Another Look at Yield Spreads*: The role of liquidity

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Abstract

Liquidity plays an important role in explaining how banks determine their allocation of funds. This paper examines whether this fact can explain yield spreads and the term structure of interest rates. The paper models banks' demand for liquidity in a manner similar to that used to study household need for liquidity, namely, by using a cash-in-advance type model. The paper finds that the shadow price of the cash-in-advance constraint plays an important role in determining yield spreads. The empirical part of the paper shows that the expectations hypothesis might be salvaged under the maintained hypothesis concerning the liquidity premium and risk premium.

Key Words: Yield Spread; Liquidity; Term Structure; Cash-in-advance Constraint
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1 Introduction

An attractive theory of the term structure of interest rates is the expectations hypothesis, which holds that the long rate equals expected future short rates over the term of the bond. Many empirical studies, such as Shiller, Campbell, and Schoenholtz (1983), Fama (1984), Mankiw and Miron (1986), Fama and Bliss (1987), Mishkin (1988), Hardouvelis (1988), Froot (1989), Simon (1989, 1990), Cook and Hahn (1990), Campbell and Shiller (1991), and Roberds, Runkle and Whiteman (1996), find that the estimated coefficients in a regression of the change in the expected future short-term interest rates on the yield spread are significantly less than the value of unity predicted by the expectations hypothesis and differ as the forecast horizon varies.[1] Even though Fama (1984), Mishkin (1988), Hardouvelis (1988), and Simon (1990) have found yield spreads do help predict future rates, the coefficient appears inconsistent with the expectations hypothesis.

Several studies, such as Mankiw and Miron (1986), McCallum (1994), and Rudebusch (1995), have shown that even if the expectations theory did hold, it would be hard to use it for forecasting due to interest rate smoothing by the Fed. Mankiw and Miron (1986) argue that the negligible predictive power of the spread after the founding of the Fed did not reflect a failure of the expectations theory. Instead, they suggest that the Fed 'stabilized' short-term rates, such as the three-month rate, and by inducing a random-walk behavior eliminated any predictable variation. McCallum (1994) proposes that the empirical failure of the rational expectations theory of the term structure of interest rates can be rationalized with the expectations theory by recognition of an exogenous random term premium plus the assumption that monetary policy in-

[1] Rudebusch (1995) refers to 'U-shaped' pattern of the predictability of the yield curve. Roberds and Whiteman (1999) state the existence of a "predictability smile" in the term structure of interest rates: spreads between long maturity rates and short rates predict subsequent movements in interest rates provided the long horizon is three months or less or if the long horizon is two years or more, but not for immediate maturities.

volves smoothing of an interest rate instrument–the short-term rate–together with the responses to the prevailing level of the spread. Rudebusch (1995) states that Federal Reserve interest rate targeting accompanied by the maintained rational expectations hypothesis explains the varying predictive ability of the yield curve.

Previous studies have also focused on the possibility of a time-varying risk premium and concluded that a time-varying risk premium can help explain the failures of the expectations theory. Examples include Engle, Lilien and Robins (1987), Simon (1989, 1990), Friedman and Kuttner (1992) and Lee (1995), among others.

However, Evans and Lewis (1994) argue that a time-varying risk premium alone is not sufficient to explain the time-varying term premium in the Treasury bill. Dotsey and Otrok (1995) suggest that a deeper understanding of interest rate behavior will be produced by jointly taking into account the behavior of the monetary authority along with a more detailed understanding of what determines term premia.

Recently, Bansal and Coleman (1996) argue that some assets other than money play a special role in facilitating transactions, which affects the rate of return that they offer. In their model, securities that back checkable deposits provide a transaction service return in addition to their nominal return. Since short-term government bonds facilitate transactions by backing checkable deposits, this results in equilibrium in a lower nominal return for these bonds. Such a view implies that liquidity plays an important role in determining the returns of various securities.

In general, liquidity refers to the ease with which an asset can be bought or sold. Asset purchases or sales are subject to transaction costs and the liquidity of an asset decreases as the costs incurred in buying and selling it increase. So, if liquidity is an important factor determining the returns of financial assets, liquidity may be important for yield spreads and the term structure of interest rates. But how do investors consider liquidity in allocating their funds between securities of different terms? Since commercial banks are principal investors and primary dealers in instruments such as Federal funds, commercial paper, and Eurodollar CDs, a study of liquidity demand by commercial banks may provide the key to answering this question. Stigum (1990) states that "in the money market, in particular, banks are players of such major importance that any serious discussion of the various markets that comprise the money market must be prefaced with a careful look at banking." Cook and La Roche (1993) also emphasize that commercial banks play an important role in the money market.

In terms of banks, liquidity means having ready cash (i.e., reserves) in all currencies to pay the bills, to fund the drawdowns of loan commitments, to meet depositor withdrawals, to honor cash calls on foreign exchange contracts and guarantees, and to meet reserve requirements (Abboud (1987)). If a bank might at some point be unable to turn its assets into ready cash, the bank faces a liquidity risk. Liquidity is a crucial fact of life for banks, and for this reason may have an implication for yield spreads and the term structure of interest rates. For example, since banks' loans are relatively illiquid long-term assets, they are not useful for purposes of bank's liquidity management. By contrast, short-term government securities such as Treasury bills are very liquid. Stigum (1990) emphasizes that all banks hold government securities for liquidity and profit.

In addition, since banks' liquidity can vary as a result of policy of the Fed, financial market conditions, the individual bank's specific demand for reserves, and so on, banks' liquidity might play an important role in explaining time-varying term premia. Most previous studies, however, have not focused on banks as the main investors in financial markets and thus, bank's liquidity.

This paper attempts to answer the following question: Can the fact that liquidity plays an important role in explaining how banks determine their allocation of funds explain yield spreads and help provide an explanation for the failure of the expectations hypothesis?

The paper begins by developing a model of a bank's optimal behavior. This model incorporates the cash-in-advance constraint (liquidity constraint) of Clower (1967), Lucas (1982), Svensson (1985), Lucas and Stokey (1987) and Bansal and Coleman (1996) into a model of bank decision-making similar to Cosimano (1987), Cosimano and Van Huyck (1989), Elyasiani, Kopecky and Van Hoose (1995), and Kang (1997). In addition, this model incorporates a time-varying risk premium. The paper finds that the shadow price of the cash-in-advance constraint has an important role in determining yield spreads and the term structure of interest rates. The empirical part of the paper shows that the expectations hypothesis might be salvaged under the maintained hypothesis concerning the liquidity premium and risk premium.

The plan of this paper is as follows. Section 2 develops a model that incorporates liquidity into banks' optimal behavior and examines the determination of yield spreads and the term structure of interest rates. Section 3 provides a brief empirical test of the simple expectations hypothesis of the term structure and obtains empirical results for the theoretical model developed in Section 2. A brief summary and concluding remarks are given in Section 4.

2 The Model

This section develops a model that incorporates liquidity into banks' optimal behavior. The first part presents a simplified model in which bank loans are two-period assets and Federal funds lent are one-period assets. The second part generalizes to the case when bank loans are n-period assets and Federal funds are one-period assets. The key results are the same in both cases.

