Aid Volatility, Human Capital, and Growth

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Abstract

This paper studies the effect of aid volatility on growth, in a model where the decision to invest in skills is endogenous. The analysis focuses on a low-income economy where the cost of acquiring education benefits from public subsidies, which are partly financed through foreign aid. Thus, aid plays a critical role in determining the distribution of skills across workers. By creating uncertainty about the net return to education, a high degree of aid volatility mitigates agents' incentives to invest in skills. If savings and growth depend on the composition of the labor force, and if more able workers are more productive, aid volatility may have an adverse effect on the mean growth rates of investment and output. Aid volatility may therefore contribute to the persistence of a stagnation equilibrium.

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1 Introduction

The adverse effect of aid volatility on economic growth has been documented in a number of empirical studies, including Chauvet and Guillaumont (2009), Chervin and van Wijnbergen (2009), Neanidis and Varvarigos (2009), Markandya et al. (2010), Aldashev and Verardi (2012), Kathavate and Mallik (2012), Kodama (2012), and Museru et al. (2014).[1] This effect—controlling for the level of aid and the endogeneity of aid flows—appears to be particularly significant for project aid, which is designed to promote directly or indirectly investment in physical and human capital.

Conceptually, the negative impact of aid volatility on growth (and possibly welfare) can result from a variety of channels. As discussed in Agénor and Aizenman (2010), a key channel through which the lack of predictability in project aid disbursements may adversely affect growth relates to the fact that it makes it difficult for recipient governments to formulate medium-term spending plans to spur growth. If aid finances a large fraction of infrastructure investment, as is often the case in low-income countries, and if creating public capital requires time (as a result of a "time to build" assumption, for instance), an aid shortfall could bring the process to a halt if no alternative sources of financing are available. This is indeed consistent with the evidence which suggests that aid shortfalls are often accompanied by cuts in public investment (Celasun and Walliser (2008)) and that volatility in government

[1] Various causes of aid volatility have been identified in the literature (see Desai and Kharas (2010) and Hudson (2015)). Aid (especially emergency aid) can be volatile for good reasons, for instance when responding countercyclically to exogenous shocks, such as terms of trade or natural disasters. This is especially the case for low-income countries, which tend to be disproportionately prone to this type of shocks. Volatility may also reflect a recipient country's political status as well as its governance and macroeconomic performance, which are to some extent endogenous to the recipient country's actions. Finally, volatility can also be a manifestation of budget cycles in donor economies.

spending has an adverse effect on economic growth (Kose et al. (2005, Table 6)). In addition, if in response to high aid volatility countries opt to reduce the desired level of investment (and thus their funding requirements), donors may misinterpret this policy adjustment as a signal of absorption problems. As a result, they may effectively reduce aid commitments—making the initial concerns about lower assistance self-fulfilling and possibly contributing to the perpetuation of a stagnation equilibrium. Putting in place a contingency fund or accumulating foreign reserves may lead to a similar outcome if (as a result of a moral hazard effect or an adverse signaling problem regarding absorption capacity) future aid is dependent on the size of the fund or the stock of reserves. In such conditions, precautionary public savings may not be able to mitigate the adverse effects of fluctuations in foreign aid on government spending and eventually on economic growth.

This paper explores an alternative, and possibly complementary, channel through which aid volatility may adversely affect growth (and possibly welfare), in a model where the decision to invest in skills is endogenous. Specifically, the analysis focuses on a low-income economy where the cost of acquiring education benefits from public subsidies, which are partly financed through domestic taxes and partly through aid. This is consistent with the evidence which suggest that, in addition to funding investment, foreign aid is often used to finance recurrent expenditures like education and health spending. The low level of income and limited capacity to enforce compliance with the law imply that policymakers have limited ability to adjust tax rates to finance their expenditures. Individuals cannot borrow to invest in education because human capital provides inadequate collateral and (consistent with the evidence for many low-income countries) credit markets function poorly. Thus, public subsidies play a critical role in determining how many individuals choose to acquire advanced education and the skill composition of the labor force.

The key results of the paper can be summarized as follows. While a positive aid shock lowers the threshold level of ability above which individuals choose to acquire skills, thereby increasing the effective supply of skilled labor, an increase in aid volatility has the opposite effect: it mitigates individuals' incentives to acquire skills. The reason is that higher aid volatility translates into higher volatility in the subsidy rate, and thus greater uncertainty about the average relative return from investing in education. In addition, an increase in aid volatility reduces the economy's average growth rate. Intuitively, aid shocks affect mean output growth through two channels—an education incentive-human capital channel and a phyical capital channel. Both of these effects operate in the same direction. Because skilled labor is more productive than unskilled labor, greater volatility in wages and the composition of the labor force translates into lower mean output as well as lower mean savings, and thus lower investment. Through both channels a mean-preserving spread of the aid shock causes a decrease in the average growth rate of output. While the effect of aid volatility on the distribution of the work force can be derived analytically, the investment and growth effects must be established numerically; nevertheless, the results are shown to be robust for a range of plausible parameter values, especially with respect to the (effective) wage premium and the sensitivity of training costs to ability.

The remainder of the paper is organized as follows. Section 2 presents the model, which is a two-period overlapping generations (OLG) model of endogenous growth with learning by doing and (as noted earlier) endogenous skills acquisition. To keep the analysis as simple as possible, it is first assumed that only skilled workers are taxed. The balanced growth equilibrium is defined in Section 3 and the stochastic equilibrium is presented in Section 4. Section 5 studies the effects of higher aid volatility on labor allocation and economic growth, using a combination of analytical results and numerical simulations. Section 6 extends the basic model to consider the case where both categories of workers are subject to taxation, and the case where the domestic tax rate is endogenous. The final section draws together the policy implications of the analysis and offers some concluding remarks.

2 The Environment

The economy is populated by individuals with different innate abilities, who live for two periods: adulthood and old age. Population is constant at $\bar{N}$. Each individual is endowed with one unit of time in each period of life; in old age, all time is devoted to leisure. There are two categories of labor in the economy, skilled and unskilled. Individuals are born unskilled and must decide at the beginning of adulthood whether to become skilled or remain unskilled for the rest of their adult life. Becoming skilled involves both a time cost and a pecuniary cost, with the latter benefiting from a public subsidy. The production of goods, which can be used either for consumption or investment, requires both types of labor as well as physical capital. The government finances its spending through taxation and foreign aid.

