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Optimal Taxation and Growth with Public Goods and Costly Enforcement

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Abstract

This paper studies optimal direct and indirect taxation in an endogenous growth framework with a productive public good and costly tax collection. Optimal (growth-maximizing) tax rules are derived under exogenous collection costs. The optimal direct-indirect tax ratio is shown to be negatively related to the administrative costs of collecting these taxes, as documented in cross-country data. This result also holds under endogenous collection costs (with these costs inversely related to administrative spending on tax enforcement), but for these to generate significant effects on tax collection requires implausibly high degrees of efficiency in spending, or the allocation of a large fraction of resources to tax enforcement. Depending on how it is financed, the latter policy may entail adverse effects on growth. Improving “tax culture” and the sense of civic duty through greater budgetary transparency may be a more effective policy to improve tax collection and promote economic growth.

JEL Classification Numbers: E62, H21, O41
1 Introduction

An important feature of the tax system in many developing countries is the prevalence of taxes on consumption or, more generally, indirect taxes. Personal income taxes are rarely comprehensive and often do not amount to much more than withholding taxes on labor income in the formal sector. Similarly, among corporate firms, taxes are collected mainly from those who are highly dependent on the formal financial sector. As documented for instance by Gordon and Li (2009), consumption taxes account for more than half of total government revenues in poor countries, whereas personal income taxes represent about 30 percent and corporate income taxes 13 percent of total revenues—compared to 33 percent, 54 percent, and 10 percent, respectively, in richer countries.1 Thus, poorer countries tend to rely relatively more than richer ones on indirect taxes.

Several observers have argued that the existence of high collection costs on personal and capital income may explain why the tax structure in many developing countries is dominated by indirect taxes. In a broad sense, these costs include not only those associated with collecting revenues, enforcing payments, and implementing audits, but also the budgetary costs incurred in preparing and promulgating tax laws. As noted by Bird and Zolt (2005), whereas developed countries devote roughly one percent of tax revenues to cover the budgetary costs of tax collection, the costs of tax administration are significantly higher in developing countries—almost three percent, according to Gallagher (2005). Figure 1, based on available data, shows a scatter diagram relating the ratio of direct-to-indirect tax revenue to the administrative cost of tax collection (measured in proportion of net tax revenue) in a sample of 41 countries over the period 2001-07. The graph suggests a negative relationship, which implies indeed that countries where tax collection costs are high (as is often the case for direct taxes) tend to rely more on indirect taxes.

Most of the existing literature on the economic effects of collection costs has been conducted in a static, partial equilibrium setting, often with an exogenous revenue

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1 Adding revenue from seigniorage and trade duties, indirect tax revenue is more than twice as large (as a fraction of total revenue) in developing countries compared to developed countries, at 76 versus 35 percent.
constraint to avoid modeling the expenditure side of the government budget. Aspects of taxation that have been studied in that context include the optimality of Friedman’s inflation tax rule, the desirability of positive tariffs, the link between the capacity to collect domestic taxes and foreign borrowing (see Aizenman (1987), Végh (1989), De Fiore (2000), and Aizenman and Marion (2004)), and the trade-off between raising revenue through higher tax rates or greater expenditures on enforcement (see Kaplow (1990) and Besfamille and Siritto (2009)). Regarding the latter, higher tax rates may seem preferable intuitively, given that enforcement directly consumes resources. Moreover, as with higher nominal rates, greater enforcement tends to distort private behavior because it raises the marginal effective tax rate. As demonstrated by Kaplow (1990), however, raising tax rates and increasing enforcement expenditure typically do not cause the same degree of distortion; in the cases that he considers, some expenditure on enforcement may be optimal even when enforcement is costly and increases (everything else equal) distortions in private behavior, and even if other revenue sources with no enforcement costs are available.

The fact that tax enforcement entails direct resource costs also has important implications for growth. In a general equilibrium setting, a binding budget constraint implies that trade-offs exist in the allocation of spending—implying that higher expenditure on tax collection may be detrimental to growth, if alternative uses of public resources have a positive impact on the productivity of private inputs or investment. At the same time, however, government spending on tax enforcement may be productive if it contributes to lowering collection costs—thereby freeing more public resources for the provision of productivity-enhancing services. But somewhat surprisingly, the literature on optimal taxation has not examined the implications of tax collection costs for the structure and level of tax rates in a growth context.

In this paper, we attempt to breach this gap by analyzing the growth-maximizing structure of taxes, viewed as consisting of indirect (consumption) taxes and direct (income) taxes, in the presence of collection costs. We do so in an endogenous growth

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2The reason why we focus on growth rather than welfare maximization is twofold. First, although at the conceptual level welfare maximization is naturally viewed as the primary objective of benevolent governments, in practice imperfect knowledge about household preferences makes adopting it as a first-best strategy difficult; because changes in income are easier to measure than welfare, a policy of growth
model with productive government spending, as in the recent line of research on taxation and growth. In this type of models, a natural trade-off arises regarding direct taxation: on the one hand, it distorts incentives to work and save, thereby lowering growth; on the other, it increases the marginal productivity of private inputs, thereby enhancing growth—and possibly welfare. This is the key insight of Barro’s (1990) contribution, which was extended in many subsequent studies. Our rationale for using an endogenous growth framework, however, is different: our goal is to highlight the trade-off that may arise between spending on tax enforcement and spending on productivity-enhancing items, and its implications for long-run growth.

More generally, and in line with the evidence discussed earlier regarding the tax structure in developed and developing countries, our goal is to examine whether the existence of a skewed distribution of tax collection costs may explain the observed bias toward indirect taxation at low levels of development. In that sense, therefore, we provide a positive, rather than normative, analysis (that is, we abstract from equity considerations). We begin by considering the case where the parameter characterizing the degree of inefficiency in collection costs is constant. We then consider the case where this parameter is endogenous and inversely related to the resources spent by the government on tax enforcement (relative to the relevant tax base). This is important because one of the main reasons why some categories of taxes are harder to collect than others in developing countries (especially the poorest ones) is the lack of administrative capacity. For instance, the relative importance of import duties in some of these countries (relative to total taxes as well as taxes on domestic consumption) is often attributed to the fact that collection costs for that category of taxes are relatively small, which in turn stems from the fact that controlling points of entry and exit of foreign goods requires more limited administrative resources.

The remainder of the paper is organized as follows. Section II presents the basic framework, which assumes constant collection costs. We abstract from human capital maximization represents a second-best strategy. Second, our goal is positive rather than normative—it is to explain the observed composition of revenues in low-income countries. In the working paper version of this article (available upon request) we analyze the welfare-maximizing solutions as well, but general analytical results are difficult to derive.
accumulation and keep labor supply exogenous. However, although the supply of raw labor is fixed, effective labor (that is, labor in efficiency units) is endogenous, in part because government spending affects productivity. Section III derives the balanced-growth path of the decentralized economy. Section IV derives the growth-maximizing tax structure. Section V endogenizes collection costs, by relating them to spending on tax enforcement. The last section offers some concluding remarks and discusses some research perspectives.

2 Basic Framework

Consider an economy populated by an infinitely-lived representative household and where a single homogeneous good is produced. The price of the good (which can be used for consumption or investment) is fixed. The government provides productivity-enhancing services (health, education, access to roads, and the like), at no charge to users. Moreover, these services are not subject to congestion. Productivity-enhancing services are thus a pure (that is, non-rival, non-excludable) public good. To finance its expenditure, the government levies direct and indirect taxes, in the form of a proportional tax on income and consumption.

2.1 Production

Goods, in quantity $Y$, are produced with private capital, $K$, and effective labor, given by the product of the number of workers (which is constant and normalized to unity)

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3 As argued by García-Peñalosa and Turnovsky (2005, p. 1052), abstracting from labor-leisure choices is a reasonable approximation when it comes to developing countries—particularly the poorest ones. Given the low levels of consumption to begin with in these countries, it is unlikely that much leisure is consumed. In addition, we do not discuss the composition of direct taxes.