2.1 Basic Model

2.1.1 Banks' Optimal Behavior Subject to Cash-in-advance Constraint

There are many banks in the banking system. Banks have an infinite horizon. Each period consists of two sessions, the beginning of period t and the end of period t. We assume that the public prefers demand deposits to cash, so all the cash is deposited in the bank at the end of the period. The reserve supply in the banking system does not change unless the Fed changes it. When borrowers do not repay loan principal as well as loan interest rate payment, banks face risks on loans (default risk). We assume that this default risk increases as the quantity of loans or the maturity of loans increases. Suppose that a representative bank has the following profit function:

$$\Pi_t = r_t^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2) + r_{t-1}^L L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2) + r_t^F F_t,$$

where L_t is the quantity of new two-period loans made in the beginning of period t, r_t^L is the yield on two-period loans made at time t, r_t^F is the yield on Federal funds lent or borrowed at time t, and F_t is the Federal funds lent or borrowed at time t. The δs' are non-negative constants. The two expressions in parentheses represent the risk on loans each period.

The bank chooses the level of new loans, L_t, and lends it to the public at the beginning of period t. The bank will get back the loan at the beginning of period $t+2$. At the end of period t, it chooses the quantity of Federal funds to lend, F_t. These choices, along with some other exogenous or predetermined factors, determine the level of reserves, R_t, with which the bank will end the period. These other factors are (1) the bank's level of demand deposits, $\overline{D}_t$, which is taken to be exogenous, with a positive value for $\overline{D}_t - \overline{D}_{t-1}$ increasing the bank's end-of-period reserve position; (2) the repayment of the Federal funds the bank lent the previous period, F_{t-1}; and (3) the repayment of the loans the bank made two periods previously, L_{t-2}. The bank's end-of-period reserves thus evolve according to[2]:

$$R_t = R_{t-1} + L_{t-2} - L_t + F_{t-1} + \overline{D}_t - \overline{D}_{t-1} - F_t. \tag{2.1}$$

In addition, the bank must satisfy the reserve requirement

$$R_t \geq \theta \overline{D}_t, \tag{2.2}$$

where θ is the required reserve ratio.

In the beginning of period t, a representative bank starts with reserve balances given by $R_{t-1} + L_{t-2}$, the transferred previous reserve balances plus the repayment of the loans made two periods previously. Given the loan rate and the Federal funds rate, a representative bank must choose its loan supply, L_t, before knowing the deposits, $\overline{D}_t$. This choice is subject to predetermined holdings $R_{t-1} + L_{t-2}$ of reserve balances. That is, in our model, a borrower from the bank wants cash, and the bank can only extend a loan to such a customer if it has cash (reserve) on hand equal to the amount of the loan. In this case, the bank's need for liquidity (sufficient reserves) can be transaction purpose as well as precautionary purpose for meeting depositor withdrawals or reserve requirement.

We can model the bank's demand for liquidity in the same way that the cash-in-advance (hereafter CIA) literature has modeled demand for liquidity by private households. In the conventional CIA formulation, goods must be purchased with cash, and a consumer can only obtain goods if he has cash on hand sufficient to pay for them.

[2] We assume that interest income less default loss is paid out as stockholder dividends and so these terms do not affect the level of reserves. In practice, Fed funds interest is paid by banks a week after the loan and term loan interest is paid much later. Abstracting from the effects of interest payments on reserves seems a useful simplification which is unlikely to matter for the results presented here.

In this case, a household's demand for cash is mainly for the purpose of transaction. In complete analogy, for the bank, loans are funded with reserves and the bank can only have loan commitments with sufficient reserve balances. This kind of transaction reserve model can be compatible with Hamilton (1996, 1998) in which banks would want to hold reserves even if there were no reserve requirements, for the same reason that members of the public hold cash: good funds are needed to effect transactions. Reserves are useful to banks beyond the purpose of reserve requirements in the sense that reserves provide banks with the transaction service return. In this case, even though banks are subject to the required reserve constraint, the true value of the liquidity services reserve provides might not be captured with the only shadow price of required reserve constraint.

How should we think of the CIA constraint in the light of borrowed reserves? Cosimano and Sheehan (1994) show from the study of discount window borrowing by weekly reporting banks disaggregated by Federal Reserve District that any single bank seldom visits the discount window and argue that this behavior is consistent either with banks not aggressively managing their discount window borrowings or more plausibly with the presence of considerable harassment costs imposed by the discount window officer. Hamilton (1998) states that banks act as if they faced a cost function for borrowing reserves from the Fed and the cost of borrowing includes nonpecuniary costs of borrowing in the form of additional regulation, supervision, and inferior credit standing with other banks. In this point of view, banks should not place excessive reliance on the discount window to obtain reserves and fund their loans to the public and thus, they might need to hold excess reserves for liquidity yield purpose. Hence, the CIA constraint which the bank faces might reflect bank's demand for liquidity.

However, in either application, the strict CIA requirement ignores such real-life institutions as credit cards available to consumers or within-day overdraft privileges available to banks on their accounts with the Fed. Even so, the requirement of needing actual cash on hand for certain transactions seems to capture the key idea of what is meant by liquidity and has proven a useful framework for thinking about liquidity demand by private households. We propose that it may also be fruitful for seeing how the need for liquidity may make a difference for understanding the rates of return on assets of different maturities held by banks. Thus, we propose that a representative bank faces a CIA or liquidity constraint such as the following:

$$L_t \leq R_{t-1} + L_{t-2}, \tag{2.3}$$

where R_{t-1} is the reserve balance transferred from the previous period to the start of period t. In this case, we assume that the repayment of outstanding loans at the beginning of period t contributes to the bank's liquidity balances at the beginning of period t. Thus, the bank enters period t with predetermined holdings of reserves as the liquidity balances and the bank's new loans must obey the CIA constraint.

2.1.2 The Equilibrium

A representative bank faces the choice of new loans at the beginning of period t and the choice of Federal funds at the end of period t. The state variables that are relevant for the bank's decision of the quantity of Federal funds to lend at the end of period t are $L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}$, and $\overline{D}_t$. Consider the following value function formulation of the decision at the end of period t:

$$\begin{aligned}
& U_t(L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \overline{D}_t) \\
= \; & \max_{\{F_t\}} \{ r_t^F F_t + \beta E_t V_{t+1}(L_t, L_{t-1}, F_t, R_t, \overline{D}_t) \},
\end{aligned}$$

subject to

$$R_{t-1} + L_{t-2} - L_t + F_{t-1} + \overline{D}_t - \overline{D}_{t-1} - F_t \geq \theta \overline{D}_t,$$

where $U_t(.)$ denotes the lifetime value of the bank's optimal program as of the second session of period t whereas $V_{t+1}(L_t, L_{t-1}, F_t, R_t, \overline{D}_t)$ is the value as of the first session of period $t+1$ and β denotes the discount factor. The choice of Federal funds is subject to equations (2.1) and (2.2).

In the beginning of period t, a representative bank starts with reserve balances given by $R_{t-1} + L_{t-2}$. Given the loan rate and the Federal funds rate, a representative bank must choose its loan supply, L_t, before knowing the deposits, $\overline{D}_t$. The state variables for the decision at the beginning of period t are $L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}$, and $\overline{D}_{t-1}$. The value function of the beginning of period at time t is:

$$\begin{aligned} V_t(L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \overline{D}_{t-1}) &= \max_{\{L_t\}} \{ r_t^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2) \\ &\quad + r_{t-1}^L L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2) \\ &\quad + E_t U_t(L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \overline{D}_t) \} \end{aligned}$$

subject to

$$L_t \leq R_{t-1} + L_{t-2},$$

where the choice of loan supply is subject to the CIA constraint, equation (2.3).

