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Aid Allocation, Growth and Welfare with Productive Public Goods

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Abstract

This paper develops an open-economy intertemporal growth model with endogenous relative prices and an imperfect world capital market. The government provides two categories of public services, infrastructure and health, which are both productive. Externalities associated with infrastructure in the production of health services are also accounted for. The model is calibrated for a "typical" low-income country and used to examine the growth and welfare effects of both permanent and temporary, tied and untied, increases in aid. Dynamic trade-offs between the short- and the long-run effects of aid shocks on growth, welfare, and the real exchange rate are shown to depend crucially on the composition of aid flows.

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Contents

1 Introduction

The macroeconomic and growth effects of foreign aid and public investment have been the subject of renewed attention in recent years. Some studies have focused on the impact of external assistance on domestic savings and the government budget. Fiscal response models, for instance, have been used to examine the impact of aid on incentives—or lack thereof—to control public spending and collect taxes.[1] Another set of (mostly empirical) studies has focused on the link between aid and economic growth, with largely inconclusive results (see Doucouliagos and Paldam (2005) and Roodman (2007)). Yet another area of investigation has been the Dutch disease effect. The argument, essentially, is that if aid is at least partially spent on nontraded goods, it may put upward pressure on domestic prices and lead to a real exchange rate appreciation. In turn, a real appreciation may induce a reallocation of labor toward the nontraded goods sector, thereby raising real wages in terms of the price of tradables. The resulting deterioration in competitiveness may lead to a decline in export performance, unsustainable current account deficits, and an adverse effect on growth. It has also been argued, however, that if there is learning by doing (that is, endogenous productivity gains) and learning spillovers between production sectors, or if aid raises public investment in infrastructure, then the longer-run effect on the real exchange rate may be ambiguous (see Torvik (2001), Adam and Bevan (2006), and Agénor et al. (2008)). Once dynamic considerations are taken into account, therefore, the Dutch "disease" does not have to be a terminal illness; longer-run, supply-side effects may eventually outweigh short-term, adverse demand-side effects on the real exchange rate.

In parallel with the literature on the macroeconomic effects of aid, much research has focused on the role of public investment in the growth process. "Conventional" effects tend to emphasize the productivity, complementarity, and crowding-out effects associated with public investment; by contrast, more recent research has focused on the externalities associated with public infrastructure, in terms of its impact on education and health outcomes.[2]

[1] See Franco-Rodriguez (2000) and McGillivray and Morrissey (2004) for an overview of the literature, Ouattara and Pinto Moreira (2007) for a recent application to a low-income country, and Gupta et al. (2003) for a cross-country analysis. Kimbrough (1986) also offers a conceptual analysis of the implications of foreign aid for fiscal policy.

[2] See Agénor and Neanidis (2006) and Agénor and Moreno-Dodson (2007) for a more detailed discussion of these channels.

Access to clean water and sanitation, for instance, helps to improve health and thereby worker productivity. By reducing the cost of boiling water, and reducing the need to rely on smoky fuels (such as wood and charcoal) for cooking, access to electricity may also help to improve health and the ability to work. Better access to transportation make it easier for individuals to seek health care and to attract qualified workers to medical facilities in remote areas, which in turn may raise productivity. In addition, as discussed by Agénor (2006), if production externalities are associated with the stock of infrastructure assets itself (as a result of network effects), a "Big Push" in public investment (possibly financed by a large cut in unproductive expenditure) may facilitate the shift from a low growth equilibrium, characterized by poor health, low productivity, and low savings, to a high growth steady state.

Few studies, however, have attempted to consider jointly the links between foreign aid, public investment, and growth in the type of intertemporal optimizing models that are now commonly used in open-economy macroeconomics. Exceptions are those of Chatterjee, Sakoulis and Turnovsky (2003) and Chatterjee and Turnovsky (2005, 2007), who analyzed the impact of aid tied to public investment in infrastructure on private capital formation and growth in an open economy. However, despite some important insights, to which we will return later on, these studies suffer from several limitations. First, they all consider a single (traded) good and therefore do not explain endogenously changes in relative prices between domestic and foreign goods. As a result, their discussion of the short- and long-term effects of aid shocks does not account for the possibility that Dutch disease effects may alter the dynamic path of the economy. Second, in these models there is only one productive public good, so that the impact of changes in the composition of (tied) aid on growth and welfare, and thus dynamic tradeoffs between spending allocations, cannot be examined. Third, households are assumed to be able to borrow (at a premium) on world capital markets; this assumption is not very realistic for most poor countries—many of which have no access whatsoever to these markets. For others, access is often limited to governments; borrowing then tends to be collateralized and subject to a (sizable) risk premium.

This paper attempts to contributes to the existing literature by addressing all of these points. We do so in an open-economy intertemporal model of endogenous growth where relative prices are endogenous, so that the possible trade-off between the short- and the longer-run effects of aid shocks on the real exchange rate (and thus the economy's adjustment path) can be addressed. In addition, we also assume that the government provides two categories of public services, infrastructure and health, which are both productive. As a result, we are able to examine not only changes in the level of aid, when it is tied to either one of these two categories of spending, but also changes in the composition of foreign assistance. We also account, in line with the evidence alluded to earlier, for externalities associated with infrastructure in the production of health services.

The remainder of the paper is organized as follows. Section II presents the framework. In addition to the features highlighted above, the model departs from existing contributions by Chatterjee, Sakoulis and Turnovsky (2003) and Chatterjee and Turnovsky (2005, 2007) by assuming that private agents cannot borrow on world capital markets, and that the government can do so only at a premium that depends negatively on the fraction of its infrastructure assets that it can pledge as collateral. As a result, current account dynamics are driven essentially by public debt accumulation. Section III characterizes briefly the solution of the model (the derivation of the balanced growth path) under alternative assumptions about the form of aid (tied and untied). Because the reduced-form dynamic system (derived in Appendices A and B) involves five highly nonlinear differential equations, we resort to numerical techniques to solve it. Calibration is thus discussed in Section IV. Experiments, related to both permanent and temporary aid shocks, are presented in Section V. Sensitivity analysis, involving changes in the degree of efficiency of public investment, the elasticity of production with respect to public infrastructure, and the existence of network effects, is performed in Section VI. The final section offers some concluding remarks.

2 The Framework

We consider a small open economy populated by an infinitely-lived representative household-producer (or household, for short). A single traded commodity, which can be used for consumption, investment, or exports, is produced with labor and capital. The government has no access to seigniorage but can issue bonds to finance its deficit. It collects a proportional tax on output, invests in infrastructure, and spends on health services. It also services its debt and provides lump-sum transfers to households. Infrastructure and health services (which are produced by the government) are provided free of charge.

2.1 Production

Commodities are produced, in quantity Y, with private capital, K_P, public capital in infrastructure, K_I, and "effective" labor, defined as the product of the quantity of "raw" labor and productivity, A. The net growth rate of the population is zero. Normalizing the population size to unity and assuming that the technology is Cobb-Douglas yields[3]

$$Y = K_I^{\alpha} A^{\beta} K_P^{1-\alpha-\beta}, \tag{1}$$

where $\alpha, \beta \in (0,1)$. Thus, although the supply of raw labor is exogenous, effective labor is endogenous.

Labor productivity depends solely on the availability of health services provided by the public sector.[4] For simplicity, we assume that the flow supply of these services is proportional to the stock of health capital, K_H. Thus, the relationship between A and K_H is also one of strict proportionality, so that $A = K_H$. Using this result with (1) yields

$$Y = (\frac{K_I}{K_P})^{\alpha}(\frac{K_H}{K_P})^{\beta} K_P. \tag{2}$$

2.2 Household Optimization

The representative household's optimization problem can be specified as

$$\max_C V = \int_0^{\infty} \frac{(CK_H^{\kappa})^{1-1/\sigma}}{1-1/\sigma} \exp(-\rho t)dt, \quad \sigma \neq 1, \; \kappa > 0, \tag{3}$$

where C is consumption, ρ the discount rate, σ is the elasticity of intertemporal substitution, and and $\kappa > 0$ measures the contribution of health to utility. Again, in (3) the flow supply of health services is assumed to be proportional to the stock of public capital in health.[5]

[3] In what folllows, time subscripts are omitted for simplicity, and a dot over a variable is used to denote its time derivative.

[4] In line with the discussion in the introduction, labor productivity could also be made to depend on access to public infrastructure, such as roads.

[5] To ensure that the instantaneous utility function has the appropriate concavity properties, we impose the restrictions $\kappa(1-1/\sigma) < 1$ and $1 > (1-1/\sigma)(1+\kappa)$.

The household's resource constraint is given by

$$\dot{K}_P = (1-\tau)Y + T - C - \delta_P K_P, \tag{4}$$

where T is lump-sum transfers (taken as given by the household), $\tau \in (0,1)$ is the tax rate on income and $\delta_P \in (0,1)$ is the rate of depreciation of private capital. We assume that domestic households do not have access to world financial markets, so private foreign borrowing is zero.

The household takes public policies and the depreciation rate as given when choosing the optimal path of consumption. Using (4), the current-value Hamiltonian for problem (3) can be written as

$$L = \frac{(CK_H^{\kappa})^{1-1/\sigma}}{1-1/\sigma} + \chi[(1-\tau)Y + T - C - \delta_P K_P],$$

where χ is the costate variable associated with constraint (4). From the first-order condition $dH/dC = 0$ and the costate condition $-dH/dK_P = \dot{\chi} - \rho\chi$, optimality conditions for this problem take the form

$$K_H^{\kappa}(CK_H^{\kappa})^{-1/\sigma} = \chi, \tag{5}$$

$$\dot{\chi} = \chi[\rho + \delta_P - (1-\tau)(1-\alpha-\beta)\frac{Y}{K_P}], \tag{6}$$

together with the budget constraint (4) and the transversality condition

$$\lim_{t\to\infty} \chi K_P \exp(-\rho t) = 0. \tag{7}$$

Condition (5) can be rewritten as

$$C = \chi^{-\sigma} K_H^{\sigma\kappa(1-1/\sigma)}.$$

Taking logs and differentiating this expression with respect to time yields

$$\frac{\dot{C}}{C} = -\sigma(\frac{\dot{\chi}}{\chi}) + v(\frac{\dot{K}_H}{K_H}),$$

which, using (6) can be written as

$$\frac{\dot{C}}{C} = \sigma s(\frac{Y}{K_P}) - \sigma(\rho + \delta_P) + v(\frac{\dot{K}_H}{K_H}), \tag{8}$$

where $s \equiv (1-\tau)(1-\alpha-\beta)$ and $v \equiv \sigma\kappa(1-1/\sigma)$.

We further assume that the household maximizes a sub-utility function in order to distinguish between domestic and imported consumption. Assuming a Cobb-Douglas form, the household's problem is

$$\max_{C_D, C_M} U = C_D^{\epsilon} C_M^{1-\epsilon}, \tag{9}$$

where $\epsilon \in (0,1)$, subject to

$$P \cdot C = P_D C_D + E C_M,$$

where C_D and C_M denote consumption of domestic and imported goods, respectively, P_D is the price of domestic goods supplied to the domestic market, E the nominal exchange rate, and P_M the world price of imports, normalized to unity. Straightforward optimization yields the familiar conditions

$$C_D = \epsilon C, \quad C_M = (1-\epsilon) C z^{-1}, \tag{10}$$

where $z = E/P_D$ is the relative price of imported goods, or equivalently here, the real exchange rate.

2.3 Government

The government invests G_I in infrastructure capital and G_H in health. It also spends on "unproductive" (or, perhaps more accurately, "not directly productive") public consumption, G_U, transfers to households, T, and interest payments on its foreign debt. Investment in infrastructure and health involves installation costs, denoted Ω_I and Ω_H. To finance its expenses, the government collects a proportional tax on output at the rate τ (as noted earlier) and receives aid, A, which takes the form of either a pure (untied) transfer or is tied to investment in infrastructure or health. Thus, in foreign-currency terms, the government budget constraint is given by

$$\dot{D}^* = z^{-1} \left\{ G_U + \sum_{h=I,H} (1+\Omega_i) G_i + T - \tau Y \right\} + r D^* - A, \tag{11}$$

where D^* is public external debt and r the interest rate on that debt.

We begin with the assumption that the recipient country receives aid as a certain percentage of its output, a, so that

$$A = a Y z^{-1}. \tag{12}$$

Next, we assume that before engaging in domestic spending, the government first services its outstanding debt out of domestic tax revenues. Thus, foreign lenders have a "seniority claim" on domestic resources, which may be justified by ruling out the option to default because of a credible threat to seize domestic assets. Thus, when $A = 0$, all components of spending are fixed fractions of total tax revenues, net of interest payments:

$$G_h = \upsilon_h(\tau Y - rD), \quad h = I, H, U, T \tag{13}$$

where $D = zD^*$ is the domestic-currency value of the public debt and $\upsilon_h \in (0, 1)$.