4 We could of course assume that the share of productive spending is also determined optimally; however, this issue has been discussed at length in the literature; see Agénor and Neanidis (2011) and the literature therein.

5 Assuming that productivity-enhancing services are nonexcludable rules out the imposition of an explicit user fee by the government, as for instance in Ott and Turnovsky (2006). However, because we assume that the implicit rent generated by the public good accrues to the household, the direct tax rate can be thought of as playing in part the role of a user fee.
and productivity, $A$. Assuming a Cobb-Douglas technology yields\(^6\)

$$Y = A^{\beta_Y} K^{1-\beta_Y},$$

where $\beta_Y \in (0, 1)$. In turn, productivity is assumed to depend on the supply of public services, $G_H$, and the private capital stock, through a standard Arrow learning-by-doing effect:

$$A = G_H^\alpha K^{1-\alpha},$$

where $\alpha \in (0, 1)$. Combining these two equations yields

$$Y = (\frac{G_H}{K})^{\beta_Y} K,$$

where $\beta = \beta_Y \alpha$. Thus, as long as $G_H/K$ is constant, output is linear in the capital stock.

### 2.2 Household Optimization

The household maximizes the discounted present value of utility

$$\max_C V = \int_0^\infty \frac{C^{1-1/\sigma}}{1-1/\sigma} \exp(-\rho t) dt, \quad \sigma > 1,$$

where $C$ is consumption and $\rho > 0$ the discount rate, and $\sigma > 0$ the intertemporal elasticity of substitution.\(^7\)

The household-producer spends on consumption, which is taxed at the rate $\tau_C \in (0, 1)$, and accumulates capital. Its income is taxed at the uniform rate $\tau_Y \in (0, 1)$. The representative household’s resource constraint is thus given by

$$\dot{K} = (1 - \tau_Y) Y - (1 + \tau_C) C,$$

where for simplicity we assume that private capital does not depreciate.

\(^6\)In what follows, time subscripts are omitted for simplicity. Also, $\dot{x} \equiv dx/dt$ is used to denote the time derivative of any variable $x$.

\(^7\)Productivity-enhancing services could be taken to affect also household utility. However, as demonstrated in the working paper of this article, this would complicate the analysis without altering the thrust of our results.
The household takes tax rates, productivity, and government services as given when choosing the optimal sequence of consumption. Using (1), (2), and (3), optimality conditions for this problem yield the familiar result
\[
\frac{\dot{C}}{C} = \sigma s \left( \frac{Y}{K} \right) - \sigma \rho,
\]
where \( s \equiv (1-\tau_Y)(1-\beta) \), together with the budget constraint (3) and the transversality condition
\[
\lim_{t \to \infty} \frac{(1 + \tau_C)K}{C^{1/\sigma}} \exp(-\rho t) = 0.
\]

2.3 Government

The government taxes both income and consumption and spends on productivity-enhancing services and administration, in quantities \( G_H \) and \( G_A \), respectively. We assume for the moment that \( G_A \) is unproductive (also with no effect on utility, for simplicity) and will consider later the case where it affects the tax enforcement technology.

As suggested by the evidence, collecting both categories of taxes entails a loss of resources. We assume that tax collection costs are measured by a term \( \phi_h \tau_h^2/2 \), where \( \phi_h \geq 0 \) is the inefficiency parameter, for \( h = C, Y \). Thus, collection costs are assumed to be a convex function of the relevant tax rate, as for instance in Aizenman (1987) and Kaplow (1990). The wedge \( \tau_h - \phi_h \tau_h^2/2 \) represents therefore the effective tax rate, which falls with increases in \( \phi_h \).

The government cannot use seigniorage or issue debt claims, and its budget must be balanced continuously. Thus, the government flow budget constraint is given by
\[
G_H + G_A = (\tau_Y - \frac{\phi_Y}{2} \tau_Y^2)Y + (\tau_C - \frac{\phi_C}{2} \tau_C^2)C.
\]

Both components of public spending are taken to be constant fractions of tax revenue, so that
\[
G_h = v_h \left\{ (\tau_Y - \frac{\phi_Y}{2} \tau_Y^2)Y + (\tau_C - \frac{\phi_C}{2} \tau_C^2)C \right\}, \quad h = H, A
\]
\(^8\)To ensure that revenues from tax \( h \) are positive requires imposing \( \tau_h < 2/\phi_h \).
\(^9\)See for instance Turnosky (1996a) for a model with government debt. However, in his model, debt plays no welfare-enhancing role. Ferreira (1999) and Holman and Neanidis (2006) develop endogenous growth models where public spending is financed by the inflation tax. In addition, given our focus on possible trade-offs between distortionary taxes, we do not account for lump-sum transfers.
with \( u_h \in (0, 1) \). The government budget constraint can thus be rewritten as

\[
v_H + v_A = 1. \tag{8}
\]

### 3 The Decentralized Equilibrium

In the present setting, the decentralized (competitive) equilibrium, in which households take government spending and tax policies (including tax collection costs) as independent of their decisions, can be defined as a set of infinite sequences for the quantities \( \{C, K\}_{t=0}^{\infty} \) such that the path \( \{C, K\}_{t=0}^{\infty} \) satisfies equation (3), (4), and (8), with constant spending shares, \( v_H \) and \( v_A \), and constant tax rates, \( \tau_C \) and \( \tau_Y \).

From (1) and (7), it can be shown that

\[
\frac{Y}{K} = (\theta v_H)^\beta/\Omega, \tag{9}
\]

where \( \Omega = 1 - \beta \), and

\[
\theta = \tau_Y - \frac{\phi_Y}{2} \tau_Y^2 + (\tau_C - \frac{\phi_C}{2} \tau_C^2)(\frac{C}{Y}),
\]

which corresponds to the tax revenue-output ratio.

This result can be combined with the budget constraint (3) to give

\[
\frac{\dot{K}}{K} = \left[ 1 - \left( \tau_Y + \tau_C(C/Y) \right) \right] (\theta v_H)^\beta/\Omega - c, \tag{10}
\]

where \( c = C/K \).

 Subtracting (10) from (4) and using (9) yields the following nonlinear differential equation in \( c \):

\[
\frac{\dot{c}}{c} = \Gamma (\theta v_H)^\beta/\Omega + c - \sigma \rho, \tag{11}
\]

where \( \Gamma = \sigma s - \left[ 1 - \left( \tau_Y + \tau_C(C/Y) \right) \right] \).

Equation (11) and the transversality condition (5) determine the dynamics of the decentralized economy. It is easy to verify that the equilibrium is globally unstable; to be on the balanced growth path (along which \( \dot{c} = 0 \)), the economy must start there. Thus, the model displays no transitional dynamics.
From (4), and using (9), the steady-state growth rate $\gamma$ is
\[
\gamma = \sigma \left\{ sv_H^{\beta/\Omega} \left[ \tau_Y - \frac{\phi_Y}{2} \tau_Y^2 + (\tau_C - \frac{\phi_C}{2} \tau_C^2) \left( \frac{\bar{C}}{\bar{Y}} \right) \right]^{\beta/\Omega} - \rho \right\},
\] (12)

where $\bar{C}/\bar{Y}$ is the constant, steady-state consumption-output ratio. Expression (12) shows that the steady-state growth rate of the economy depends directly, in general, on both tax rates and their associated collection costs, as well as the spending share on productivity-enhancing services, $v_H$.