A representative bank's problem is to maximize the value function $V_t(.)$ subject to the CIA constraint at the first session of period t and the value function $U_t(.)$ subject to the balance-sheet constraint (2.1) and reserve requirement (2.2). When we derive first-order and envelope conditions, it is straightforward to characterize the equilibrium as satisfying the following equations:

$$r_t^F = \beta E_t \eta_{t+1} + \lambda_t, \tag{2.4}$$

$$\begin{aligned} & r_t^L - (\delta_0 + \delta_1 L_t) + \beta r_t^L - \beta(\delta_0 + \delta_1 L_t) \\ = \; & r_t^F + \beta E_t r_{t+1}^F + \eta_t - \beta E_t \eta_{t+1} \end{aligned} \tag{2.5}$$

where η_t and λ_t denote the multiplier of the CIA constraint and the multiplier of the required reserve constraint respectively. The one represents the shadow price of the CIA constraint while the other is the implicit price of required reserve constraint. Equation (2.5) can be rewritten as follows:

$$\begin{aligned} r_t^L \; = \; & \frac{1}{(1+\beta)}[r_t^F + \beta E_t r_{t+1}^F] + (\delta_0 + \delta_1 L_t) \\ & + \frac{1}{(1+\beta)}[\eta_t - \beta^2 E_t \eta_{t+2}]. \end{aligned} \tag{2.6}$$

The derivation of equations (2.4) - (2.6) is handled in Apendix A.

A representative bank chooses F_t (and hence a value for λ_t and $E_t \eta_{t+1}$) so as to satisfy (2.4). In equilibrium, F_t must be zero, and the exogenous supply of reserves and demand for loans will determine λ_t and $E_t \eta_{t+1}$, which together with (2.4), will determine r_t^F. The interest rate adjusts to clear the market. The equation (2.4) bears an analogy with Svensson's (1985) paper. It states that the current Federal funds rate is the sum of the discounted expected value of next period's shadow price of the CIA constraint and the shadow price of the required reserve constraint at time t. Even though the required reserve constraint is non-binding and the shadow price of it is zero, the existence of a binding liquidity constraint drives a positive Federal funds rate. So against Federal funds, reserve is held for the future liquidity services it provides, and the value of these liquidity services is the value of relaxing the future liquidity constraint. Equation (2.5) is an Euler equation and an optimal condition between the loan market and the Federal funds market. The equation implies that the marginal benefit of increasing the volume of loans is equal to the marginal benefit of lending Federal funds.

Equation (2.6) states that the loan rate is the weighted average of the current Federal funds rate and expected future Federal funds rate plus the cost resulting from the risk (or transaction costs) on loans and the cost of loss of the liquidity benefit. The parenthesis of the right-hand side is the risk premium while the second bracket of the right-hand side is the liquidity premium. The liquidity premium depends on the difference between the current shadow price of the CIA constraint and the expected shadow price of CIA constraint at time $t+2$. If the bank expects the liquidity benefit of the loan lent at time t to be higher at the beginning of time $t+2$ when the bank gets back the loan lent at the beginning of time t, the bank doesn't require as much compensation for liquidity loss. On the other hand, if the bank expects the liquidity benefit of the loan lent today to be lower at the beginning of time $t+2$, the bank will require more compensation for liquidity loss.

Multiplying equation (2.5) by β and taking the expectation at time $t-1$ and rearranging it, we get:

$$\begin{aligned} E_{t-1}r_t^L &= \frac{1}{\beta(1+\beta)}[\beta E_{t-1}r_t^F + \beta^2 E_{t-1}r_{t+1}^F] + (\delta_0 + \delta_1 E_{t-1}L_t) \\ &\quad + \frac{1}{\beta(1+\beta)}[\beta E_{t-1}\eta_t - \beta^3 E_{t-1}\eta_{t+2}]. \end{aligned} \tag{2.7}$$

Equation (2.7) is the term structure model that incorporates a time-varying liquidity premium and risk premium given information at time $t-1$. One point to make is that if the CIA constraint is binding, monetary policy draining or injecting reserve balances can have an impact not only on the Federal funds rate and the loan rate, but also on the term premium and thus, monetary policy can affect the term structure of interest rates.

2.2 General Model

2.2.1 The Equilibrium

So far, we focused on the simple case in which banks' loans were two-period assets and Federal funds lent one-period assets. In this section, we will extend our model to the case in which banks' loans are n-period assets and Federal funds lent remain one-period assets. The basic set-up is the same. A representative bank's profit function is as follows:

$$\begin{aligned}\Pi_t &= r^L_{n,t}L_t - (\delta_0 L_t + \frac{\delta_1}{2}L_t^2) + r^L_{n,t-1}L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2}L_{t-1}^2) + ... \\ &\quad + r^L_{n,t-1}L_{t-1} - (\delta_0 L_{t-n+1} + \frac{\delta_1}{2}L_{t-n+1}^2) + r_t^F F_t,\end{aligned}$$

where $r^L_{n,t}$ is the yield on n-period loan made at time t. In this case, a representative bank has two options at time $t-1$. One option is for the bank to hold reserves at the end of period $t-1$ in order to lend them over n periods at the beginning of period t. The other option is that the bank rolls over reserves as Federal funds for n periods. If a representative bank lends the public loans over n periods, the bank faces risks on loans and this default risk increases as the quantity of loans or the maturity of loans increases. The expressions in parentheses represent the risk on loans each period.

At the end of period t, the bank chooses F_t, the quantity of Federal funds to lend. These choices, along with some other exogenous or predetermined factors, determine the level of reserves with which the bank will end the period. These other factors are the same for (1) and (2) in the basic model and (3) the repayment of the loans the bank made n periods previously (L_{t-n}). The bank's end of period reserves thus evolve according to:

$$R_t = R_{t-1} + L_{t-n} + F_{t-1} + \overline{D}_t - \overline{D}_{t-1} - L_t - F_t. \tag{2.8}$$

In addition, the bank must satisfy the reserve requirement:

$$R_t \geq \theta \overline{D}_t.$$

In the beginning of period t, a representative bank starts with reserve balances given by $R_{t-1} + L_{t-n}$. Given the loan rate and the Federal funds rate, a representative bank must choose its loan supply, L_t, before knowing the deposits, $\overline{D}_t$. As in the basic model, the bank faces a CIA constraint:

$$L_t \leq R_{t-1} + L_{t-n}. \tag{2.9}$$

The state variables that are relevant for the bank's decision of the quantity of Federal funds to lend at the end of period t are $L_t, L_{t-1}, ..., L_{t-n}, F_{t-1}, R_{t-1}$, and $\overline{D}_t$. The value function formulation of the decision for the end of the period at time t is as follows:

$$\begin{aligned} & U_t(L_t, L_{t-1}, L_{t-2}, ..., L_{t-n}, F_{t-1}, R_{t-1}, \overline{D}_t) \\ = \; & \max_{\{F_t\}} \{ r_t^F F_t + \beta E_t V_{t+1}(L_t, L_{t-1}, ..., L_{t-n+1}, F_t, R_t, \overline{D}_t) \}, \end{aligned}$$

subject to

$$R_{t-1} + L_{t-n} + F_{t-1} + \overline{D}_t - \overline{D}_{t-1} - L_t - F_t \geq \theta \overline{D}_t.$$