2.1 Individuals

Individuals have identical preferences but are born with different abilities, indexed by $x \in (0, 1)$. Ability follows a uniform distribution with density function $f(x)$, a cumulative distribution function $F(x) = x$, and mean 0.5.[2]

[2]For simplicity, the *ex ante* distribution of innate ability is assumed not change over the course of generations.

Each individual maximizes utility and decides whether to enter the labor force as an unskilled worker or (after undergoing training) as a skilled worker. The individual's ability partly determines his relative cost of acquiring skills.

Specifically, an adult with ability x can enter the labor force at the beginning of period t as an unskilled worker and earn the wage w_t^U, which is independent of the worker's ability. Alternatively, the individual may choose to spend first a fraction of time $\varepsilon \in (0,1)$ of his time endowment at the beginning of adulthood in training (during which time no income is earned) and enter the labor force for the remainder of the period as a skilled worker, earning the (average) wage w_t^S. Training involves a direct pecuniary cost, which is partly financed by a government subsidy.

Let $c_{t+j}^{h,t}$ denote consumption at period $t+j$ of an individual working at wage w_t^h, where $h = U, S$, born at the beginning of period t, with $j = 0, 1$. The individual's discounted utility function is given by

$$U_t^h = \eta_C \ln c_t^{h,t} + \frac{\ln c_{t+1}^{h,t}}{1+\rho}, \quad h = U, S \tag{1}$$

where $\rho > 0$ is the discount rate and $\eta_C > 0$ a preference parameter.

For the moment, only skilled workers are subject to taxation. The period-specific budget constraints are given by

$$c_t^{U,t} + s_t^U = w_t^U, \tag{2}$$

$$c_t^{S,t} + s_t^S = (1-\tau)[(1-\varepsilon)w_t^S - e_t], \tag{3}$$

$$c_{t+1}^{h,t} = (1 + r_{t+1})s_t^h, \quad h = U, S \tag{4}$$

where s_t^h is savings of type-h worker, $1 + r_{t+1}$ the rate of return on holding assets between periods t and $t+1$, $\tau \in (0,1)$ the tax rate, and e_t the cost of training. Thus, income is taxed net of education expenditure.

The training cost is proportional to the wage that a skilled worker makes once training is completed and he becomes employed:[3]

$$e_t = (\frac{\omega - \mu_t}{x^\chi})(1-\varepsilon)w_t^S, \tag{5}$$

where $\omega, \chi \in (0,1)$ and $0 \leq \mu_t < \omega$ is the share of the training cost covered by government subsidies. The training cost is assumed to depend inversely on ability because more able individuals need less time to learn or, equivalently, a higher innate ability facilitates the acquisition of skills.[4]

As shown in the Appendix, solving the individual optimization problem for $h = U, S$ gives the optimal levels of consumption in periods t and $t+1$. Substituting these results in (1) gives the indirect utility functions V^h, $h = U, S$. Let x_t^C denote the threshold level of ability which is such that all individuals with ability $x > x_t^C$ choose to become skilled. This critical value is thus obtained by setting $V^S = V^U$, which gives

$$x_t^C = \frac{(\omega - \mu_t)^{1/\chi}}{0.5}[1 - \frac{w_t^U}{(1-\tau)(1-\varepsilon)w_t^S}]^{-1/\chi} - 1. \tag{6}$$

This equation describes an increasing and convex relationship between the wage ratio w_t^U/w_t^S and x_t^C.

The productivity of unskilled workers is equal to unity, independently of each worker's ability. The relative supply of unskilled labor, θ_t^U, is thus equal to

$$\theta_t^U = F(x_t^C) = \int_0^{x_t^C} f(x)dx = x_t^C. \tag{7}$$

The proportion of raw supply of skilled labor is $1 - x_t^C$, and the average productivity of workers with ability $x \in (x_t^C, 1)$, who have therefore completed training, equals $(x_t^C + 1)/2$. Thus, the relative (effective) supply of

[3]This assumption, which is similar to Blankenau and Camara (2009) for instance, reflects the view that the education sector employs skilled labor—namely, teachers.

[4]Note also that, for simplicity, the training cost is assumed to be paid *after* training is completed and the individual has successfully entered the skilled labor force.

skilled labor, θ_t^S, is given by[5]

$$\theta_t^S = \frac{(1 + x_t^C)}{2}(1 - x_t^C) = \frac{1 - (x_t^C)^2}{2}. \tag{8}$$

2.2 Production

Production of goods, Y_t, requires the use of a composite labor input in efficiency units, L_t, and private capital, K_t:

$$Y_t = K_t^{\alpha}(Z_t L_t)^{1-\alpha}, \tag{9}$$

where $\alpha \in (0, 1)$ and Z_t is a labor-enhancing productivity variable that captures learning-by-doing effects. In turn, the composite labor input is defined as

$$L_t = \gamma L_t^S + L_t^U, \tag{10}$$

where γ is a productivity parameter and L_t^S and L_t^U are given by

$$L_t^S = (1 - \varepsilon)\theta_t^S \bar{N}, \qquad L_t^U = \theta_t^U \bar{N}. \tag{11}$$

To ensure that skilled labor (adjusted for time worked) is more productive than unskilled labor, the restriction $\gamma(1 - \varepsilon) > 1$ (which therefore implies that $\gamma > 1$) is imposed.

In standard Arrow-Romer fashion, the learning-by-doing effect relates overall labor productivity to the capital-worker ratio:

$$Z_t = \frac{K_t}{\bar{N}}. \tag{12}$$

Profit maximization with respect to L_t^S and L_t^U leads to, using (12),

$$w_t^U = (1 - \alpha)[\gamma(1 - \varepsilon)\theta_t^S + \theta_t^U]^{-\alpha} Z_t, \tag{13}$$

[5] Note that, because the supply of skilled labor is measured in efficient units of human capital, the equality $\theta_t^S + \theta_t^U = 1$ does not hold. This is because the number of skilled workers is adjusted for average ability, as measured by $(x_t^C + 1)/2$. Thus, $\theta_t^S + \theta_t^U$ is less than unity as long as $x_t^C < 1$..

$$w_t^S = (1-\alpha)[\gamma(1-\varepsilon)\theta_t^S + \theta_t^U]^{-\alpha} Z_t \gamma, \tag{14}$$

which implies that the skilled wage is a fixed markup over the unskilled wage:

$$w_t^S = \gamma w_t^U. \tag{15}$$

Combining (9) to (12), output is given by

$$Y_t = [\gamma(1-\varepsilon)\theta_t^S + \theta_t^U]^{1-\alpha} K_t, \tag{16}$$

which shows that, in standard AK fashion, output is linear in private capital.