The presence of collection costs—or, more generally, waste due to tax inefficient administration—generates not only a reduction in government revenues but also a resource loss for the economy, as can be seen by consolidating the budget constraints (3) and (6):
\[
C + \dot{K} + G_H + G_A = (1 - \frac{\phi_Y}{2} \tau_Y^2)Y - \frac{\phi_C}{2} \tau_C^2 C.
\] (13)

To determine $\bar{C}/\bar{Y}$, divide through the representative household’s resource constraint equation (3) by $K$, to get
\[
\frac{\dot{K}}{K} = (1 - \tau_Y) \frac{Y}{K} - (1 + \tau_C) \frac{C Y}{\bar{Y} \bar{K}}.
\]

Along the balanced growth path, $\dot{K}/K = \gamma$; solving the above expression for $\bar{C}/\bar{Y}$ and noting that from (12) $(\bar{Y}/\bar{K})^{-1} = \sigma[s - \rho(\bar{Y}/\bar{K})^{-1}]^{\beta/\Omega}$ yields
\[
\frac{\bar{C}}{\bar{Y}} = \frac{1}{1 + \tau_C} \left\{ 1 - \tau_Y - \frac{1}{s - \rho(\bar{Y}/\bar{K})^{-1}} \right\},
\] (14)

with $\bar{Y}/\bar{K} = (\tilde{b} v_H)^{\beta/\Omega}$ from (9). Substituting (14) in (12), and applying the implicit function theorem, it can be established that the growth rate depends ambiguously on $\tau_Y$ and $\tau_C$. For $\tau_Y$, this reflects not only the standard trade-off identified by Barro (1990), but also changes in the long-run consumption-output ratio and changes in the cost of collecting direct taxes. For $\tau_C$, the intuition is that the direct effect of an increase in the tax rate on consumption is to raise revenue and promote growth, by allowing the government to spend more resources on productivity-enhancing services. At the same time, however, a higher $\tau_C$ increases the cost of collecting indirect taxes and lowers the tax base (as a proportion of output), implying that the net effect on revenue (and thus growth) is in general ambiguous.
4 Constant Inefficiency in Collection Costs

Suppose now that the government sets fiscal policy so as to maximize growth. We begin with the case of constant inefficiency, and examine next the case where inefficiency can be reduced by administrative spending on tax enforcement.

With the use of equations (12), (14) and (9), the following result can be established:

**Proposition 1.** With constant inefficiency, the growth-maximizing tax structure is given by

\[
\frac{\phi_C}{2} (\tau_C^*)^2 + \phi_C \tau_C^* - 1 = 0, \quad (15)
\]

\[
\tau_Y^* \left[ 1 - (1 + \beta) \frac{\phi_C}{2} \tau_Y^* + \beta \phi_Y \right] + \left[ \tau_C^* - \frac{\phi_C}{2} (\tau_C^*)^2 \right] \left( \frac{\bar{C}}{Y} \right) = \beta. \quad (16)
\]

The solution is thus recursive. Equation (15) implies that there are two solutions to the optimal consumption tax rate; by Descartes’ rule of sign, these solutions are of opposite sign. A positive tax rate on consumption has two opposite effects on growth: on the one hand, it increases revenues and allows more productive spending; on the other, it increases collection costs and reduces the consumption tax base (as a proportion of output), as noted earlier. But because these costs are always positive (given their quadratic nature) a negative tax on consumption (a subsidy) cannot exploit this trade-off, as can be verified from (12). We therefore focus our discussion on a non-negative consumption tax rate.\(^{10}\) As shown in the Appendix, for the optimal consumption tax rate to be positive requires \(\phi_C > 0\); to obtain a solution that is both positive and bounded requires \(\phi_C > 2/3\). Intuitively, the degree of inefficiency must be high enough to ensure a genuine trade-off between the “revenue effect” and the “collection cost effect” when setting the optimal tax rate.

The following result can also be established:

**Corollary 1.1.** With constant inefficiency, the growth-maximizing consumption tax rate varies inversely with \(\phi_C \left( \frac{\partial \tau_C^*}{\partial \phi_C} < 0 \right)\) and \(\lim_{\phi_C \to -\infty} \tau_C^* = 0\).

Thus, an increase in the “own” degree of inefficiency associated with collecting indirect taxes lowers its optimal value. Table 1 illustrates this result numerically for

\(^{10}\)In addition, our aim is to explain an actual fact—greater reliance on consumption taxes, rather than subsidies (see Figure 1).
values of \( \phi_C \) in the range \((1, 10)\). Based on the data presented by Gordon and Li (2009, Table 2), and using VAT rates as a proxy for the consumption tax rate, the results show that values of \( \phi_C \) in the range \((4, 6)\) give estimates that are close to those observed in developed countries, whereas values in excess of 6 produce estimates closer to VAT rates observed in developing countries.

The inverse relationship described in Corollary 1.1 is similar to the result derived by Aizenman (1987) and others, in a static, partial equilibrium context. That this result continues to hold in a growth context is important because most of the literature on taxation and growth can generate a distortionary consumption tax only when labor supply decisions are endogenous (see for instance Milesi-Ferretti and Roubini (1998b) and Turnovsky (2000)). In our model, by contrast, labor supply (“raw” or “effective”) plays no role in the results; the optimal consumption tax depends solely on the magnitude of the relevant collection cost.

The optimality condition (16) is a nonlinear equation in \( \tau_Y^* \) and \( \tau_C^* \), and in general no explicit solution can be derived. The two tax rates act as substitutes, given that according to this condition the government can generate revenue (as a fraction of output) equal to \( \beta \), with a combination of the two tax rates. However, given that \( \tau_C^* \) is solely a function of \( \phi_C \), which is constant, we can use the implicit function theorem to evaluate the effect of \( \phi_Y \) and \( \phi_C \) on \( \tau_Y^* \). From the derivations in the Appendix, the following result can then be established:

**Corollary 1.2.** With constant inefficiency, and if the initial income tax rate is not too large, an increase in the degree of inefficiency in collecting direct taxes decreases the growth-maximizing income tax rate \( (\partial \tau_Y^* / \partial \phi_Y < 0) \), whereas an increase in the degree of inefficiency in collecting indirect taxes raises the growth-maximizing income tax rate \( (\partial \tau_Y^* / \partial \phi_C > 0) \).

The condition on the initial income tax rate is \( \tau_Y < \beta / (1 + \beta) \), which implies \( \tau_Y < \beta \). Intuitively, it means that initially the economy must be positioned sufficiently below the optimal rate \( \beta \) and on the left-hand side of the “Laffer curve” that prevails in the absence of collection costs, for an increase in the own degree of inefficiency (which raises collection costs) to reduce the optimal tax rate (which offsets the negative effect of a higher \( \phi_Y \) on tax revenues). The condition \( \tau_Y < \beta / (1 + \beta) \) is not particularly restrictive; assuming a value of \( \beta = 0.36 \), the condition on the initial \( \tau_Y \) requires the
income tax rate to be less than 0.26.\footnote{Recall that $\beta$, which is equal to $\beta_Y \alpha$, is the net elasticity of output with respect to productivity-enhancing public services. It reflects both the elasticity of output with respect to labor, $\beta_Y$, and the marginal benefits of a variety of public services—including infrastructure, education, and health—on productivity, $\alpha$. Using a standard value of $\beta_Y = 0.6$, and assuming a value of $\alpha = 0.6$ (which may be on the low side for a low-income country if public services consist of health and education, in particular) gives $\beta = 0.36$.} This restriction is largely satisfied for most low-income countries, where the average ratio of (personal) income taxes to GDP is around 0.02. For Africa, for instance, the ratio is 0.022 for the period 1975-2001, and for India it is 0.014 for the period 1998-2002 (see Bird and Zolt (2005, Tables 1 to 3)). Thus, under a fairly plausible condition, an increase in collection costs associated with direct (indirect) taxation tends to reduce (increase) the optimal income tax rate.