If $A > 0$ and aid is tied to productive investment in infrastructure and health, then G_I and G_H become

$$G_h = \upsilon_h(\tau Y - rD) + \lambda_h aY, \quad h = I, H \tag{14}$$

where $\lambda_h \in (0, 1)$ denotes the share of infrastructure and health in tied aid (so that $\lambda_I + \lambda_H = 1$), whereas unproductive spending and transfers continue to be given by

$$G_h = \upsilon_h(\tau Y - rD). \quad h = U, T \tag{15}$$

By contrast, if $A > 0$ but aid is a pure transfer, then all components of spending are given by

$$G_h = \upsilon_h \left[(\tau + a)Y - rD\right]. \quad h = I, H, U, T \tag{16}$$

In line with the literature on country risk (see Turnovsky (1997), for instance), the cost of borrowing abroad for the government includes a premium, PR, above and over the risk-free world interest rate, r_f. We define the premium as a positive function of the foreign debt-to-infrastructure capital ratio:

$$PR = \frac{\gamma}{2}(\frac{D}{\chi K_I})^2, \tag{17}$$

where $\chi \in (0, 1)$ denotes the extent to which infrastructure capital can serve as collateral. The convexity of this function implies that the premium demanded by the markets increases at a faster rate as the liability position of the government worsens.[6] Using (17), the cost of foreign borrowing by the

[6] We assume that health capital, which consists essentially of hospital buildings, dispensaries, and so on, cannot be pledged as collateral, unlike telecommunications systems or airline assets abroad, for instance.

government can thus be written as

$$r = r_f + \frac{\gamma}{2}(\frac{D}{\chi K_I})^2. \qquad (18)$$

As in Chatterjee and Turnovsky (2005, 2007), installation costs are taken to depend on the ratios of investment in infrastructure and health to their respective capital stocks:

$$\Omega_I = \frac{\Sigma_I}{2}(\frac{G_I}{K_I}), \quad \Omega_H = \frac{\Sigma_H}{2}(\frac{G_H}{K_H}), \qquad (19)$$

where Σ_I, $\Sigma_H > 0$.

The stocks of public capital in infrastructure and health evolve over time according to

$$\dot{K}_I = \varphi_I G_I - \delta_I K_I, \qquad (20)$$

$$\dot{K}_H = K_I^{\mu}(\varphi_H G_H)^{1-\mu} - \delta_H K_H, \qquad (21)$$

where $\delta_I, \delta_H \in (0,1)$ are constant depreciation rates and $\varphi_I, \varphi_H \in (0,1)$ are efficiency parameters that measure the extent to which investment flows translate into actual accumulation of public infrastructure and health capital, respectively. When $\varphi_h < 1$, investment outlays are subject to inefficiencies, which tend to limit their positive impact on the accumulation of public assets. Using a broad measure of public capital, Arestoff and Hurlin (2005), for instance, estimate the value of these coefficients to vary between 0.4 and 0.6 for a group of developing countries.[7]

In addition, in (21), where $\mu \in (0,1)$, the accumulation of public capital in health services requires combining both government spending on health and public capital in infrastructure. This captures the idea, alluded to in the introduction, that access to infrastructure (such as electricity) is essential to build and operate public capital in health (for instance, hospitals).

Finally, we do not allow the government to run Ponzi games, so that

$$\lim_{t\to\infty} D \exp[-\int_0^{\infty} r_u du] = 0. \qquad (22)$$

[7]This specification is also consistent with the results in Rajkumar and Swaroop (2008), who found that good governance (as measured by low levels of corruption and high quality of bureaucracy) improves the impact of government spending on education and health on outcomes in both sectors—possibly because it reduces waste and creates more productive capital.

2.4 Relative Price Adjustment

Because the domestic good is an imperfect substitute to the imported good, the issue of what determines its equilibrium price must be addressed. As noted earlier, this is essential to discuss Dutch disease effects.

Let AD denote aggregate demand for the domestic good and suppose that private investment, I_P, takes the form of spending only on domestic goods. Assuming that the government imports a fraction $\zeta \in (0, 1)$ of its total expenditure, we have

$$AD = C_D + I_P + (1 - \zeta)G, \tag{23}$$

where

$$G = G_U + \sum_{h=I,H} (1 + \Omega_i)G_i.$$

Similarly, let AS denote aggregate supply of the domestically-produced good to the domestic market. Assuming that the country exports a fraction $x = \varpi z^{\theta}$ domestic production, we have

$$AS = (1 - \varpi z^{\theta})Y, \tag{24}$$

where $\theta > 0$ and $\varpi > 0$ is normalized to ensure that $x \in (0, 1)$ at all times. Thus, a depreciation raises exports and lowers the supply of domestic goods on the domestic market.[8]

With full price flexibility, the equality between (23) and (24) can be solved for P_D, the price of the domestic good. To impart more realistic behavior to prices, however, we assume that domestic prices adjust only gradually. Specifically, we adopt a symmetric Walrasian price adjustment specification, in which domestic prices increase (fall) with excess demand (supply) of domestic goods. Assuming, in addition, full adjustment to changes in prices of imported goods implies that changes in the real exchange rate can be specified as

$$\frac{\dot{z}}{z} = -\psi(\frac{AD - AS}{AS}), \tag{25}$$

[8] Rather than simply postulating the existence of an export function, an alternative approach would be to assume that there is a constant elasticity of transformation (CET) function between sales on the domestic market and abroad, as for instance in Agénor, Bayraktar, and El Aynaoui (2008). However, this would not make any difference qualitatively.

where $\psi > 0$ is the speed of adjustment.

From (4), $I_P = (1-\tau)Y + T - C$. Substituting this result together with (10), (23) and 24) into (25) yields

$$\frac{\dot{z}}{z} = \psi\left\{1 - \frac{(\epsilon-1)C + (1-\tau)Y + T + (1-\zeta)G}{(1-\varpi z^{\theta})Y}\right\}. \qquad (26)$$

3 Decentralized Equilibrium

Appendices A and B show how the decentralized equilibrium of the economy is determined when aid is tied to productive government spending, as in (14), and untied, as in (16). Essentially, in both cases the model can be condensed into a first-order nonlinear differential equation system in $c = C/K_P$, $k_I = K_I/K_P$, $k_H = K_H/K_P$, $d = D/K_P$ and z. These five equations, together with the initial conditions k_I^0, k_H^0, d^0, z^0 and the transversality conditions (7) and (22), characterize the dynamics of the economy.

The BGP is a set of functions $\{c, k_I, k_H, d, z\}_{t=0}^{\infty}$ such that the reduced-form equations given in Appendices A and B, and the transversality conditions (7) and (22), are satisfied, and consumption, the stocks of public capital in infrastructure and health, the stock of private capital, the domestic-currency and foreign-exchange value of the government's foreign debt, all grow at the same constant rate γ, and the real exchange rate is constant.[9] Let $\tilde{x}$ denotes the stationary value of x; from the results in the Appendices, this growth rate is given by, in the case of tied aid,

$$\gamma^T = \sigma s\tilde{k}_I^{\alpha}\tilde{k}_H^{\beta} - \sigma(\rho + \delta_P),$$

$$+\upsilon\left\{\tilde{k}_H^{-1}\varphi_H\left[\upsilon_H(\tau\tilde{k}_I^{\alpha}\tilde{k}_H^{\beta} - r\tilde{d}) + (1-\lambda_I)a\tilde{k}_I^{\alpha}\tilde{k}_H^{\beta}\right] - \delta_H\right\},$$

whereas with untied aid,

$$\gamma^U = \sigma s\tilde{k}_I^{\alpha}\tilde{k}_H^{\beta} - \sigma(\rho + \delta_P)$$

$$+\upsilon\left\{\tilde{k}_H^{-1}\tilde{k}_I^{\mu}\left[\varphi_H\upsilon_H\{(\tau+a)\tilde{k}_I^{\alpha}\tilde{k}_H^{\beta} - r\tilde{d}\}\right]^{1-\mu} - \delta_H\right\}.$$

[9] γ is also the rate of growth of output of commodities, given the assumption of constant returns to scale.

4 Calibration

Due to the complexity of the reduced-form system derived in Appendices A and B, we analyze the dynamics of the model through numerical simulations. Parameter values are chosen as much as possible to represent a small low-income, aid-receiving country. To assess the robustness of our results, we perform in the next section some sensitivity analysis around some key parameters.

We start with the production of commodities and set $\alpha = 0.1$ and $\beta = 0.55$ as the elasticities of infrastructure and health capital respectively, implying that the elasticity of output with respect to private capital is 0.35, which is a fairly conventional choice. This value is also consistent with a number of empirical estimates for developing countries, such as Cole and Neumayer (2006, p. 925). For the health technology, the value of μ is difficult to pin down, because the evidence is microeconomic in nature; for our benchmark equilibrium, we choose a low value and set $\mu = 0.1$, as in Agénor (2005*a*). The depreciation rates of of public capital in infrastructure and health, δ_I and δ_H, are both set as 4.5 percent, whereas private capital is assumed to depreciate at 6.5 percent, in line with the estimate of Bu (2006, Table 8) for a group of low-income countries.

The rate of time preference, ρ, is set at 3.5 percent, and the intertemporal elasticity of substitution, σ, is taken to be 0.2, consistent with the evidence for low-income countries (see Ogaki, Ostry, and Reinhart (1996) and Agénor and Montiel (2008, Chapter 13)). We set the parameter characterizing the impact of health capital on utility, κ, to 0.25, as in Turnovsky (2004). The share of domestic consumption in total consumption, ϵ, is set at 0.6, so that imported consumption accounts for 40 percent of total consumption. The government is assumed to import the same amount of its total expenditure so ς is taken to be 0.4.

The tax rate on output, τ, is set at 0.18. This value is in line with actual ratios for many low-income countries, where taxation (which is essentially indirect in nature) provides a more limited source of revenue than in higher-income countries (see Agénor and Neanidis (2007)). The initial shares of government spending on infrastructure services and health services, υ_I and υ_H are set at 0.2 and 0.1, respectively. The share of "unproductive" spending, υ_U (which includes also public wages and salaries, although they are not explicitly accounted for) is set at 0.4 thus leaving the share of transfers, $\upsilon_T = 1 - \upsilon_I - \upsilon_H - \upsilon_U$, as 0.3. The estimates used here can be viewed as representing the case of a government that can only use limited amount of funds for productive government spending and therefore the initial levels of infrastructure and health stocks are low, as expected in most aid-receiving countries. Put differently, the fact that the initial position is characterized by relatively low spending shares on productive outlays is consistent with the very assumption that the economy is poor and "stuck" initially in a low-growth steady state.

For the efficiency of public investment in infrastructure and health, we refer to Arestoff and Hurlin (2005) who estimate φ_I between 0.4 and 0.6 for a group of developing countries. For the benchmark equilibrium, we thus use $\varphi_I = \varphi_H = 0.5$ and then perform sensitivity analysis with lower and higher values for both parameters. We set the scaling coefficients for installation costs, Σ_I and Σ_H, to unity.[10] We further set $\chi = 0.75$, so that three quarters of infrastructure capital can serve as collateral to foreign borrowing, and the scaling coefficient for the premium, Γ, to 0.01. The world risk-free interest rate, r_f, is taken to be 4 percent and the elasticity of exports to the exchange rate, θ, is set at 0.5 with $\varpi = 0.2$. Finally, the speed of adjustment of the exchange rate to its equilibrium value, ψ, is set at 0.5 and we assume that the government does not receive any aid initially, so that $a = 0$.

Table 1 summarizes the parameter values used in the benchmark case. Substituting these values in the reduced-form system yields the benchmark equilibrium for the economy, which is summarized in Table 2. As can be inferred from the first row of the table, the equilibrium values are $k_I = 0.184$, $k_H = 0.131$, $d = 0.125$, $z = 3.142$, and $c = 0.453$. The debt-output ratio stands at 19.6 percent and the consumption-output ratio at 71.3 percent, and the economy grows at a constant rate of 1.38 percent. The equilibrium interest rate is 4.81 percent, implying that the country pays a premium of 0.81 percent above the risk-free interest rate. These figures are illustrative of a low-income country with scarce infrastructure and health capital (as mentioned earlier), low savings, and sluggish growth.

[10] We could of course also explore the implications of differences in Σ_I and Σ_H for our results. However, because there is limited information on the nature of these costs in poor countries, we did not feel compelled to pursue this line of investigation.

5 Simulations of Aid Shocks

We now examine two types of experiments: a permanent increase in aid, and a temporary (ten-period only) increase in aid. We contrast the results when aid is tied and untied.

5.1 Permanent Increase in Aid

Consider first a permanent increase in the flow of foreign aid. We assume that the country receives aid equal to 1 percent of commodity output in every period and we first distinguish between the two cases where aid is either tied to productive government spending (project-based aid), or takes the form of a pure transfer, which we label untied aid. In the case of tied aid, we further distinguish between two scenarios, where all aid is tied either to infrastructure investment, in which case $\lambda_I = 1$, or to investment in health, so that $\lambda_H = 1$. Although a 1 percent increase in aid does not seem to be large in magnitude, in the benchmark equilibrium it implies about a 30 percent increase in infrastructure investment when $\lambda_I = 1$ and a more than 50 percent increase in investment in health when $\lambda_H = 1$ every period.

The second, third and fourth rows of Tables 1a and 1b present the long-run and immediate short-run effects of an aid shock on the key variables for the three cases defined earlier. The last column in Table 1a shows the percentage change in total welfare (ΔW) whereas the last column in Table 1b presents the instantaneous change in welfare ($\Delta W(0)$) at $t = 0$ following the shock. In both cases welfare changes are measured as the percentage change in the initial health capital stock necessary to achieve the after-shock total welfare calculated through the transition to the new equilibrium (see Appendix C).