2.3 Government

The government levies (as noted earlier) a tax on skilled wages at the constant rate $\tau \in (0,1)$ and receives foreign aid, which is subject to a random component. Public sector resources are used to provide subsidies to education, for a total G_t^E, and to spend on other (unproductive) items, in amount G_t^U.[6] It cannot issue debt claims and must therefore run a balanced budget:

$$G_t^E + G_t^U = (\tau + a_t)(1-\varepsilon) w_t^S N_t^S. \tag{17}$$

Thus, foreign aid takes the form of a transfer a_t, which for simplicity is assumed to be proportional to the tax base.[7]

The total cost of subsidies to education is given by $\mu_t(1-\varepsilon)w_t^S\theta_t^S\bar{N}$. Let $\upsilon \in (0,1)$ denote the constant share of public resources spent on subsidies; using (11) and (17), it must therefore be that $\mu_t(1-\varepsilon)w_t^S\theta_t^S = \upsilon(\tau + a_t)(1-\varepsilon)w_t^S\theta_t^S$, or equivalently

$$\mu_t = \upsilon(\tau + a_t). \tag{18}$$

[6] In contrast to some of the literature on fiscal response models, the tax rate does not depend directly on the level of aid. This is consistent with the evidence reviewed in Morrissey (2015), which suggests that aid flows do not appear to be systematically associated with a reduction in taxation effort.

[7] Assuming instead that foreign aid is fixed in proportion of output per worker would not qualitatively affect the analysis.

Thus an increase in the aid share raises the subsidy rate to education.

To account for aid volatility, the aid share a_t is taken to be given by

$$a_t = a + \delta_t, \tag{19}$$

where $a > 0$ is the unconditional mean of the aid share and δ_t is a shock with zero mean and a density function $g(\delta_t)$, defined over the interval $(-\delta, \delta)$ with $\delta > 0$ and $\delta < a$ (to ensure that aid remains positive if $\delta_t = -\delta$).[8] For simplicity, it will be assumed that δ_t follows also a uniform distribution so that the variance of a_t is $\sigma_a^2 = \delta^2/3$. Thus, an increase in δ captures a mean-preserving spread in aid volatility.

Because aid is volatile, the subsidy rate μ_t (as implied by (18)) is also a random variable. By implication, given that public policies must be adjusted to satisfy the budget constraint after an aid shock, volatility of public sector resources feeds into each individual's decision to acquire skills.

2.4 Savings-Investment Equilibrium

The equilibrium condition between aggregate savings and investment requires the capital stock in $t+1$ to be equal to savings in period t by individuals born in $t-1$:

$$K_{t+1} = s_t^U \theta_t^U \bar{N} + s_t^S \theta_t^S \bar{N}. \tag{20}$$

3 Stochastic Equilibrium

In this economy, a *stochastic dynamic competitive equilibrium* is a sequence of quantities $\{c_t^{h,t}, c_{t+1}^{h,t}, s_t^h, K_{t+1}, r_{t+1}, x_t^C, \theta_t^U, \theta_t^S, Y_t, \mu_t\}_{t=0}^{\infty}$, for $h = U, S$, such that for $K_0 > 0$ given, *a*) $\{c_t^{h,t}, c_{t+1}^{h,t}, s_t^h\}_{t=0}^{\infty}$ solve household h's optimization

[8]Note that a is also subject to an upper bound to ensure that $\mu_t < \omega$, as noted earlier.

problem, given $\{\mu_t\}_{t=0}^{\infty}$; *b*) $\{K_{t+1}, \theta_t^U, \theta_t^S, Y_t\}_{t=0}^{\infty}$ solves the representative producer's profit maximization problem, given $\{x_t^C, \mu_t\}_{t=0}^{\infty}$; *c*) r_{t+1}, x_t^C, θ_t^U and θ_t^S are stationary; *d*) the government budget is balanced; and *e*) the goods and labor markets clear in each period.

Given this definition, the solution of the model proceeds as follows. As shown earlier (see equation (6)), the threshold level of ability x_t^C above which individuals choose to acquire skills depends on the wage ratio, which, from (15), is constant. Thus, in equilibrium the stochastic threshold level of ability (or equivalently the share of unskilled workers) is given by

$$x_t^C = \frac{(\omega - \mu_t)^{1/\chi}}{0.5}[1 - \frac{1}{(1-\tau)(1-\varepsilon)\gamma}]^{-1/\chi} - 1. \tag{21}$$

Substituting (18) in this equation yields

$$x_t^C = g(a_t) = \Lambda[\omega - \upsilon(\tau + a_t)]^{1/\chi} - 1, \tag{22}$$

where

$$\Lambda \equiv \frac{1}{0.5}[1 - \frac{1}{(1-\tau)(1-\varepsilon)\gamma}]^{-1/\chi} > 0.$$

It is easy to verify that $g' < 0$ and $g'' > 0$. Thus, a transitory positive (negative) aid shock δ_t lowers (raises) the threshold level of ability above which individuals choose to acquire skills, thereby increasing (reducing) the effective supply of skilled labor. The reason of course is that a positive aid shock raises the subsidy rate, thereby lowering the cost of education.