To illustrate that the condition on the size of initial $\tau_Y$ is not particularly restrictive, we run a series of numerical simulations with respect to the optimality condition \eqref{eq:16}. In the simulations, we calculate the optimal income tax rate $\tau_Y^*$ for various values of $\phi_Y$ and $\phi_C$ (recall that the latter solely determines $\tau_C^*$), once we set parameter values for $\beta$, $\rho$, $\sigma$, and $\nu_H$. For $\beta$, we use the value 0.36. For $\sigma$ and $\rho$ we use values of 0.2 and 0.04, respectively. The first value is consistent with estimates for low-income countries, as documented by Ogaki, Ostry, and Reinhart (1996) and Agénor and Montiel (2008), whereas the second is fairly standard. For $\nu_H$, we use a value of 0.09, which corresponds to the share of total tax revenue toward public expenditure for the group of 63 low-income countries (as defined in the World Bank Atlas Classification, averaged over 1970-2000). The estimated values of $\tau_Y^*$ are presented in Table 2. These values support Corollary 1.2 as to the effect of $\phi_Y$ and $\phi_C$ on $\tau_Y^*$. The size of $\tau_Y^*$ is also consistent with the low income tax rates observed in poor countries, as documented in Bird and Zolt (2005).\footnote{The quantitative effect of $\phi_Y$ and $\phi_C$ on $\tau_Y^*$ illustrated in Table 2 is not very sensitive to the parameter values used for $\beta$, $\rho$, $\sigma$, and $\nu_H$, and neither is the pattern identified in the table.}

Combining the results in Corollaries 1.1 and 1.2, we obtain the relationship between the optimal (growth-maximizing) ratio of direct to indirect tax rates ($\tau_Y^*/\tau_C^*$) and the two associated types of collection costs ($\phi_Y$ and $\phi_C$). The expressions are given by

$$\frac{d(\tau_Y^*/\tau_C^*)}{d\phi_Y} = \frac{1}{\tau_C^*} \frac{d\tau_Y^*}{d\phi_Y} < 0,$$

\hfill (17)
\[
\frac{d(\tau_C^*/\tau_C^*)}{d\phi_C} = \frac{\tau_C^* \frac{d\tau_C^*}{d\phi_C} - \tau_Y^* \frac{d\tau_Y^*}{d\phi_C}}{(\tau_C^*)^2} > 0.
\] (18)

Equations (17) and (18) suggest that collection costs associated with both consumption and income taxes induce a “substitution effect” in the tax structure. In this sense, a greater cost of collecting one of the taxes leads to greater reliance on the other tax, in relative terms. This is illustrated in Table 3, which combines Table 1 and Table 2.

Given our interest in replicating the phenomenon observed in Figure 1, we now examine how tax collection costs (as defined in our model) are related to the optimal ratio of direct to indirect tax revenue, rather than to the ratio of tax rates. Then, taking a further step, we express the collection costs in terms of their definition in OECD (2008), used to develop Figure 1. This amounts to transforming tax collection costs as a percentage of collecting 100 units of revenue. This exercise allows a direct quantitative comparison between the simulated estimates of our model and the actual country experiences documented in the data.

The way we formulated tax collection costs in the government flow budget constraint (6), offers an easy calculation of the ratio of direct to indirect tax revenue; it is given by

\[
\frac{(\tau_Y - \frac{\phi_Y}{2} \tau_Y^2)Y}{(\tau_C - \frac{\phi_C}{2} \tau_C^2)C} = \frac{(\tau_Y - \frac{\phi_Y}{2} \tau_Y^2)}{(\tau_C - \frac{\phi_C}{2} \tau_C^2)Y}.
\] (19)

Along the balanced growth path, using equations (15) and (16), with the expressions for \(\tilde{C}/\tilde{Y}\) from (14) and for \(\tilde{Y}/\tilde{K}\) from (9), we calculate the optimal ratio of direct to indirect tax revenue for various values of \(\phi_Y\) and \(\phi_C\); once again for given parameter values of \(\beta, \rho, \sigma,\) and \(\nu_H\). These are shown in Table 4, which depicts the “substitution effect” of tax collection costs in the revenue-generating structure of the two taxes.

Our formulation of tax collection costs offers an additional advantage, as it can be easily transformed to the definition used by OECD (2008). The definition considers the administrative cost of collecting direct (personal income) and indirect (value added and excise) tax revenue expressed as a fraction of net revenue collected. In other words, it measures how much it costs to collect 100 units of revenue. In terms of our model,
this measure corresponds to

\[
Cost \ of \ Tax \ Collection = \frac{\phi_Y \tau_Y^2 Y}{(\tau_Y - \frac{\phi_Y \tau_Y}{2})Y} + \frac{\phi_C \tau_C^2 C}{(\tau_C - \frac{\phi_C \tau_C}{2})C},
\]

or

\[
Cost \ of \ Tax \ Collection = \frac{\phi_Y \tau_Y}{1 - \frac{\phi_Y \tau_Y}{2}} + \frac{\phi_C \tau_C}{1 - \frac{\phi_C \tau_C}{2}}. \quad (20)
\]

Using equations (19) and (20), along the balanced growth path, we can show the relationship between the cost of collecting taxes and the optimal ratio of direct to indirect tax revenue generated by our model. This relationship is displayed in Figure 2, for values of \(\phi_C\) in the range of (3,10), and for \(\phi_Y\) in the range of (5,50). The circles and the blue line represent the actual observations from Figure 1, while the squares and the red line reflect the points generated by our model. In general, the simulated values seem to do a good job at replicating the data. They capture the negative relationship between the two variables, and the calibrated linear trend matches the actual trend. A notable point, though, is that the generated values are not as dispersed as the actual data. This could be explained by a number of factors or parameters that could influence the relationship between collection costs and the revenue ratio, but are not incorporated in our model.\(^{13}\) Moreover, the simulated values are generated along the balanced growth path, whereas the actual data correspond to economies (a mix of developed and developing countries) that could be in transition toward their steady-state path.

In spite of that, our model does track the data pretty well by showing that tax collection costs lead to adjustments in the ratio of (direct to indirect) tax revenue, even when these costs are treated as exogenously determined. Put differently, with constant inefficiency, the growth-maximizing ratio of direct-to-indirect tax revenue is inversely related to the total administrative costs of collecting these taxes. This result, therefore, provides support to our main conjecture—that the observed bias in the tax structure of developing countries toward indirect taxes may be the result of (or, more precisely, the optimal response to) high collection costs associated with income taxes.

\(^{13}\)These factors may include corrupt practices by tax officials, tax evasion by households, rules of punishment for bureaucrats and households who fail to abide by the tax law, and so on.
Before turning to the evaluation of this result in the presence of endogenous collection costs, we digress for a moment and focus on the growth-maximizing solution of the income tax rate in equation (16), which exhibits multiple equilibria. Suppose that the government faces a situation where $\phi_C \to \infty$, that is, the cost of collecting the consumption tax becomes prohibitive. Then, as noted earlier, $\tau_C^* = 0$. The optimal condition for determining the growth-maximizing income tax rate (16) now simplifies to

$$
\tau_Y^* - (1 + \beta)\frac{\phi_Y}{2}(\tau_Y^*)^2 = \beta(1 - \phi_Y \tau_Y^*),
$$

or equivalently $F(\tau_Y^*) = G(\tau_Y^*)$. Setting $\Delta = 1 + \beta$, we have\footnote{Rewriting (21) as a monic polynomial, Descartes’ rule of sign implies that this equation has either two positive real roots, or none at all, depending on parameter values. For obvious economic reasons, we focus on the first case and consider only bounded solutions, that is, $\tau_Y \in (0, 1)$.}

$$
F(0) = 0,
$$

$$
F(1) = 1 - 0.5\phi_Y \Delta \Rightarrow \left\{ \begin{array}{ll}
F(1) > 0 & \text{for } \phi_Y < 2/\Delta \\
F(1) < 0 & \text{for } \phi_Y > 2/\Delta
\end{array} \right.
$$

$$
\frac{\partial F}{\partial \tau_Y} = 1 - \phi_Y \tau_Y^* \Delta \Rightarrow \left\{ \begin{array}{ll}
> 0 & \text{for } \phi_Y < 1/\tau_Y^* \Delta \\
< 0 & \text{for } \phi_Y > 1/\tau_Y^* \Delta
\end{array} \right.
$$

$$
\frac{\partial^2 F}{\partial \tau_Y^2} = -\phi_Y \Delta < 0,
$$

$$
G(0) = \beta > 0,
$$

$$
G(1) = \beta(1 - \phi_Y) \Rightarrow \left\{ \begin{array}{ll}
G(1) > 0 & \text{for } \phi_Y < 1 \\
G(1) < 0 & \text{for } \phi_Y > 1
\end{array} \right.
$$

$$
\frac{\partial G}{\partial \tau_Y} = -\beta \phi_Y < 0, \quad \frac{\partial^2 G}{\partial \tau_Y^2} = 0.
$$

Thus, $F(\tau_Y)$ is either an increasing or a decreasing concave function of $\tau_Y$, whereas $G(\tau_Y)$ is a decreasing (non-concave, non-convex) function of $\tau_Y$. As a result, the optimal value of the direct tax rate may not be unique.