The state variables for the decision at the beginning of period t are $L_{t-1}, L_{t-2}, ..., L_{t-n}, F_{t-1}, R_{t-1}$, and $\overline{D}_{t-1}$. The value function of the beginning of period t is as follows:

$$
\begin{aligned}
& V_t(L_{t-1},..,L_{t-n},F_{t-1},R_{t-1},\overline{D}_{t-1}) \\
= & \max_{\{L_t\}}\{r^L_{n,t}L_t-(\delta_0 L_t+\frac{\delta_1}{2}L_t^2)+r^L_{n,t-1}L_{t-1}-(\delta_0 L_{t-1}+\frac{\delta_1}{2}L_{t-1}^2)+... \\
& +r^L_{n,t-n+1}L_{t-n+1}-(\delta_0 L_{t-n+1}+\frac{\delta_1}{2}L_{t-n+1}^2)+E_tU_t(L_t,L_{t-1},...,L_{t-n},F_{t-1},R_{t-1},\overline{D}_t)\}
\end{aligned}
$$

subject to

$$L_t \leq R_{t-1}+L_{t-n}.$$

From first-order and envelope conditions, it is straightforward to characterize the equilibrium as satisfying the following equations:

$$r_t^F=\beta E_t\eta_{t+1}+\lambda_t, \tag{2.4}$$

$$
\begin{aligned}
& (1+\beta+...+\beta^{n-1})r^L_{n,t}-(1+\beta+...+\beta^{n-1})(\delta_0+\delta_1 L_t) \\
= & (r_t^F+\beta E_t r^F_{t+1}+...+\beta^{n-1}E_t r^F_{t+n-1})+\eta_t-\beta^n E_t\eta_{t+n}.
\end{aligned} \tag{2.10}
$$

For n period loan and one period Federal funds, we get the same equilibrium condition for the Federal funds market as the equation (2.4) in the basic model. (See Appendix B for the derivation of equilibrium conditions (2.4) and (2.10)). Equation (2.10) can be rewritten as follows:

$$
\begin{aligned}
r^L_{n,t} = & \frac{1}{(1+\beta+...+\beta^{n-1})}[r_t^F+\beta E_t r^F_{t+1}+...+\beta^{n-1}E_t r^F_{t+n-1}] \\
& +(\delta_0+\delta_1 L_t)+\frac{1}{(1+\beta+...+\beta^{n-1})}[\eta_t-\beta^n E_t\eta_{t+n}].
\end{aligned} \tag{2.11}
$$

Equation (2.11) states that the n-period loan rate is the weighted average of the one-period current Federal funds rate and expected future Federal funds rates over n periods plus the cost resulting from the risk on loans and the cost from the loss of liquidity benefit. Multiplying equation (2.10) by β, taking the expectation at time $t-1$, and rearranging it, we get:

$$\begin{aligned} E_{t-1}r^L_{n,t} \quad &= \quad \frac{1}{\beta(1+\beta+...+\beta^{n-1})}[\beta E_{t-1}r^F_t + ... + \beta^n E_{t-1}r^F_{t+n-1}] + (\delta_0 + \delta_1 E_{t-1}L_t) \\ &+\frac{1}{\beta(1+\beta+...+\beta^{n-1})}[\beta E_{t-1}\eta_t - \beta^{n+1}E_{t-1}\eta_{t+n}]. \end{aligned} \tag{2.12}$$

2.2.2 Term Structure Implication

Equation (2.12) determines the term structure of interest rates between the n-period loan rate and the one-period Federal funds rate.[3] This term structure model incorporates a time-varying liquidity premium and risk premium. Since banks' liquidity can vary over time, the liquidity difference shows up on the term structure model as the liquidity premium. Hence, banks' liquidity plays an important role in explaining the time-varying term premium and thus can help to explain the widespread rejection of the expectations hypothesis. Hence, a term structure model that incorporates banks' liquidity demand into banks' optimal behavior might provide an alternative to the simple expectations hypothesis.

Assuming $\beta = 1$, equation (2.11) can be rewritten as follows:

$$r^L_{n,t} = \frac{1}{n}E_t\sum_{i=0}^{n-1} r^F_{t+i} + (\delta_0 + \delta_1 L_t) + \frac{1}{n}[\eta_t - E_t\eta_{t+n}]. \tag{2.13}$$

[3] Most of the empirical term-structure literature concentrates on the term structure for government securties. In this point of view, the term structure developed in our model might be limited for general application. However, even so, the key idea of what is meant by liquidity seems to provide an significant implication for the term structure.

To explore whether this model can quantitatively match the feature of the expectations hypothesis under the maintained hypothesis concerning the liquidity premium and risk premium, we need to get quantitative magnitudes of the shadow prices. Recall from equation (2.4) that in equilibrium, the current Federal funds rate is the expected value of next period's discounted shadow price of the CIA constraint, assuming the required reserve constraint is not binding.[4] Thus, we can take conditional expectations of both sides of equation (2.13) based on information available at $t-1$ as in (2.12) to use the previous Federal funds rate as the expected value of the discounted shadow price of the CIA constraint. Similarly, we can assume $E_{t-1}r^F_{t+n-1} = E_{t-1}\eta_{t+n}$. Then, equation (2.13) can be rewritten:

$$\begin{aligned} E_{t-1}r^L_{n,t} &= \frac{1}{n}E_{t-1}\sum_{i=0}^{n-1} r^F_{t+i} + (\delta_0 + \delta_1 E_{t-1}L_t) \\ &\quad + \frac{1}{n}[r^F_{t-1} - E_{t-1}r^F_{t+n-1}]. \end{aligned} \tag{2.14}$$

Further assuming rational expectations, let $v_{t+j}, j = 0,1,2,...,n-1,\ e_t$, and ξ_t denote the following forecast errors orthogonal to information available at time $t-1$:

$$v_{t+j} = r^F_{t+j} - E_{t-1}r^F_{t+j},\ j = 0,1,2,...,n-1, \tag{2.15}$$

$$e_t = r^L_{n,t} - E_{t-1}r^L_{n,t}, \tag{2.16}$$

$$\xi_t = L_t - E_{t-1}L_t. \tag{2.17}$$

[4] Frost (1970) shows that banks hold excess reserves because the cost associated with constantly adjusting reserve positions is greater than the interest earned on short-term securties and the profitability of holding excess reserves when interest rates are very low makes the banks' demand for excess reserves kinked at a low rate of interest. In practice, excess reserves from the data for depository institution which are taken from Statistical Release provided by the Federal Reserve Board of Governors are always positive during the sample period of our empirical study. Therefore, our assumption is consistent with the U.S. data over this sample period.

We also assume that e_t, ξ_t, and v_{t+j} for all j's are serially uncorrelated and mutually independent. Substituting equations (2.15), (2.16), and (2.17) into (2.14) and rearranging it, we get:

$$\begin{aligned} r^L_{n,t} &= \frac{1}{n}\sum_{i=0}^{n-1} r^F_{t+i} + (\delta_0 + \delta_1 L_t) + \frac{1}{n}[r^F_{t-1} - r^F_{t+n-1}] \\ &\quad +(e_t - \frac{1}{n}\sum_{j=0}^{n-2} v_{t+j} - \delta_1 \xi_t). \end{aligned} \tag{2.18}$$

Equation (2.18) provides plausible parameter values for the risk premium and liquidity premium under above assumption. The second term on the right-hand side of (2.18), $\delta_0 + \delta_1 L_t$, and the third term, $\frac{1}{n}(r^F_{t-1} - r^F_{t+n-1})$, capture the risk premium and the liquidity premium, respectively. Subtracting r^F_t from both sides on equation (2.18) and rearranging it result in:

$$\begin{aligned} \frac{1}{n}\sum_{i=0}^{n-1} r^F_{t+i} - r^F_t &= -\delta_0 + (r^L_{n,t} - r^F_t) - \delta_1 L_t - \frac{1}{n}[r^F_{t-1} - r^F_{t+n-1}] \\ &\quad -(e_t - \frac{1}{n}\sum_{j=0}^{n-2} v_{t+j} - \delta_1 \xi_t). \end{aligned} \tag{2.19}$$

The model (2.19) implies that the simple expectations hypothesis doesn't hold because of the liquidity premium and the risk premium. We can estimate the model and examine if the expectations hypothesis might be salvaged under the maintained hypothesis concerning the liquidity premium and the risk premium.