The Appendix shows that, in equilibrium, the stochastic growth rate of capital (the investment rate) is given by, using (18),

$$\frac{K_{t+1}}{K_t} = \frac{\sigma(1-\alpha)}{\{\gamma(1-\varepsilon)0.5[1-(x_t^C)^2] + x_t^C\}^{\alpha}} \tag{23}$$

$$\times \left\{ x_t^C + [1 - \frac{\omega - \upsilon(\tau + a_t)}{[0.5(1 + x_t^C)]^{\chi}}]\gamma(1-\tau)(1-\varepsilon)0.5[1-(x_t^C)^2] \right\},$$

where $\sigma = 1/\left[1 + (1+\rho)\eta_C\right] < 1$ is the savings rate. The stochastic growth rate of output is given by

$$\frac{Y_{t+1}}{Y_t} = \left\{ \frac{\gamma(1-\varepsilon)0.5[1-(x_{t+1}^C)^2] + x_{t+1}^C}{\gamma(1-\varepsilon)0.5[1-(x_t^C)^2] + x_t^C} \right\}^{1-\alpha} \frac{K_{t+1}}{K_t}. \tag{24}$$

Inserting the solutions obtained from (21) and (22) in (23) and (24) yields the solutions for the investment rate and the growth rate of output. Thus, investment and output growth are both nonstationary processes. In addition, the growth rate of output depends on aid shocks occurring in both t and $t+1$.

4 Aid Volatility and Growth

Substituting $x_t^C = g(a_t)$ in (23) and (24), the growth rates of capital and output can be written as

$$\frac{K_{t+1}}{K_t} = f^K(a_t) = \frac{\sigma(1-\alpha)}{\{\gamma(1-\varepsilon)0.5[1-g(a_t)^2] + g(a_t)\}^{\alpha}} \tag{25}$$

$$\times \left\{ g(a_t) + [1 - \frac{\omega - \upsilon(\tau + a_t)}{[0.5(1+g(a_t))]^{\chi}}]\gamma(1-\tau)(1-\varepsilon)0.5[1-g(a_t)^2] \right\},$$

and

$$\frac{Y_{t+1}}{Y_t} = f^Y(a_t, a_{t+1}) = \left\{ \frac{\gamma(1-\varepsilon)0.5[1-g(a_{t+1})^2] + g(a_{t+1})}{\gamma(1-\varepsilon)0.5[1-g(a_t)^2] + g(a_t)} \right\}^{1-\alpha} f^K(a_t) \tag{26}$$

From (22), the mean value of the ability threshold can be approximated by[9]

$$\mathbb{E}x_t^C \simeq \Lambda[\omega - \upsilon(\tau + a)]^{1/\chi} \left\{ 1 + \frac{0.5}{[\omega - \upsilon(\tau + a)]^2} \frac{\upsilon^2}{\chi} (\frac{1}{\chi} - 1) \right\} \sigma_a^2. \tag{27}$$

[9] Taking a second-order approximation of $g()$ around a yields $g() \simeq g(a) + g'(a_t - a) + 0.5g''(a_t - a)^2$. The expected value of this expresion is thus $\mathbb{E}[g()] \simeq g(a) + 0.5g''\sigma_a^2$.

This expression implies that an increase in the volatility of foreign aid raises the mean threshold level of ability above which individuals choose to acquire skills. Intuitively, more volatility in aid translates into more volatility in the subsidy rate, and thus greater uncertainty about the relative return from investing in education, that is, the wage ratio. This uncertainty mitigates the motivation for individuals to invest in training and results in a lower mean ability threshold.

From (25) and (26), the mean growth rates of capital and output can be approximated by[10]

$$\mathbb{E}(\frac{K_{t+1}}{K_t}) \simeq f^K(a) + 0.5\frac{d^2 f^K}{da_t^2}\sigma_a^2, \tag{28}$$

$$\mathbb{E}(\frac{Y_{t+1}}{Y_t}) \simeq f^Y(a,a) + 0.5(\frac{\partial^2 f^Y}{\partial a_t^2} + \frac{\partial^2 f^Y}{\partial a_{t+1}^2})\sigma_a^2, \tag{29}$$

which show that the average growth rates of output and investment both depend on aid volatility. An increase in δ will decrease the mean growth rate of both variables if the coefficients of σ_a^2 in both expressions are negative, that is, if the second-order derivative of $f^K()$, and the sum of the direct second-order derivatives of $f^Y()$, are both negative.[11] Intuitively, when this condition holds, the gain associated with a more qualified pool of workers as a result of a favorable aid shock (and thus a higher subsidy rate to education) less than compensates the loss resulting from an unfavorable shock, so that,

[10]For the first equation, the approximation proceeds as in the case of $g(a_t)$. For the second, a second-order approximation of $f^Y()$ around a in (26) yields $f^Y() \simeq f^Y(a,a) + f_1^Y(a_t - a) + f_2^Y(a_{t+1} - a) + f_{12}^Y(a_t - a)(a_{t+1} - a) + 0.5f_{11}^Y(a_t - a)^2 + 0.5f_{22}^Y(a_{t+1} - a)^2$. The expected value of this expresion is thus $\mathbb{E}[f^Y()] \simeq f^Y(a,a) + 0.5(f_{11}^Y + f_{22}^Y)\sigma_a^2$.

[11]As shown by Rothschild and Stiglitz (1970), the expected value of a concave (convex) function of a random variable is decreased (increased) by a mean-preserving spread of that variable. This is the case here for $f^K()$. The result for $f^Y()$ follows from a simple generalization to the case where there are two statistically independent random variables with the same means and the same variances.

on average, the growth rates of investment and output are both decreased by a mean-preserving spread in the distribution of the shock. In effect, the economy would be better off if it were to obtain the amount of aid a with certainty, rather than getting the same amount on average, but with a non-zero variance.

