich in biodiversity, use of modern agricultural inputs would lead to biodiversity degradation. By contrast, where the zero-input boundary intersects the stable branch of the saddle point *to the left of the steady state equilibrium*, i.e. at low levels of *Z*, agricultural production enhances biodiversity conservation (for $Z(t) \leq Z^{\infty}$). This implies that the adverse impact of agriculture on biodiversity is more pronounced in biodiversity-rich systems, as by the nature of agricultural production there is inherent competition between agriculture growth and biodiversity enhancement. The following proposition summarises a key implication of these results:

Proposition 3: *Aggregate utility is maximised when biodiversity conservation activities are segregated from economic activity (agriculture) in areas with high biodiversity, while these activities are integrated in areas that are poor in biodiversity.*

6. Summary and Conclusions

The EKC remains a controversial issue in environmental economics. While the most optimists see empirical regularities in the inverted U-shape, sceptics also abound based on analyses of empirical data. This paper has focused on a theoretical analysis which shows the possibility that the relationship between agricultural growth and biodiversity loss may be described as a reverse EKC.

This relationship holds in the model given the specification of the social welfare function and the role of biodiversity in agricultural production. Here biodiversity is a utility yielding scarce good. The model predicts that there is a turning point for biodiversity conservation in intensified agricultural economies. At low levels of agricultural output, reducing biodiversity loss requires an increase in agricultural input intensification. By contrast, when output levels have been optimally increased to high levels, reductions of input intensity are needed to reduce biological loss.

EKC advocates base the validity of the turning point in environmental degradation in any of three main fronts: environment being seen as a luxury good, moving from agricultural and industrial growth patterns to a thriving service economy, and through technical progress alongside economic growth. The model shows that the reason for the U-shape relationship is a fast decrease of marginal utility of agricultural output with increases in food consumption in the early stage and subsequently a fast increase of marginal disutility of biodiversity loss. Thus the reverse EKC-type dynamics is due to the preference structure and the role of biodiversity as natural capital stock.

These results are consistent with the idea that agro-ecosystems may be described as *semi-natural* habitats. In contrast to *natural* habitats, the ecological dynamics of these systems should be understood as inextricably linked and co-evolving with agricultural activities. This is now being acknowledged by policy makers who argue that the deterioration in the quality of many semi-natural habitats occur as former types of management are abandoned or replaced with more intensive systems (DEFRA, 2002). The analysis of this paper has aimed to illuminate these linkages.

As a final note, a limitation of the present analysis ought to be acknowledged. The role of biodiversity has been simplified. For instance, the model has ruled out non-convexities even though discontinuities in the supply of agricultural output due to marginal changes in biodiversity are likely. The results of the model should therefore be interpreted with care. Nevertheless, despite of its simplicity, the model provides a framework to evaluate the connection between agricultural input intensification and biodiversity loss in complex ecological-economic systems.

Appendix :

The Partial Derivatives Of the Implicit Function, ***X(Z,Y):***

Eq (6) is relabelled as A1:

$$U_Y\left[\delta F_X - \gamma\right] + \delta U_B B_X = 0 \qquad (A1)$$

Eq (*A1)* is differentiated w.r.t *Z*, and rearranging:

$$X_Z = \frac{-\delta U_{BB} B_Z B_X}{\left[\delta U_Y F_{XX} + \delta U_{BB}\left(B_X\right)^2 + \delta U_B B_{XX}\right]} > 0 \qquad (A2)$$

By differentiating (A1) w.r.t *Y* and rearranging:

$$X_Y = \frac{\left[\delta F_X - \gamma\right]U_{YY}}{\left[-\delta U_Y F_{XX} - \delta U_{BB}\left(B_X\right)^2 - \delta U_B B_{XX}\right]} < 0 \qquad (A3)$$

The denominator is positive and the numerator is negative, as $\left(\delta\, \partial F/\partial X - \gamma\right) > 0$ by Eq. (5c).