5.1.1 Long-Run Effects

We first consider the case where aid is solely tied to investment in health capital and therefore $\lambda_H = 1$. As the second row of Table 2a shows, the ratio of health capital to private capital almost doubles in this case as a result of high investment in health, increasing from 0.131 to 0.226. This large increase in health capital also triggers a large increase in consumption (due to the complementarity effect in the utility function) and crowds out private investment, increasing the consumption-private capital ratio from 0.453 to

0.640 in the new equilibrium. The crowding-out of private capital also leads to a slight increase in the public debt-private capital ratio to 0.134, but the high productivity effect of health in output more than compensates for the crowding-out effect and the debt-output ratio falls instead, from 0.196 to 0.153. For the same reason, the increase in the consumption-output ratio is milder compared to the consumption-private capital ratio, from 0.713 to 0.734 in the new steady state. The public infrastructure-private capital ratio increases slightly to 0.215 due to the indirect effect of an increase in the health capital stock on infrastructure investment through output. In terms of the exchange rate, aid has conflicting effects. While part of the increase in consumption is met by imports and is not reflected in the increase in domestic consumption, private investment (that is, savings) falls by the total amount of the increase in consumption (on both domestic and imported goods) due to the inability of private agents to borrow on international financial markets. Therefore, the overall effect of an increase in consumption is a fall in aggregate domestic demand. By contrast, aid increases government spending on domestic goods, both through investment in health capital and the additional installation costs associated with that investment, whereas increases in the health stock and infrastructure capital have supply-side effects through increases in labor productivity and the availability of production inputs. Thus, the net effect of aid on the exchange rate depends on which of these effects dominates. When $\lambda_H = 1$, the large increase in consumption and the high productivity effect of health capital dominate in the long run and the exchange rate *depreciates* from 3.142 to 3.145. In this case, the country does not experience a (long-run) Dutch disease effect, and grows at a rate of 2.5 percent in the new steady state—characterized by higher infrastructure and health capital ratios. The welfare effect of the increase in aid is also very high in this case, as the large increases in consumption and health capital lead to a 92.5 percent increase in total welfare.

We next consider the other corner case where aid is completely tied to infrastructure investment and $\lambda_I = 1$. The long-run effects of a 1 percent increase in aid are displayed in the third row of Table 2a. As expected, aid now has a larger effect on the rate of accumulation of infrastructure assets, and the public infrastructure-private capital ratio increases to 0.243. However, the direct and indirect effects on the health capital stock are now smaller due to the low elasticities of output and health with respect to infrastructure (α and μ); as a result, the health capital-private capital ratio increases only slightly, to 0.137. Consequently, the effect on consumption is also relatively small compared to the case $\lambda_H = 1$ and the consumption-output ratio increases to 0.717. Moreover, the low productivity of infrastructure cannot create a large-enough supply-side effect to compensate for the increase in installation costs and domestic government spending due to the additional investment outlays, and the real exchange rate now *appreciates* to 3.063. For this reason, the debt-output ratio rises to 0.218 despite the appreciation of the real exchange rate. The growth and welfare effects are also smaller than in the previous case, as growth rises to 1.53 percent only and the welfare gain stands at a modest 5.5 percent.

Third, we experiment with a 1 percent increase in untied aid, which simply increases total resources available to the government by a similar amount. Put differently, untied aid is distributed among the government spending categories according to the initial spending shares.[11] As the fourth row of Table 1a shows, the results illustrate an intermediate case between the two corner cases of tied aid described above. The public infrastructure-private capital ratio rises to 0.201 whereas the effect on health is slightly larger than tied aid with $\lambda_I = 1$ as the health capital-private capital ratio increases to 0.142. Because aid contributes less now to public investment in infrastructure and health, it also creates less additional debt through installation costs. Further, private investment also rises through the direct increase in transfers and therefore although the fall in the interest rate is less than tied aid, debt-private capital ratio increases only marginally to 0.128. However, the debt-output ratio is higher than in the case of tied aid with $\lambda_H = 1$ because of the high productivity of health compared to private and infrastructure capital and the partial loss of aid through unproductive government spending. The real appreciation is slightly larger compared to the case $\lambda_I = 1$, as z falls to 3.057. The growth effect is only marginally higher than with $\lambda_I = 1$, with a steady-state growth rate now of 1.54 percent, whereas the larger increases in consumption and health yield a 10.1 percent welfare gain.

5.1.2 Transitional Dynamics

Table 2b and Figure 1 display the immediate short-run responses and the transition paths to the new equilibrium following the shock. As the second row of Table 2 reveals, when $\lambda_H = 1$, the growth rate of health capital jumps

[11] United aid has therefore the same effects as (partially) tied aid with $\lambda_I = 0.2$, $\lambda_H = 0.1$ (because $v_I = 0.2$ and $v_H = 0.1$), 10 percent of the aid allocated to private investment because $v_T = 0.1$, and the rest to unproductive spending ($v_U = 0.4$).

to 4.4 percent on impact. Because all aid is allocated to investment in health, the growth rate of the infrastructure capital stock remains unchanged. By contrast, the consumption-output ratio jumps to 0.796, and this large jump in consumption leads to a dramatic fall in the growth of private capital to -3.87 percent. Overall, the growth rate of output falls from the initial value of 1.38 percent to 1.2 percent. When $\lambda_I = 1$, the growth rate of infrastructure capital increases now to 3.1 percent on impact, whereas the growth rate of the health stock (and thus the growth rate of consumption, as implied by (8)). The initial jump in consumption and the consumption-output ratio is less than the case when $\lambda_H = 1$, and therefore the adverse affect on the accumulation of private capital is smaller as the growth rate of private capital falls to 0.85 percent. The overall effect on the growth of output is a marginal fall to 1.36 percent. When aid is untied, the immediate effect is again an intermediate case as the growth rates of public capital in infrastructure and health jump to 1.72 and 1.68 percent, respectively. However, the larger initial jump in consumption leads to a slightly larger drop in the growth rate of private capital, compared to tied aid with $\lambda_I = 1$. Overall, tied aid with $\lambda_I = 1$ and untied aid induce the smallest drop in the growth rate of output, as it falls marginally in both cases.

In terms of welfare, the immediate responses show the same pattern with long-run welfare gains. The large initial jump in consumption when aid is tied to investment in health leads to an immediate increase of 52.5 percent in welfare, whereas when $\lambda_I = 1$, the short-run welfare gain is only 4.9 percent and untied aid delivers a 5.8 percent rise in instantaneous welfare.

The transition paths to the new steady-state equilibrium are displayed in Figure 1 for all three cases considered. The consumption-private capital ratio increases smoothly to the new equilibrium value following the initial jump in all cases. For the public infrastructure-private capital ratio, transition is smooth and monotonic for $\lambda_I = 1$ and untied aid as the growth rate of infrastructure capital stays above the growth rate of private capital. By contrast, when aid is tied and $\lambda_H = 1$, the growth rate of private capital falls excessively initially, so that the infrastructure-private capital ratio rises above the new equilibrium level before stabilizing. The same pattern is observed (but to a much smaller extent) for the health-private capital ratio when $\lambda_I = 1$, and the transition is monotonic for both $\lambda_H = 1$ and now with untied aid, as the growth rate of health capital remains above the growth of private capital throughout the transition. When aid is tied to investment in health, the real exchange rate depreciates on impact by more than its new equilibrium level. It remains higher than that level throughout the transition. It appreciates monotonically to the new equilibrium when aid is untied and falls initially below the new steady-state level before stabilizing when $\lambda_I = 1$, as the supply-effect never dominates the aggregate demand effect. The interest rate increases slightly initially when $\lambda_H = 1$ as the growth rate of debt exceeds the growth rate of the infrastructure capital stock. It also converges at a slow speed when $\lambda_I = 1$, because continuous investment in infrastructure first leads to a fall below the new equilibrium interest rate before converging. In the case of untied aid, the interest rate falls monotonically to the new equilibrium but convergence is at a much slower rate than $\lambda_I = 1$. Furthermore, the significant crowding-out effect on private capital and the initial increase in the interest rate, coupled with the real appreciation, leads to a sharp initial increase in the debt-private capital ratio above the new equilibrium, when $\lambda_H = 1$. The same pattern is also true for untied aid at a smaller scale but this time convergence is even slower, whereas when $\lambda_I = 1$, the transition is smooth and monotonic. However, due to the high elasticity of output with respect to health capital, $\lambda_H = 1$ performs much better in terms of the debt-output ratio and converges faster, with the largest fall in the equilibrium value of that variable.

5.2 Temporary Increase in Aid

We now consider the case where the increase in aid is only for a temporary period of time, rather than permanent. In particular, we assume that the country receives aid (tied or untied) equivalent to 1 percent of output for only the first 10 periods and that the shock is reversed after 10 periods.

The dynamics associated with this shock are displayed in Figure 2. As expected, the consumption-private capital, infrastructure-private capital, and health stock-private capital ratios all rise above their initial equilibrium levels following the shock. The increase in the consumption-private capital ratio is highest when aid is tied to investment in health, as in the case of a permanent shock. However, a major difference is that when aid is totally allocated to health capital, the infrastructure-private capital ratio falls now *below* its initial steady-state value, after the aid is cut back. The reason is that the increase in consumption stabilizes and the growth rate of private capital rises above that of infrastructure capital. When aid is tied to infrastructure investment or untied, the exchange rate *appreciates* through the first 10 periods and then converges back to the initial equilibrium after the shock is reversed, the speed of convergence being higher when $\lambda_I = 1$ than with untied aid. By contrast, when $\lambda_H = 1$, the real exchange rate initially *depreciates*, as consumption increases and the supply-side effect of health is strong, but falls below its equilibrium value before stabilizing. The same is also true for the debt-private capital ratio when $\lambda_H = 1$, as the real appreciation and the recovery in the growth rate of the private capital stock after 10 periods pulls the debt-private capital ratio below the equilibrium value.

The dynamics of the interest rate with temporary aid is also similar to the permanent case. When $\lambda_I = 0$, the interest rate initially increases and then falls slightly below the equilibrium before converging. By contrast, when aid is tied to infrastructure investment or untied, the interest rate falls through the first 10 periods, and the magnitude of the drop is larger for $\lambda_H = 1$ compared to the untied aid case. The debt-output ratio falls in the first 10 periods if $\lambda_I = 0$ or aid is untied but it increases slightly initially if $\lambda_H = 1$, falling for a short time—as the real exchange rate appreciates more and the interest rate falls more toward the tenth period—but then increasing again when the increase in aid is reversed. Finally, the welfare gains from a temporary shock are very similar to the permanent case, with $\lambda_H = 1$ yielding a 67.2 percent welfare gain while untied aid increases total welfare by 7.17 percent and $\lambda_I = 1$ by 4.48 percent.

Although the country receives aid for only 10 periods, the higher growth experienced during the first 10 periods has permanent effects on the levels of infrastructure, health and private capital stocks, as well as output and the instantaneous welfare of the representative household. Figure 3 presents these permanent effects, calculated as the ratio of the after-shock levels to the levels that would have prevailed had the country stayed on its initial growth path. As the first panel shows, the highest change in instantaneous welfare is achieved when aid is tied to investment in health, before. By contrast, although untied aid delivers a larger total welfare gain than $\lambda_I = 1$ (as in the permanent case), the relative change in instantaneous welfare is highest for $\lambda_I = 1$ in the long run when aid is temporary. Similarly, temporary aid tied to infrastructure investment results in a higher long-run level of private capital, as well as infrastructure capital, than untied aid. With regards to health capital, untied aid behaves only marginally better than $\lambda_I = 1$, but overall, the positive effect of infrastructure capital accumulated through the first 10 periods with $\lambda_I = 1$ dominates and the relative change in output is higher when the temporary aid is allocated to infrastructure rather than untied. When $\lambda_H = 1$, the relative change in health capital is larger due to the high elasticity of output with respect to (effective) labor; the relative change in output is significantly larger than with $\lambda_I = 1$ and untied aid.

6 Sensitivity Analysis

In order to check the robustness of the results above, we now perform some sensitivity analysis with respect to some key parameters and variables. We concentrate on three important features of the model: the degree of efficiency of public investment, the elasticity of production (of commodities as well as health services) with respect to public infrastructure, and the existence of network effects. In all experiments, we consider only permanent shocks.

6.1 Efficiency of Public Investment

Table 3 displays the sensitivity of the long-run responses of the variables, growth and welfare to the efficiency of public investment in infrastructure and health (φ_I, φ_H). The case $\varphi_I = \varphi_H = 0.5$ corresponds to the benchmark solution and the figures show, as before, percentage deviations from baseline values. One of the most interesting results appears in the second row of Table 3. If the degrees of efficiency of public investment and health are both 0.45 and the country starts with lower ratios of health and infrastructure stocks with respect to private capital, the impact of aid is much larger as the infrastructure-private capital ratio increases by 22.1 percent compared to the benchmark value of 16.6 percent and the change in the health capital-private capital ratio rises to 81.8 percent. Similarly, as Table 4 shows, the growth and welfare effects are also larger. However, aid now has a larger impact on the debt-output ratio as the fall in that variable (as shown in Table 3) reaches 8.3. The reduction in the debt burden, and associated interest payments, increases the resources that the government can allocate to noninterest outlays and raises aggregate demand for the domestic good.[12] This, in turn, reverses the depreciation of exchange rate observed in the benchmark equilibrium to a marginal appreciation. Similar results also hold for the cases $\lambda_I = 1$ and untied aid; with lower efficiency of public investment, the appreciation of the real exchange rate is larger, and the growth and welfare effects are higher, than in the benchmark equilibrium.