However, the second-order derivatives appearing in (28) and (29) are too complex to be derived analytically.[12] The restrictions needed for concavity to hold must be determined numerically. To do so the following values are used as a benchmark case. The savings rate, σ, is set at 0.12, which is the value reported by Agénor and Dinh (2015) for the private savings rate in low-income countries. The fraction of time spent in acquiring skills, ε, is initially set at 0.15. Thus, assuming that the period of adulthood in the model corresponds to 25 years, implies that the average number of years needed to undergo advanced training is about 4 years. The cost of education, ω, measured in proportion of the after-tax skilled wage is set at 0.1, to account for the fact that such costs are high in a low-income economy. The elasticity parameter χ, which reflects the impact of ability on the efficiency of training, is set at 0.7. On the production side, the elasticity of output with respect to capital, α, is set at the conventional value of 0.3, whereas he markup of the skilled wage over the unskilled wage, γ, is set at 1.3. Thus, the effective wage premium, given by $\gamma(1-\varepsilon)$, is about 11 percent. On the government side, the share of spending on higher education in total government resources (including aid), υ, is set at 0.05, consistent with the data discussed in Agénor and Alpaslan (2013) for low-income countries. The tax rate, τ, is also calculated as in that paper, by dividing the effective tax

[12] From (21) and (26), it is clear that $f^Y()$ is concave in a_{t+1}, but the expressions for d^2f^K/da_t^2 and d^2f^Y/da_t^2 cannot be signed unambiguously.

rate on wages estimated by Baldacci et al. (2004, Table 1) by (to match the model's definition) the average share of labor income for developing countries estimated by Guerriero (2012); this gives $\tau = 0.22$.

Based on these values, Mathematica is used to derive symbolically, and evaluate numerically, the second-order derivatives appearing in (28) and (29). The results show that they are all negative, which imply indeed that an increase in aid volatility has a negative effect on both the investment and output growth rates.[13] Intuitively, aid shocks affect average output growth through two channels: an education incentive-human capital channel and a physical capital channel. In the present setting, both of these effects operate in the same direction. Because aid volatility translates into volatility of the subsidy rate to education, it leads to greater volatility in wages and lowers the mean value of the relative supply of skilled workers. In turn, this translates into a lower mean growth rate of output, both directly and indirectly, because higher volatility in wages leads to lower mean savings and thus lower mean investment. This, in turn, magnifies the direct effect on aid volatility on the mean growth rate of output.

The adverse effect on savings, obtained here in a model in which agents are risk neutral, is in contrast with those obtained from models in which agents are risk averse. From the results in De Hek (1999) for instance, it can be inferred that, by contributing to overall macroeconomic volatility, aid volatility would lead to more precautionary saving if agents are risk averse; this, in turn, would exert a positive effect on the mean growth rate of output. This result is also at variance with models such as those of Arellano et al. (2009), Neanidis and Varvarigos (2009), and Carter (2015), where aid

[13] Note that negative values for both $\partial^2 f^Y / \partial a_t^2$ and $\partial^2 f^Y / \partial a_{t+1}^2$ is a sufficient, bot not necessary, condition for their sum to be negative.

volatility may have a positive effect on investment and growth through its impact on household incentives to smooth consumption.

To assess the robustness of these results a sensitivity analysis is performed, with a focus on two parameters for which the empirical evidence is not as strong, the productivity parameter, γ, and the sensitivity of training costs to ability, χ.[14] These parameters are varied over the ranges $(1.3, 1.45)$ and $(0.7, 0.85)$, respectively. These values, together with those of the second-order derivative $d^2 f^K/da_t^2$ and the sum of the second-order partial derivatives $(\partial^2 f^Y/\partial a_t^2)+(\partial^2 f^Y/\partial a_{t+1}^2)$, are reported in Figures 1 and 2. The figures show that both expressions are indeed negative over those ranges; in fact, $\partial^2 f^Y/\partial a_t^2$ and $\partial^2 f^Y/\partial a_{t+1}^2$ are both negative. Moreover, the shape of the curves in the figures show clearly that higher or lower values of γ and χ would not affect the sign of the derivative expressions displayed on the vertical axis. Thus, these experiments show that the adverse effect of aid volatility on the mean growth rates of investment and output are robust for a plausible range parameter values.

5 Extensions

In this section two extensions of the analysis are considered: first, the case where the subsidy rate remaining endogenous but both categories of workers are subject to taxation, and the case where the subsidy rate is fixed. the domestic tax rate is endogenous.

[14] Sensitivity analysis within plausible ranges with respect to the other parameters (for which there is more consensus in the literature) did not change the results.

5.1 Taxation of Unskilled Workers

In the foregoing discussion it was assumed that only skilled workers are taxed. Suppose that now wages of both types of workers are taxed at the same rate τ, so that the government budget constraint (17) is replaced by $G_t^E + G_t^U = (\tau + a_t)\left[w_t^U N_t^U + (1-\varepsilon) w_t^S N_t^S\right]$. It can be easily established that in equilibrium the ability threshold level and the subsidy rate, equations (21) and (22), are now replaced by

$$x_t^C = \Lambda(\omega - \mu_t)^{1/\chi} - 1, \tag{30}$$

$$\mu_t = \upsilon(\tau + a_t)\left\{1 + [\frac{2x_t^C}{\gamma(1-\varepsilon)[1-(x_t^C)^2]}]\right\}, \tag{31}$$

As before, x_t^C implies a negative relationship between x_t^C and μ_t. However, the subsidy rate is no longer independent of the ability threshold; equation (31) implies now that that μ_t is positively related to both a_t and x_t^C; an increase in the relative supply of skilled workers (a fall in x_t^C) reduces that rate. Indeed, equations (30) and (31) represent now a system in x_t^C and μ_t. Substituting (31) in (31) yields

$$G(x_t^C, a_t) = x_t^C - \Lambda\left\{\omega - \upsilon(\tau + a_t)\left\{1 + [\frac{2x_t^C}{\gamma(1-\varepsilon)[1-(x_t^C)^2]}]\right\}\right\}^{1/\chi} + 1 = 0. \tag{32}$$

Assuming that $G()$ is differentiable and applying the implicit function theorem, it can be shown that, as before, a positive aid shock raises the subsidy rate, thereby lowering the cost of education and raising incentives to acquire skills. In addition, the reduction in x_t^C and the concomitant increase in the number of skilled individuals lowers now the unskilled-effective skilled labor ratio, $x_t^C/[1-(x_t^C)^2]$, which tends (all else equal) to reduce the subsidy rate. However, this effect is not large enough to offset the direct effect.

Equation (23) is now replaced by

$$\frac{K_{t+1}}{K_t} = \frac{\sigma(1-\tau)(1-\alpha)}{\{\gamma(1-\varepsilon)0.5[1-(x_t^C)^2] + x_t^C\}^\alpha}$$

$$\times \left\{ x_t^C + [1 - \frac{\omega - \mu_t}{[0.5(1+x_t^C)]^\chi}]\gamma(1-\varepsilon)0.5[1-(x_t^C)^2] \right\},$$

while equation (24) remains the same.