Intuitively, the reason why the solution to the growth-maximizing problem may lead to multiple equilibria is because an increase in the income tax rate exerts two adverse effects on growth: it not only reduces incentives to save and invest, but it also increases tax collection costs—at a rate that depends on the tax rate itself. Depending on how inefficient tax collection is, this additional adverse effect may significantly alter the government’s optimal choice. If $\phi_Y$ is not large, the optimal tax rate will not
depart much from the benchmark value of the Barro rule, $\beta$. But if $\phi_Y$ is sufficiently large, multiple solutions may emerge—the government may maximize the growth rate by setting either a high tax rate (which generates large positive externalities but also entails large collection costs) or a low tax rate (which may entail low collection costs but generates only mild spillovers).

Possible outcomes are illustrated in Figures 3 and 4. Consider first Figure 3. It is clear from the above results that $G(0) > F(0)$, while, in general, $G(1) \leq F(1)$. Given, however, that for multiple equilibria to emerge $F(\tau_Y)$ must cut $G(\tau_Y)$ from above, a necessary condition for multiplicity is $F(1) < G(1) < 0$. From the above results, this implies that $\phi_Y$ must be greater than 2.\(^{15}\)

Equilibrium values are those for which $F(\tau_Y)$ and $G(\tau_Y)$ intersect. The decreasing line corresponding to $G(\tau_Y)$ can take two forms, as determined by the sign of $G(1)$, shown in blue ($G_A$) and light blue ($G_B$) in the figure. As for $F(\tau_Y)$, three possibilities are displayed:

- Curve $F_A$, shown in red in the figure, corresponds to the case where $F(1) > 0$, $F(1) > G(1)$, and $F' > 0$. There is a single feasible equilibrium value for $\tau_Y^*$ at point $A$ when $G(1) > 0$ and at point $A'$ when $G(1) < 0$.

- Curve $F_B$, shown in dotted black in the figure, corresponds to the case where $0 > F(1) > G(1)$, and $F' < 0$. There is a single feasible equilibrium value for $\tau_Y^*$ at point $B$.

- Curve $F_C$, shown in dashed purple in the figure, corresponds to the case where $F(1) < G(1) < 0$, and $F'' < 0$. There are now two feasible solutions at points $C$ and $C'$.

Figure 3 illustrates also these possible outcomes, relating the optimal tax rate to the inefficiency parameter $\phi_Y$ and for $\beta = 0.36$. The figure considers values of $\phi_Y$ varying between 0 and 10 and shows that the optimal direct tax rate is 0.36 (that is, $\beta$) when $\phi_Y = 0$. For $0 < \phi_Y \leq 2$, equation (21) yields a single value of $\tau_Y^*$, while for

\(^{15}\)From (6), it is possible to have $\phi_Y > 2$ with direct tax revenues remaining positive, given that $\tau_Y^* \in (0,1)$. 

16
there can now be two equilibria, in line with the above reasoning. In addition, it is clear in both the cases of single and multiple equilibria that as $\phi_Y$ increases, $\tau^*_Y$ declines, thereby illustrating the negative association between the two.

## 5 Endogenous Inefficiency in Collection Costs

We now consider the case where collection costs are endogenous, in the sense that the inefficiency parameters $\phi_C$ and $\phi_Y$ depend on the amount of resources spent by the government on administration, $G_A$, in order to improve tax collection (through better monitoring of taxpayers, for instance), relative to the base of each tax. In general, it is plausible to assume that this relationship is convex, which would reflect increasing marginal benefits of improved efficiency; for tractability, however, we will take it to be linear. In addition, the linear relationship allows a direct comparison with the benchmark case of exogenous inefficiency in tax collection. Formally, therefore

$$
\phi_Y = \phi_{Y0} - \frac{\psi_Y \varepsilon G_A}{Y}, \quad (22)
$$

$$
\phi_C = \phi_{C0} - \frac{\psi_C (1 - \varepsilon) G_A}{C}, \quad (23)
$$

where $\phi_{C0}, \phi_{Y0} \geq 0$ and $\varepsilon \in (0, 1)$. Equations (22) and (23) can be viewed as treating the inefficiency associated with the collection of taxes ($\phi_C$ and $\phi_Y$) as an outcome of two elements, one “cultural” and one policy-related. The cultural part of tax inefficiency is captured by $\phi_{C0}$ and $\phi_{Y0}$ and corresponds to what can be described generically as the tax culture, or the good citizenry of the population with respect to their tax obligations. If taxpayers are convinced that fulfilling their tax obligations will effectively help to improve the provision of public goods and services, they may be more inclined to do so. A good tax culture, or high sense of civic duty with respect to tax obligations, can be captured by relatively low values of $\phi_{C0}$ and $\phi_{Y0}$. By contrast, if taxpayers feel that the government is not using its tax proceeds properly (due to, say, corrupt activities or poor allocation of funds), they may be less inclined to fulfill their tax obligations, by evading tax payments. This can be captured by assuming high values of $\phi_{C0}$ and $\phi_{Y0}$.
The policy-oriented element of expressions (22) and (23) is the amount of expenditure the government allocates toward tax enforcement, $G_A$, and the share of administrative spending allocated to improving collection of direct versus indirect taxes, $\varepsilon$ and $1 - \varepsilon$. Thus, the higher the amount of resources spent on tax enforcement (relative to the base of each tax), the lower the degree of inefficiency. An increase in the parameter $\psi_h$, which measures the marginal impact of administrative spending on the degree of inefficiency, corresponds to an improvement in the tax enforcement technology. In low-income countries, we would expect $\psi_Y$ to be relatively low to begin with (a reflection of poor overall administrative capacity). A value of $\psi_h = 0$ or $G_A = 0$ corresponds to the benchmark case of constant collection costs.

The budget constraint remains (6) and the spending shares are given by (7). Because administrative spending is a function of overall tax revenues (as defined in (7)), the specification of the endogenous collection cost functions (22) and (23) depends directly on both tax rates. Solving the model as before, it can be established that

$$\frac{Y}{K} = \left[ \tau_Y - \frac{\phi_Y0}{2} \frac{\tau_Y^2}{Y} + (\tau_C - \frac{\phi_C0}{2} \frac{\tau_C^2}{C}) \right]^{\beta/\Omega},$$

and the steady-state growth rate is given by

$$\gamma = \sigma \left\{ \frac{suH^{\beta/\Omega}}{(1 - 0.5\pi v_A)^{\beta/\Omega}} \left[ \tau_Y - \frac{\phi_Y0}{2} \frac{\tau_Y^2}{Y} + (\tau_C - \frac{\phi_C0}{2} \frac{\tau_C^2}{C}) \right]^{\beta/\Omega} - \rho \right\},$$

where $\pi$ is defined as

$$\pi \equiv \psi_Y \varepsilon \tau_Y^2 + \psi_C (1 - \varepsilon) \tau_C^2.$$

It can be noted that $v_A$ now has a direct positive effect on growth; administrative spending becomes, in a sense, productive. At the same time, because equation (14) continues to apply for $\tilde{C}/\tilde{Y}$, with the use of $\tilde{Y}/\tilde{K}$ from (24) along the balanced growth path, $v_A$ also has an indirect effect on growth. A condition for positive steady-state growth emerges as $1 - 0.5\pi v_A > 0$.