[12] Recall that, in all experiments, the government allocates its resources to various spending categories after servicing its debt.

The opposite scenario, where efficiency of public investment is higher, also has important consequences for the impact of aid. Because higher efficiency results in higher initial stocks of health and infrastructure capital, percentage deviations from the initial equilibrium fall and the impact of aid diminishes as efficiency increases. Although the fall in the debt-output ratio also gets smaller with higher efficiency (thereby reducing the impact of aid on domestic government spending), our results show that the negative effect of diminishing returns dominate and the exchange rate tends to appreciate as efficiency increases. In this sense, Table 3 shows that the exchange rate is more responsive to the efficiency of health investment (and thus the initial stock of health capital) due to the high elasticity of output with respect to labor and the effect of health on utility. If the initial stock of public capital in health is small to begin with, then aid will have a greater impact on consumption and at the same time have a larger supply-side effect because the returns to health capital are very high. Therefore, as Table 4 shows, the gains in total welfare also fall as the efficiency of public investment increases.

However, sensitivity of growth to public investment efficiency displays a different pattern. If the degree of efficiency is below a certain level ($\varphi_I = \varphi_H = 0.45$), the impact on growth is larger than in the benchmark case. Moreover, increasing the efficiency parameter above the benchmark values also increases the impact of aid on growth, as less and less resources are now wasted in the process of capital accumulation. The increase in the growth rate of output is largest when $\varphi_I = \varphi_H = 0.65$ if aid is tied to health investment. This is because the positive effect of less waste in health investment dominates the negative effect of diminishing returns due to the high productivity of health. Further, the last column of Table 4 also shows that $\lambda_I = 1$ has a marginally larger growth effect, than untied aid if the efficiency of public investment is high enough.

Finally, although the long-run effects of aid are stronger for all variables (except the real exchange rate), measured in terms of percentage deviations from baseline values, with low efficiency of public investment, it must be noted that for values of φ_I and φ_H below 0.45, the initial equilibrium is no longer stable. The reason is that in that case the amount of public waste is too high and the country pays too large a premium over the world interest rate. As a result, unsustainable debt dynamics tend to develop and translate into spending levels that are too low and eventually become negative.

6.2 Production Elasticities

One of the main reasons why untied aid performs better than aid tied to infrastructure investment in the benchmark equilibrium is the relatively low productivity of infrastructure in the production of goods and health capital. Therefore, we now experiment with higher elasticities of output of commodities and health capital with respect to infrastructure capital. To save space, Table 5 displays the sensitivity to α and μ of the long-run effects of aid on growth, welfare and the real exchange rate only. In line with the empirical evidence, these two parameters are varied within the range 0.1-0.2 (see Agénor (2005*a*)). The impact of aid on growth when $\lambda_I = 1$ increases significantly with higher values of α and μ, and aid tied to infrastructure performs better than untied aid both in terms of growth if $\alpha = \mu = 0.15$, and both in terms of growth and welfare if $\alpha = \mu = 0.2$. However, as α and μ increase, the relative productivity of health with respect to infrastructure capital is lower and $\lambda_H = 1$ tends to yield a real appreciation and lower growth.

6.3 Network Effects of Infrastructure

We now assume that the efficiency of public capital in the production of commodities is endogenously related to the infrastructure-private capital ratio, as in Agénor (2006), as a result of network effects. Formally, (1) is now replaced by

$$Y = (\pi K_I)^{\alpha} K_H^{\beta} K_P^{1-\alpha-\beta}, \tag{27}$$

where $\pi = \Lambda k_I^{\eta}$, with Λ denoting a shift parameter. Because we are dealing with a low-income country, where public infrastructure is scarce to begin with, we will focus on the case where the efficiency of public capital displays increasing returns, so that $\eta > 1$.[13]

Table 6 shows the sensitivity of the permanent responses of growth, welfare and the real exchange rate to weak and strong network effects of infrastructure capital, for slightly different values of the tax rate on output. If the network effects are not strong ($\eta = 1.5$), then allocating aid only to investment in infrastructure translates into an appreciation of the real exchange rate, regardless of the tax rate. However, even with weak network

[13] More mature economies, by contrast, would be characterized by $\eta < 1$. The case considered previously, of course, corresponds to $\eta = 0$.

effects, $\lambda_I = 1$ now has significantly higher growth rates than untied aid. Moreover, because health capital is now relatively less productive compared to infrastructure capital, the real exchange rate appreciates as well in the long run when $\lambda_H = 1$. The magnitude of the appreciation is smaller and growth and welfare gains are highest for $\tau = 0.16$, and as the tax rate increases, the positive effects of aid fall because the government now uses more resources to spend on unproductive expenses and crowds out more private capital accumulation.

By contrast, if there are strong network effects associated with infrastructure capital and $\eta = 4$, allocating aid to investment in infrastructure does not yield appreciation any more with $\tau = 0.18$ and performs significantly better than untied aid in terms of welfare and growth.Similarly, $\lambda_H = 1$ yields a much stronger depreciation of the real exchange rate than in the benchmark equilibrium, as the indirect positive effect on output through accumulation of infrastructure capital now dominates the negative impact due to the relatively less productive effect of health capital mentioned above. However, with strong network effects, the initial equilibrium is not stable with a tax rate of $\tau = 0.16$. So the existence of network effects puts a lower bound on the tax rate for the economy to be stable initially.

Figure 4 shows how sensitive the transition to the new steady state is with respect to network externalities. To save space, the figure shows parametric plots relating only to two of the endogenous variables in the model, the real exchange rate and the public infrastructure-private capital ratio. As the figures reveal, when network effects are weak, allocating aid to infrastructure investment leads to a monotonic appreciation of the exchange rate to the new steady state (as in the benchmark equilibrium with no network effects), whereas the dynamics in the case of $\lambda_H = 1$ and untied aid and are not altered either—the real exchange rate first depreciates and then appreciates in real terms when $\lambda_H = 1$, and monotonically appreciates when aid is untied. However, if the network effects are strong, $\lambda_I = 1$ implies initially a depreciation of the real exchange rate above its new equilibrium level, as infrastructure capital accumulates. Although untied aid also has similar dynamics (because part of the aid is spent on infrastructure and health), the slope of the parametric plot is steeper for $\lambda_I = 1$ and the magnitude of the initial real depreciation is larger.

7 Concluding Remarks

This paper studied the impact of aid shocks in an intertemporal model of endogenous growth in a small open developing economy. In the model, the government provides two categories of public services, infrastructure and health, which are both productive. Because domestically-produced goods are imperfect substitutes for imported goods, relative prices (or, equivalently here, the real exchange rate) is endogenous. Externalities associated with infrastructure in the production of health services are also accounted for, in light of the evidence discussed in Agénor and Moreno-Dodson (2007). These features make the model well suited to discuss the short- and longer run effects of aid shocks on the real exchange rate (the so-called Dutch disease effects) and the allocation of aid among alternative spending categories. The model also departs from existing contributions on aid and growth by Chatterjee and Turnovsky (2005, 2007) by assuming that private agents cannot borrow internationally, and that the government can borrow only at a premium that depends on its collateralizable infrastructure assets.

The model was calibrated for a low-income country and used to examine both permanent and temporary, tied and untied, increases in aid. Sensitivity analysis, with respect to the degree of efficiency of public investment, the elasticity of production (of both goods and health services) with respect to infrastructure, and the existence of network effects, was also conducted. The analysis highlights the existence of dynamic trade-offs between the short- and the longer-run effects of aid on the real exchange rate.[14] Moreover, such effects depend not only on the level of aid, but also on its composition.

The analytical framework presented in this paper could be extended in various directions. First, the model could be endogenized to account for endogenous "raw" labor supply and a more general production technology for commodities and health services, as for instance in Chatterjee and Turnovsky (2007). As shown in that study, these extensions would provide additional channels through which aid shocks affect growth and welfare. Second, the model could be modified to account for the moral hazard effects of aid, by introducing an inverse relation between the level of aid, as a share of GDP, and the tax rate. This would allow a link between our model and fiscal response models, which have been used to examine the impact of aid on

[14] At the same time, our results provide only partial support for the evidence by Radelet, Clemens, and Bhavnani (2006), which suggests that aid targeted to infrastructure may generate quick growth payoffs.

taxes and government expenditure—that is, the degree of fungibility of aid. As noted in the introduction, some of these studies have shown indeed that an increase in aid may lead to a decline in public savings through lower tax revenues, as governments reduce their tax collection effort. Alternatively, as documented by Chatterjee, Giuliano, and Kaya (2007), increases in aid may translate into a shift in the composition of government spending away from investment and toward consumption. In turn, reduced incentives to mobilize domestic resources, or shifts away from productive spending, may mitigate the benefits of sustained increases in aid for economic growth and welfare.

Another important issue to address is the impact of aid volatility on growth, welfare, and tax policy, as discussed in Agénor and Aizenman (2007). Studies by Bulir and Hamann (2006) and Hudson and Mosley (2006) have found that the volatility of aid is much larger than the volatility of domestic tax revenues, with coefficients of variation in the range of 40-60 percent of mean aid flows. Both studies also found that aid volatility has actually increased since the late 1990s, as does Kharas (2007) for a large group of aid recipients. By their very nature, some types of aid (such as emergency aid or, to a lower extent, program aid) should indeed exhibit a high degree of volatility. By contrast, project aid should be relatively stable, given that it is designed to promote (directly or indirectly) investment in physical and human capital. Volatility in that category of aid could make it difficult for recipient governments to formulate medium-term investment programs to spur growth and be very detrimental to long-term economic and social development in these countries. Accounting for aid volatility in a stochastic version of the present framework would therefore be a very fruitful extension.

Appendix A
Decentralized Equilibrium with Tied Aid

In order to determine the decentralized equilibrium in the economy when aid is tied to productive government spending, we start with dividing (4) by K_P and using (15)to get

$$\frac{\dot{K}_P}{K_P} = [(1-\tau)]\frac{Y}{K_P} + \upsilon_T(\tau\frac{Y}{K_P} - rd) - c - \delta_P,$$

where $c = C/K_P$ and $d = D/K_P$. Substituting (2) into this equation gives

$$\frac{\dot{K}_P}{K_P} = [(1-\tau)]\, k_I^\alpha k_H^\beta + \upsilon_T(\tau k_I^\alpha k_H^\beta - rd) - c - \delta_P, \tag{A1}$$

with $k_I = K_I/K_P$ and $k_H = K_H/K_P$. Next, dividing (20) by K_I and substituting (14) yields

$$\frac{\dot{K}_I}{K_I} = \varphi_I \left[\upsilon_I(\tau\frac{Y}{K_I} - r\frac{D}{K_I}) + \lambda_I a\frac{Y}{K_I}\right] - \delta_I,$$

which can be rewritten as

$$\frac{\dot{K}_I}{K_I} = k_I^{-1}\varphi_I \left[\upsilon_I(\tau\frac{Y}{K_P} - rd) + \lambda_I a\frac{Y}{K_P}\right] - \delta_I.$$

Using (2) as before, we get

$$\frac{\dot{K}_I}{K_I} = k_I^{-1}\varphi_I \left[\upsilon_I(\tau k_I^\alpha k_H^\beta - rd) + \lambda_I a k_I^\alpha k_H^\beta\right] - \delta_I. \tag{A2}$$

Performing the same steps for K_H, the growth rate of health capital can also be obtained as

$$\frac{\dot{K}_H}{K_H} = k_H^{-1}\varphi_H k_I^\mu \left[\upsilon_H(\tau k_I^\alpha k_H^\beta - rd) + (1-\lambda_I)a k_I^\alpha k_H^\beta\right]^{1-\mu} - \delta_H. \tag{A3}$$

From (11), (18), and $A = aYz^{-1}$, we get

$$\frac{\dot{D}^*}{D^*} = (D^* z)^{-1}\left[G_U + (1+\Omega_I)G_I + (1+\Omega_H)G_H + T - (\tau + a)Y\right] + r_f + \frac{\gamma}{2}(\frac{D}{\chi K_I})^2.$$

Substituting (14) and (15) into this equation yields

$$\frac{\dot{D}^*}{D^*} = D^{-1}\left\{(\upsilon_U + \upsilon_T)(\tau Y - rD) + (1+\Omega_I)\left[\upsilon_I(\tau Y - rD) + \lambda_I a Y\right]\right. \tag{A4}$$

$$\left. + (1+\Omega_H)\left[\upsilon_h(\tau Y - rD) + (1-\lambda_I)aY\right] - (\tau + a)Y\right\} + r_f + \frac{\gamma}{2}(\frac{D}{\chi K_I})^2.$$

From (14) and (19), installation costs for infrastructure are given by

$$\Omega_I = \frac{\Sigma_I}{2}(\frac{\upsilon_I(\tau Y - rD) + \lambda_I a Y}{K_I}), \tag{A5}$$

which, using (2), can be rewritten as

$$\Omega_I = \frac{\Sigma_I}{2}k_I^{-1}\left[\upsilon_I(\tau k_I^\alpha k_H^\beta - rd) + \lambda_I a k_I^\alpha k_H^\beta\right]. \tag{A6}$$