Assuming that the implicit function $G()$ in equation (32) has continuous partial derivatives, then there is a unique, continuously differentiable function $x_t^C = g(a_t)$ with $g' < 0$ as before. However, determining the impact of aid volatility on the threshold ability level (which involves calculating the sign of g'') must also be done numerically. Using the same benchmark parameter values described earlier, these calculations show again that an increase in aid volatility has an adverse effect on the mean values of the distribution of the labor force, the investment rate, and the growth rate of output. Moreover, these results are robust to the same range of parameter values for γ and χ.

5.2 Endogenous Taxation

In the foregoing analysis it was assumed that the domestic tax rate is constant and that the subsidy rate adjusted endogenously to equilibrate total subsidies and the share of public resources allocated to education. Suppose now instead that the subsidy rate is constant and that the tax rate is the equilibrating variable; thus, $\mu(1-\varepsilon)w_t^S\theta_t^S = \upsilon(\tau_t + a_t)(1-\varepsilon)w_t^S\theta_t^S$, or equivalently, instead of (18),

$$\tau_t = \mu\upsilon^{-1} - a_t.$$

with again $\mu < \omega$ and, to ensure that the tax rate remains positive, $\mu > \upsilon(a - \delta)$. Equation (22) is replaced by

$$x_t^C = g(a_t) = \Lambda[1 - \frac{1}{(1-\mu\upsilon^{-1} + a_t)(1-\varepsilon)\gamma}]^{-1/\chi} - 1,$$

where now $\Lambda = 2(\omega - \mu)^{1/\chi} > 0$. It can readily be verified that once again $g' < 0$ and $g'' > 0$. A positive aid shock leads to a lower tax rate, which means a higher after-tax skilled wage; this, in turn, promotes investment in skills.

Equation (25) is now also replaced by

$$\frac{K_{t+1}}{K_t} = f^K(a_t) = \frac{\sigma(1-\alpha)}{\{\gamma(1-\varepsilon)0.5[1-g(a_t)^2] + g(a_t)\}^\alpha}$$

$$\times \left\{ g(a_t) + [1 - \frac{\omega - \mu}{[0.5(1+g(a_t))]^\chi}]\gamma[(1 - \mu\upsilon^{-1} + a_t)(1-\varepsilon)0.5[1-g(a_t)^2] \right\},$$

whereas equation (26), the growth rate of output, remains the same.

As before, and because $g'' > 0$, the mean ability threshold increases with a mean-preserving spread of the distribution of aid shocks. Increased aid volatility once raises volatility in the relative (net) return to education—this time because of increased volatility in the after-tax skilled wage. The result is a lower mean value of individuals acquiring skills.

With endogenous taxation the effects of higher aid volatility on the mean growth rates of capital and output must again be studied numerically. Setting $\mu = 0.02$, and using the same methodology as described earlier, the results remain qualitatively the same: an increase in aid volatility has a negative effect in both cases.

Of course, when the tax rate is endogenous, the human capital channel would not operate if both skilled and unskilled workers are taxed at the same rate; in that case, the wage ratio is independent of the tax rate. However, as long as taxation is progressive (with unskilled workers taxed at a lower rate than skilled workers) and the equilibrium between resources allocated to, and spending on, education is maintained through adjustment in the tax rate of skilled workers, the analysis would continue to hold.

6 Concluding Remarks

The purpose of this paper was to examine the longer-run implications of aid volatility in a growth model where the decision to acquire skills is endogenous and foreign aid, which is subject to random shocks, serves to finance training subsidies. Using a combination of analytical derivations and numerical experiments, the analysis showed that, by creating uncertainty about the net return to education, a high degree of aid volatility mitigates agents' incentives to invest in skills. If savings and growth depend on the composition of the labor force, and if more skilled workers are more productive, aid volatility may therefore have an adverse effect on the mean growth rates of investment and output. Because consumption is linear in wages in the model, aid volatility is also bad for the welfare of skilled households (directly) and unskilled households (indirectly, through output volatility). Thus, aid volatility has an adverse effect on social welfare as well, in addition to possibly contributing to the persistence of a stagnation equilibrium.

The education incentive-human capital and physical capital channels highlighted in this paper offer an alternative view regarding the potential impact of aid volatility on growth in low-income countries, in addition to the infrastructure channel emphasized by Agénor and Aizenman (2010) for instance. A possible extension, as in that paper, would be to consider whether a contingency fund financed partly through domestic taxation and partly through aid proceeds, can mitigate the adverse effect of aid volatility on long-term growth. In a growth context, the trade-off between the moral hazard effect of a contingency fund (the fact that the level of aid may be dependent on the size of the fund) and the benefits of a more stable flow of resources in terms of human capital accumulation, investment and growth may take a different form than the static one considered in Agénor and Aizenman (2010). However, such analysis should also take into account the cost of raising taxes in an environment where administrative capacity is weak. The policy problem could therefore be viewed as a two-stage process—first, determine the optimal share of total government resources that should be allocated to the fund, and second determine how much should come from domestic taxes and how much from foreign aid.

Another extension would be to consider the impact of aid volatility on income distribution, in a setting with ability across individuals is heterogeneous. In the present model where (effective) skilled labor and unskilled labor are perfect substitutes, the wage ratio was shown to be fixed in equilibrium. A more general treatment would help to address the question of whether aid volatility, by worsening income distribution, can make it more difficult to pledge collateral to finance education—with potentially negative effects on the mean growth rates of physical investment and output.