---

\[16\] We would also expect $\psi_Y$ to be substantially lower than $\psi_C$, as a result of the ease with which consumption taxes can be raised. However, this inequality has no bearing on our results.

\[17\] In the working paper version of this article (available upon request), it is shown that, with endogenous collection costs, an increase in the efficiency of the tax enforcement technology increases the growth-maximizing share of spending on tax enforcement.
From (25), (14) and (24), it can be established that the growth-maximizing tax structure is determined by

\[ M_1(\tau^*_C, \tau^*_Y; v_A, \psi_Y, \psi_C, \varepsilon) = 0, \]  
\[ M_2(\tau^*_C, \tau^*_Y; v_A, \psi_Y, \psi_C, \varepsilon) = 0, \]  

where the functions \( M_1(\cdot) \) and \( M_2(\cdot) \) are defined in the Appendix. In general, these conditions cannot be solved explicitly for the optimal tax rates. For this reason we resort to numerical simulations.

Our goal is to examine whether, and to what degree, the policy-oriented element of expressions (22) and (23) bears an effect on the inefficiency of tax collection, \( \phi_C \) and \( \phi_Y \). This amounts to an evaluation of the sensitivity of tax inefficiency to the share of government revenue toward tax enforcement \( (v_A) \) and the effectiveness of administrative spending \( (\psi_h) \).\(^{18}\) The calibration uses equations (27) and (28), with the expressions for \( \tilde{C}/\tilde{Y} \) from (14) and \( \tilde{Y}/\tilde{K} \) from (24), to calculate the optimal ratio of direct to indirect tax revenue for various values of \( v_A \) and \( \psi_h \). This necessitates setting parameter values not only for \( \beta, \rho, \sigma, \) and \( v_H \), which are the same as before, but also for \( \phi_{C0}, \phi_{Y0} \). We use values of \( \phi_{C0} = 7 \) and \( \phi_{Y0} = 20 \) so that when \( v_A = 0 \) the benchmark optimal tax rates are in line with the existing evidence, as documented for instance by Bird and Zolt (2005) and Gordon and Li (2009). These correspond to \( \tau^*_C = 0.133 \) and \( \tau^*_Y = 0.039 \) in tables 1 and 2, and give rise to an optimal tax revenue ratio of 0.390 in Table 4; these values appear in bold in the tables.

The results of this exercise are reported in Table 5. The first set of results sets \( v_A \) to a low value (0.01), consistent with the evidence reported in Bird and Zolt (2005), and uses two values for \( \psi_h \), 50 and 200.\(^{19}\) Regarding the magnitude of the simulated effects, the outcomes do not vary significantly from the benchmark optimal tax revenue ratio of 0.390, which corresponds to \( \psi_h = 0 \). The second set of results retains \( v_A = 0.01 \) but substantially increases the values for \( \psi_h \) to 3\( \times \)10\(^3\) and 4\( \times \)10\(^3\). An increase in \( \psi_h \) now

\(^{18}\)We could also consider how tax inefficiency responds to the allocation of public spending on tax enforcement between the two taxes \( \varepsilon \), but we choose to treat both taxes with an equal weight; \( \varepsilon = 0.5 \).

\(^{19}\)As there is no guidance in the literature on the magnitude of tax enforcement technology, \( \psi_h \), we experimented with various values, ranging from zero to a few thousands.
yields more pronounced effects on the optimal ratio of tax revenue. The magnitude of these changes, however, is relatively small, given the sizeable increase in $\psi_h$. The final set of results, restores the lower values of $\psi_h$, but concerns itself with the value of $v_A$ required to get findings similar to the second set of outcomes. This requires a very high value of $v_A = 0.2$. Put differently, a government would need to spend an implausibly high share of its revenue toward tax enforcement, in order to observe a significant change in the optimal tax revenue ratio.20

Regarding the direction of the effects illustrated in Table 5, these accord well with intuition. Across the rows (columns) of the table, an improvement in the marginal efficiency with which indirect (direct) taxes are collected always reduces (increases) the optimal tax revenue ratio, because the cost of collecting indirect (direct) taxes declines. Therefore, it becomes more efficient to collect the type of tax for which the enforcement technology improves.

In general, these findings support the argument that tax culture plays a far more important role on the degree of (in)efficiency with which taxes are collected, compared to policies that attempt to improve tax collection by spending directly on tax administration—through enhanced auditing procedures and prosecution of offenders. If policies could instead target directly improvements in tax culture, and lead to an improved sense of civic duty, then results could be more pronounced. Such policies could include improving transparency in tax collection and government spending (by putting on government websites lists of tax offenders, terms of procurement contracts, and using medias to provide concrete details about the execution and efficiency of government projects, and so on).

6 Concluding Remarks

The purpose of this paper has been to analyze, in a growth context, the optimal setting of indirect (consumption) taxes and direct (income) taxes in the presence of collection

\footnote{Note that the increase in $v_A$ is assumed to be financed by a cut in a category of government spending that has no output-enhancing effects. In terms of our model, this means that an increase in $v_A$ is not funded by an equivalent decrease in $v_H$. Otherwise, if public services are highly productive (that is, if $\alpha$ is high), a shift in spending toward tax administration would also have a large, adverse effect on growth, as illustrated in a related context by Agénor and Neanidis (2011).}
costs. To do so we considered a model that accounts for productivity-enhancing public services. The first part presented the basic framework, with exogenous collection costs. With constant inefficiency, it was shown that the growth-maximizing tax rate on consumption is inversely related to the “own” degree of inefficiency associated with tax collection. This is in contrast with most of the existing literature on taxation and growth, which is able to justify a distortionary consumption tax only when (raw) labor supply is endogenous, but in line with microeconomic and partial equilibrium results. The growth-maximizing direct tax rate was shown to be negatively (positively) related to the collection cost on direct (indirect) taxes. However, in this case multiple solutions may arise. More importantly, the results also imply that the optimal direct-indirect tax revenue ratio is negatively related to the administrative costs of tax collection, as documented in cross-country data.

We next considered the case of endogenous inefficiency, by defining a tax enforcement technology that relates government spending (measured in proportion of the relevant tax base) and the degree of inefficiency in collecting taxes. In general, it is not possible to derive unambiguous analytical results; we therefore resorted to numerical analysis. Our results show that for changes in administrative spending on tax enforcement to generate significant effects on tax collection costs, they require implausibly high degrees of efficiency in spending, or the allocation of a large fraction of resources to tax enforcement. Moreover, if the latter policy is financed by a cut in productive spending, it may entail adverse effects on growth. Improving “tax culture” and the sense of civic duty through greater budgetary transparency may thus be a more effective policy to improve tax collection and promote growth.

The analysis presented in this paper can be extended in several directions. First, the model can be extended to focus not only on the issue of indirect versus direct taxation but also, within direct taxation, on taxation of wages and capital income. This issue has received considerable attention in the growth literature. In neoclassical models with exogenous growth for instance, a key result is the Chamley-Judd proposition, according to which in the long run zero taxation of capital income and positive taxation of labor income is optimal.\(^{21}\) The main argument is that capital income taxation

\(^{21}\)See Judd (1985) and Chamley (1986). Chari and Kehoe (1999) provide an overview of research
lowers the private return to capital, and hence discourages savings, investment, and economic growth. By contrast, in models of endogenous growth where externalities are generated through the accumulation of physical or human capital (see Pecorino (1993), Corsetti and Roubini (1996), Ortigueira (1998), and Milesi-Ferretti and Roubini (1998a, 1998b)), a common result is that labor income taxation may discourage the accumulation of human capital—in a manner similar to the negative effect of capital income taxation on savings and investment—thereby reducing the growth rate in the long term.\textsuperscript{22} Hence, zero taxation of labor is also optimal. Examining these issues in a model with public goods and non zero collection costs, as was done here, would shed additional light on these issues. Indeed, the introduction of collection costs may help to explain why, as documented by Gordon and Li (2009), industrial countries raise about five times as much from personal income taxes than from corporate income taxes, whereas their relative importance is roughly the same in developing countries.