Similarly, installation costs for health are

$$\Omega_H = \frac{\Sigma_H}{2}k_H^{-1}\left[\upsilon_H(\tau k_I^\alpha k_H^\beta - rd) + (1-\lambda_I) a k_I^\alpha k_H^\beta\right]. \tag{A7}$$

Equation (A4) can be rewritten as

$$\frac{\dot{D}^*}{D^*} = d^{-1}\left\{(\upsilon_U + \upsilon_T)(\tau\frac{Y}{K_P} - rd) + (1+\Omega_I)\left[\upsilon_I(\tau\frac{Y}{K_P} - rd) + \lambda_I a\frac{Y}{K_P}\right]\right.$$

$$\left. + (1+\Omega_H)\left[\upsilon_h(\tau\frac{Y}{K_P} - rd) + (1-\lambda_I)a\frac{Y}{K_P}\right] - (\tau + a)\frac{Y}{K_P}\right\} + r_f + \frac{\Gamma}{2}(\frac{d}{\chi k_I})^2,$$

or equivalently, using (2),

$$\frac{\dot{D}^*}{D^*} = d^{-1}\left\{(\upsilon_U + \upsilon_T)(\tau k_I^\alpha k_H^\beta - rd) + (1+\Omega_I)\left[\upsilon_I(\tau k_I^\alpha k_H^\beta - rd) + \lambda_I a k_I^\alpha k_H^\beta\right]\right. \tag{A8}$$

$$\left. + (1+\Omega_H)\left[\upsilon_h(\tau k_I^\alpha k_H^\beta - rd) + (1-\lambda_I) a k_I^\alpha k_H^\beta\right] - (\tau + a)k_I^\alpha k_H^\beta\right\} + r_f + \frac{\gamma}{2}(\frac{d}{_I\chi k})^2$$

From (14), (15), and (26), the exchange rate evolves according to

$$\frac{\dot{z}}{z} = -\psi\left\{(\epsilon - 1)\frac{c}{k_I^\alpha k_H^\beta} + (1-\tau) + \upsilon_T(\tau - \frac{rd}{k_I^\alpha k_H^\beta})\right. \tag{A9}$$

$$+(1-\zeta)\left[+\upsilon_U(\tau-\frac{rd}{k_I^\alpha k_H^\beta})+(1+\Omega_I)(\upsilon_I(\tau-\frac{rd}{k_I^\alpha k_H^\beta})+\lambda_I a)\right.$$

$$\left.+(1+\Omega_H)(\upsilon_H(\tau-\frac{rd}{k_I^\alpha k_H^\beta})+(1-\lambda_I)a)\right](1-\varpi z^\theta)^{-1}-1\Bigg\}.$$

Because $D = D^* z$, we have

$$\dot{D}/D = \dot{D}^*/D^* + \dot{z}/z. \tag{A10}$$

Finally, using (2), (8),and (A3),

$$\frac{\dot{C}}{C} = \sigma s k_I^\alpha k_H^\beta - \sigma(\rho + \delta_P) \tag{A11}$$

$$+v\left\{k_H^{-1}\varphi_H\left[\upsilon_H(\tau k_I^\alpha k_H^\beta - rd) + (1-\lambda_I) a k_I^\alpha k_H^\beta\right] - \delta_H\right\}.$$

Equations (A2),(A3), (A8), (A9), (A10), and (A11) can be further condensed into a first-order nonlinear differential equation system in $c = C/K_P$, $k_I = K_I/K_P, k_H = K_H/K_P$, $d = D/K_P$ and z.

Appendix B
Decentralized Equilibrium with Untied Aid

To determine the decentralized equilibrium in the economy with untied aid, we start with dividing (4) with K_P and using (15)to get

$$\frac{\dot{K}_P}{K_P} = [(1-\tau)]\frac{Y}{K_P} + \upsilon_T\left[(\tau+a)\frac{Y}{K_P} - rd\right] - c - \delta_P, \tag{B1}$$

where $c = C/K_P$ and $d = D/K_P$. Substituting (2) into this equation gives

$$\frac{\dot{K}_P}{K_P} = [(1-\tau)]\,k_I^{\alpha}k_H^{\beta} + \upsilon_T\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right] - c - \delta_P, \tag{B2}$$

with $k_I = K_I/K_P$ and $k_H = K_H/K_P$. Next, dividing (20) by K_I and substituting (16) yields

$$\frac{\dot{K}_I}{K_I} = k_I^{-1}\varphi_I\left\{\upsilon_I\left[(\tau+a)\frac{Y}{K_P} - rd\right]\right\} - \delta_I. \tag{B3}$$

Using (2) as before, we get

$$\frac{\dot{K}_I}{K_I} = k_I^{-1}\varphi_I\left\{\upsilon_I\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right]\right\} - \delta_I. \tag{B4}$$

Performing the same steps for K_H, the growth rate of health capital can also be obtained as

$$\frac{\dot{K}_H}{K_H} = k_H^{-1}k_I^{\mu}\left\{\varphi_H\upsilon_H\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right]\right\}^{1-\mu} - \delta_H. \tag{B5}$$

From (11), (18), and $A = aYz^{-1}$, we get as with tied aid

$$\frac{\dot{D}^*}{D^*} = (D^*z)^{-1}\left[G_U + (1+\Omega_I)G_I + (1+\Omega_H)G_H + T - (\tau+a)Y\right] + r_f + \frac{\gamma}{2}\left(\frac{D}{\chi K_I}\right)^2.$$

Substituting (15) and (16) into this equation yields

$$\frac{\dot{D}^*}{D^*} = D^{-1}\left\langle(\upsilon_U+\upsilon_T)\left[(\tau+a)Y - rD\right] + (1+\Omega_I)\left\{\upsilon_I\left[(\tau+a)Y - rD\right]\right\}\right. \tag{B6}$$

$$+ (1+\Omega_H)\left\{\upsilon_H\left[(\tau+a)Y - rD\right]\right\} - (\tau+a)Y\rangle + r_f + \frac{\gamma}{2}(\frac{D}{\chi K_I})^2.$$

From (16) and (19), installation costs for infrastructure are given by

$$\Omega_I = \frac{\Sigma_I}{2}\left\{\frac{\upsilon_I\left[(\tau+a)Y - rD\right]}{K_I}\right\}, \tag{B7}$$

which, using (2), can be rewritten as

$$\Omega_I = \frac{\Sigma_I}{2}k_I^{-1}\left\{\upsilon_I\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right]\right\}. \tag{B8}$$

Similarly, installation costs for health are

$$\Omega_H = \frac{\Sigma_I}{2}k_H^{-1}\left\{\upsilon_H\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right]\right\}. \tag{B9}$$

Equation (B6) can be rewritten as

$$\frac{\dot{D}^*}{D^*} = d^{-1}\left\langle(\upsilon_U+\upsilon_T)\left[(\tau+a)\frac{Y}{K_P} - rd\right] + (1+\Omega_I)\left\{\upsilon_I\left[(\tau+a)\frac{Y}{K_P} - rd\right]\right\}\right.$$

$$\left. + (1+\Omega_H)\left\{\upsilon_H\left[(\tau+a)\frac{Y}{K_P} - rd\right]\right\} - (\tau+a)\frac{Y}{K_P}\right\rangle + r_f + \frac{\gamma}{2}(\frac{d}{\chi k_I})^2,$$

or equivalently, using (2),

$$\frac{\dot{D}^*}{D^*} = d^{-1}\left\langle(\upsilon_U+\upsilon_T)\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right] + (1+\Omega_I)\left\{\upsilon_I\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right]\right\}\right. \tag{B10}$$

$$\left. + (1+\Omega_H)\left\{\upsilon_H\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - rd\right]\right\} - (\tau+a)k_I^{\alpha}k_H^{\beta}\right\rangle + r_f + \frac{\gamma}{2}(\frac{d}{\chi k_I})^2$$

From (15),(16) and (26), the exchange rate evolves according to

$$\frac{\dot{z}}{z} = -\psi\left\langle(\epsilon-1)\frac{c}{k_I^{\alpha}k_H^{\beta}} + (1-\tau) + \upsilon_T\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - \frac{rd}{k_I^{\alpha}k_H^{\beta}}\right]\right. \tag{B11}$$

$$+(1-\zeta)\left\{\upsilon_U\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - \frac{rd}{k_I^{\alpha}k_H^{\beta}}\right] + (1+\Omega_I)\upsilon_I\left[(\tau+a)k_I^{\alpha}k_H^{\beta} - \frac{rd}{k_I^{\alpha}k_H^{\beta}}\right]\right.$$

$$+(1+\Omega_H)\upsilon_H\left[(\tau+a)k_I^{\alpha}k_H^{\beta}-\frac{rd}{k_I^{\alpha}k_H^{\beta}}\right]\Bigg\}(1-\varpi z^{\theta})^{-1}-1\Bigg\rangle$$

Because $D = D^* z$, we have

$$\dot{D}/D = \dot{D}^*/D^* + \dot{z}/z. \tag{B12}$$

Finally, using (2), (8),and (B5),

$$\frac{\dot{C}}{C} = \sigma s k_I^{\alpha} k_H^{\beta} - \sigma(\rho + \delta_P) \tag{B13}$$

$$+v\left\{k_H^{-1}k_I^{\mu}\left[\varphi_H\upsilon_H\{(\tau+a)k_I^{\alpha}k_H^{\beta}-rd\}\right]^{1-\mu}-\delta_H\right\}.$$

Equations (B2),(B4), (B5), (B10), (B11), and (B13) can be further condensed into a first-order nonlinear differential equation system in $c = C/K_P$, $k_I = K_I/K_P$, $k_H = K_H/K_P$, $d = D/K_P$ and z.

Appendix C
Measuring Welfare Gains

In order to determine the welfare gains following the shock, let us first calculate the baseline level of welfare. For this, we first define $\Pi = 1 - 1/\sigma$. Then, using (3), the baseline total welfare becomes

$$W_b = \frac{1}{\Pi}\int_0^{\infty} (C_b K_{Hb}^{\kappa})^{\Pi} \exp(-\rho t) dt. \tag{C1}$$

Using the fact that the constant baseline growth rate is γ, equation (C1) can be rewritten as

$$W_b = \frac{1}{\Pi}\int_0^{\infty} (C_0 K_{H0}^{\kappa})^{\Pi} \exp\left\{[\gamma\Pi(1+\kappa) - \rho]\, t\right\} dt. \tag{C2}$$

Evaluating this integral yields

$$W_b = \frac{1}{\Pi}\frac{(C_0 K_{H0}^{\kappa})^{\Pi}}{[\gamma\Pi(1+\kappa) - \rho]} = W_b(C_0, K_{H0}). \tag{C3}$$

where C_0 is a constant term and K_{H0} is the level of health capital stock at $t = 0$.

Total welfare following the shock is given by

$$W_p = \frac{1}{\Pi}\int_0^{\infty} \left[C(t) K_H^{\kappa}(t)\right]^{\Pi} \exp(-\rho t) dt = W_p[C(t), K_H(t)], \tag{C4}$$

where $C(t)$ and $K_H(t)$ are the time paths for consumption and stock of health capital during the transition and thus are time-dependent. In order to compare these two levels of welfare, we want to determine the necessary percentage change in the initial level of health stock (K_{H0}) that would yield the same level of total utility following the shock. In other words, we want to find a coefficient Θ such that $W_b(C_0, \Theta K_{H0}) = W_p[C(t), K_H(t)]$.This calculation yields

$$W_b(C_0, \Theta K_{H0}) = \frac{1}{\Pi}\int_0^{\infty} \left[C_0 (\Theta K_{H0})^{\kappa}\right]^{\Pi} \exp\left\{[\gamma\Pi(1+\kappa) - \rho]\, t\right\} dt, \tag{C5}$$

which simplifies to

$$W_b(C_0, \Theta K_{H0}) = \frac{1}{\Pi}\frac{\left[C_0 (\Theta K_{H0})^{\kappa}\right]^{\Pi}}{[\gamma\Pi(1+\kappa) - \rho]} = \Theta^{\kappa\Pi} W_b(C_0, K_{H0}). \tag{C6}$$

Imposing $\Theta^{\kappa\Pi} W_b = W_p$, we find that

$$\Theta - 1 = (\frac{W_p}{W_b})^{1/(\kappa\Pi)} - 1. \tag{C7}$$

This condition gives the necessary percentage change in the initial health capital stock that would yield the same level of total utility following the shock.

In (C5), we implicitly assume that the initial level of consumption remains constant at its baseline level. Analogously, short-run welfare gains are calculated by

$$\Theta - 1 = (\frac{V_p}{V_b})^{1/(\kappa\Pi)} - 1, \tag{C8}$$

where $V_b = V_b(C_b, K_{Hb})$ is the instantaneous welfare at the baseline steady-state equilibrium and $V_p = V_p[C(0), K_H(0)]$ is instantaneous welfare at time $t = 0$ following the shock.