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Appendix

From (2) and (4), the household's consolidated budget constraint is, for unskilled individuals,

$$c_t^{U,t} + \frac{c_{t+1}^{U,t}}{1+r_{t+1}} = w_t^U, \tag{A1}$$

whereas from (3) and (4), for individuals who become skilled,

$$c_t^{S,t} + \frac{c_{t+1}^{S,t}}{1+r_{t+1}} = (1-\tau)[(1-\varepsilon)w_t^S - e_t],$$

or equivalently, using (5) to substitute out for e_t and replacing x by the average ability of skilled workers,

$$c_t^{S,t} + \frac{c_{t+1}^{S,t}}{1+r_{t+1}} = (1-\tau)\left\{1 - \frac{\omega - \mu_t}{[0.5(1+x_t^C)]^\chi}\right\}(1-\varepsilon)w_t^S. \tag{A2}$$

Each individual maximizes (1) subject to his or her intertemporal budget constraint, either (A1) or (A2). The first-order conditions give the standard Euler equation

$$\frac{c_{t+1}^{h,t}}{c_t^{h,t}} = \frac{1+r_{t+1}}{(1+\rho)\eta_C}. \quad h = U, S \tag{A3}$$

Substituting this result in (A1) and (A2), together with (**??**), yields

$$c_t^{U,t} = [\frac{(1+\rho)\eta_C}{1+(1+\rho)\eta_C}]w_t^U,$$

$$c_t^{S,t} = [\frac{(1+\rho)\eta_C}{1+(1+\rho)\eta_C}](1-\tau)\left\{1 - \frac{\omega - \mu_t}{[0.5(1+x_t^C)]^\chi}\right\}(1-\varepsilon)w_t^S,$$

or equivalently

$$c_t^{U,t} = (1-\sigma)w_t^U, \tag{A4}$$

$$c_t^{S,t} = (1-\sigma)(1-\tau)\left\{1 - \frac{\omega - \mu_t}{[0.5(1+x_t^C)]^\chi}\right\}(1-\varepsilon)w_t^S, \tag{A5}$$

where $\sigma = 1/[1+(1+\rho)\eta_C] < 1$. Thus, from (2) and (3),

$$s_t^U = \sigma w_t^U, \tag{A6}$$

$$s_t^S = \sigma \left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \tau)(1 - \varepsilon)w_t^S. \tag{A7}$$

Substituting (A6) and (A7) for savings in (4) for $h = U, S$ yields

$$c_{t+1}^{U,t} = (1 + r_{t+1})\sigma w_t^U, \tag{A8}$$

$$c_{t+1}^{S,t} = (1 + r_{t+1})\sigma \left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \tau)(1 - \varepsilon)w_t^S. \tag{A9}$$

Now, substituting (A4) and (A5) as well as (A8) and (A9) in (1) yields the indirect utility functions

$$V_t^U = \eta_C \ln[(1 - \sigma)w_t^U] + \frac{1}{1 + \rho}\ln[(1 + r_{t+1})\sigma w_t^U],$$

$$V_t^S = \eta_C \ln\left[(1 - \sigma)(1 - \tau)\left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \varepsilon)w_t^S\right]$$

$$+\frac{1}{1 + \rho}\ln\left[(1 + r_{t+1})\sigma(1 - \tau)\left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \varepsilon)w_t^S\right].$$

Setting $V_t^U = V_t^S$ and simplifying yields

$$\eta_C\left[\ln w_t^U - \ln\{\left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \tau)(1 - \varepsilon)w_t^S\}\right]$$

$$+\frac{1}{1 + \rho}\left[\ln w_t^U - \ln\{\left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \tau)(1 - \varepsilon)w_t^S\}\right] = 0,$$

or

$$(\eta_C + \frac{1}{1 + \rho})\ln\left\{\frac{w_t^U}{\left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1 - \tau)(1 - \varepsilon)w_t^S}\right\} = 0.$$

Given that $\eta_C + 1/(1 + \rho) > 0$, this expression implies that

$$\ln\left\{\frac{1}{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}}[\frac{w_t^U}{(1 - \tau)(1 - \varepsilon)w_t^S}]\right\} = \ln 1,$$

or

$$\frac{w_t^U}{(1 - \tau)(1 - \varepsilon)w_t^S} = 1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}.$$

This equation can be rearranged to give

$$\frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi} = 1 - \frac{w_t^U}{(1-\tau)(1-\varepsilon)w_t^S},$$

or

$$(1 + x_t^C)^\chi = \frac{\omega - \mu_t}{0.5^\chi}[1 - \frac{w_t^U}{(1-\tau)(1-\varepsilon)w_t^S}]^{-1},$$

that is

$$x_t^C = \frac{(\omega - \mu_t)^{1/\chi}}{0.5}[1 - \frac{w_t^U}{(1-\tau)(1-\varepsilon)w_t^S}]^{-1/\chi} - 1, \tag{A10}$$

which is shown as equation (6) in the text. Substituting (15) in (A10) yields

$$x_t^C = \frac{(\omega - \mu_t)^{1/\chi}}{0.5}[1 - \frac{1}{\gamma(1-\tau)(1-\varepsilon)}]^{-1/\chi} - 1, \tag{A11}$$

which is equation (21) in the text.

With taxation of both skilled and unskilled workers, substituting (15) in (18) yields

$$\mu_t = \upsilon(\tau + a_t)\left\{1 + [\frac{\theta_t^U}{\gamma(1-\varepsilon)\theta_t^S}]\right\},$$

that is, using (7) and (8):

$$\mu_t = \upsilon(\tau + a_t)\left\{1 + [\frac{2x_t^C}{\gamma(1-\varepsilon)[1 - (x_t^C)^2]}]\right\}, \tag{A12}$$

which is shown as equation (-) in the text.

To study the dynamics, substitute first (A6) and (A7) in (20); this gives

$$K_{t+1} = \sigma w_t^U \theta_t^U \bar{N}$$

$$+\sigma(1-\tau)\left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}(1-\varepsilon)w_t^S\theta_t^S\bar{N},$$

which can be rewritten as, using (15),

$$K_{t+1} = \sigma\left[\theta_t^U + \left\{1 - \frac{\omega - \mu_t}{[0.5(1 + x_t^C)]^\chi}\right\}\gamma(1-\tau)(1-\varepsilon)\theta_t^S\right]w_t^U\bar{N}.$$

Substituting for Z_t in (13) using (12) and substituting for w_t^U in the expression above yields

$$\frac{K_{t+1}}{K_t} = \frac{\sigma(1-\alpha)}{[\gamma(1-\varepsilon)\theta_t^S + \theta_t^U]^\alpha} \left\{ \theta_t^U + [1 - \frac{\omega - \mu_t}{[0.5(1+x_t^C)]^\chi}]\gamma(1-\tau)(1-\varepsilon)\theta_t^S \right\}. \tag{A13}$$

Substituting (7) and (8) in this expression gives (23) in the text.