3 Estimation of the Term Structure Model

3.1 The Data

The weekly data set we use runs from February 1, 1984 to December 27, 2000, which gives us 882 observations.[5] The interest rates are taken from Statistical Release provided by the Federal Reserve Board of Governors. We also take quantities of loans from item H.8 (assets and liabilities of all commercial banks in the USA) of the Federal Reserve Statistical Release.[6] Since this series refers to outstanding loans at each period, we use the change in this series as a proxy for the volume of new loans extended. We consider Federal funds rates as a short term rate and 1- and 3-month commercial paper (CP) rates as long-term rates. All interest rates are averages of 7 calendar days ending on Wednesday and annualized using a 360-day year.[7] Here, the CP rate is viewed as a substitute for the rate on bank's loans to financial and industrial companies.

Figure 1 shows movements of the Federal Funds rate and the 1-month and 3-month CP rates during this sample. One interesting feature is that the Federal Funds rate fluctuated around the CP rates before 1990 U.S. recession but after the recession the CP rates were higher than the Federal Funds rate. During the sample period, averages of the Federal Funds rate, 1-month and 3-month CP rates are 6.23%, 6.28% and 6.29% respectively, and thus the 1-month and 3-month CP rates are on average higher by 5 and 6 basis points respectively, than the Federal Funds rate. However, the standard deviation of the Federal Funds rate is 1.93, higher than those of the 1-month and 3-

[5] Since the Fed changed from lagged reserve accounting to contemporaneous reserve accounting in February 1984, we use only data after February 1984.

[6] These quantities of loans are loans and leases in bank credit by weekly reporting banks. These quantities are slightly different depending on all commerical banks, domestic commercial banks, and large commercial banks. However, the estimation results were quantitatively and qualitatively similar.

[7] The original CP rates are business-daily averages of offering rates on commercial paper placed by several leading dealers for firms whose bond rating is AA or the equilvalent. After we multiply Friday's interest rate by 3 and use the value of the previous business day for holidays, we construct weekly averages of 7-days series.

month CP rates, 1.84 and 1.82, respectively which implies that the volatility of the Federal Funds rate is somewhat higher than those of the CP rates.

3.2 A Test of the Expectations Hypothesis

We start from the test of the simple expectations hypothesis, which implies that the long-term rate is the weighted average of the current short-term rate and expected future short-term rates and that the current spread between the long-term rate and short-term rate predicts the change in future short-term rates. That is,

$$r_{n,t}^{L} = \frac{1}{n} E_t \sum_{i=0}^{n-1} r_{t+i}^{F}, \tag{3.1}$$

where $r_{n,t}^{L}$, r_t^F are the n-period CP rate (our substitute for the n-period loan rate) and one-period Federal funds rate respectively. Assuming rational expectations, one can rearrange equation (3.1) to yield the following relationship as the term structure regression for empirical investigation:

Model I: $\delta_0 = \delta_1 = 0, \eta_t = E_t \eta_{t+n}$

$$\frac{1}{n} \sum_{i=0}^{n-1} r_{t+i}^{F} - r_t^F = \alpha + \phi(r_{n,t}^{L} - r_t^F) + \varepsilon_t, \tag{3.2}$$

where $\varepsilon_t = \frac{1}{n} \sum\limits_{i=0}^{n-1} r_{t+i}^{F} - \frac{1}{n} E_t \sum\limits_{i=0}^{n-1} r_{t+i}^{F}$ and should be uncorrelated with any variable known at time t. Here, n corresponds to 4 or 12 weeks for one- and three-month commercial paper, respectively.

Equation (3.2) can be estimated by OLS with autocorrelation-heteroskedasticity consistent errors. According to the simple expectations hypothesis, $\alpha = 0$, and $\phi = 1$. This test can be nested within our term structure model (equation (2.19)) by imposing the restrictions $\delta_0 = \delta_1 = 0$, and $\eta_t = E_t \eta_{t+n}$. Table 1 shows the result for estimation of (3.2).

The estimated coefficient on the spread is significantly less than unity and different from zero at conventional significance levels. In addition, the estimated coefficients on the constant are significantly less than zero. These results are very similar to previous empirical studies.[8]

3.3 Test of the Expectations Hypothesis with Liquidity Premium and Risk Premium

Our model developed in Section 2 implies that the simple expectations hypothesis does not hold because of the liquidity premium and the risk premium. Since banks' optimal behavior is subject to a CIA constraint, bank's liquidity causes the shadow price of the CIA constraint to play an important role in yield spreads and the term structure of interest rates. Thus, our model suggests that we need to incorporate a liquidity premium and a risk premium into the simple expectations hypothesis. The model (2.19) forms the basis of the tests of the term structure that we focus on. Subtracting $\frac{1}{n} r^F_{t+n-1}$ from both sides of (2.19) to avoid including ex post future interest rate in the equation as a regressor, we get:

$$\begin{aligned} \frac{1}{n}\sum_{i=0}^{n-2} r^F_{t+i} - r^F_t &= -\delta_0 + (r^L_{n,t} - r^F_t) - \delta_1 L_t - \frac{1}{n} r^F_{t-1} \\ &\quad + (\frac{1}{n}\sum_{j=0}^{n-2} v_{t+j} + \delta_1 \xi_t - e_t). \end{aligned} \tag{3.3}$$

Then, equation (3.3) involves running the regression:

[8] Rudebusch (1995) provides an excellent survey of previous empirical results.

Model II: $\delta_0 \neq 0, \delta_1 \neq 0, E_{t-1}\eta_t = r^F_{t-1}, E_{t-1}\eta_{t+n} = E_{t-1}r^F_{t+n-1}$

$$\frac{1}{n}\sum_{i=0}^{n-2} r^F_{t+i} - r^F_t = \alpha + \phi(r^L_{n,t} - r^F_t) + \gamma_1 r^F_{t-1} + \gamma_2 L_t + \varepsilon_t, \tag{3.4}$$

where L_t is the quantity of new loans at time t, and $\varepsilon_t = \frac{1}{n}\sum_{j=0}^{n-2} v_{t+j} + \delta_1\xi_t - e_t$. The regression (3.4) differs from all tests of expectations hypothesis in the existing literature where regressand is not $\frac{1}{n}\sum_{i=0}^{n-2} r^F_{t+i} - r^F_t$ but $\frac{1}{n}\sum_{i=0}^{n-1} r^F_{t+i} - r^F_t$. Equation (3.4) cannot be estimated by OLS because ε_t is correlated with the regressors $r^L_{n,t}$ and r^F_t. Rational expectations requires ε_t to be uncorrelated with anything known to banks at time $t-1$ but $r^L_{n,t}$ and r^F_t are not in the date $t-1$ information set. Equation (3.4) can be estimated by instrumental variables using valid instruments. We consider two-stage least squares (2SLS) with a constant, lagged Federal funds rates, and lagged quantities of new loans as instruments. We employ Hansen's (1982) methods in order to check the overidentifying restrictions and thus, test these conjectures about what is the correct set of instruments. Hansen's test statistic has an asymptotic χ^2 distribution with $r-k$ degrees of freedom if the model is correctly specified, where r is the number of instruments and k is the number of estimated coefficients. According to Model II, $\alpha = -\delta_0, \phi = 1, \gamma_1 = -n^{-1}$, and $\gamma_2 = -\delta_1$. Table 2 shows the results.