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Table 1
Parameter Values: Benchmark Case

	Benchmark Values
Production	$\alpha = 0.1, \beta = 0.55,\ \mu = 0.1$ $\delta_I = 0.045, \delta_H = 0.045, \delta_P = 0.065$
Preferences	$\sigma = 0.2, \rho = 0.035, \kappa = 0.25, \epsilon = 0.6$
Government	$\upsilon_U = 0.6, \upsilon_T = 0.1, \upsilon_I = 0.2, \upsilon_H = 0.1$ $\varphi_I = \varphi_H = 0.5, \Sigma_I = \Sigma_H = 1,$ $\varsigma = 0.4, \chi = 0.75$
Other	$\theta = 0.5, \varpi = 0.2, \psi = 0.5,\ r_f = 0.04, a = 0$

Table 2

a) Long-Run Effects of 1% Increase in Aid

	c	k_I	k_H	d	z	r %	Growth %	C/Y	D/Y	ΔW %
Benchmark Equilibrium	0.453	0.184	0.131	0.125	3.142	4.81	1.38	0.713	0.196	-
Tied Aid $\lambda_H = 1$	0.640	0.215	0.226	0.134	3.145	4.69	2.50	0.734	0.153	92.5
Tied Aid $\lambda_I = 1$	0.480	0.243	0.137	0.146	3.063	4.63	1.53	0.717	0.218	5.5
Untied Aid	0.482	0.201	0.142	0.128	3.057	4.72	1.54	0.718	0.191	10.1

b) Short-Run Effects of 1% Increase in Aid

	γ_C	γ_D	γ_{KH}	γ_{KI}	γ_{KP}	γ_Y	C(0)/Y(0)	ΔW(0)
Benchmark Equilibrium	1.38	1.38	1.38	1.38	1.38	1.38	0.713	-
Tied Aid $\lambda_H = 1$	0.77	1.91	4.40	1.38	-3.87	1.20	0.796	52.5
Tied Aid $\lambda_I = 1$	1.38	1.54	1.38	3.10	0.85	1.36	0.751	4.9
Untied Aid	1.31	1.54	1.68	1.72	0.79	1.37	0.723	5.8

Table 3
Sensitivity of Permanent Responses to Efficiency of Public Investment
(p_I, p_H)

	φ_I =0.45, φ_H =0.45					φ_I= 0. 5, φ_H= 0. 5					φ_I = 0. 65, φ_H = 0. 5					φ_I = 0. 5, φ_H = 0. 65					φ_I = 0. 65, φ_H =0. 65				
	C/Y	k_I	k_H	z	D/Y	C/Y	k_I	k_H	z	D/Y	C/Y	k_I	k_H	z	D/Y	C/Y	k_I	k_H	z	D/Y	C/Y	k_I	k_H	z	D/Y
Tied $\lambda_H = 1$	3.65	22.1	81.8	-0.1	-8.3	2.1	16.6	72.3	0.1	-4.3	2.1	14.4	69.0	0.07	-2.3	1.7	13.1	67.0	-0.4	-2.2	1.5	11.9	65.0	-0.6	-1.2
Tied $\lambda_I = 1$	0.5	34.6	5.7	-2.5	2.1	0.4	32.2	4.5	-2.5	2.2	0.3	31.5	4.4	-2.6	1.5	0.3	31.0	3.9	-2.6	2.1	0.3	30.7	4.0	-2.7	1.5
Untied Aid	0.7	10.7	10.3	-2.8	-1.6	0.5	8.8	8.5	-2.7	-0.5	0.4	8.3	8.0	-2.7	-0.2	0.4	8.0	7.7	-2.8	0	0.4	7.8	7.5	-2.8	0

Table 4
Sensitivity of Growth and Welfare to Efficiency of Public Investment

	φ_I=0.45, φ_H=0.45		φ_I = 0. 5, φ_H = 0. 5		φ_I = 0. 65, φ_H = 0.5		φ_I = 0. 5, φ_H = 0.65		φ_I = 0. 65, φ_H = 0. 65	
	ΔGrowth %	ΔW %	ΔGrowth %	ΔW %	ΔGrowth %	ΔW %	ΔGrowth %	ΔW %	ΔGrowth %	ΔW %
Tied $\lambda_H = 1$	1.11	112.7	1.13	92.5	1.15	84.1	1.26	76.1	1.3	71.2
Tied $\lambda_I = 1$	0.17	7.2	0.16	5.5	0.17	4.9	0.18	4.3	0.192	3.9
Untied	0.18	11.8	0.17	10.1	0.17	9.3	0.18	8.6	0.187	8.2

Table 5
Sensitivity of Growth, Welfare and Exchange Rate to Elasticity of Output and Health to Infrastructure
φ_I = 0. 5, φ_H = 0. 5 (Benchmark)

	α=0.1 μ=0.1			α=0.15 μ=0.15			α=0.2 μ=0.2		
	ΔGrowth %	ΔW %	Δz %	ΔGrowth %	ΔW %	Δz %	ΔGrowth %	ΔW %	Δz %
Tied $\lambda_H = 1$	1.13	92.5	0.1	1.0	87.2	-0.5	0.98	81.4	-1.2
Tied $\lambda_I = 1$	0.16	5.5	-2.5	0.26	8.8	-2.3	0.36	12.5	-2.3
Untied	0.17	10.1	-2.7	0.18	10.8	-2.7	0.2	11.7	-2.8

Table 6
Sensitivity of Growth, Welfare and Exchange Rate to Infrastructure Externality and Tax Rate
$\varphi_I = 0.5$, $\varphi_H = 0.5$ (Benchmark)

η=1.5

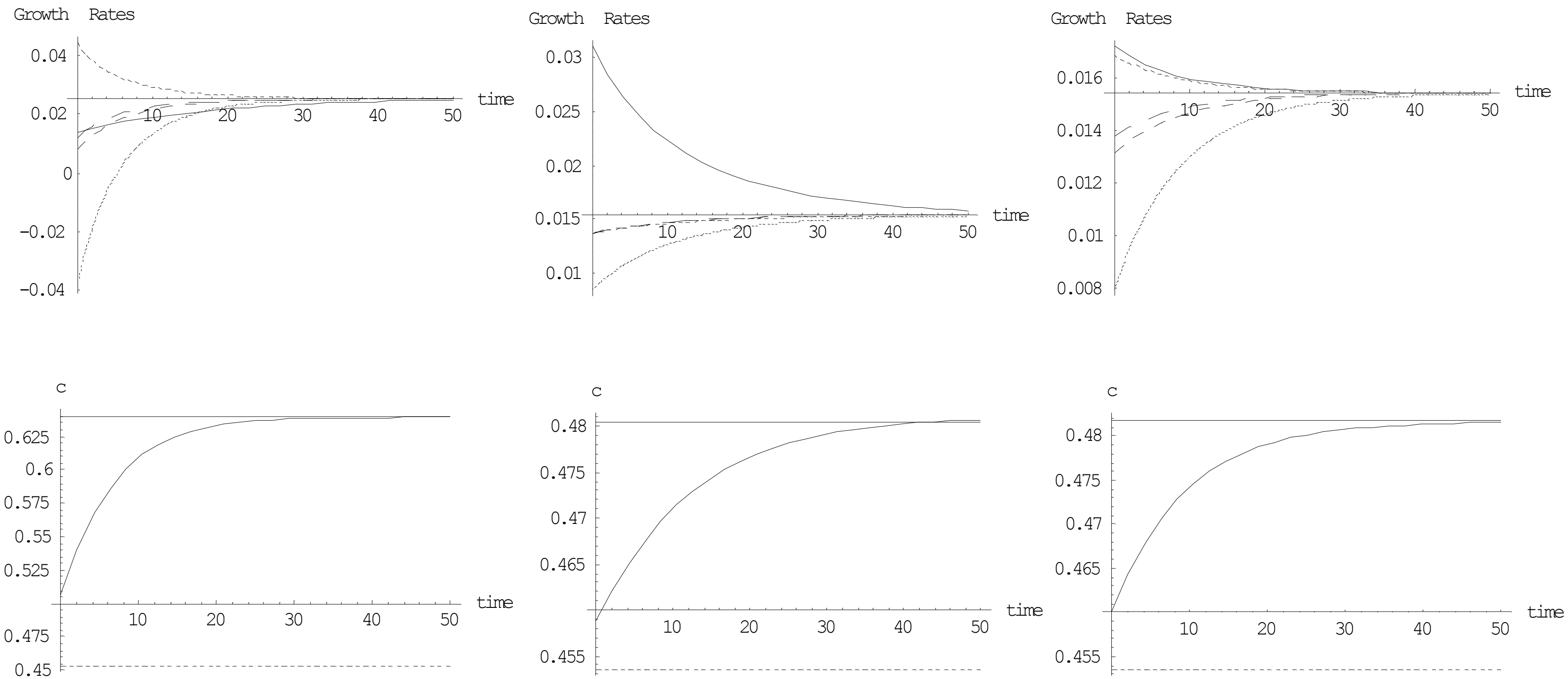
	$\tau = 0.16$			$\tau = 0.18$			$\tau = 0.2$		
	ΔGrowth %	ΔW %	Δz %	ΔGrowth %	ΔW %	Δz %	ΔGrowth %	ΔW %	Δz %
Tied $\lambda_H = 1$	1.80	91.3	-0.8	1.80	71.2	-1.5	1.72	59.1	-1.9
Tied $\lambda_I = 1$	0.56	12.9	-2.3	0.54	10.4	-2.6	0.53	8.9	-2.8
Untied	0.34	12.7	-2.8	0.34	10.7	-2.9	0.33	9.36	-3.0

η=4

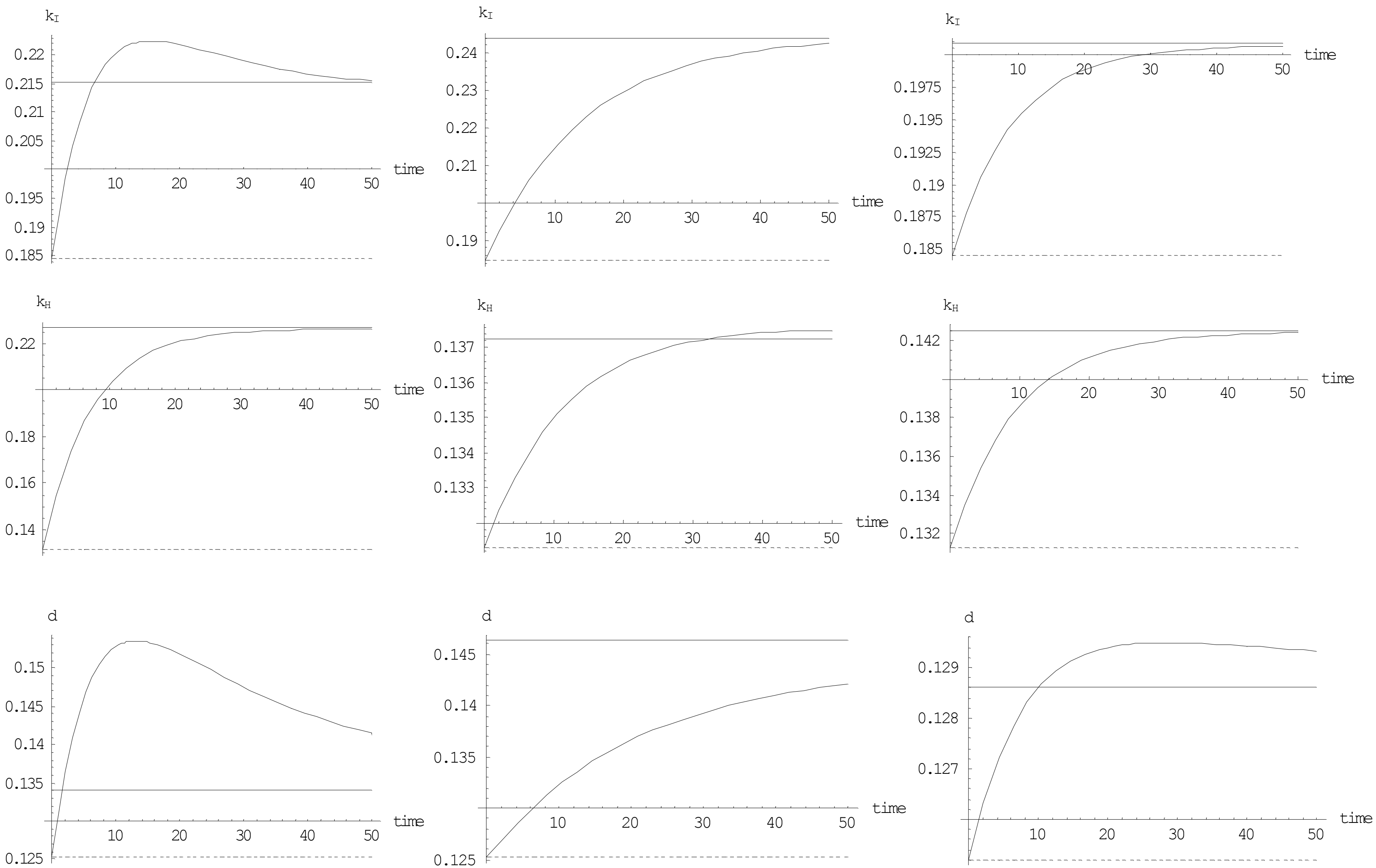
	$\tau = 0.16$			$\tau = 0.18$			$\tau = 0.2$		
	ΔGrowth %	ΔW %	Δz %	ΔGrowth %	ΔW %	Δz %	ΔGrowth %	ΔW %	Δz %
Tied $\lambda_H = 1$	Not Stable			1.64	330.4	1.6	1.49	159.7	-0.1
Tied $\lambda_I = 1$	Not Stable			1.02	116.5	0.06	0.89	58.2	-1.2
Untied	Not Stable			0.41	32.8	-2.0	0.38	23.2	-2.5