Second, our model could be extended to account for compliance costs incurred by private agents (such as costs associated with collecting and transmitting the data required by the tax agency) and tax evasion. As shown by Kaplow (1990), it may be optimal to allocate at least some fraction of government revenues to tax enforcement, despite the fact that this entails direct resource costs, instead of raising tax rates. However, greater enforcement itself may increase distortions; it may lead the private sector to devote more resources to either comply with the legislation or avoid getting caught. As pointed out by Bird and Zolt (2005, p. 936), tax systems in developing countries typically impose large compliance costs on taxpayers, over and above the costs of actually paying taxes. Some studies have found that, on average, compliance costs in these countries may be up to four or five times larger than the direct administrative on the issue of capital taxation. The Chamley-Judd result is derived under the assumption that households can fully insure against idiosyncratic risk. Some contributions have shown that if, by contrast, idiosyncratic risk is not insurable, positive capital taxation may be optimal. Even if insurance markets are complete, or equivalently households face no idiosyncratic risk, financial market frictions (in the form of borrowing constraints) may make the taxation of capital income desirable.\textsuperscript{22} This result hinges upon the assumption that the accumulation of human capital requires the use of both human and physical capital. As the analysis in Milesi-Ferretti and Roubini (1998a, 1998b) indicates, the growth implications of factor income taxation in this type of models is sensitive to the technology in the human capital producing sector. If human capital is not a reproducible factor in production, the optimal tax on labor income may be zero.
costs incurred by governments.\textsuperscript{23} Tax evasion is also an endemic problem in many countries. In an early, partial equilibrium contribution, Boadway, Marchand, and Pestieau (1994) found that if different taxes have different evasion characteristics, an optimal tax mix emerges naturally. In one of the few contributions (in a growth context) to this issue, Chen (2003) found that the equilibrium (uniform) tax rate is higher and the growth rate smaller in an economy with tax evasion, compared to an otherwise identical economy without tax evasion. A fruitful area of research that would combine Chen’s analysis and the specification adopted in the present contribution would be to study the optimal allocation of administrative spending (the parameter $\varepsilon$ in (22) and (23)) between improving collection of direct and indirect taxes, assuming different abilities to evade the two categories of taxes. Such analysis would also need to take into account the fact that a key difference between tax evasion and collection costs is that although they both reduce government revenues, the former does not necessarily entail a net resource loss for the economy.

\textsuperscript{23}This is also an important issue for industrial countries. For the United States, for instance, De Fiore (2000, p. 28) estimates that in the late 1990s collection costs accounted for 0.6 percent of revenues, whereas compliance costs amounted to 9.1 percent.
References


Appendix
(Not necessarily for publication)

In Proposition 1, the optimality condition with respect to $\tau_C^*$,

$$\frac{\phi_C}{2}(\tau_C^*)^2 + \phi_C\tau_C^* - 1 = 0,$$

(A1)
can be rewritten as $0.5(\tau_C^*)^2 + \tau_C^* - 1/\phi_C = 0$. The discriminant of this quadratic equation is $1+2/\phi_C$; the two solutions are therefore $-1\pm (1+2/\phi_C)^{1/2}$. One solution is always negative; to obtain a positive solution requires $(1+2/\phi_C) > 1$, so that $2/\phi_C > 0$, which implies $\phi_C > 0$. In order to get a value of $\tau_C^* < 1$, we need $-1+(1+2/\phi_C)^{1/2} < 1$, or $1+2/\phi_C < 4$, so that, we need $\phi_C > 2/3$. Finally, if $\phi_C \to \infty$, $\tau_C^* = 0$.

The optimality condition with respect to $\tau_Y^*$ in Proposition 1 is

$$\tau_Y^* \left[ 1 - (1 + \beta)\frac{\phi_Y}{2}\tau_Y^* + \beta\phi_Y \right] + \left[ \tau_C^* - \frac{\phi_C}{2}(\tau_C^*)^2 \right] \left( \frac{\dddot{C}}{Y} \right) = \beta.$$  

(A2)

To determine the steady-state impact of the inefficiencies in collecting taxes on the growth-maximizing tax rates, equations (A1) and (A2) can be manipulated to yield

$$\begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} d\tau_C \\ d\tau_Y \end{bmatrix} = \begin{bmatrix} b_{13} & b_{14} \\ b_{23} & b_{24} \end{bmatrix} \begin{bmatrix} d\phi_C \\ d\phi_Y \end{bmatrix},$$

(A3)

where

$$b_{11} = \phi_C\tau_C^* + \phi_C > 0,$$

$$b_{12} = 0,$$

$$b_{13} = -\left( \frac{(\tau_C^*)^2}{2} + \tau_C^* \right) < 0,$$

$$b_{14} = 0,$$

$$b_{21} = (1 - \phi_C\tau_C^*) \left( \frac{\dddot{C}}{Y} \right) + \left[ \tau_C^* - \frac{\phi_C}{2}(\tau_C^*)^2 \right] \frac{\partial \left( \frac{\dddot{C}}{\tau} \right)}{\partial \tau_C},$$

$$b_{22} = 1 - (1 + \beta)\phi_Y\tau_Y^* + \beta\phi_Y + \left[ \tau_C^* - \frac{\phi_C}{2}(\tau_C^*)^2 \right] \frac{\partial \left( \frac{\dddot{C}}{\tau} \right)}{\partial \tau_Y},$$

$$b_{23} = \frac{(\tau_C^*)^2}{2} \left( \frac{\dddot{C}}{Y} \right) - \left[ \tau_C^* - \frac{\phi_C}{2}(\tau_C^*)^2 \right] \frac{\partial \left( \frac{\dddot{C}}{\phi_C} \right)}{\partial \phi_C},$$

$$b_{24} = (1 + \beta)\frac{(\tau_Y^*)^2}{2} - \beta\tau_Y^* - \left[ \tau_C^* - \frac{\phi_C}{2}(\tau_C^*)^2 \right] \frac{\partial \left( \frac{\dddot{C}}{\phi_Y} \right)}{\partial \phi_Y}.$$
Using equation (14), one can show that
\[
\frac{\partial}{\partial \tau^*_C} \left( \frac{\bar{C}}{Y} \right) < 0, \quad \frac{\partial}{\partial \tau^*_Y} \left( \frac{\bar{C}}{Y} \right) < 0, \quad \frac{\partial}{\partial \phi_C} \left( \frac{\bar{C}}{Y} \right) > 0, \quad \frac{\partial}{\partial \phi_Y} \left( \frac{\bar{C}}{Y} \right) > 0.
\]

Substituting the expressions of these partial derivatives into \( b_{2i}, i = 1 - 4 \), we get
\[
b_{21} = 0, \\
b_{23} > 0,
\]
while a sufficient condition for \( b_{22} > 0 \) and \( b_{24} < 0 \) is that
\[
\tau_Y < \frac{\beta}{1 + \beta}.
\]

Under these restrictions, solving the system of equations (A3) yields, noting that
\[
b_{11}b_{22} - b_{12}b_{21} = b_{11}b_{22} > 0,
\]
\[
\quad \frac{d \tau^*_C}{d \phi_C} = \frac{b_{13}b_{22} - b_{12}b_{23}}{b_{11}b_{22} - b_{12}b_{21}} = \frac{b_{13}}{b_{11}} < 0,
\]
\[
\quad \frac{d \tau^*_C}{d \phi_Y} = \frac{b_{14}b_{22} - b_{12}b_{24}}{b_{11}b_{22} - b_{12}b_{21}} = 0,
\]
\[
\quad \frac{d \tau^*_Y}{d \phi_C} = \frac{b_{11}b_{24} - b_{14}b_{21}}{b_{11}b_{22} - b_{12}b_{21}} = \frac{b_{24}}{b_{22}} < 0,
\]
\[
\quad \frac{d \tau^*_Y}{d \phi_Y} = \frac{b_{11}b_{23} - b_{13}b_{21}}{b_{11}b_{22} - b_{12}b_{21}} = \frac{b_{23}}{b_{22}} > 0.
\]

These results are used to establish Corollaries 1.1 and 1.2.