In both cases, the estimated coefficients on the spread are significantly different from zero and not significantly different from unity, in contrast with the estimated coefficients on the spread in Model I. The t-test for the estimated coefficient on the spread shows that the null hypothesis that the coefficient of the spread is unity is not rejected at the 5% level in both cases. In addition, following Hansen's (1982) method, χ^2_1 for the 1-month CP rate and χ^2_2 for the 3-month CP rate are 1.288 and 5.816 respectively, so the null hypothesis that Model II is correctly specified is accepted at the 5% level. These results imply that these instruments are valid.[9]

All the estimated coefficients have the signs predicted by the theoretical model developed in Section 2 and all estimated coefficients are statistically significant at the conventional level except on the coefficients on L_t. In particular, the estimated coefficients on the liquidity premium are close to the values that the theoretical model implies (-0.25 for the 1-month CP rate and -0.084 for the 3-month CP rate) and statistically significant, indicating that the liquidity plays an important role in explaining the term premium. However, among two components reflecting the risk premium, the constant is statistically significant whereas the estimated coefficients on L_t are not significantly different from zero.

3.4 Robustness

Since we didn't have a direct measure of the loan rate, we used the CP rates as a proxy rate of bank loan rate instead. However, as pointed out in Kashyap et al. (1993), bank loans are special and the commercial papers might be imperfect substitutes. In addition, Stigum (1990) and Cook and LaRoche (1993) state that historically the CP market has been remarkably free of default risk in contrast to bank loans. In this point of view, the CP rate might not be an good choice of proxy. To investigate this issue, we consider the Euro-dollar (ED) rate as an another proxy. Even though the ED rate is a deposit rate and a liability, it is subject to default risk and can fluctuate in accordance to bank's liquidity demand. The 1-month and 3-month ED rates are taken from Statistical Release provided by the Federal Reserve Board of Governor for the

[9] When we included lagged CP rate as an instrument on the estimation of equation (3.4), we rejected the null hypothesis of Hansen's test, which implies that lagged CP rates are not valid instruments. In addition, we estimated the equation using more lagged Federal Funds rates and lagged quantities of new loans as instruments. In some cases, we rejected the null hypothesis that the model is correctly specified but overall the estimated results were similar.

sample. Figure 2 plots movements in these ED rates and the Federal Funds rate. The ED rates show very similar movements to the CD rates displayed in Figure 1.

Table 3 shows estimation results for Model I and Model II. For Model I, the results are very similar to those of the CP rates. For Model II, there is no significant difference between the ED rates and the CP rates. The estimated coefficients on the spread are significantly different from zero and not significantly different from unity. The estimated coefficients on the liquidity premium are statistically significant and quite close to the values expected by the theoretical model. Hansen's test statistics are 0.7 for the 1-month ED rate and 10.29 for the 3-month ED rate respectively, and we do not reject the null, indicating that Model II is correctly specified. However, the estimated coefficients on the risk premium components are not significantly different from zero in contrast to the case of the CP rates.

4 Conclusion

This paper has focused on commercial banks as the main investors in financial markets and banks' liquidity as an important component to determine yield spreads over different term securities. For this end, we have developed a term structure model that incorporates the liquidity demand by commercial banks into banks' optimal behavior. The paper has shown that the shadow price of the cash-in-advance constraint plays an important role in determining the yield spread. Moreover, the empirical study has provided evidence that when we incorporate the liquidity premium and risk premium resulting from transaction activities into the term structure of interest rates, the expectations hypothesis of the term structure of interest rates might be salvaged.

The results of the paper have an important implication. As most households' transaction activities are subject to their liquidity conditions, so are banks' transaction activities. When banks allocate their funds into financial securities of different maturities, they incorporate information about liquidity as well as risk into their portfolio management decisions. Investors know that long-term assets are relatively less liquid than short-term assets and the difference in liquidity among these financial assets is incorporated into their returns. This might be one reason why previous studies have not produced a consensus about the empirical failure of the simple expectations hypothesis. We feel that the story presented here provides a useful alternative to the simple expectations hypothesis.

Appendix A

Here, we derive equations (2.4) and (2.5), where bank loans are two-period assets and Federal funds are one-period assets. Consider the following value function formulation of the decision at the end of period t:

$$\begin{aligned} & U_t(L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \overline{D}_t) \\ = \;& \max_{\{F_t\}} \{ r_t^F F_t + \beta E_t V_{t+1}(L_t, L_{t-1}, F_t, R_t, \overline{D}_t) \}, \end{aligned}$$

subject to

$$R_{t-1} + L_{t-2} - L_t + F_{t-1} + \overline{D}_t - \overline{D}_{t-1} - F_t \geq \theta \overline{D}_t.$$

The first-order conditions are as follows:

$$r_t^F + \beta E_t \frac{\partial V_{t+1}}{\partial F_t} = \beta E_t \frac{\partial V_{t+1}}{\partial R_t} + \lambda_t, \tag{A1}$$

$$R_t \geq \theta \overline{D}_t, \; \textit{with equality if } \lambda_t > 0. \tag{A2}$$

The value function of the beginning of period at time t is:

$$\begin{aligned} V_t(L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \overline{D}_{t-1}) \;=\; & \max_{\{L_t\}} \{ r_t^L L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2) \\ & + r_{t-1}^L L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2) \\ & + E_t U_t(L_t, L_{t-1}, L_{t-2}, F_{t-1}, R_{t-1}, \overline{D}_t) \} \end{aligned}$$

subject to

$$L_t \leq R_{t-1} + L_{t-2}.$$

The first-order conditions are as follows:

$$r_t^L - (\delta_0 + \delta_1 L_t) + E_t \frac{\partial U_t}{\partial L_t} = \eta_t, \tag{A3}$$

$$L_t \leq R_{t-1} + L_{t-2},\ with\ equality\ if\ \eta_t > 0. \tag{A4}$$

By using the envelope condition from the value function of the end of period t, we get:

$$\frac{\partial V_{t+1}}{\partial F_t} = \beta E_{t+1} \frac{\partial V_{t+2}}{\partial R_{t+1}} + \lambda_{t+1}, \tag{A5}$$

$$\frac{\partial V_{t+1}}{\partial R_t} = \beta E_{t+1} \frac{\partial V_{t+2}}{\partial R_{t+1}} + \eta_{t+1} + \lambda_{t+1}. \tag{A6}$$