Transversality Conditions:

We verify that the optimal path satisfies the following transversality conditions:

$$\lim_{t\to\infty}\mu(t) = 0 \quad \text{i.e. } \mu(t) \to 0 \text{ as } t \to \infty \qquad \text{(A4.a)}$$

Eq. (5b) gives the solution path of $\mu(t)$ as

$$\mu^* = \frac{1}{\delta}\frac{\partial U}{\partial Y^*}e^{-\rho t} \qquad \text{(A4.b)}$$

Condition (A6a) implies that for U_Y to tend infinite, *Y* must approach to zero. However, in this model *Y** does not tend to zero as *t* approaches infinity since setting $\partial H_C \big/ \partial Y = 0$ rules out any corner solution The exponential term tends to zero, as *t* goes to infinite. Therefore, the first transversality condition is satisfied.

The second transversality condition is

$$\lim_{t\to\infty} H = 0 \qquad \text{i.e. } H \to 0 \text{ as } t \to \infty \qquad \text{(A4.c)}$$

For the specified problem, the solution path of H is

$$H^* = U(Y^*, B^*)e^{-\rho t} + \mu^*(\alpha Z^* + \delta F(.) - \delta Y^* - \gamma X^*) \qquad \text{(A4.d)}$$

$U(.)e^{-\rho t}$ tends to zero as when *t* goes to infinite. The bracketed expression in the second term, i.e. the state equation, is zero by the definition of the steady state. Therefore, the second transversality condition (ii) is also satisfied.

Sufficient Conditions for optimality

The Mangasarian sufficiency theorem requires that: (i) the Hamiltonian is concave in (Z, Y, X), (ii) μ(t) is nonnegative (i.e., the transversality condition), and (iii) $\lim_{t\to\infty}\mu(t)[Z(t)-Z^*(t)]\geq 0$, where *Z*(t)* denotes the optimal state path and *Z(t)* is any other admissible state path. However, if the Hamiltonian is strictly concave, then the maximum principle is sufficient for a unique global maximum.

The Hessian matrix of the Hamiltonian is

$$H=\begin{bmatrix} H_{XX} & H_{XY} & H_{XZ} \\ H_{YX} & H_{YY} & H_{YZ} \\ H_{ZX} & H_{ZY} & H_{ZZ} \end{bmatrix} \qquad \text{(A5.a)}$$

where:

$$H_{XX}=U_{BB}(B_X)^2+U_B B_{XX}+\varphi\delta F_{XX}<0$$
$$H_{XY}=0$$
$$H_{XZ}=U_{BB}B_X B_Z$$
$$H_{YX}=0$$
$$H_{YY}=U_{YY}$$
$$H_{YZ}=0$$
$$H_{ZX}=U_{BB}B_X B_Z>0$$
$$H_{ZY}=0$$
$$H_{ZZ}=U_{BB}(B_Z)^2+U_B B_{ZZ}+\varphi\delta F_{ZZ}<0$$
$$H_{ZZ}=U_{BB}(B_Z)^2+U_B B_{ZZ}+\varphi\delta F_{ZZ}<0$$

After rearranging, the Hessian determinant is given by:

$$|H|=[U_B B_{XX}+\varphi\delta F_{XX}][U_{YY}U_{BB}(B_Z)^2+U_{YY}U_B B_{ZZ}+\varphi\delta U_{YY}F_{ZZ}] +U_{BB}(B_X)^2[U_{YY}U_B B_{ZZ}+\varphi\delta U_{YY}F_{ZZ}]<0 \qquad \text{(A5.b)}$$

To verify that the supplementary condition, $\lim_{t\to\infty}\mu(t)[Z(t)-Z^*(t)]\geq 0$, also hold, it was shown earlier that the transversality condition $\lim_{t\to\infty}\mu(t)=0$ was satisfied. So regardless whether the term (*Z (t)* – *Z*(t)*) is bounded or tends to zero as *t* goes to infinity, this condition is satisfied as an equality since $\mu(t)$ tends to zero as *t* tend to infinity. Consequently, given the concavity of the Hamiltonian, the maximum principle is sufficient for an global maximum.