FIGURE 1
Permanent Responses to 1% Increase in Aid

Tied Aid $\lambda_H = 1$ | **Tied Aid $\lambda_l = 1$** | **Untied Aid**



Tied Aid $\lambda_H = 1$ | **Tied Aid $\lambda_l = 1$** | **Untied Aid**



Tied Aid $\lambda_H = 1$ **Tied Aid $\lambda_l = 1$** **Untied Aid**

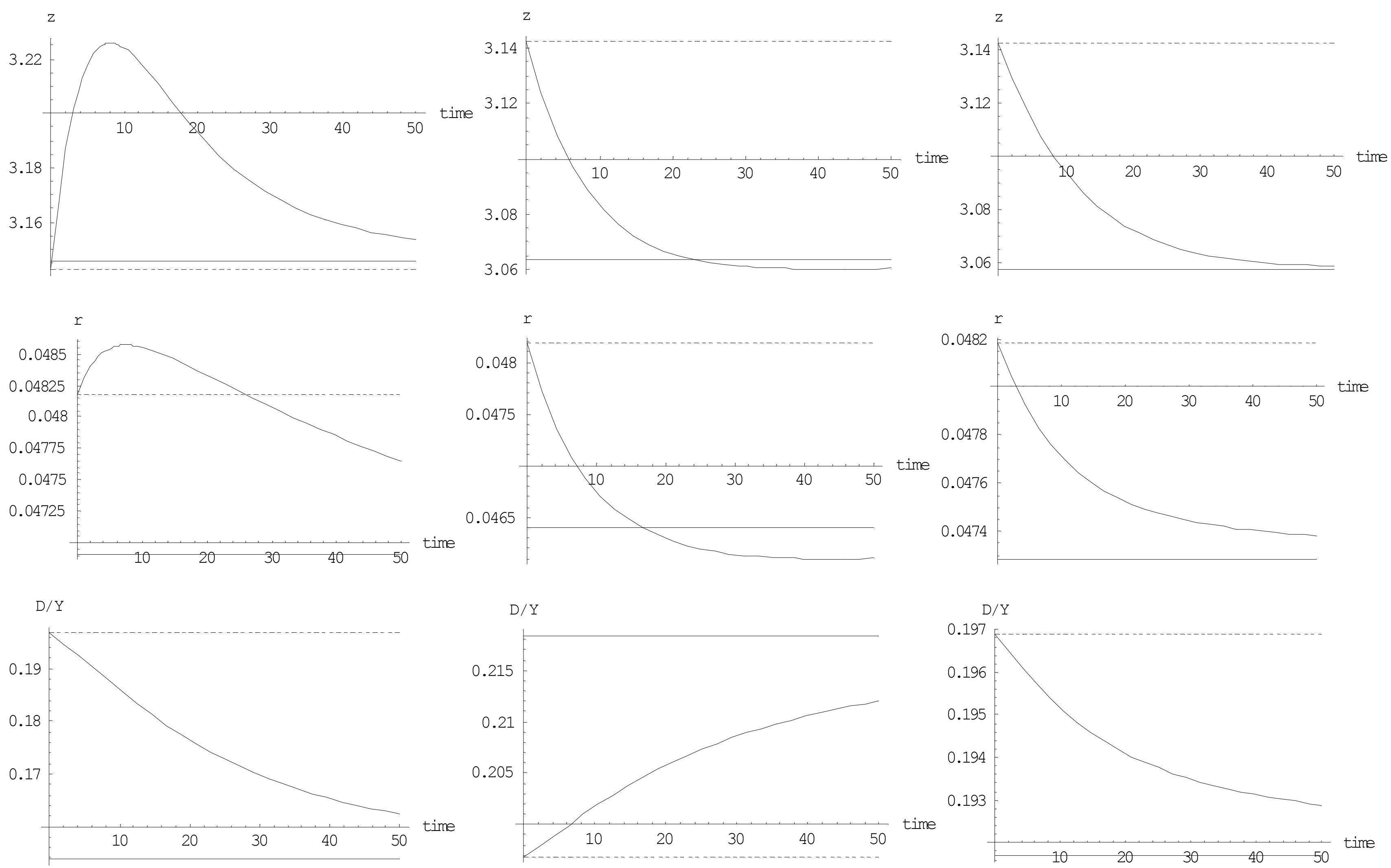
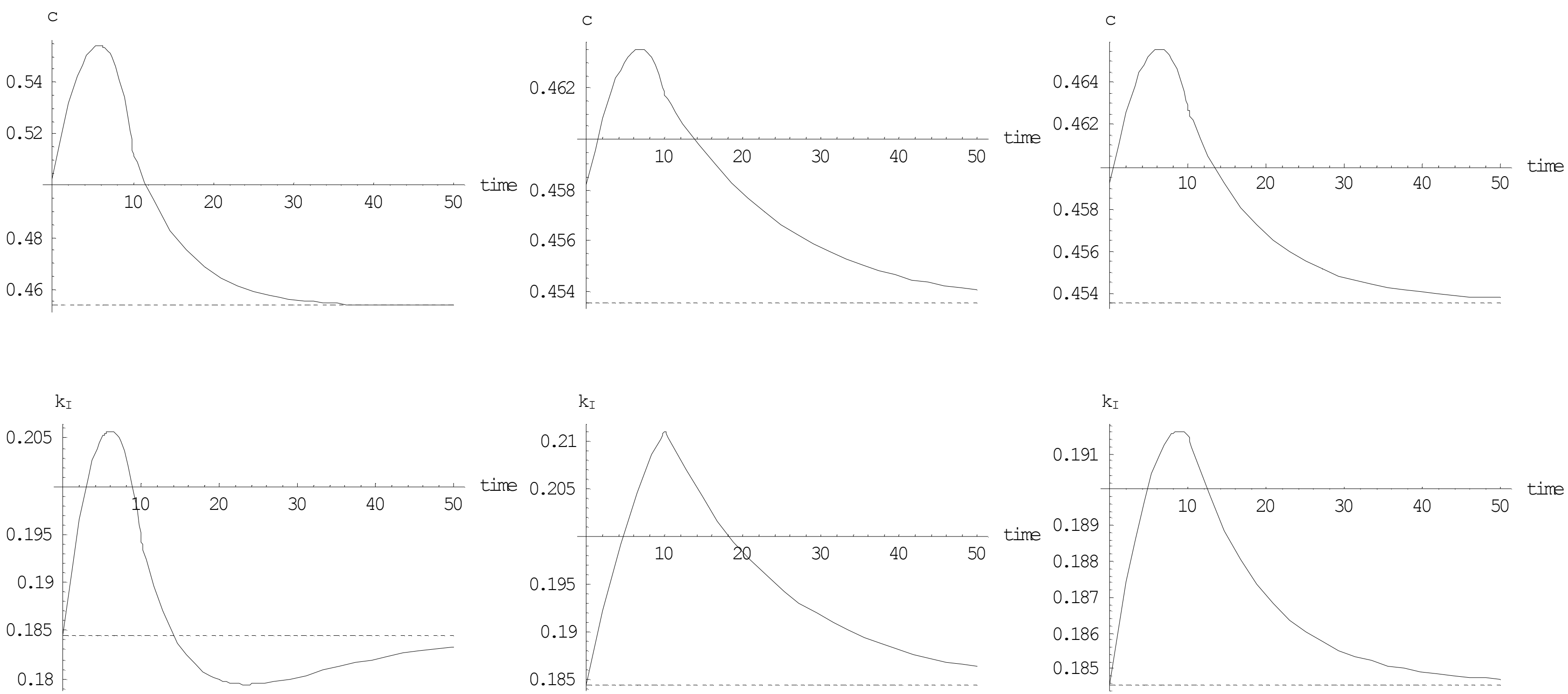
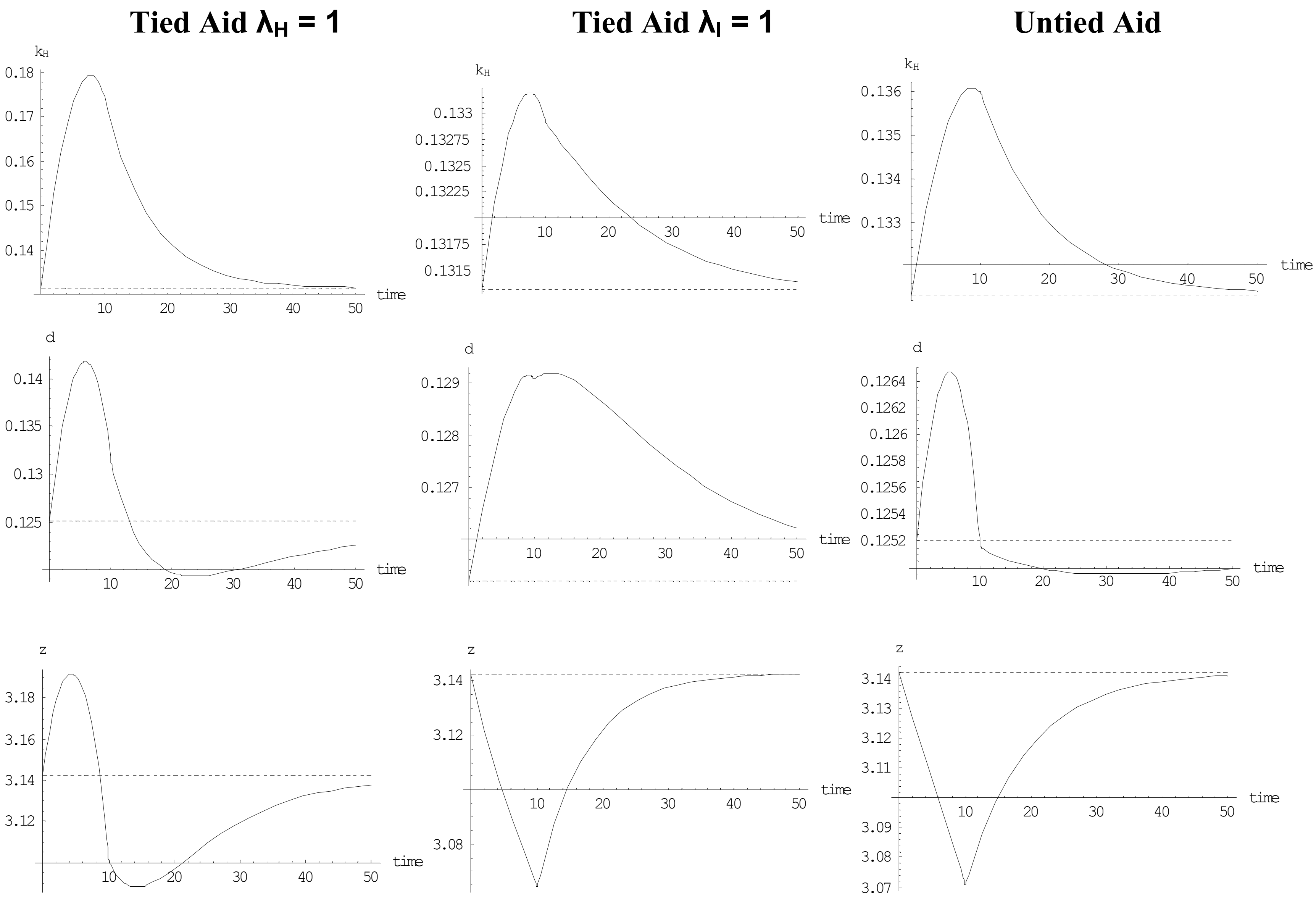