Substituting equations (A5) and (A6) into equation (A1) and simplifying it provide the equation (2.4):

$$r_t^F = \beta E_t \eta_{t+1} + \lambda_t, \tag{2.4}$$

In addition, we can obtain the following further result by using the envelope condition for the value function at the beginning of period t:

$$\begin{aligned}
\frac{\partial U_t}{\partial L_t} &= \beta E_t r_t^L - \beta(\delta_0 + \delta_1 E_t L_t) + \beta^3 E_t \frac{\partial V_{t+3}}{\partial R_{t+2}} + \beta^2 E_t \lambda_{t+2} + \beta^2 E_t \eta_{t+2} \\
&\quad -\beta E_t \eta_{t+1} - \beta E_t \lambda_{t+1} - \beta^3 E_t \frac{\partial V_{t+3}}{\partial R_{t+2}} - \beta^2 E_t \eta_{t+2} - \beta^2 E_t \lambda_{t+2} - \lambda_t \\
&= \beta E_t r_t^L - \beta(\delta_0 + \delta_1 E_t L_t) - (\beta E_t \eta_{t+1} + \lambda_t) - \beta E_t \lambda_{t+1}
\end{aligned}$$

$$\frac{\partial U_t}{\partial L_t} = \beta E_t r_t^L - \beta(\delta_0 + \delta_1 E_t L_t) - r_t^F - \beta E_t r_{t+1}^F + \beta^2 E_t \eta_{t+2}, \tag{A7}$$

since $\beta E_t \eta_{t+1} + \lambda_t = r_t^F,\ \beta E_t \lambda_{t+1} = \beta E_t r_{t+1}^F - \beta^2 E_t \eta_{t+2}$. Substituting equation (A7) into equation (A3) results in equation (2.5):

$$\begin{aligned} & r_t^L - (\delta_0 + \delta_1 L_t) + \beta r_t^L - \beta(\delta_0 + \delta_1 L_t) \\ = \; & r_t^F + \beta E_t r_{t+1}^F + \eta_t - \beta E_t \eta_{t+1}. \end{aligned} \tag{2.5}$$

Appendix B

In a more general case, banks loans are n-period assets and Federal funds remain one-period assets and we derive equations (2.4) and (2.10). The value function formation of the decision for the end of the period at time t is as follows:

$$\begin{aligned} & U_t(L_t, L_{t-1}, L_{t-2}, ..., L_{t-n}, F_{t-1}, R_{t-1}, \overline{D}_t) \\ = \; & \max_{\{F_t\}} \{ r_t^F F_t + \beta E_t V_{t+1}(L_t, L_{t-1}, ..., L_{t-n+1}, F_t, R_t, \overline{D}_t) \}, \end{aligned}$$

subject to

$$R_{t-1} + L_{t-n} + F_{t-1} + \overline{D}_t - \overline{D}_{t-1} - L_t - F_t \geq \theta \overline{D}_t.$$

The first-order conditions are as follows:

$$r_t^F + \beta E_t \frac{\partial V_{t+1}}{\partial F_t} = \beta E_t \frac{\partial V_{t+1}}{\partial R_t} + \lambda_t, \tag{B1}$$

$$R_t \geq \theta \overline{D}_t, \; with \; equality \; if \; \lambda_t > 0. \tag{B2}$$

The value function of the beginning of period t is as follows:

$$
\begin{aligned}
& V_t(L_{t-1}, .., L_{t-n}, F_{t-1}, R_{t-1}, \overline{D}_{t-1}) \\
= \; & \max_{\{L_t\}} \{ r^L_{n,t} L_t - (\delta_0 L_t + \frac{\delta_1}{2} L_t^2) + r^L_{n,t-1} L_{t-1} - (\delta_0 L_{t-1} + \frac{\delta_1}{2} L_{t-1}^2) + ... \\
& + r^L_{n,t-n+1} L_{t-n+1} - (\delta_0 L_{t-n+1} + \frac{\delta_1}{2} L_{t-n+1}^2) + E_t U_t(L_t, L_{t-1}, ..., L_{t-n}, F_{t-1}, R_{t-1}, \overline{D}_t) \}
\end{aligned}
$$

subject to

$$
L_t \leq R_{t-1} + L_{t-n}.
$$

The first-order conditions are as follows:

$$
r^L_{n,t} + \frac{\partial U_t}{\partial L_t} - (\delta_0 + \delta L_t) = \eta_t, \tag{B3}
$$

$$
L_{t-n} + R_{t-1} \geq L_t, \; with \; equality \; if \; \eta_t > 0. \tag{B4}
$$

Using the envelope condition and equations (A5) and (A6), and substituting them into equation (B1) result in the same equilibrium condition for the Federal funds market as the equation (2.4) in the basic model.

In addition, we can obtain the following further result by using the envelope condition:

$$
\begin{aligned}
\frac{\partial U_t}{\partial L_t} &= \beta E_t r^L_{n,t} - \beta(\delta_0 + \delta_1 L_t) + \beta^2 E_t r^L_{n,t} - \beta^2(\delta_0 + \delta_1 L_t) + ... + \\
& \beta^{n-1} E_t r^L_{n,t} - \beta^{n-1}(\delta_0 + \delta_1 L_t) + \beta^{n+1} E_t \frac{\partial V_{t+n+1}}{\partial R_{t+n}} + \beta^n E_t \lambda_{t+n} + \beta^n E_t \eta_{t+n} \\
& -\lambda_t - \beta E_t \lambda_{t+1} - ... - \beta^n E_t \lambda_{t+n} \\
& -\beta E_t \eta_{t+1} - \beta^2 E_t \eta_{t+2} - ... - \beta^n E_t \eta_{t+n} - \beta^{n+1} E_t \frac{\partial V_{t+n+1}}{\partial R_{t+n}} \\
&= (\beta + \beta^2 + ... + \beta^{n-1}) E_t r^L_{n,t} - (\beta + \beta^2 + ... + \beta^{n-1})(\delta_0 + \delta_1 L_t) \\
& -\beta E_t \eta_{t+1} - ... - \beta^{n-1} E_t \eta_{t+n-1} - \lambda_t - \beta E_t \lambda_{t+1} - ... - \beta^{n-1} E_t \lambda_{t+n-1} \\
\frac{\partial U_t}{\partial L_t} &= (\beta + \beta^2 + ... + \beta^{n-1}) E_t r^L_{n,t} - (\beta + \beta^2 + ... + \beta^{n-1})(\delta_0 + \delta_1 L_t) \\
& -(r^F_t + \beta E_t r^F_{t+1} + ... + \beta^{n-1} E_t r^F_{t+n-1}) + \beta^n E_t \eta_{t+n}, \qquad \text{(B5)}
\end{aligned}
$$

since $\beta E_t \eta_{t+1} + \lambda_t = r^F_t, ..., \beta^{n-1} E_t \eta_{t+n-1} + \beta^{n-2} E_t \lambda_{t+n-2} = \beta^{n-2} E_t r^F_{t+n-2}$, and $\beta^{n-1} E_t \lambda_{t+n-1} = \beta^{n-1} E_t r^F_{t+n-1} - \beta^n E_t \eta_{t+n}$. Substituting equation (B5) into equation (B3) results in equation (2.10):

$$
\begin{aligned}
& (1 + \beta + ... + \beta^{n-1}) r^L_{n,t} - (1 + \beta + ... + \beta^{n-1})(\delta_0 + \delta_1 L_t) \\
= \; & (r^F_t + \beta E_t r^F_{t+1} + ... + \beta^{n-1} E_t r^F_{t+n-1}) + \eta_t - \beta^n E_t \eta_{t+n}. \qquad \text{(2.10)}
\end{aligned}
$$

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Table 1 The expectation hypothesis test without liquidity premium and risk premium

$$\frac{1}{n}\sum_{i=0}^{n-1} r_{t+i}^{F} - r_{t}^{F} = \alpha + \phi(r_{n,t}^{L} - r_{t}^{F}) + \varepsilon_{t},$$

maturity	$\widehat{\alpha}$	$\widehat{\phi}$	R^2
1-month CP	-0.020^{**} (0.009)	0.297^{***} (0.083)	0.169
3-month CP	-0.052^{*} (0.028)	0.488^{***} (0.082)	0.218

Note: The numbers in parenthesis are Newey and West's (1987) autocorrelation-heteroscedasticity consistent standard errors corrected with four lags for 1-month CP rate and twelve lags for the 3-month CP rate. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level in a two-tailed test respectively.