Local Stability Analysis:

The characteristic roots of the linearised differential-equation system ($\dot{Z}$, $\dot{Y}$) are examined:

$$\dot{Z}=\alpha Z+\delta F(.)-\delta Y-\gamma X \qquad \text{(A6.a)}$$

$$\dot{Y} = -\frac{U_Y}{U_{YY}}\left[\alpha - \rho + \delta F_Z - [\delta F_X - \gamma]\frac{B_Z}{B_X}\right] \tag{A6.b}$$

The linearized system of the differential through a Taylor expansion around the steady state (Z^{∞}, Y^{∞}), where t $J_d \begin{bmatrix} Z^{\infty} \\ Y^{\infty} \end{bmatrix}$ is constant is given by:

$$\begin{bmatrix} \dot{Z} \\ \dot{Y} \end{bmatrix} - \begin{bmatrix} \dot{Z}_Z & \dot{Z}_Y \\ \dot{Y}_Z & \dot{Y}_Y \end{bmatrix}\begin{bmatrix} Z \\ Y \end{bmatrix} = \begin{bmatrix} \dot{Z} \\ \dot{Y} \end{bmatrix} - J_d \begin{bmatrix} Z \\ Y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \tag{A7}$$

where J_d is the Jacobian matrix for this system. The two differential equations are assumed to be functionally independent i.e. $|J_d| \neq 0$.[10] The behaviour of the phase trajectories near the equilibrium point depends on the sign of the characteristic roots of the Jacobian matrix.

$$\dot{Z}_Z = \alpha + \delta F_Z + [\delta F_X - \gamma] X_Z \tag{A8.a}$$

$$\dot{Z}_Y = -\delta + [\delta F_X - \gamma] X_Y < 0 \tag{A8.b}$$

$$\dot{Y}_Z = -\frac{U_Y}{U_{YY}}\left\{\delta F_{ZZ} - \left[\frac{[\delta F_X - \gamma]\delta B_X B_{ZZ} + \delta B_X B_Z F_{XX} X_Z - [\delta F - \gamma] B_Z B_{XX} X_Z}{(B_X)^2}\right]\right\} < 0 \tag{A8.c}$$

$$\dot{Y}_Y = -\frac{U_Y}{U_{YY}} \frac{\left[-\delta B_X B_Z F_{XX} + [\delta F_X - \gamma] B_Z B_{XX}\right] X_Y}{(B_X)^2} > 0 \tag{A8.d}$$

If $|J_d| < 0$, it implies that the steady state is locally a saddle point (Figure 1a).

If $|J_d| > 0$, we need to check the sign of the trace of the Jacobian determinant, trJ_d, to determine the type of equilibrium. If trJ_d >0 at least one of the characteristic roots is positive which means that the steady state is not asymptotically stable.

However, the maximum principle solution satisfies the sufficiency conditions. Therefore, the dynamic system of this model is expected to a saddle point equilibrium, which it is often the case with autonomous infinite-horizon problems (Leonard and Long, 1992).

[10] If $|J_d| = 0$, the $\dot{Z} = 0$ and $\dot{Y} = 0$ would coincide and give a lineful of equilibrium points.

Diagrams

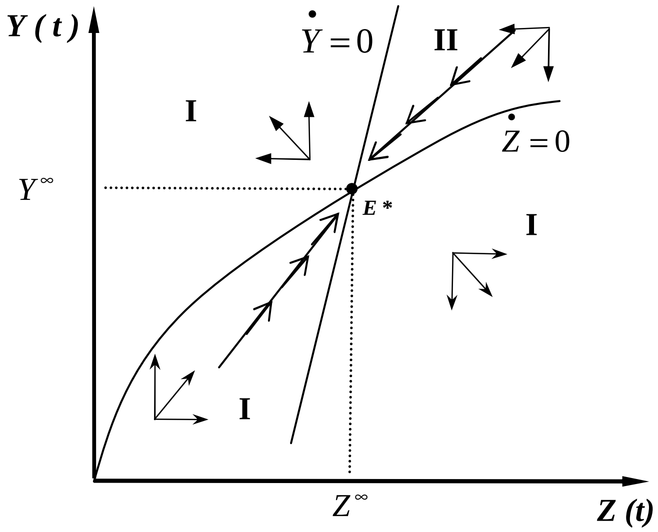

Figure 1a: E as a saddle point equilibrium with the slope of the output isocline flatter than the biodiversity isocline