Consider now the case of endogenous inefficiency. The optimality conditions under growth maximization are given by
\[
\frac{v_A \psi_C (1 - \varepsilon) \tau^*_C}{1 - 0.5\pi v_A} = \frac{[1 - \phi C_0 (1 + 0.5\tau^*_C)] (\bar{C} / \bar{Y})}{\tau^*_Y - 0.5\phi_0 (\tau^*_Y)^2 + [\tau^*_C - 0.5\phi C_0 (\tau^*_C)^2] (\bar{C} / \bar{Y})},
\]
\[
\frac{v_A \psi_Y \varepsilon \tau^*_Y}{1 - 0.5\pi v_A} = \frac{1 - \Omega}{1 - \tau^*_Y} \beta + \frac{[\tau^*_C - 0.5\phi C_0 (\tau^*_C)^2] (\bar{C} / \bar{Y}) - (1 - \phi Y_0 \tau^*_Y)}{\tau^*_Y - 0.5\phi Y_0 (\tau^*_Y)^2 + [\tau^*_C - 0.5\phi C_0 (\tau^*_C)^2] (\bar{C} / \bar{Y})},
\]
which define the two implicit functions \( M_1(\cdot) \) and \( M_2(\cdot) \) in the text. These expressions are highly nonlinear, and as a result explicit solutions for the optimal tax rates, and of their relationship with the marginal impact of administrative spending on the degree of collection inefficiency, \( \psi_h \), cannot be obtained. For this reason, we rely on numerical simulations, as discussed in the text.
Table 1  
Effect of Consumption Tax Collection Costs on the Growth-Maximizing Consumption Tax Rate  
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<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
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<td>0.291</td>
<td>0.224</td>
<td>0.183</td>
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<td><strong>0.133</strong></td>
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Table 2  
Effects of Tax Collection Costs on the Growth-Maximizing Income Tax Rate  
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<th>6</th>
<th>7</th>
<th>8</th>
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<tbody>
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<td>0.1259</td>
<td>0.1300</td>
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<td>10</td>
<td>0.0685</td>
<td>0.0715</td>
<td>0.0737</td>
<td>0.0755</td>
<td>0.0769</td>
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</tr>
<tr>
<td>20</td>
<td>0.0363</td>
<td>0.0379</td>
<td><strong>0.0390</strong></td>
<td>0.0399</td>
<td>0.0407</td>
<td>0.0413</td>
</tr>
<tr>
<td>30</td>
<td>0.0246</td>
<td>0.0257</td>
<td>0.0265</td>
<td>0.0271</td>
<td>0.0276</td>
<td>0.0280</td>
</tr>
<tr>
<td>40</td>
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<td>0.0194</td>
<td>0.0200</td>
<td>0.0205</td>
<td>0.0208</td>
<td>0.0211</td>
</tr>
<tr>
<td>50</td>
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<td>0.0156</td>
<td>0.0161</td>
<td>0.0164</td>
<td>0.0167</td>
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</table>

Table 3  
Effects of Tax Collection Costs on the Growth-Maximizing Ratio of Direct-to-Indirect Tax Rates  
<table>
<thead>
<tr>
<th>$\phi_Y$</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_Y/\tau_C$</td>
<td>0.658</td>
<td>0.813</td>
<td>0.970</td>
<td>1.127</td>
<td>1.285</td>
<td>1.443</td>
</tr>
<tr>
<td>10</td>
<td>0.373</td>
<td>0.462</td>
<td>0.550</td>
<td>0.639</td>
<td>0.728</td>
<td>0.818</td>
</tr>
<tr>
<td>20</td>
<td>0.198</td>
<td>0.244</td>
<td><strong>0.291</strong></td>
<td>0.338</td>
<td>0.385</td>
<td>0.432</td>
</tr>
<tr>
<td>30</td>
<td>0.134</td>
<td>0.166</td>
<td>0.198</td>
<td>0.229</td>
<td>0.261</td>
<td>0.293</td>
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<tr>
<td>40</td>
<td>0.102</td>
<td>0.125</td>
<td>0.149</td>
<td>0.173</td>
<td>0.197</td>
<td>0.222</td>
</tr>
<tr>
<td>50</td>
<td>0.082</td>
<td>0.101</td>
<td>0.120</td>
<td>0.139</td>
<td>0.159</td>
<td>0.178</td>
</tr>
</tbody>
</table>

Table 4  
Effects of Tax Collection Costs on Growth-Maximizing Ratio of Direct-to-Indirect Tax Revenue  
<table>
<thead>
<tr>
<th>$\phi_Y$</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_Y/\tau_C$</td>
<td>1.174</td>
<td>1.409</td>
<td>1.643</td>
<td>1.872</td>
<td>2.104</td>
<td>2.334</td>
</tr>
<tr>
<td>10</td>
<td>0.587</td>
<td>0.698</td>
<td>0.808</td>
<td>0.916</td>
<td>1.024</td>
<td>1.131</td>
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<tr>
<td>20</td>
<td>0.289</td>
<td>0.342</td>
<td><strong>0.390</strong></td>
<td>0.444</td>
<td>0.495</td>
<td>0.545</td>
</tr>
<tr>
<td>30</td>
<td>0.191</td>
<td>0.226</td>
<td>0.259</td>
<td>0.292</td>
<td>0.325</td>
<td>0.358</td>
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<tr>
<td>40</td>
<td>0.143</td>
<td>0.168</td>
<td>0.193</td>
<td>0.218</td>
<td>0.242</td>
<td>0.265</td>
</tr>
<tr>
<td>50</td>
<td>0.114</td>
<td>0.134</td>
<td>0.154</td>
<td>0.173</td>
<td>0.192</td>
<td>0.211</td>
</tr>
</tbody>
</table>
Table 5
Effects of Decreasing Tax Collection Costs
on Growth-Maximizing Ratio of Direct-to-Indirect Tax Revenue

<table>
<thead>
<tr>
<th>$\frac{r_Y - \frac{1}{2}(r_Y)^2}{r_C - \frac{1}{2}(r_C)^2}$</th>
<th>$\psi_C$</th>
<th>$\psi_Y$</th>
<th>$\psi_C$</th>
<th>$\psi_Y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$v_A = 0.01$</td>
<td>$50$</td>
<td>$200$</td>
<td>$3 \cdot 10^3$</td>
<td>$4 \cdot 10^3$</td>
</tr>
<tr>
<td>$v_A = 0.01$</td>
<td>$0.3934$</td>
<td>$0.3898$</td>
<td>$0.3309$</td>
<td>$0.1861$</td>
</tr>
<tr>
<td>$v_A = 0.2$</td>
<td>$0.3947$</td>
<td>$0.3911$</td>
<td>$0.3404$</td>
<td>$0.1871$</td>
</tr>
</tbody>
</table>
Figure 1
Ratio of Direct to Indirect Tax Revenue against Administrative Cost of Tax Collection
(Cross section of 41 countries, average over 2001-2007)

Notes: Direct tax revenue is defined as the revenue from the taxation of personal income (as percentage of total country tax revenue). Indirect tax revenue is defined as the sum of domestic value added and excise tax revenue (as percentage of total country tax revenue). Administrative cost of tax collection is defined as the ratio of the annual costs of administration incurred by a revenue authority, with the total revenue collected over the course of a fiscal year, expressed as a percentage or as the cost of collecting 100 units of revenue. The country sample is comprised by 41 OECD and selected non-OECD countries made available in the OECD documents “Tax Administration in OECD and Selected Non-OECD Countries: Comparative Information Series” (2004, 2006, 2008). The USA is excluded from the sample as it is the only country from those surveyed that does not use a form of VAT for indirect taxation, instead relying largely on retail sales taxes that are administered independently by most states.
Figure 2
Ratio of Direct to Indirect Tax Revenue against Administrative Cost of Tax Collection
(Cross section of 41 countries, average over 2001-2007)

Notes: As in Table 1. The calibrated values of the model appear in red squares. These values are generated by varying the inefficiency parameter associated with the collection of consumption tax rates ($\phi_C$) in the range of 3-10 and the inefficiency parameter associated with the collection of income tax rates ($\phi_Y$) in the range of 5-50.
Figure 3
Growth-Maximizing Income Tax Rate
with Constant Own Inefficiency Parameter

$F(\tau_Y), G(\tau_Y)$
Figure 4
Growth-Maximizing Income Tax Rate for Varying Values of the Own Inefficiency Parameter