FIGURE 2
Temporary Responses to 1% Increase in Aid

Tied Aid $\lambda_H = 1$ | **Tied Aid $\lambda_I = 1$** | **Untied Aid**





Tied Aid $\lambda_H = 1$
Tied Aid $\lambda_l = 1$
Untied Aid
k_H
d
z
time

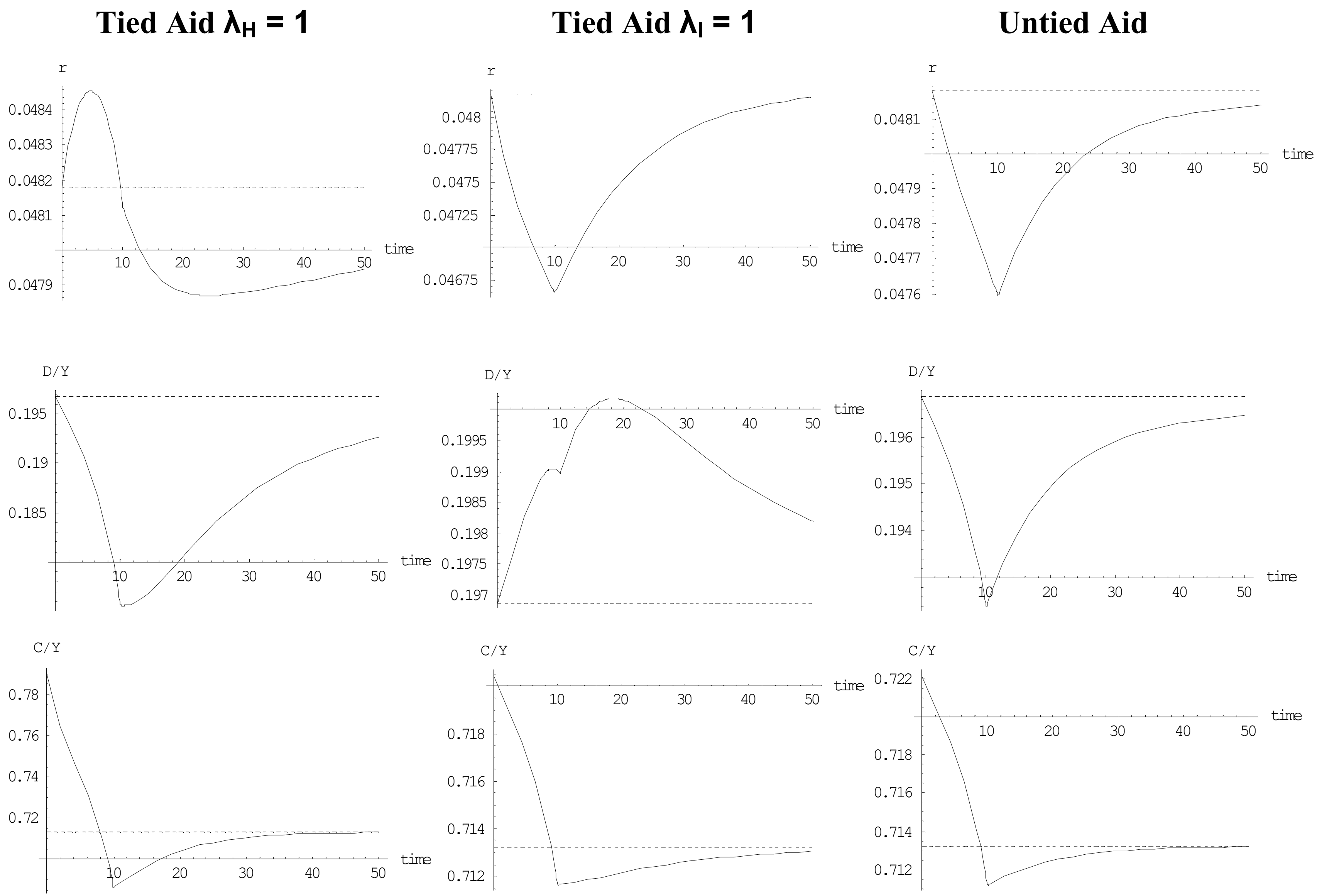
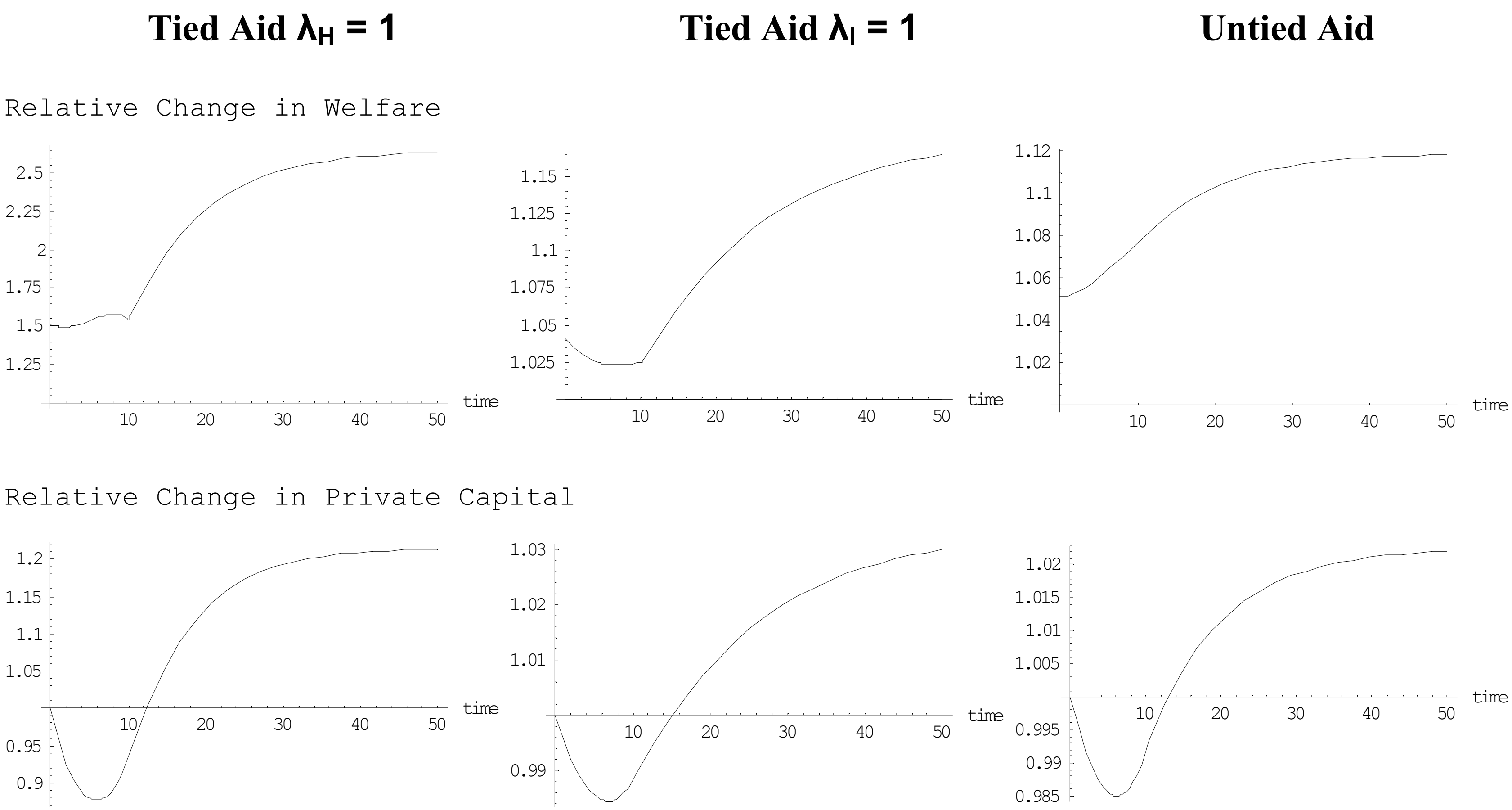

Tied Aid λ_H = 1
Tied Aid λ_l = 1
Untied Aid
r
0.0484
0.0483
0.0482
0.0481
0.0479
time
10 20 30 40 50
r
0.048
0.04775
0.0475
0.04725
0.04675
time
10 20 30 40 50
r
0.0481
0.0479
0.0478
0.0477
0.0476
time
10 20 30 40 50
D/Y
0.195
0.19
0.185
time
10 20 30 40 50
D/Y
0.1995
0.199
0.1985
0.198
0.1975
0.197
time
10 20 30 40 50
D/Y
0.196
0.195
0.194
time
10 20 30 40 50
C/Y
0.78
0.76
0.74
0.72
time
10 20 30 40 50
C/Y
0.718
0.716
0.714
0.712
time
10 20 30 40 50
C/Y
0.722
0.718
0.716
0.714
0.712
time
10 20 30 40 50

Figure 3
Permanent Effects of 1 % Temporary Increase in Aid



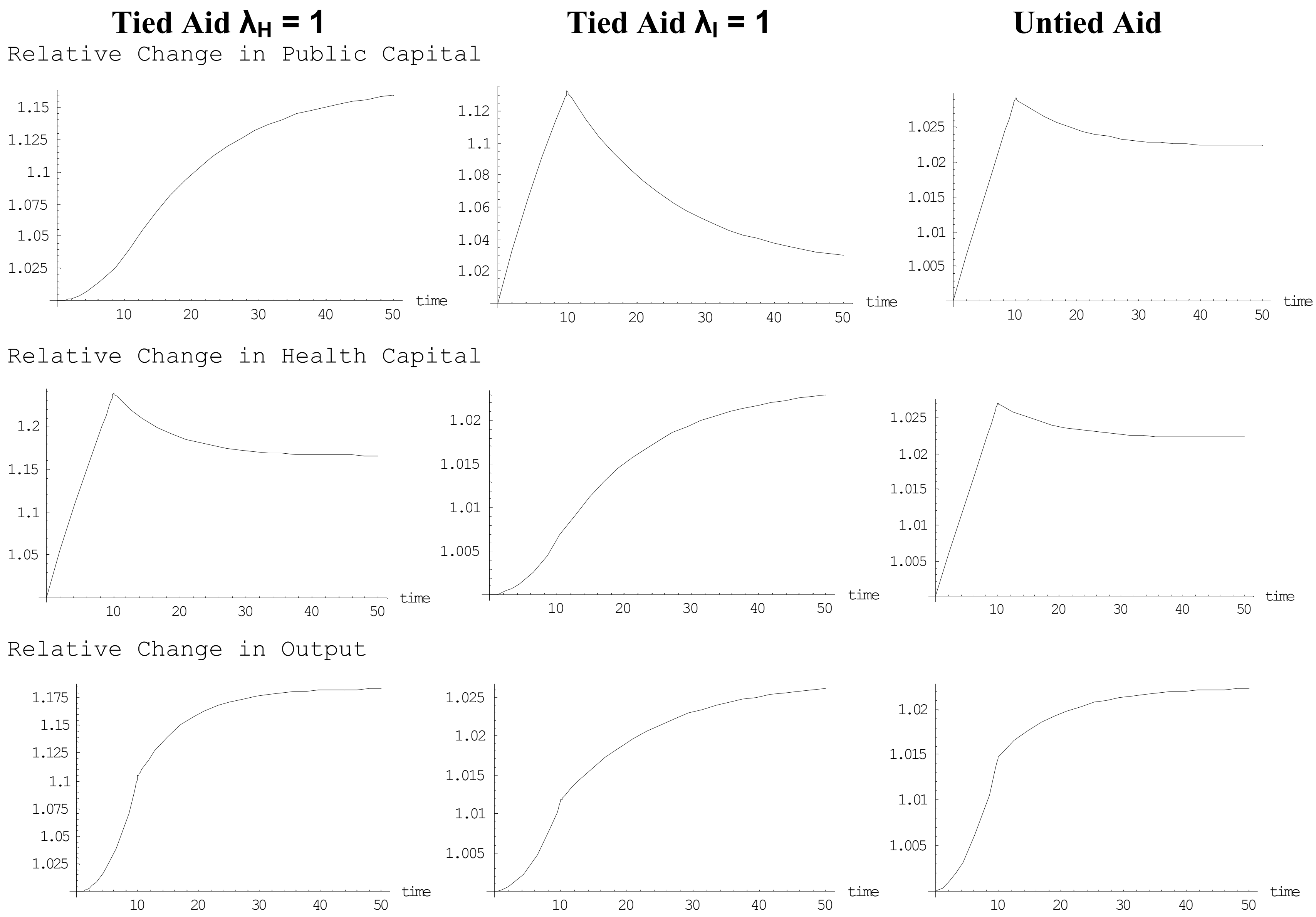
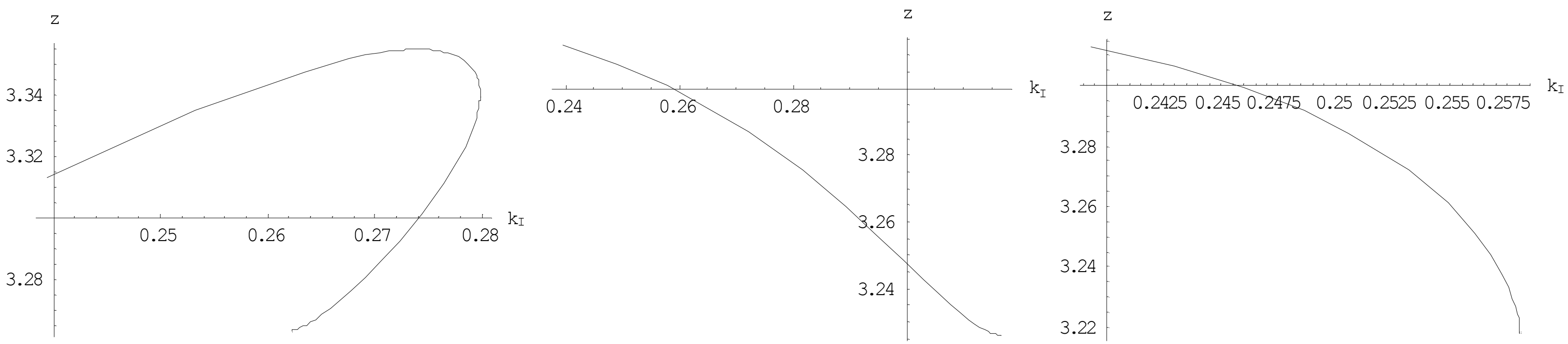

Tied Aid $\lambda_H = 1$
Tied Aid $\lambda_I = 1$
Untied Aid
Relative Change in Public Capital
Relative Change in Health Capital
Relative Change in Output
time

Figure 4
Sensitivity of Transition to Infrastructure Externality

η = 1.5

Tied Aid λ_H = 1 | **Tied Aid λ_I = 1** | **Untied Aid**



η = 4

Tied Aid λ_H = 1 | **Tied Aid λ_I = 1** | **Untied Aid**