Table 2 The expectations hypothesis test with liquidity premium and risk premium: Two stage least squares estimation

$$\frac{1}{n}\sum_{i=0}^{n-2} r_{t+i}^{F} - r_{t}^{F} = \alpha + \phi(r_{n,t}^{L} - r_{t}^{F}) + \gamma_1 r_{t-1}^{F} + \gamma_2 L_t + \varepsilon_t$$

maturity	$\widehat{\alpha}$	$\widehat{\phi}$	$\widehat{\gamma}_1$	$\widehat{\gamma}_2$	Instrument
1-month CP	-0.308^{***} (0.069)	0.903^{***} (0.172)	-0.206^{***} (0.010)	-0.005 (0.011)	constant,$r_{t-2}^{F},..,r_{t-4}^{F},L_{t-1}$
3-month CP	-0.385^{***} (0.129)	0.989^{***} (0.208)	-0.047^{**} (0.021)	0.027 (0.019)	constant,$r_{t-1}^{F},...,r_{t-4}^{F},L_{t-2}$

Note: The numbers in parenthesis are Newey and West's (1987) autocorrelation-heteroscedasticity consistent standard errors corrected with four lags for the 1-month CP rate and twelve lags for the 3-month CP rate. ***, and ** denotes statistical significance at the 1% and 5% level in a two-tailed test respectively.

Table 3. The expectations hypothesis test with liquidity premium and risk premium: Federal Funds rate and the ED rates

model	maturity	$\widehat{\alpha}$	$\widehat{\phi}$	$\widehat{\gamma}_1$	$\widehat{\gamma}_2$	instrument
M. I	1-month	-0.033^{***} (0.009)	0.334^{***} (0.094)			
	3-month	-0.107^{***} (0.024)	0.457^{***} (0.094)			
M. II	1-month	-0.080 (0.050)	0.813^{***} (0.148)	-0.250^{***} (0.006)	0.004 (0.011)	constant, $r^F_{t-2}, .., r^F_{t-4}, L_{t-1}$
	3-month	-0.023 (0.085)	0.972^{***} (0.105)	-0.114^{***} (0.015)	0.005 (0.006)	constant, $r^F_{t-2}, .., r^F_{t-12}, L_{t-2}$

Note: The numbers in parenthesis are Newey and West's (1987) autocorrelation-heteroscedasticity consistent standard errors corrected with four lags for the 1-month ED rate and twelve lags for the 3-month ED rate. *** denotes statistical significance at the 1% level in a two-tailed test respectively. M.I and M.II denote the Model I in equation (3.2) and the Model II in equation (3.4) respectively.

Figure 1. Weekly Federal funds rate and 1-month and 3-month Commercial Paper rates

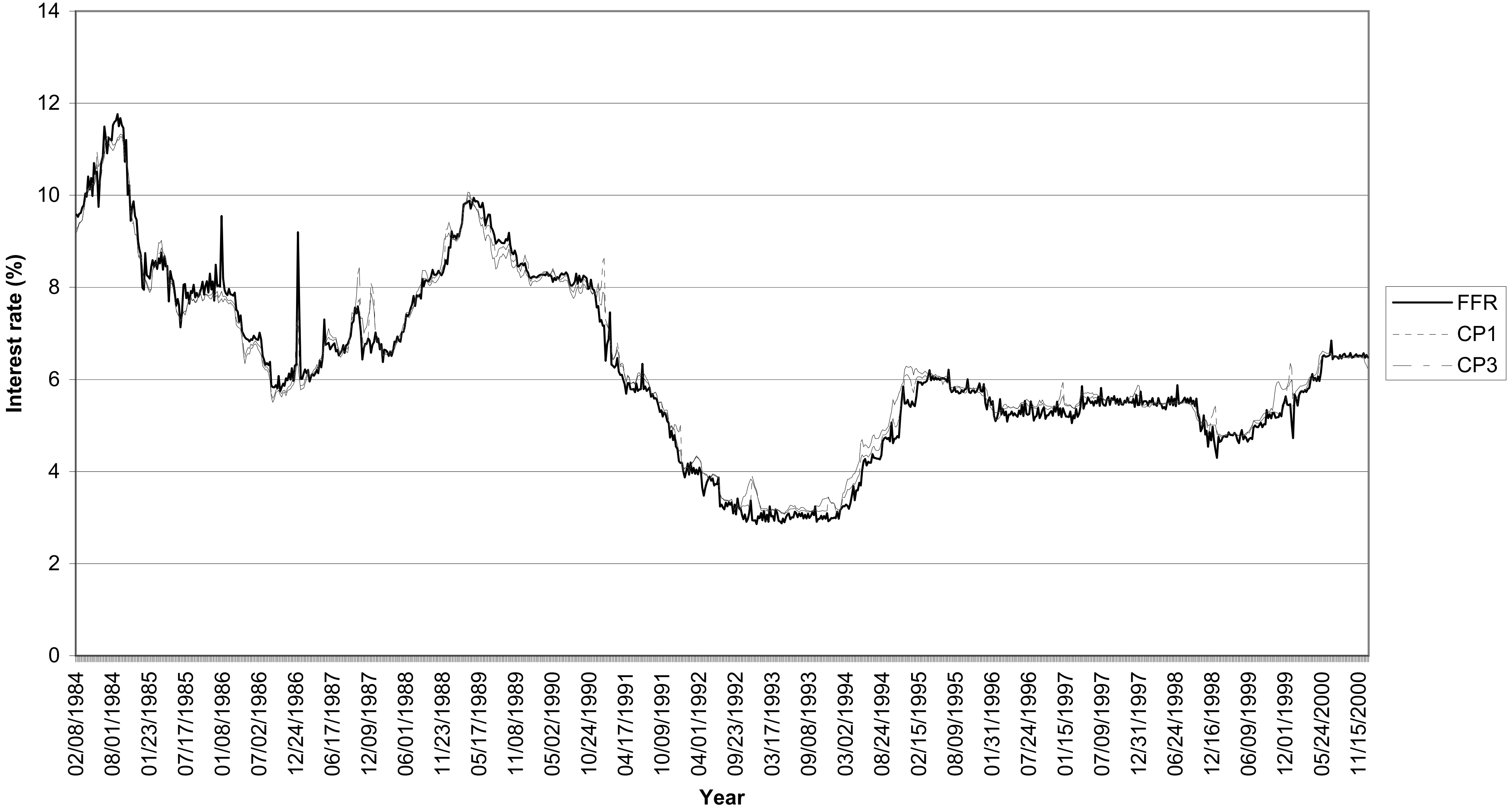

Figure 2. Weekly Federal Funds rate and 1-month and 3-month Euro-dollar rates