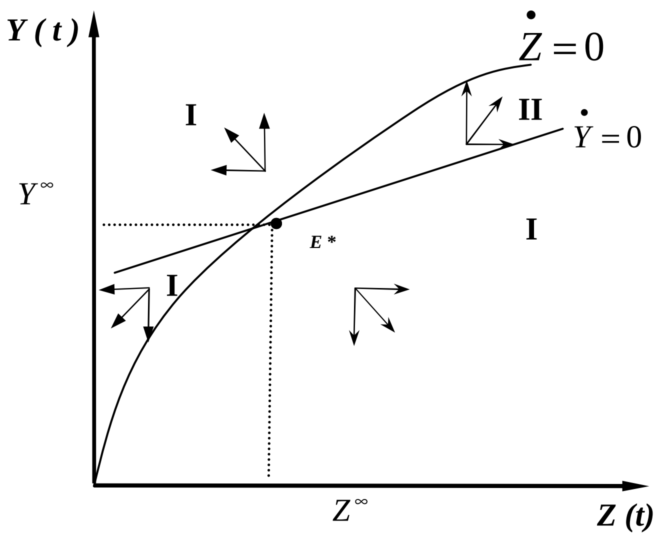

Figure 1b: E as a saddle point equilibrium with the slope of the output isocline steeper than the biodiversity isocline

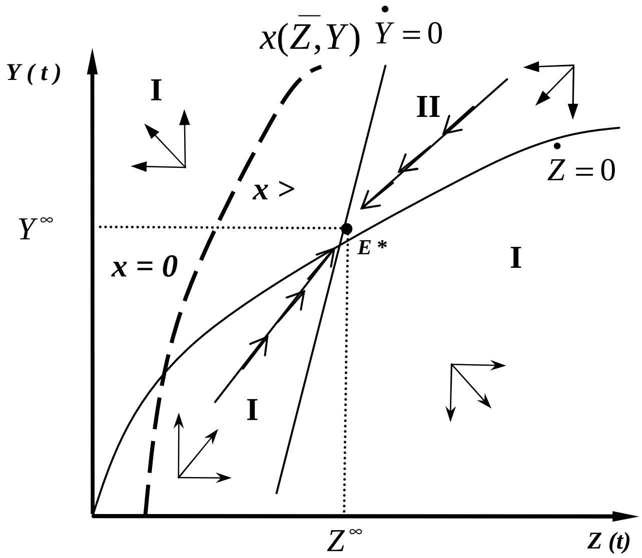

Figure 2a: Zero input level intersects the saddle path to the let of the equilibrium point

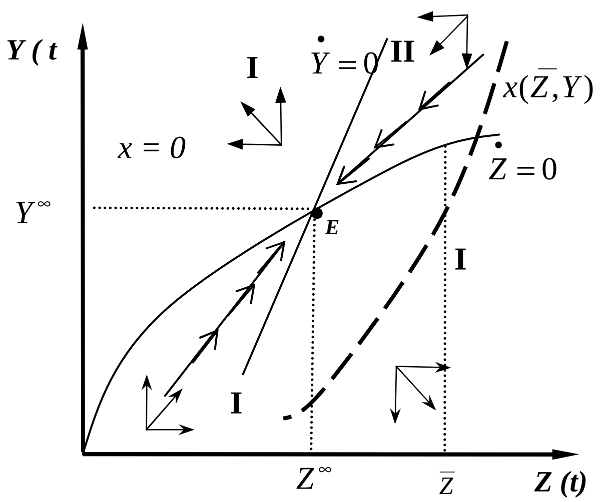

Figure 2b: Zero input level intersects the saddle path to the right of the equilibrium point

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