From (16),

$$Y_{t+1} = [\gamma(1-\varepsilon)\theta_{t+1}^S + \theta_{t+1}^U]^{1-\alpha} K_{t+1},$$

which implies that

$$\frac{Y_{t+1}}{Y_t} = \left\{ \frac{\gamma(1-\varepsilon)\theta_{t+1}^S + \theta_{t+1}^U}{\gamma(1-\varepsilon)\theta_t^S + \theta_t^U} \right\}^{1-\alpha} \frac{K_{t+1}}{K_t}. \tag{A14}$$

Substituting (7) and (8) in this expression gives (24) in the text.

In the nonstochastic steady state, $\theta_{t+1}^h = \theta_t^h = \tilde{\theta}^h$, $h = U, S$. Thus, the deterministic steady-state growth rate of output—which, from (A14), is the same as the growth rate of capital—is given by

$$1 + g = \frac{\sigma(1-\alpha)}{[\gamma(1-\varepsilon)\tilde{\theta}^S + \tilde{\theta}^U]^\alpha} \left[\tilde{\theta}^U + \left\{ 1 - \frac{\omega - \tilde{\mu}}{[0.5(1+\tilde{x}^C)]^\chi} \right\} \gamma(1-\tau)(1-\varepsilon)\tilde{\theta}^S \right], \tag{A15}$$

with $\tilde{x}^C$ and $\tilde{\mu}$ obtained from the joint, nonstochastic solution of (A11) and (A12):

$$\tilde{x}^C = \frac{(\omega - \tilde{\mu})^{1/\chi}}{0.5} [1 - \frac{1}{\gamma(1-\varepsilon)}]^{-1/\chi} - 1, \tag{A16}$$

$$\tilde{\mu} = \upsilon(\tau + a) \left\{ 1 + [\frac{2\tilde{x}^C}{\gamma(1-\varepsilon)[1 - (\tilde{x}^C)^2]}] \right\}, \tag{A17}$$

and with $\tilde{\mu} = \upsilon(\tau + a)$ when only skilled workers are taxed. However, as noted in the text, the stochastic competitive equilibrium may or may not converge to the deterministic equilibrium.

Figure 1

Parameters γ and χ, and Second-Order Derivative of $f^K()$

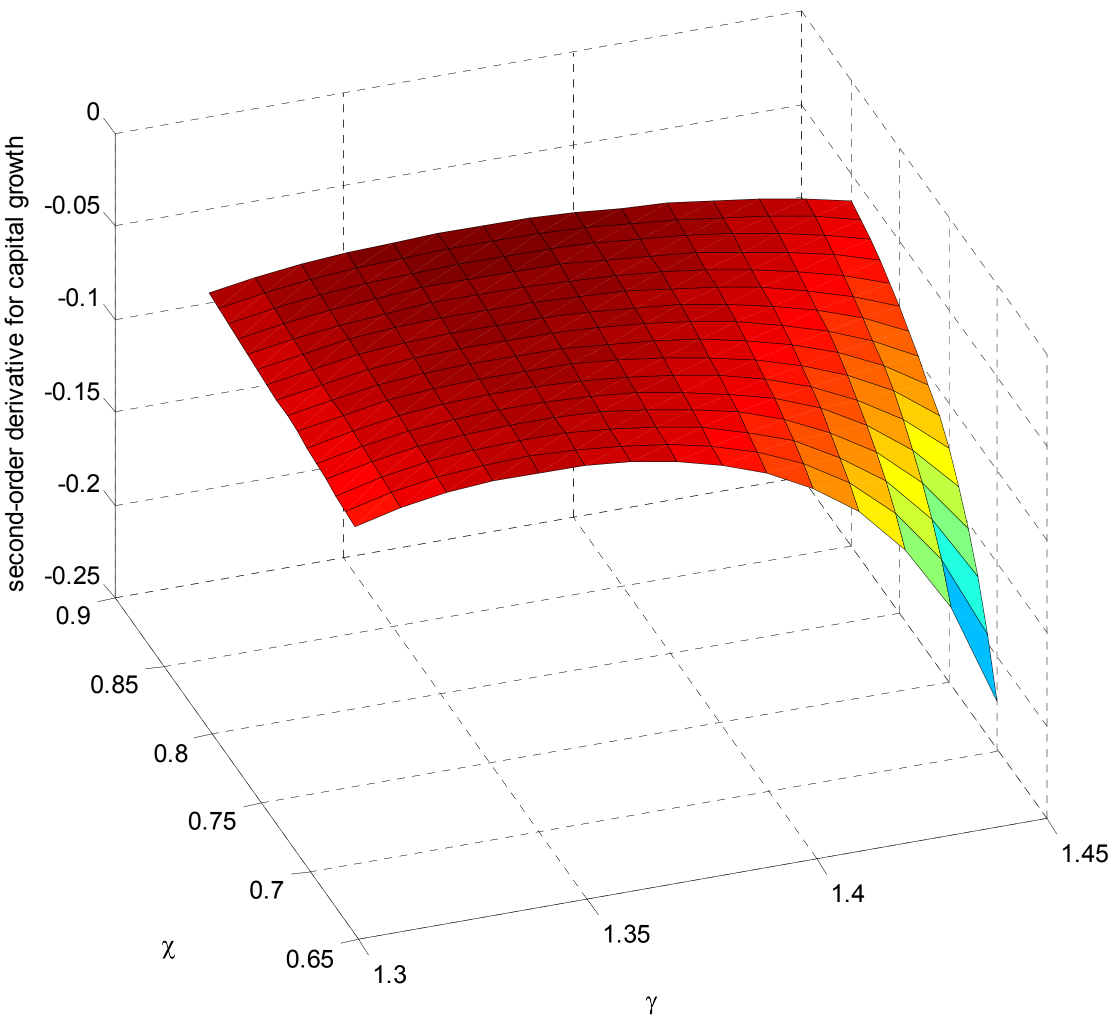

Figure 2

Parameters γ and χ, and Sum of Second-Order Derivatives of $f^Y()$